Linear Optics Measurements by TBT Analysis at Fermilab Booster

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Focusing errors and Optics Measurements

Errors in the focusing elements lead to optics perturbation with bad consequences:

- *unpredictable response* to machine parameter change
- *uncontrolled beam size* (aperture, luminosity...)

Today BPM systems allow sophisticated techniques and several methods for measuring the linear optics and fitting measurement to a model have been developed.

Two main *philosophies*:

- Closed Orbit response to the excitation of correctors (not the topic of this talk). The (usually) large number of constraints allows to compute accurately the unknown parameters at BPMs and correctors by simple computations. It is time consuming, Tevatron is the largest machine were it is applied .
- Analysis of beam oscillations excited by single kicks or AC dipoles (TBT analysis); data acquisition is fast and, unlike the previous method, it may be applied in fast cycling machines.

Optics from Fourier Analysis of TBT data (single kick)

The Fourier analysis of TBT data has been first applied at LEP in 1992 as a tool for measuring the *uncoupled linear optics*, the very first measurements of *phase advance* between 6 tune monitors from TBT dating back to 1988 at LEAR (CERN).

At Fermilab this technique is mainly used for measuring

- Tevatron linear coupling at *shot set-up* set up and, when needed, along the ramp;
- Booster tunes and linear coupling.

Linear Coupling through TBT Analysis

In the presence of *coupling*, the excitation of one of the two modes excite an oscillation in the other mode too.

Resulting vertical motion (first order approximation) following a horizontal kick

$$y_n^j = \left[\sqrt{\beta_y^j}\left(\mathrm{e}^{-i\Phi_y^j} w_+^j - \mathrm{e}^{i\Phi_y^j} w_-^j\right) - \sqrt{\beta_x^j}\mathrm{e}^{i\Phi_x^j}\sin\chi_j\right] A_x \mathrm{e}^{iQ_x(\theta_j + 2\pi n)} + c.c.$$

where

$$\chi_j \equiv \text{tilt of } j\text{th BPM}$$

$$\Phi_z \equiv \int_0^\theta d\theta' R/\beta_z - Q_z\theta \quad \text{(periodic phase function)}$$

Horizontal motion following a vertical kick

$$x_n^j = \left[\sqrt{\beta_x^j}\left(\mathrm{e}^{-i\Phi_x^j} w_+^j + \mathrm{e}^{i\Phi_x^j} w_-^{*j}\right) + \sqrt{\beta_y^j}\mathrm{e}^{i\Phi_y^j}\sin\chi_j\right] A_y \mathrm{e}^{iQ_y(\theta_j + 2\pi n)} + c.c.$$

The (periodic) functions $w_\pm$ are related to the distribution of coupling sources by

$$w_\pm(\theta) = -\int_0^{2\pi} d\theta' \frac{C^\pm(\theta')}{4\sin \pi Q_\pm} \mathrm{e}^{-iQ_\pm[\theta-\theta'-\pi \mathrm{sign}(\theta-\theta')]}$$

with $Q_\pm \equiv Q_x \pm Q_y$ and

$$C^\pm(\theta) \equiv \frac{R\sqrt{\beta_x\beta_y}}{2B\rho}\Big\{\Big(\frac{\partial B_x}{\partial x} - \frac{\partial B_y}{\partial y}\Big) + B_\theta\Big[\Big(\frac{\alpha_x}{\beta_x} - \frac{\alpha_y}{\beta_y}\Big) - i\Big(\frac{1}{\beta_x} \mp \frac{1}{\beta_y}\Big)\Big]\Big\}\mathrm{e}^{i(\Phi_x \pm \Phi_y)}$$

($R \equiv$ machine radius). The functions $\tilde{w}^\pm \equiv w_\pm \mathrm{e}^{iQ_\pm\theta}$

- are constant in coupler free regions
- experience a discontinuity $-iC^\pm\ell/2R$ at coupler locations (diagnostics tool).
- are constant on the resonances $Q_x \pm Q_y = int$.

The FFT of y^j at Q_x, $Y^j(Q_x)$, for a horizontal kick ($X^j(Q_y)$ for a vertical one) is proportional to the *coupling functions* $w_\pm(\theta_j)$.

Assuming BPMs tilt small or already known [a] the Fourier analysis of the TBT data gives 2 real equations in 4 unknowns for each BPM.

When between two consecutive monitors there are no strong source of coupling, the four equations can be solved in favor of $w_\pm(\theta_j) = w_\pm(\theta_{j+1})$.

A discontinuity in the values computed by using 3 adjacent BPMs reveals the presence of a coupling source.

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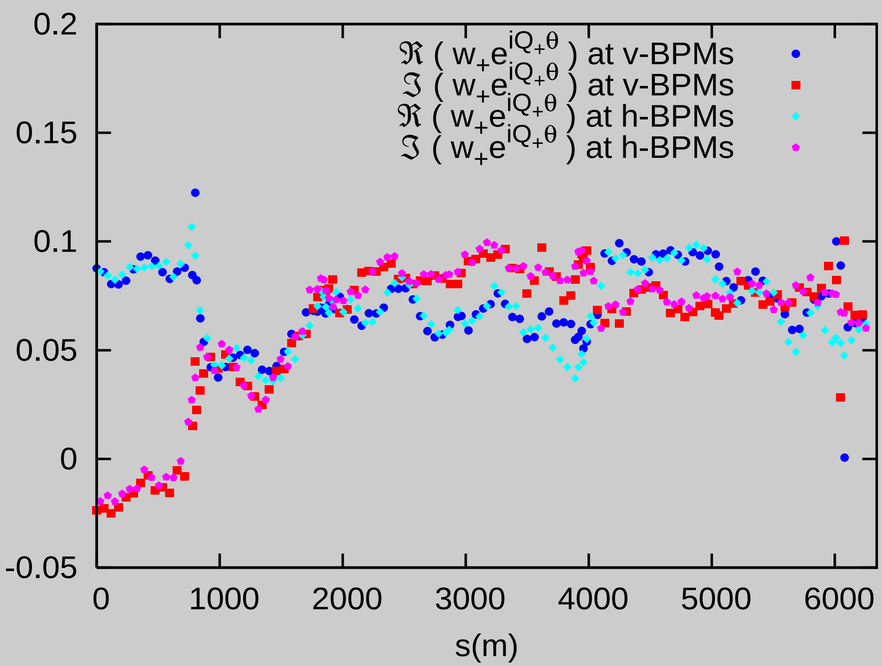
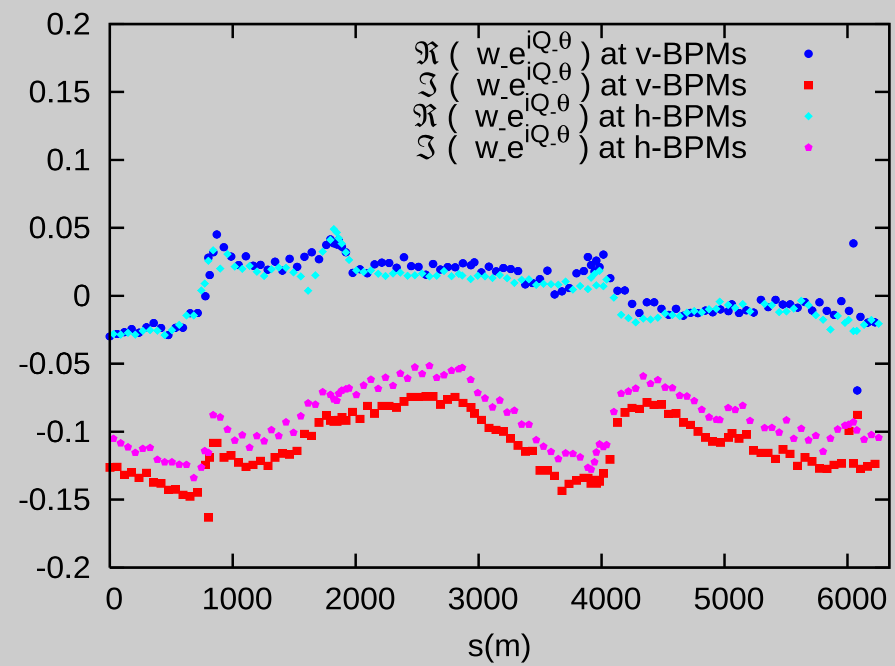
[a] under some circumstances it could be evaluated afterwards by requiring results to be *"smooth"*

