Model-Independent Analysis of turn-by-turn BPM measurements in storage rings

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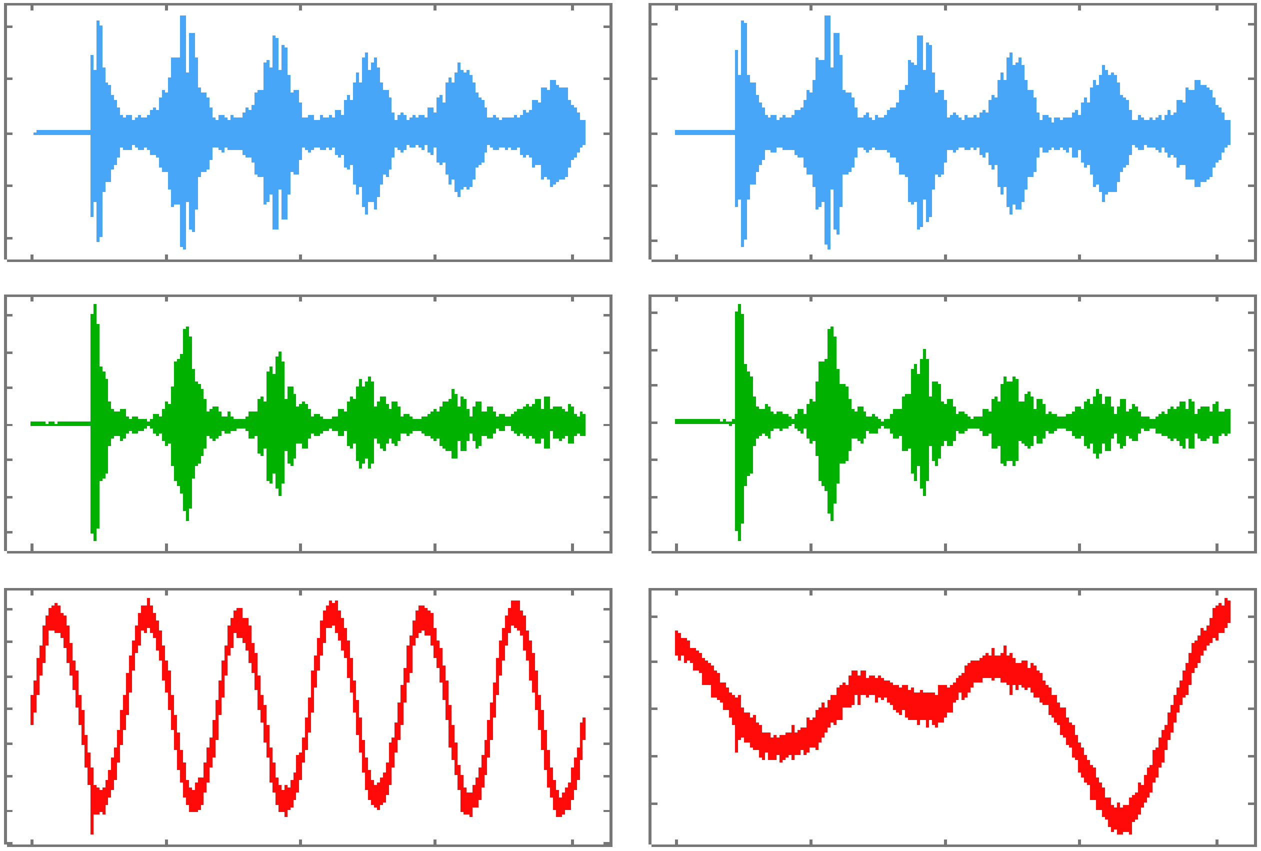


Model-Independent Analysis of BPM measurements

Basic idea of MIA is to represent all measured BPM signals as a linear combination of small number of independent sources:

N beam turns, recorded at *M* monitors:

Singular Value Decomposition (SVD):

$$B = \begin{bmatrix} x_1(t_1) & x_2(t_1) & \cdots & x_M(t_1) \\ x_1(t_2) & x_2(t_2) & \cdots & x_M(t_2) \\ \vdots & \vdots & \ddots & \vdots \\ x_1(t_N) & x_2(t_N) & \cdots & x_M(t_N) \end{bmatrix} = \overset{\hat{U}}{\begin{bmatrix} u_{11} & \cdots & u_{1M} \\ \vdots & \ddots & \vdots \\ u_{N1} & \cdots & u_{NM} \end{bmatrix}} \overset{\Sigma}{\begin{bmatrix} \sigma_1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \sigma_M \end{bmatrix}} \overset{\hat{V}^T}{\begin{bmatrix} v_{11} & \cdots & v_{1M} \\ \vdots & \ddots & \vdots \\ v_{1M} & \cdots & v_{MM} \end{bmatrix}^T}$$

Orthogonal | Diagonal | Orthogonal

In other words, each BPM readout can be represented as a sum of components with rapidly declining amplitudes:

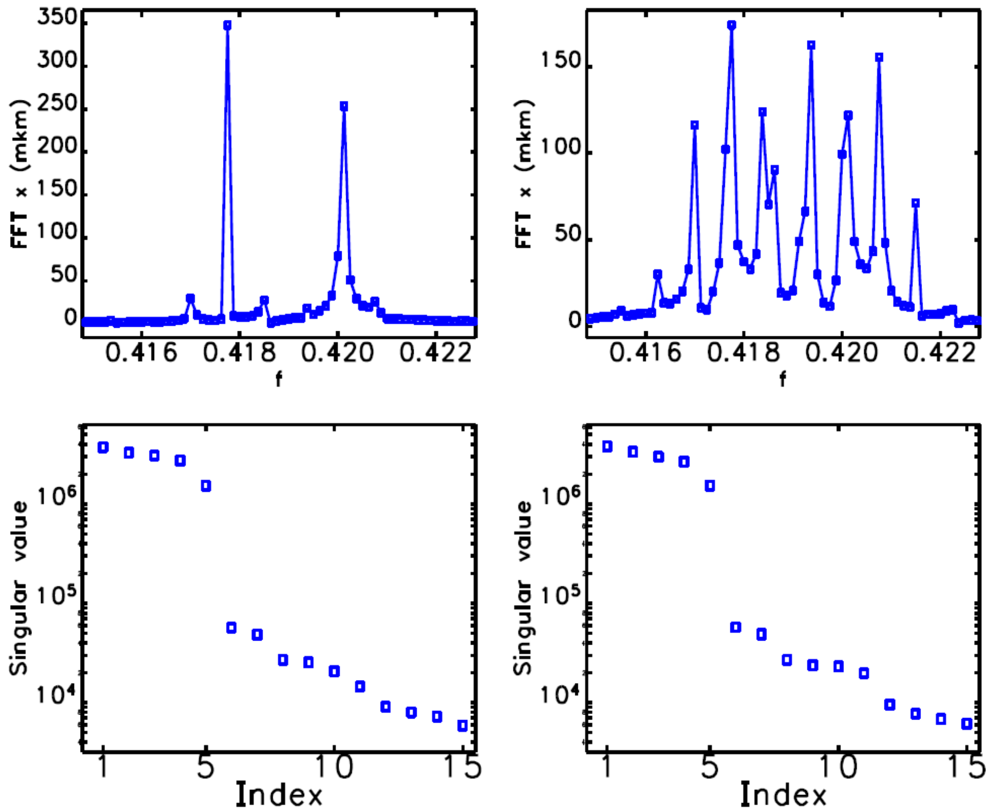
$$\begin{bmatrix} x_m(t_1) \\ x_m(t_2) \\ \vdots \\ x_m(t_N) \end{bmatrix} = \sum_{j=1}^{M} \sigma_j v_{mj} \begin{bmatrix} u_j(t_1) \\ u_j(t_2) \\ \vdots \\ u_i(t_N) \end{bmatrix}$$

$u_j(t)$ – temporal modes

$v_j(s)$ – spatial modes

$$\langle u_j u_k \rangle_t = \sum_t u_j(t) u_k(t) = \begin{cases} 0, & j \neq k \\ 1, & j = k \end{cases}$$

Simulated betatron+synchrotron oscillations in the Tevatron
with different sextupole settings:



$$\boldsymbol{X}(t,s) \approx R(s)\,\boldsymbol{X}_0(t) + \boldsymbol{D}(s)\,\delta(t),$$

$$\boldsymbol{X}(t,s) = (x, x', y, y')^T,$$

$R(s) - 4\times 4$ transport matrix
from 0 to point *s*,

$\boldsymbol{D}(s)$ – dispersion function,

$\delta(t) = \Delta p / p,$

$\boldsymbol{X}_0(t) = \boldsymbol{X}(t,0) - \boldsymbol{D}(0)\,\delta(t).$

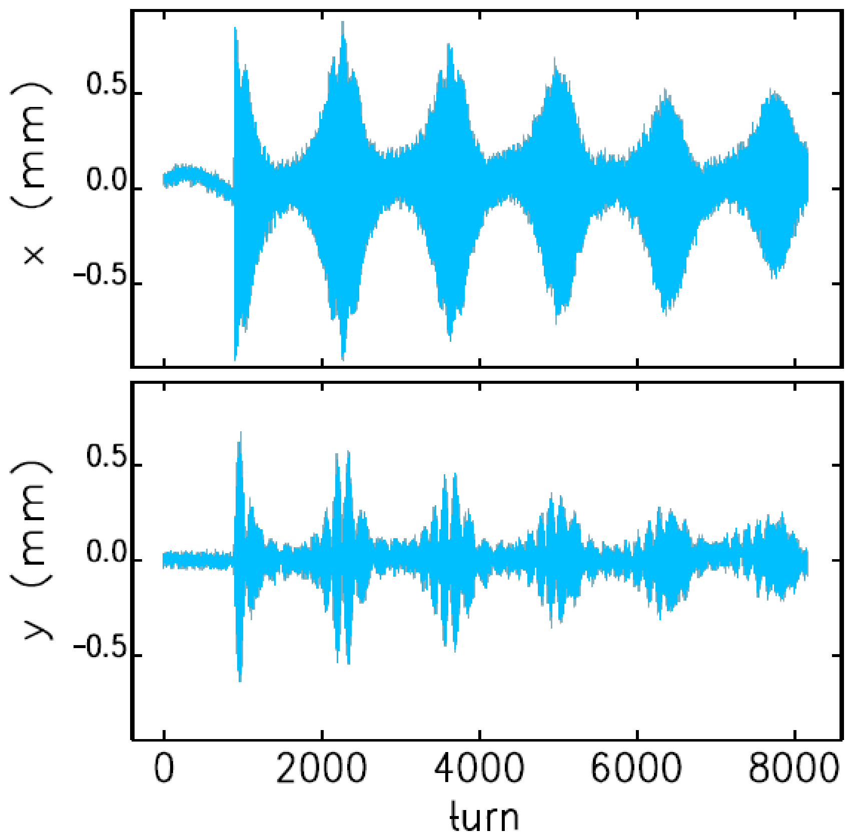
$$B = \begin{bmatrix} \vdots & \vdots & \vdots & \vdots & \vdots \\ x_0(t) & x'_0(t) & y_0(t) & y'_0(t) & \delta(t) \\ \vdots & \vdots & \vdots & \vdots & \vdots \end{bmatrix} \begin{bmatrix} \vdots & \vdots & \vdots & \vdots & \vdots \\ R_{11}(s) & R_{12}(s) & R_{13}(s) & R_{14}(s) & D_x(s) \\ \vdots & \vdots & \vdots & \vdots & \vdots \end{bmatrix}^T$$

