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# Violazione del sapore leptonico in modelli di grande unificazione

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*Main motivations for studying LFV processes:*

•Neutrino oscillations shows that lepton family numbers are not conserved

•Consequence (in general) of new physics at the TeV scale,

while in the SM is higly suppressed

•Model dependent and so complementary to direct searches of new physics

*Experiments:*

Running: **BaBar**, **Belle**

**MEG** (first data in 2007)

Future: **SuperKEKB** (2011)

**PRISM/PRIME** (next decade)

**Super Flavour factory** (?)

*Main motivations for studying LFV processes:*

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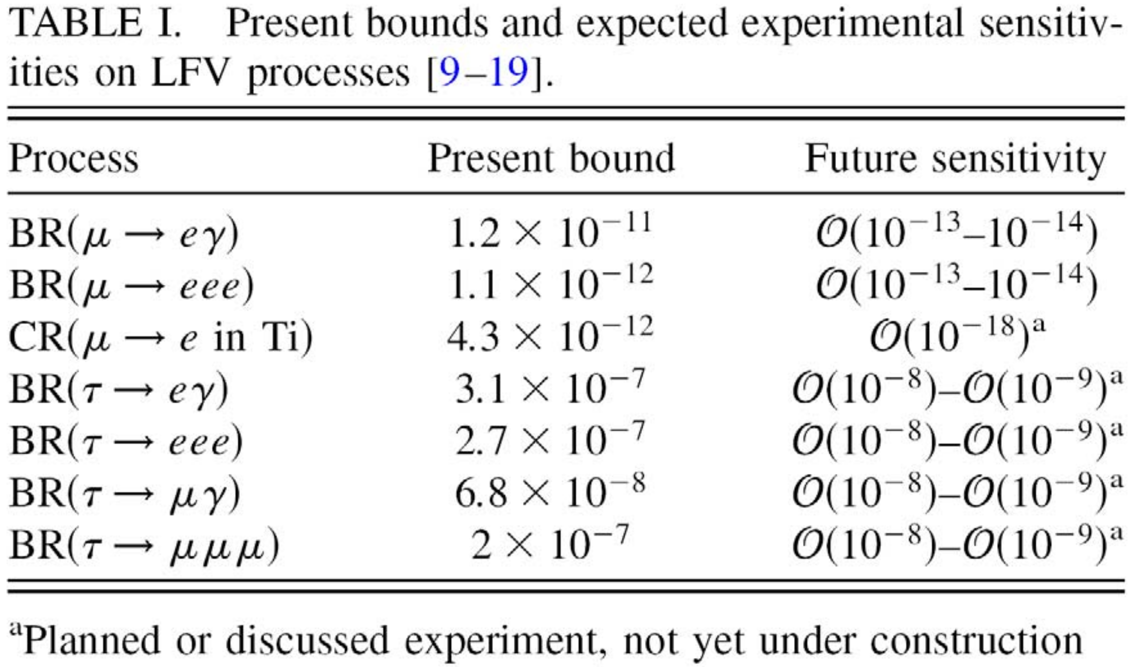
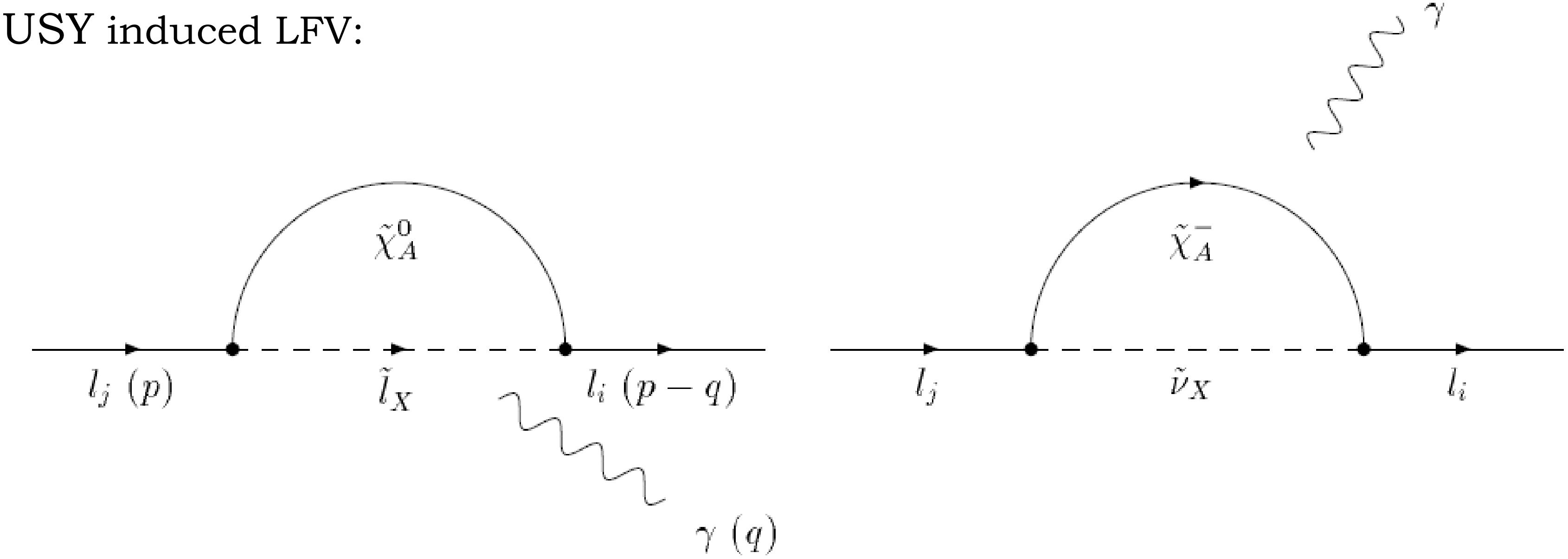


TABLE I. Present bounds and expected experimental sensitivities on LFV processes [9–19].

| Process | Present bound | Future sensitivity |
|---|---|---|
| $\mathrm{BR}(\mu \to e\gamma)$ | $1.2 \times 10^{-11}$ | $\mathcal{O}(10^{-13}\text{–}10^{-14})$ |
| $\mathrm{BR}(\mu \to eee)$ | $1.1 \times 10^{-12}$ | $\mathcal{O}(10^{-13}\text{–}10^{-14})$ |
| $\mathrm{CR}(\mu \to e \text{ in Ti})$ | $4.3 \times 10^{-12}$ | $\mathcal{O}(10^{-18})^{\mathrm{a}}$ |
| $\mathrm{BR}(\tau \to e\gamma)$ | $3.1 \times 10^{-7}$ | $\mathcal{O}(10^{-8})\text{–}\mathcal{O}(10^{-9})^{\mathrm{a}}$ |
| $\mathrm{BR}(\tau \to eee)$ | $2.7 \times 10^{-7}$ | $\mathcal{O}(10^{-8})\text{–}\mathcal{O}(10^{-9})^{\mathrm{a}}$ |
| $\mathrm{BR}(\tau \to \mu\gamma)$ | $6.8 \times 10^{-8}$ | $\mathcal{O}(10^{-8})\text{–}\mathcal{O}(10^{-9})^{\mathrm{a}}$ |
| $\mathrm{BR}(\tau \to \mu\mu\mu)$ | $2 \times 10^{-7}$ | $\mathcal{O}(10^{-8})\text{–}\mathcal{O}(10^{-9})^{\mathrm{a}}$ |

[a] Planned or discussed experiment, not yet under construction

# SUSY induced LFV:



$$\mathcal{M}^2_{\tilde{l}} = \begin{pmatrix} (\mathbf{m^2_{\tilde{l}}})_{\mathbf{ij}} + (m^2_l)_{ij} + \mathcal{O}(g^2)\delta_{ij} & (\mathbf{A_l})_{\mathbf{ji}} v_d - (m_l)_{ji}\mu\tan\beta \\ (\mathbf{A_l})_{\mathbf{ij}} v_d - (m_l)_{ij}\mu\tan\beta & (\mathbf{m^2_{\tilde{e}}})_{\mathbf{ij}} + (m^2_l)_{ij} + \mathcal{O}(g^2)\delta_{ij} \end{pmatrix}$$

**GUT** effect (e.g. *SU(5)* ) if $M_X > M_{GUT}$

$$(\Delta_{RR})_{i\neq j} = -3 \cdot \frac{3m_0^2 + a_0^2}{16\pi^2} Y_t^2\, V_{i3} V_{j3} \ln\left(\frac{M_X^2}{M_{GUT}^2}\right)$$

**See-saw**:

$$m_\nu = -Y_\nu \hat{M}_R^{-1} Y_\nu^T \langle H_u \rangle^2$$

$$(\Delta_{LL})_{i\neq j} = -\frac{3m_0^2 + A_0^2}{16\pi^2} Y_{\nu\ i3} Y_{\nu\ j3} \ln\left(\frac{M_X^2}{M_{R_3}^2}\right)$$

The most general renormalizable *SO(10)* superpotential, relevant to fermion masses:

$$W_{SO(10)} = Y_{ij}^{10} 16_i 16_j 10 + Y_{ij}^{126} 16_i 16_j 126 + Y_{ij}^{120} 16_i 16_j 120$$

$$\Rightarrow \begin{cases} m_D^\nu = M_{10} - 3M_{126} + M_{120} \\ m_u = M_{10} + M_{126} + M_{120} \end{cases}$$

*At least one of the eigenvalues of the neutrino Yukawa matrix in* $Y_\nu = m_D^\nu / v_u$ *has to be as large as the top Yukawa.*

And what about the *mixing angles*? We consider two benchmark cases:

“Minimal” mixing (CKM):

$$Y^\nu = Y^u \Rightarrow Y^\nu = V_{\rm CKM}^T Y^u_{diag} V_{\rm CKM}$$

“Maximal” mixing (PMNS):

$$Y^\nu = U_{\rm PMNS} Y^u_{diag}$$

CKM case:

$$W_{SO(10)} = (Y_u)_{ij}\mathbf{16}_i\mathbf{16}_j\mathbf{10}_u + (Y_d)_{ii}\mathbf{16}_i\mathbf{16}_i\mathbf{10}_d + (Y_R)_{ij}\mathbf{16}_i\mathbf{16}_j\mathbf{126}$$

