Reduction of one-loop amplitudes at the integrand level

Giovanni Ossola

NCSR “Demokritos”, Athens

Napoli, 11-13 Aprile 2007
IFAE 2007
Incontri sulla Fisica delle Alte Energie

Outline

1. Introduction: Wishlists and Troubles
2. OPP Reduction
3. Numerical Tests
   - 4-photon amplitudes
   - 6-photon amplitudes

Introduction: LHC needs NLO

- The experimental programs of LHC require high precision predictions for multi-particle processes
- In the last years we have seen a remarkable progress in the theoretical description of multi-particle processes at tree-order, thanks to very efficient recursive algorithms
- The current need of precision goes beyond tree order. At LHC, all analyses require at least next-to-leading order calculations (NLO)
- The search and the interpretation of new physics requires a precise understanding of the Standard Model. We need accurate predictions and reliable error estimates
- As a result,a big effort has been devoted by several groups to the problem of an efficient computation of one-loop corrections for multi-particle processes!

NLO WISHLIST

Les Houches 2005

process ($V \in \{Z, W, \gamma\}$)	relevant for
1. $pp \to V\,V$+ jet	$t\bar{t}H$, new physics
2. $pp \to H + 2$ jets	H production by Vector Boson Fusion
3. $pp \to t\bar{t}\,b\bar{b}$	$t\bar{t}H$
4. $pp \to t\bar{t} + 2$ jets	$t\bar{t}H$
5. $pp \to V\,V\,b\bar{b}$	VBF$\to H \to VV$, $t\bar{t}H$, new physics
6. $pp \to V\,V + 2$ jets	VBF$\to H \to VV$
7. $pp \to V + 3$ jets	various new physics signatures
8. $pp \to V\,V\,V$	SUSY trilepton searches

Processes for which a NLO calculation is both desired and feasible

NLO TROUBLES

Problems arising in NLO calculations

- Large Number of Feynman diagrams
- Reduction to Scalar Integrals (or sets of known integrals)
- Numerical Instabilities (inverse Gram determinants, spurious phase-space singularities)
- Extraction of soft and collinear singularities (we need virtual and real corrections)

Many methods are available...

- Traditional Methods: Feynman Diagrams & Passarino-Veltman Reduction
- Semi-Numerical (Algebraic/Partly Numerical) Approaches
- Numerical (Numerical/Partly Algebraic) Approaches
- Twistor-inspired Unitarity-based Methods

Many methods are available...

- Traditional Methods: Feynman Diagrams & Passarino-Veltman Reduction
- Semi-Numerical (Algebraic/Partly Numerical) Approaches
- Numerical (Numerical/Partly Algebraic) Approaches
- Twistor-inspired Unitarity-based Methods

⋆ Integrand-level reduction [OPP]

Many methods are available...

- Traditional Methods: Feynman Diagrams & Passarino-Veltman Reduction
- Semi-Numerical (Algebraic/Partly Numerical) Approaches
- Numerical (Numerical/Partly Algebraic) Approaches
- Twistor-inspired Unitarity-based Methods

⋆ Integrand-level reduction [OPP]

Ellis, Giele, Glover, Zanderighi; Denner, Dittmaier; Del Aguila, Pittau; Van Hameren, Vollinga, Weinzierl; Binoth, Guillet, Heinrich, Schubert; GRACE Group (Belanger et al.). Ferroglia, Passera, Passarino, Uccirati; Binoth, Heinrich; Anastasiou, Daleo; Nagy, Soper. Berger, Bern, Dixon, Forde, Kosower; Cachazo, Svrcek, Witten; Britto, Feng, Mastrolia. Ossola, Papadopoulos, Pittau; Belanger, Boudjema, Fujimoto, Ishikawa, Kaneko, Kato, Shimizu; van Neerven, Vermaseren; Su, Xiao, Yang, Zhu; Many more [please, accept my apologies...]

