Multijet Cross Sections

and Resummations

Physics at LHC
Princeton, March 22, 2007

George Sterman

- I. Perturbative resummations
- II. What makes jet cross sections infrared safe (sometimes)?
- III. Describing the jets: angularities
- IV. Nonperturbative effects
- V. Resummation with color exchange (with a surprise)

Start with some fundamentals . . .

⋆ **I. Infrared Safety & Resummation: Why? When? How?**

- Logarithmic corrections
- Structure of jets and interjet radiation
- Applications to backgrounds and signals (sensitivity to color flow)
- Patterns and effects of power corrections

- **Infrared safety** & **asymptotic freedom**:

$$
\begin{aligned}
Q^2\ \hat{\sigma}_{\rm SD}(Q^2,\mu^2,\alpha_s(\mu)) &= \sum_n c_n(Q^2/\mu^2)\ {\alpha_s}^n(\mu) + \mathcal{O}\left(\frac{1}{Q^p}\right) \\
&= \sum_n c_n(1)\ {\alpha_s}^n(Q) + \mathcal{O}\left(\frac{1}{Q^p}\right)
\end{aligned}
$$

- $\mathrm{e}^+\mathrm{e}^-$ total; jets: a sum over collinear rearrangements and soft emission organizes all long-time transitions, which must sum to ≤ 1 by unitarity. But not always as simple as it seems.

- **Generalization: factorization**

$$Q^2\sigma_{\mathrm{phys}}(Q,m) = \omega_{\mathrm{SD}}(Q/\mu,\alpha_s(\mu)) \otimes f_{\mathrm{LD}}(\mu,m) + \mathcal{O}\left(1/Q^p\right)$$

$\mu =$ **factorization scale**; $m=$ **IR scale** (m may be perturbative)

- **New physics** in ω_{SD}; f_{LD} "universal"

- Deep-inelastic ($p = 2$), $\mathrm{p\bar{p}} \to Q\bar{Q}$ . . .

- Decays: $B \to \pi\pi$ and "elastic" limits: $\mathrm{e^+e^-} \to \mathrm{JJ}$ as $m_J \to 0$

⋆ When Can We Resum?

- **Whenever there is factorization, there is evolution**

$$0 = \mu \frac{d}{d\mu} \ln \sigma_{\rm phys}(Q, m)$$

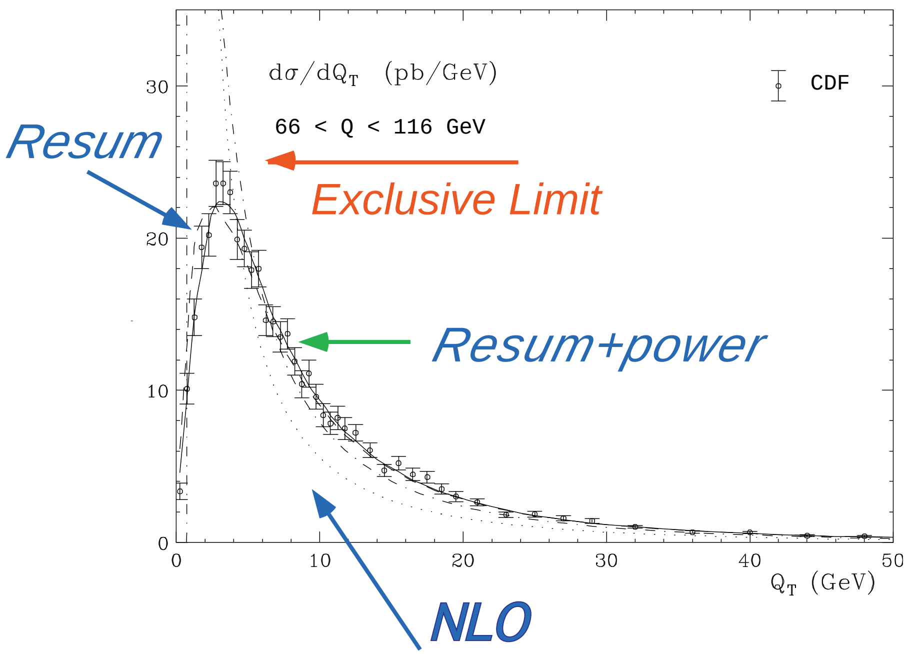
$$\mu \frac{d \ln f}{d\mu} = -P(\alpha_s(\mu)) = -\mu \frac{d \ln \omega}{d\mu}$$

