Rediscovering GEANE as track follower for the VMC: results from PANDA

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- Brief review of the package GEANE.
- Update of TGeant3 and the new C++ interface to GEANE in FairROOT.
- Update of the physics models in GEANE for use with low density materials: Coulomb multiple scattering and straggling in energy loss.
- Results from the PANDA Straw Tube Tracker: pull distributions.
- Application for a GEANE-based Kalman Filter.

What is GEANE?

- Package to calculate the **average trajectories** of particles through **dense materials** and to calculate the **transport matrix** as well as the propagated errors **covariance matrix** in a given track representation.

- It is a **track follower**: it predicts the trajectory of a charged particle in terms of mean values and errors both in forward and in backward direction. **Three effects** are taken into account:

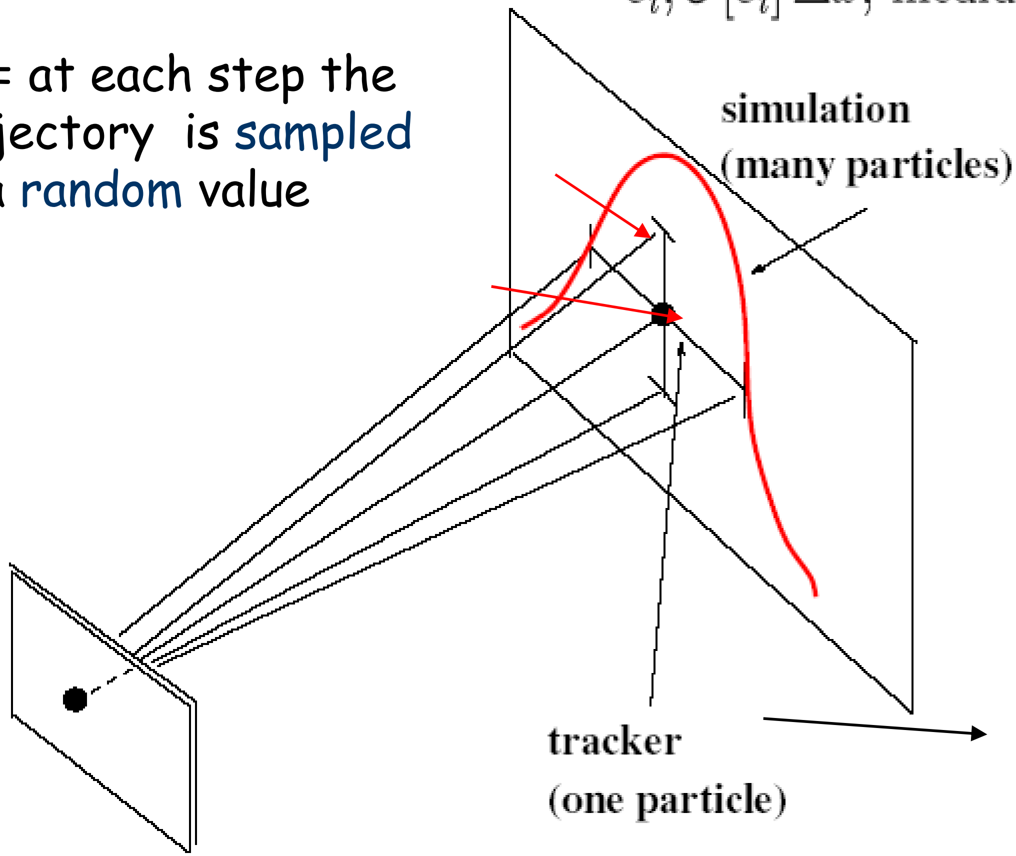
  - energy loss (affects mean values and errors)
  - Coulomb multiple scattering (affects errors only)
  - magnetic field (affects mean values only)

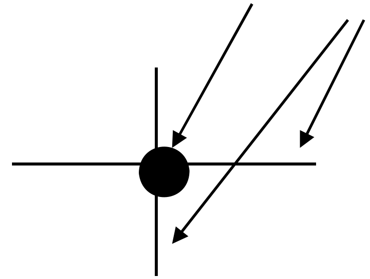
This has to be checked against a real MC where one generates an ensemble of particles and compares the mean and sigma of the distributions with the predicted ones (**pull distributions**).

Tracking vs MC

$$e_i, \boldsymbol{\sigma}[e_i]\,\Delta x,\ \text{medium} \rightarrow \text{TRACKER}i,j \rightarrow e_j,\ \boldsymbol{\sigma}[e_j]$$

MC= at each step the trajectory is sampled as a random value





Tracking= at each step the trajectory is calculated as a mean value with an associate error

Energy loss affects both tracking (averages) and error propagation (covariance matrix), multiple scattering affects the error propagation only.

Track following tools

Two approaches:

- the GEANT3-GEANE chain: the tracking banks and routines are the same as in MC.

  The user gives the starting and ending planes or volumes to a routine and the tracking is done automatically.

  Only the MC geometry file is needed!

  It works very well and it is used in many experiments (see the CERN Report W5013 GEANE, 1991 by Innocente,Maire and Nagy).

It is independent from the experiment!

- private solution: in the software are implemented some tracking classes:

  input: x_i, T_i , s_i, step, medium, magnetic field

  output: new x_i, T_i, s_i

  The user has to manage geometry,medium and detector interfaces to each geometry change!

It is dependent on the experiment!

With the VMC concept and the TGeant3 class, **GEANE is already available in ROOT** and it is awaiting to be used!

Track parameters in GEANE

Three different **track representations** (beam directed along x axis):

- **Central System (SC, representation I)**
  1/p, λ, ϕ, y, z
  Usually applied in the overall reference frame

- **Detector System (SD, representation II)**
  1/p, v', w', v, w
  where (u,v,w) orthonormal reference system with vw plane coincident with the detector plane.
  Usually applied when the trajectory has to be evaluated at different detector planes (colliding beam experiments where planes can take great variety of directions).

- **Spline System (SP, representation III)**
  1/p, y'=py/px, z'=pz/px, y, z
  Usually applied in fixed target experiments where the trajectory is evaluated on successive parallel planes (perpendicular to x axis)

GEANE provides routines to **switch** from one representation to the other.