OPP Reduction - Intro

Any m-point one-loop amplitude can be written as

$$A(\bar{q}) = \frac{N(q)}{\bar{D}_0 \bar{D}_1 \cdots \bar{D}_{m-1}}$$

A bar denotes objects living in $n = 4 + \epsilon$ dimensions

$$\bar{D}_i = (\bar{q} + p_i)^2 - m_i^2$$

$$\bar{q}^2 = q^2 + \tilde{q}^2$$

$$\bar{D}_i = D_i + \tilde{q}^2$$

External momenta p_i are 4-dimensional objects

OPP "MASTER" FORMULA - I

Let us rewrite the 4-dim $N(q)$ at the integrand level in terms of D_i

$$
\begin{aligned}
N(q) = & \sum_{i_0<i_1<i_2<i_3}^{m-1} \left[d(i_0 i_1 i_2 i_3) + \tilde{d}(q; i_0 i_1 i_2 i_3)\right] \prod_{i\neq i_0, i_1, i_2, i_3}^{m-1} D_i \\
+ & \sum_{i_0<i_1<i_2}^{m-1} \left[c(i_0 i_1 i_2) + \tilde{c}(q; i_0 i_1 i_2)\right] \prod_{i\neq i_0, i_1, i_2}^{m-1} D_i \\
+ & \sum_{i_0<i_1}^{m-1} \left[b(i_0 i_1) + \tilde{b}(q; i_0 i_1)\right] \prod_{i\neq i_0, i_1}^{m-1} D_i \\
+ & \sum_{i_0}^{m-1} \left[a(i_0) + \tilde{a}(q; i_0)\right] \prod_{i\neq i_0}^{m-1} D_i \\
+ & \tilde{P}(q) \prod_{i}^{m-1} D_i
\end{aligned}
\qquad (1)
$$

OPP "MASTER" FORMULA - II

$$N(q) = \sum_{i_0<i_1<i_2<i_3}^{m-1} \left[d(i_0 i_1 i_2 i_3) \qquad \right] \prod_{i\neq i_0,i_1,i_2,i_3}^{m-1} D_i + \sum_{i_0<i_1<i_2}^{m-1} [c(i_0 i_1 i_2) \qquad] \prod_{i\neq i_0,i_1,i_2}^{m-1} D_i$$
$$+ \sum_{i_0<i_1}^{m-1} \left[b(i_0 i_1) \qquad \right] \prod_{i\neq i_0,i_1}^{m-1} D_i + \sum_{i_0}^{m-1} [a(i_0) \qquad] \prod_{i\neq i_0}^{m-1} D_i \tag{2}$$

- The quantities $d(i_0 i_1 i_2 i_3)$ are the coefficients of 4-point functions with denominators labeled by i_0, i_1, i_2, and i_3.
- $c(i_0 i_1 i_2)$, $b(i_0 i_1)$, $a(i_0)$ are the coefficients of all possible 3-point, 2-point and 1-point functions, respectively.

OPP "MASTER" FORMULA - II

$$N(q) = \sum_{i_0<i_1<i_2<i_3}^{m-1} \left[d(i_0 i_1 i_2 i_3) + \tilde{d}(q; i_0 i_1 i_2 i_3)\right] \prod_{i\neq i_0, i_1, i_2, i_3}^{m-1} D_i + \sum_{i_0<i_1<i_2}^{m-1} \left[c(i_0 i_1 i_2) + \tilde{c}(q; i_0 i_1 i_2)\right] \prod_{i\neq i_0, i_1, i_2}^{m-1} D_i$$
$$+ \sum_{i_0<i_1}^{m-1} \left[b(i_0 i_1) + \tilde{b}(q; i_0 i_1)\right] \prod_{i\neq i_0, i_1}^{m-1} D_i + \sum_{i_0}^{m-1} \left[a(i_0) + \tilde{a}(q; i_0)\right] \prod_{i\neq i_0}^{m-1} D_i \qquad (2)$$

The quantities $\tilde{d}$, $\tilde{c}$, $\tilde{b}$, $\tilde{a}$ are the "spurious" terms

- They still depend on q (integration momentum)
- They should vanish upon integration

What is the explicit expression of the spurious term?

