lattice QCD   a personal view in 20 minutes. . .

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Naples, 11-04-2007

- lattice QCD is the only reliable way to perform a precision test of the standard model at the non-perturbative level
- $\leq$2006: lattice QCD has been used in order to *estimate* the hadronic inputs to the CKM Unitarity Triangle Analysis (UTA) since these where *unknowns*.
- the UTA can be now performed without the need of lattice inputs (UT_{fit} Collaboration JHEP **0610** (2006) 081)
- $\geq$2007: to the lattice it is asked to provide, let's say, ΔM_s within the standard model with an error of the same order of the one quoted by the CDF collaboration: i.e. all the systematics have to be under control beyond any reasonable doubt!
- different groups are following different strategies to reach the same goal and in the end, hopefully, we will have the best we can ask: the same result confirmed by different calculations
- presently we still have numbers affected by largely different systematics . . .

UNC: quenching

- quenching is definitively bad
  - the effect of the quenching is observable dependent and impossible to quantify without unquenched simulations in the same range of parameters
- at the same time quenched simulations have been able to predict $\sin 2\beta$

1995	$\sin(2\beta)$	$=$	0.650 ± 0.120	Ciuchini et al Z. Phys. C **68** (1995) 239
2000	$\sin(2\beta)$	$=$	0.698 ± 0.066	Ciuchini et al JHEP **0107** (2001) 013
exp	$\sin(2\beta)$	$=$	0.687 ± 0.032	

  - the effectiveness of quenching may be due to a sort of “matching”: one has to tune the quark masses and the lattice spacing by using experimental inputs

- staggered fermions are introduced on the lattice by simulating the following quark action (actually in its improved version):

$$\bar{\chi} D_{\text{stag}} \chi = \sum_n \bar{\chi}_n \left[\sum_\mu \frac{\eta_{n,\mu}}{2} \left(U_{n,\mu} \chi_{n+\mu} - U^\dagger_{n-\mu,\mu} \chi_{n-\mu} \right) + m_0 \chi_n \right]$$

affected by doubling, i.e. it has $2^4 = 16$ one-component fermions

- rooting means that gauge configurations are generated according to the following partition function:

$$Z^{\text{root}}_{N_f=3} = \int DU e^{-S_g} \left\{ \det[D_{\text{stag}}(m_u)] \det[D_{\text{stag}}(m_d)] \det[D_{\text{stag}}(m_s)] \right\}^{1/4}$$

S. R. Sharpe@LATTICE 2006 [PoS **LAT2006** (2006) 022]:

Q: **“Rooted staggered fermions: Good, bad or ugly?”**

A: **ugly**! in the sense that are affected by unphysical contributions at regulated stage that need a complicate analysis to be removed

other sources of systematics

- chiral pathologies ($\chi\mathrm{E}$)
  - f_B, for example, is expected to diverge in the quenched chiral limit
  - use of NLO χPT formulas to extrapolate results gives complete control or large(reliable) errors
- cutoff effects ($a\mathrm{E}$)
  - single and coarse lattice spacing
  - extrapolations with 3 or more lattice spacings gives complete control or large(reliable) errors
- finite volume effects ($L\mathrm{E}$)
  - may be the dominant source of uncertainty in unquenched calculations of M_π, f_π, etc.

lattice calculations of heavy–light systems, issues

- on currently affordable lattice sizes (at least in unquenched simulations) one has

$$am_b > 1 \qquad Lm_d > 1$$
$$am_b < 1 \qquad Lm_d < 1$$

- we are able to simulate "relativistic" beauty–light systems on
  - big volumes with big cutoff effects
  - small volumes with big finite volume effects

- so we have to devise smart strategies to cope with this *two-scales problem*. we can divide the different approaches in
  - big volume strategies
  - finite volume strategies

HQET one can resort to the static approximation that can also be non–perturbatively renormalized

E Eichten et al Phys. Lett. B **234** (1990) 511

J Heitger et al JHEP **0402** (2004) 022

HQET EXT one can simulate the relativistic theory with heavy masses around the physical charm mass and extrapolate (or interpolate with the static) to the beauty mass relying on HQET predictions

