



# Misura di $\Delta m_s$ , $\Delta \Gamma_s$ e $\phi_s$ a Tevatron

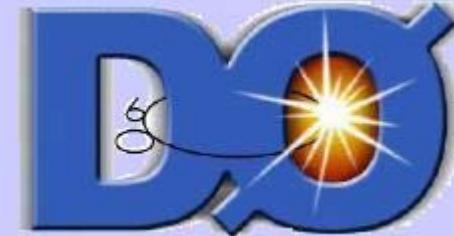
Simone Pagan Griso  
University of Padova

IFAE 2007

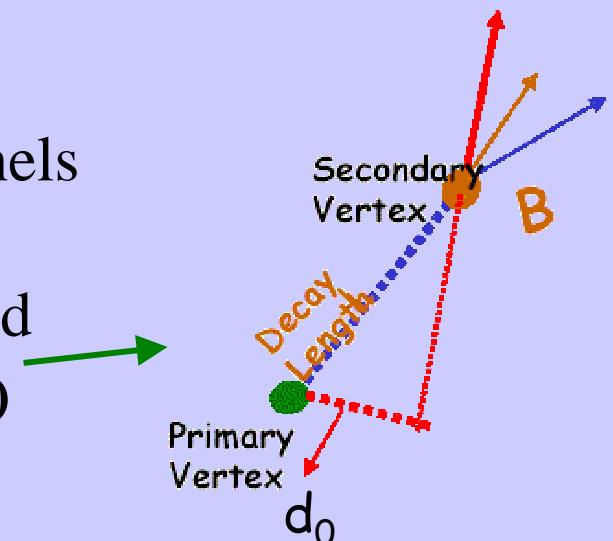
Napoli, 11 Aprile 2007



# B-physics @ Tevatron

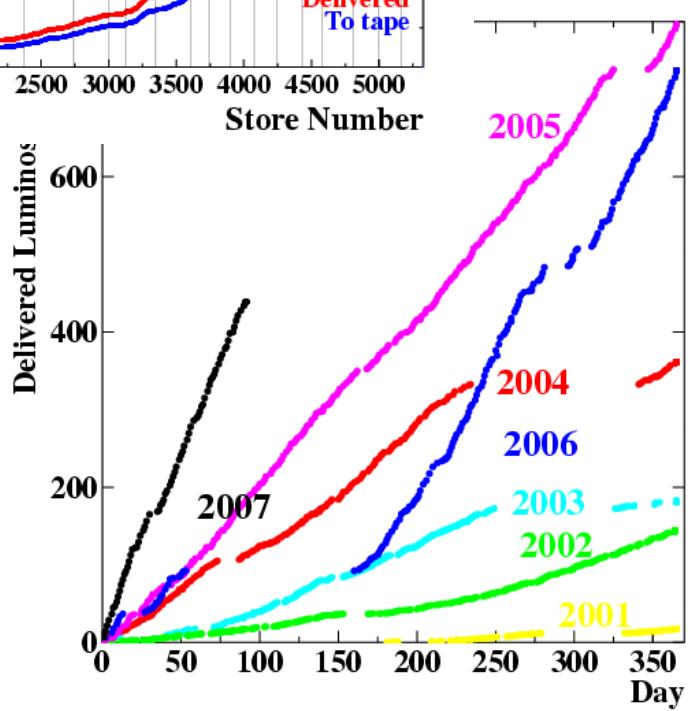
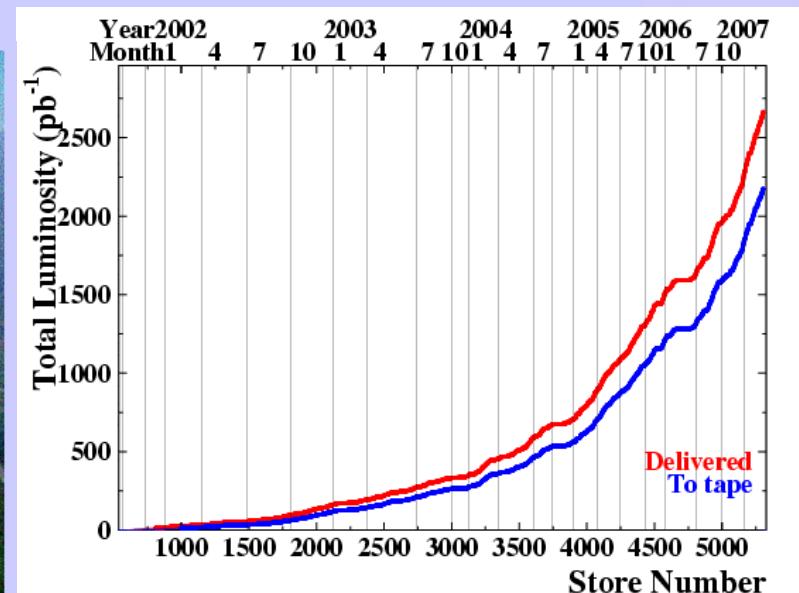
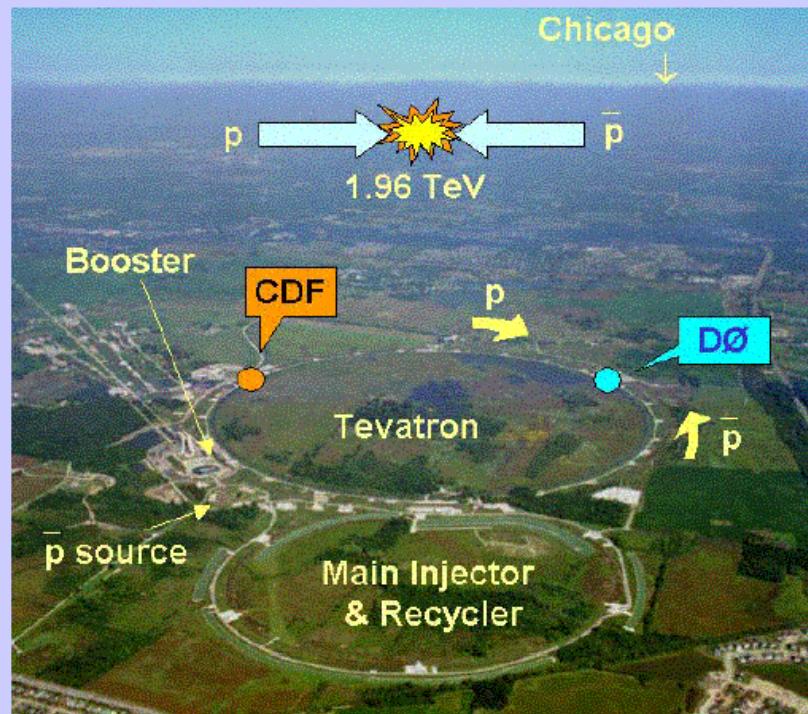


- Hadron colliders are a difficult environment...
  - lots of inelastic background ( $S/N \approx 1/1000$ )
- ... but have many advantages:
  - large x-section for b production ( $\sigma \approx 150 \mu\text{b}$  @ 2TeV)
  - all B hadron species are accessible ( $B^{0,+}, B_s, \Lambda_b, \Sigma_b, \dots$  )
- Key ingredient: High purity triggers!
  - D0: Trigger on single and di-muon channels  
**(semileptonic B decays + di-lepton final states)**
  - CDF: Silicon tracking to trigger displaced vertices ( $\sigma_{d_0} \approx 48 \mu\text{m}$ )  
**(all hadronic final states)**





# Tevatron @ FNAL



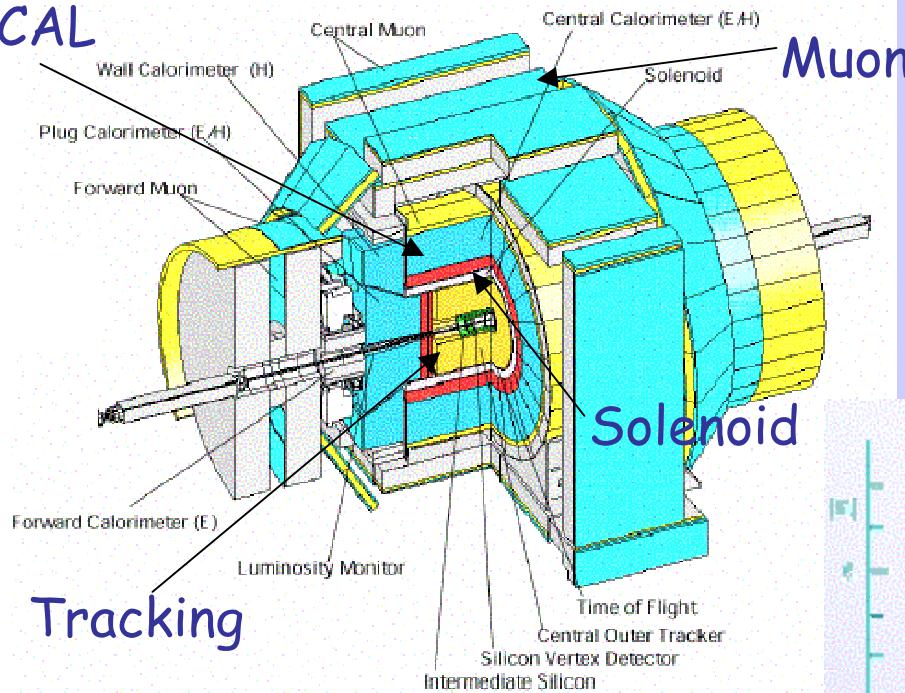
- $> 2 \text{ fb}^{-1}$  to tape! (current analysis  $\approx 1 \text{ fb}^{-1}$ )
- Luminosity still increasing!  
(Record inst. lum.  $2.9 \cdot 10^{32} \text{ cm}^{-2}\text{s}^{-1}$ )