- **Wherever there is evolution there is resummation**

$$\ln \sigma_{\rm phys}(Q, m) = \exp \left\{ \int_q^Q \frac{d\mu'}{\mu'} P\left(\alpha_s(\mu')\right) \right\}$$

Explicit Logs: p_T distributions, event shapes, BFKL

$$\frac{d\sigma(Q)}{dQ_1} \propto \frac{1}{Q_1} \sum_n C_n \alpha_s^n \ln^{an+b}\left(\frac{Q}{Q_1}\right) \quad \Lambda \ll Q_1 \ll Q$$



(from Kulesza, G.S., Vogelsang (2002))

– maximum then decrease near “exclusive” limit (parton model kinematics) replaces divergence

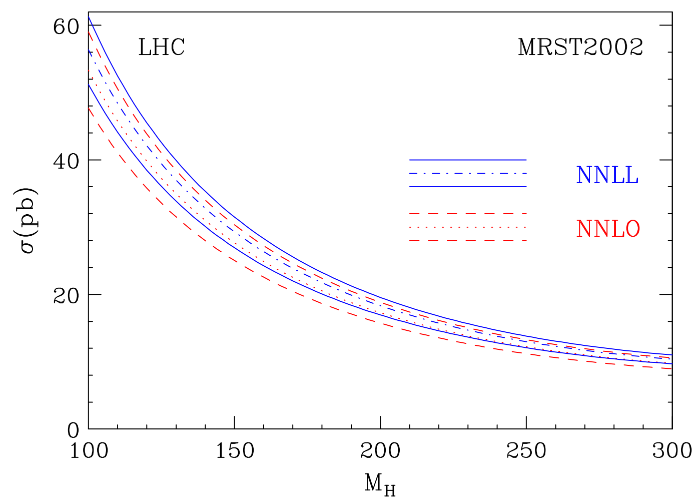
– soft but perturbative radiation broadens distribution

– typically NP correction necessary for quantitative description of data.

– recover fixed order away from exclusive limit

Implicit logs: threshold resummations, 1PI high-p_T

$$\sigma(Q) \propto \int \frac{dQ_1}{Q_1} F(Q_1) \sum_n C_n \alpha_s^n \ln^{an+b}\left(\frac{Q}{Q_1}\right) \quad F(0) = 0$$



(from Catani, de Florian, Grazzini, Nason (2003))

– Modest change, scale improvement $\leftrightarrow$ increased confidence

- Factorization structure and proofs:

$$\frac{d\sigma(Q, a+b \to N_{\rm jets})}{dQ} = H_{IJ} \otimes \prod_{c=a,b} \mathcal{P}_{c'/c} \times S_{JI} \times \prod_i J_i$$

  - A story with only these pieces:

    * Evolved incoming partons $\mathcal{P}_{a'/a}$, $\mathcal{P}_{b'/b}$ collide at H_{IJ}, I, J label color exchange in M and M^*;

    * Outgoing jets J_i and coherent soft emission S_{JI}.

    * Holds to any fixed α_s^n, all $\ln^a \mu/Q$ to $\sim E_{\rm soft}/E_{\rm jet}$.

- Where it comes from . . .
  - Cancellation of final-state interactions
  - Incoherence of initial-state dynamics for relative velocities $\sim c$.
  - Requires large momentum transfer to distinguish initial and final states.
  - For jets or event shapes, require no worse than logarithmic singularities.

II. Special point: what makes a jet-finding algorithm infrared safe?

– A set of collinear particles and soft particles define a subspace of loop/phase space momenta

