Acceptance measurement in SIS-18 by means of transverse beam excitation with noise

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Outline

- Introduction
- Analytical beam loss model
- Diffusion driven by white noise
- Tracking with realistic noise
- Evaluation of experimental data
- Summary

Introduction

Motivation:

- Acceptance of SIS-18 was never measured
- Help in controlling beam. In particular important during high current operation within FAIR project

General Method:

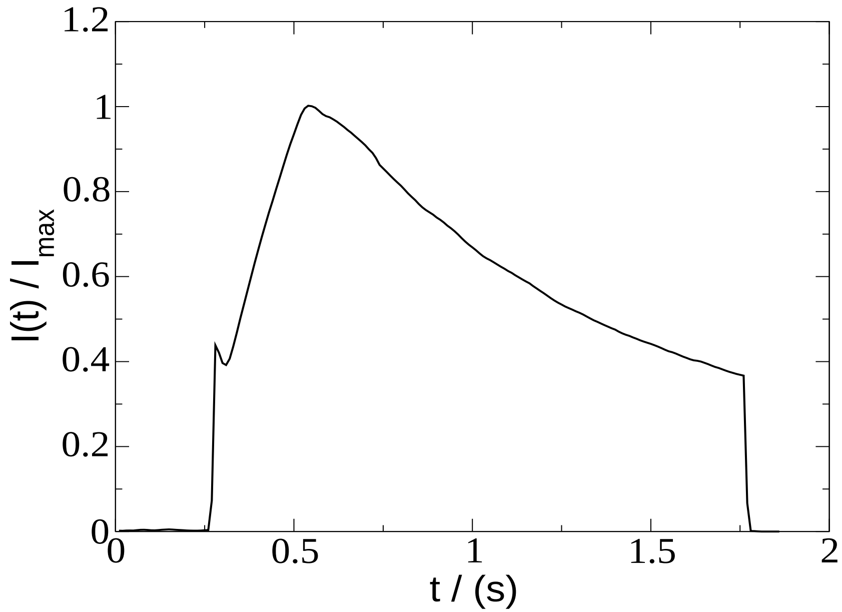
- Transverse beam excitation with noise $\longrightarrow$ Diffusion and growth of beam width
- Existing method: Measure time from beginning of excitation until start of beam loss when beam edge reaches acceptance.
- Need to know initial beam width

Introduction

Determine acceptance of SIS-18 from time evolution of beam current.[1]

- **Diffusion and growth of beam width due to transverse beam excitation**
- **Beam loss when beam width exceeds acceptance.**
- **Measure of beam current and evaluate it by comparison with simulation results**

Ta^{61+} at $E = 100$ MeV



After sufficiently long time transverse beam shape and time evolution of beam current become independent of initial beam shape.

[1] S. Sorge, G. Franchetti, and A. Parfenova, Phys. Rev. ST-AB 14, 052802 (2011).

Introduction

Main experimental parameters

Circumference of SIS-18 C	216.72 m
Ion	Ta^{61+}
Initial particle number $N_{p,0}$	$\sim 10^9$
Working point, (ν_x, ν_y)	$(4.17, 3.29)$
Energy E	100 MeV/u
RMS momentum spread σ_p	$\approx 5.0 \times 10^{-4}$
Nominal vertical chromaticity $\xi_{y,nat}$	-1.4647

For the moment only vertical acceptance measured.

Analytical beam loss model

- Increase of beam width and resulting particle loss is driven by **diffusion**.

- **Diffusion:** particle transport from phase space regions of high density to those with low density
$$\vec{j} = -C\nabla f$$
where $\vec{j}, C, \nabla, f$ defined in vertical phase space.

- Diffusion equation for particle distribution dependent only on emittance[1]:
$$\frac{\partial f}{\partial t} = \left(\frac{\mathrm{d}\epsilon_{av}}{\mathrm{d}t}\right)\frac{\partial}{\partial \epsilon}\left(\epsilon\frac{\partial f}{\partial \epsilon}\right)$$
  - Needs to be solved for $f(\epsilon, t=0) = f(\epsilon),\ f(\epsilon \geq \epsilon_{lim}, t) = 0$.
  - ϵ_{av} – averaged emittance, ϵ_{lim} – limiting emittance = **acceptance**.