Track parameters in GEANE

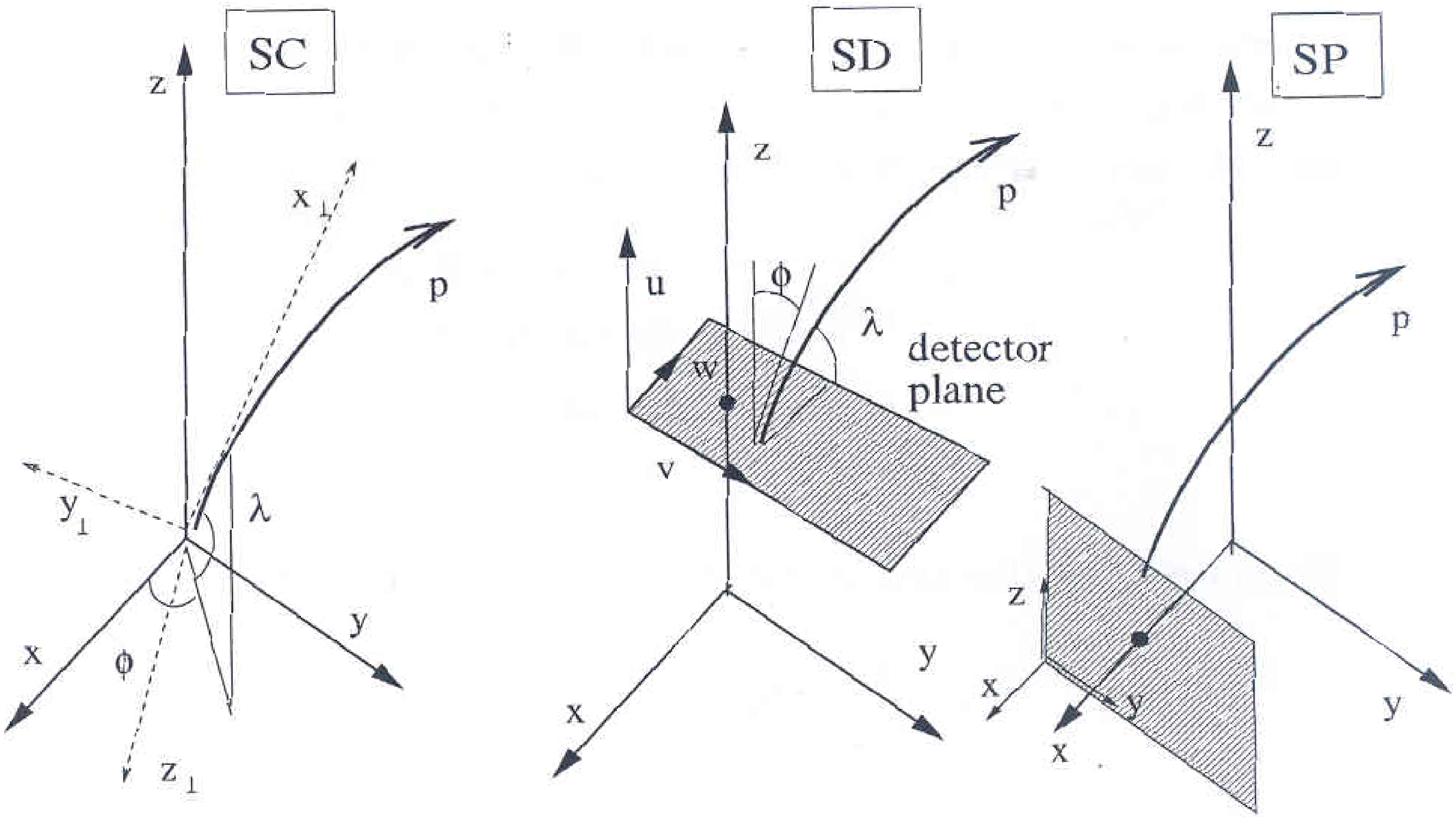


Figure 1.2: Systems of reference for the track following

Old GEANE Input and Output

Three different **requests** can be made for a given representation.
Only two functions to be called (`Eufil(p/l/v)+Ertrak`):

- Extrapolation of the track parameters to a given plane (repr. III)

  Function `Eufilp(N,Ein,Pli,Plf)`

  ```
  N: number of predictions (<=10)
  Ein: initial error matrix
  Pli: initial plane
  Plf: final plane
  ```

- Extrapolation of the track parameters at a given track length (repr. I)

  Function `Eufill(N,Ein,Xlf)`

  ```
  Xlf: track length
  ```

- Extrapolation of the track parameters entering/exiting a given volume (repr. I)

  Function `Eufilv(N,Ein,Namv,Numv,Iovl)`

  ```
  Namv: volume name
  Numv: volume copy number
  Iovl: entering/exiting flag
  ```

Old GEANE Input and Output

- Track parameters evaluation:

Function `Ertrak(x1,p1,x2,p2,ip,chopt)`

```
x1: initial position
p1: initial momentum
x2: final position
p2: final momentum
ip: particle code (à la Geant)
chopt: extrapolation options
       'L': tracking until length reached
       'P': tracking until plane reached
       'V': tracking until volume reached
       'O': no error calculation
       'E': exact error calculation
       'B': backward tracking (energy
            loss added to current energy)
```

The output parameters are stored in the Fortran COMMONs.

Update of TGeant3

- The class only contains the structures for Geane input/output and the function **ertrak()**.
- Structures for Geane input/output (Geane COMMONs) are set as public so that the user can access them:

```
Ertrio_t *fErtrio
Eropts_t *fEropts
Eroptc_t *fEroptc
Erwork_t *fErwork
Trcom3_t *fTrcom3
```

- Geane interface functions are added:

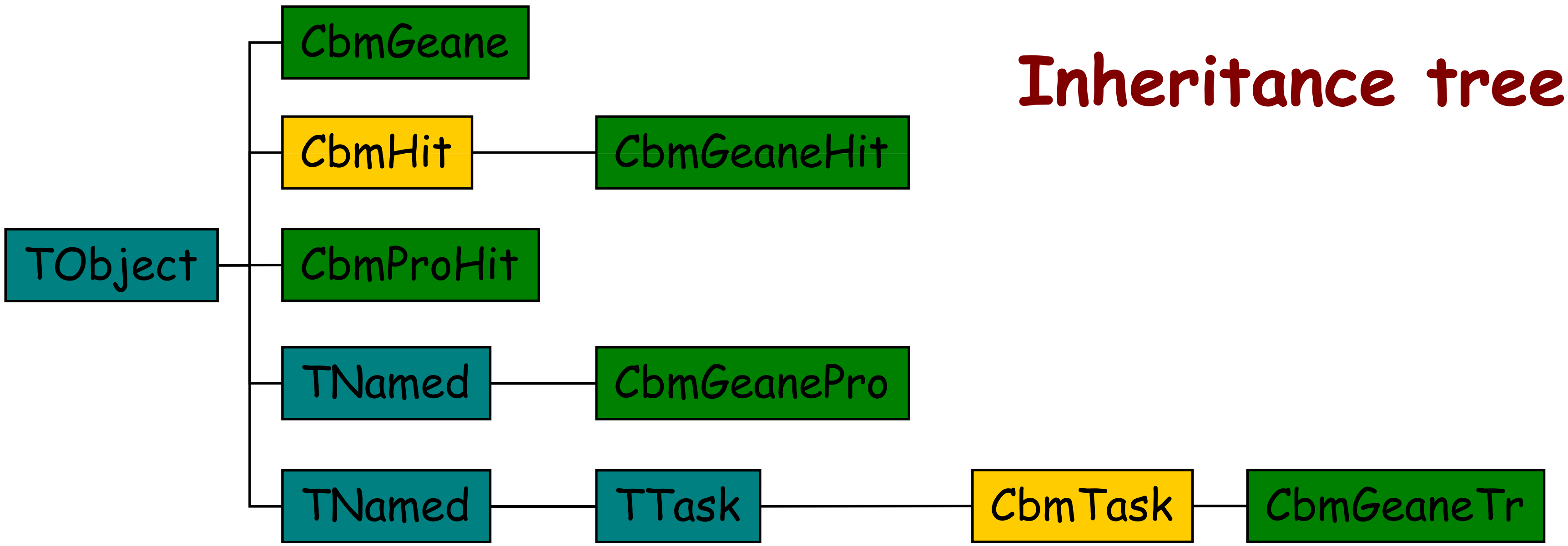
```
void eufill(n, ein, xlf);
void eufilp(n,ein,pli,plf);
void eufilv(n,ein,namv,numv,iovl);
void trscsd(pc,rc,pd,rd,h,ch,ierr,spu,dj,dk);
void trsdsc(pd,rd,pc,rc,h,ch,ierr,spu,dj,dk);
void trscsp(ps,rs,pc,rc,h,ch,ierr,spx);
void trspsc(ps,rs,pc,rc,h,ch,ierr,spx);
```

In this way the GEANE functionality has been reintroduced.

The new classes in FairROOT

- **CbmGeane**: configuration + import geometry from MC (**TGeo**) + import magnetic field map
- **CbmGeaneHit**: store track params + errors before and after extrapolation (persistent, saved to output tree)
- **CbmProHit**: stores extrapolation results (transient, memory only)
- **CbmGeanePro**: performs the extrapolation (calls to EufilV/P/L and to Ertrak)
- **CbmGeaneTr**: hit producer (performs the track following)