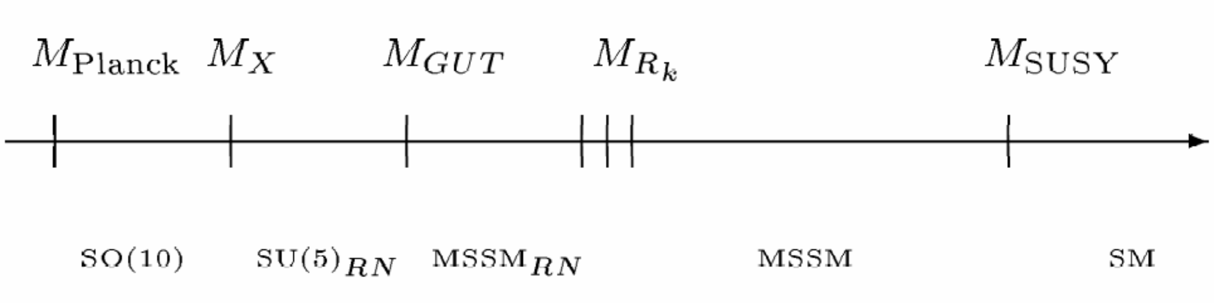
PMNS case:

$$W_{SO(10)} = (Y_u)_{ij}\mathbf{16}_i\mathbf{16}_j\mathbf{10}_u + (Y_d)_{ii}\mathbf{16}_i\mathbf{16}_i\frac{\langle\mathbf{45}\rangle}{M_{\rm Planck}}\mathbf{10}_d + (Y_R)_{ij}\mathbf{16}_i\mathbf{16}_j\mathbf{126}$$

Just frameworks to compute the RG effects in the soft masses sector,

not complete fermion masses & mixings models!

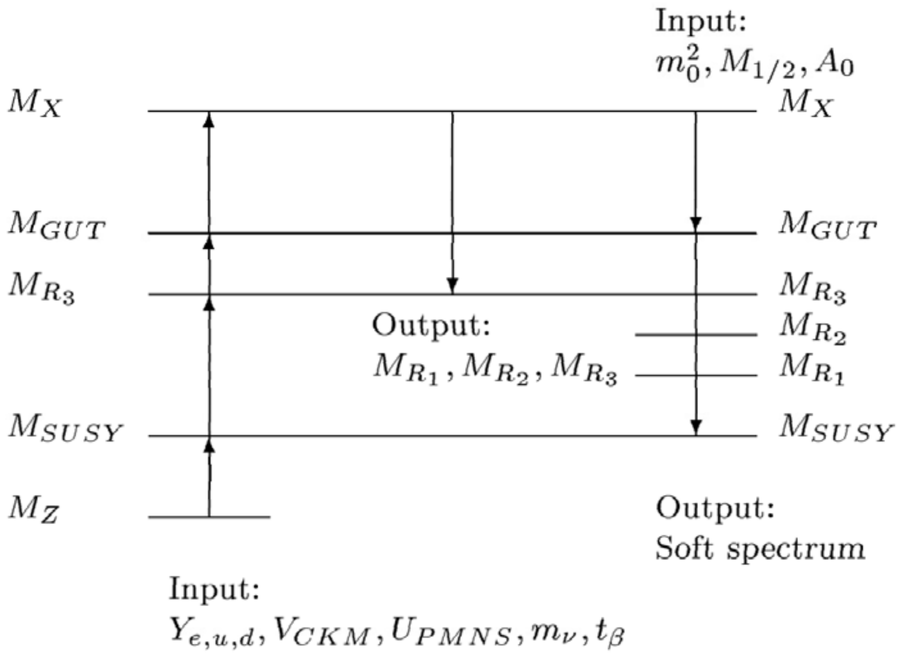
# Scheme of the RG running and of the energy scales involved



How does the routine work:

- Running of the low energy fermion param.
- Definition of $Y_\nu$
- $M_R$ from see-saw
- RG evolution of the soft parameters
- SUSY spectrum
- Th. & Exp. checks
- Computation of the LFV processes rates through the exact SUSY masses and mixings

Sketch of the running routine



# Scanning the mSUGRA parameter space:

$0 < m_0 < 5000$ GeV

$0 < M_{1/2} < 1500$ GeV

$-3m_0 < A_0 < +3m_0$

$\tan\beta = 10, 40$

*$\mu$ and $B\mu$ fixed by EWB*

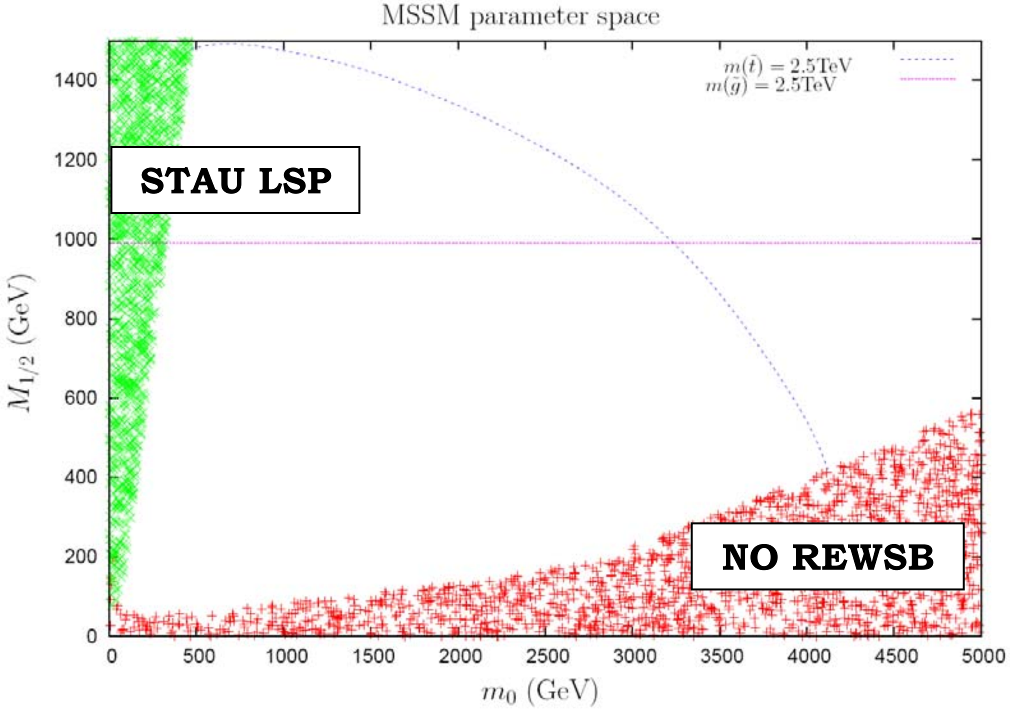
*Theoretical constraints:*

- REWSB
- No tachyonic particles
- Neutral LSP

*Experimental constraints:*

- LEP limit on Higgs mass
- Limits on SUSY particles

→ The region where at least one squark has mass below 2.5 TeV is ‘our’ LHC accessible region.



**MSSM**

$\tan\beta = 30;\ A_0 = 0;\ m_t = 173$ GeV

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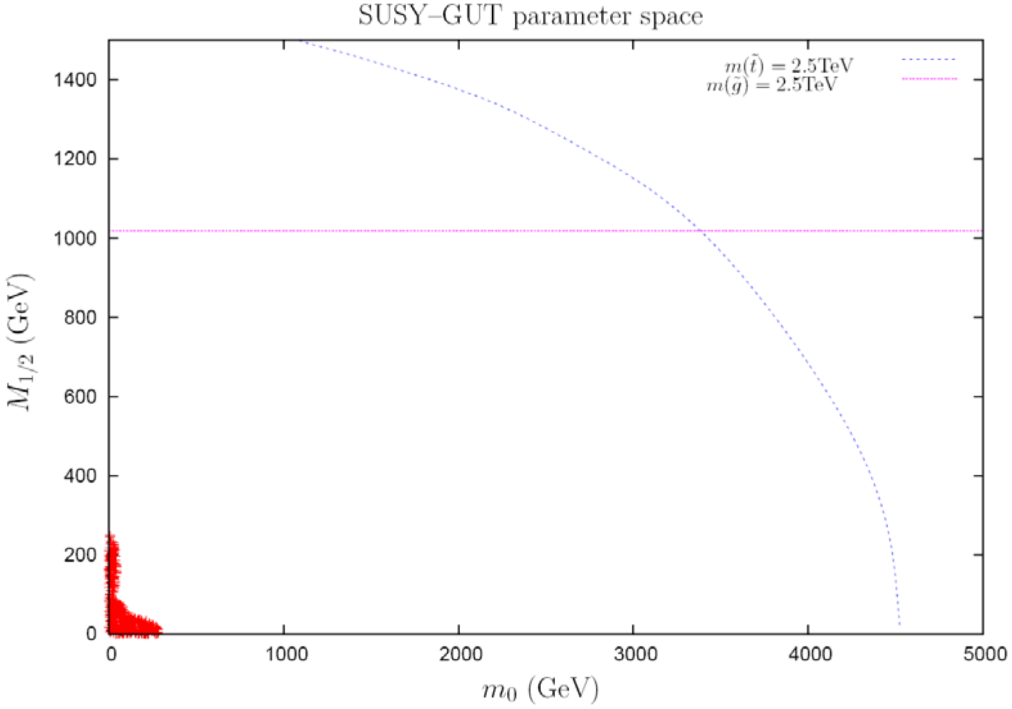