Spurious Terms - I

Following F. del Aguila and R. Pittau, arXiv:hep-ph/0404120

- Express any q in $N(q)$ as

$$q^\mu = -p_0^\mu + \sum_{i=1}^{4} c_i\, \ell_i^\mu \ , \ \ {\ell_i}^2 = 0$$

$$k_1 = \ell_1 + \alpha_1 \ell_2 \,, \quad k_2 = \ell_2 + \alpha_2 \ell_1 \,, \ k_i = p_i - p_0$$
$${\ell_3}^\mu = < \ell_1|\gamma^\mu|\ell_2] \,, \ {\ell_4}^\mu = < \ell_2|\gamma^\mu|\ell_1]$$

- The coefficients c_i either reconstruct denominators D_i

$\rightarrow$ They give rise to d, c, b, a coefficients

Spurious Terms - I

Following F. del Aguila and R. Pittau, arXiv:hep-ph/0404120

- Express any q in $N(q)$ as

$$q^\mu = -p_0^\mu + \sum_{i=1}^4 c_i\,\ell_i^\mu \;,\; {\ell_i}^2 = 0$$

$$k_1 = \ell_1 + \alpha_1 \ell_2\,, \quad k_2 = \ell_2 + \alpha_2 \ell_1\,,\; k_i = p_i - p_0$$
$${\ell_3}^\mu =< \ell_1|\gamma^\mu|\ell_2]\,,\; {\ell_4}^\mu =< \ell_2|\gamma^\mu|\ell_1]$$

- The coefficients c_i either reconstruct denominators D_i or vanish upon integration

→ They give rise to d, c, b, a coefficients
→ They form the spurious $\tilde{d}$, $\tilde{c}$, $\tilde{b}$, $\tilde{a}$ coefficients

Spurious Terms - II

- $\tilde{d}(q)$ term (only 1)

$$\tilde{d}(q) = \tilde{d}\, T(q)\,,$$

where $\tilde{d}$ is a constant (does not depend on q)

$$T(q) \equiv Tr[(\not{q} + \not{p}_0)\not{\ell}_1\not{\ell}_2\not{k}_3\gamma_5]$$

Spurious Terms - II

- $\tilde{d}(q)$ term (only 1)

$$\tilde{d}(q) = \tilde{d}\, T(q)\,,$$

where $\tilde{d}$ is a constant (does not depend on q)

$$T(q) \equiv Tr[(\not{q} + \not{p}_0)\not{\ell}_1 \not{\ell}_2 \not{k}_3 \gamma_5]$$

- $\tilde{c}(q)$ terms (they are 6)

$$\tilde{c}(q) = \sum_{j=1}^{j_{max}} \{ \tilde{c}_{1j} [(q+p_0)\cdot \ell_3]^j + \tilde{c}_{2j} [(q+p_0)\cdot \ell_4]^j \}$$

In the renormalizable gauge, $j_{max} = 3$

Spurious Terms - II

- $\tilde{d}(q)$ term (only 1)

$$\tilde{d}(q) = \tilde{d}\, T(q)\,,$$

where $\tilde{d}$ is a constant (does not depend on q)

$$T(q) \equiv Tr[(\not{q} + \not{p}_0)\not{\ell}_1 \not{\ell}_2 \not{k}_3 \gamma_5]$$

- $\tilde{c}(q)$ terms (they are 6)

$$\tilde{c}(q) = \sum_{j=1}^{j_{max}} \{\tilde{c}_{1j}[(q+p_0)\cdot \ell_3]^j + \tilde{c}_{2j}[(q+p_0)\cdot \ell_4]^j\}$$