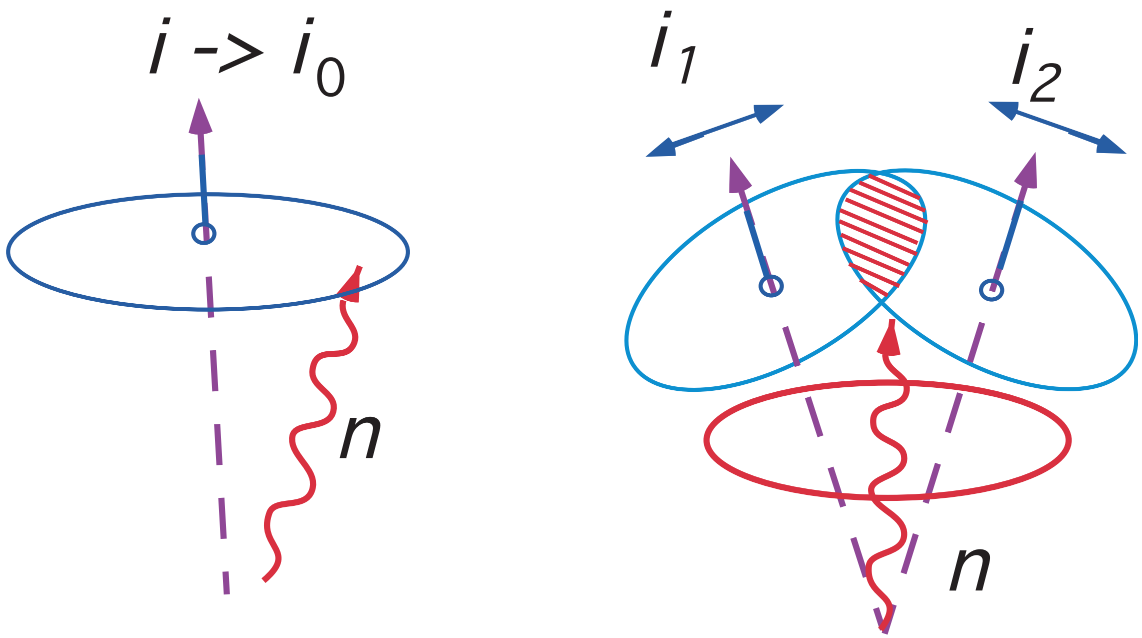
– "Normal variables" ($n =$ soft gluon energies, relative k_T for collinear particles, etc.) determine where the subspace is.

– "Instrinsic variables" ($i =$ hard particle energy, direction, etc.) parameterize the subspace. Number of jets changes at $i = i_0$.

– Generic cancellation between virtual and real states:

$$\int \frac{di}{Q} \int_0 \frac{dn}{n} \, (\, F(n) - F(0) \,)$$

– Example on the left.

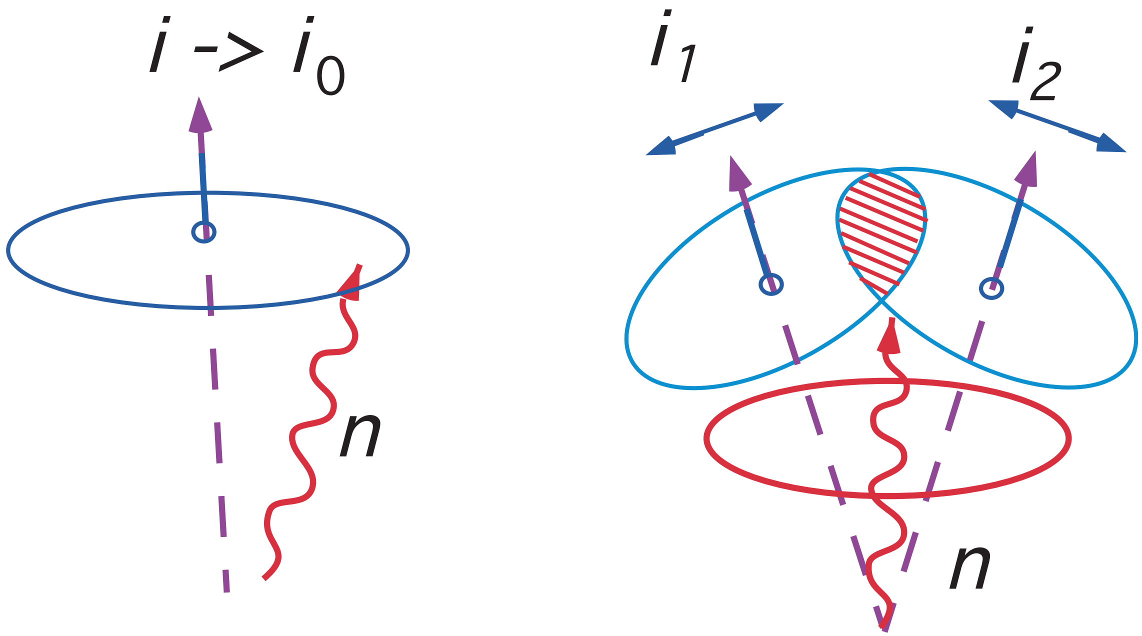


– For energy i below minimum value i_0, soft gluon emission required to make up the difference near the subsurface $i = i_0$.

$$\int^{i_0} \frac{di}{Q} \left(\int_{i-i_0} \frac{dn}{n} F(0) \right) = - \int_{i_0} \frac{di}{Q} \ln(i - i_0)\, F(0)$$

– OK if singularities aren't worse than logarithmic.

– Example on the right:



– Soft gluon on the right reduces number of jets any energy n on a subspace of the same dimension as the singular surface.

$$\int \frac{di}{Q} \int \frac{dn}{n} \, F(0) \to \infty$$

- IR safety requires that jet number change only on subspace of lower dimension than the relevant singular surface.