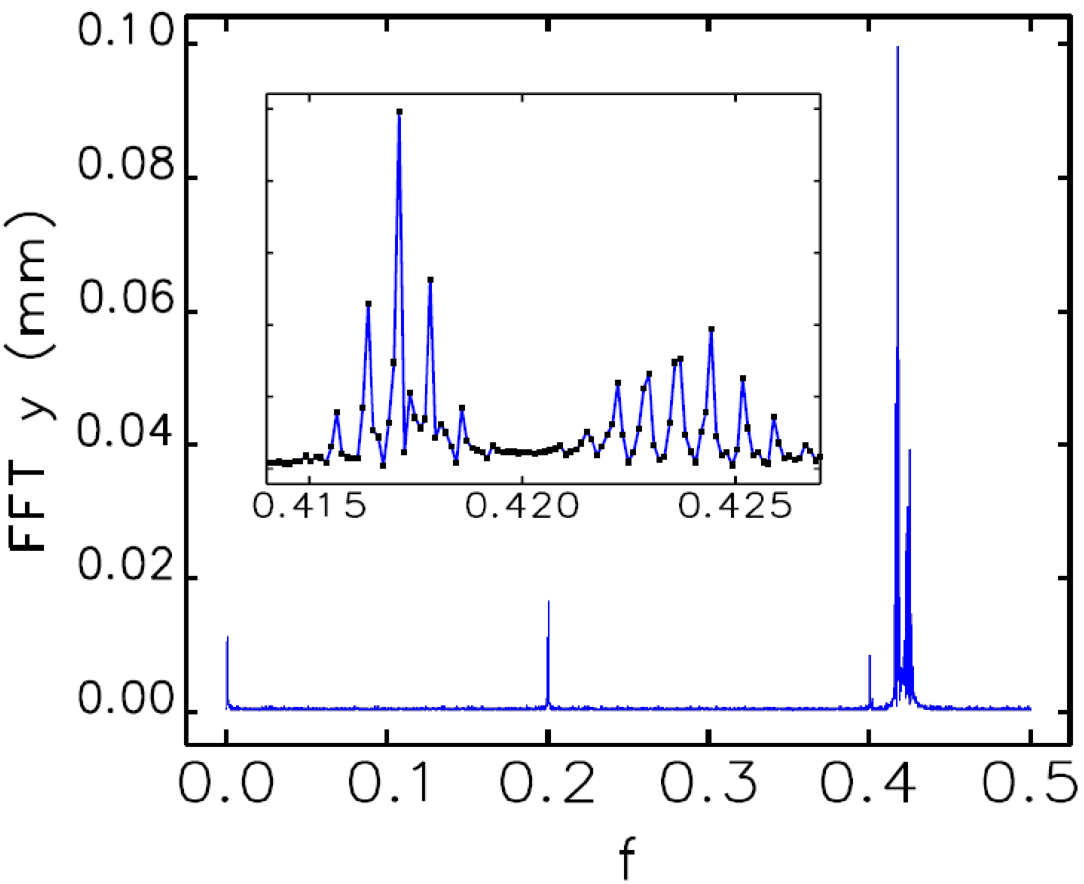
I. e. main temporal modes – linear combinations of $x_0(t), x'_0(t), y_0(t), y'_0(t), \delta(t)$,

and main spatial modes – linear combinations of $R_{11}(s), R_{12}(s), R_{13}(s), R_{14}(s), D_x(s)$

(are determined only by linear optics).

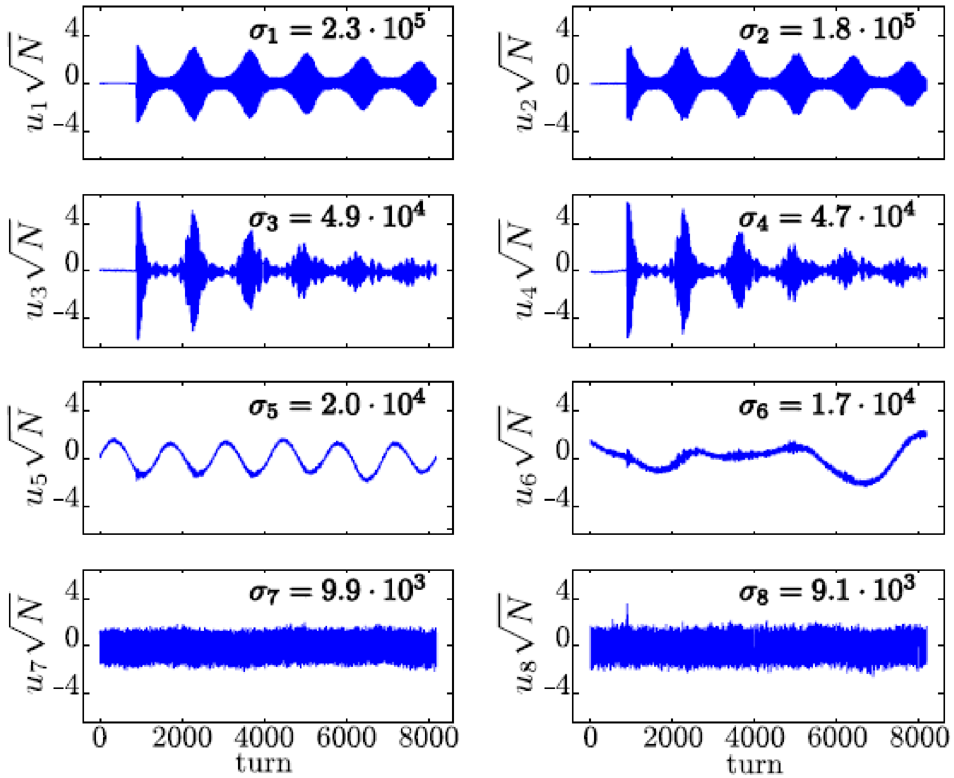
Measured BPM signals



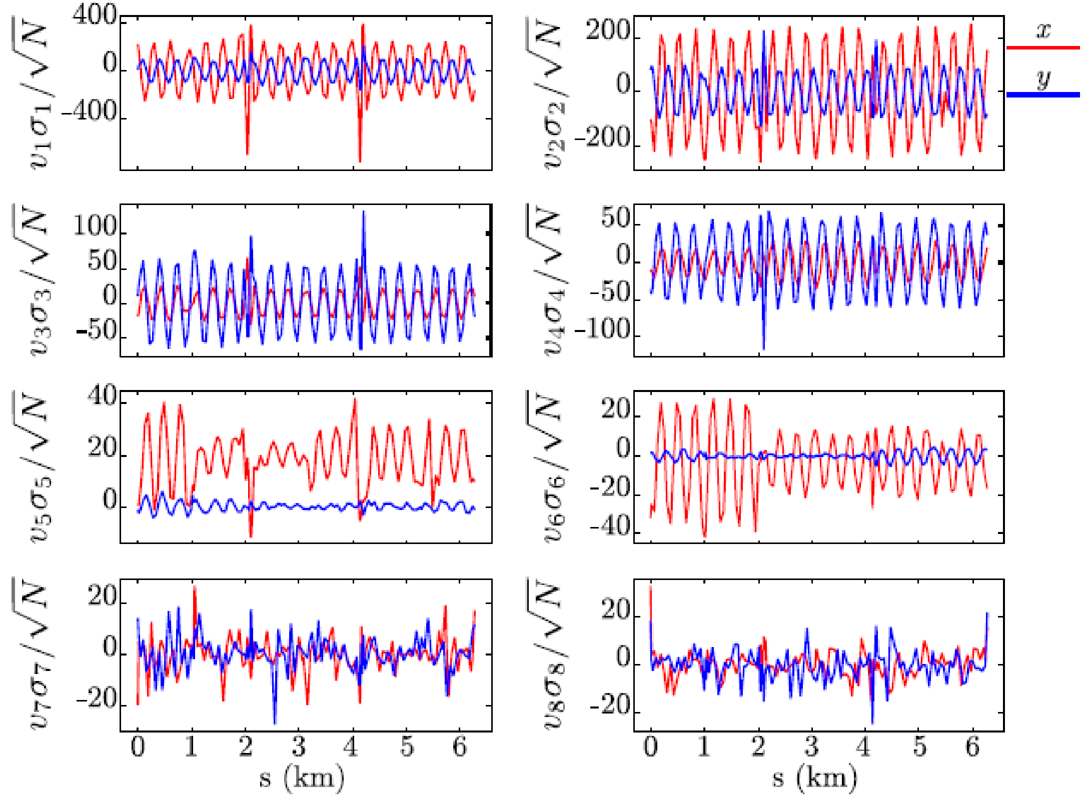


Kicked beam oscillations in the Tevatron

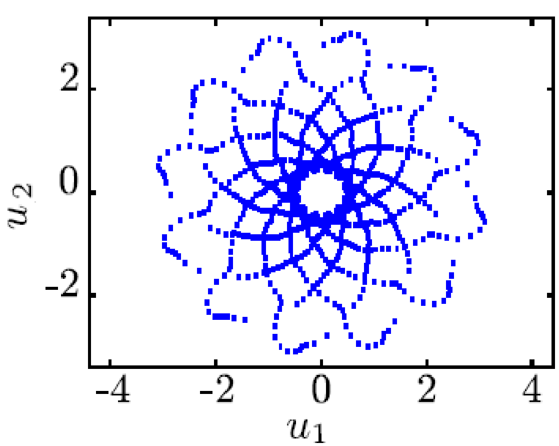
Temporal modes
(mutually independent components)