Tevatron Coupling functions (November 2005 data)

$\tilde{w}^{+}$



$\tilde{w}^{-}$



Discontinuities are visible around 1000 m (skew quad SQA0 location) and 4000 m (D16) where a badly tilted quadrupole was found.

Booster

The Fermilab Booster is a 474 m long machine accelerating protons from 0.4 to 8 GeV kinetic energy in 33 ms (about 22000 turns). Transition energy is crossed at about turn #9480.

As at most low energy fast ramping synchrotrons measurements are not easy.

Nominal Booster tunes are Q_x=6.75 and Q_y=6.85.

- Correction of the *difference linear coupling* resonance is necessary for setting the tunes close to each other, thus providing room for space charge detuning in presence of large beam intensity demanded by the neutrino physics program.
- The *sum linear coupling* resonance is believed to be source of *vertical emittance growth*.

For these reasons is desirable to have a reliable tool for on-line measurament and possibly compensation of linear coupling.

The application for measuring the Booster tunes from TBT data has been improved by introducing

- *phased sum* for determining beam tunes
- *two peaks* analysis for identifying peaks.

Two sets of measurement are performed, by kicking the beam horizontally and vertically. In each plane the tune is determined by analyzing the signal in the plane of kick.

Phased Sum

The oscillation propagates around the ring as

$$z_n^{(k)} = \frac{1}{2}\, a_z^k\, e^{i[2\pi Q_x(n-1)+\mu_z^k+\psi_{z0}]} + c.c. \qquad (z = x, y)$$

($k \equiv$ BPM index). When the quantity

$$\tilde{z}_n \equiv \Sigma_{k=1}^{M} z_n^{(k)} e^{-i\mu_z^{(k)}}$$

($M \equiv$ number of BPMs) is Fourier-analysed, rather than $z_n^{(k)}$, signal-to-noise ratio is improved by a factor $\sqrt{M}$.

Two Peaks Algorithm

A second problem is the *tune identification*. Suggested algorithm:

- phased sum is Fourier analyzed;
- spectrum is interpolated and oscillation damping time is evaluated;
- largest peak is subtracted from original signal, taking into account damping;
- the new signal is again Fourier analyzed.

As a result for each plane a set of two tune "candidates", with corresponding amplitudes $|\boldsymbol{A_{x,1}}|$, $|\boldsymbol{A_{x,2}}|$, $|\boldsymbol{A_{y,1}}|$ and $|\boldsymbol{A_{y,2}}|$, is found.

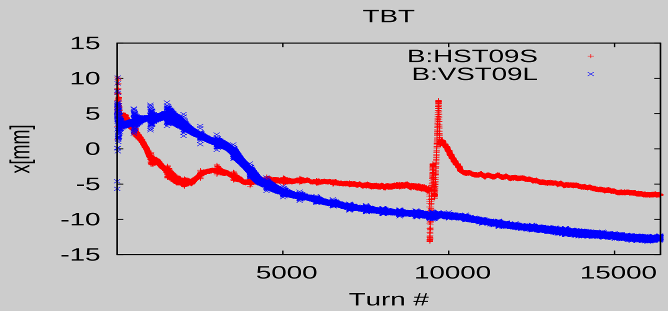
- Is the distance of the two largest peaks, $|\boldsymbol{A_{x,1}}|$ and $|\boldsymbol{A_{y,1}}|$ larger then the resolution of the CFT? Tunes are identified: $\boldsymbol{q_x^{true}} = \boldsymbol{q_{x,1}}$ and $\boldsymbol{q_y^{true}} = \boldsymbol{q_{y,1}}$
- Is it smaller? 2th peaks are compared:

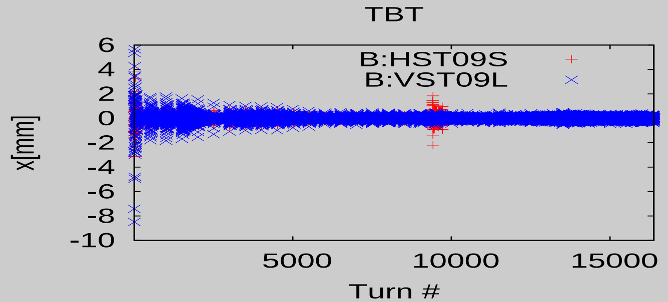
$$|\boldsymbol{A_{x,2}}|/|\boldsymbol{A_{x,1}}| > |\boldsymbol{A_{y,2}}|/|\boldsymbol{A_{y,1}}| \Rightarrow \boldsymbol{q_x^{true}} = \boldsymbol{q_{x,2}} \text{ and } \boldsymbol{q_y^{true}} = \boldsymbol{q_{y,1}}$$
$$|\boldsymbol{A_{x,2}}|/|\boldsymbol{A_{x,1}}| < |\boldsymbol{A_{y,2}}|/|\boldsymbol{A_{y,1}}| \Rightarrow \boldsymbol{q_x^{true}} = \boldsymbol{q_{x,1}} \text{ and } \boldsymbol{q_y^{true}} = \boldsymbol{q_{y,2}}$$

- The Booster has 48 self-triggered BPMs (stripline devices) measuring beam position in both horizontal and vertical planes. TBT acquisition has improved but it is not fully reliable.

- Acquisition is done during ramping. The beam is kicked either horizontally or vertically each 500 turns; the kicker voltage is constant along the ramp.

- Orbit has a large excursions; closed orbit is piecewise computed and subtracted from raw data.