# CDFII and DO



CAL



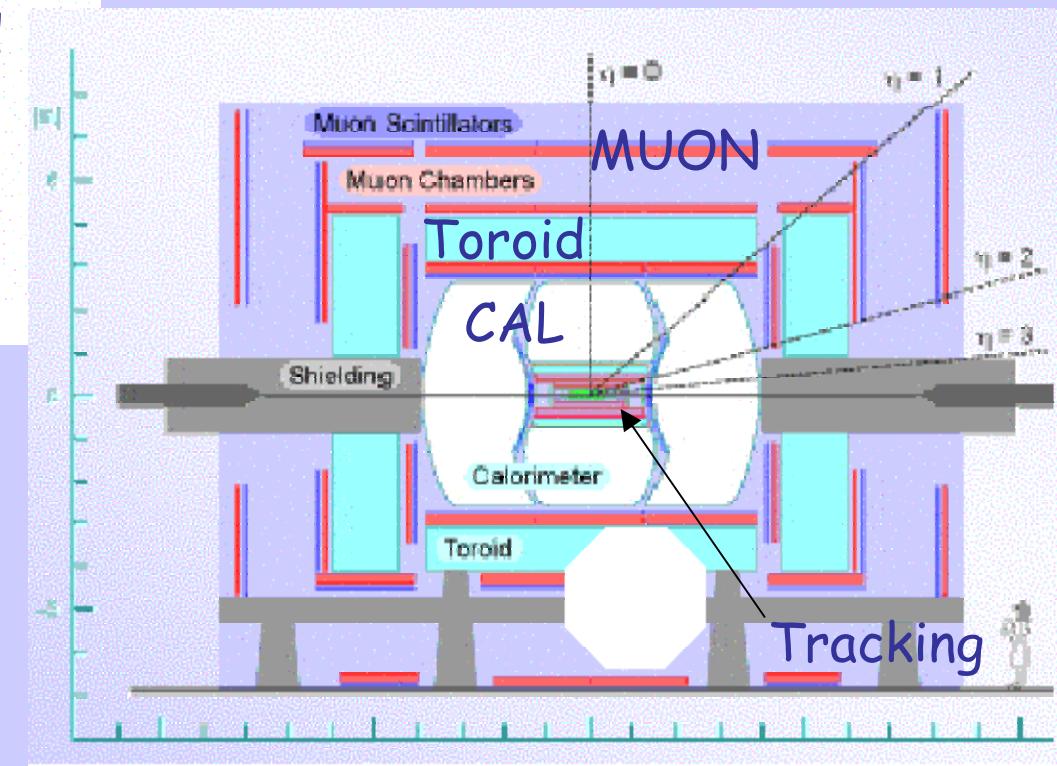
Tracking

CDFII

- Proper time resolution:  $\sigma_{ct} \approx 26\mu\text{m}$  (fully reconstructed  $B_s$  decays)
- $P_t$  res.  $\sigma_{Pt}/P_t \approx 0.07\% P_t [\text{GeV}/c]^{-1}$  using Silicon + Drift chamber  
( $\Rightarrow \sigma$  mass  $\approx 15\text{MeV}$  for  $J/\Psi \rightarrow \mu\mu$ )

D0

- Muon coverage up to  $|\eta| < 2$
- Forward tracking:
  - Silicon Microstrip Tracker up to  $|\eta| < 3$
  - Central Fiber Tracker up to  $|\eta| < 2.5$





# Outline



- $\Delta m_s$  → oscillation frequency of  $B_s - \bar{B}_s$  system
    - CDFII measures dominates
  - $\Delta\Gamma_s$  → lifetime difference on  $B_s - \bar{B}_s$  mass eigenstates
    - D0: 3 untagged measures approach
  - $\phi_s$  → SM CP violating phase  $\equiv \arg\left(\frac{V_{ts}V_{tb}^*}{V_{ts}^*V_{tb}} \frac{V_{cs}^*V_{cb}}{V_{cs}V_{cb}^*}\right)$ 
    - only D0 measure up to now
- ⇒ Observables really sensible to New Physics contributions

# $\Delta m_s$ measurement

$B_{s,L}, B_{s,H}$  mass eigenstates of  $B_s$ - $\bar{B}_s$  system

$$\Delta m_s = m(B_{s,H}) - m(B_{s,L}) \propto |V_{ts}|^2$$

$$\frac{\Delta m_s}{\Delta m_d} = \frac{m_{Bs}}{m_{Bd}} \frac{|V_{ts}|^2}{|V_{td}|^2} \xi^2$$

From lattice QCD  
 ≈ 4% error on  $|V_{ts}|/|V_{td}|$

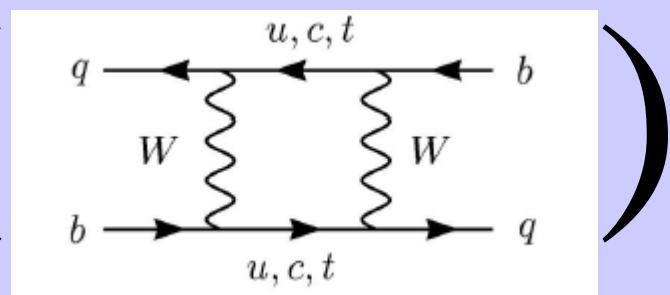
Strong check for SM, but also for new physics:

New Physics [Lenz, Nierste; hep-ph/0612167]  
 (model indip.)

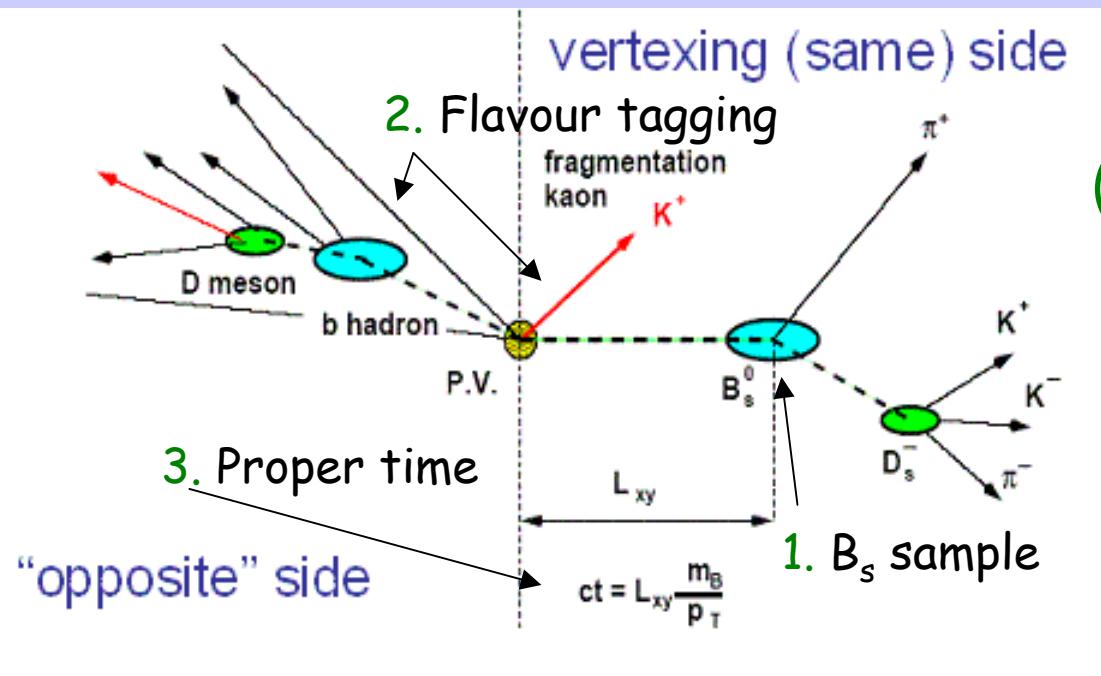
$$M_{12} = M_{12}^{SM} \cdot |\Delta_s| e^{i\phi_s^\Delta}$$

$$M_{12}^{SM} \approx \text{Re} \left( \right.$$

$$\Delta m_s = m_H - m_L = \Delta m_s^{SM} \cdot |\Delta_s|$$



# $\Delta m_s$ measurement



$$A(t) = \frac{P^{nomix} - P^{mix}}{P^{nomix} + P^{mix}} = D \cos(\Delta m_s t)$$

$$\frac{1}{\sigma_A} = \sqrt{\frac{\varepsilon D^2 S}{2}} e^{-\frac{(\Delta m_s \sigma_t)^2}{2}} \sqrt{\frac{S}{S+B}}$$