**SU(5)$_{\mathbf{RN}}$**

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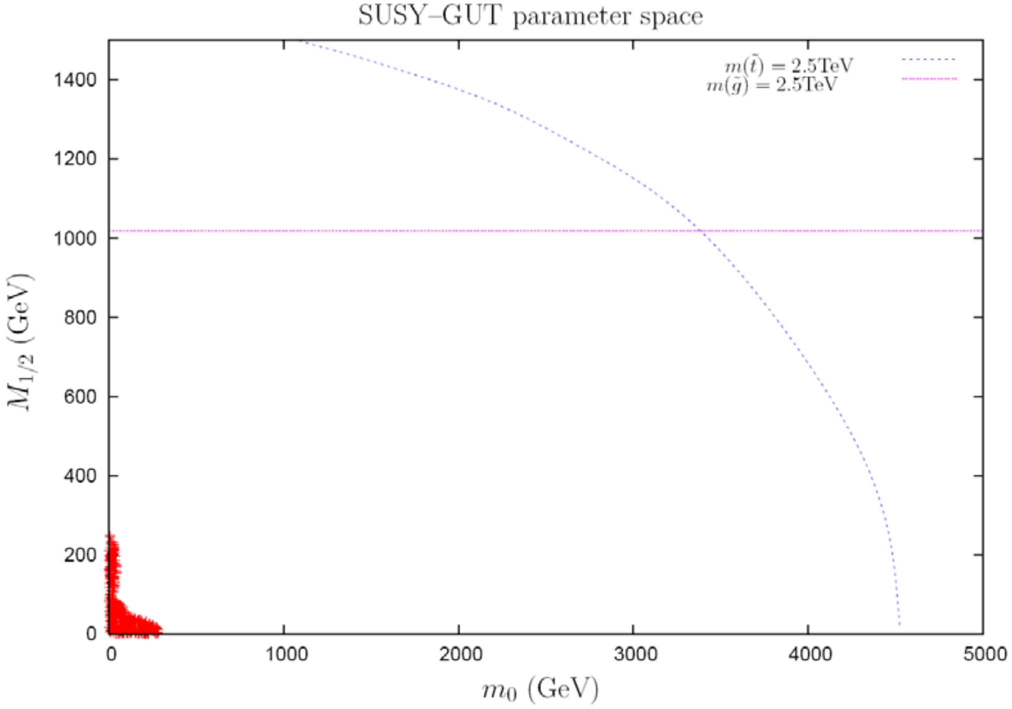

**REWSB:**

$$|\mu|^2 = \frac{m^2_{H_d} - m^2_{H_u}\tan^2\beta}{\tan^2\beta - 1} - \frac{1}{2}m^2_Z$$

$$\sin 2\beta = \frac{2B\mu}{m^2_{H_u} + m^2_{H_d} + 2\mu^2},$$

# Scanning the mSUGRA parameter space:

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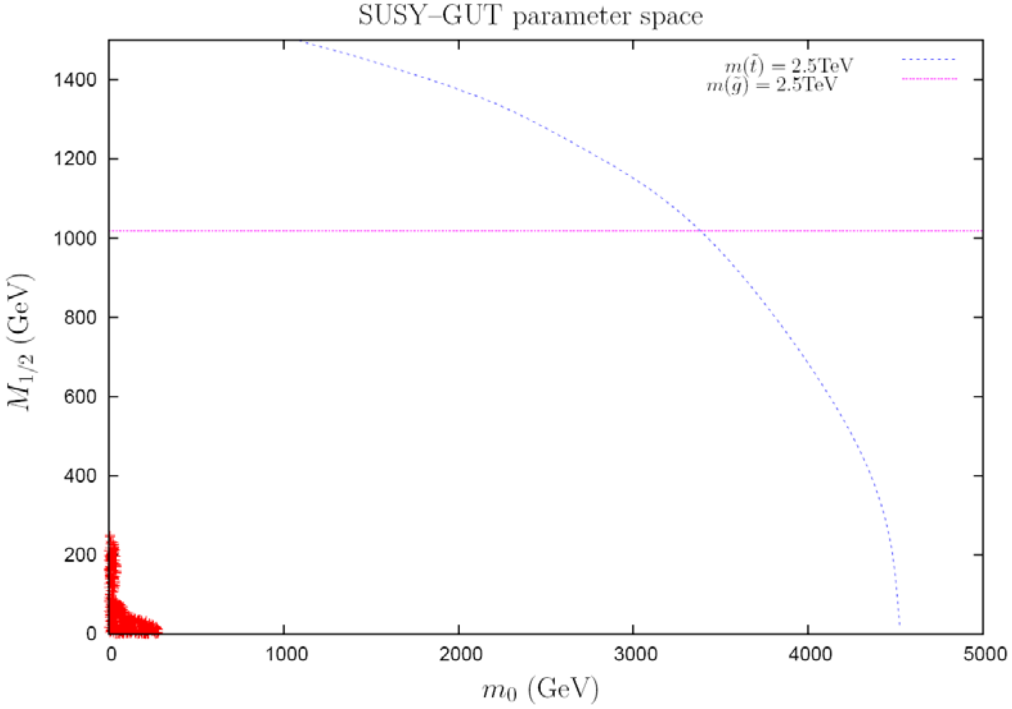

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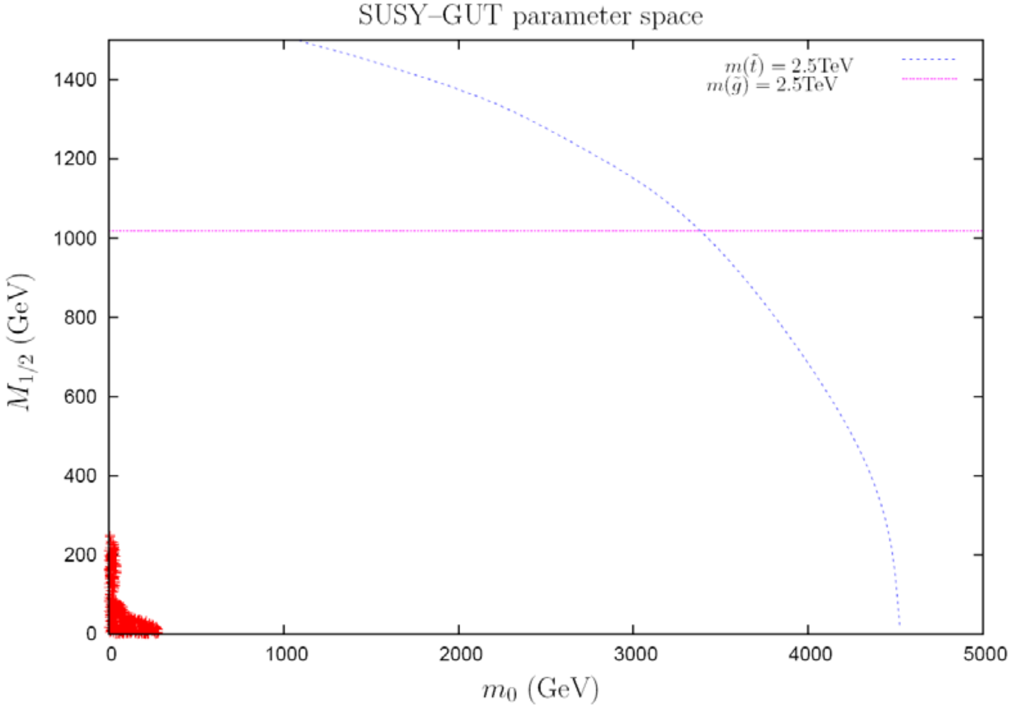


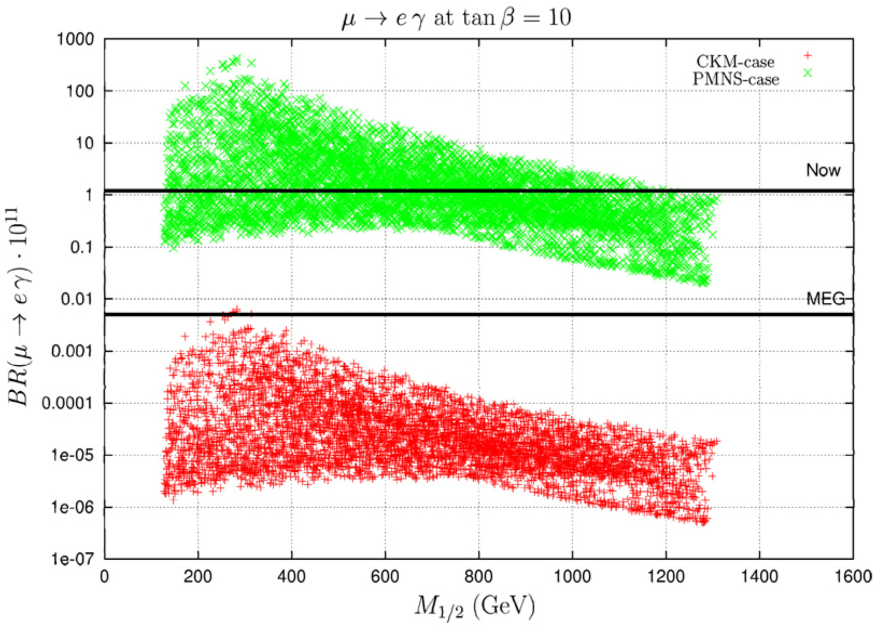
**LSP:**

$$m^2_{\tilde{\tau}_R}(M_{GUT}) = \frac{96}{80\pi^2} M^2_{1/2} \ln\left(\frac{M_X}{M_{GUT}}\right) \approx 0.4 M^2_{1/2}$$

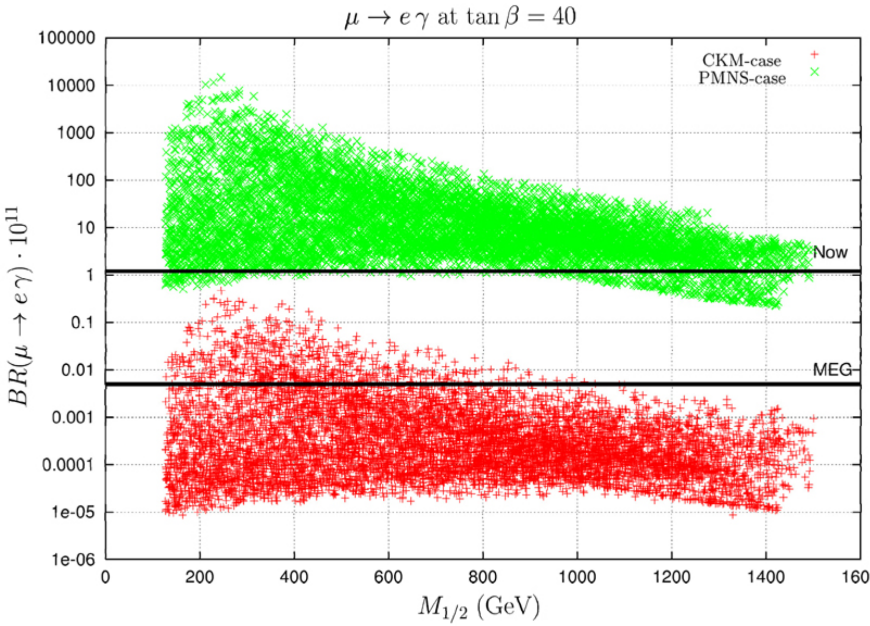
(right stau mass for $m_0 = 0$ )

→ The region where at least one squark has mass below 2.5 TeV is 'our' LHC accessible region.