Inheritance tree



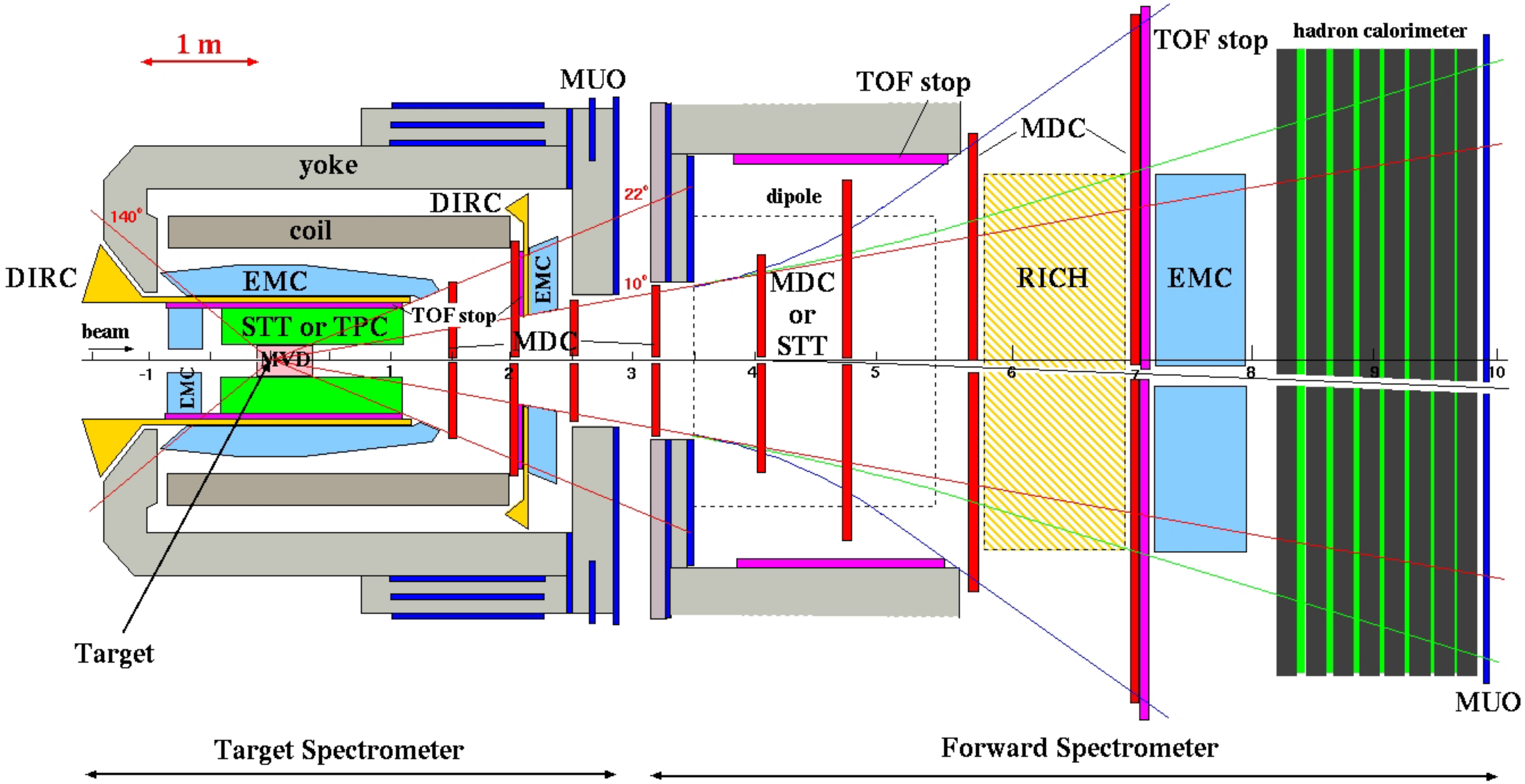
Integration with FairROOT

- Geane is now fully integrated in PandaROOT as a package: definition of classes `CbmGeane, CbmGeaneHit, CbmGeanePro,CbmProHit, CbmGeaneTr`
- Simplified and intuitive user interface: only four methods in `CbmGeanePro` class

```
Propagate(…);
PropagateToPlane(v0, v1, v2);
PropagateToVolume(VolName, CopyNo , option);
PropagateToLength(length);
```

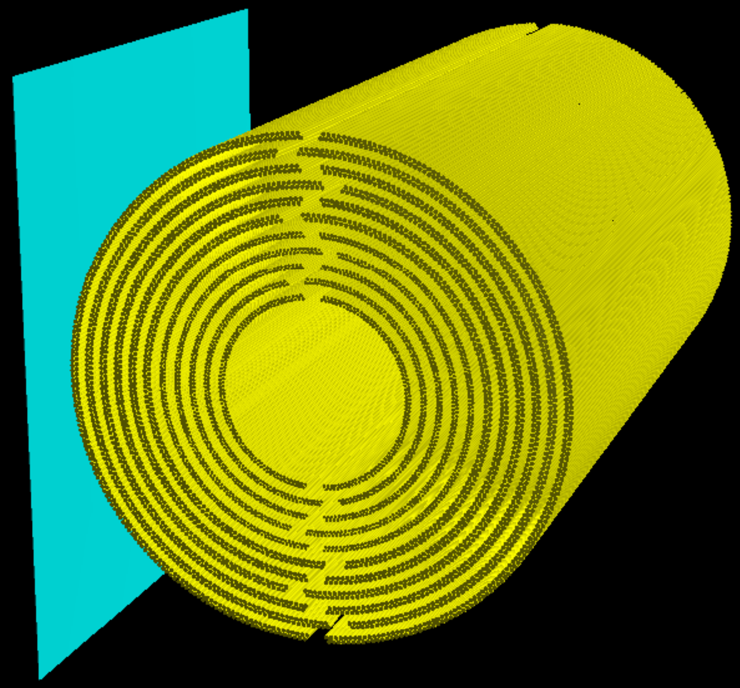
- Track following can be done from macro or compiled code.
- The exact geometry and field used by the simulation are taken into account **automatically** by the track follower.
- GEANE works as a track follower in the TGeometry modeler and with both Geant3 and Geant4 as MC engines!
- Results are stored as a **tree** in the ROOT file.
- Pull distributions calculation is straighforward using `TTree::Draw()`.

The PANDA detector



Pulls for the STT

We have defined a plane to which we extrapolate the track parameters.



Important issue: how GEANE treat multiple scattering and straggling in energy loss for gaseous materials?

Multiple scattering

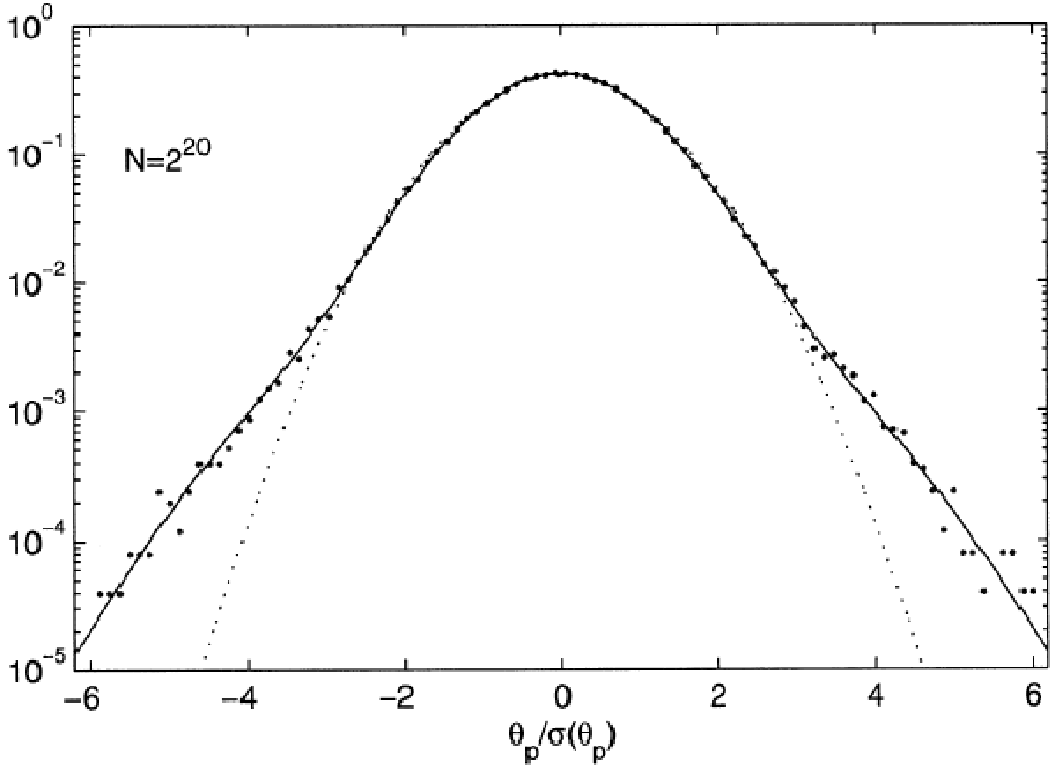
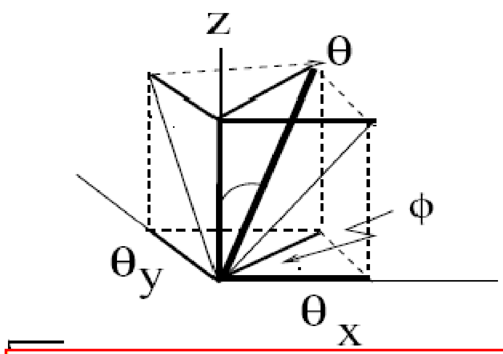


Fig. 3. The density of the projected multiple scattering angle in carbon, in standard measure, for $N = 2^{10}$ (top) and $N = 2^{20}$ (bottom). The dots are the frequencies of a simulated sample obtained by summing over single scatters. The dotted line is the density of a standard Gaussian.