4

3

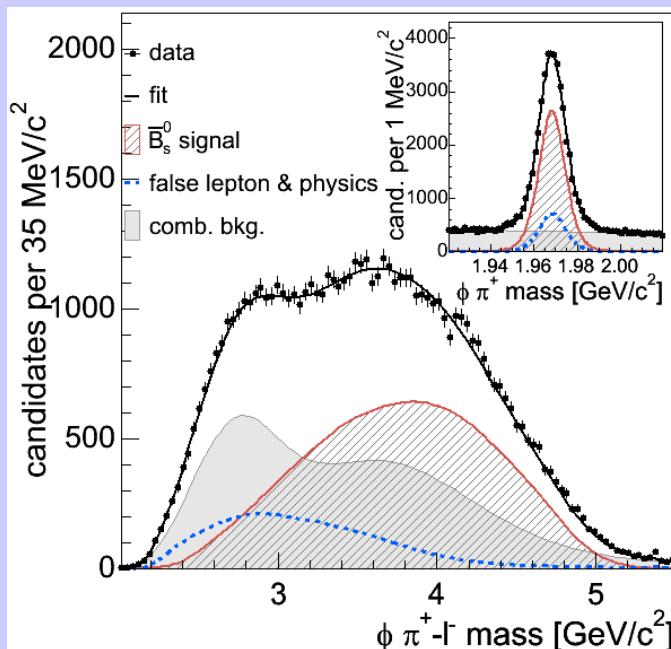
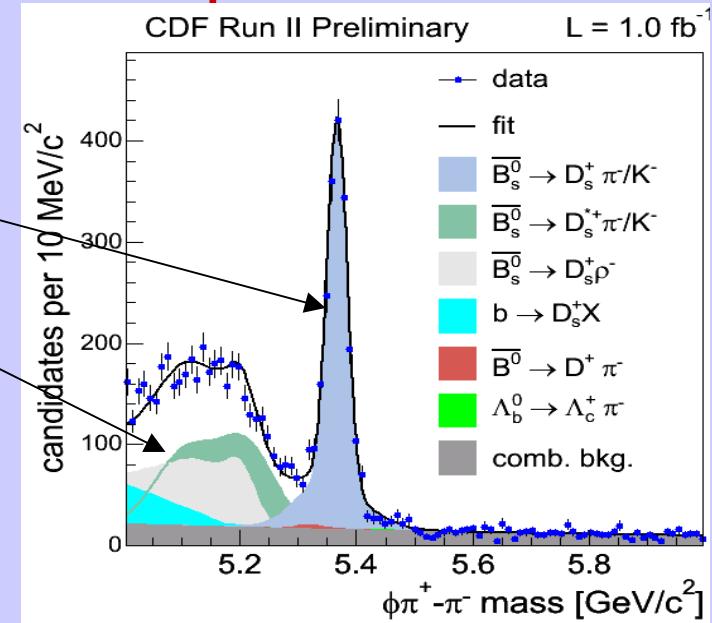
2

1

1.  $B_s$  signal (gives flavour at decay)
2. Flavour at production
3. Proper time
4. Fit for mixed and unmixed distribution

# $\Delta m_s$ @ CDFII - Samples

| Sample ( $L \approx 1\text{fb}^{-1}$ )  | Yield |
|---|-------|
| Fully reconstructed<br>( $B_s \rightarrow D_s(3)\pi$ , $D_s \rightarrow \phi\pi$ , $K^*K$ , $3\pi$ )          | 5600  |
| Partially reconstructed<br>( $B_s \rightarrow D_s^*\pi$ , $B_s \rightarrow D_s\rho$ missing $\pi^0, \gamma$ ) | 3100  |
| Semileptonic<br>( $B_s \rightarrow D_s^{(*)} l \nu_l$ , $l=e,\mu$ )   | 61500 |



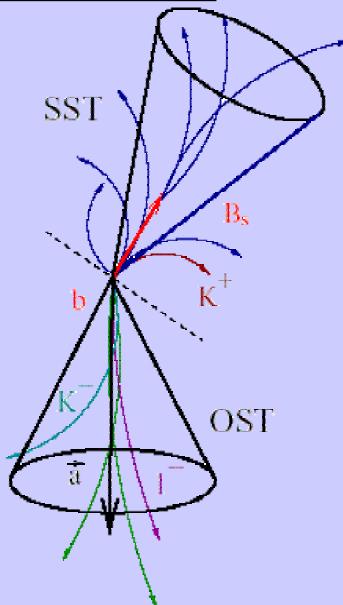
## Some highlights:

- NN kinematical selections
- PID ( $dE/dx + \text{TOF}$ ) for  $K/\pi$  separation (reject bg)

# $\Delta m_s$ @ CDFII - Flavour Tagging

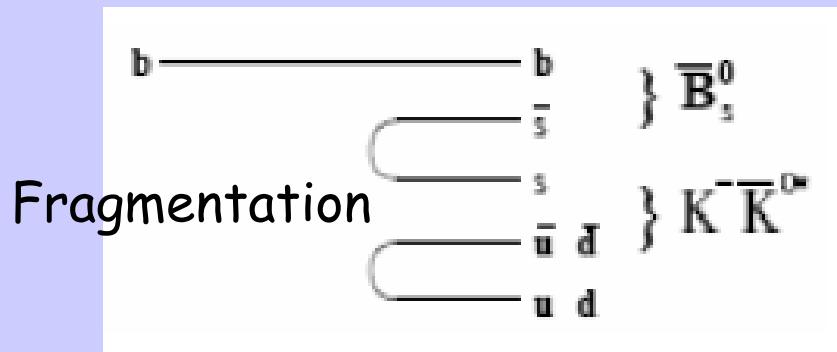
## Opposite Side Tagger

- $e, \mu$  charge
- Jet charge
- $K$  charge



## Same Side Tagger

- $B_s$  will often be produced with a  $K$



- Combine PID & kinematics

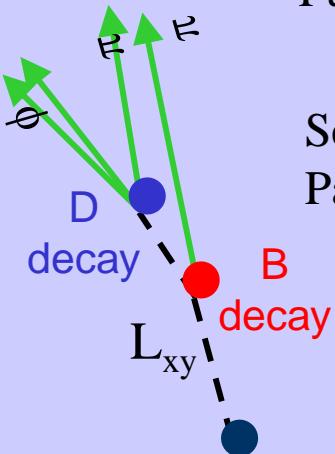
| Tagger          | $\varepsilon D^2 (\%)$ |
|-----------------|------------------------|
| OST NN          | $1.8 \pm 0.1$          |
| SST<br>hadronic | $3.7 \pm 0.9$          |
| semileptonic    | $4.8 \pm 1.2$          |