FERMILAB the FERMILAB approach consists in simulating the following action with $am_0 > 1$

$$S = \sum_n \bar{\psi}_n \left[m_0 + \gamma_0 D_0 + \zeta \vec{\gamma} \cdot \vec{D} - r_t \frac{aD_0^2}{2} - r_s \frac{a\vec{D}^2}{2} + c_B \frac{i\sigma_{ij} F_{ij}}{4} + c_E \frac{i\sigma_{0i} F_{0i}}{2} \right] \psi_n$$

i.e. the Symanzik effective action for quarks with $|a\vec{p}| \ll 1$ with *mass dependent* coefficients usually computed perturbatively

A X El-Khadra et al Phys. Rev. D **55** (1997) 3933

S Aoki et al Prog. Theor. Phys. **109** (2003) 383

N H Christ et al hep-lat/0608006

NRQCD another possibility consists in simulating one among the possible lattice discretizations of the NRQCD lagrangian at a given order in v^2 and α_s; for example

$$
\begin{aligned}
H &= -\frac{\Delta^2}{2m_b} + \delta H \\
\delta H &= -c_1 \frac{(\Delta^2)^2}{8m_b^3} + c_2 \frac{ig}{8m_b^2}(\vec{\Delta}\cdot\vec{E} - \vec{E}\cdot\vec{\Delta}) \\
&\quad - c_3 \frac{g}{8m_b^2}\vec{\sigma}\cdot(\vec{\Delta}\times\vec{\tilde{E}} - \vec{\tilde{E}}\times\vec{\Delta}) \\
&\quad - c_4 \frac{g}{8m_b^2}\vec{\sigma}\cdot\vec{\tilde{B}} + c_5 \frac{a^2\Delta^{(4)}}{24m_b} - c_6 \frac{a(\Delta^2)^2}{16m_b^2}
\end{aligned}
$$

A Gray et al Phys. Rev. D **72** (2005) 094507

the theory is non–renormalizable, can be matched to QCD only perturbatively, the continuum limit cannot be taken (heavy–light?)

the bag parameter is defined as the matrix element of the following operator

$$O_1 = \bar{\psi}^i \gamma_\mu (1-\gamma_5) q^i \bar{\psi}^j \gamma_\mu (1-\gamma_5) q^j = O_{VV+AA} - O_{VA+AV}$$

$$\langle \bar{K}^0 | \hat{O}_1 | K^0 \rangle = \langle \bar{K}^0 | \hat{O}_{VV+AA} | K^0 \rangle = \frac{8}{3} M_K^2 f_K^2 B_K(\mu)$$

Wilson fermions: $\hat{O}_{[VV+AA]} = Z_{11} \left[O_{[VV+AA]} + \Delta_{12} O_{[VV-AA]} + \Delta_{13} O_{[SS-PP]} + \Delta_{14} O_{[SS+PP]} + \Delta_{15} O_{[TT]} \right]$

M Guagnelli et al JHEP **0603** (2006) 088

ALPHA 06: P Dimopoulos et al. Nucl. Phys. B **749** (2006) 69

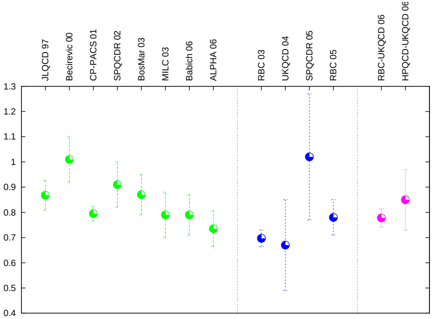
D Becirevic et al Phys. Lett. B **487** (2000) 74

$SPQ_{CD}R$ 02: D Becirevic et al. Nucl. Phys. Proc. Suppl. **119** (2003) 359

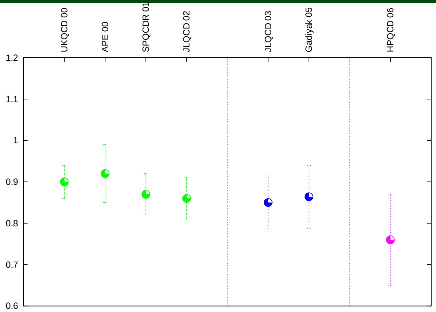
$SPQ_{CD}R$ 05: F Mescia et al. PoS **LAT2005** (2006) 365

the results do not show a significant dependence

- upon the number of dynamical flavors
- upon the renormalization procedure
- upon the finite size effects