[1] D. A. Edwards and M. Syphers, "An introduction to the Physics of High Energy Accelerators"

Analytical beam loss model

Solution to diffusion equation $\longrightarrow$ series:

$$f(\epsilon, t) = \sum_{n=1}^{\infty} c_n J_0\left(\lambda_n \sqrt{\frac{\epsilon}{\epsilon_{lim}}}\right) \exp\left[-\frac{\lambda_n^2}{4}\left(\frac{\mathrm{d}\epsilon_{av}}{\mathrm{d}t}\right)\frac{t}{\epsilon_{lim}}\right]$$

Analytical beam loss model

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- Fortunately, λ_1 is smallest zero of Bessel function. 1st term remains

$$f(\epsilon,t) \propto J_0\left(\lambda_1\sqrt{\frac{\epsilon}{\epsilon_{lim}}}\right)\exp\left[-\frac{\lambda_1^2}{4}\left(\frac{\mathrm{d}\epsilon_{av}}{\mathrm{d}t}\right)\frac{t}{\epsilon_{lim}}\right]$$

  and

$$N_p(t) \propto \exp\left[-\frac{\lambda_1^2}{4}\left(\frac{\mathrm{d}\epsilon_{av}}{\mathrm{d}t}\right)\frac{t}{\epsilon_{lim}}\right]$$

  when time sufficiently large.

- f is only solution if $\mathrm{d}\epsilon_{av}/\mathrm{d}t = \mathrm{const}$ $\longrightarrow$ proof for white noise.

Diffusion driven by white noise

- Search for time evolution of averaged or beam emittance

$$\epsilon_{av}(t = nT_0) = \frac{1}{N_p} \sum_{p=1}^{N_p} \epsilon_p(nT_0)$$

- Assume particle motion given by

$$\begin{pmatrix} y_{p,n+1} \\ y'_{p,n+1} \end{pmatrix} = M \cdot \begin{pmatrix} y_{p,n} \\ y'_{p,n} + \Delta y'_{p,n} \end{pmatrix}$$

- Lattice represented by linear one turn map

$$M = \begin{pmatrix} \cos\mu + \alpha \sin\mu & \beta \sin\mu \\ -\frac{1+\alpha^2}{\beta} \sin\mu & \cos\mu - \alpha \sin\mu \end{pmatrix}$$

with α, β – Twiss functions, $\mu = 2\pi\nu$ – phase advance per turn.

- Kicks $\Delta y'_{p,n}$ uncorrelated with respect to turn and particle (white noise)

Diffusion driven by white noise

Analytic expression for beam emittance

1. Beam emittance is linear function of time $t = nT_0$:

$$\epsilon_{av}(nT_0) = \epsilon_{av,0} + n\beta\sigma^2_{\Delta y'} \qquad \Rightarrow \qquad \frac{\mathrm{d}\epsilon_{av}}{\mathrm{d}t}\Big|_{t=nT_0} = \frac{\beta\sigma^2_{\Delta y'}}{T_0} = \mathrm{const}$$

   $\sigma_{\Delta y'}$: rms momentum kick strength.

2. Use this scheme for particle tracking:

   Provides first benchmark of code.

Diffusion driven by white noise

Beam emittance: tracking vs. analytic model

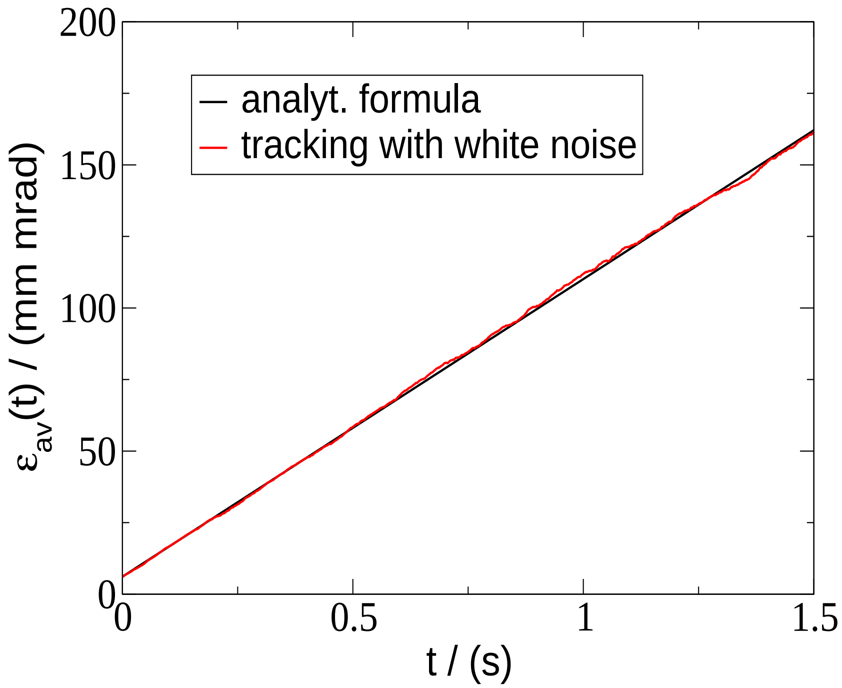
Conditions:

- **White noise with**

  $\boldsymbol{\sigma_{\Delta y'} = 5 \times 10^{-6}}$ **rad**

- **2000 test particles**

- $\boldsymbol{\epsilon_{lim} = 45}$ **mm mrad**



Diffusion driven by white noise

Particle number: tracking vs. analytic model

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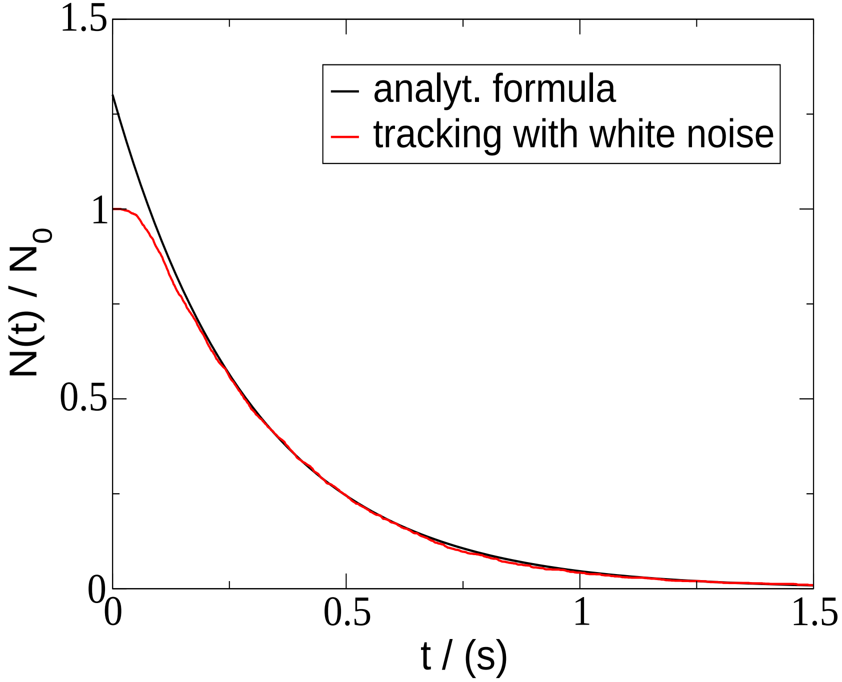

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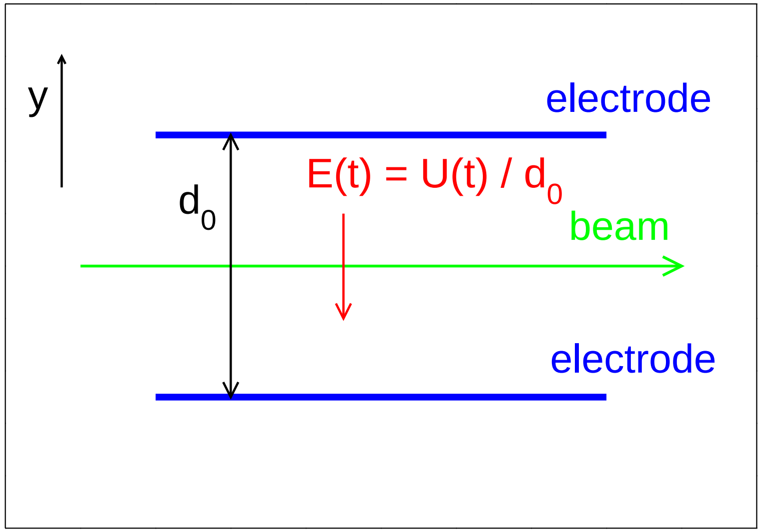
Tracking with realistic noise

Noise used in experiment:

- Generated by an exciter with

  length $l_0 = 750$ mm

  vertical gap $d_0 = 70$ mm



- Transverse RF voltage

$$U(t) = U_a \sin\left[2\pi f_C t + \phi(t)\right].$$

  Carrier frequency: $f_C = \nu_{frac}/T_0$

- $\phi(t)$ follows random bit sequence, where $\phi = 0$ when bit status is 0 and $\phi = \pi$ when bit status is 1, ("Pseudorandom phase shift signal").

- Duration of a bit status is $T_S \sim 100\ T_0$.