$\varepsilon$  = tagging efficiency

$$D = \frac{N^{right} - N^{wrong}}{N^{right} + N^{wrong}}$$

Combined in the final likelihood

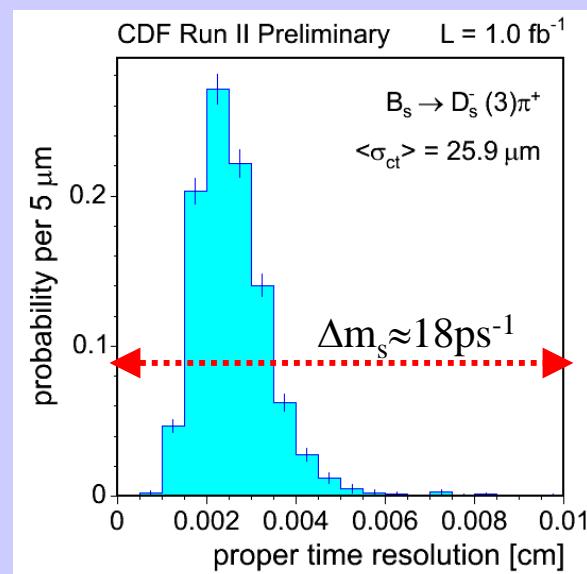
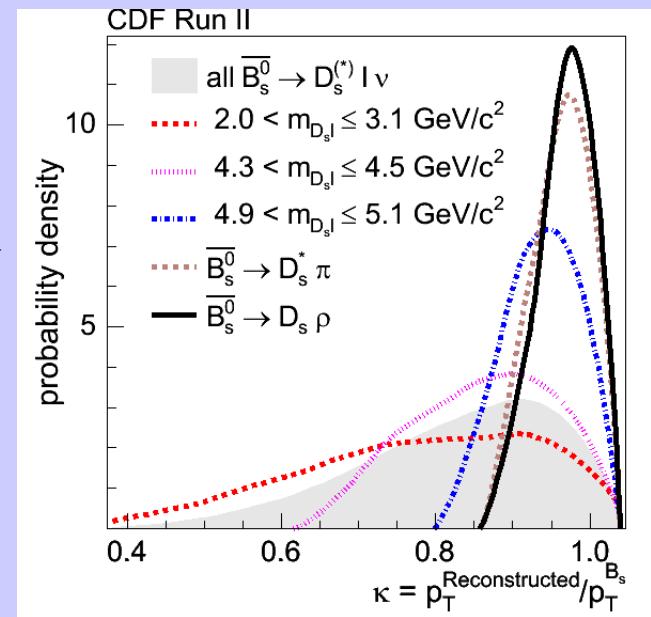
# $\Delta m_s$ @ CDFII - Proper time



Fully reconstructed  $c\tau = m(B_s) \frac{L_{xy}(B_s)}{p_T(B_s)}$

Semileptonic  
Part. reco'ed  $c\tau = m(B_s) \frac{L_{xy}(B_s / D_s l)}{p_T(B_s / D_s l)} K$

$$\frac{\sigma(c\tau)}{c\tau} = \frac{\sigma_{p_T}}{p_T} \oplus \left( \frac{\sigma_{L_{xy}}}{L_{xy}} \left[ \oplus \frac{\sigma_K}{K} \right] \right)$$

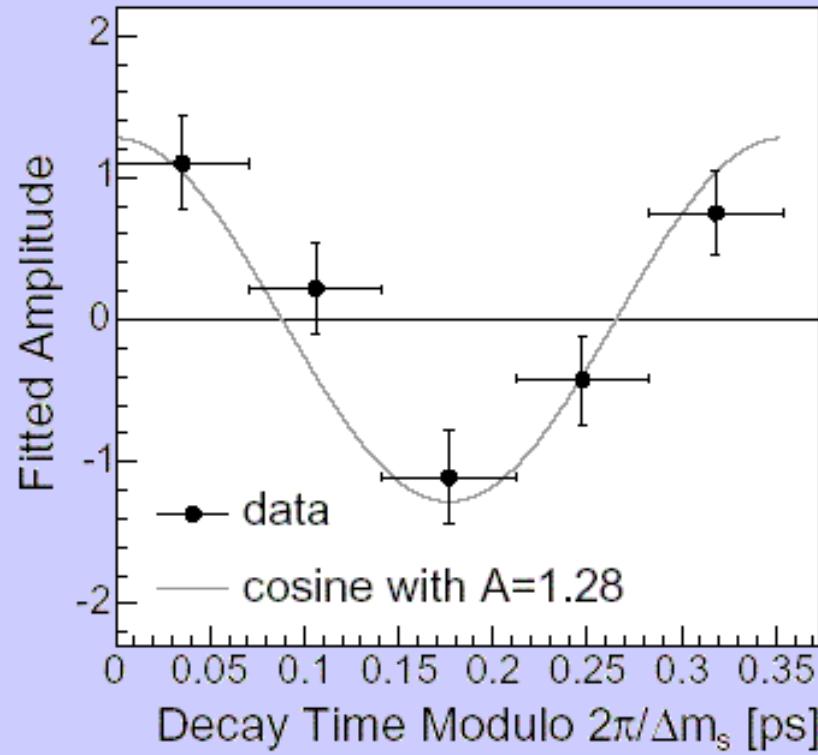
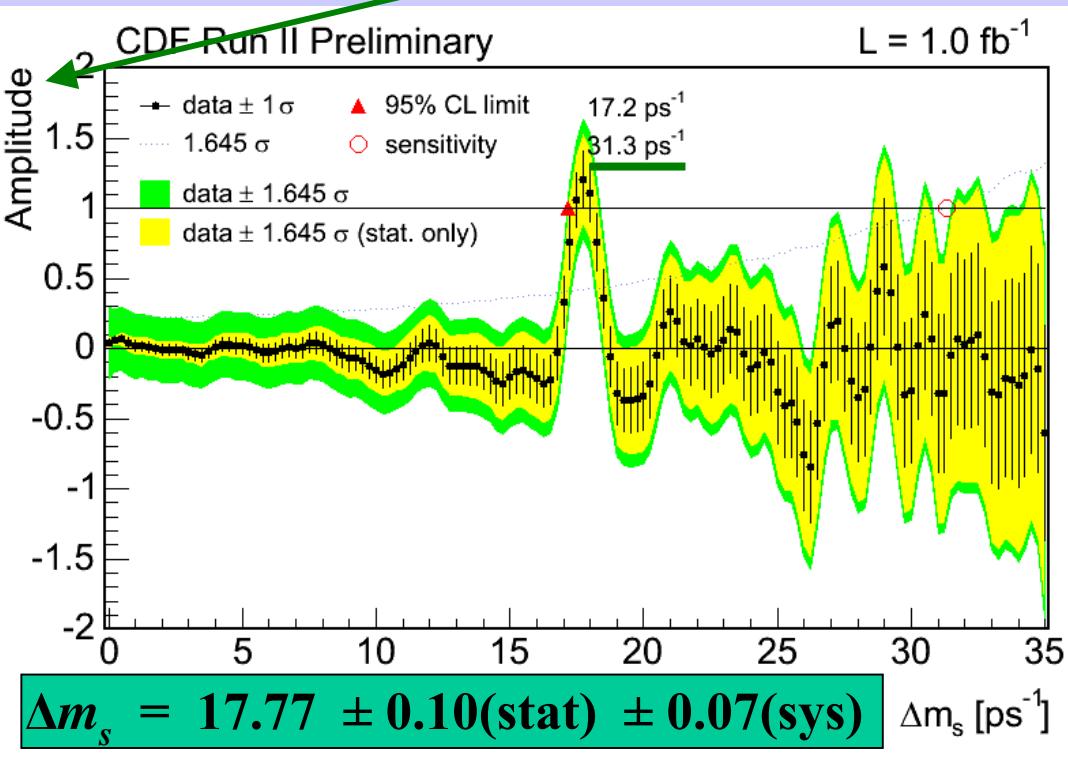


| Sample                  | $\langle \sigma(c\tau) \rangle$ |
|-------------------------|---------------------------------|
| Fully reconstructed     | 26 $\mu\text{m}$                |
| Partially reconstructed | 29 $\mu\text{m}$                |
| Semileptonic            | 45 $\mu\text{m}$                |

# $\Delta m_s$ @ CDFII - Result

[CDF Collaboration; PRL 97, 242003 2006 ]

$$P(t; A)_{B_s \rightarrow \bar{B}_s} = \frac{1}{2\tau} e^{-\frac{t}{\tau}} (1 \pm A \cos(\Delta m_s t))$$