In the renormalizable gauge, $j_{max} = 3$

- $\tilde{b}(q)$ and $\tilde{a}(q)$ give rise to 8 and 4 terms, respectively

General strategy

Now we know the form of the spurious terms:

$$N(q) = \sum_{i_0<i_1<i_2<i_3}^{m-1} \left[d(i_0 i_1 i_2 i_3) + \tilde{d}(q; i_0 i_1 i_2 i_3)\right] \prod_{i\neq i_0, i_1, i_2, i_3}^{m-1} D_i + \sum_{i_0<i_1<i_2}^{m-1} [c(i_0 i_1 i_2) + \tilde{c}(q; i_0 i_1 i_2)] \prod_{i\neq i_0, i_1, i_2}^{m-1} D_i$$
$$+ \sum_{i_0<i_1}^{m-1} \left[b(i_0 i_1) + \tilde{b}(q; i_0 i_1)\right] \prod_{i\neq i_0, i_1}^{m-1} D_i + \sum_{i_0}^{m-1} [a(i_0) + \tilde{a}(q; i_0)] \prod_{i\neq i_0}^{m-1} D_i \quad (3)$$

Our calculation is now reduced to an algebraic problem

General strategy

Now we know the form of the spurious terms:

$$\begin{aligned} N(q) &= \sum_{i_0<i_1<i_2<i_3}^{m-1} \left[d(i_0 i_1 i_2 i_3) + \tilde{d}(q; i_0 i_1 i_2 i_3)\right] \prod_{i \neq i_0, i_1, i_2, i_3}^{m-1} D_i + \sum_{i_0<i_1<i_2}^{m-1} \left[c(i_0 i_1 i_2) + \tilde{c}(q; i_0 i_1 i_2)\right] \prod_{i \neq i_0, i_1, i_2}^{m-1} D_i \\ &+ \sum_{i_0<i_1}^{m-1} \left[b(i_0 i_1) + \tilde{b}(q; i_0 i_1)\right] \prod_{i \neq i_0, i_1}^{m-1} D_i + \sum_{i_0}^{m-1} \left[a(i_0) + \tilde{a}(q; i_0)\right] \prod_{i \neq i_0}^{m-1} D_i \end{aligned} \tag{3}$$

Our calculation is now reduced to an algebraic problem

Extract all the coefficients by evaluating N(q) for a set of values of the integration momentum q

General strategy

Now we know the form of the spurious terms:

$$
\begin{aligned}
N(q) = & \sum_{i_0<i_1<i_2<i_3}^{m-1} \left[d(i_0 i_1 i_2 i_3) + \tilde{d}(q; i_0 i_1 i_2 i_3)\right] \prod_{i \neq i_0, i_1, i_2, i_3}^{m-1} D_i + \sum_{i_0<i_1<i_2}^{m-1} \left[c(i_0 i_1 i_2) + \tilde{c}(q; i_0 i_1 i_2)\right] \prod_{i \neq i_0, i_1, i_2}^{m-1} D_i \\
& + \sum_{i_0<i_1}^{m-1} \left[b(i_0 i_1) + \tilde{b}(q; i_0 i_1)\right] \prod_{i \neq i_0, i_1}^{m-1} D_i + \sum_{i_0}^{m-1} \left[a(i_0) + \tilde{a}(q; i_0)\right] \prod_{i \neq i_0}^{m-1} D_i
\end{aligned} \tag{3}
$$

Our calculation is now reduced to an algebraic problem

Extract all the coefficients by evaluating N(q) for a set of values of the integration momentum q

There is a very good set of such points: Use values of q for which a set of denominators D_i vanish → The system becomes "triangular": solve first for 4-point functions, then 3-point functions and so on

Example: d coefficients of 4-point function

$$N(q) = d + \tilde{d}(q) + \sum_{i=0}^{3} [c(i) + \tilde{c}(q; i)] D_i + \sum_{i_0<i_1}^{3} \left[b(i_0 i_1) + \tilde{b}(q; i_0 i_1)\right] D_{i_0} D_{i_1}$$

$$+ \sum_{i_0=0}^{3} [a(i_0) + \tilde{a}(q; i_0)] D_{i\neq i_0} D_{j\neq i_0} D_{k\neq i_0}$$

We look for a q of the form $q^\mu = -p_0^\mu + x_i \ell_i^\mu$ such that

$$D_0 = D_1 = D_2 = D_3 = 0$$

$\rightarrow$ we get a system of equations in x_i that has two solutions $q_0^\pm$