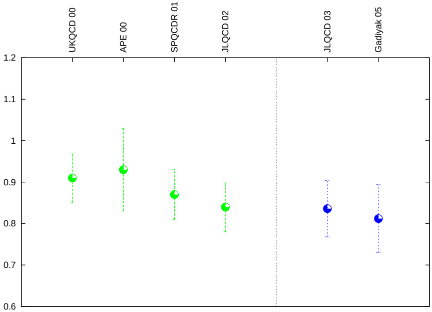
$B_{B_s}(m_b)$:



the observable does not seem to depend upon

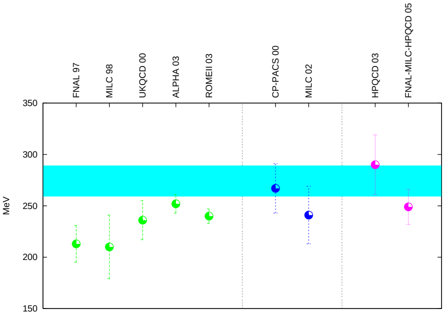
- the number of dynamical flavors
- the renormalization systematics
- the heavy quark "technology"

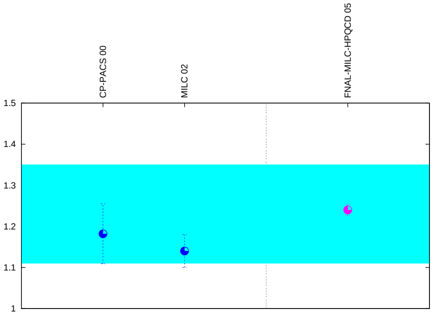
$B_B(m_b)$:



- there isn't a sizable dependence even on the quark mass
- all the "dependencies" are factorized in the vacuum saturation approximation: decay constants. . .

lattice calculations of $f_{D_q} \leq 2006$





CLEO: $f_{D_s} = 274(13)(7)$ MeV

BaBar: $f_{D_s} = 283(17)(7)(14)$ MeV

BaBar+CLEO: $\frac{f_{D_s}}{f_D} = 1.27(14)$

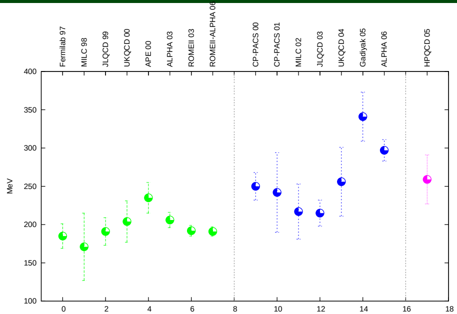
CLEO: $\frac{f_{D_s}}{f_D} = 1.23(11)(4)$

B. Aubert et al hep-ex/0607094
M. Artuso et al arXiv:0704.0629

M Artuso et al Phys. Rev. Lett. **95** (2005) 251801

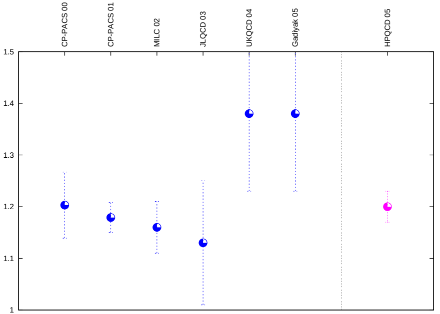
lattice calculations of $f_{B_q} \leq 2006$

f_{B_s}:



- the quenched results are very precise but significantly smaller than the unquenched ones
- this may signal a sizable dependence of this observable upon the number of dynamical flavors

f_{B_s}/f_B:



- the static numbers by Gadiyac et al 06, UKQCD 04 are identical and one sigma higher than the others