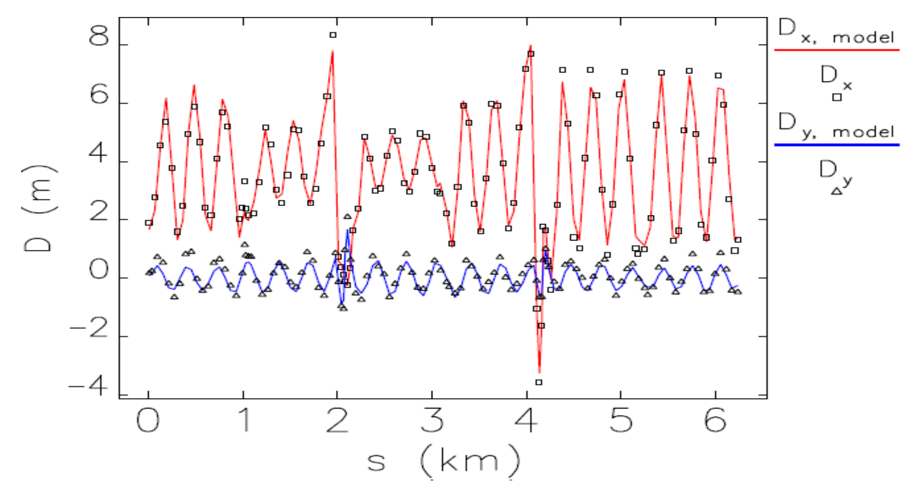
Spatial modes
(amplitude of temporal mode vs BPM position)



Beam centroid trace in phase space:



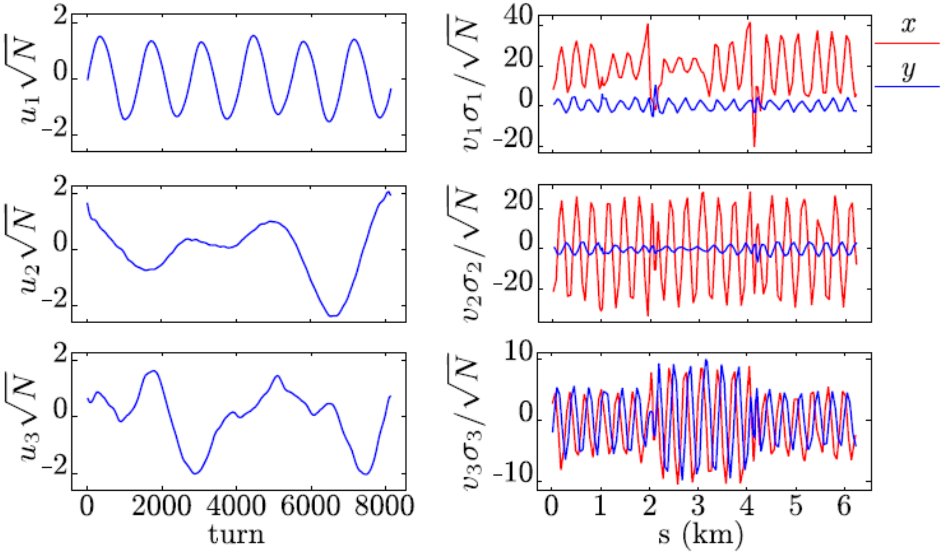
Lattice functions:



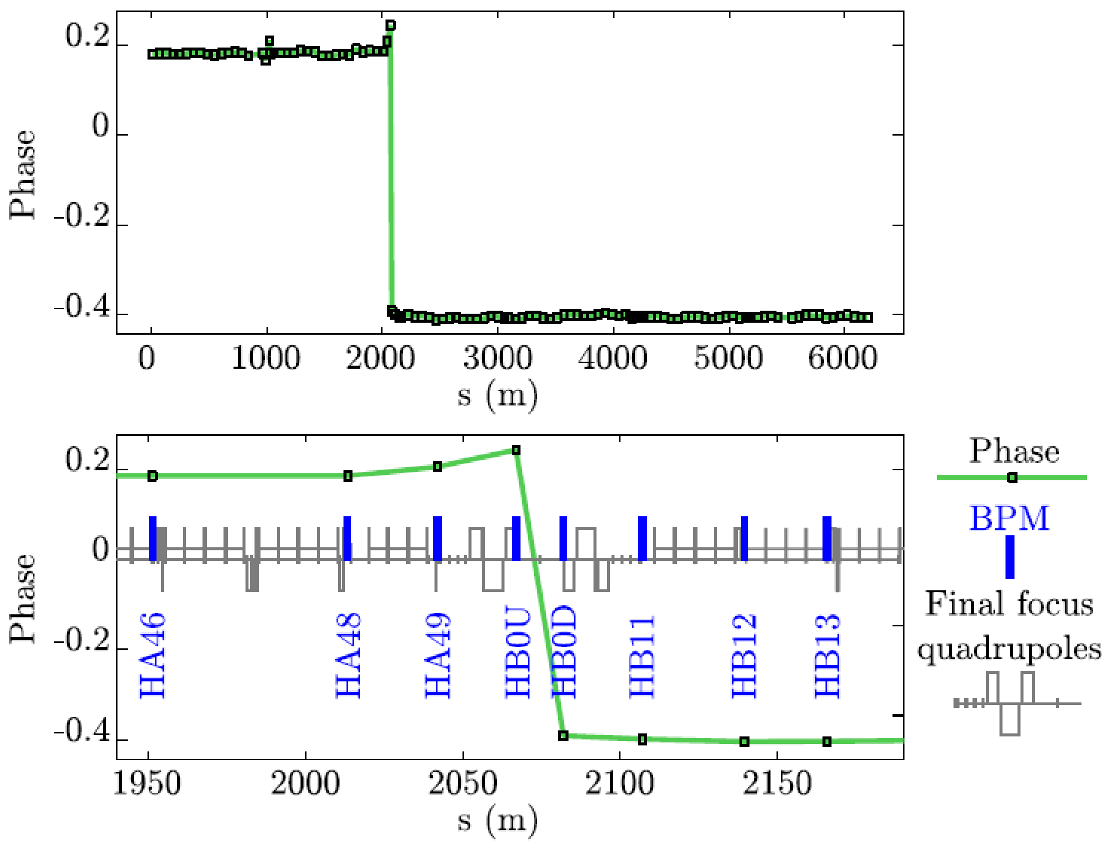
Horizontal and vertical dispersion function

Finding the origin of some modes

Slow modes (after low-pass Fourier filter)



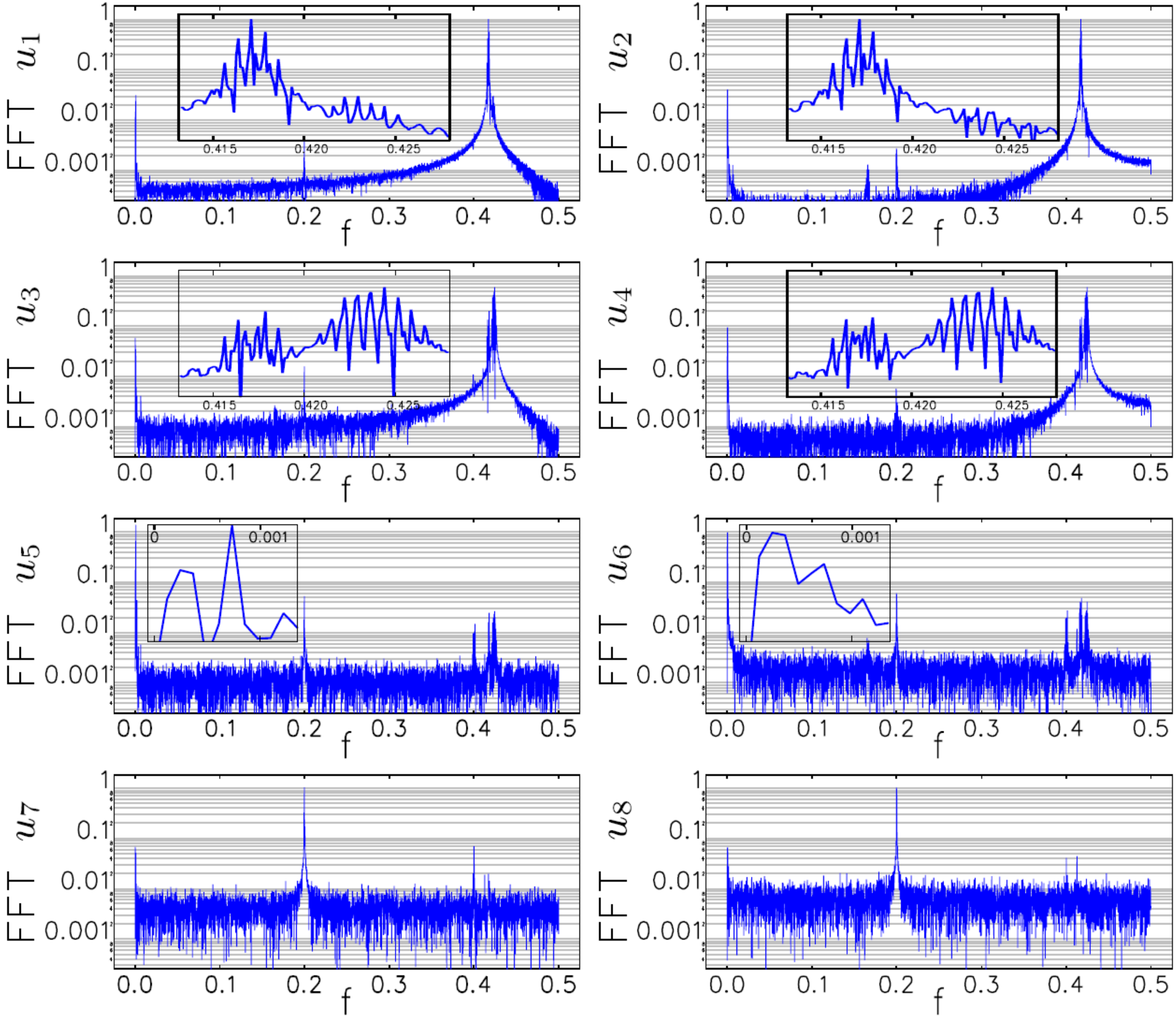
"Betatron" phase of mode 2 (vibrational mode)