Example: d coefficients of 4-point function

$$N(q) \quad = \quad d + \tilde{d}(q)$$

Our "master formula" for $q = q_0^{\pm}$ is:

$$N(q_0^{\pm}) = [d + \tilde{d}\; T(q_0^{\pm})]$$

→ solve to extract the coefficients d and $\tilde{d}$

Example: d coefficients of 4-point function

$$N(q) - d - \tilde{d}(q) = \sum_{i=0}^{3} [c(i) + \tilde{c}(q;i)] D_i + \sum_{i_0<i_1}^{3} \left[b(i_0 i_1) + \tilde{b}(q; i_0 i_1)\right] D_{i_0} D_{i_1}$$
$$+ \sum_{i_0=0}^{3} [a(i_0) + \tilde{a}(q; i_0)] D_{i\neq i_0} D_{j\neq i_0} D_{k\neq i_0}$$

Then we can move to the extraction of c coefficients using

$$N'(q) = N(q) - d - \tilde{d}\, T(q)$$

and setting to zero three denominators (ex: $D_1 = 0$, $D_2 = 0$, $D_3 = 0$)

Rational Terms - I

- Let's go back to the integrand

$$A(\bar{q}) = \frac{N(q)}{\bar{D}_0 \bar{D}_1 \cdots \bar{D}_{m-1}}$$

- Insert the expression for $N(q) \to$ we know all the coefficients

$$N(q) = \sum_{i_0<i_1<i_2<i_3}^{m-1} \left[d + \tilde{d}(q)\right] \prod_{i \neq i_0, i_1, i_2, i_3}^{m-1} D_i + \sum_{i_0<i_1<i_2}^{m-1} [c + \tilde{c}(q)] \prod_{i \neq i_0, i_1, i_2}^{m-1} D_i + \cdots$$

- Finally rewrite all denominators using

$$\frac{D_i}{\bar{D}_i} = \bar{Z}_i \,, \quad \text{with} \quad \bar{Z}_i \equiv \left(1 - \frac{\tilde{q}^2}{\bar{D}_i}\right)$$

Rational Terms - II

$$
\begin{aligned}
A(\bar{q}) \;=\; & \sum_{i_0<i_1<i_2<i_3}^{m-1} \frac{d(i_0 i_1 i_2 i_3) + \tilde{d}(q; i_0 i_1 i_2 i_3)}{\bar{D}_{i_0} \bar{D}_{i_1} \bar{D}_{i_2} \bar{D}_{i_3}} \prod_{i \neq i_0, i_1, i_2, i_3}^{m-1} \bar{Z}_i \\
+ & \sum_{i_0<i_1<i_2}^{m-1} \frac{c(i_0 i_1 i_2) + \tilde{c}(q; i_0 i_1 i_2)}{\bar{D}_{i_0} \bar{D}_{i_1} \bar{D}_{i_2}} \prod_{i \neq i_0, i_1, i_2}^{m-1} \bar{Z}_i \\
+ & \sum_{i_0<i_1}^{m-1} \frac{b(i_0 i_1) + \tilde{b}(q; i_0 i_1)}{\bar{D}_{i_0} \bar{D}_{i_1}} \prod_{i \neq i_0, i_1}^{m-1} \bar{Z}_i \\
+ & \sum_{i_0}^{m-1} \frac{a(i_0) + \tilde{a}(q; i_0)}{\bar{D}_{i_0}} \prod_{i \neq i_0}^{m-1} \bar{Z}_i \qquad (4)
\end{aligned}
$$

The rational part is produced, after integrating over $d^n q$, by the $\tilde{q}^2$ dependence in $\bar{Z}_i$

$$\bar{Z}_i \equiv \left(1 - \frac{\tilde{q}^2}{\bar{D}_i}\right)$$

Summary

Calculate $N(q)$

- We do not need to repeat this for all Feynman diagrams. We can group them and solve for (sub)amplitudes directly
- Calculate $N(q)$ numerically via recursion relations
- Just specify external momenta, polarization vectors and masses and proceed with the reduction!