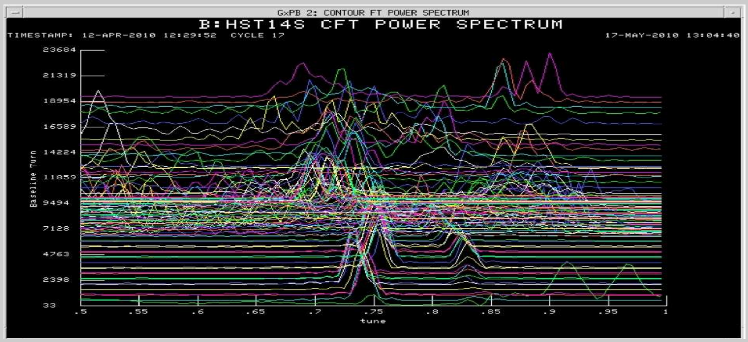


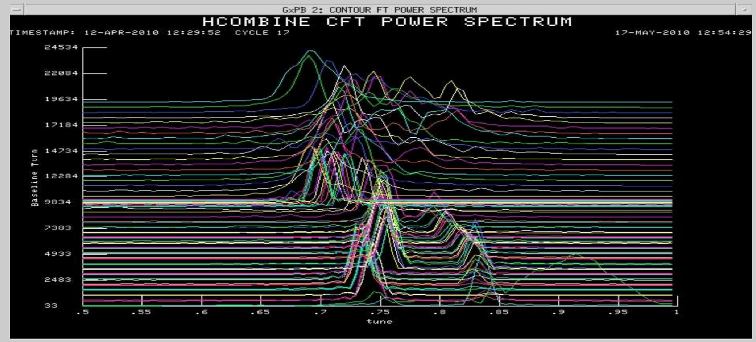


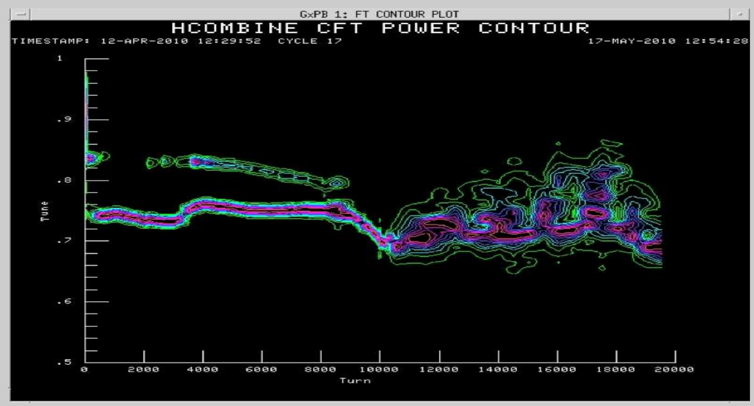
Phased-sum analysis allows to detect the beam tunes in conditions where other algorithms fail.