At any pair of (horizontal) BPMs any spatial mode can be represented as a linear combination of two independent orbits:

$$\begin{cases} v_{\text{vibr}}(s_1) = C_1 v_{\text{b1}}(s_1) + C_2 v_{\text{b2}}(s_1) \\ v_{\text{vibr}}(s_2) = C_1 v_{\text{b1}}(s_2) + C_2 v_{\text{b2}}(s_2) \end{cases}$$

tan(Phase) = C_1/C_2



Fourier amplitudes of temporal MIA modes
Residual mode mixing limits many practical applications (like measurements of beta-functions)

Betatron functions for coupled motion and MIA modes

$x(t,s)=Re[a_1 f_{1x}(s)e^{i\mu_1 t}+a_2 f_{2x}(s)e^{i\mu_2 t}]$ – linear betatron oscillations,

$a_1=|a_1|e^{i\psi_1}, \quad f_{1x}(s)=|f_{1x}(s)|e^{i\psi_{1x}(s)}$

if $a_1=a_2=1,$ then

$$x(t,s)=|f_{1x}(s)|\cos(\psi_{1x}(s)+\mu_1 t)+|f_{2x}(s)|\cos(\psi_{2x}(s)+\mu_2 t)$$

$$x(t,s) = \begin{bmatrix} \cos\mu_1 t \\ \sin\mu_1 t \\ \cos\mu_2 t \\ \sin\mu_2 t \end{bmatrix}^T \begin{bmatrix} |f_{1x}(s)|\cos\psi_{1x}(s) \\ -|f_{1x}(s)|\sin\psi_{1x}(s) \\ |f_{2x}(s)|\cos\psi_{2x}(s) \\ -|f_{2x}(s)|\sin\psi_{2x}(s) \end{bmatrix}$$

$$B = \begin{bmatrix} \cdots & \cos\mu_1 t & \cdots \\ \cdots & \sin\mu_1 t & \cdots \\ \cdots & \cos\mu_2 t & \cdots \\ \cdots & \sin\mu_2 t & \cdots \end{bmatrix}^T \begin{bmatrix} \cdots & |f_{1x}(s)|\cos\psi_{1x}(s) & \cdots \\ \cdots & -|f_{1x}(s)|\sin\psi_{1x}(s) & \cdots \\ \cdots & |f_{2x}(s)|\cos\psi_{2x}(s) & \cdots \\ \cdots & -|f_{2x}(s)|\sin\psi_{2x}(s) & \cdots \end{bmatrix} = U_f V_f^T$$

If $N|\mu_1-\mu_2|/2\pi \gg 1, \quad U_f$ – orthogonal, $U_f^T U_f \approx \frac{N}{2} I_{4\times 4}$

$V_f=\hat{V}_f \Sigma_f \hat{O}_f,$ then $B=\left(\sqrt{\frac{2}{N}} U_f \hat{O}_f\right)\left(\sqrt{\frac{N}{2}}\Sigma_f\right)\hat{V}_f^T$ In order to calculate optical functions it is necessary to find $\hat{O}_f$

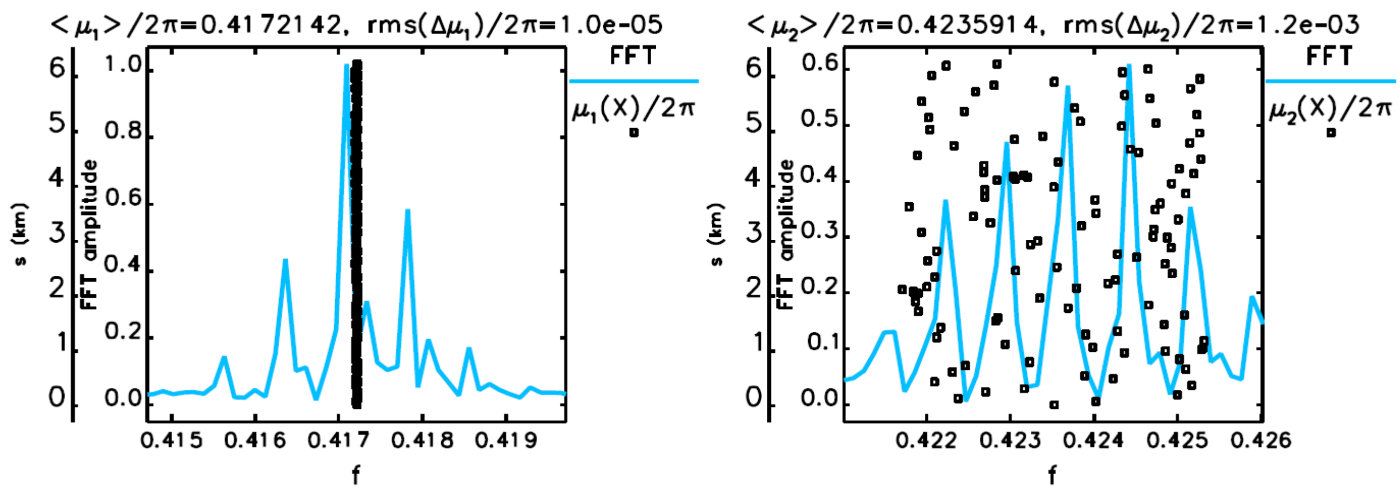
Criterion of betatron mode separation:

Suppose that instead of each BPM we have 2 BPMs which are exactly 1 turn apart:

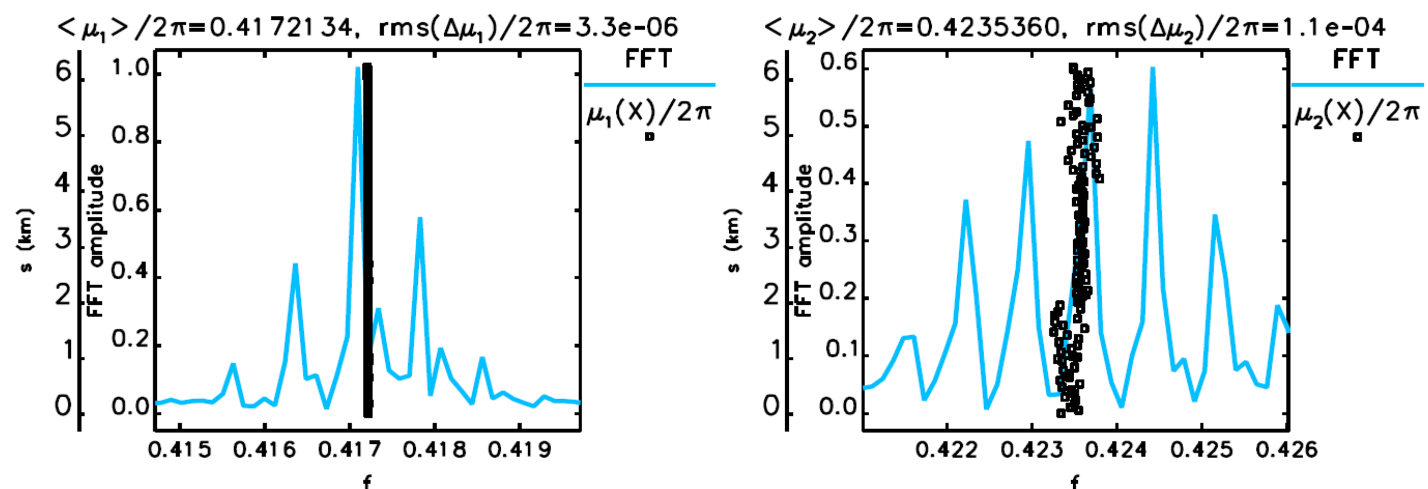
$$B_{new} = \left[\begin{array}{ccc|ccc} x_1(t_1) & \cdots & x_M(t_1) & x_1(t_2) & \cdots & x_M(t_2) \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ x_1(t_{N-1}) & \cdots & x_M(t_{N-1}) & x_1(t_N) & \cdots & x_M(t_N) \end{array} \right]$$

Betatron phase advances between such BPMs are equal to betatron tunes

Before mode separation:

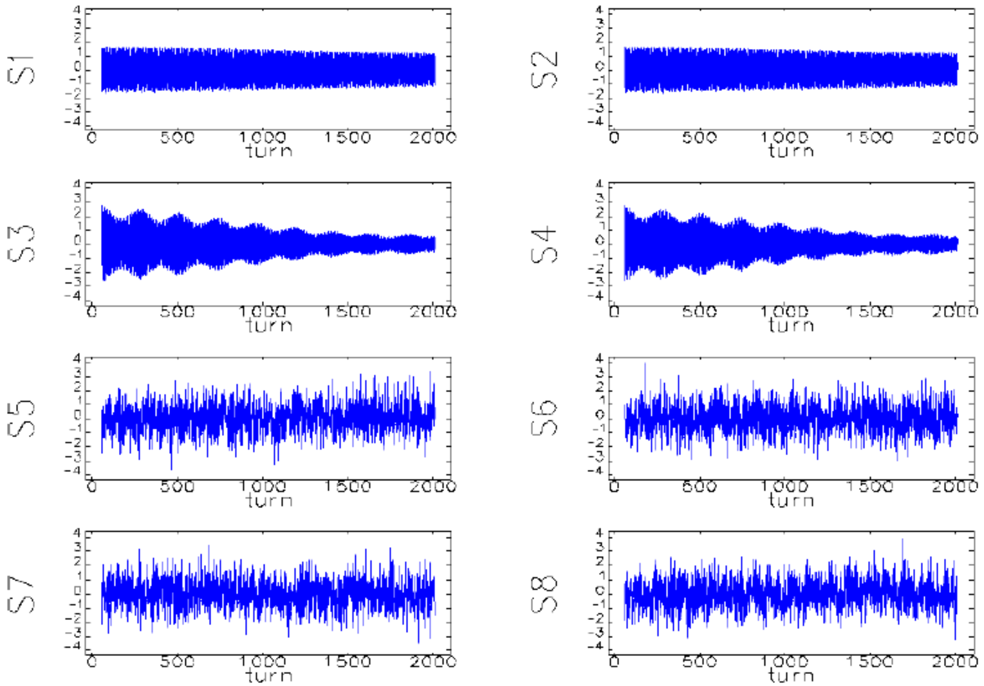


After mode separation (via additional rotation transformation):

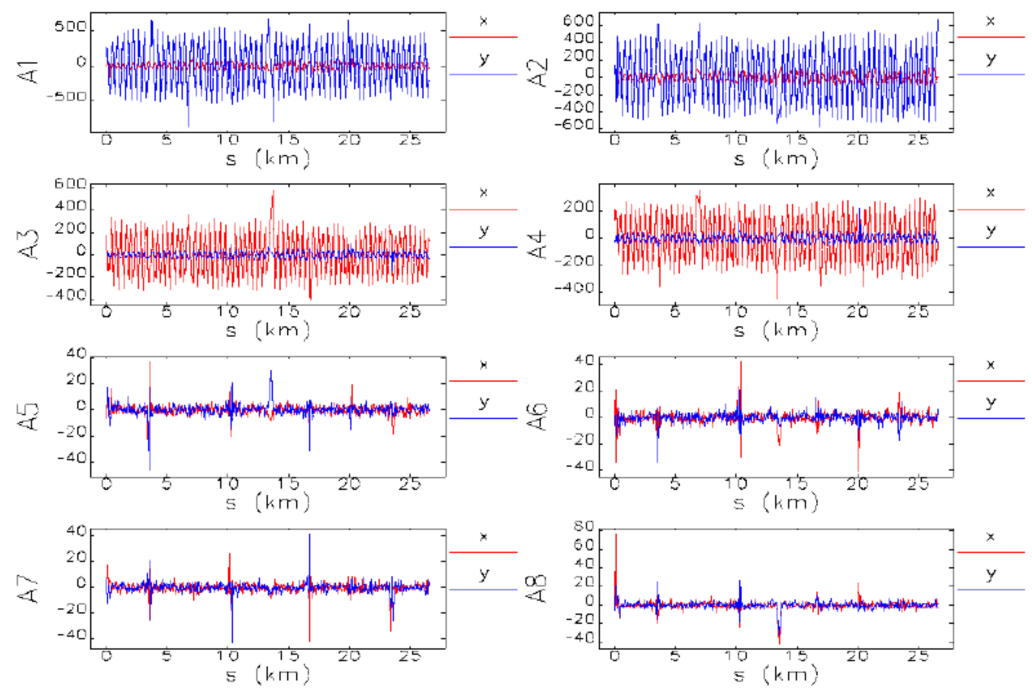


MIA applied to the LHC turn-by-turn data (courtesy R. Miyamoto):

Temporal modes:



Spatial modes:



Elements of transport matrix between BPMs

Using any 4 linearly independent orbits (e. g. spatial modes of betatron oscillations) it is possible to calculate some of the transport matrix elements between BPMs:

$$(x_1^a x_2^b - x_2^a x_1^b)/Q_{12} + (x_3^a x_4^b - x_4^a x_3^b)/Q_{34} = \mathcal{R}_{12}^{ab}$$

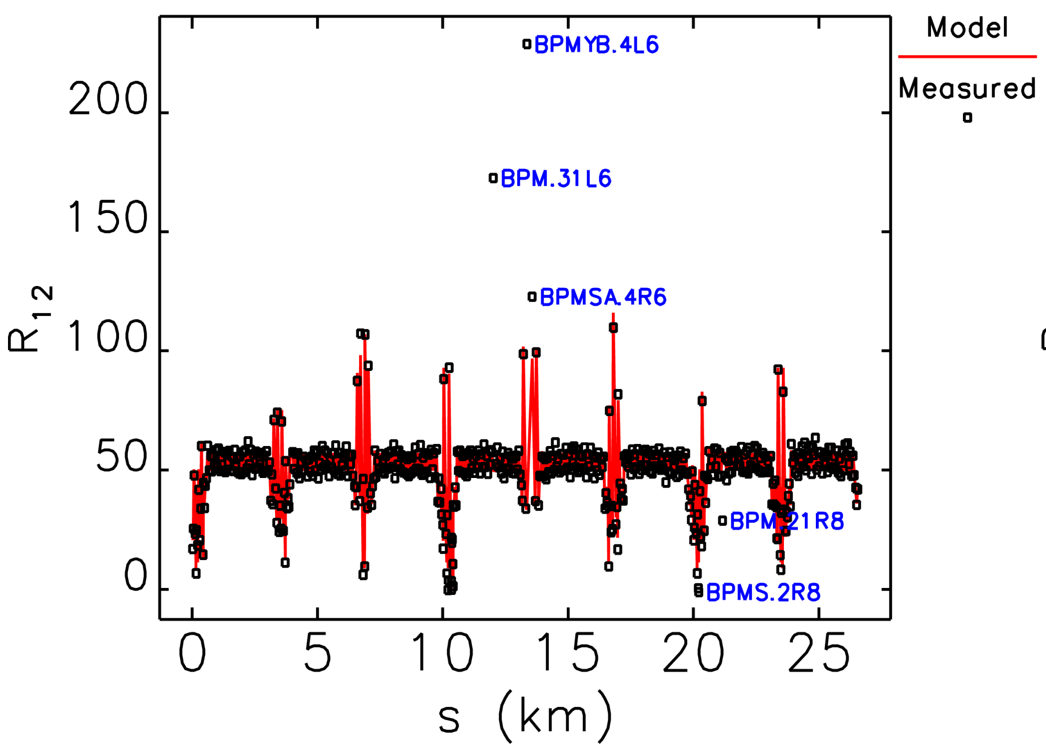
$$(x_1^a y_2^b - x_2^a y_1^b)/Q_{12} + (x_3^a y_4^b - x_4^a y_3^b)/Q_{34} = \mathcal{R}_{32}^{ab}$$

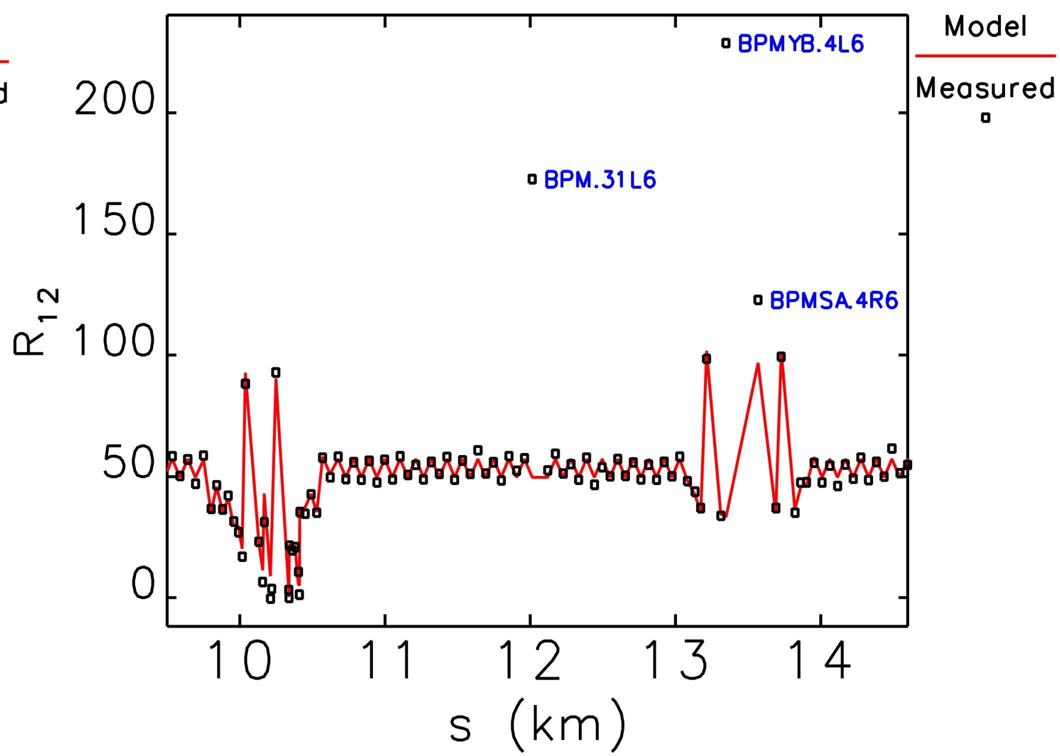
$$(y_1^a x_2^b - y_2^a x_1^b)/Q_{12} + (y_3^a x_4^b - y_4^a x_3^b)/Q_{34} = \mathcal{R}_{14}^{ab}$$