Compute all coefficients

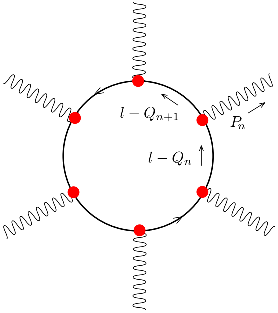
- by evaluating N(q) at certain values of integration momentum

Evaluate scalar integrals

- massive integrals → FF [G. J. van Oldenborgh]
- massless integrals → OneLOop [A. van Hameren]

4-photon and 6-photon amplitudes

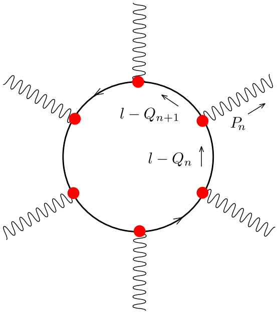
As an example we present 4-photon and 6-photon amplitudes
(via fermionic loop of mass m_f)



Input parameters for the reduction:

- External momenta p_i
- Masses of propagators in the loop
- Polarization vectors

As an example we present 4-photon and 6-photon amplitudes (via fermionic loop of mass m_f)



Input parameters for the reduction:

- External momenta p_i → in this example massless, i.e. $p_i^2 = 0$
- Masses of propagators in the loop → all equal to m_f
- Polarization vectors → various helicity configurations

Four Photons – Comparison with *Gounaris et al.*

$$\frac{F^f_{++++}}{\alpha^2 Q_f^4} = -8 \qquad (5)$$

Rational Part

Four Photons – Comparison with *Gounaris et al.*

$$
\begin{aligned}
\frac{F^{f}_{++++}}{\alpha^2 Q_f^4} &= -8 + 8\left(1+\frac{2\hat{u}}{\hat{s}}\right)B_0(\hat{u}) + 8\left(1+\frac{2\hat{t}}{\hat{s}}\right)B_0(\hat{t}) \\
&\quad - 8\left(\frac{\hat{t}^2+\hat{u}^2}{\hat{s}^2}\qquad\qquad\right)[\hat{t}\,C_0(\hat{t}) + \hat{u}\,C_0(\hat{u})] \\
&\quad - 4\left[\qquad\qquad\frac{\hat{t}^2+\hat{u}^2}{\hat{s}^2}\qquad\qquad\right]D_0(\hat{t},\hat{u})
\end{aligned}
\tag{5}
$$

Massless four-photon amplitudes

$$
\begin{aligned}
\frac{F^f_{++++}}{\alpha^2 Q_f^4} = & -8 + 8\left(1+\frac{2\hat{u}}{\hat{s}}\right) B_0(\hat{u}) + 8\left(1+\frac{2\hat{t}}{\hat{s}}\right) B_0(\hat{t}) \\
& - 8\left(\frac{\hat{t}^2+\hat{u}^2}{\hat{s}^2} - \frac{4m_f^2}{\hat{s}}\right)[\hat{t}\,C_0(\hat{t}) + \hat{u}\,C_0(\hat{u})] \\
& - 4\left[4m_f^4 - (2\hat{s}\,m_f^2 + \hat{t}\,\hat{u})\frac{\hat{t}^2+\hat{u}^2}{\hat{s}^2} + \frac{4m_f^2\hat{t}\,\hat{u}}{\hat{s}}\right] D_0(\hat{t},\hat{u}) \\
& + 8m_f^2(\hat{s} - 2m_f^2)[D_0(\hat{s},\hat{t}) + D_0(\hat{s},\hat{u})] \qquad (5)
\end{aligned}
$$