”un-rooted” unquenched algorithms

- in the last few years there has been a dramatic improvement in non-staggered unquenched algorithms

M. Lüscher Comput. Phys. Commun. **165** (2005) 199

C. Urbach et al Comput. Phys. Commun. **174**, 87 (2006)

M. A. Clark et al Phys. Rev. Lett. **98**, 051601 (2007)

- all these algorithms are based on the HMC and make use of multiple time step integrators

S. Duane et al Phys. Lett. B **195**, 216 (1987)

J. C. Sexton et al Nucl. Phys. B **380**, 665 (1992)

$$Z = \int dU\, e^{-S_G[U]} \det\left(D^\dagger[U]D[U]\right)$$

$$= \int dUdP\, d\phi^\dagger d\phi\, e^{\frac{P^2}{2} - S_G[U] - \int d^4x\, \phi^\dagger(x) D^\dagger[U]D[U]\phi(x)}$$

$$\frac{dP}{d\tau} = -F_G[U] - \sum_i \mathcal{F}_i[\mathcal{M}_i^{-1}]$$

$$D^\dagger[U]D[U] = \prod_i \mathcal{M}_i[U]$$

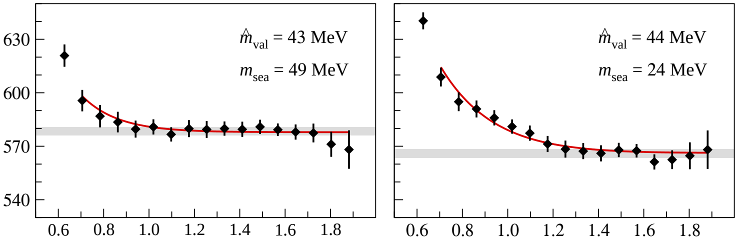
$$\frac{dU}{d\tau} = P\ U$$

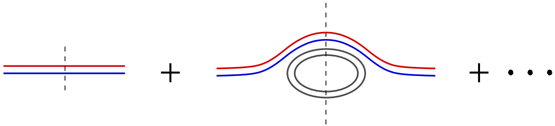
$$= \int dUdP \sum_i d\phi_i^\dagger d\phi_i\, e^{\frac{P^2}{2} - S_G[U] - \sum_i \int d^4x\, \phi^\dagger(x)\mathcal{M}_i[U]\phi(x)}$$

light Wilson(-like) quarks: unquenching effects

- large scale projects of unquenched simulations with light Wilson quarks have been started
- genuine unquenching effects show up by studying the large time behavior of two-point correlation functions

L. Del Debbio, L. Giusti, M. Luscher, R. Petronzio and N. T. JHEP **0702** (2007) 056



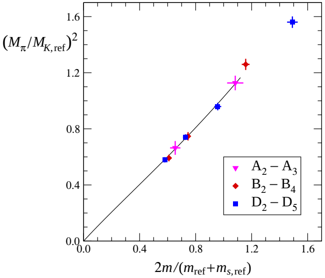


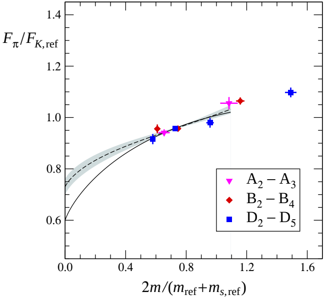
$$M_{eff}(t) = M_0 + c\, e^{-2M_\pi t} + \ldots$$

$N_f = 2$ **Wilson**: $a \simeq 0.07$ fm, $L \simeq 1.7$ fm — L. Del Debbio, L. Giusti, M. Luscher, R. Petronzio and N.T. JHEP **0702** (2007) 056

$N_f = 2$ **Wilson**: $a \simeq 0.05$ fm, $L \simeq 1.7$ fm — L. Del Debbio, L. Giusti, M. Luscher, R. Petronzio and N.T. JHEP **0702** (2007) 082

$N_f = 2$ **SW-Wilson**: $a \simeq 0.08$ fm, $L \simeq 1.9$ fm





$$M_\pi^2 = M^2 + \frac{M^4}{32\pi^2 F^2} \ln\left(\frac{M^2}{\Lambda_3^2}\right) + \ldots \qquad M^2 = 2Bm$$