Tracking with realistic noise

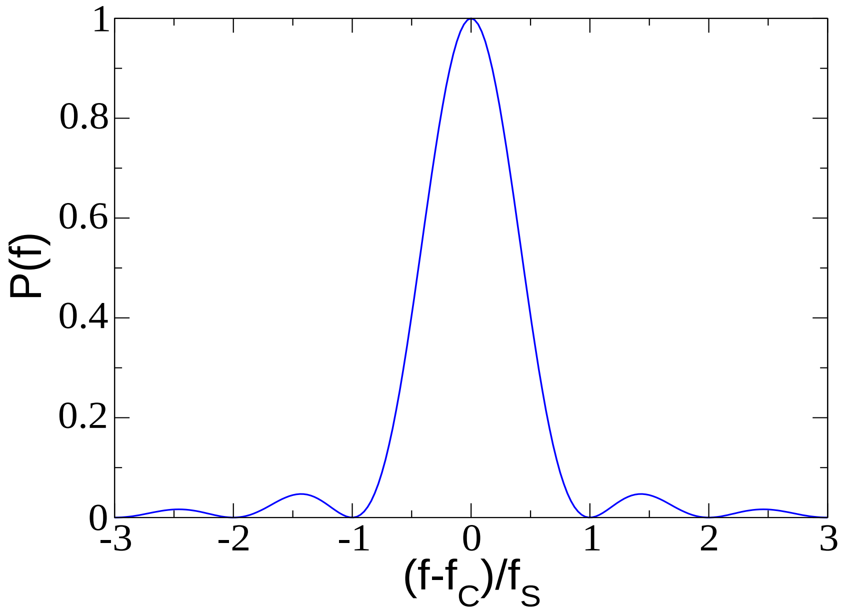
Resulting power spectrum (for positive frequencies)

$$P(f) \propto \frac{\sin^2[\pi(f - f_C)/f_S]}{[(f - f_C)/f_S]^2}$$

with width $f_S = 1/T_S$,

which has to cover tune spread

arising from momentum spread.



Usage of exciter voltage requires particle tracking → model shown before.

Tracking with realistic noise

- Inclusion of tune shift due to momentum spread requires modification of M:

$$M = \begin{pmatrix} \cos\mu_p + \alpha \sin\mu_p & \beta \sin\mu_p \\ -\frac{1+\alpha^2}{\beta} \sin\mu_p & \cos\mu - \alpha \sin\mu_p \end{pmatrix}$$

with phase advance of particle p: $\mu_p = 2\pi\nu(1 + \xi\delta_p)$

- Momentum kick to all particles as function of turn number:

$$\Delta y'(n) = \frac{e}{m_u c^2 \beta_0^2 \gamma_0} \frac{Z}{A} \frac{U_a l_0}{d_0} \sin\left[2\pi\nu_{frac} n + \phi_0(n)\right].$$

with

- Z, A charge state and mass number of ions
- U_a, l_0, d_0 voltage amplitude, and length and gap width of the exciter
- ν_{frac} fractional part of the betatron tune

Tracking with realistic noise

Unfortunately, exciter signal provides the same momentum kick to all particles

$$\Downarrow$$

Coherent motion of all particles in phase space independent of particle density:

$\longrightarrow$ **No diffusion**, instead random oscillation of beam centre.

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$\longrightarrow$ **No diffusion**, instead random oscillation of beam centre.

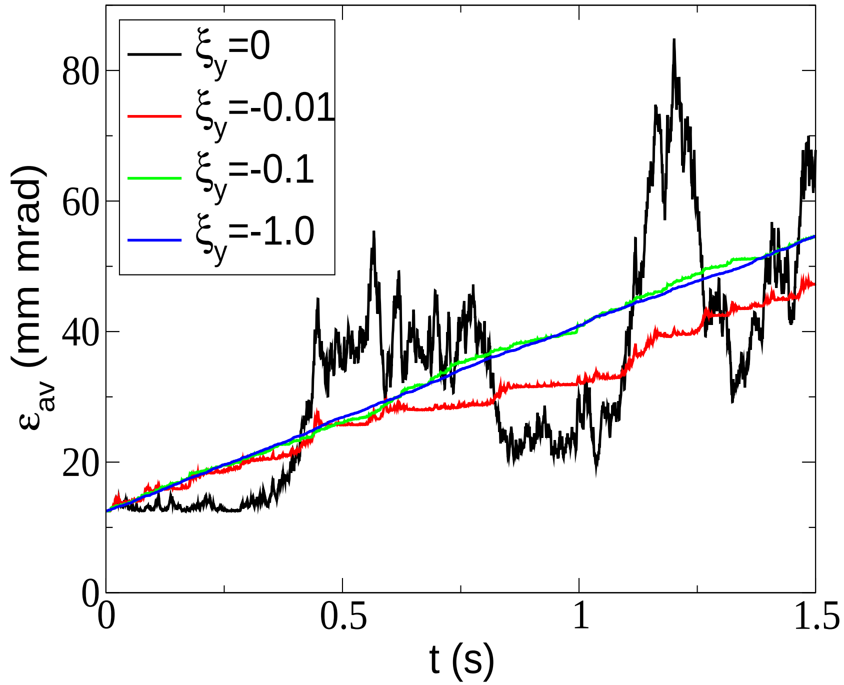
On the other hand,

- Momentum spread with σ_p

  $\longrightarrow$ tune spread $\Delta\nu_{rms} = \sigma_p \xi \nu$

- If chromaticity ξ is sufficiently large, oscillation of beam centre is damped

- **Beam emittance ϵ_{av} becomes linear function of time $\rightarrow$ Diffusion.**