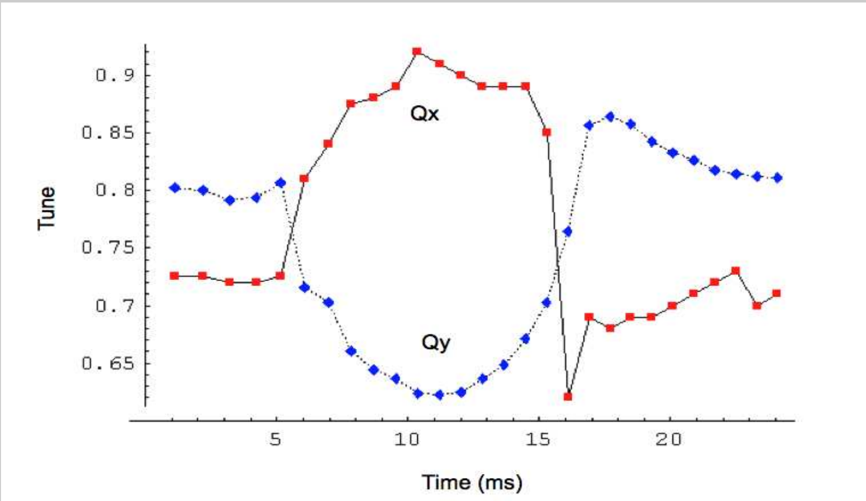
Single BPM FT

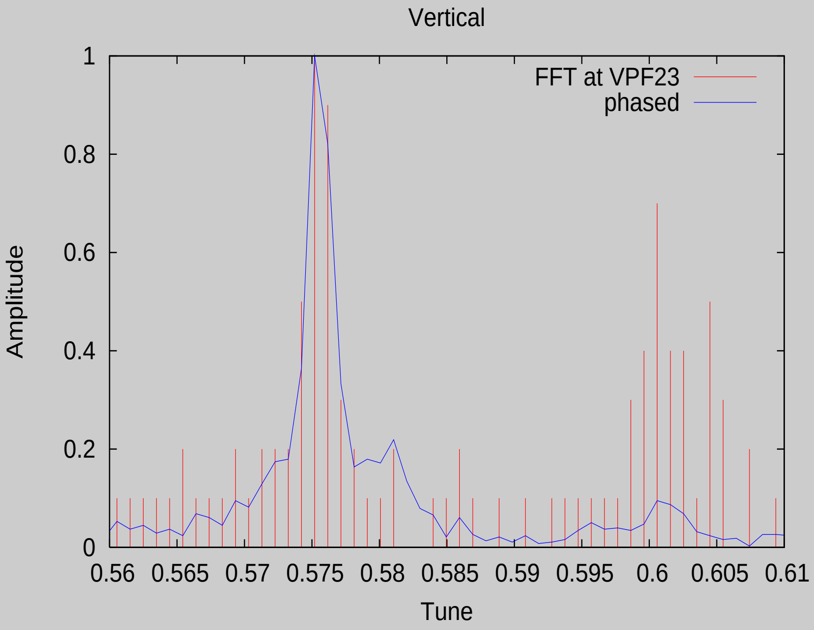
Phased sum FT







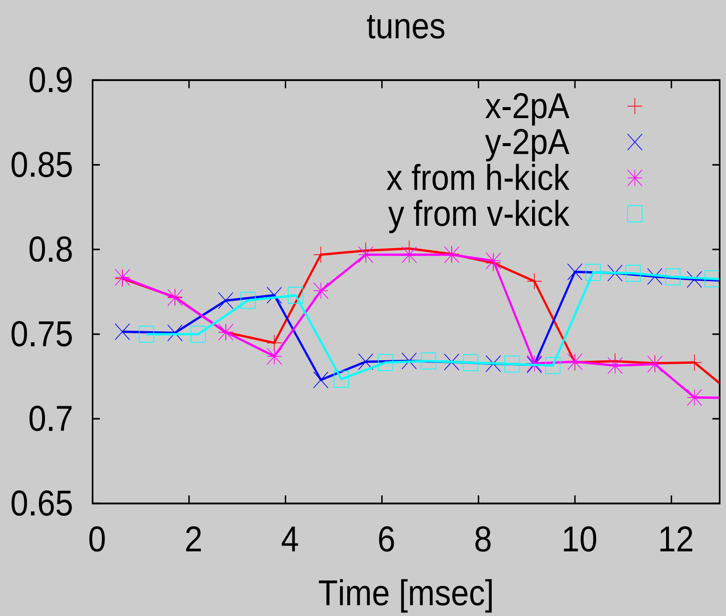


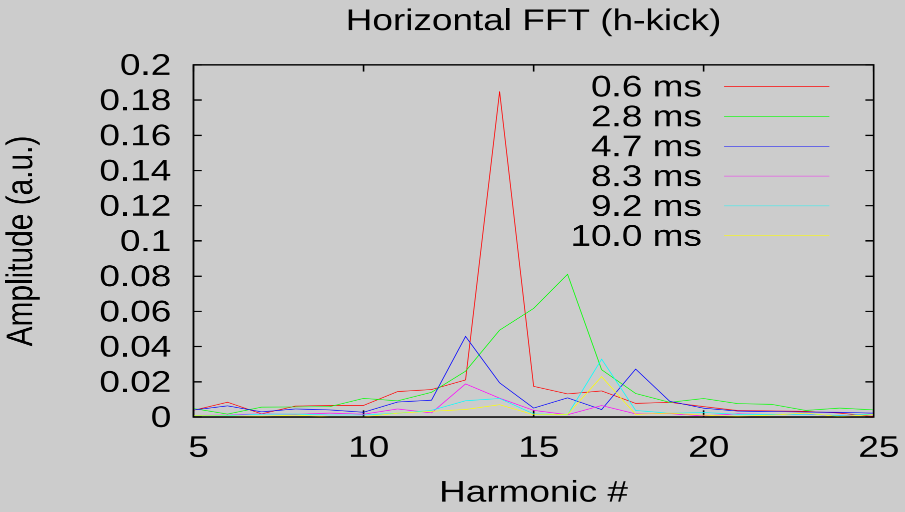


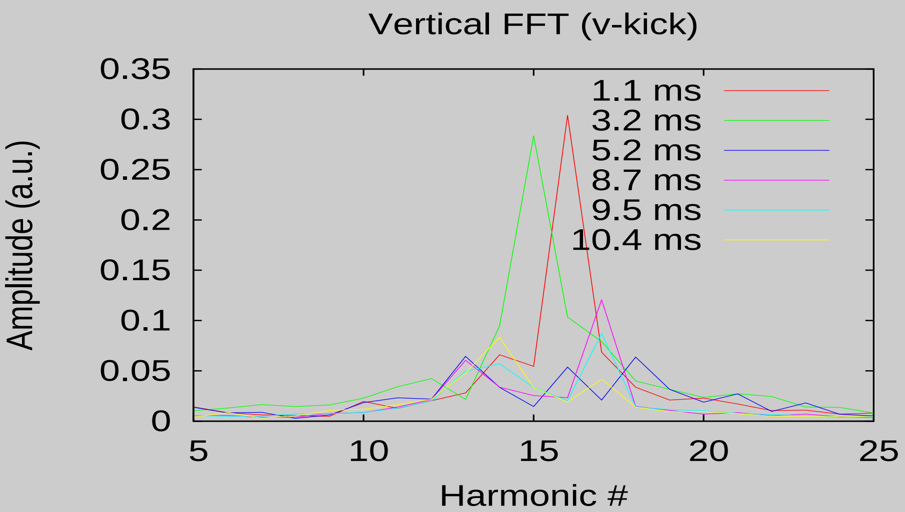
Combined CFT allowed detecting a mis-steering of the tunes during Booster set up after a shut-down.

Phased analysis for Tevatron TBT data helps resolving the vertical tune (at 0.581) for this data set taken after kicking horizontally on the ramp.

“2-peaks-algorithm” picks the right value for q_x at 9.2 ms.