- Problem if trial cones are centered only on particles (seeds).
  Seymour (1998)

- "Midpoint algorithm" deals with the example above but the problem is more general unlesss all cones are sampled.

- Very recently, "Practical seedless cone algortithm": G. Salam and G. Soyez (2007) identify cones with hard particles "at the edges".

III. Describing the jets: angularities

- Flexible event shapes (C.F. Berger, Kúcs, GS (2003), Berger, Magnea (2004))

$$
\begin{aligned}
\tau_a &= \frac{1}{Q} \sum_{i \,\mathrm{in}\, N} p_{Ti}\, e^{-(1-a)|\eta_i|} \\
&= \frac{1}{Q} \sum_{i \,\mathrm{in}\, N} E_i\, (\sin\theta_i)^a\, (1 - |\cos\theta_i|)^{1-a}
\end{aligned}
$$

  - p_{Ti}, η_i relative to thrust $(a = 0)$ axis (can be chosen jet-by-jet).

  - Broadening: $a = 1$; inclusive limit $a \to \infty$.

  - For multijet final states, define η_i relative to closest jet.

– Cross section is a convolution in contributions of each jet and a soft radiation function

$$\sigma\left(\tau_a, Q, a\right) \;=\; H_{IJ} \int dt_s \prod_{\mathrm{jets}\ i} \int dt_i\, S_{JI}(t_s) \prod_i J_i(t_i, p_{Ji})$$
$$\times\, \delta\Big(\sum_i t_i + t_s - \tau_a\Big)$$

– Thus, general resummed cross section can be written as an inverse transform

$$\sigma\left(\tau_a, Q, a\right) \;=\; \int_C d\nu\, \mathrm{e}^{\nu\,\tau_a}\, H_{IJ}\, S_{JI}(\nu) \prod_i J_i(\nu, p_{Ji})$$

in terms of $f(\nu) = \int_0^\infty dt\, e^{-\nu t} f(t)$.

– The jet in transform space

$$J_i(\nu, p_{Ji}) \;=\; \int_0 d\tau_a \, \mathrm{e}^{-\nu \tau_{Ji}} \; J_i(\tau_{Ji}, p_{Ji}) = \mathrm{e}^{\frac{1}{2} E(\nu, Q, a)}$$

$$\begin{aligned} E(\nu, Q, a) \;=\; & 2 \int_0^1 \frac{du}{u} \Bigg[\int_{u^2Q^2}^{uQ^2} \frac{dp_T^2}{p_T^2} A\left(\alpha_s(p_T)\right) \left(\mathrm{e}^{-u^{1-a} \nu (p_T/Q)^a} - 1 \right) \\ & + \frac{1}{2} B\left(\alpha_s(\sqrt{u}Q)\right) \left(\mathrm{e}^{-u(\nu/2)^{2/(2-a)}} - 1 \right) \Bigg] \end{aligned}$$

– Expansion in $\alpha_s(Q)$ finite at all orders. A depends on q vs. G.

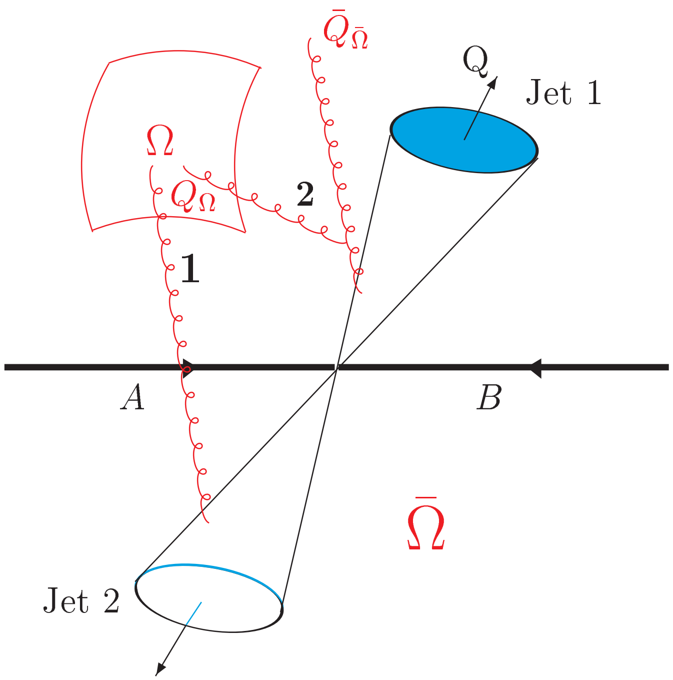
- Jet shapes in DIS, pp similar if overall final state limited (global)
  (Dasgupta and Salam (2000, 2002))

- Semi-numerical resummation (flexibility)
  Banfi, Salam, Zanderighi (2002, 2003)

A caution . . .