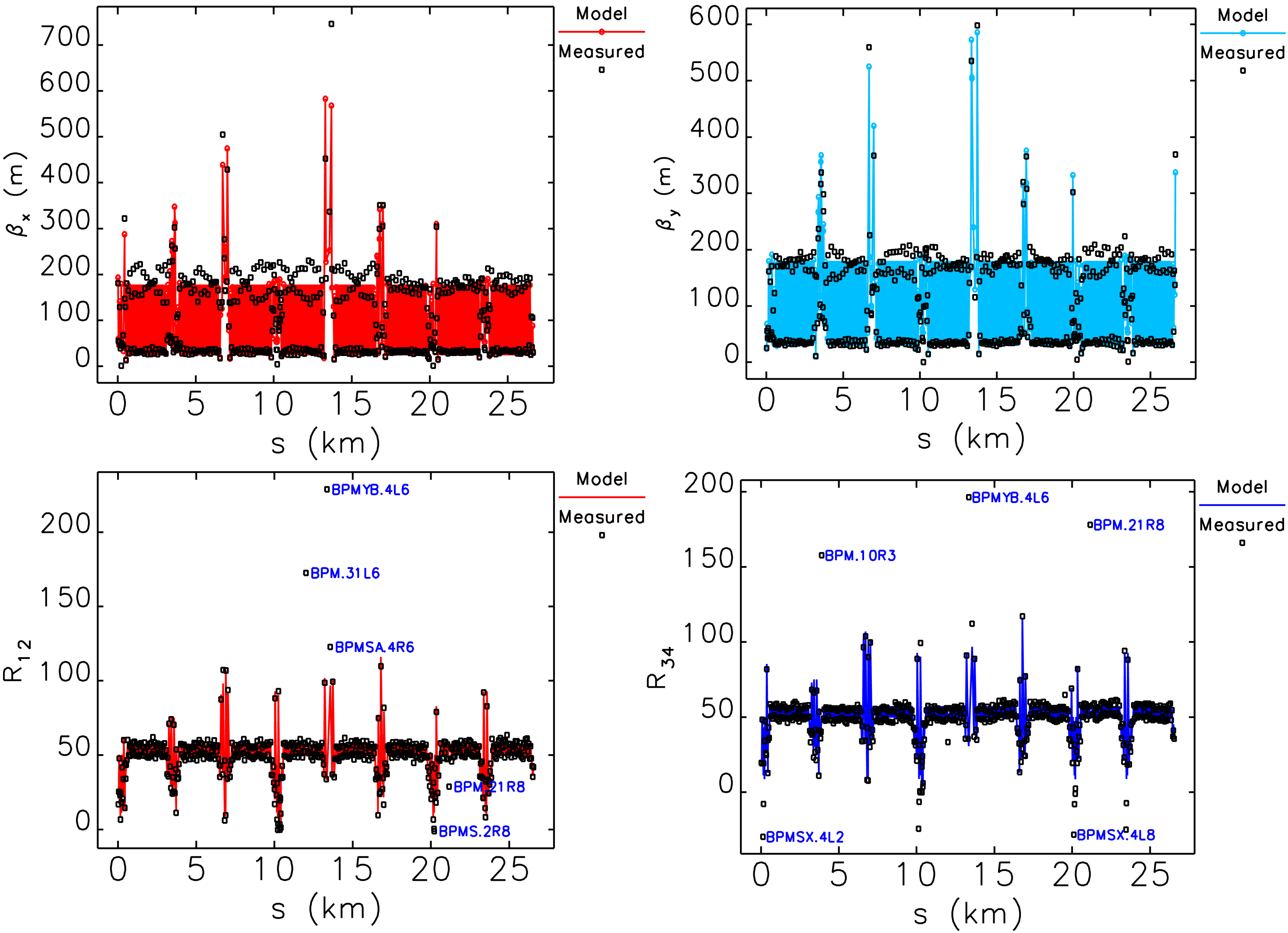
$$(y_1^a y_2^b - y_2^a y_1^b)/Q_{12} + (y_3^a y_4^b - y_4^a y_3^b)/Q_{34} = \mathcal{R}_{34}^{ab}$$

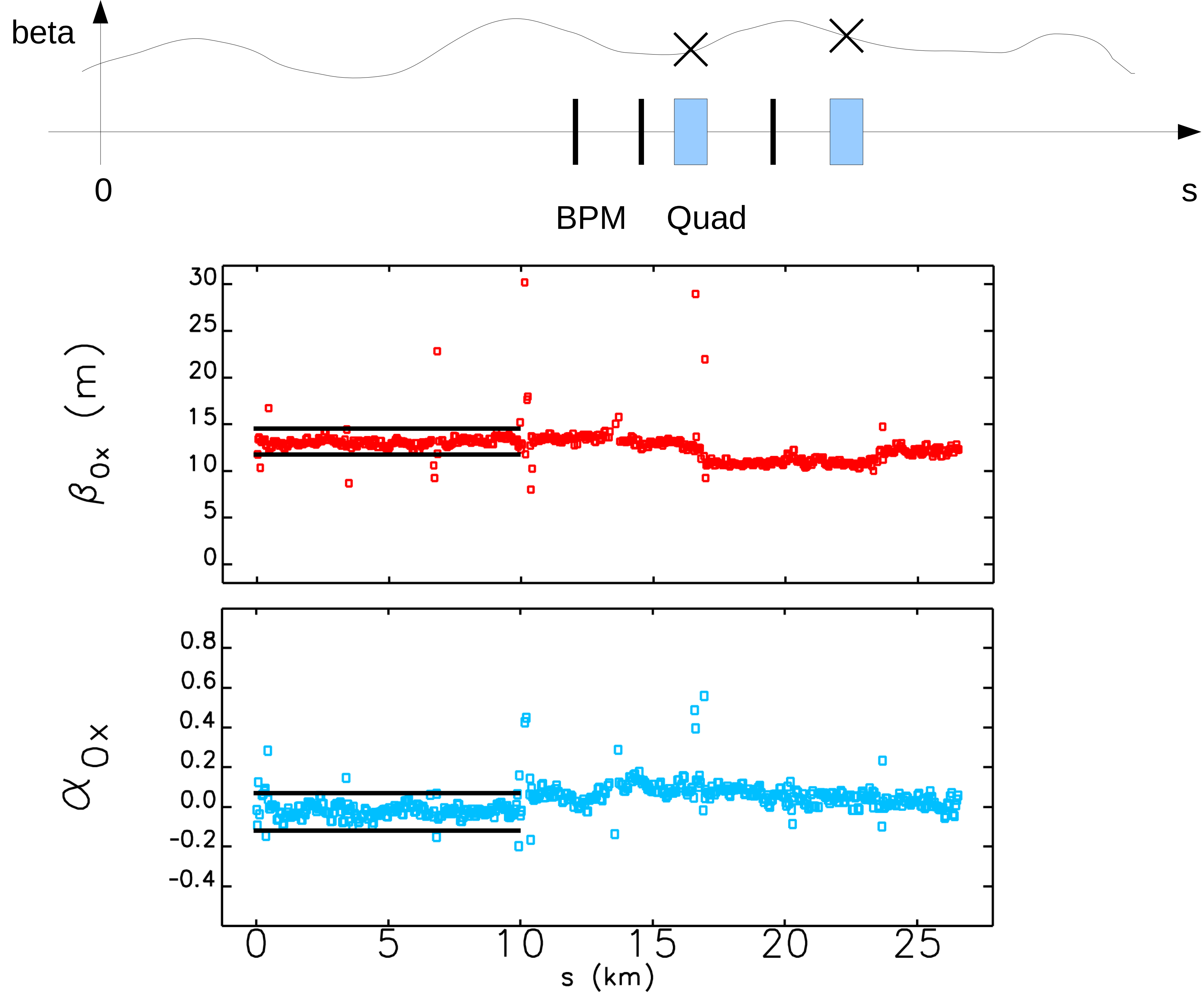
J. Irwin and Y. T. Yan, "*Beamline Model Verification Using Model Independent Analysis*", Proceedings of EPAC 2000, Vienna, Austria.

R_{12} calculated from turn-by-turn measurements at the LHC:

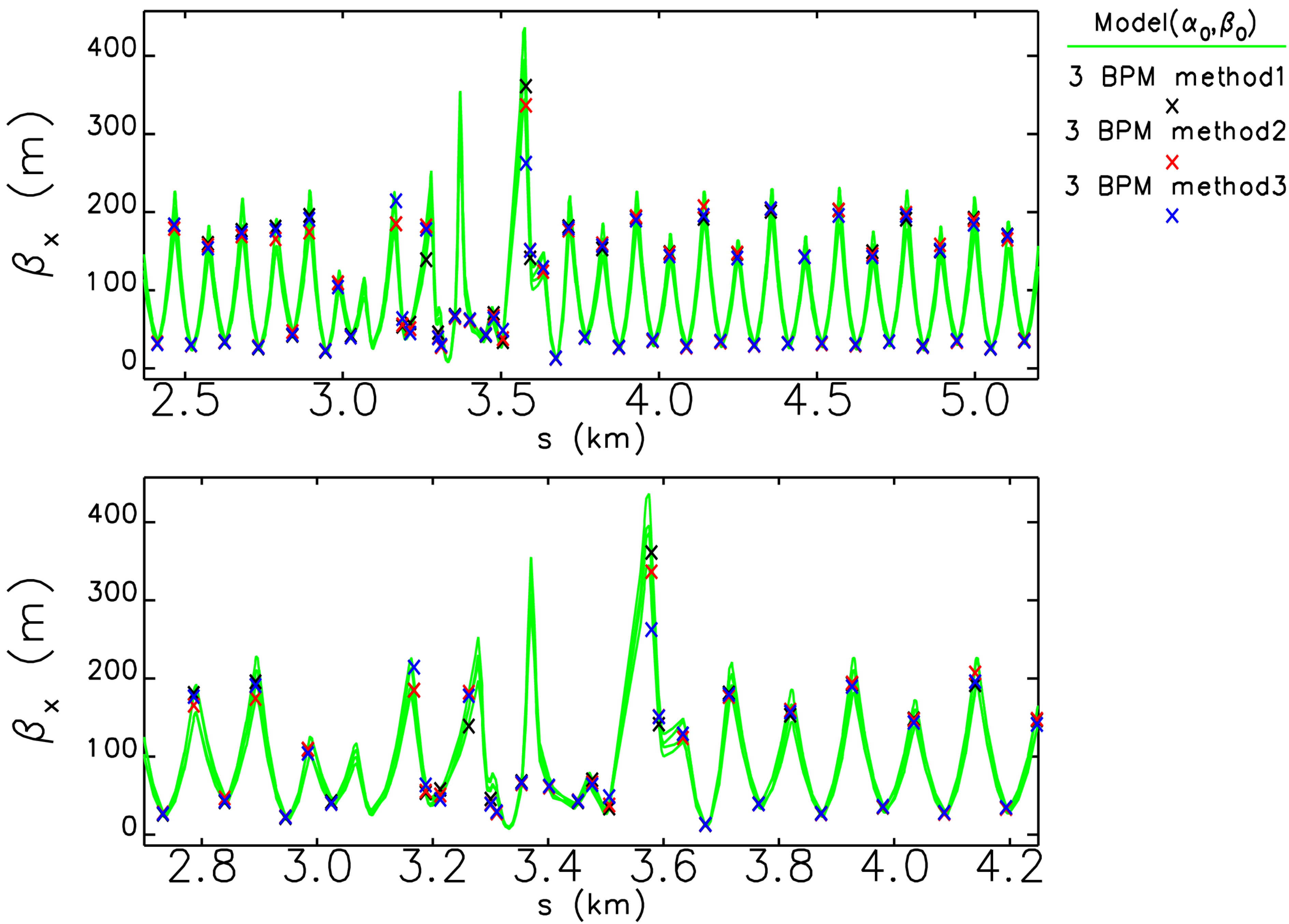





Model
Measured
β_x (m)
s (km)
β_y (m)
R_12
R_34
BPMYB.4L6
BPM.31L6
BPMSA.4R6
BPM.21R8
BPMS.2R8
BPM.10R3
BPMSX.4L2
BPMSX.4L8



beta
0
s
BPM
Quad
β_{0x} (m)
30
25
20
15
10
5
0
α_{0x}
0.8
0.6
0.4
0.2
0.0
-0.2
-0.4
0
5
10
15
20
25
s (km)



Model(α_0,β_0)
3 BPM method1
3 BPM method2
3 BPM method3
β_x (m)
s (km)
0
100
200
300
400
2.5
3.0
3.5
4.0
4.5
5.0
2.8
3.2
3.4
3.6
3.8
4.2

Conclusion

It is possible to use MIA for coupled beta-function measurement even if tunes are so close that their synchrobetatron sidebands overlap

MIA may also be used to identify the origin of some unexpected components of BPM readings, like in the case of vibrating final focus quadrupoles in the Tevatron.

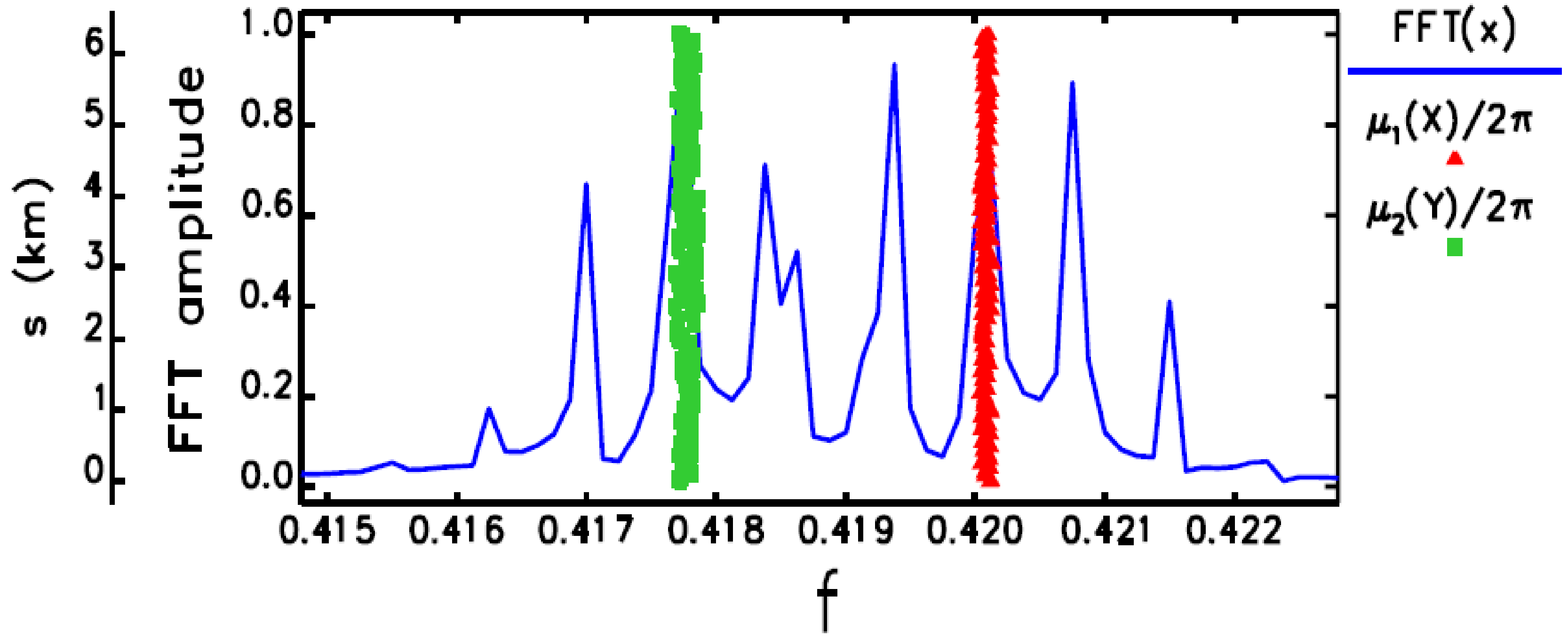
MIA applied to the LHC data identified multiple noisy BPMs. Also several large deviations from the design model were located with the local transport matrix method.

Simple method of focusing error identification immune to BPM gain errors is proposed.

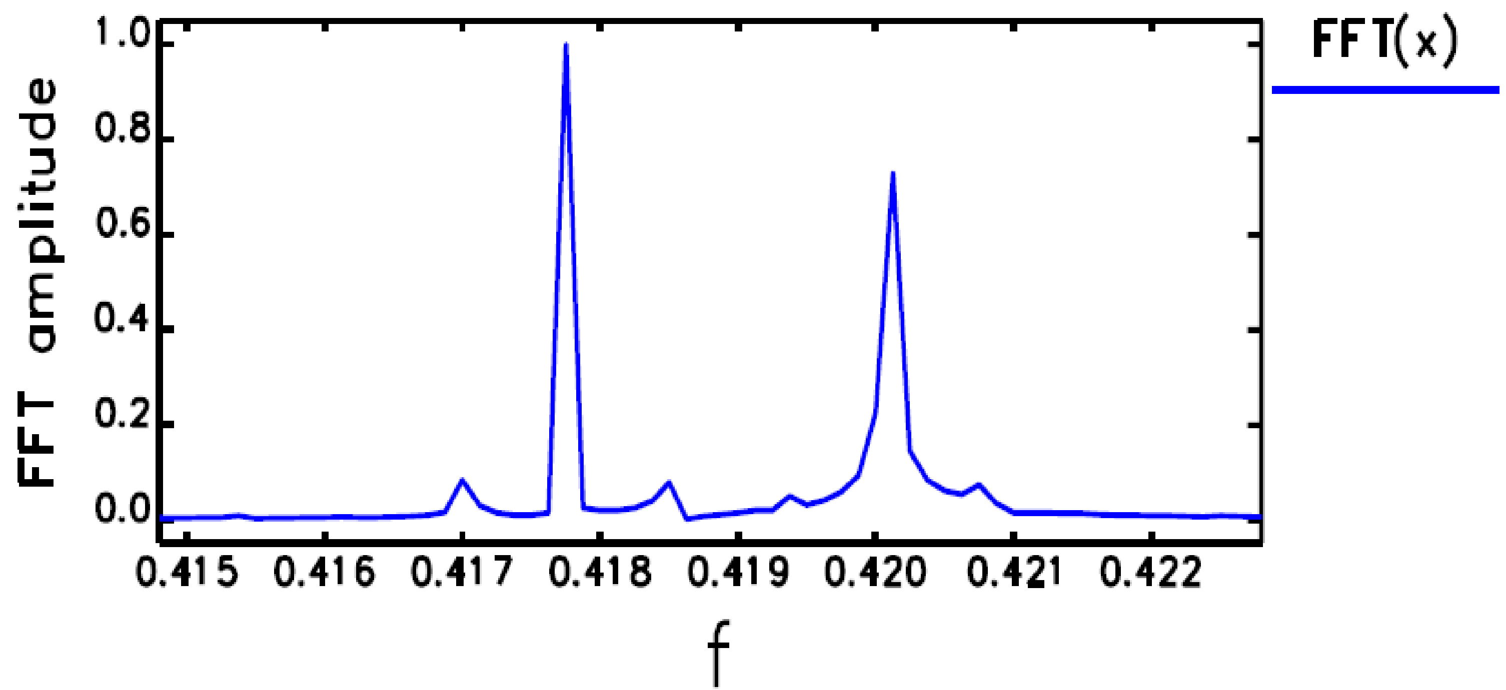
Many thanks to:
V. Lebedev, A. Valishev, Yu. Maltseva, Yu. Alexahin, M. Borland, R. Miyamoto

Test of mode unmixing in the case of overlaping synchrobetatron sidebands

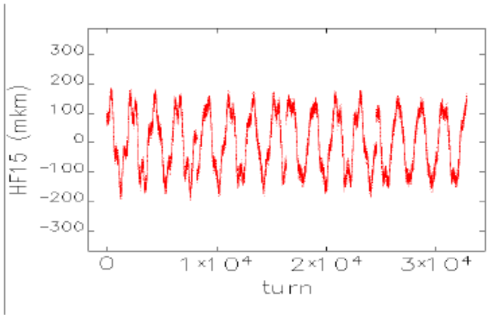
Typical Tevatron chromaticity (tracking simulation)