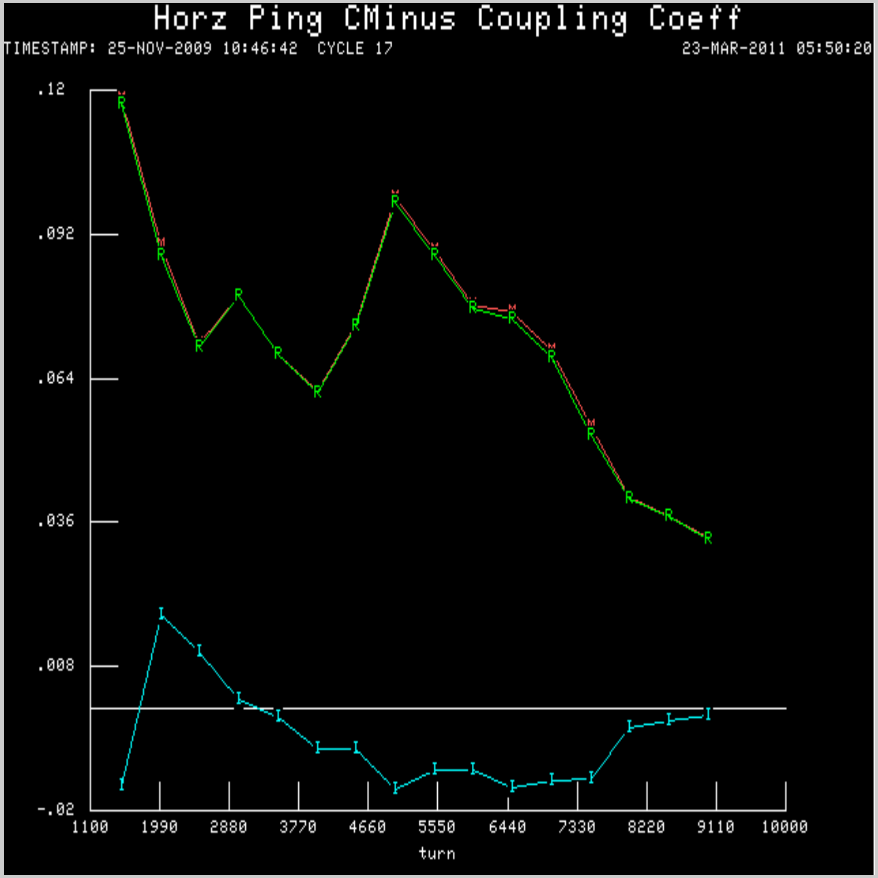


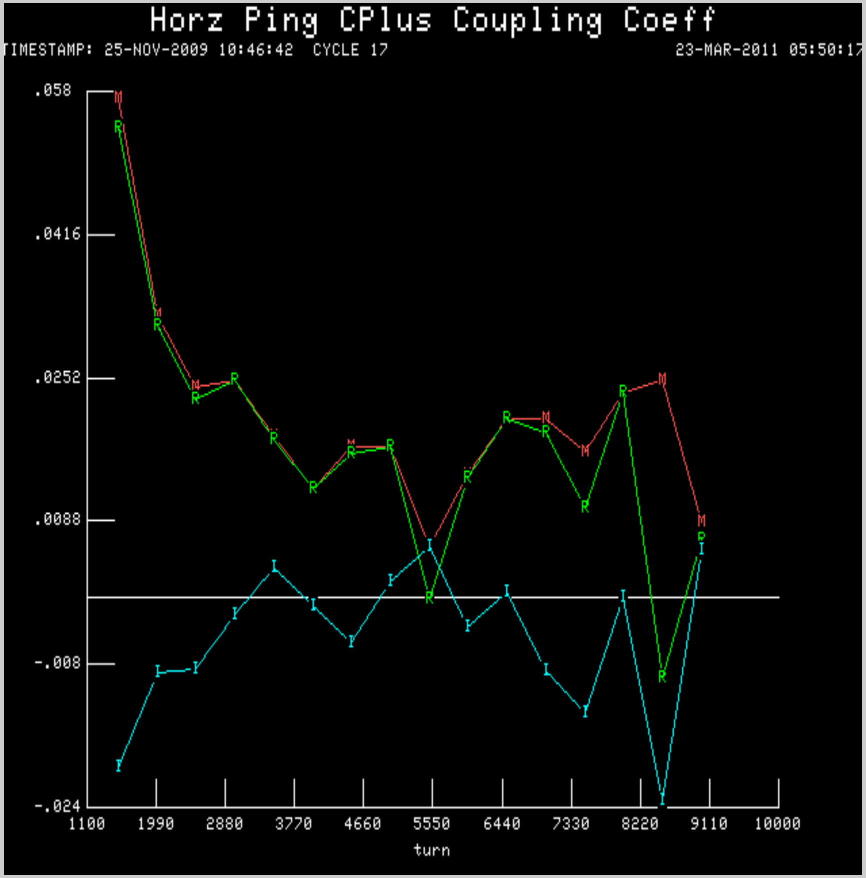




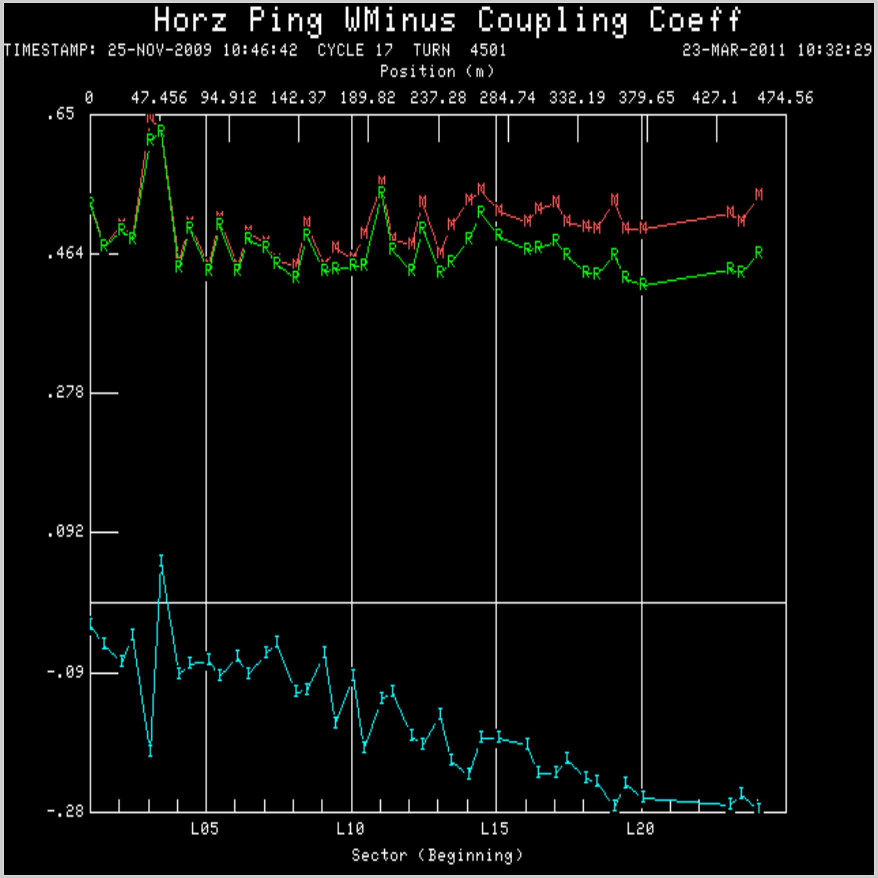
BOOSTER COUPLING MEASUREMENT

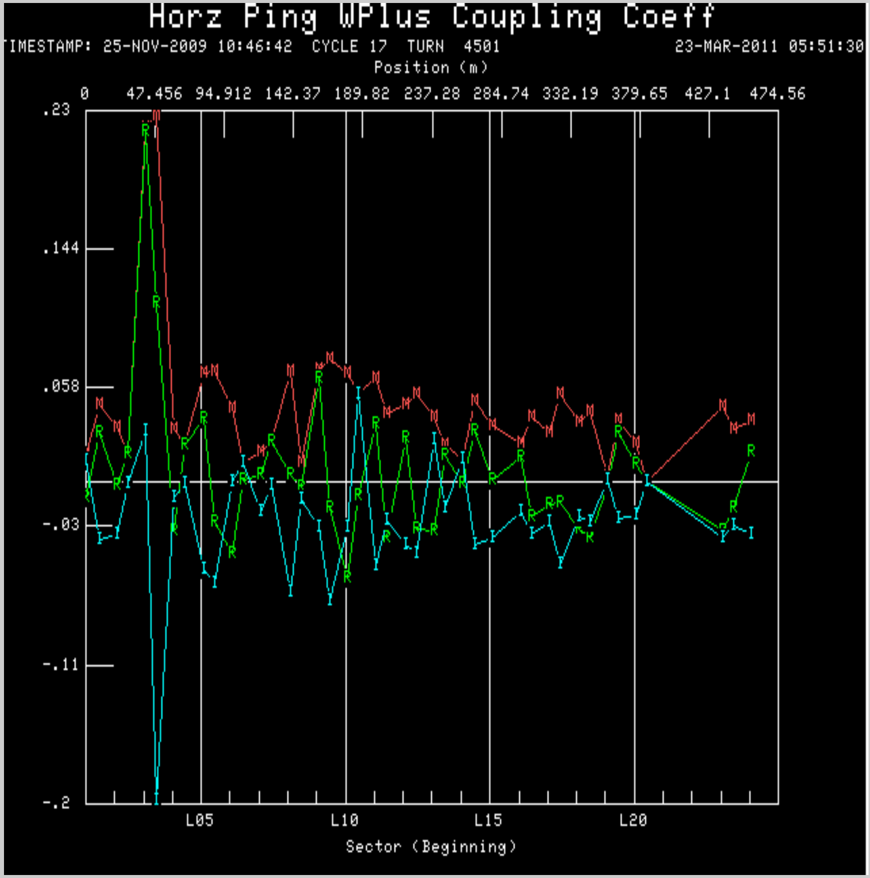
Coupling coefficients on ramp





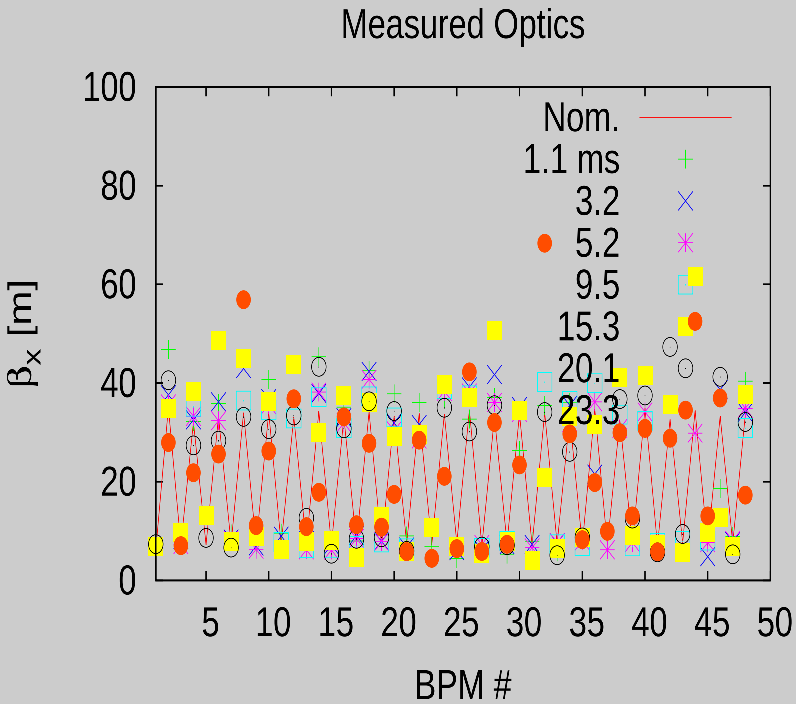
Coupling functions vs. position