Non-global logs: color and energy flow

(Dasgupta & Salam (2001))



- Observations about non-global logs.

- a) If cross section is inclusive in $\bar{\Omega}$ $\rightarrow$ Number of jets not fixed!

- b) Correlation with event shape $\tau_a \ldots$:
  fixes number of jets $\rightarrow$ factorization
  (C.F. Berger, Kúcs, GS (2003), Dokshitzer, Marchesini (2003))

- c) Conjecture: If only number of very forward jets is not fixed,
  non-global effects can be "universal", and subtracted out
  with pile-up, etc.

- d) Still work to be done ((Forshaw, Kyrieleis, Seymour (2006))

IV. Nonperturbative effects in angularities

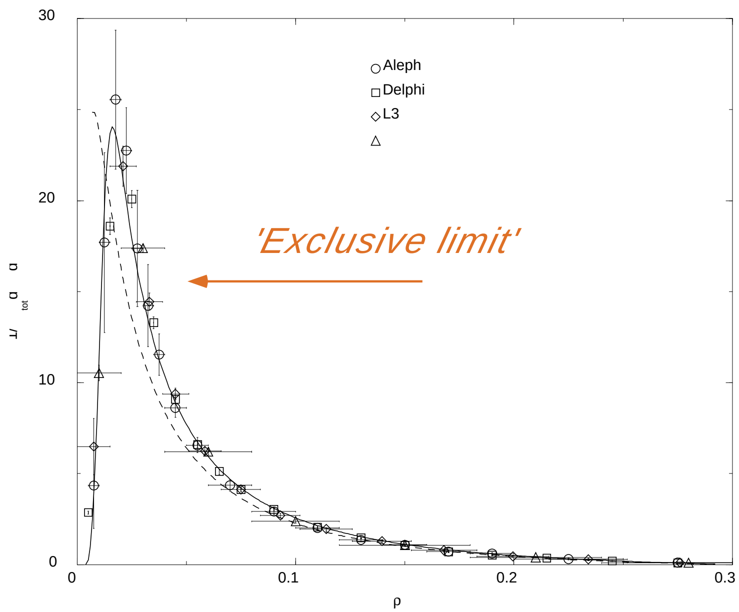
- How to interpret expressions like

$$E(\nu, Q, a) = 2\int_0^1 \frac{du}{u}\Bigg[\int_{u^2Q^2}^{uQ^2} \frac{dp_T^2}{p_T^2} A\left(\alpha_s(p_T)\right)\left(\mathrm{e}^{-u^{1-a}\nu(p_T/Q)^a} - 1\right)$$
$$+\frac{1}{2}B\left(\alpha_s(\sqrt{u}Q)\right)\left(\mathrm{e}^{-u(\nu/2)^{2/(2-a)}} - 1\right)\Bigg]$$

Enter: nonperturbative scales in resummmed PT

- Necessary to describe the data for resum of these "explicit" logs.

- Example: Heavy jet distribution at the LEP Z pole ($\sim \tau_0$) (Korchemsky and Tafat (2000)).



  - Dashed line: NLL resummed; solid line: NP "shape function" fit
  - What's that?

– Organizing the NP corrections: shape function approach.

– $p_T > \kappa$, PT, $p_T < \kappa$, expand exponentials

– Low p_T replaced by $f_{\rm NP}$ "shape function"

$$
\begin{aligned}
E(\nu, Q, a) \;&=\; E_{\rm PT}(\nu, Q, \kappa, a) \\
&\quad + \frac{2}{1-a} \sum_{n=1}^{\infty} \frac{1}{n\, n!} \left(-\frac{\nu}{Q}\right)^{n} \int_0^{\kappa^2} \frac{dp_T^2}{p_T^2}\, p_T^n\, A\left(\alpha_s(p_T)\right) + \ldots \\
&\equiv\; E_{\rm PT}(\nu, Q, \kappa, a) + \ln \tilde{f}_{a,\rm NP}\left(\frac{\nu}{Q}, \kappa\right)
\end{aligned}
$$

- Separate, energy-dependent factor for each jet. (To be explored!)