Multiple scattering

Changes to the ermcsc.f routine

$$p(x,\theta_p;d) = \frac{2\sqrt{3}}{\pi}\frac{1}{<\theta_p^2> d^2}\exp\left[-\frac{4}{<\theta_p^2>}\left(\frac{\theta_p^2}{d} - \frac{3x\theta_p}{d^2} + \frac{3x^2}{d^3}\right)\right]$$

$$<\theta_p^2>= \frac{(0.0136)^2\ d}{p^2\,\beta^2 X_0}\left[1 + 0.038\ \ln\left(\frac{d}{X_0}\right)\right]^2$$

PDG: wrong

$$<\theta_p^2>= \frac{184.96\cdot 10^{-6}}{p^2}\frac{d}{\beta^2 X_0}\ .$$

GEANE: obsolete

$$<\theta_p^2>= \frac{225\cdot 10^{-6}}{p^2}\frac{d}{\beta^2 X_s}\ , \qquad X_s = X_0\,\frac{Z+1}{Z}\,\frac{\ln(287\,Z^{-1/2})}{\ln(159\,Z^{-1/3})}$$

R. Frühwirth and M. Regler, Nucl. Instr. and Meth. A456(2001)369

Modification of GEANE for PANDA

σ(λ)
pull

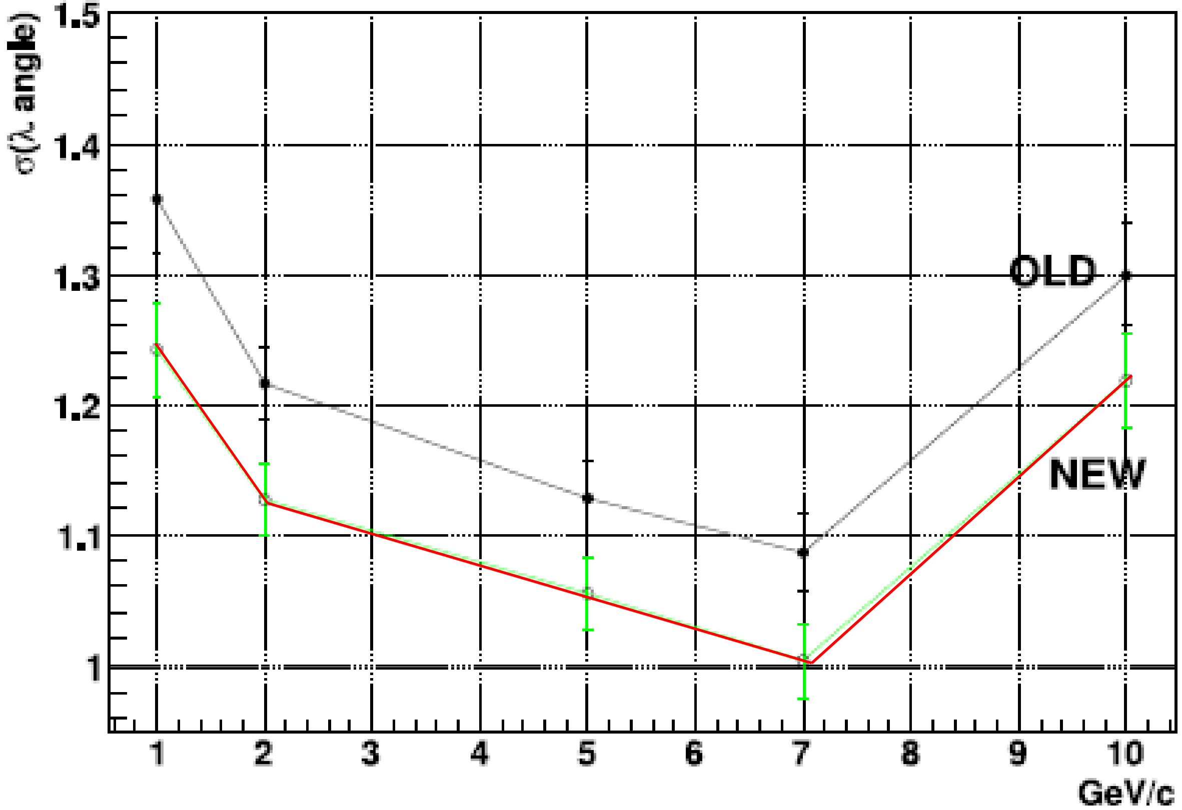


Figure 9: Behaviour with energy of the σ of the pull distribution of the dip angle λ. OLD with eq. (47), NEW with eq. (45).

Straggling of energy loss

The fluctuations in ionization for one particle of charge z, mass m, velocity β, are characterized by the parameter κ,

$$\kappa = \frac{\xi}{E_{\text{max}}} \;, \tag{60}$$

which is proportional to the ratio of mean energy loss to the maximum allowed energy transfer E_{max} in a single collision with an atomic electron:

$$E_{\text{max}} = \frac{2m_e\beta^2\gamma^2}{1 + 2\gamma m_e/m + (m_e/m)^2} \;, \tag{61}$$

where $\gamma = 1/\sqrt{1-\beta^2} = E/m$ and m_e is the electron mass. The parameter ξ comes from the Rutherford scattering cross section and is defined as [11]:

$$\xi = 153.4 \frac{z^2 Z}{\beta^2 A} \rho d \quad (\text{keV}) \;, \tag{62}$$

where ρ, d, Z and A are the density (g/cm^3), thickness, atomic and mass number of the medium.

Straggling of energy loss

1. for heavy absorbers, $\kappa > 10$ and the distribution is gaussian;

$$\sigma_{11}^2(l) = \frac{E^2}{p^6}\,\sigma^2(E) = \frac{E^2}{p^6}\,\xi\,E_{\max}(1-\beta^2/2)$$

2. for moderate absorbers, $0.01 < \kappa < 10$ and the distribution follows the function of Vavilov [15], that tends smoothly to the gaussian by increasing the thickness;

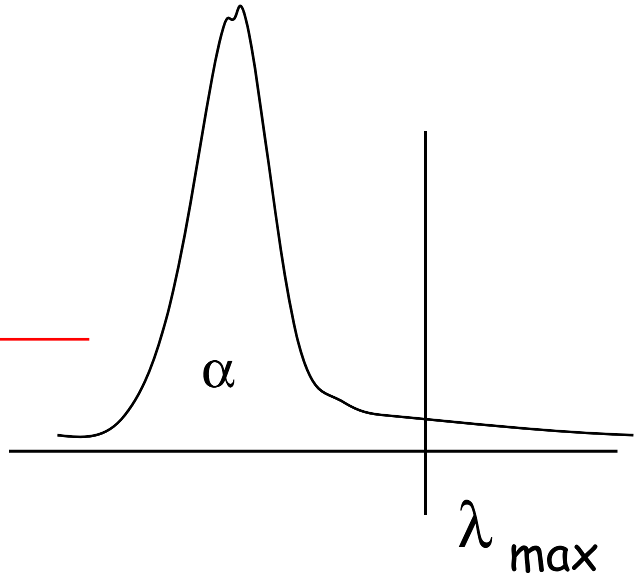
3. when $\kappa < 0.01$, we are in the presence of thin absorbers. When the number of collisions $N_c > 50$, the distribution follows the Landau function [15];

4. for very thin absorbers, $N_c < 50$ (the condition $\kappa \ll 0.01$ is implicitly fulfilled) and there are no universal straggling functions, but only approximated models [14]. We will call this as the *sub-Landau* condition or regime; it is the dominant one in gaseous detectors at PANDA energies.

Energy loss fluctuactions: there is **no rigorous solution** for track following

Landau has infinite variance!!

λ_{max}	α	Mean	σ_α
11.1	0.90	1.61	2.83
22.4	0.95	2.40	4.23
110.0	0.99	4.19	10.16
200.0	0.995	4.82	13.88
339.0	0.997	5.37	18.19
507.0	0.998	5.78	22.33
1007.0	0.999	6.48	31.59



sampling gives (unlikely) strong fluctuations: for example, for 1 GeV pions in 1 cm Ar we have $E_{\rm med} \simeq 2.5$ keV, $E_{\rm max} \simeq 66$ MeV and a standard deviation of about 100 keV due to the δ-electrons tail.