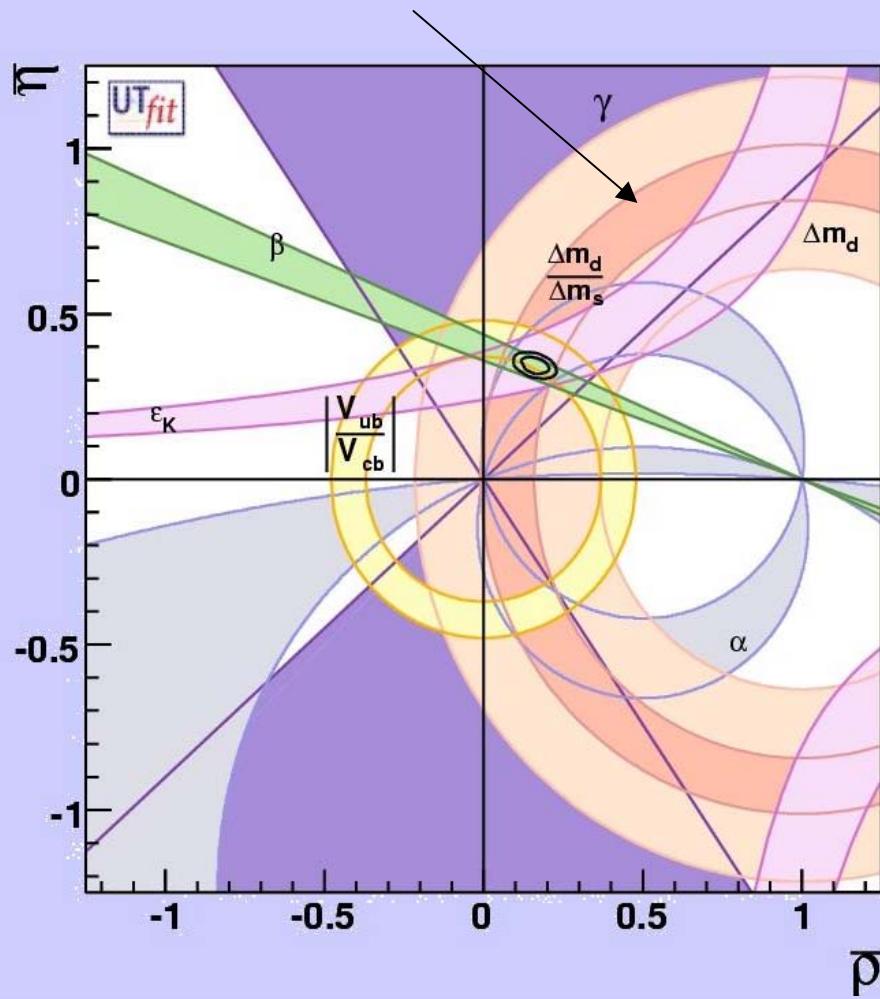


Compatible with SM prediction:  
 $\Delta m_s^{\text{SM}} = 19.30 \pm 6.68 \text{ ps}^{-1}$   
 [Lenz, Nierste; hep-ph/0612167]

p-value =  $8 \cdot 10^{-8} > 5 \sigma$

# $\Delta m_s$ @ CDFII - Unitarity triangle

$$\frac{|V_{td}|}{|V_{ts}|} = 0.2060 \pm 0.0007(\text{exp}) \quad {}^{+0.0081}_{-0.0060}(\text{theor})$$



→ UTFit

[<http://utfit.roma1.infn.it/ckm-results/ckm-results.html>]

CKMFitter has similar results

[<http://www.slac.stanford.edu/xorg/ckmfitter/>]

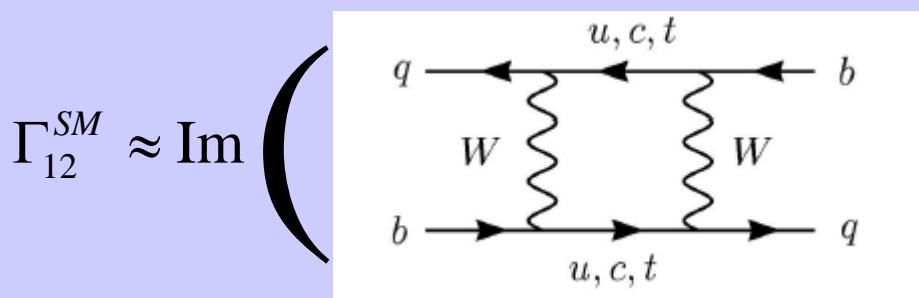
- Most precise measure of  $|V_{td}|/|V_{ts}|$
- Bigger uncertainty is theoretical
- SM still compatible with exp.

# Angular analysis of $B_s \rightarrow J/\Psi \Phi$

[D0 Collaboration, hep-ex/0701012]



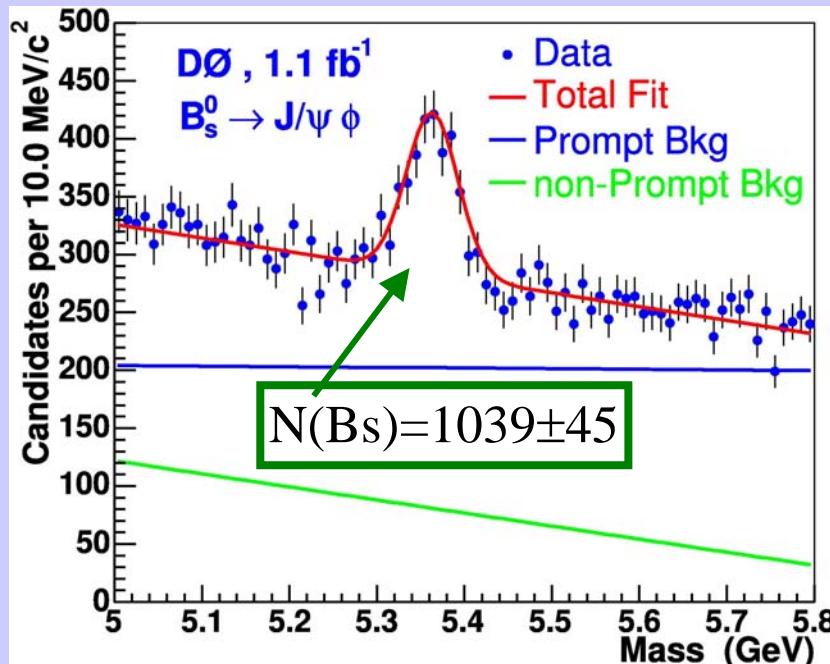
- $B_{s,L/H}$  have different lifetimes:  $\Delta\Gamma_s = \Gamma_L - \Gamma_H = 2|\Gamma_{12}^s| \cdot \cos(\phi_s + \phi_s^\Delta)$



$$\Delta\Gamma_s = 2|\Gamma_{12}^s| \cdot \cos(\phi_s + \phi_s^\Delta)$$

Arrows point from the terms  $\phi_s$  and  $\phi_s^\Delta$  in the equation to green circles, which are then connected by arrows to the text "NP CPV phase" and "SM CPV phase".

- Pseudo-Scalar→Vector-Vector decay:
  - CP-even and CP-odd final states
  - Angular distribution of  $B_{s,L/H}$  decay products can disentangle them
  - $J/\Psi \rightarrow \mu\mu$ ,  $\phi \rightarrow KK$
  - CP even/odd interference term  $\propto \sin \phi_s$

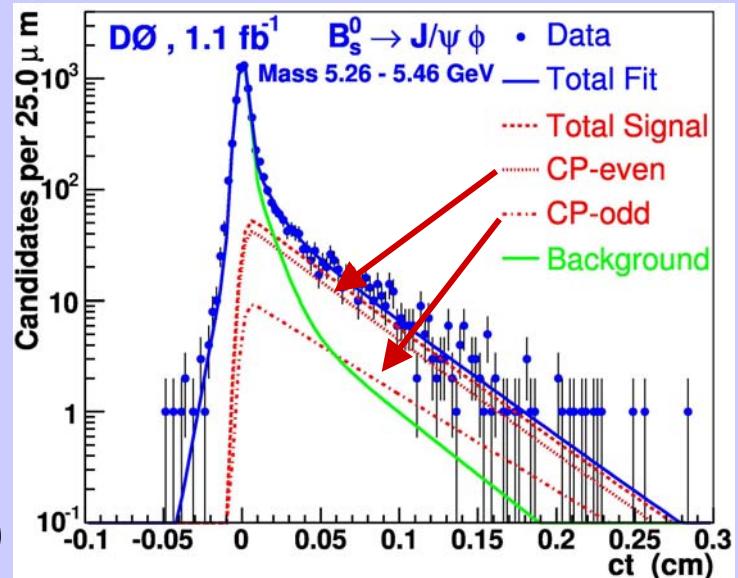


# $B_s \rightarrow J/\Psi \Phi$ – Results

Simultaneous fit to mass, decay length and 3 angles with an unbinned maximum likelihood fit

Assuming no CPV in  $B_s$  mixing

Allowing for CP violation ( $\phi_s$  free)



|                       | $\Delta\Gamma_s$ [ps $^{-1}$ ]  | $\phi_s$ [rad]                   | $\langle\tau\rangle$ [ps]       |
|-----------------------|---------------------------------|----------------------------------|---------------------------------|
| CDF (260 pb $^{-1}$ ) | $0.47^{+0.19}_{-0.24} \pm 0.01$ | ---                              | $1.40^{+0.15}_{-0.13} \pm 0.02$ |
| D0 (CP)               | $0.12^{+0.08}_{-0.10} \pm 0.02$ | ---                              | $1.52 \pm 0.08^{+0.01}_{-0.04}$ |
| D0 (CPV)              | $0.17 \pm 0.08 \pm 0.02$        | $-0.79 \pm 0.56^{+0.01}_{-0.14}$ | $1.49 \pm 0.08^{+0.01}_{-0.04}$ |

SM predictions:

[Lenz, Nierste; hep-ph/0612167]

$$\Delta\Gamma_s^{SM} = 0.096 \pm 0.039 \text{ ps}^{-1}$$

$$\phi_s^{SM} = (4.2 \pm 1.4) \cdot 10^{-3}$$

# Flavour charge asymmetries



[Phys. Rev. D 74, 092001 (2006)] [hep-ex/0701007 (2007)]

$$A_{SL}^{s,SM} = \frac{\Delta\Gamma_s}{\Delta m_s} \tan\phi_s = (2.06 \pm 0.57) \cdot 10^{-5}$$

Very sensible to CVP from new physics

[Lenz, Nierste; hep-ph/0612167]

Two D0 measures on untagged  $B_s$  samples

D0:  $B_s \rightarrow D_s \mu^\pm X$  ( $\mathcal{L} \approx 1.3 \text{ fb}^{-1}$ )

$$A_{SL}^{s,unt} = \frac{N(D_s^- \mu^+) - N(D_s^+ \mu^-)}{N(D_s^- \mu^+) + N(D_s^+ \mu^-)}$$

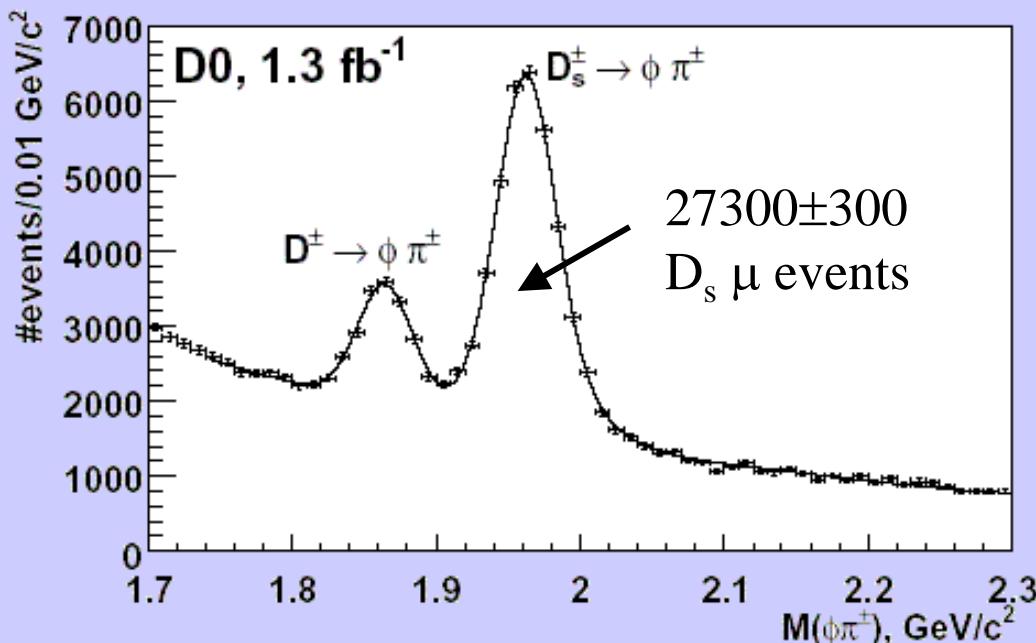
$$A_{SL}^s = 0.0245 \pm 0.0193 \pm 0.0035$$

D0:  $p\bar{p} \rightarrow b\bar{b} \rightarrow \mu^\pm \mu^\pm$  ( $\mathcal{L} \approx 1.0 \text{ fb}^{-1}$ )

$$A_{SL}^{\mu\mu} = \frac{N(b\bar{b} \rightarrow \mu^+ \mu^+ X) - N(b\bar{b} \rightarrow \mu^- \mu^- X)}{N(b\bar{b} \rightarrow \mu^+ \mu^+ X) + N(b\bar{b} \rightarrow \mu^- \mu^- X)}$$

About 580K di-muon events

$$A_{SL}^s = -0.0064 \pm 0.0101$$



- Both detector and physics contribute to charge asymmetry
- Flip B-field @ 0.1% regularly reduce detector systematics



# $\Delta\Gamma_s$ , $\phi_s$ and the 5M

Redo  $B_s \rightarrow J/\Psi \phi$  fit with the constraint

$$\Delta\Gamma_s \cdot \tan\phi_s = A_{SL}^s \cdot \Delta m_s$$

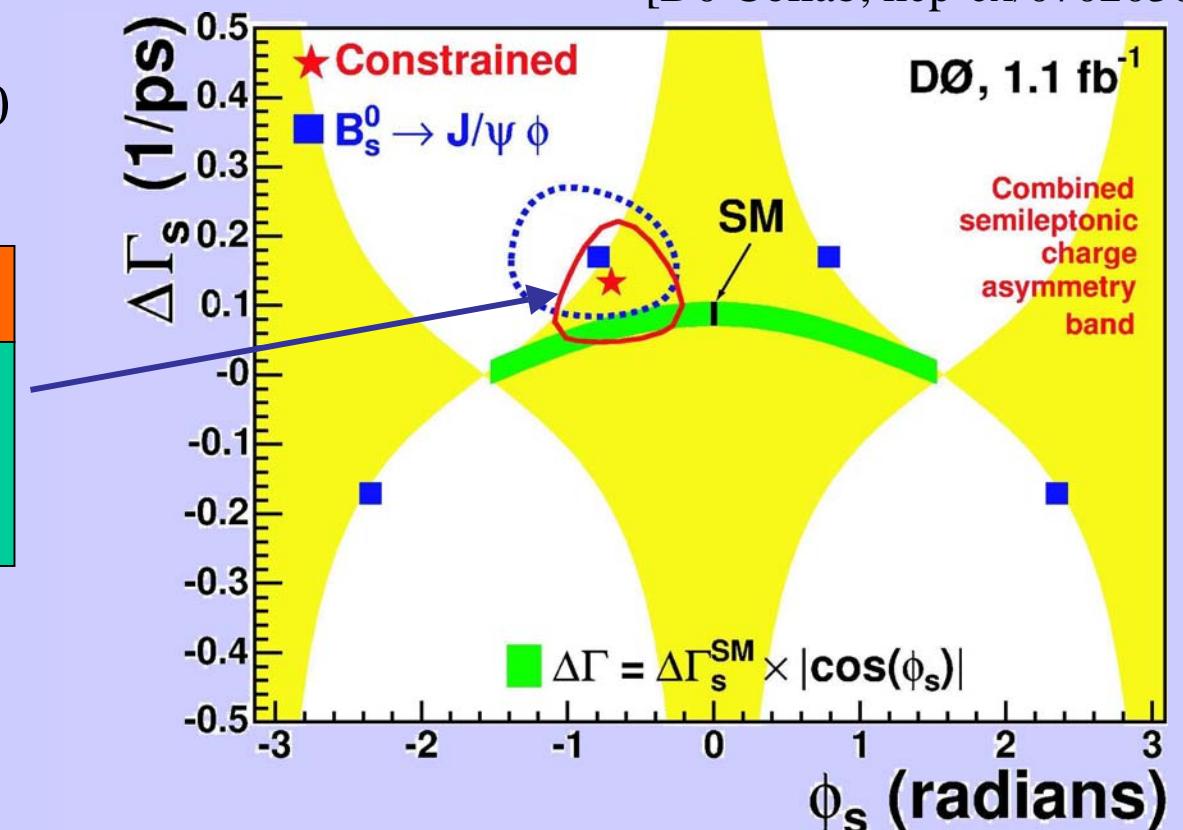
$$A_{SL}^{s,combined} = 0.0001 \pm 0.0090$$

Fit results:

$$\Delta\Gamma_s = 0.13 \pm 0.09 \text{ ps}^{-1}$$

$$\phi_s = -0.70^{+0.47}_{-0.39}$$

[D0 Collab, hep-ex/0702030]



SM still compatible, but room for new physics!

# Conclusions

- Described more significant measures for  $\Delta m_s$ ,  $\Delta \Gamma_s$  and  $\phi_s$  at Tevatron
- SM still compatible with experimental results but...
  - Upcoming new CDF results ( $1\text{fb}^{-1}$ )
  - Both CDF and D0 already have  $2\text{fb}^{-1}$  to analyze, more to come
  - Preparing for tagged  $B_s \rightarrow J/\Psi \Phi$  analysis  
(both CDF and D0, roughly  $\approx 0.3$  sensitivity on  $\phi_s$ )
  - Future LHCb measures  
(sensitivity of 0.02 on  $\phi_s$  with  $2\text{fb}^{-1}$ )

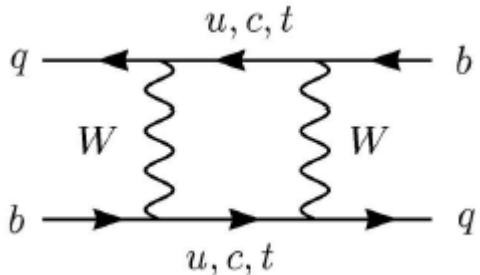
[Peter Vankov, Lake Louise Winter Institute 2007]

will be able to further check the SM prediction.