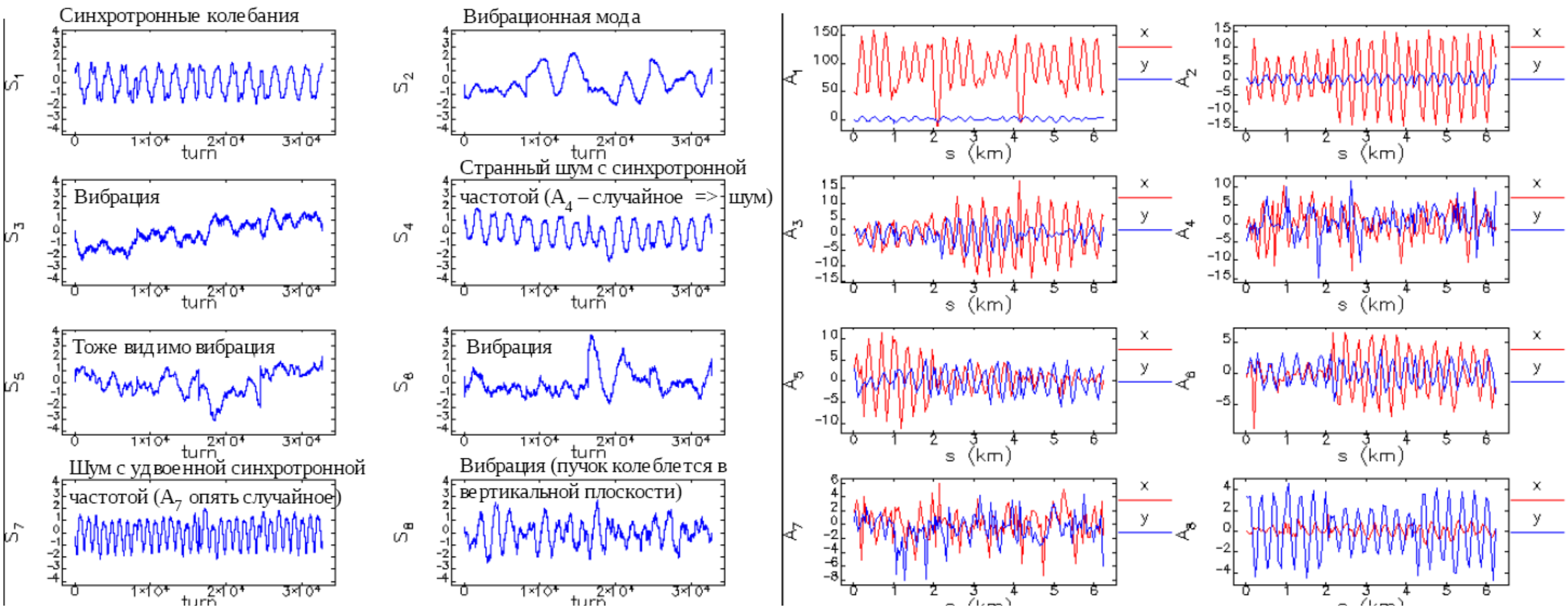


Small chromaticity (only sextupoles are changed)



BPM measurements without kicks:
4 measurements are combined:





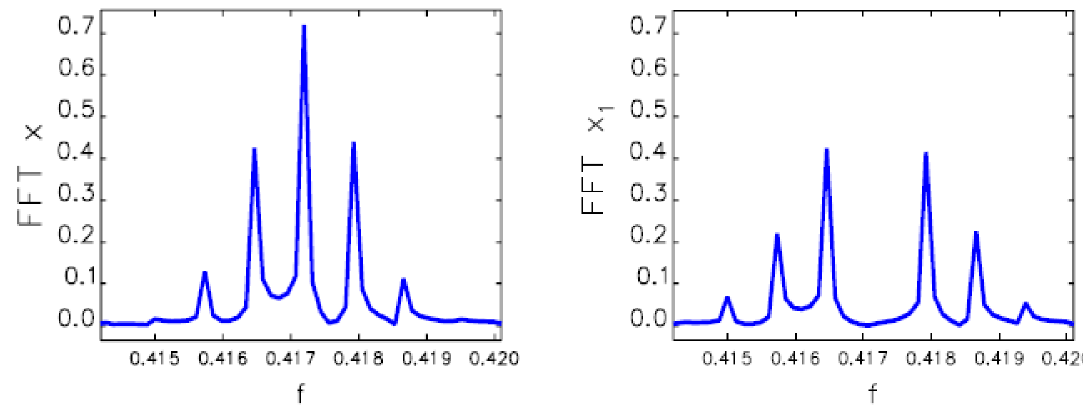


FIG. 10: Typical FFT amplitudes of both high-frequency terms in Eq. 10. $x(t) = \cos(\mu_x t + \zeta_x \sin\mu_s t)$, $x_1(t) = x(t)\cos\mu_s t$.

Appendix: Chromatic effects

Consider single particle undergoing betatron and synchrotron oscillations in a ring without betatron coupling. According to the perturbation theory [*Appendix?*] if the unperturbed solution (without synchrotron oscillations) written in the Floquet form is

$$x(s,t) = \mathrm{Re}\left[a_x f_x(s) e^{i\mu_x t}\right],$$

then the solution that takes into account synchrotron oscillations will be

$$\begin{aligned} x(s,p) &= D_x(s)\delta(t) + \\ &+ \mathrm{Re}\left\{ a_x \left[f_x(s) + g_x(s)\delta(t)\right] e^{i\int\limits_0^t \mu_x(\delta)dt} \right\}, \end{aligned}$$

where $D_x(s)$ is the dispersion function, δ is the fractional momentum deviation and $\mu_x(\delta) = \mu_x + \xi_x\delta(t)$ is the chromatic dependence of horizontal betatron tune. For linear synchrotron oscillations

$$\delta(t) = \delta_0 \cos(\mu_s t + \psi_s),$$

where μ_s is the tune of synchrotron oscillations, δ_0 and ψ_s — its amplitude and initial phase. Finally, assuming for simplicity $\psi_s = 0$,

$$\begin{aligned} x(s,t) &= D_x(s)\delta_0 \cos\mu_s t + \\ &+ \mathrm{Re}\left[a_x f_x(s) e^{i\mu_x t + i\zeta_x \sin\mu_s t}\right] + \\ &+ \mathrm{Re}\left[a_x g_x(s) e^{i\mu_x t + i\zeta_x \sin\mu_s t}\right]\delta_0\cos\mu_s t, \quad (10) \end{aligned}$$

where $\zeta_x = \delta_0\xi_x/\mu_s$.

Typical Fourier spectra of turn-by-turn signal components correstonding the second and third term in Eq. 10 are shown in Fig. 10.

MIA provides a way to estimate the chromaticity of lattice functions given by the third term in Eq. 10. To observe the effect of this term we need to apply a narrow Fourier filter to data and leave only horizontal BPM readings in the beam history matrix B. Fig. 11 shows the results of such data analysis. Amplitudes of spatial MIA modes shown in Fig. 11 mean that the chromatic term in this case is around 1 % of the second term in Eq. 10.

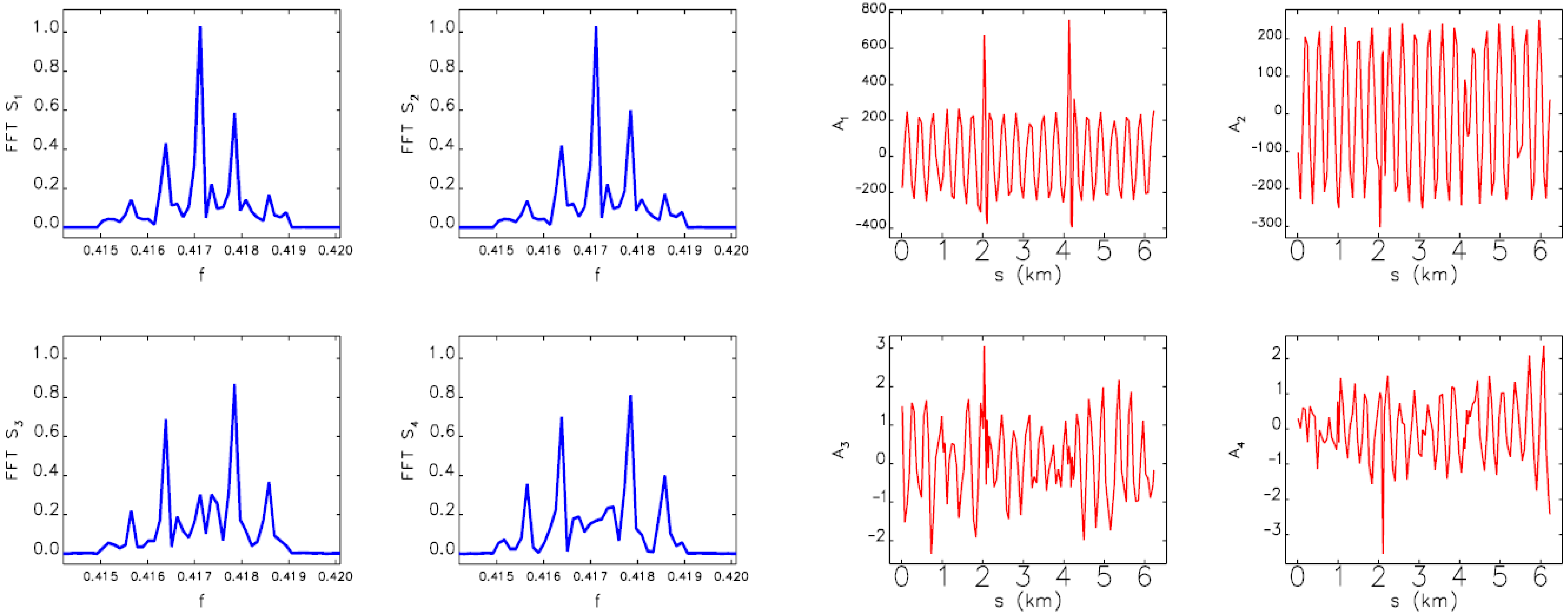


FIG. 11: *Chromatic modes...* Narrow Fourier filter (from 0.414 to 0.420) was applied to data before MIA. Readings only from horizontal monitors were included into the beam history matrix B.

AC-dipole measurements::