Evaluation of experimental data

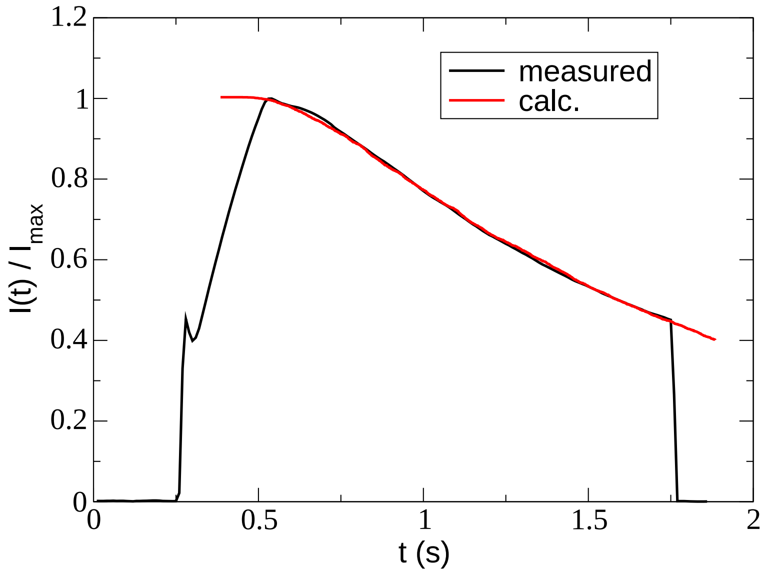
- Acceptance determined with trial-and-error procedure, i.e. calculate $N(t)$ with many different test acceptances, until it fits measured $N(t)$
- Evaluated measurements for $E = 100$ MeV/u, $U_a = 29$ V, and two f_S.
- Obtained acceptance:

$f_S = 0.01/T_0$ **(figure) :**

$\epsilon_{lim} = 46$ mm mrad

$f_S = 0.005/T_0$:

$\epsilon_{lim} = 45$ mm mrad



Similar acceptance values for different conditions.

Evaluation of experimental data

Error estimation: find upper limit for acceptance error

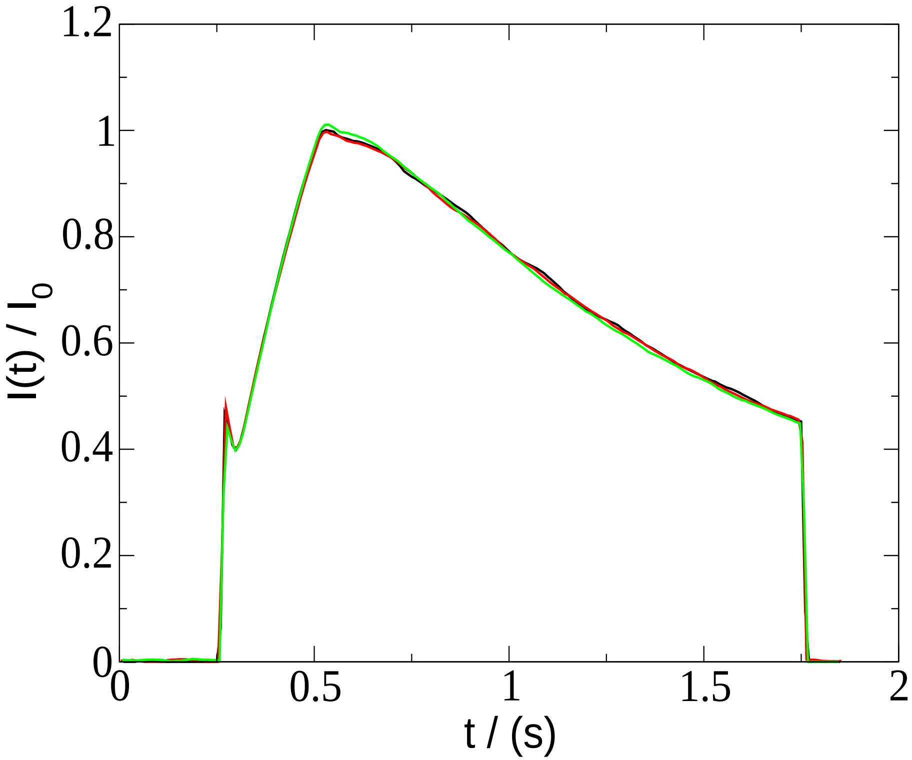
Distinguish between

1. Random errors:
   - Lead to random deviations in measured current although machine settings are not changed.
2. Systematic errors:
   - Parameters used in evaluation are standard machine parameters or from MAD-X calculations.
   - They may deviate from parameters in the real synchrotron.