**STAY TUNED!**  


# BACKUP

# The $B_s$ - $\bar{B}_s$ system



Assuming CPT invariance

$$-i \frac{d}{dt} \begin{pmatrix} |B_s\rangle \\ |\bar{B}_s\rangle \end{pmatrix} = \left[ \begin{pmatrix} M_{11} & M_{12} \\ M_{12}^* & M_{22} \end{pmatrix} - \frac{i}{2} \begin{pmatrix} \Gamma_{11} & \Gamma_{12} \\ \Gamma_{12}^* & \Gamma_{22} \end{pmatrix} \right] \begin{pmatrix} |B_s\rangle \\ |\bar{B}_s\rangle \end{pmatrix}$$

Mass eigenstates:

$$B_{S,L} = p|B_s\rangle \pm q|\bar{B}_s\rangle$$

Also CP eigenstates if  $|p| / |q| = 1$

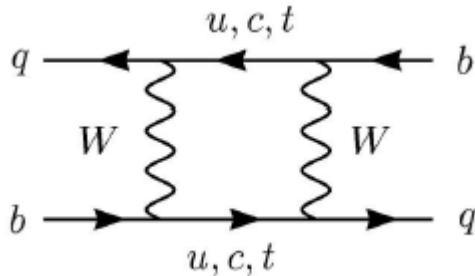
Many interesting observables in order to explore  $B_s$  system properties:

$$\Delta m_s = M_H - M_L \cong 2|M_{12}| \quad \Delta\Gamma_s = \Gamma_L - \Gamma_H \cong 2|\Gamma_{12}| \cos\phi_s \quad \phi_s = \arg\left(-\frac{M_{12}}{\Gamma_{12}}\right)$$

$$A_{fs}^s = \text{Im} \frac{\Gamma_{12}}{M_{12}} = \frac{|\Gamma_{12}|}{|M_{12}|} \sin\phi_s = \frac{\Delta\Gamma_s}{\Delta m_s} \tan\phi_s = \frac{N(B_s \rightarrow f) - N(\bar{B}_s \rightarrow \bar{f})}{N(B_s \rightarrow f) + N(\bar{B}_s \rightarrow \bar{f})}$$

(with  $\bar{B}_s \rightarrow f$  and  $B_s \rightarrow \bar{f}$  forbidden;  $\bar{f}$  is the CP conjugate )

# NP in $B_s$ - $\bar{B}_s$ system



Lowest Order

Dispersive part  $\rightarrow M_{12}$

Absorptive part  $\rightarrow \Gamma_{12}$

$\Gamma_{12}$  dominated by tree-level  $b \rightarrow \bar{c} \bar{c} s$

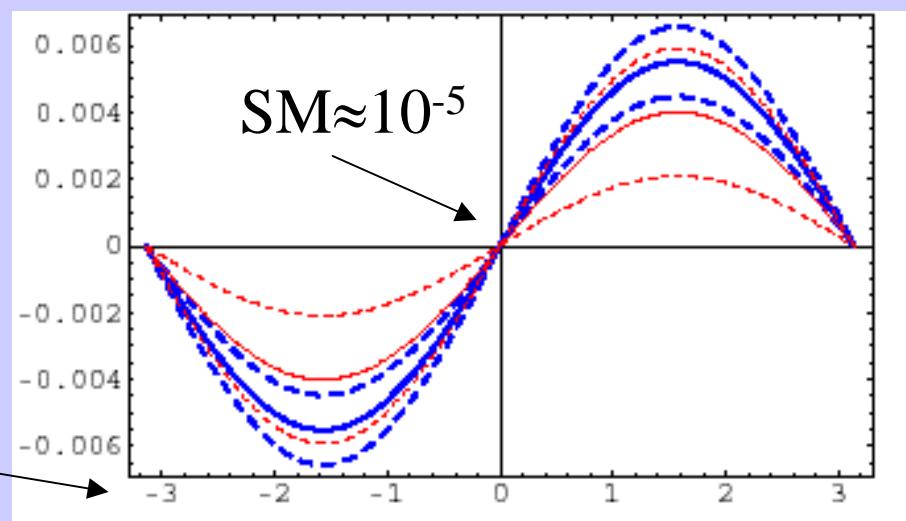
$M_{12}$  sensible to new physics; model independent approach:

$$M_{12} = M_{12}^{SM} \Delta_s = M_{12}^{SM} |\Delta_s| e^{i\phi_s^\Delta} \quad [\text{Lenz, Nierste; hep-ph/0612167}]$$

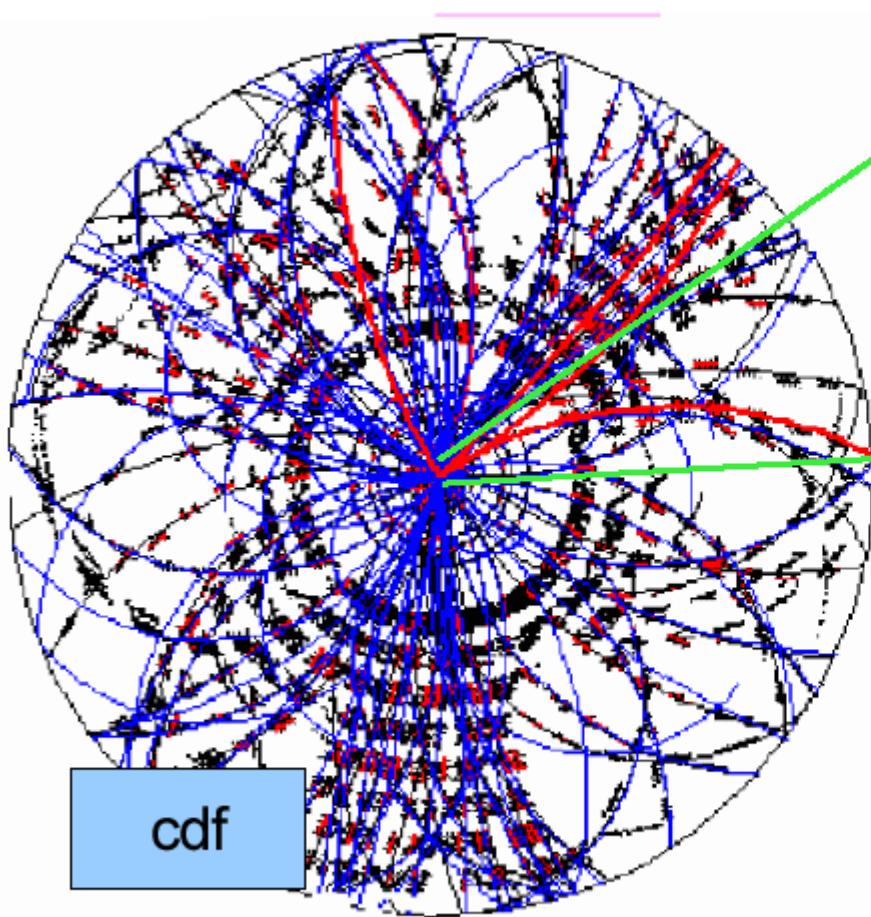
$\Rightarrow \Delta m_s$  sensible to  $|\Delta_s|$

$\Rightarrow \Delta \Gamma_s$  and  $a_{fs}^s$  also to  $\phi_s^\Delta$

e.g. sensibility of  $a_{fs}^s$  to  $\phi_s^\Delta$

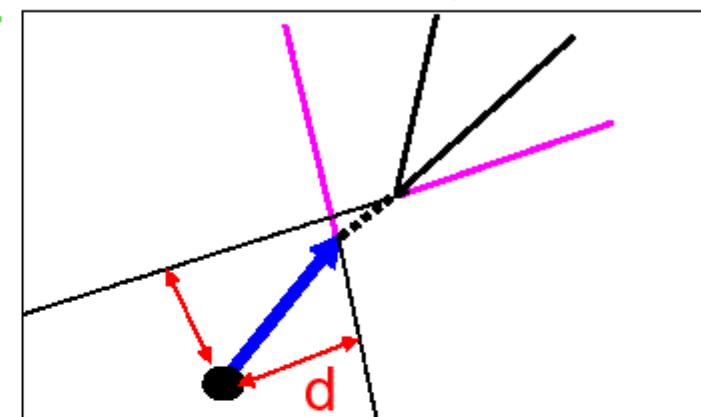


A typical B event at a hadron collider

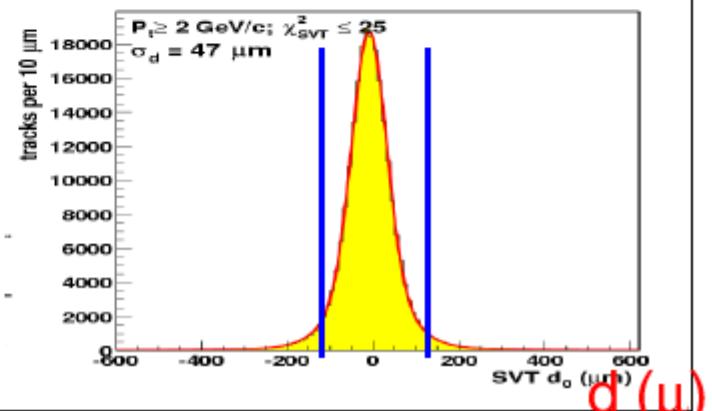


=> look for displaced tracks

Trigger on events with two displaced ( $d > 120 \mu\text{m}$ ) tracks



very fast reconstruction of silicon data at L2 (20 $\mu\text{s}$  latency) by dedicated hardware: SVT