Massive four-photon amplitudes

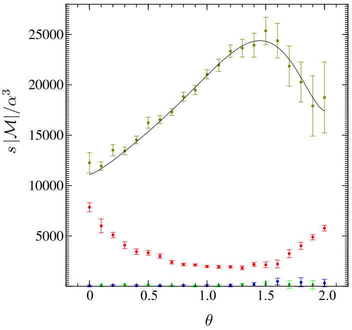
Four Photons – Comparison with *Gounaris et al.*

$$\frac{F^f_{++++}}{\alpha^2 Q_f^4} = -8 + 8\left(1+\frac{2\hat{u}}{\hat{s}}\right)B_0(\hat{u}) + 8\left(1+\frac{2\hat{t}}{\hat{s}}\right)B_0(\hat{t})$$
$$- 8\left(\frac{\hat{t}^2+\hat{u}^2}{\hat{s}^2} - \frac{4m_f^2}{\hat{s}}\right)[\hat{t}C_0(\hat{t}) + \hat{u}C_0(\hat{u})]$$
$$- 4\left[4m_f^4 - (2\hat{s}m_f^2 + \hat{t}\hat{u})\frac{\hat{t}^2+\hat{u}^2}{\hat{s}^2} + \frac{4m_f^2\hat{t}\hat{u}}{\hat{s}}\right]D_0(\hat{t},\hat{u})$$
$$+ 8m_f^2(\hat{s} - 2m_f^2)[D_0(\hat{s},\hat{t}) + D_0(\hat{s},\hat{u})] \quad (5)$$

Massive four-photon amplitudes

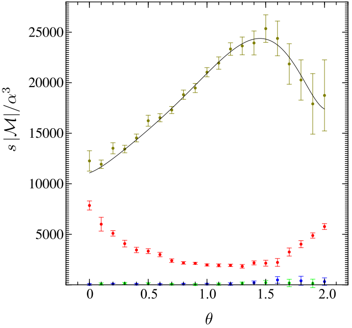
Results also checked for F^f_{+++-} and F^f_{++--}

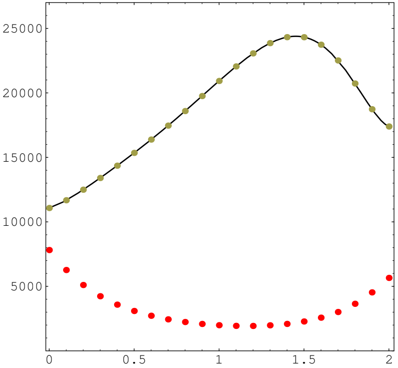
Massless case: $[+ + - - - -]$ and $[+ - - + + -]$



Plot presented by Nagy and Soper hep-ph/0610028

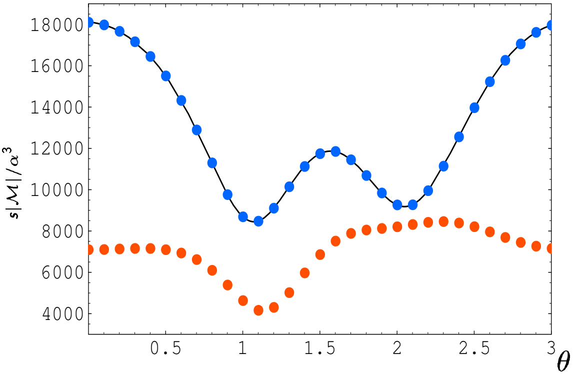
Massless case: $[++----]$ and $[+--++-]$





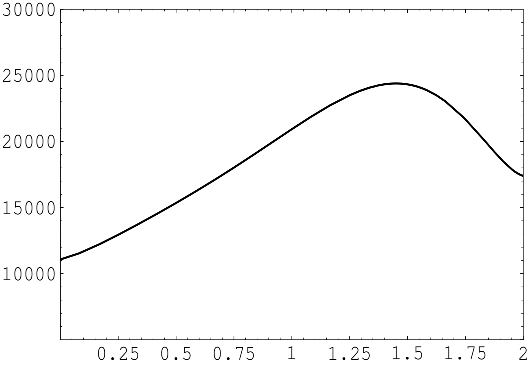
Analogous plot produced with OPP reduction