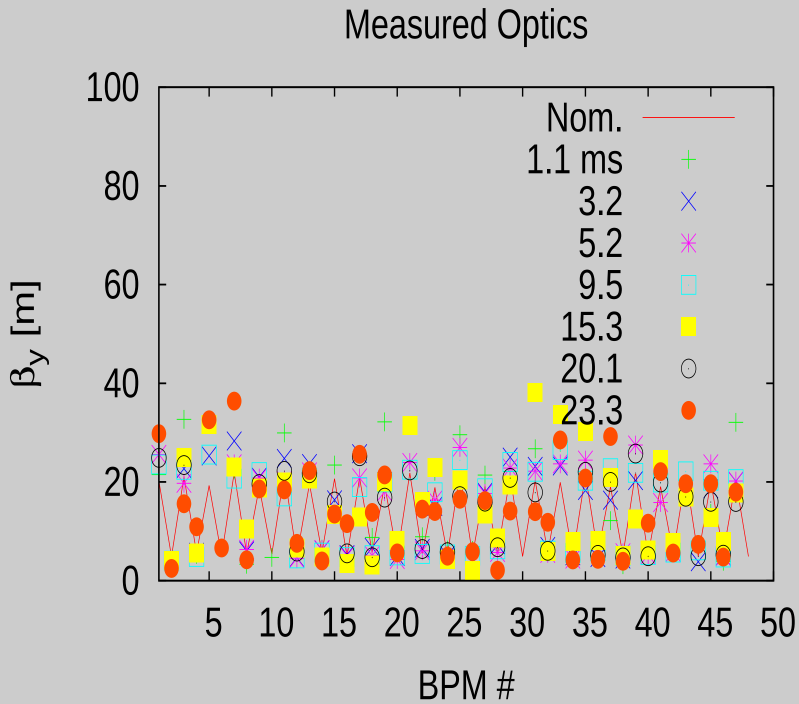




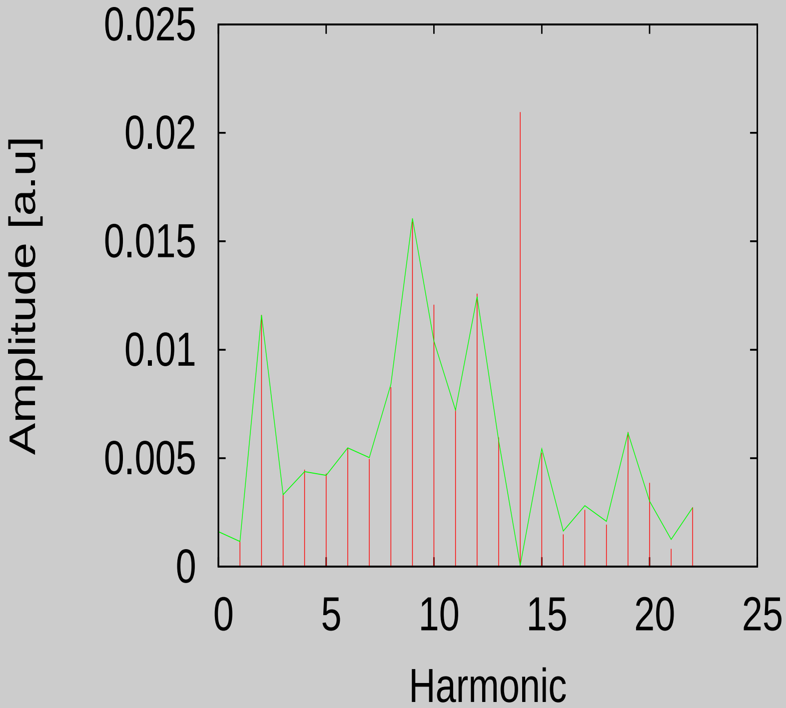
Linear Optics

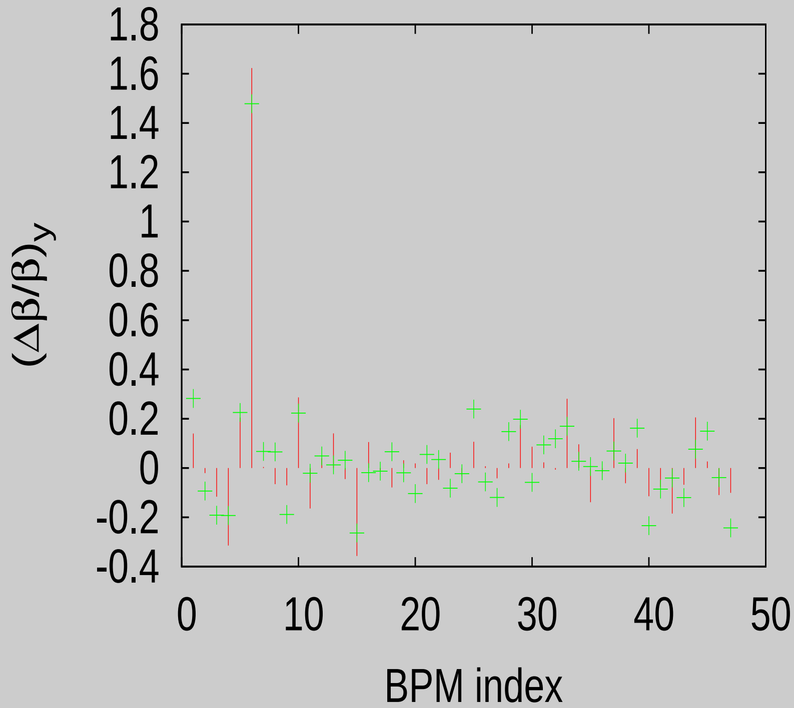
Twiss functions on ramp





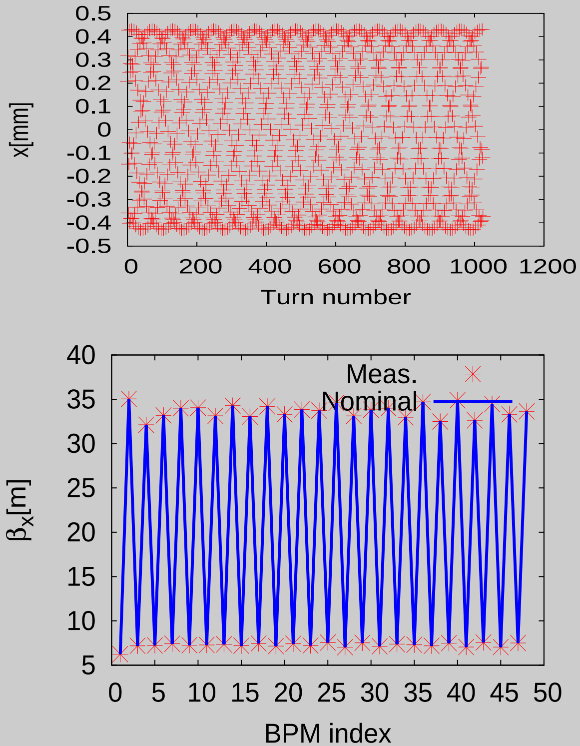
Spatial Fourier Analysis for a *good* data set shows a peak where expected