Evaluation of experimental data

Error estimation: random errors

$f_S = 0.01/T_0$



- Random errors

  $\Downarrow$

  spread in curves for $I(t)$

- Search for upper limit for errors

- Spread always $< 1\ \%$, corresponds to $|\Delta\epsilon_{lim,random}| < 1$ mm mrad or

$$\delta\epsilon_{lim,random} := \frac{|\Delta\epsilon_{lim,random}|}{\epsilon_{lim}} < 2\ \%$$

Evaluation of experimental data

Error estimation: systematic errors

1. Assume maximum relative errors to estimate upper limit of acceptance error

Variable	unperturbed value X_0	maximum relative error $\lvert \Delta X / X_0 \rvert$
β_y	7.0 m	10 %
ξ_y	-1.4647	10 %
σ_p	5×10^{-4}	10 %

2. Add each error separately to the corresponding unperturbed value and use trial-and-error procedure to find acceptance $\epsilon_{lim}(X + \Delta X)$ so that $N(t)$ fits $N(t)$ calculated with $\epsilon_{lim}(X)$.

Evaluation of experimental data

Error estimation: systematic error in β_y

Uncertainty due to $\Delta\beta_y/\beta_y$ can be predicted from analytic expression

$$\epsilon_{av}(n) = \epsilon_{av,0} + \frac{\beta_y}{N_p}\sum_{p=1}^{N_p}\sum_{k,l=0}^{n-1}\cos[(k-l)\mu_p]\Delta y_k'\Delta y_l'$$

because

$$\epsilon_{lim} \propto \frac{d\epsilon_{av}}{dt} \propto \beta_y.$$

$$\Downarrow$$

$$\frac{\Delta\epsilon_{lim}(\Delta\beta_y)}{\epsilon_{lim}} = 0.1 \quad \text{for} \quad \frac{\Delta\beta_y}{\beta_y} = 0.1.$$

Evaluation of experimental data

Error estimation: systematic error in β_y

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$$\Downarrow$$

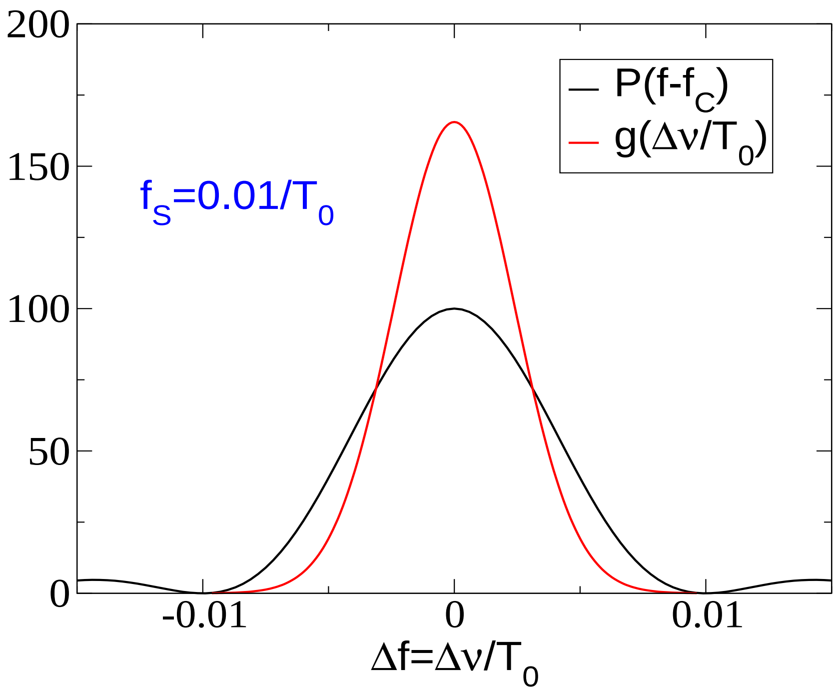
$$\frac{\Delta\epsilon_{lim}(\Delta\beta_y)}{\epsilon_{lim}} = 0.1 \quad \text{for} \quad \frac{\Delta\epsilon_{lim}(\Delta\beta_y)}{\epsilon_{lim}} = 0.1.$$

Could numerically be well confirmed.