GEANE modification for PANDA:

Changes to the ertrch.f and erland.f routines

a. for big and moderate absorbers when $\kappa > 0.005$, the variance $\sigma^2(E)$ is given by eq. (63) (old GEANE method);

$$\sigma^2(E) = \frac{\xi^2}{\kappa}(1 - \beta^2/2) = \xi \, E_{\max}(1 - \beta^2/2) \; .$$

b. for thin absorbers, $\kappa < 0.005$, when the number of collisions from eq. (67) is $N_c > 50$, $\sigma^2(E)$ is given by eq. (66) with $\alpha = 0.995$ and $\sigma_\alpha = 13.88$;

$$\sigma(E) = \xi \, \sigma_\alpha$$

Truncated Landau

New parameter alpha

c. for very thin absorbers, when $\kappa < 0.005$ and $N_c < 50$, the variance $\sigma_\alpha^2(E)$ is given by eq. (71). The default value for the area considered is $\alpha = 0.995$.

Urban model

$$\sigma^2(E) = N_1 e_1^2 + N_2 e_2^2 + N_3 \langle E_3 \rangle^2 + N_3 \sigma_\alpha^2(E_3)(N_3 + 1)$$

GEANE old

GEANE new for PANDA

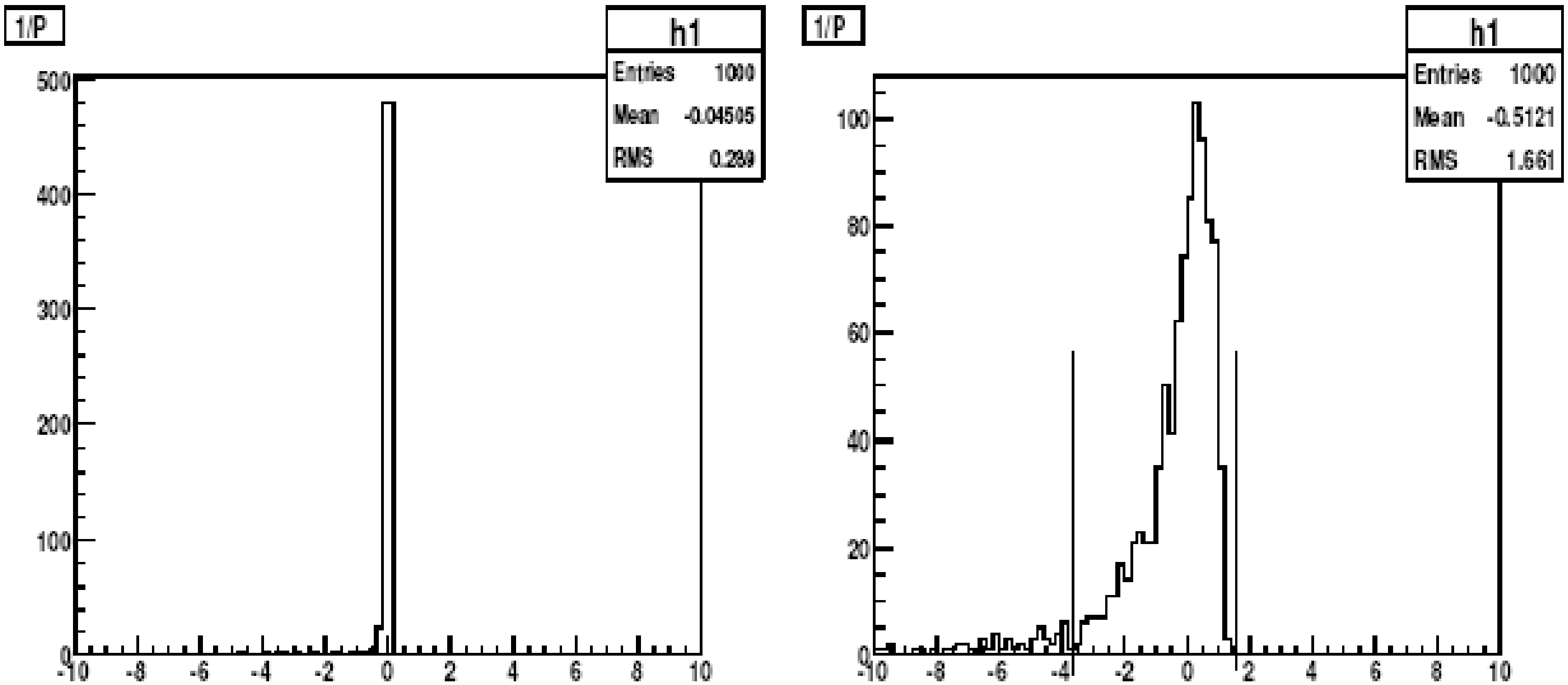


Figure 10: Pull distribution $\Delta(1/p)/\sigma$ for 1 GeV muons after passing through the PANDA straw tube detector. Left: Standard GEANE result (RMS$\simeq$ 0.3 in the displayed window); right: result after the modification with $\alpha = 0.995$ (see the text). The region between the vertical lines has RMS= 1.03.