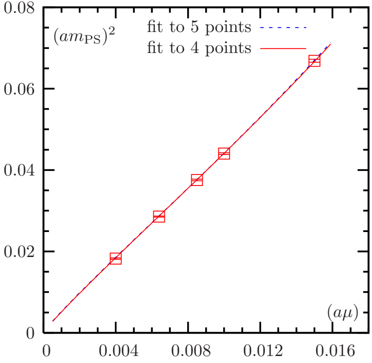
$$F_\pi = F - \frac{M^2}{16\pi^2 F} \ln\left(\frac{M^2}{\Lambda_4^2}\right) + \ldots$$

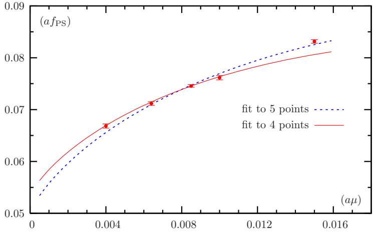
$$\bar{l}_3 = \ln\left(\frac{M^2}{\Lambda_3^2}\right)_{M=139.6\ \mathrm{MeV}} = 3.0(5)(1)$$

light Wilson(-like) quarks: comparison with χPT

$N_f = 2$ **TM-Wilson**: $a \simeq 0.09$ fm, $L \simeq 2$ fm

Ph. Boucaud et al [ETM Collaboration] arXiv:hep-lat/0701012



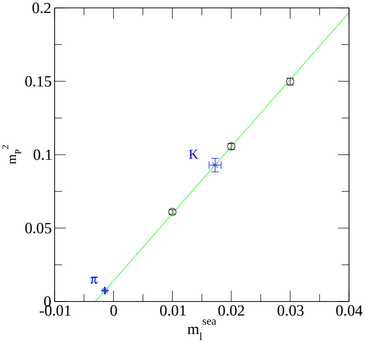


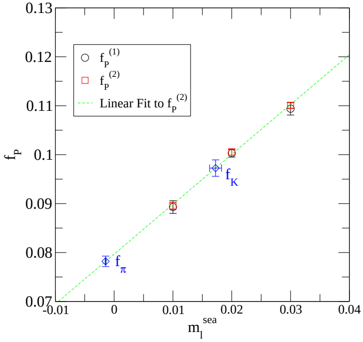
$\bar{l}_3 = 3.65(12)$

$\bar{l}_4 = 4.52(6)$

$N_f = 3$ **domain-wall**: $a \simeq 0.12$ fm, $L \simeq 2$ fm, $L_s = 16$

C. Allton et al [RBC and UKQCD Collaborations] arXiv:hep-lat/0701013





SSM the Step Scaling Method has been introduced in order to deal with two–scale problems on the lattice

M Guagnelli et al Phys. Lett. B **546** (2002) 237

on a very general ground, it is based on a simple identity

$$\mathcal{O}(E_h, E_l, \infty) = \quad \mathcal{O}(E_h, E_l, L_0) \underbrace{\frac{\mathcal{O}(E_h, E_l, 2L_0)}{\mathcal{O}(E_h, E_l, L_0)}}_{\sigma(E_h, E_l, L_0)} \underbrace{\frac{\mathcal{O}(E_h, E_l, 4L_0)}{\mathcal{O}(E_h, E_l, 2L_0)}}_{\sigma(E_h, E_l, 2L_0)} \quad \dots$$

- and on a reasonable "phenomenological assumption", i.e finite volume effects are due to the low energy scale

$$\sigma(E_h, E_l, L) \simeq \sigma(E_l, L) \qquad \frac{\partial}{\partial(\frac{1}{E_h})}\sigma(E_h, E_l, L) \simeq 0 \quad E_h \gg E_l$$

- so, provided that $E_h \gg 4E_l$, one has

$$\mathcal{O}(E_h, E_l, \infty) \simeq \quad \mathcal{O}(E_h, E_l, L_0) \quad \sigma(E_h/2, E_l, L_0) \quad \sigma(E_h/4, E_l, 2L_0) \quad \dots$$

the step scaling method, heavy-light mesons

- in the case of heavy-light systems the argument can be made rigorous by using HQET predictions
- let us take f_B as an example