Massless case: $[++----]$ and $[++--+-]$



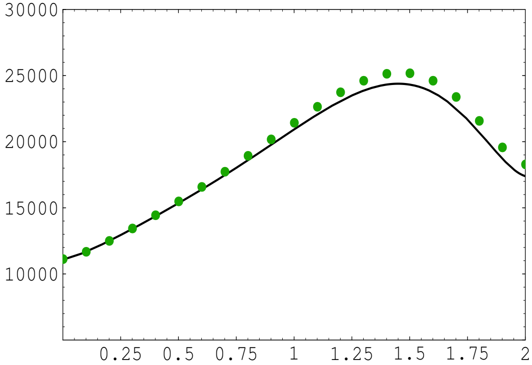
Same idea for a different set of external momenta

Six Photons with Massive Fermions



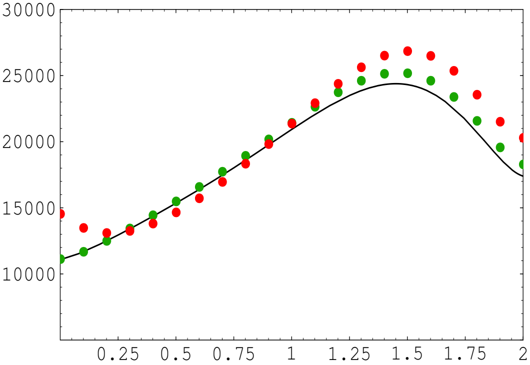
- Massless result [Mahlon]

Six Photons with Massive Fermions



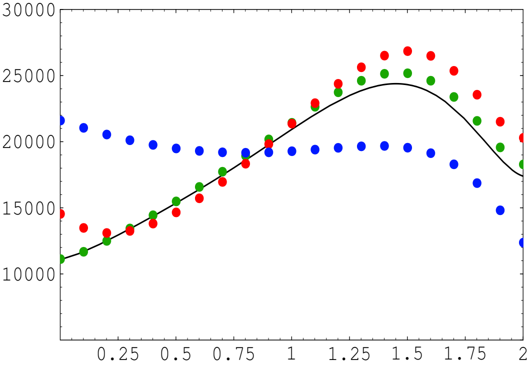
- Massless result [Mahlon]
- $m = 0.5$ GeV

Six Photons with Massive Fermions



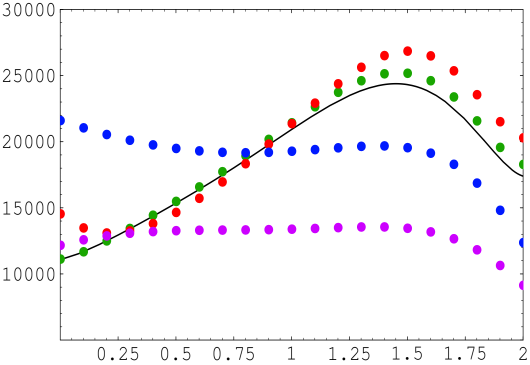
- Massless result [Mahlon]
- $m = 0.5$ GeV
- $m = 4.5$ GeV

Six Photons with Massive Fermions



- Massless result [Mahlon]
- $m = 0.5$ GeV
- $m = 4.5$ GeV
- $m = 12.0$ GeV

Six Photons with Massive Fermions



- Massless result [Mahlon]
- $m = 0.5$ GeV
- $m = 4.5$ GeV
- $m = 12.0$ GeV
- $m = 20.0$ GeV

Summary and Conclusions

LHC requires NLO calculations!

- There is a variety of interesting options available, but no universal method for one-loop multi-leg calculations

Summary and Conclusions

LHC requires NLO calculations!

- There is a variety of interesting options available, but no universal method for one-loop multi-leg calculations
- OPP is a "young" method, but it seems promising

Summary and Conclusions

LHC requires NLO calculations!

- There is a variety of interesting options available, but no universal method for one-loop multi-leg calculations
- OPP is a "young" method, but it seems promising
- More results soon!

G. O., C. G. Papadopoulos and R. Pittau
Nucl. Phys. B **763**, 147 (2007) [arXiv:hep-ph/0609007]
arXiv:0704.1271 [hep-ph]