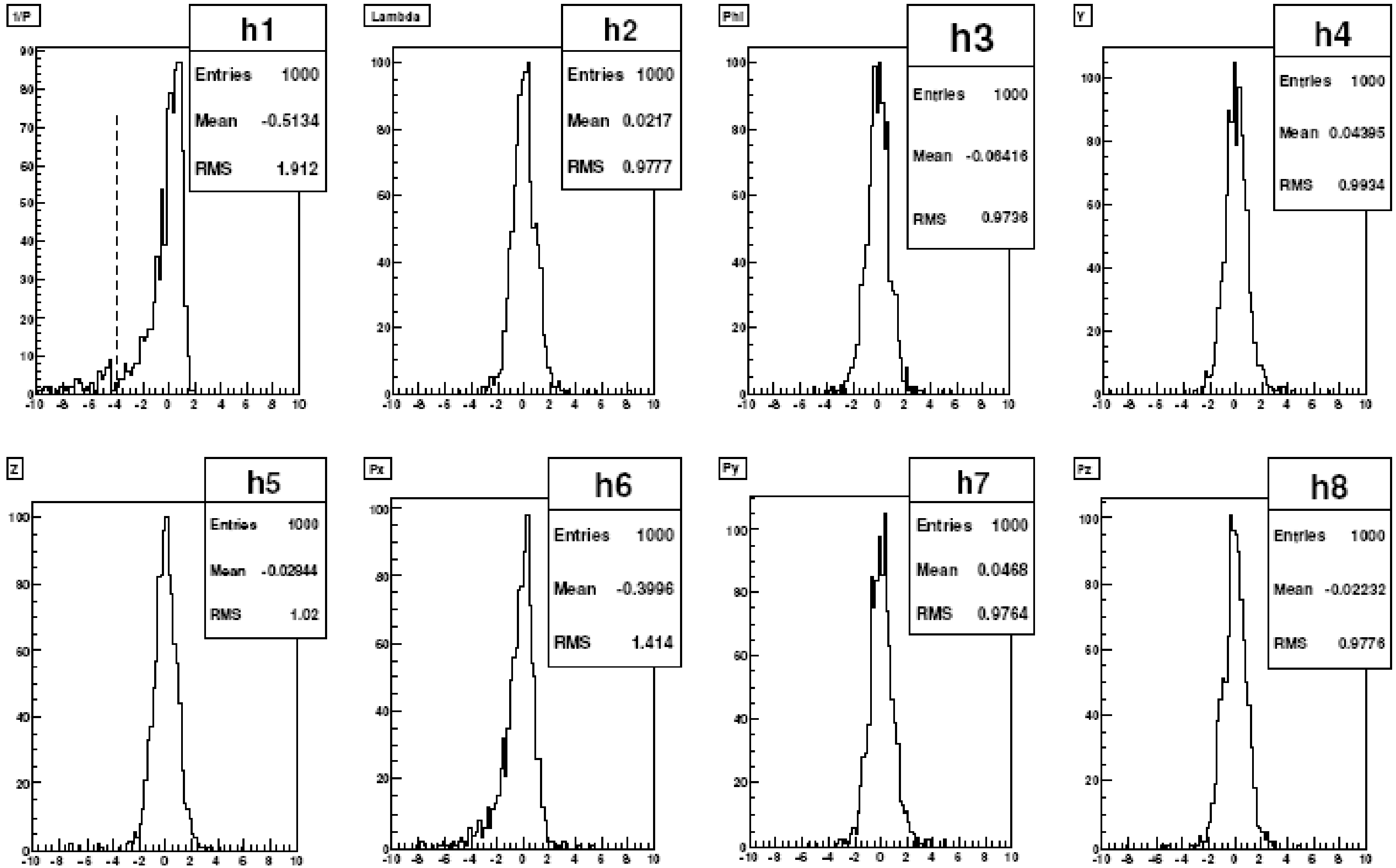
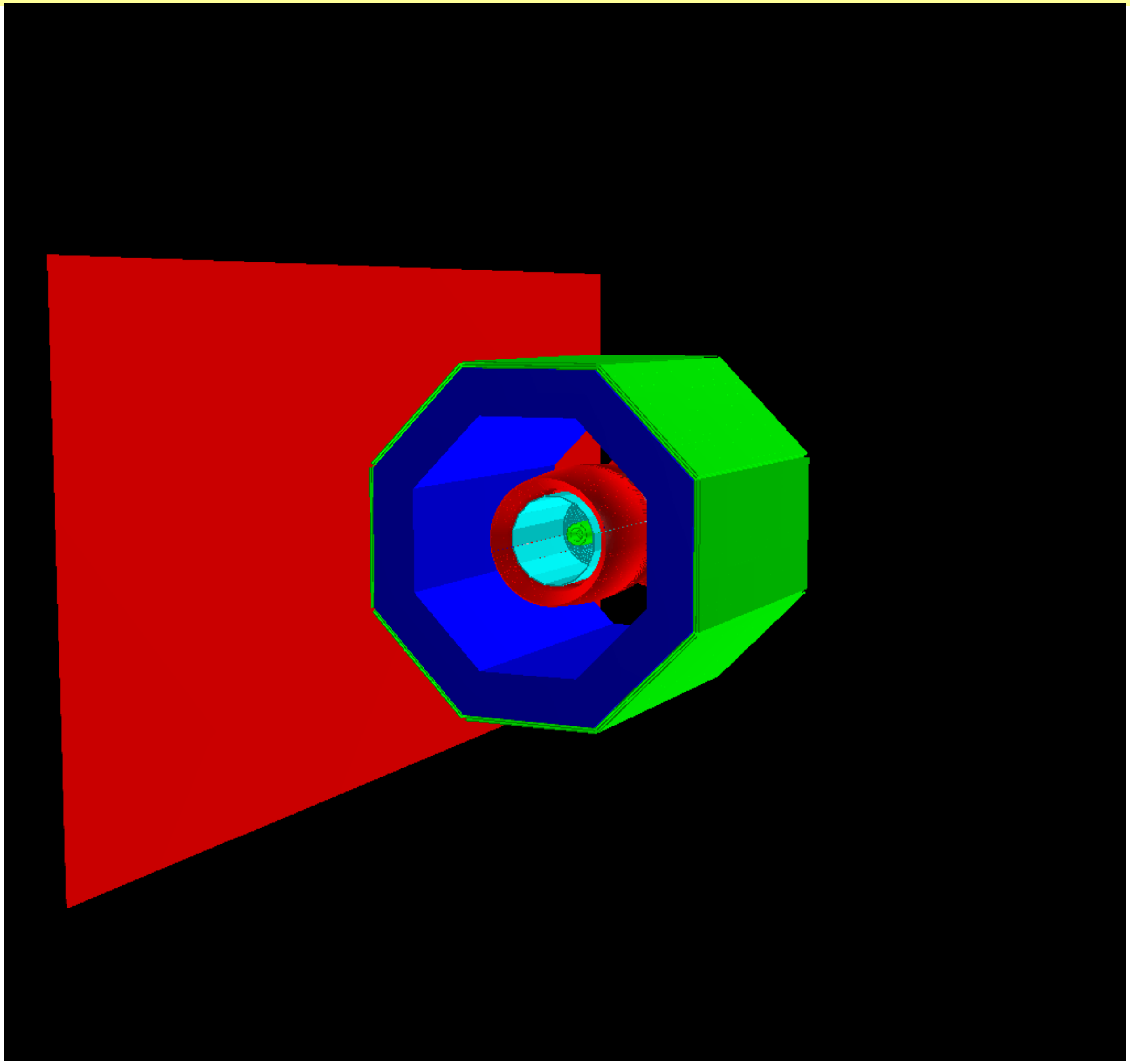


Figure 12: Pull distributions of the 5 track parameters and of p_x, p_y, p_z in the case of 2 GeV muons after passing through the 22 layers of the straw tube detector only. For the $1/p$ histogram dispersion up to the dotted line one has RMS= 1.08 (top left).

Pulls for the whole Panda detector



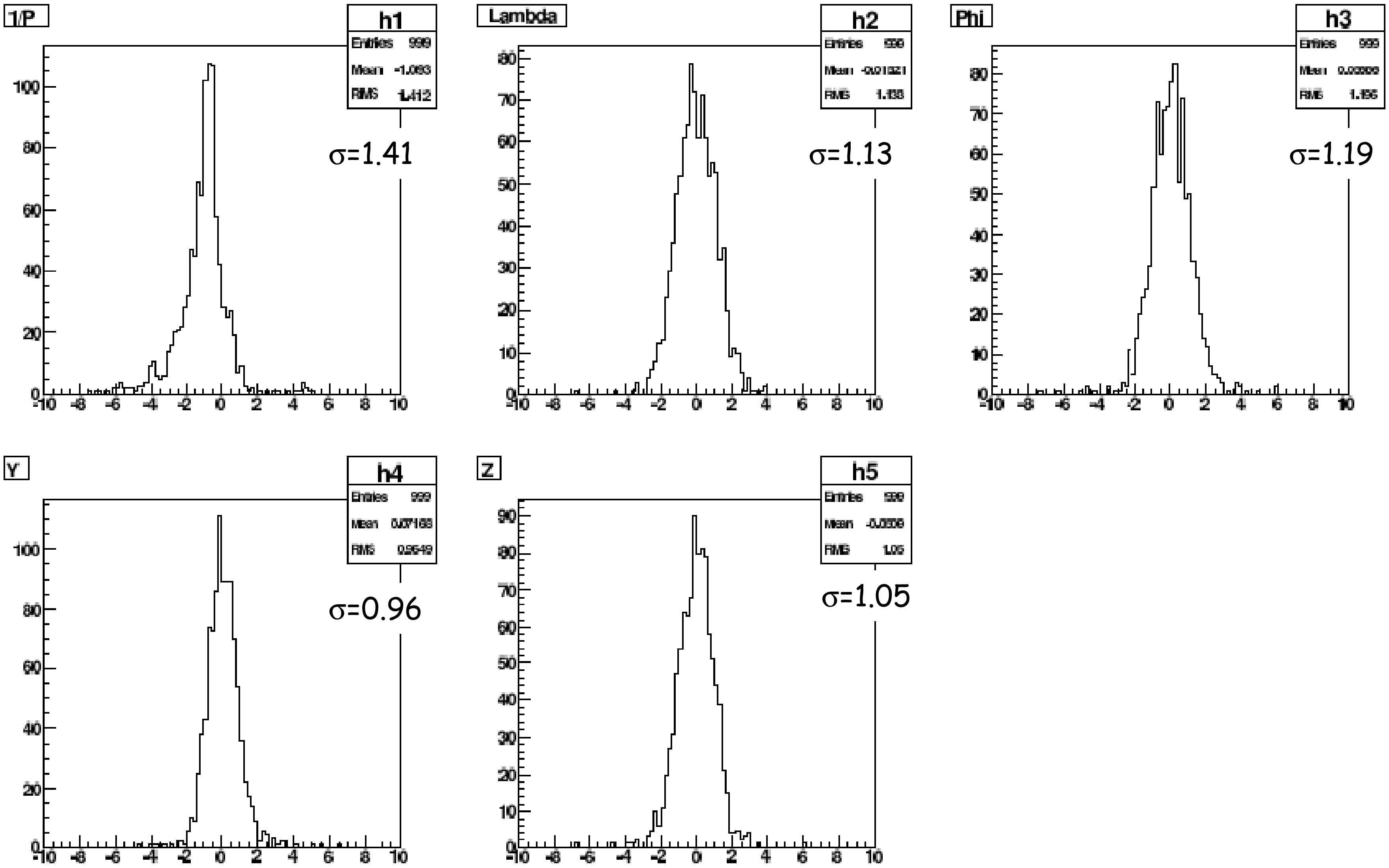
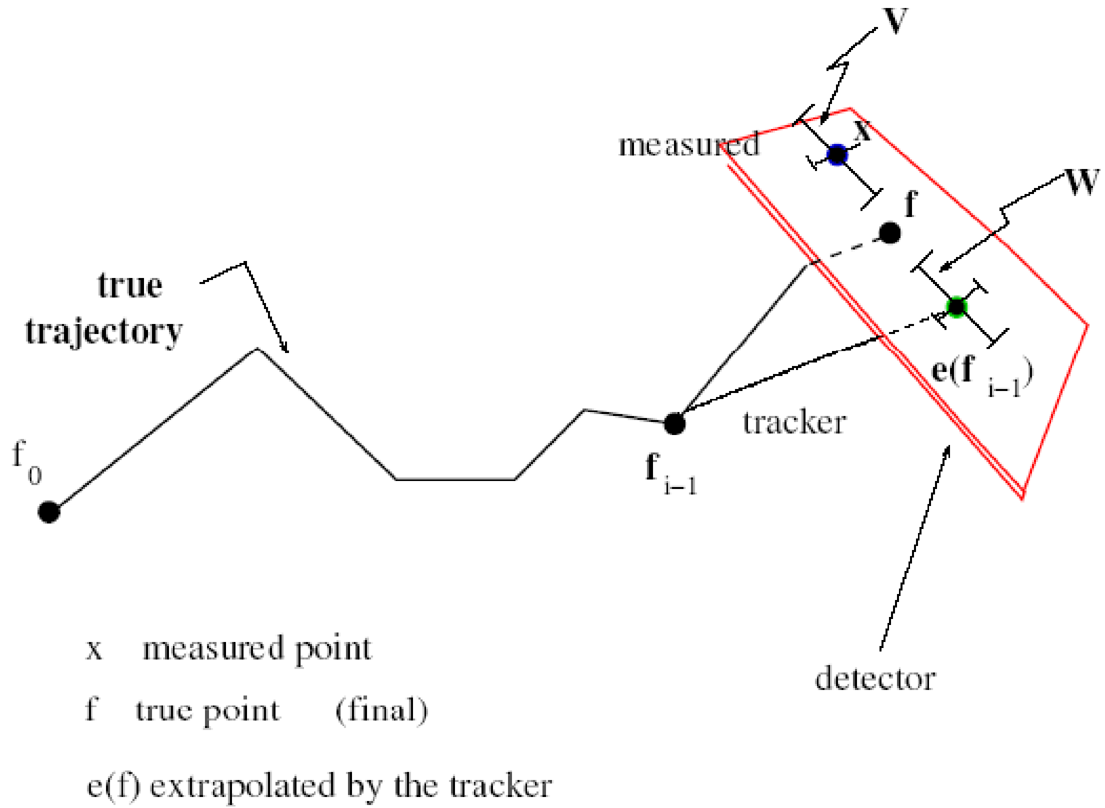