- Shape function factorizes in moments $\rightarrow$ convolution

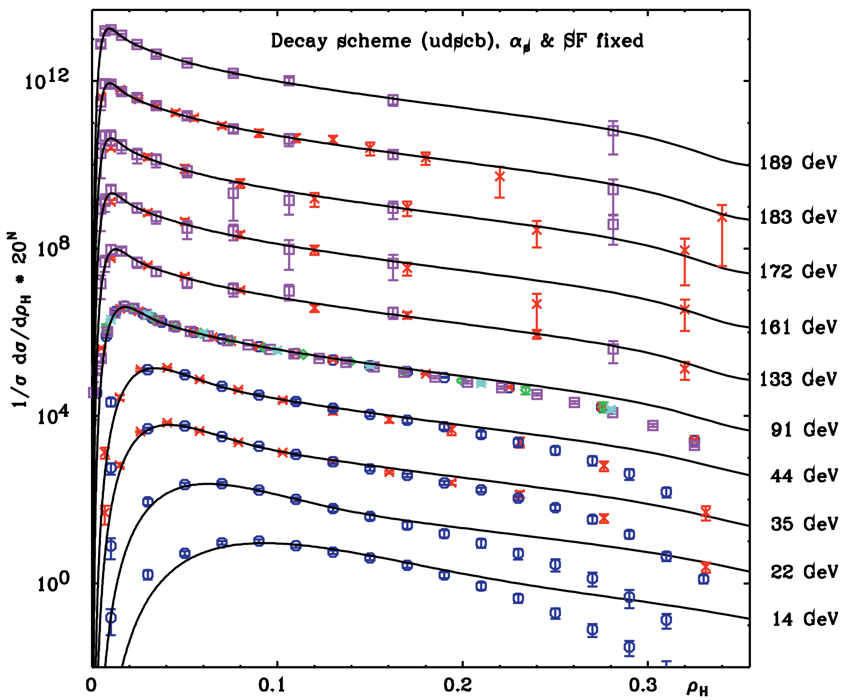
$$\sigma(\tau_a, Q) = \int d\xi \, f_{a,\mathrm{NP}}(\xi) \; \sigma_{\mathrm{PT}}(\tau_a - \xi, Q)$$

- $\mathrm{e}^+\mathrm{e}^-$: fit at $Q = M_{\mathrm{Z}}$ $\Rightarrow$predictions for all Q, any (quark) jet.

- Portable to jets in hadronic collisions.

- And will be sensitive to gluon/quark origin of the jet

- Shape function phenomenology for thrust at LEP.



(Korchemsky,GS, Belitsky; Gardi Rathsman,Magnea (1998 . . .)

- Scaling property for τ_a event shapes

  (C.F. Berger & GS (2003) Berger and Magnea (2004)

- Test of rapidity-independence of NP dynamics

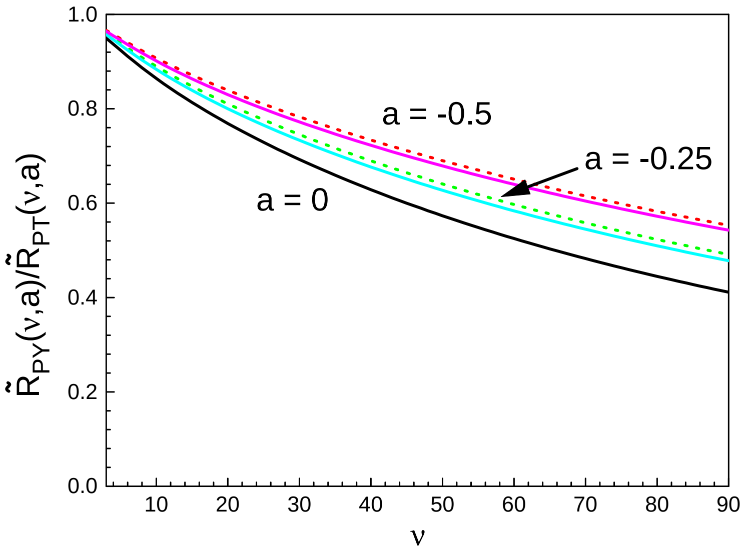
  (C. Lee, GS (2006); SCET)

$$\ln \tilde{f}_{a,\mathrm{NP}}\left(\frac{\nu}{Q},\kappa\right) = \frac{1}{1-a}\sum_{n=1}^{\infty} \lambda_n(\kappa)\left(-\frac{\nu}{Q}\right)^n$$

$$\tilde{f}_a\left(\frac{\nu}{Q},\kappa\right) = \left[\tilde{f}_0\left(\frac{\nu}{Q},\kappa\right)\right]^{\frac{1}{1-a}}$$

- What PYTHIA gives in moment space (2003)