# $\mu \to e\,\gamma$ and **MEG** sensitivity reach



$\mu \to e\,\gamma$ at $\tan\beta = 10$



$\mu \to e\,\gamma$ at $\tan\beta = 40$

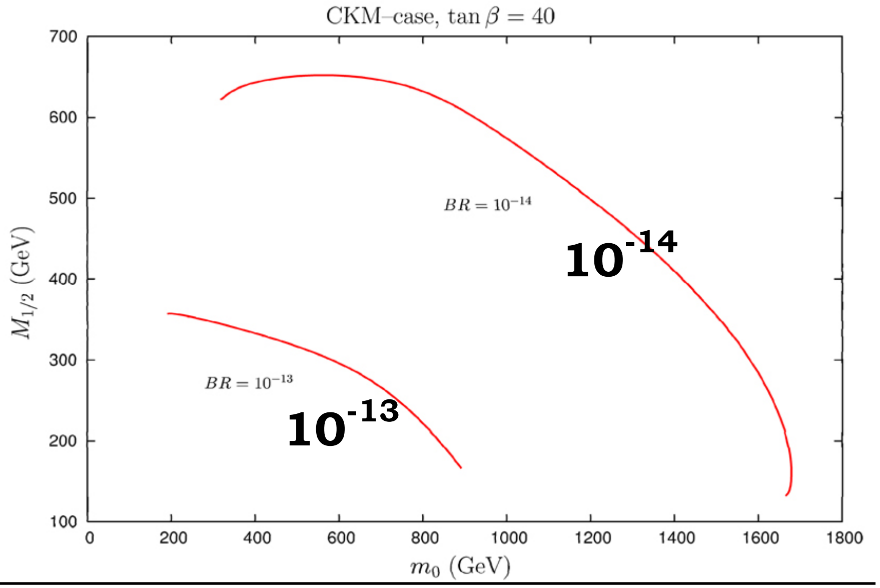
- Maximal mixing (PMNS), high tan$\beta$ case, already ruled out in the LHC accessible region.

**MEG** will test it well beyond the LHC.

- Minmal case (CKM) presently unconstrained.

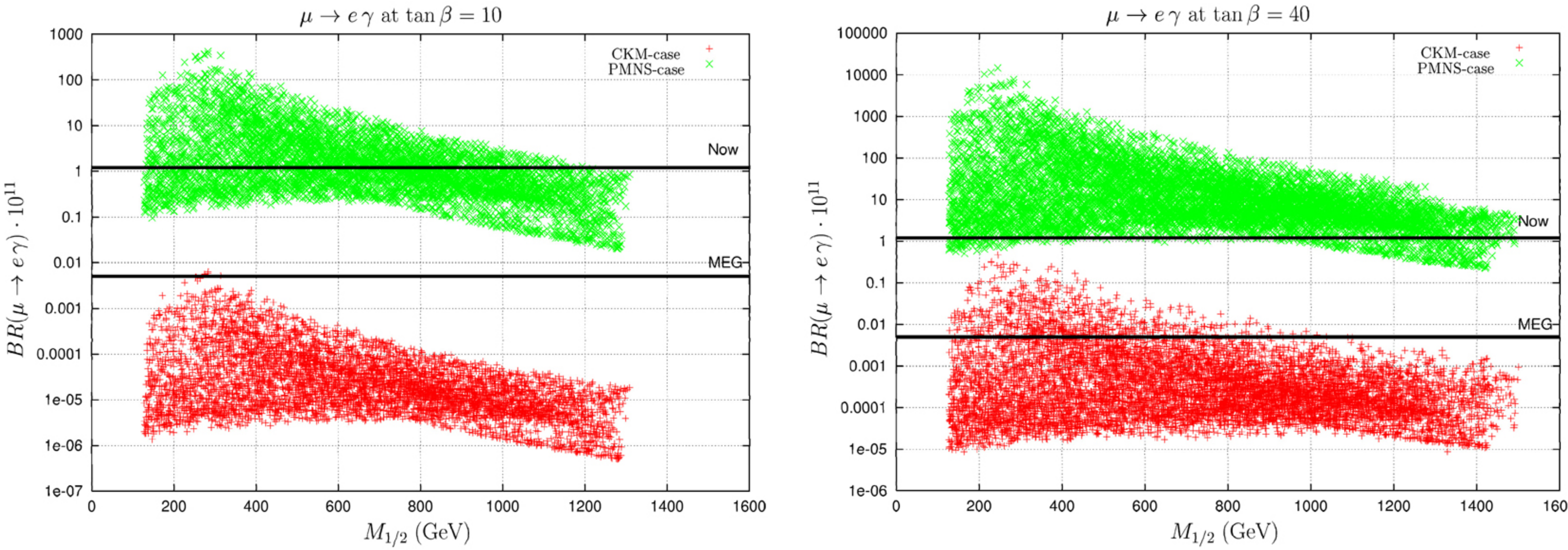
**MEG** will test, for high values of tan$\beta$, the region $(m_0, m_{\tilde{g}}) \lesssim 1$ TeV

But in the PMNS case, the rate depends on $U_{e3}\ldots$



CKM–case, $\tan\beta = 40$

# $\mu \to e\,\gamma$ and **MEG** sensitivity reach



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But in the PMNS case, the rate depends on $U_{e3}\ldots$

$$U_{\rm PMNS} = \begin{pmatrix} U_{e1} & U_{e2} & \bigcirc\!\!\!\!\!\!U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix}$$

CHOOZ limit: $|U_{e3}| \lesssim 0.14$

These scatter plots: $|U_{e3}| = 0.07$

# $U_{e3}$ and $\mathrm{BR}(\mu \to e + \gamma)$

$$(\Delta_{LL})_{i\neq j} = -\frac{3m_0^2 + A_0^2}{16\pi^2} \sum_k Y_{\nu\, ik} Y^\dagger_{\nu\, kj} \ln\left(\frac{M_X^2}{M_{R_k}^2}\right)$$

**PMNS case:**

$$Y_\nu = U_{\mathrm{PMNS}} Y_u^{diag}$$

$$(\Delta_{LL})_{12} = -\frac{3m_0^2 + A_0^2}{16\pi^2} \left( y_t^2\, \enclose{circle}{U_{e3}} U^*_{\mu 3} \ln\left(\frac{M_X^2}{M_{R_3}^2}\right) + y_c^2\, U_{e2} U^*_{\mu 2} \ln\left(\frac{M_X^2}{M_{R_2}^2}\right) + y_u^2\, U_{e1} U^*_{\mu 1} \ln\left(\frac{M_X^2}{M_{R_1}^2}\right) \right)$$

$$|U_{e3}^{lim}| \approx \frac{y_c^2}{y_t^2} \frac{|U_{e2}| \cdot |U_{\mu 2}|}{|U_{\mu 3}|} \frac{\ln M_X - \ln M_{R_2}}{\ln M_X - \ln M_{R_3}} \sim \mathcal{O}(10^{-5}),$$

$U_{e3} \approx 0$

$$\Rightarrow \quad \frac{Y_c^2 U_{\mu 2} U_{e2} \ln(M_X/M_{R_2})}{Y_t^2 V_{\mathrm{td}} V_{\mathrm{ts}} \ln(M_X/M_{R_3})} \sim \mathcal{O}(10^{-2})$$

"CKM" case

# $U_{e3}$ and $\mathrm{BR}(\mu \to e+\gamma)$

$$(\Delta_{LL})_{i\neq j} = -\frac{3m_0^2+A_0^2}{16\pi^2}\sum_k Y_{\nu\,ik}Y^\dagger_{\nu\,kj}\ln\left(\frac{M_X^2}{M_{R_k}^2}\right)$$

Bottom-up parametrization:

$$Y_\nu = \frac{1}{\langle H_u\rangle}U_{\mathrm{PMNS}}\mathcal{D}_\nu \cancel{R}\mathcal{D}_N$$

$$(\Delta_{LL})_{12} = -\frac{3m_0^2+A_0^2}{16\pi^2}\left(y_t^2\,\enclose{circle}{U_{e3}}U^*_{\mu3}\ln\left(\frac{M_X^2}{M_{R_3}^2}\right)+y_c^2\,U_{e2}U^*_{\mu2}\ln\left(\frac{M_X^2}{M_{R_2}^2}\right)+y_u^2\,U_{e1}U^*_{\mu1}\ln\left(\frac{M_X^2}{M_{R_1}^2}\right)\right)$$