G M de Divitiis et al Nucl. Phys. B **672** (2003) 372

D Guazzini et al PoS **LAT2006** (2006) 084

$$\sigma(m_h, m_d, L) = \frac{f_B^0(m_d, 2L)\left(1 + \frac{f_B^1(m_d, 2L)}{m_h} + \dots\right)}{f_B^0(m_d, L)\left(1 + \frac{f_B^1(m_d, L)}{m_h} + \dots\right)} = \sigma^{\text{stat}}(m_d, L)\left(1 + \frac{f_B^1(m_d, 2L) - f_B^1(m_d, L)}{m_h}\right)$$

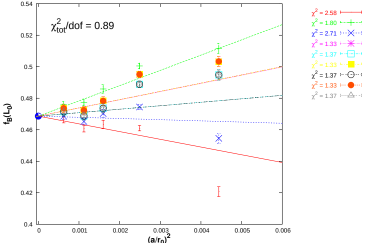
$$= \sigma^{\text{stat}}(m_d, L)\left(1 + \frac{f_B^{1,1}(m_d)}{m_h L}\right)$$

- even better in the case of the meson masses (b–quark mass calculation)

M Guagnelli et al Nucl. Phys. B **675** (2003) 309

$$\sigma(m_h, m_d, L) = \frac{M(m_h, m_d, 2L)}{M(m_h, m_d, L)} = \frac{m_h + \bar{\Lambda}(m_d, 2L) + \dots}{m_h + \bar{\Lambda}(m_d, L) + \dots} = 1 + \frac{\bar{\Lambda}(m_d, 2L) - \bar{\Lambda}(m_d, L)}{m_h} + \dots$$

the step scaling method, does it works in practice?



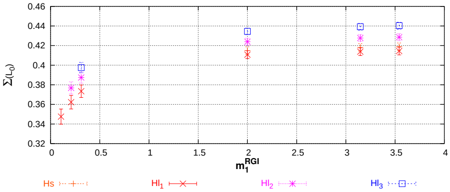
UNC: The calculation is quenched.

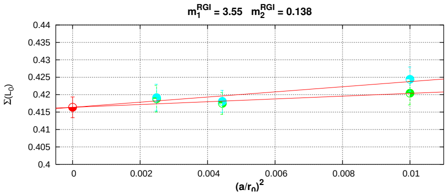
EFT: Fully non perturbative through SSM.

χE: The strange quark is under control.

***a*E:** 4 lattice spacings.

***L*E:** Naturally estimated.





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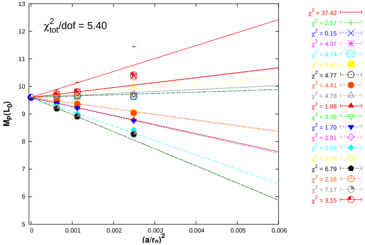


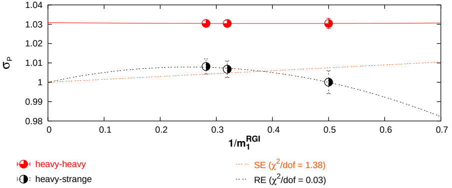
UNC: The calculation is quenched.

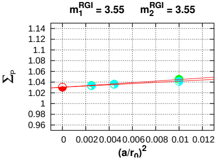
EFT: Fully non perturbative through SSM.

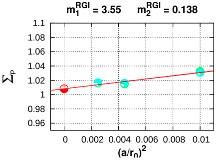
χE: The strange quark is under control.