Figure 11: Pull distributions of the 5 track parameters in the case of 2 GeV muons that have passed through the whole detector, just before the PANDA

Tracking with Kalman: application of GEANE.

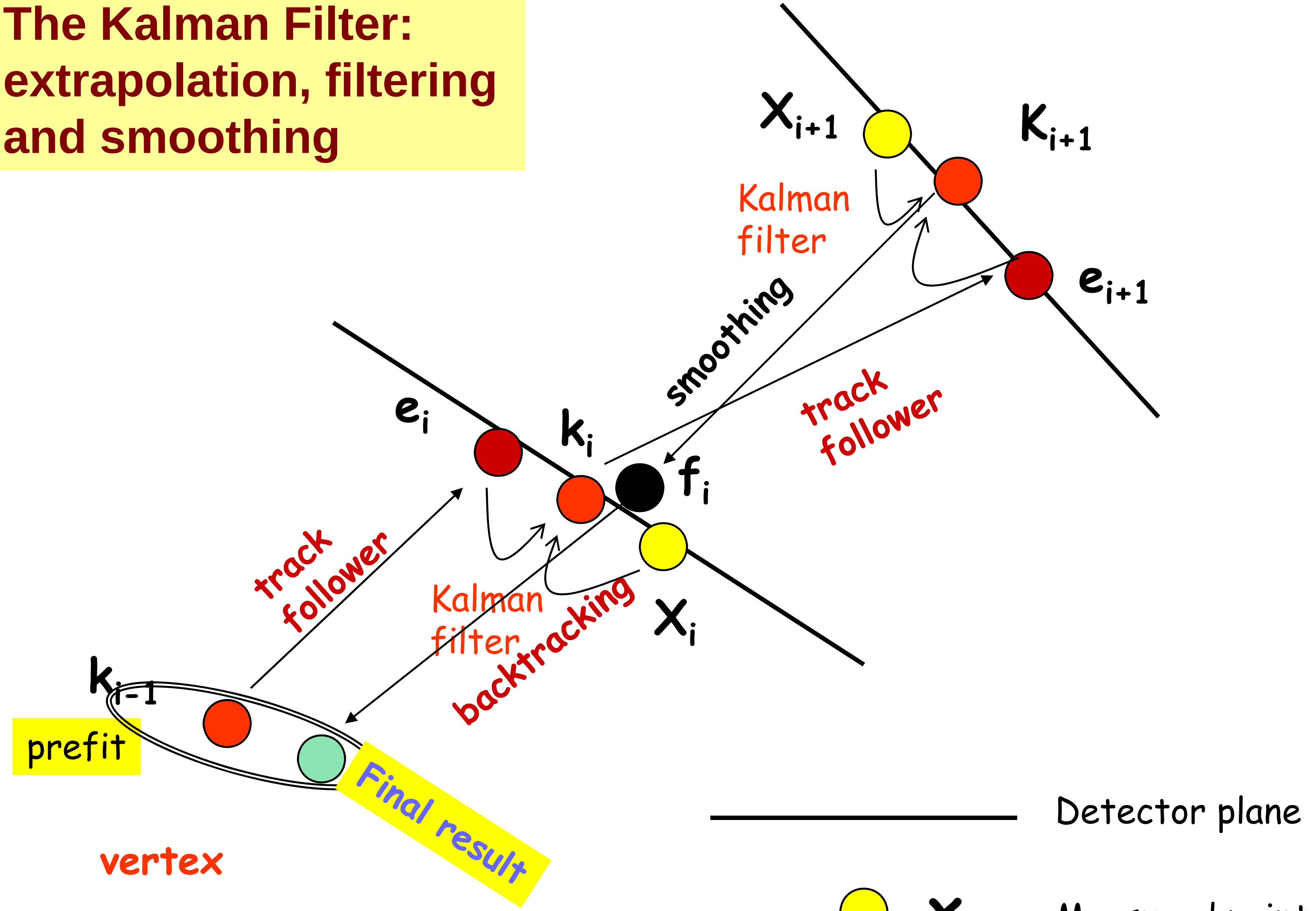


The best estimate of the track is given by minimizing w.r.t the f variables:

$$\chi^2(f) = \sum_i [(e_i[f_{i-1}] - f_i) \boldsymbol{W}_{i-1} (e_i[f_{i-1}] - f_i)] + (x_i - f_i) \boldsymbol{V}_i (x_i - f_i) \quad (1)$$

Note the $\boldsymbol{W}$ matrix associated to e_i because the extrapolation start from the true point.

The Kalman Filter: extrapolation, filtering and smoothing



Summary

- The **GEANE functionality** has been **reintroduced** in TGeant3.
- GEANE is now fully integrated in FairROOT with a **new C++ interface**: it can be used with the TGeometry modeler and with both G3 and G4!
- **Some modifications** for multiple scattering and energy loss have been introduced in GEANE to allow its use for gaseous detectors: old GEANE by default (no need to change existing code), new features only by explicit request.
- The results are good enough to allow the use of this track following in global fits (**Panda Kalman filter**).
- The energy loss part (errors on 1/p) is not completely under control (further investigations are necessary).
- All the mathematical and physical details are contained in a **technical report** from our group.

PANDA Report PV/01-07

Detailed report available at: http://www.pv.infn.it/~fontana/tracking.pdf

Track following in dense media and inhomogeneous magnetic fields

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A first general track follower for PANDA

Abstract

We collect and explain here the basic formulae of the track following and their implementation in GEANE. Some modifications for the existing code are suggested and the results on PANDA apparatus are shown.