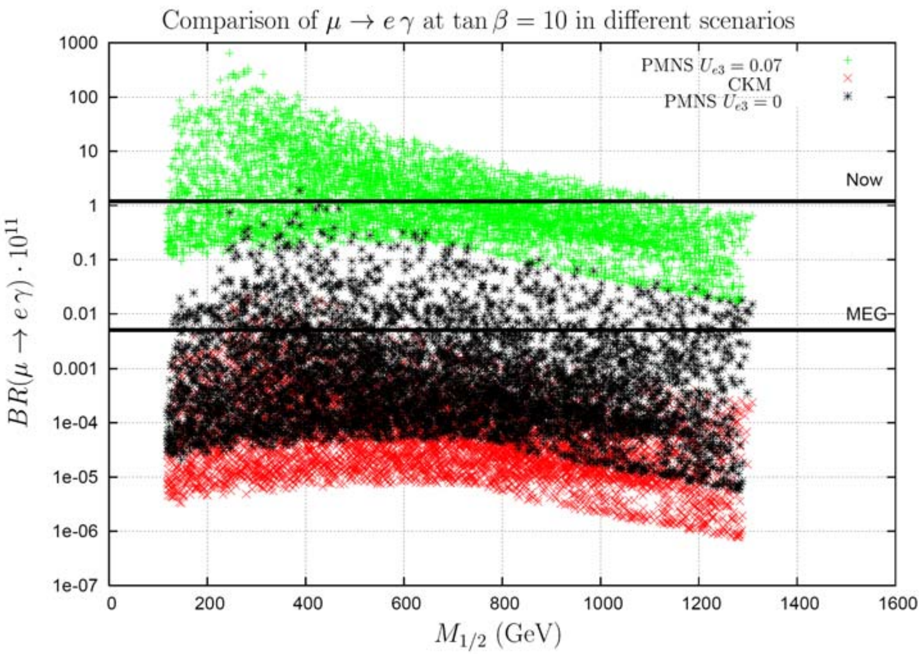
$$|U_{e3}^{lim}| \approx \frac{y_c^2}{y_t^2}\,\frac{|U_{e2}|\cdot|U_{\mu2}|}{|U_{\mu3}|}\,\frac{\ln M_X-\ln M_{R_2}}{\ln M_X-\ln M_{R_3}} \sim \mathcal{O}(10^{-5}),$$

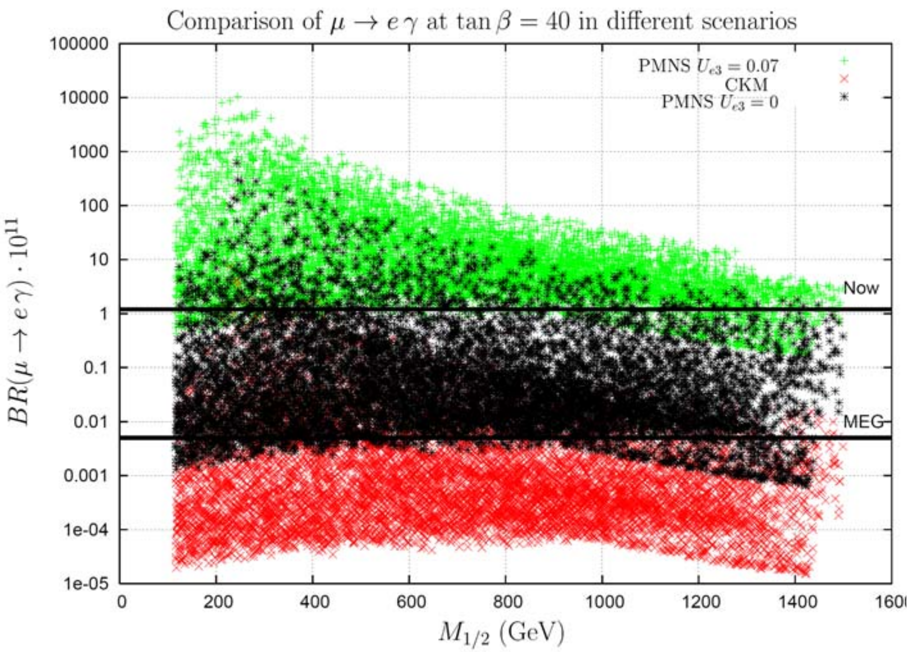
$U_{e3}\approx 0$

$$\Rightarrow \quad \frac{Y_c^2U_{\mu2}U_{e2}\ln(M_X/M_{R_2})}{Y_t^2V_{\mathrm{td}}V_{\mathrm{ts}}\ln(M_X/M_{R_3})} \sim \mathcal{O}(10^{-2})$$

“CKM” case

# $\mu \to e\gamma$ in the $U_{e3} = 0$ PMNS case





- The PMNS $U_{e3} = 0$ scenario is currently better costrained by $\tau \to \mu\gamma$
- For high values of tan$\beta$, **MEG** will probe almost all the LHC accessible parameter space
- In the case of small tan$\beta$, MEG will test up to $(m_0, m_{\tilde{g}}) \lesssim 900$ GeV

The observed enhancement is due to the interplay of different effects:

- The running of $U_{e3}$ from low energy up to the high scale where the PMNS condition is imposed
- The dominance in some regions of the paramater space of *SU(5)* generated contributions

# $\tau \to \mu\gamma$ and the **Super B** (and **Flavour**) factories

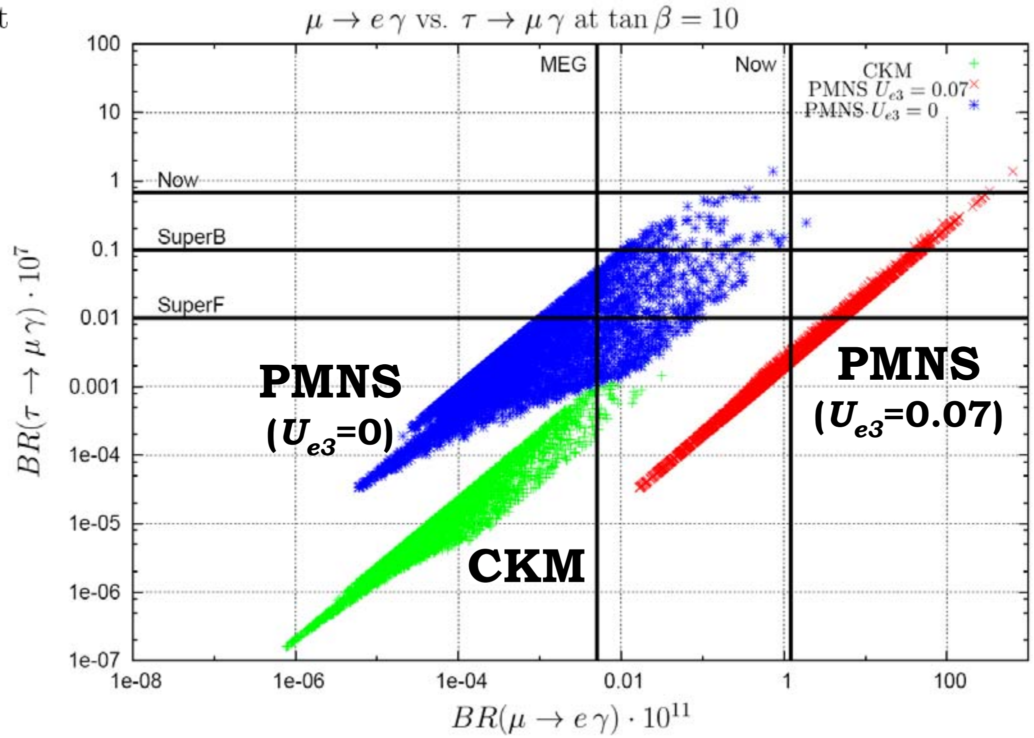
TABLE IX: Reach in $(m_0, m_{\tilde{g}})$ of the present and planned experiment from their $\tau \to \mu\gamma$ sensitivity.

| | PMNS | | CKM | |
|---|---|---|---|---|
| Exp. | $t_\beta = 40$ | $t_\beta = 10$ | $t_\beta = 40$ | $t_\beta = 10$ |
| BaBar, Belle | 1.2 TeV | no | no | no |
| SuperKEKB | 2 TeV | 0.9 TeV | no | no |
| Super Flavour [a] | 2.8 TeV | 1.5 TeV | 0.9 TeV | no |

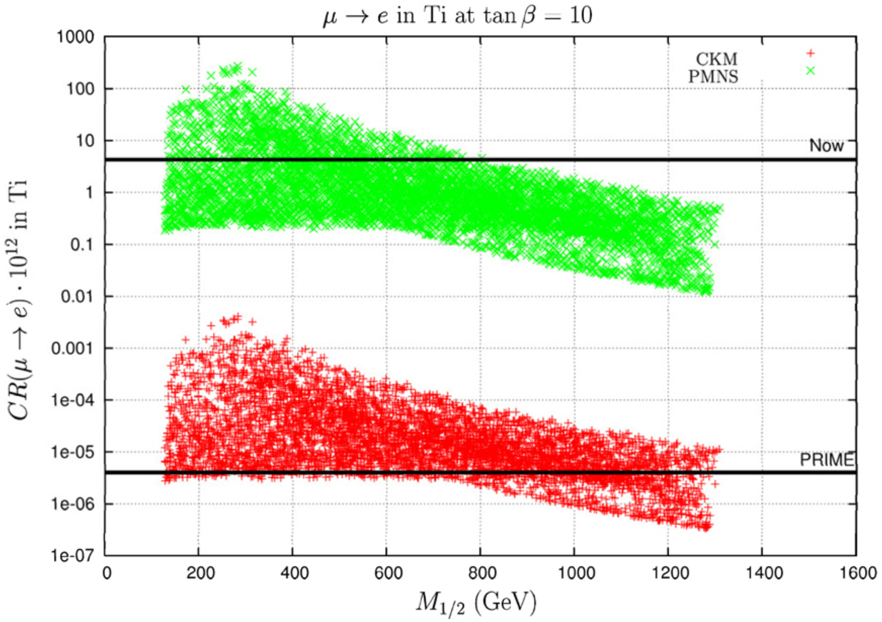
[a] Post–LHC era proposed/discussed experiment