  (Analysis with L3 data underway by Banerjee, Jindal, Kaur)



V. Evolution with Color Exchange

- What distinguishes hadron colliders.
- Multiloop scattering amplitudes in dimensional regularization

(Catani (1998) Tejeda-Yeomans & GS (2002) Kosower (2003)) Aybat, Dixon & GS (2006)

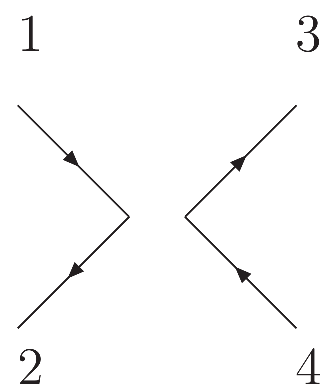
  – Amplitude for partonic process

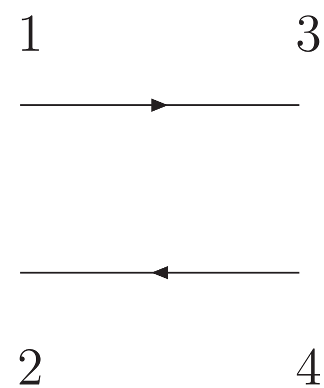
$$\mathrm{f}: \quad f_A(p_A, r_A) + f_B(p_B, r_B) \to f_1(p_1, r_1) + f_2(p_2, r_2)$$

$$\mathcal{M}^{[\mathrm{f}]}_{\{r_i\}}\left(p_j, \frac{Q^2}{\mu^2}, \alpha_s(\mu^2), \epsilon\right) = \mathcal{M}^{[\mathrm{f}]}_{L}\left(p_j, \frac{Q^2}{\mu^2}, \alpha_s(\mu^2), \epsilon\right) \, (c_L)_{\{r_i\}}$$

- Need to control poles in ϵ for factorized calculations at fixed order and for resummation. Evolution for "soft" functions S_{IJ}.

- Example: $q\bar{q}$ tensors $(c_L)_{\{r_i\}}$:

$c_1 \equiv$ 

$c_2 \equiv$ 

  – Jet/soft factorization at amplitude level. (Sen (1983)):

$$\mathcal{M}_L^{[\mathrm{f}]}\left(p_i, \frac{Q^2}{\mu^2}, \alpha_s(\mu^2), \epsilon\right) = \prod_{i=A,B,1,2} J_i^{[\mathrm{virt}]}\left(\frac{Q^2}{\mu^2}, \alpha_s(\mu^2), \epsilon\right)$$

$$\times \mathbf{S}_{LI}^{[\mathrm{f}]}\left(p_i, \frac{Q^2}{\mu^2}, \alpha_s(\mu^2), \epsilon\right) h_I^{[\mathrm{f}]}\left(\wp_i, \frac{Q^2}{\mu^2}, \alpha_s(\mu^2)\right)$$

- Soft function labelled by color exchange (singlet, octet . . .)

- Factors require dimensional regularization

- Same factorization $\longrightarrow$ resummation

- Poles at 2- and higher loops . . .

- Relation to supersymmetric Yang-Mills theories

  (Bern, Czakon, Dixon, Kosower & Smirnov (2006) verified structure to 4 loops.)

– Dimensionally-regularized jets

(Magnea & GS (1990))

$$J_i\left(\frac{Q^2}{\mu^2},\alpha_s(\mu^2),\epsilon\right) = \exp\Bigg\{\frac{1}{4}\int_0^{-Q^2}\frac{d\xi^2}{\xi^2}\Bigg[\mathcal{K}^{[i]}(\alpha_s(\mu^2),\epsilon)$$

$$+\mathcal{G}^{[i]}\left(-1,\bar{\alpha_s}\left(\frac{\mu^2}{\xi^2},\alpha_s(\mu^2),\epsilon,\right)\epsilon\right)$$

$$+\frac{1}{2}\int_{\xi^2}^{\mu^2}\frac{d\tilde{\mu}^2}{\tilde{\mu}^2}\gamma_K^{[i]}\left(\bar{\alpha_s}\left(\frac{\mu^2}{\tilde{\mu}^2},\alpha_s(\mu^2),\epsilon\right)\right)\Bigg]\Bigg\}.$$

$$J_i\left(\frac{Q^2}{\mu^2},\alpha_s(\mu^2),\epsilon\right) = \exp\left[\sum_{n=1}^{\infty}\left(\frac{\alpha_s(\mu^2)}{\pi}\right)^n\sum_{n=1}^{n+1}\frac{E_m^{[i]\,(n)}(\varepsilon)}{\varepsilon^n}+\text{finite}\right]$$