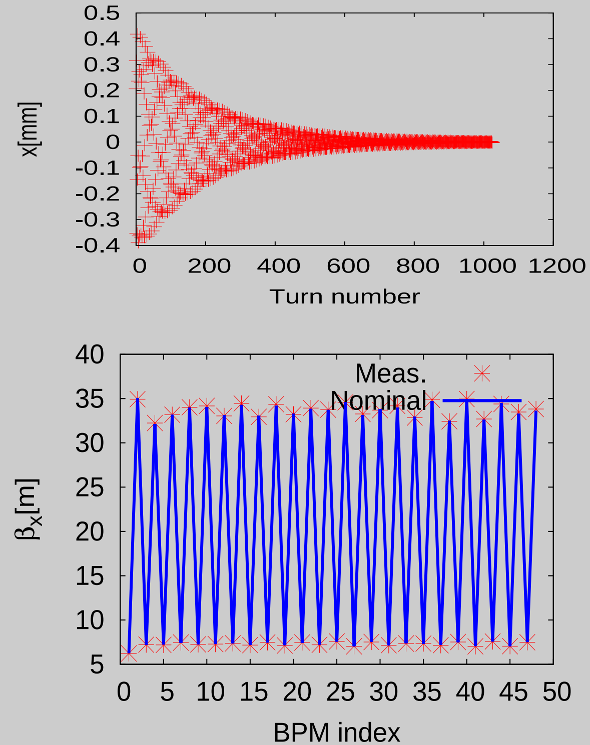


Subtracting the component h=14 the beta-beating is reduced from 26% to 10%.

Booster and Tevatron both have no AC dipole. Signal decoherence: is it a limit?



w/o damping, $\boldsymbol{A_x}$=0.17



with damping, $\boldsymbol{A_x}$=0.03

In *absence* of noise, decoherence results in a under-estimation of the oscillation amplitude with no consequences for the Twiss function measurement. Similarly for coupling evaluation.

Fourier Analysis vs. ICA

ICA uses a method for *blind source separation* applied in signal processing for recovering a set of signals of which only linear mixtures at discrete time steps are known, under the assumption of *narrowband* and *independent* sources.

Vector $\vec{X}(t_i)$ containing the measurements at M stations at time t_i is written as

$$\vec{X}(t_i) = \begin{pmatrix} x_1 \\ x_2 \\ \dots \\ x_M \end{pmatrix} = A\vec{S}(t_i) + \vec{N}(t_i)$$

$M \equiv$ number of BPMs

$N \equiv$ number of sources

$i = 1, ..P \quad P \equiv$ number of turns

where $\vec{S}(t)$ describes the N sources, $A_{M\times N}$ is the *mixing matrix* and $\vec{N}_M$ contains the measurement noise.

Logical steps:

- SVD decomposition of $\vec{X}$ covariance matrix $C^x(0)$

$$C^x(n=0) = < X_j(t_i) X_k(t_i + nT) > = \begin{pmatrix} U_1 & 0 \\ 0 & U_2 \end{pmatrix} \begin{pmatrix} \Sigma_1 & 0 \\ 0 & \Sigma_2 \end{pmatrix} \begin{pmatrix} U_1^T & 0 \\ 0 & U_2^T \end{pmatrix}$$

  and keep only the N_s singular values above threshold

- construct

$$V_{N_s \times M} \equiv \Sigma_1^{-1/2} U_1^T \quad \text{and} \quad \xi_{N_s \times P} \equiv V X$$

- $C^s(n)$ is *diagonal* and related to $C^\xi(n)$ by a similarity transformation

$$C^\xi(n) = [\Sigma_1^{-1/2} U_1^T A]\, C^s(n)\, [\Sigma_1^{-1/2} U_1^T A]^T \equiv W C^s(n) W^T$$

- find the transformation, W, *diagonalizing simultaneously* all $C^\xi(n)$, with $n \in [n_1, n_m]$

- (important part of) mixing matrix A and source matrix S are

$$A = V^{-1} W \qquad S = W^T V X$$

Betatron motion:

$$\begin{Bmatrix} S_{n,k} \\ S_{n+1,k} \end{Bmatrix} = \begin{Bmatrix} \cos 2\pi Q_n(k-1) \\ \sin 2\pi Q_n(k-1) \end{Bmatrix} \qquad \text{(sources)}$$

$$\begin{Bmatrix} A_{jn} \\ A_{j,n+1} \end{Bmatrix} = \begin{Bmatrix} a_n\sqrt{\beta_{jn}}\sin\left(\mu_{jn}+\Phi_n\right) \\ a_n\sqrt{\beta_{jn}}\cos\left(\mu_{jn}+\Phi_n\right) \end{Bmatrix} \qquad \text{(mixing)}$$

Twiss functions

$$\beta_{i,n} = a_n^2(A_{ni}^2 + A_{\bar{n}i}^2) \qquad \mu_{i,n} = \phi_n + \tan^{-1}\left(\frac{A_{ni}^2}{A_{\bar{n}i}^2}\right)$$

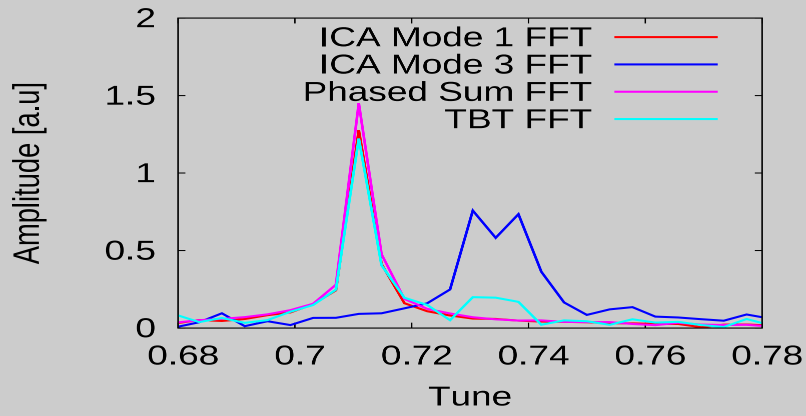
where a_n and ϕ_n are constant of motion and n and $\bar{n}$ are the indeces corresponding to the betatron motion component [a].

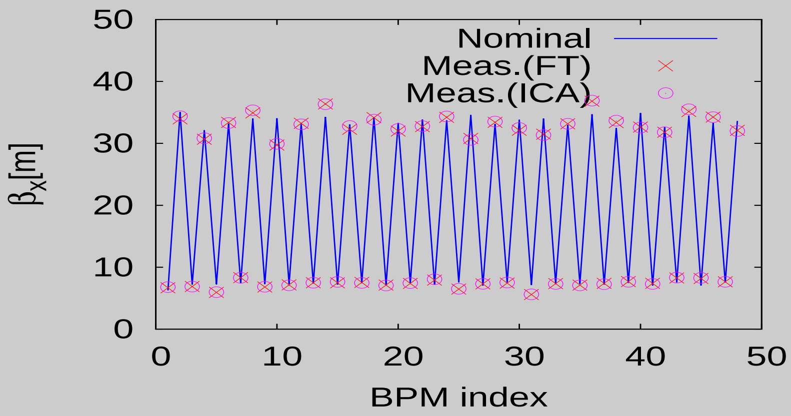
As for the Fourier analysis a *model* is needed for computing the scaling factor a.