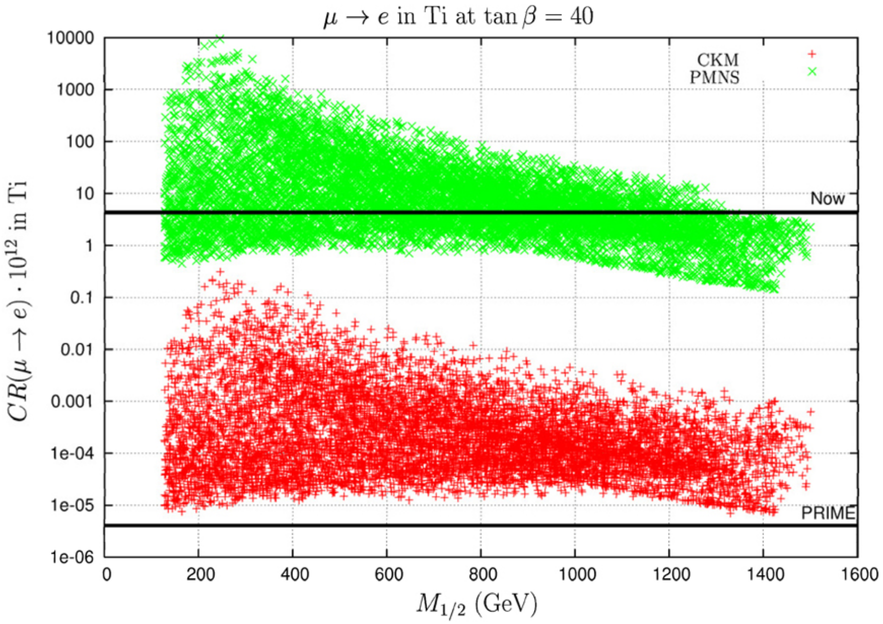
$\tau \to \mu\gamma$ vs. $\mu \to e\gamma$

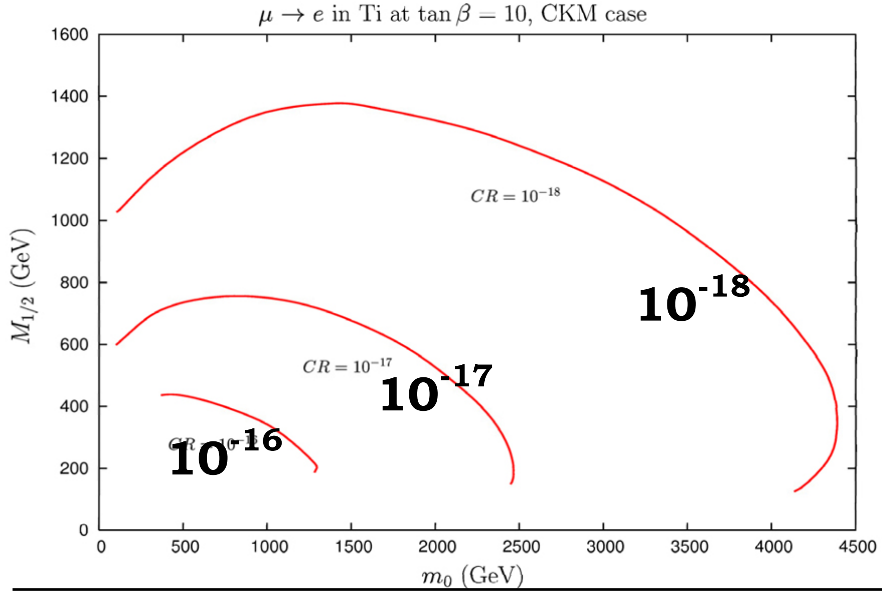
$\tan\beta = 10$

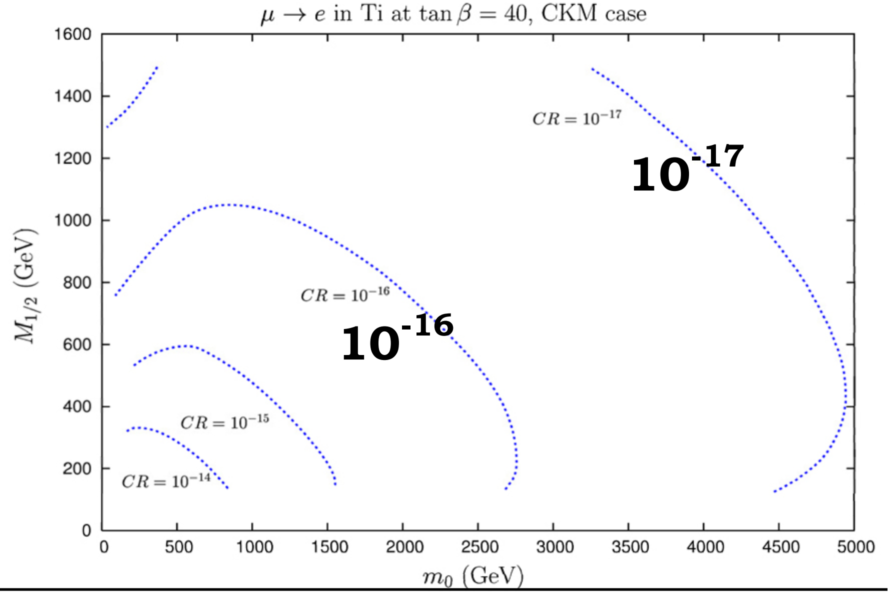


# $\mu \to e$ in Ti and **PRISM/PRIME** conversion experiment









## Conclusions

- LFV experiments are able to constrain/discriminate among different SUSY-GUTs scenarios, thus resulting highly complementary to the LHC.

*Supposing that LHC does find evidences of SUSY:*

- If they detect LFV processes, considering the interplay between different experiments, we should be able to get a deep insight into the structure of $Y_\nu$
- If MEG (and Super Flavour) happens not to see LFV, only two possibilities should be left:

a) minimal mixing, low tan$\beta$ scenario

b) mSUGRA-SO(10) see-saw without fine-tuned $Y_\nu$ is not a viable framework of new physics.

- If the planned high sensitivity of PRISM/PRIME doesn't manage to find LFV evidences, the latter conclusion should be the most feasible one.

*Moreover:*

- LFV experiments will be able to test some scenarios even in the region of the mSUGRA parameter space beyond the LHC sensitivity reach
- The correlation of $U_{e3}$ and LFV can be important in the context of SUSY-GUTs and any measurement of LFV at MEG could shed some light on either $U_{e3}$ or on the parameter space of SUSY-GUTs.

# Other slides

# “Running” $U_{e3}$ and $\mathrm{BR}(\mu \to e + \gamma)$

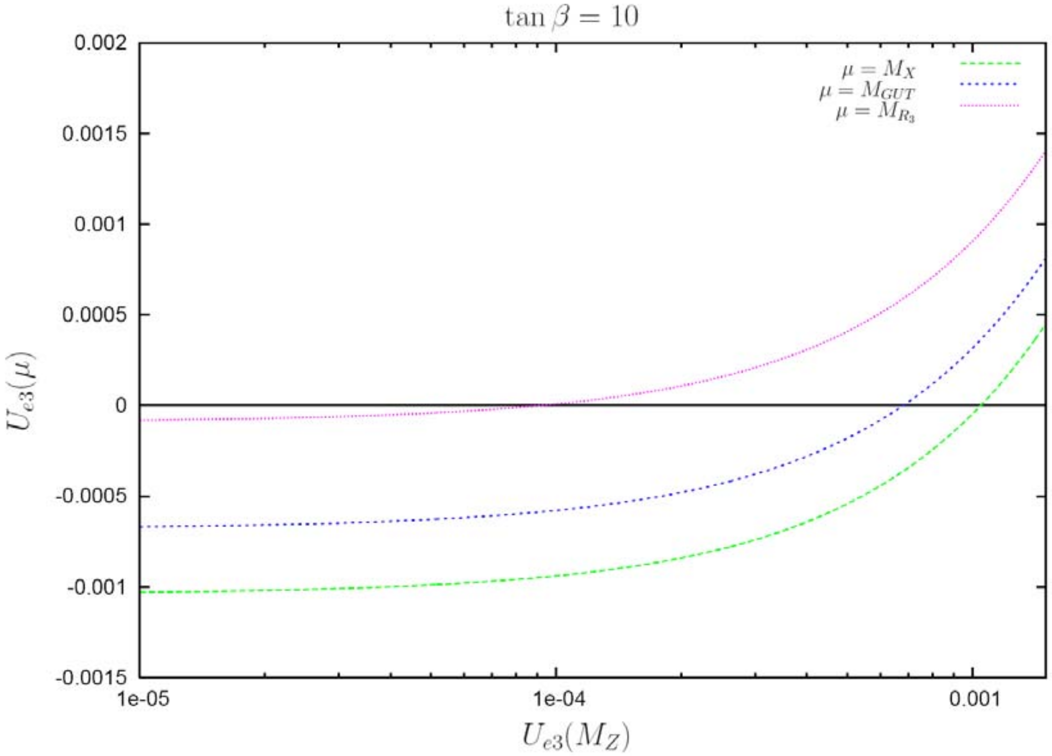
$$m_\nu(\mu) = Y_\nu(\mu) M_R^{-1}(\mu) Y_\nu^T(\mu)$$

The $U_{e3}$ evolution gives, for hierarchical neutrinos (phases set to 0):

$$\Delta U_{e3}^{hie}(M_W \to M_X) \approx -\frac{1}{16\pi^2}\left[y_\tau^2 \ln(\frac{M_X}{M_W}) + y_t^2 \ln(\frac{M_X}{M_{R_3}})\right] U_{e1}U_{e2}U_{\mu 3}U_{\tau 3}\frac{m_{\nu_2} - m_{\nu_1}}{m_{\nu_3}}$$

$$\sim -(\tan^2\beta \cdot \mathcal{O}(10^{-6}) + \mathcal{O}(10^{-3})),$$

Indipendent of $U_{e3}$!