Evaluation of experimental data

Error estimation: systematic error in ξ_y, σ_p

- $\xi_y = -1.4647, \sigma_p = 5.0 \times 10^{-4} \Rightarrow \Delta\nu_{rms} = \xi_y \sigma_p \nu_y = 2.41 \times 10^{-3}$
- $f_S \sim \Delta\nu_{rms} \Rightarrow$ Power density not uniform



Evaluation of experimental data

Error estimation: systematic error in ξ_y, σ_p

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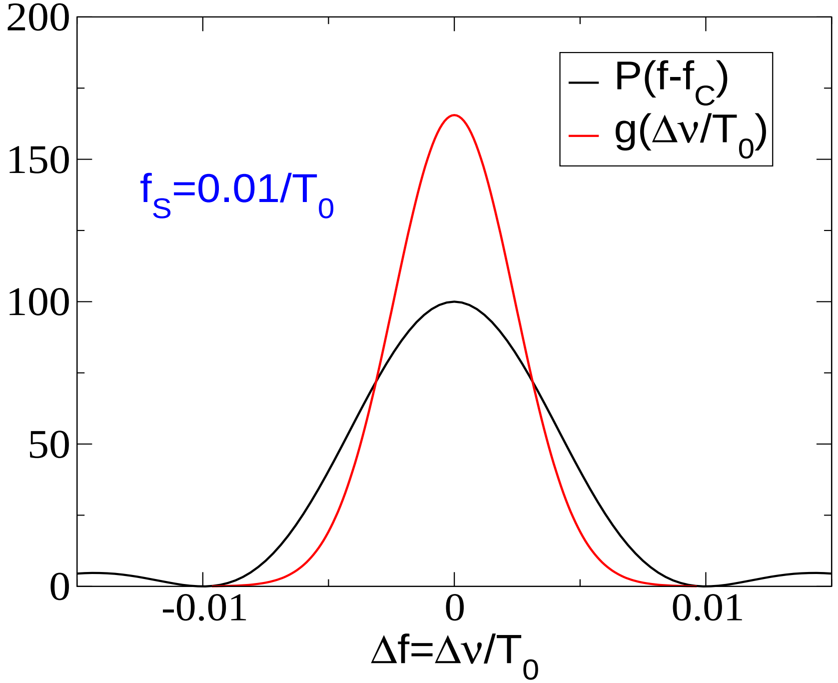
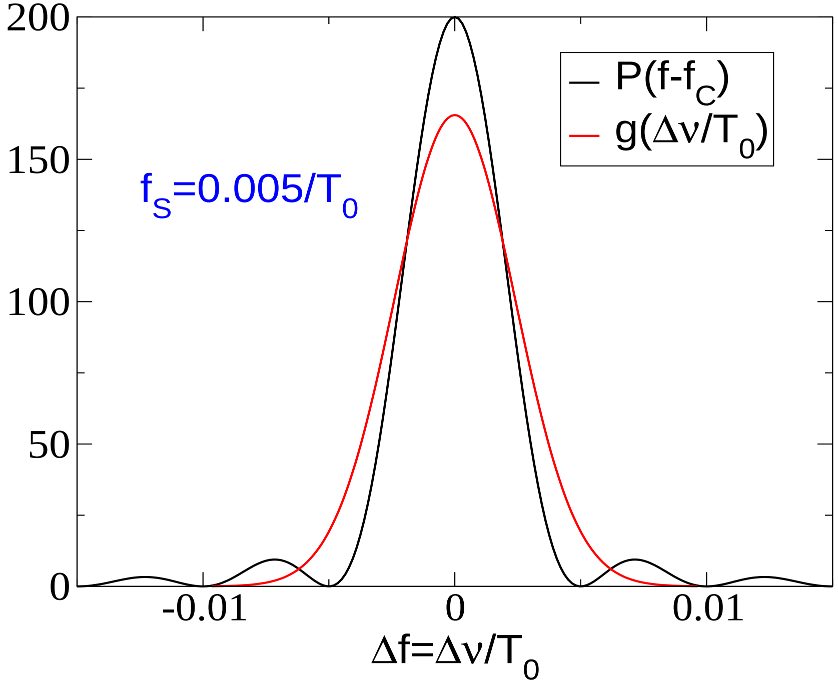
- $f_S \sim \Delta\nu_{rms} \Rightarrow$ Power density not uniform
- Less uniform for smaller f_S