***a*E:** 3 lattice spacings.

***L*E:** Naturally estimated.







the differential decay rate for the process $B \longrightarrow D\ell\nu$ is given by

$$\frac{d\Gamma(B \longrightarrow D\ell\nu)}{d\omega} = (\text{known factors})|V_{cb}|^2(\omega^2 - 1)^{\frac{3}{2}} F_D^2(\omega)$$

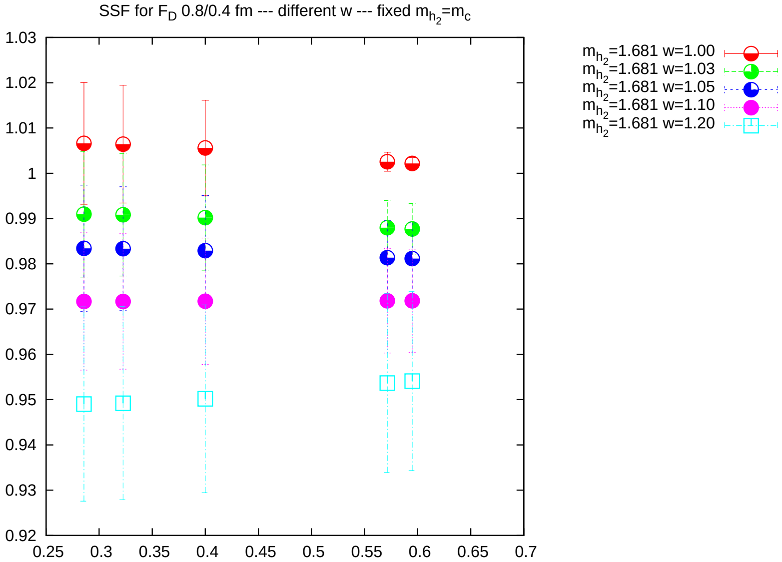
$$\omega = \frac{p_B \cdot p_D}{M_B M_D} = v_B \cdot v_D$$

we have applied the step scaling method to the calculation of $F_D(\omega)$

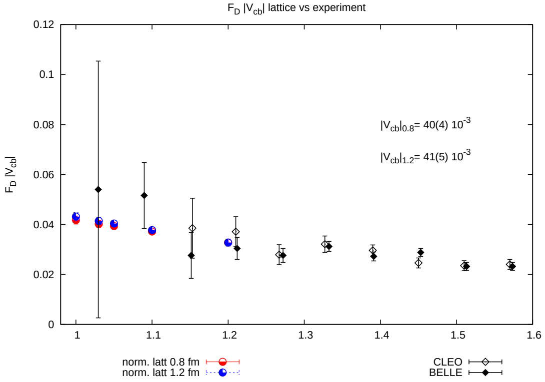
G M de Divitiis,E Molinaro,R Petronzio,N T

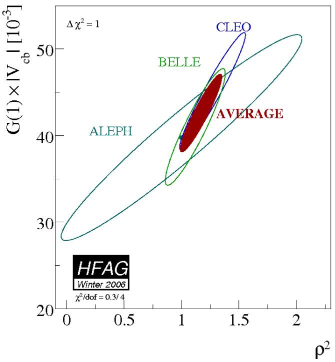
$$F_D(m_b, \infty\,|m_c, m_l, \omega) = F_D(m_b, 0.4\text{ fm}) \underbrace{\frac{F_D(m_b, 0.8\text{ fm})}{F_D(m_b, 0.4\text{ fm})}}_{\sigma_{F_D}(m_b, 0.4\text{ fm})} \cdots$$

the step scaling functions are extremely flat



experimental situation vs lattice at $\omega > 1$





how do we compare with other determinations?

FERMILAB quenched: $F_D(\omega = 1) = 1.058(20)$ unquenched: $F_D(\omega = 1) = 1.074(24)$

Tor Vergata quenched: $F_D(\omega = 1) = 1.041(40)$ $F_D(\omega = 1.2) = 0.799(33)$

outlooks

- we (at least part of us!) got the message: complete control over the systematics!
- presently we can get a “good idea” about the systematics by looking at the ”dispersion” of the different calculations
- tomorrow:
  - ”rooted” results will be checked against ”un-rooted” results (light dynamical Wilson quarks, dynamical GW quarks, etc.)
  - (hopefully) un-rooted unquenched results on heavy-light observables will be obtained by using finite volume approaches (SSM and/or HQET non-perturbatively renormalized)
  - will the quoted errors decrease?