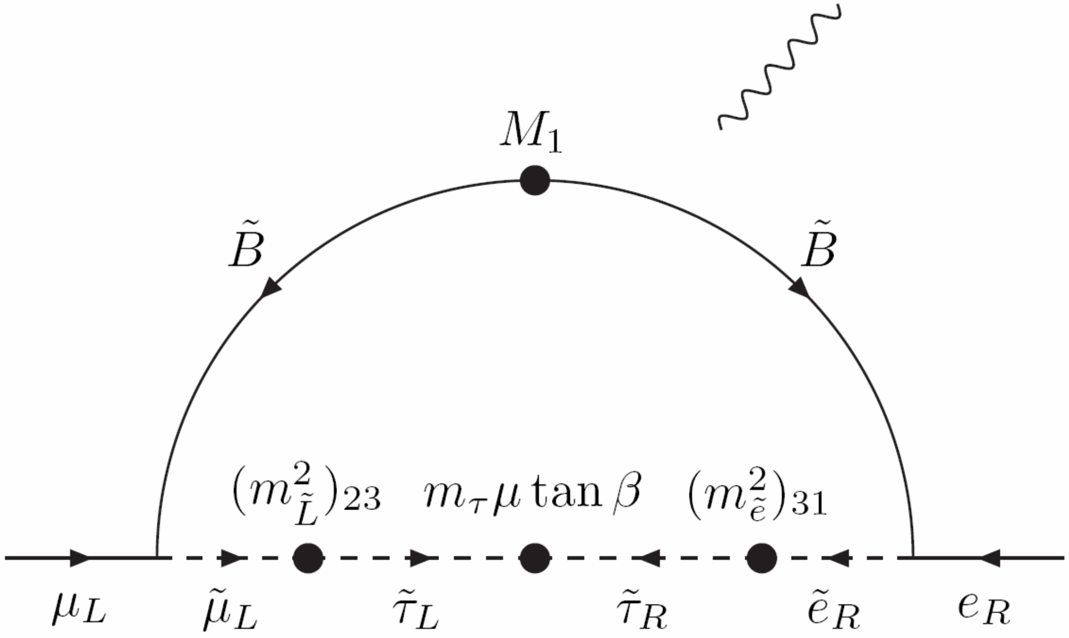


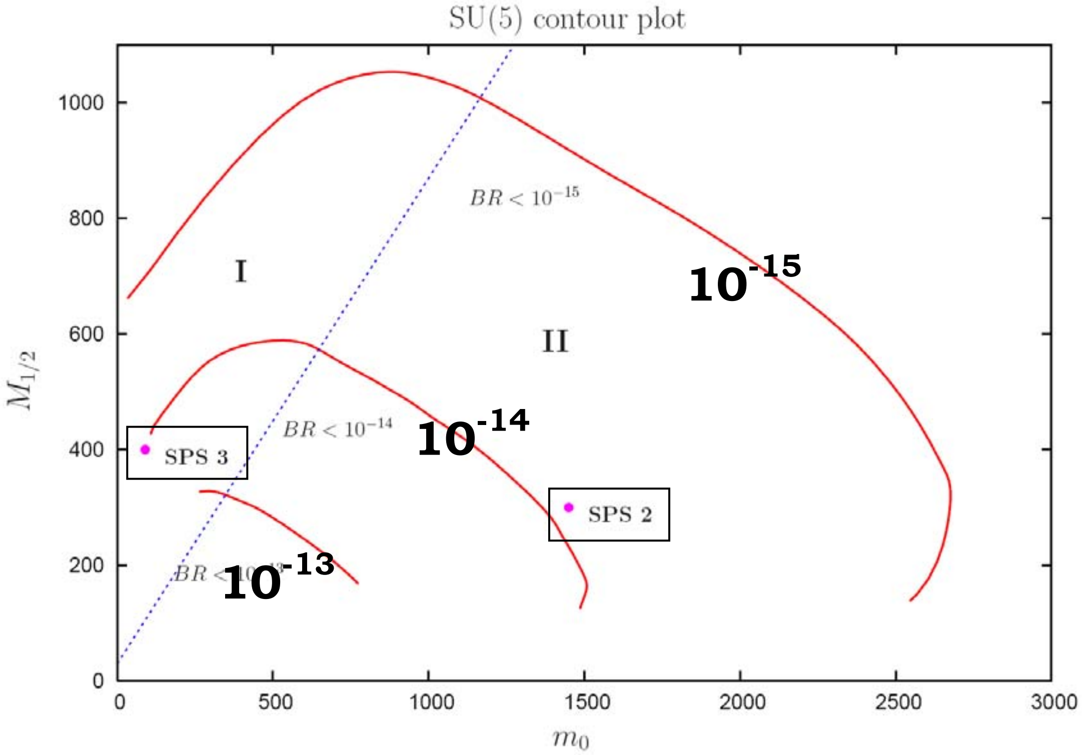
# "Pure" *SU(5)* effects

Let's turn on the *SU(5)* running ⟹ $(\Delta_{RR})_{i\neq j} = -3 \cdot \frac{3m_0^2 + a_0^2}{16\pi^2} Y_t^2 \, V_{i3} V_{j3} \ln\left(\frac{M_X^2}{M_{GUT}^2}\right)$

⟹ double MI indipendent of $U_{e3}$ (usually subdominant):

$$(\delta_{LR})_{21}^{eff} = (\delta_{LL})_{23} \cdot \mu m_\tau \tan\beta \cdot (\delta_{RR})_{31}$$





$\mathrm{BR}(\mu \to e + \gamma)$

Contour plot

$U_{e3} = 0$

$\tan\beta = 10$

$A_0 = 0$

Region **I**:

$$(\delta_{LL})_{23}(\delta_{RR})_{13}$$

**SPS 3**

$m_0$ = 90 GeV , $M_{1/2}$ =400 GeV,

$A_0$ = 0, $\tan\beta = 10$

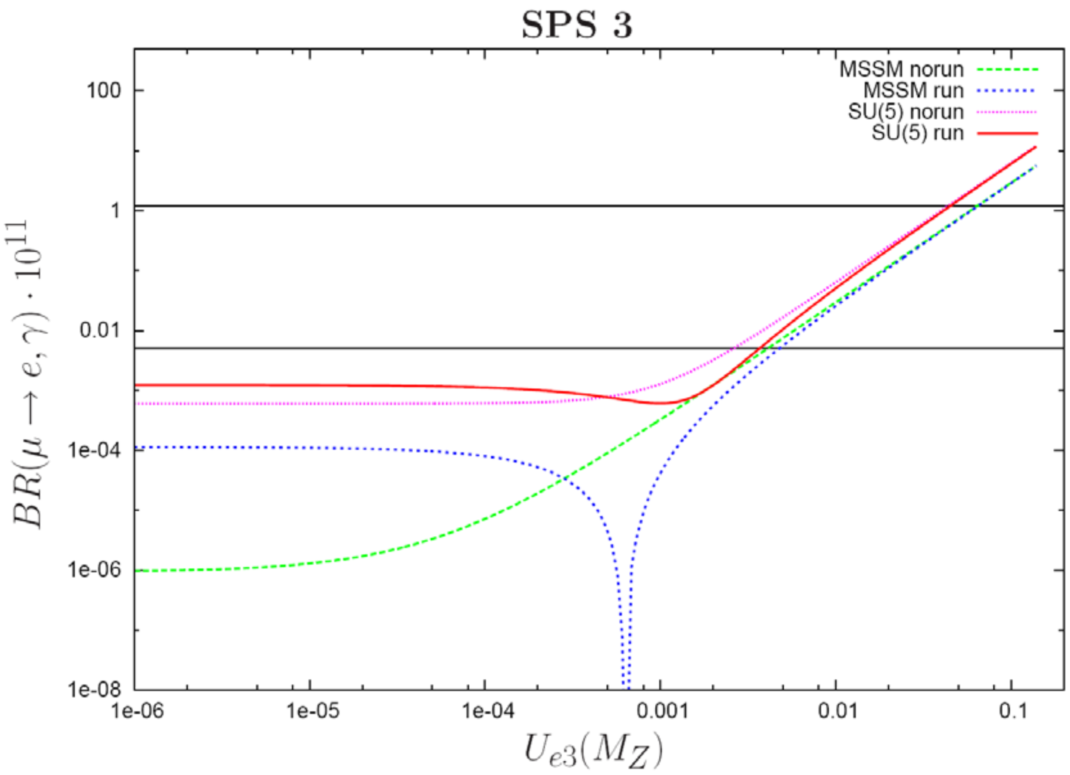
Region **II**:

$$(\delta_{LL})_{12}$$

**SPS 2**

$m_0$ = 1450 GeV , $M_{1/2}$ =300 GeV,

$A_0$ = 0, $\tan\beta = 10$



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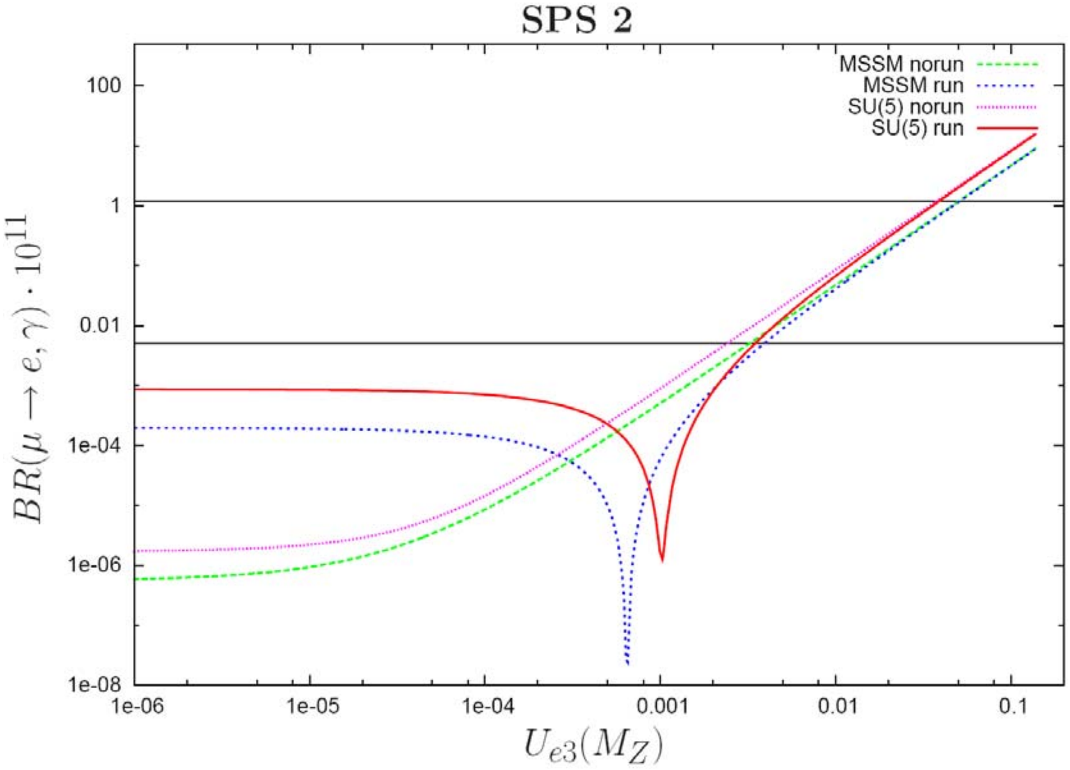
$m_0$ = 1450 GeV , $M_{1/2}$ =300 GeV,