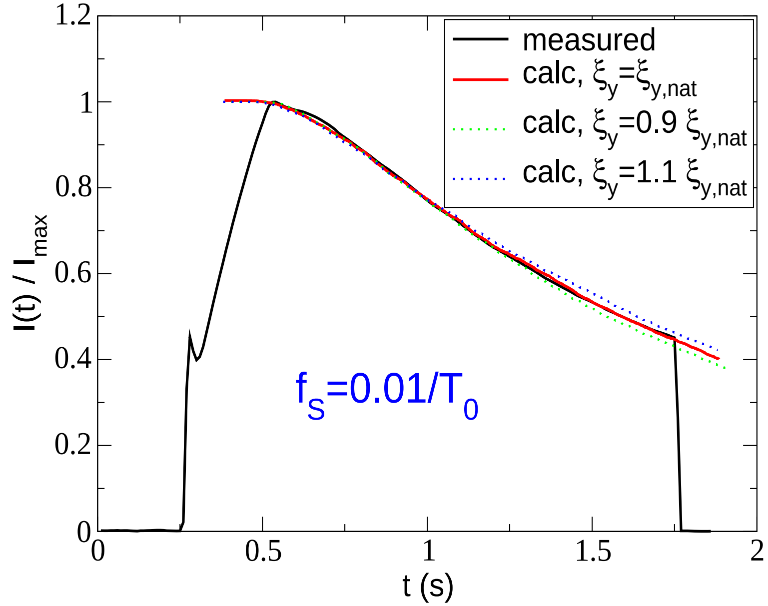


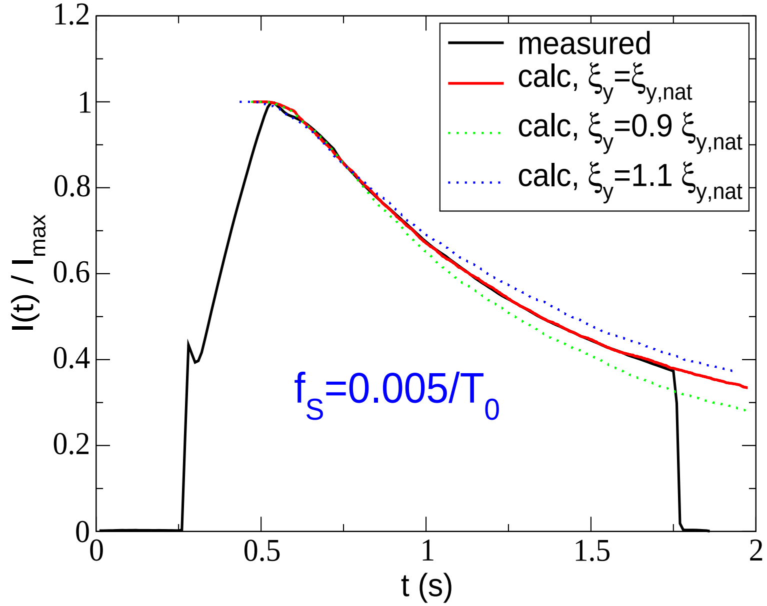


Evaluation of experimental data

Error estimation: systematic error in ξ_y, σ_p

Modification of beam loss due to error in $\Delta\nu_{rms}$ depends on f_S





Change in acceptance to make $I(t)$ for $\xi = \xi_{nat} \times (1 \pm 0.1)$ fit $I(t)$ for ξ_{nat}:

$f_S = 0.01/T_0$: $\Delta\epsilon_{lim} = (-2.5$ mm mrad, $+3.7$ mm mrad$)$

$f_S = 0.005/T_0$: $\Delta\epsilon_{lim} = \mp 6.6$ mm mrad.

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Evaluation of experimental data

Error estimation: systematic error in ξ_y, σ_p

1. Acceptance error due to relative error of $\pm 10\ \%$ in $\xi \cdot \sigma_p$ strongly depends on f_S
2. On the other hand, similar acceptances obtained for different f_S

$$\Downarrow$$

- Assumed error in ξ, σ_p too large because error configuration leading to similar acceptance values is not probable
- $\Delta(\xi\sigma_p)/(\xi\sigma_p) = -4\ \% \rightarrow$ acceptance $\epsilon_{lim} = 48$ mm mrad for both f_S

$$\Downarrow$$

$$\left| \frac{\Delta\epsilon_{lim}[\Delta(\xi\sigma_p)]}{\epsilon_{lim}} \right| \approx 7\ \%$$

Evaluation of experimental data

Error estimation

Total relative acceptance error:

$$|\Delta\epsilon_{lim,rel}| = \sqrt{[\Delta\epsilon_{lim,rel}(\textbf{random})]^2 + [\Delta\epsilon_{lim,rel}(\Delta\beta)]^2 + \{\Delta\epsilon_{lim,rel}[\Delta(\xi \cdot \sigma_p)]\}^2}$$

$$< 13\ \%.$$

Summary

- Method to measure vertical acceptance of SIS-18 based on measuring beam loss developed.
- Evaluate experimental result by particle tracking.
- Tracking code benchmarked for white noise excitation against analytical model.
- Use of realistic noise excitation to determine acceptance.
- Found similar values for different widths of noise power spectrum.
- Estimated error to be smaller than $13\ \%$.

Next steps

- Determination of horizontal acceptance.
- Extension to measurement of dynamic aperture.

  Requires usage of tracking with realistic lattice

  $\longrightarrow$ MAD-X with noise introduced by means of “update” command