It sounds as ICA could be more powerful than the simple Fourier analysis in presence of "spurious" sources as synchrotron side-bands.

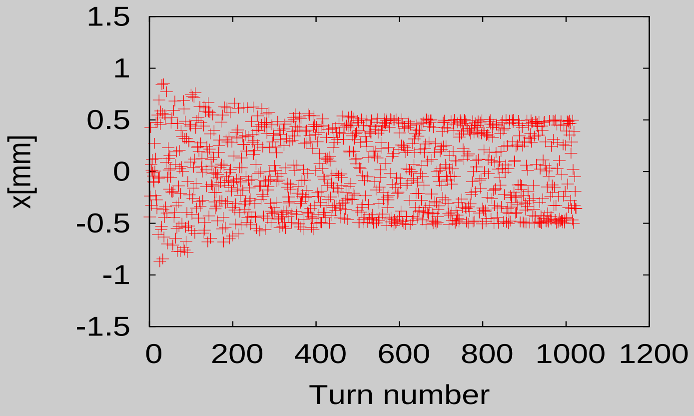
[a] recognized by a Fourier analysis of the rows of S

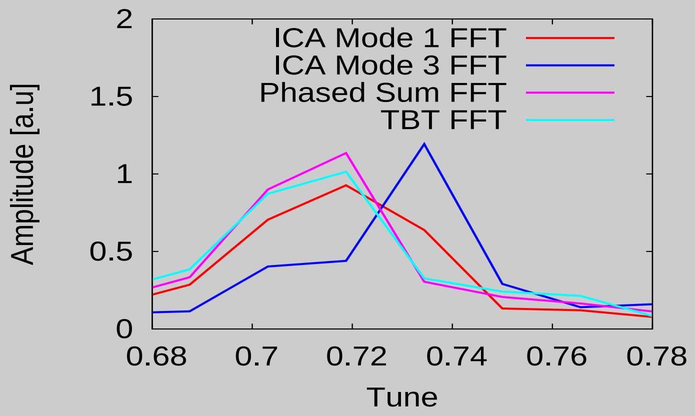
TBT with band limited noise $q = [0.73, 0.74]$ (nominal optics)
no damping, 256 turns

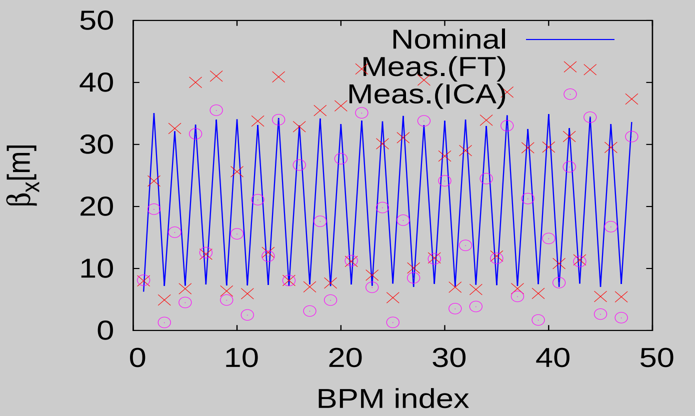




adding damping, 64 turns



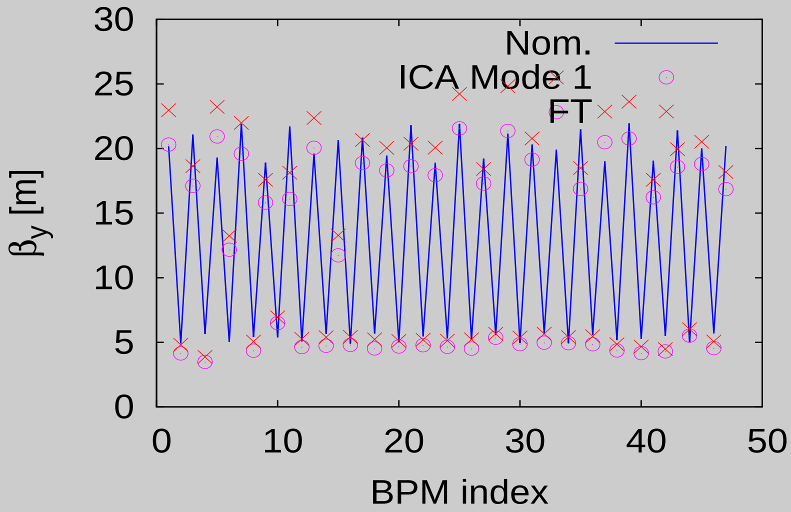


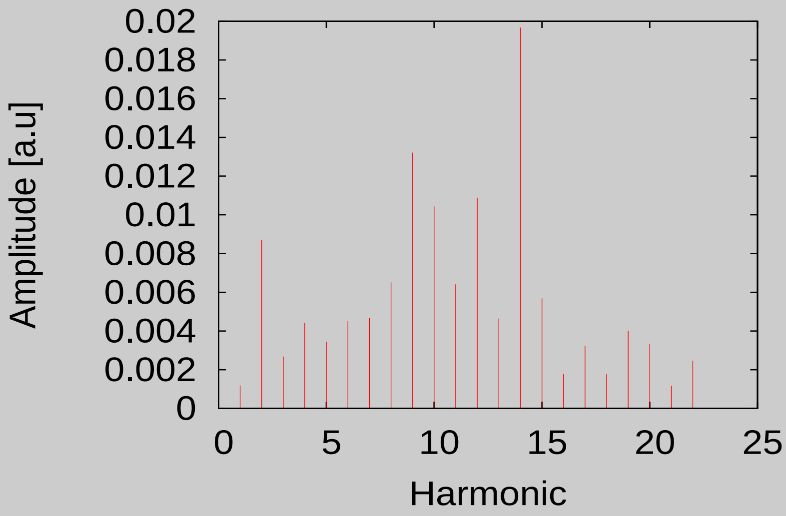


(fake) $\boldsymbol{\Delta}\beta/\beta$: 27% (FT), 40% (ICA)

ICA is able of finding the (disturbance) tunes also with few turns, no advantage for optics meaurement.

Comparison on measured data (v-ping)
(same data as in slide 18)





Spectrum of ICA $(\Delta\beta/\beta)_y$

Conclusions

For the FNAL Booster a reliable on-line tool for detecting and identifying tunes was needed. Fourier analysis of TBT data is a simple tool for measuring machine tunes, linear optics and coupling.

- Tune detection has been improved by analyzing the *phased sum*. This requires the knowledge of the phase advance, not required by ICA.
- The *2-peaks-algorithm* seems resolving the remaining unclear cases.
- The coupling measurement algorithm has been implemented in the Booster console application.
- The TBT Fourier analysis allowed detecting Booster
  - tune mis-steering;
  - correction quadrupole circuits wiring errors.
- Next step: beam-based calibration of skew quadrupole circuits and systematic correction of linear coupling.