$A_0$ = 0, $\tan\beta = 10$



Region **I**:

$$(\delta_{LL})_{23}(\delta_{RR})_{13}$$

Region **II**:

$$(\delta_{LL})_{12}$$

**SPS 3**

$m_0$ = 90 GeV , $M_{1/2}$ =400 GeV,

$A_0$ = 0, tan$\beta$ $=10$

**SPS 2**

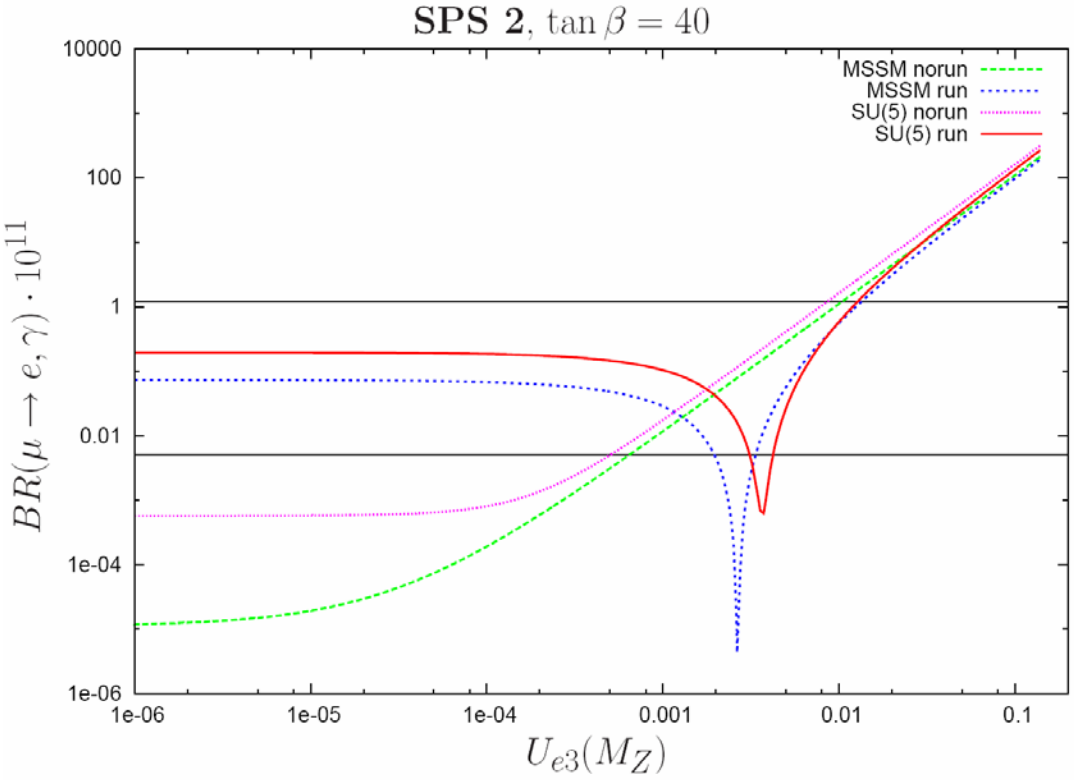
$m_0$ = 1450 GeV , $M_{1/2}$ =300 GeV,

$A_0$ = 0, tan$\beta$ $=10$

# Dependence on tan$\beta$

Region **I**: $$(\delta_{LR})^{eff}_{21} = (\delta_{LL})_{23} \cdot \mu m_\tau \tan\beta \cdot (\delta_{RR})_{31}$$

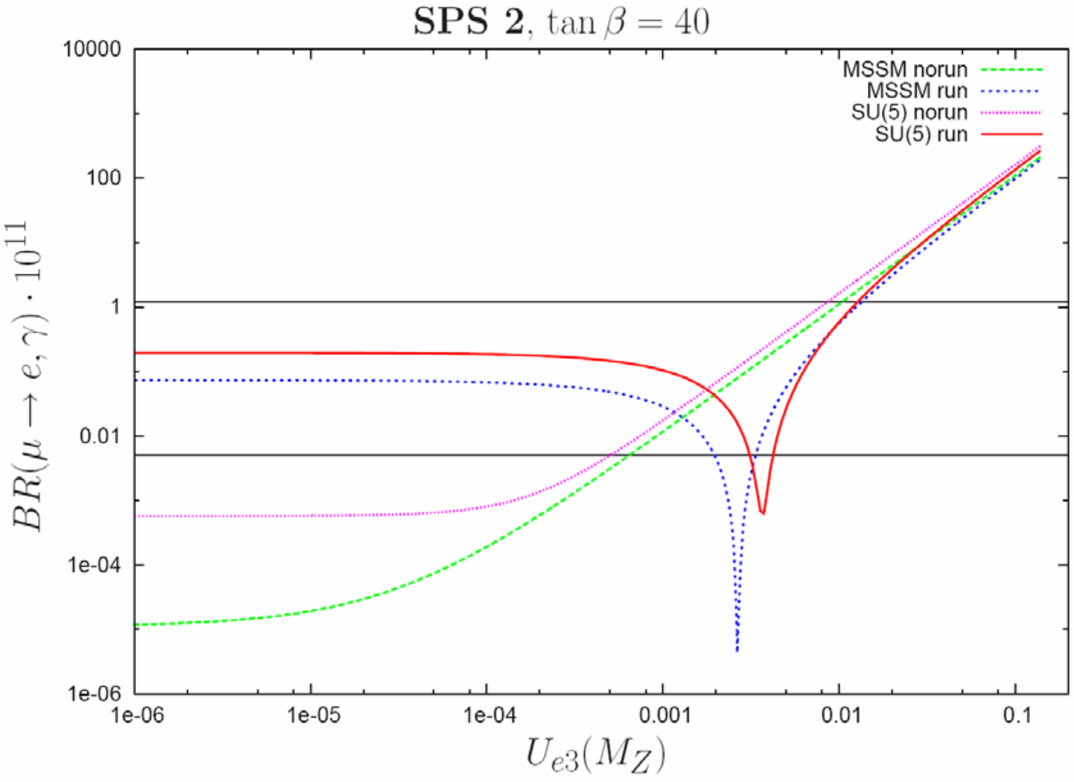
Region **II**: $$\Delta U^{hie}_{e3}(M_W \to M_X) \approx -\frac{1}{16\pi^2}\left[y_\tau^2 \ln(\frac{M_X}{M_W}) + y_t^2 \ln(\frac{M_X}{M_{R_3}})\right] U_{e1}U_{e2}U_{\mu 3}U_{\tau 3}\frac{m_{\nu_2}-m_{\nu_1}}{m_{\nu_3}}$$
$$\sim -(\tan^2\beta \cdot \mathcal{O}(10^{-6}) + \mathcal{O}(10^{-3})),$$



# Dependence on tan$\beta$

Region **I**: $$(\delta_{LR})^{eff}_{21} = (\delta_{LL})_{23} \cdot \mu m_\tau \boxed{\tan\beta} \cdot (\delta_{RR})_{31}$$

Region **II**: $$\Delta U^{hie}_{e3}(M_W \to M_X) \approx -\frac{1}{16\pi^2}\left[y_\tau^2 \ln(\frac{M_X}{M_W}) + y_t^2 \ln(\frac{M_X}{M_{R_3}})\right] U_{e1}U_{e2}U_{\mu 3}U_{\tau 3}\frac{m_{\nu_2}-m_{\nu_1}}{m_{\nu_3}}$$
$$\sim -(\boxed{\tan^2\beta} \cdot \mathcal{O}(10^{-6}) + \mathcal{O}(10^{-3})),$$



And what about the *mixing angles*? We consider two benchmark cases:

“Minimal” mixing (CKM):

$$Y^\nu = Y^u \Rightarrow Y^\nu = V^T_{\rm CKM} Y^u_{diag} V_{\rm CKM}$$

“Maximal” mixing (PMNS):

$$Y^\nu = U_{\rm PMNS} Y^u_{diag}$$

CKM case:

$$W_{SO(10)} = (Y_u)_{ij} \mathbf{16}_i \mathbf{16}_j \mathbf{10}_u + (Y_d)_{ii} \mathbf{16}_i \mathbf{16}_i \mathbf{10}_d + (Y_R)_{ij} \mathbf{16}_i \mathbf{16}_j \mathbf{126}$$

PMNS case:

$$W_{SU(5)} = \frac{1}{2}\,(Y_u)_{ii}\ \mathbf{10_i}\ \mathbf{10_i}\ \mathbf{5_u} + (Y_\nu)_{ii}\ \mathbf{\bar{5}_i}\ \mathbf{1_i}\ \mathbf{5_u} + (Y_d)_{ij}\ \mathbf{10_i}\ \mathbf{\bar{5}_j}\ \mathbf{\bar{5}_d} + \frac{1}{2}\ M^R_{ii}\ \mathbf{1_i 1_i}$$

$$V^T_{\rm CKM}\ Y_d\ U^T_{\rm PMNS} = Y_d^{\rm diag}$$