– γ_K, $\mathcal{K}$ related to A above

– Dimensionally-regularized S

$$\mathbf{S}^{[\mathrm{f}]}\left(\frac{Q^2}{\mu^2},\alpha_s(\mu^2),\epsilon\right)$$
$$= \mathrm{P}\,\exp\left[-\frac{1}{2}\int_0^{-Q^2}\frac{d\tilde{\mu}^2}{\tilde{\mu}^2}\mathbf{\Gamma}^{[\mathrm{f}]}\left(\bar{\alpha}_s\left(\frac{\mu^2}{\tilde{\mu}^2},\alpha_s(\mu^2),\epsilon\right)\right)\right]$$

$\Gamma^{[\mathrm{f}]}$: anomalous dimension; color mixing

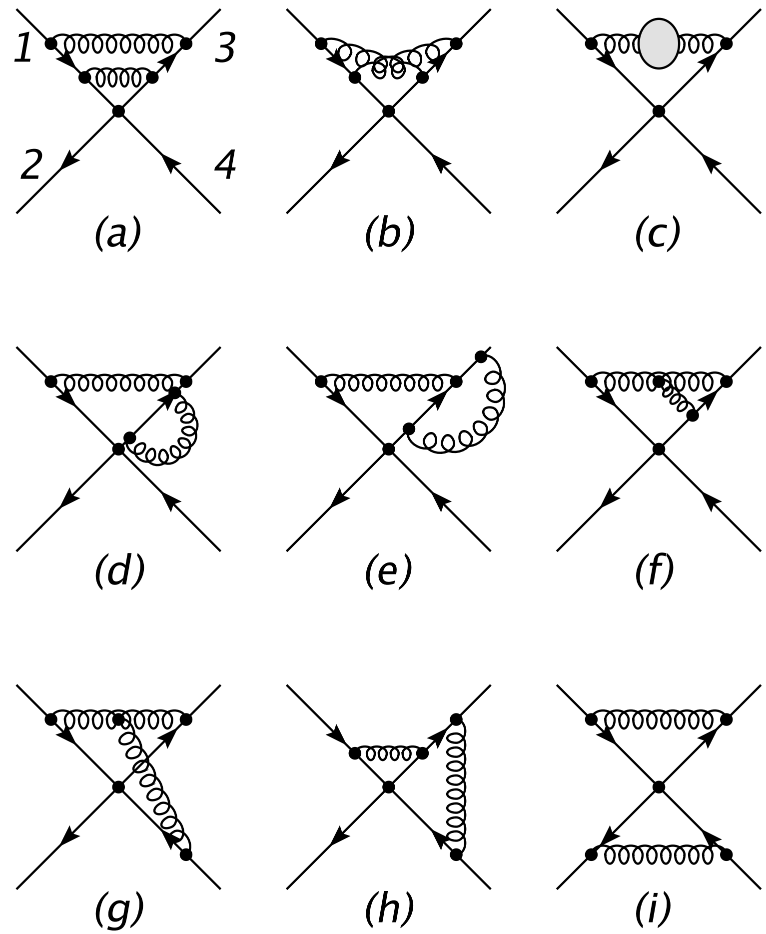
- New result for all massless $2 \to n$ processes (Aybat, Dixon, GS (2006))

$$\Gamma_S = \frac{\alpha_s}{\pi}\left(1 + \frac{\alpha_s}{\pi} K\right) \Gamma_{S'}^{(1)} + \cdots$$

$\Gamma^{(2)} = (K/2)\Gamma^{(1)}$ with same K as in the DGLAP splitting.

Related to the "CMW" or MC/bremsstrahlung scheme.

(Catani, Marchesini & Webber (1990))



The diagrams with 3g vertices vanish!

To NNLO, “single-web” exchange generalizes single gluon.

(C.F. Berger, 2002)

- The full two-loop single-pole terms $\times$ LO are simply

$$\left[\sum_{i \in \mathrm{f}} \frac{E_1^{[i]\,(2)}}{\varepsilon} + \frac{1}{4\varepsilon} \Gamma_S^{[\mathrm{f}]\,(2)} \right] \times \ \mathrm{LO}$$

- $E_1^{[i]\ (2)}$ is 2 loop single pole in Sudakov form factor
  (Ravindran, Smith, van Neerven (2005))

  Agrees with Jantzen, Kuhn, Penin, Smirnov (2005, 2006) in EW logs.

- Hints of unexpected simplicity for IR gluons.

- Increasing insight into the structure of final states.