References

- V. Innocente, M. Maire and E. Nagy, GEANE: Average Tracking and Error Propagation Package, CERN Program Library W5013-E (1991)
- V. Innocente and E. Nagy, Trajectory fit in presence of dense materials NIM A324(1993)297
- R. Fruewirth et al., Data Analysis Techniques for High Energy Physics, 2° edition, Cambridge University Press, 2000
- K. Lassilla-Perini and L. Urban, NIM A362(1995)416
- Geant3 and Geant4 manuals

Extra slides

Geane/FairROOT Interface

Class	Function	Purpose	Old GEANE correspondence	Notes
CbmPro	From_MARS_To_SC to SC system	go from lab system (MARS)	none	in preparation
CbmPro	From_SC_To_SD	go from SC to SD system	TRSCSD	in preparation
CbmPro	From_SD_To_SC	go from SD to SC system	TRSDSC	in preparation
CbmPro	From_SD1_To_SD2	go from an SD system to another SD system	TRS1S2	in preparation
CbmPro	PropagateToVolume	prepare for tracking	EUFILV	initialization
CbmPro	PropagateToPlane	prepare for tracking	EUFILP	initialization
CbmPro	PropagateToLength	prepare for tracking	EUFILL	preparation
CbmPro	Propagate (V option)	perform a tracking step	ERTRAK	tracking
CbmPro	Propagate (P option)	perform a tracking step	ERTRAK	tracking
CbmPro	Propagate (L option)	perform a tracking step	ERTRAK	tracking
CbmGeaneHit	GetP1,GetLambda1,GetPhi1 GetfY1, GetfZ1	get initial track parameters	none	output
CbmGeaneHit	GetErrorMat	get the initial 15-dim covariance matrix	none	output
CbmGeaneHit	GetP,GetLambda,GetPhi GetfY, GetfZ	get final track parameters	none	output
CbmGeaneHit	GetErrorMat	get the final 15-dim covariance matrix	none	output

Table 2: list of the possible functions for an interface with GEANE

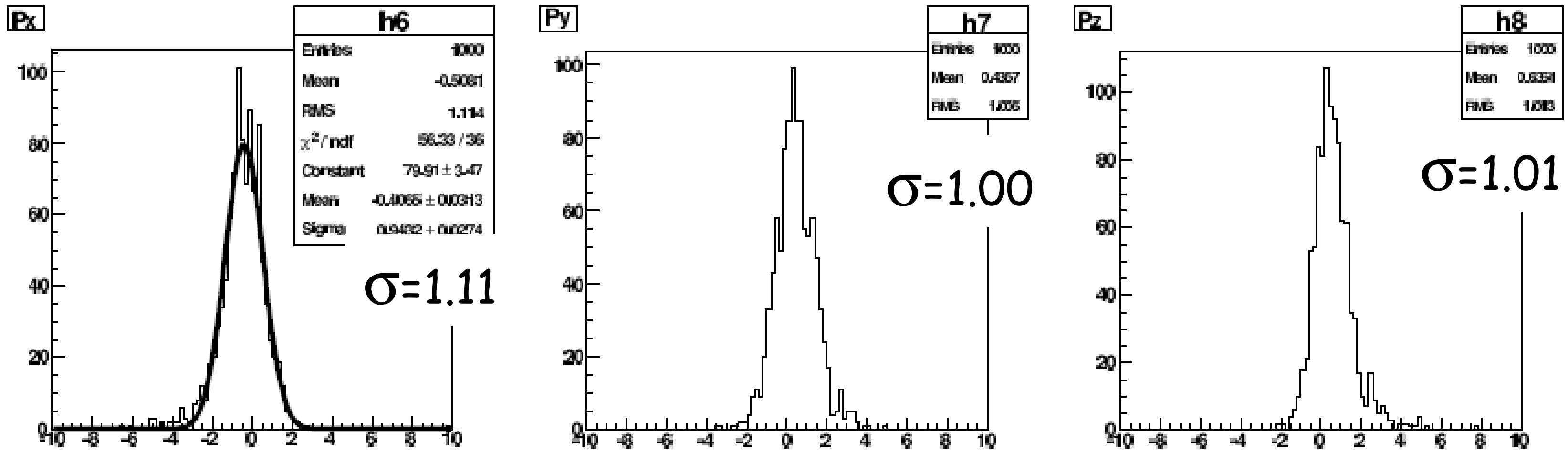

Figure 14: Pull quantities for the momentum components p_x, p_y, p_z for 5 GeV pions after the whole detector immediately before the PANDA magnet.

$$\sigma^2[p_x] = \sum_{ij} \frac{\partial p_x}{\partial x_i} \sigma[x_i, x_j] \ , \quad \sigma[x_i, x_i] \equiv \sigma^2_{x_i} \ , \qquad (78)$$

where $x_i = [p, \lambda, \phi]$ and the non diagonal elements of the tracking covariance matrix involved in the calculations are $\sigma[1/p, \lambda]$, $\sigma[1/p, \phi]$, $\sigma[\lambda, \phi]$:

$$\sigma^2_p = p^4 \sigma^2_{1/p} \ , \quad \sigma[p, \lambda] = -p^2 \sigma[1/p, \lambda] \ , \quad \sigma[p, \phi] = -p^2 \sigma[1/p, \phi] \ .$$

Tracking with Kalman

The minimization gives:

$$\frac{\partial \chi^2}{\partial f_i} = \boldsymbol{W}_{i-1,i}(e_i[f_{i-1}] - f_i) + \boldsymbol{V}(x_i - f_i) \qquad (2)$$
$$+\boldsymbol{T}_{i,i+1}\boldsymbol{W}_{i,i+1}(e_{i+1}[f_i] - f_{i+1}) = 0$$

where the last (*extra*) term comes from the extrapolation procedure (tracker).

The best way to solve eq (2) is the Kalman algorithm (Kalman, 1961). It is based on three steps:

- **EXTRAPOLATION**: calculation of e_i and $\boldsymbol{W}$. Deterministic step made by the tracker.

$$e_i = \boldsymbol{G}_{i-1,i}[k_{i-1}] \quad (3)$$

$$\boldsymbol{\sigma}^2[e_i] = \boldsymbol{T}_{i-1,i}\,\boldsymbol{\sigma}^2[k_{i-1}]\,\boldsymbol{T}^T_{i-1,i} + \boldsymbol{W}^{-1}_{i-1,i} \quad (4)$$

Square brackets mean function argument
e_i = EXTRAP. extrapolation
k_i = result of the Kalman filter
$\boldsymbol{T}_{i-1,i}$ = EXTRAP. transport matrix
$\boldsymbol{\sigma}(k)^2$ = Kalman error matrix
$\boldsymbol{\sigma}^2[e_i]$ = EXTRAP. error matrix
$\boldsymbol{W}_{i-1,i}$ = EXTRAP. energy loss and multiple scattering weight matrix
$\boldsymbol{W}^{-1}_{i-1,i}$= covariance matrix inverse of W

- **FILTERING**: minimizes the first two terms of eq. (2). It is simply the weighted mean;

$$k_i = \boldsymbol{\sigma}^2[k_i]\left(\boldsymbol{\sigma}^{-2}[e_i]\, e_i + \boldsymbol{V}_i x_i\right) \quad (5)$$

$$\boldsymbol{\sigma}^{-2}[k_i] = \boldsymbol{\sigma}^{-2}[e_i] + \boldsymbol{V}_i \quad (6)$$

x_i = measured points

k_i = Kalman average value

$\boldsymbol{\sigma}(k)^2$ = Kalman error matrix

$\boldsymbol{\sigma}^2(e)$ = EXTRAP. error matrix

$\boldsymbol{V}$ = original weight matrix of the measured points

- **SMOOTHING**: necessary to minimize a χ^2 in the presence of the extrapolation term (last term in eq. (2)).

$$f_i = k_i + \boldsymbol{A}_i \left(f_{i+1} - e_{i+1} \right) \quad (7)$$

$$\boldsymbol{\sigma}^2[f_i] = \boldsymbol{\sigma}^2[k_i] + \boldsymbol{A}_i \left(\boldsymbol{\sigma}^2[f_{i+1}] - \boldsymbol{\sigma}^2[e_{i+1}] \right) \boldsymbol{A}_i^T \quad (8)$$

$$\boldsymbol{A}_i = \boldsymbol{\sigma}^2[k_i] \, \boldsymbol{T}_{i,i+1}^T \, \boldsymbol{\sigma}^{-2}[e_{i+1}] \quad (9)$$

f_i = final average value
$\boldsymbol{\sigma}(k)^2$ = Kalman error matrix
$\boldsymbol{\sigma}^2(e)$ = EXTRAP. error matrix