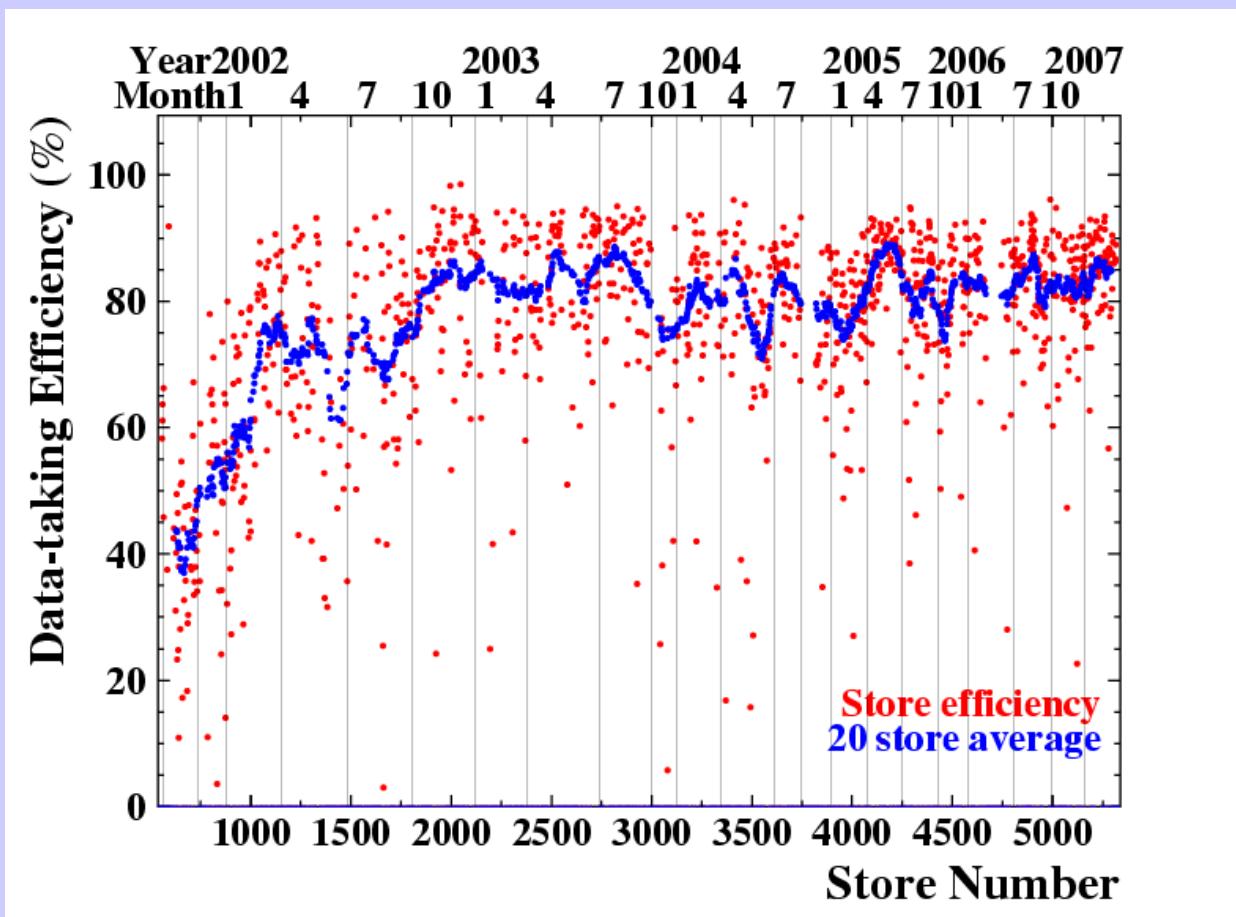
# Tevatron

CDF Expect an integrated luminosity delivered:

3-4  $\text{fb}^{-1}$  for end 2007

4-6  $\text{fb}^{-1}$  for end 2008

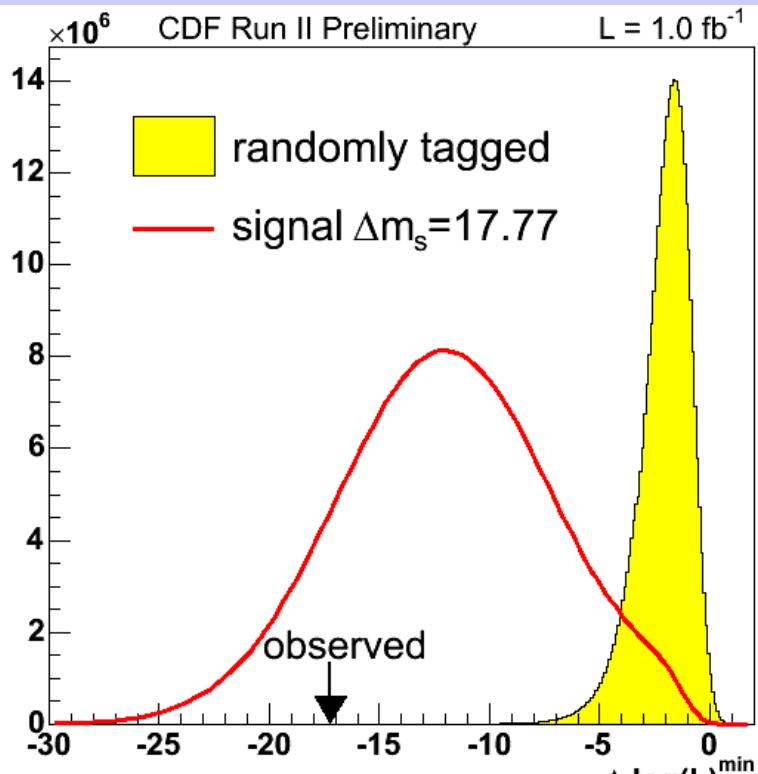
6-8  $\text{fb}^{-1}$  for end 2009



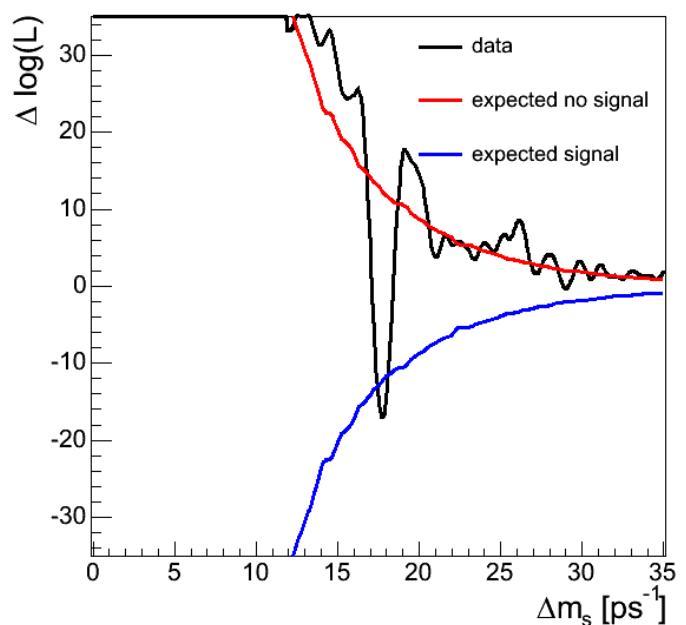
# Bs mixing - CDF

## Systematics

### P-value distribution



| Systematics          | Syst. [ps <sup>-1</sup> ] |
|----------------------|---------------------------|
| SVX Alignment        | 0.04                      |
| Track Fit Bias       | 0.05                      |
| PV bias from tagging | 0.02                      |
| All other syst.      | <0.01                     |
| <b>TOTAL</b>         | <b>0.07</b>               |



**p-value =  $8 \cdot 10^{-8} > 5 \sigma$**

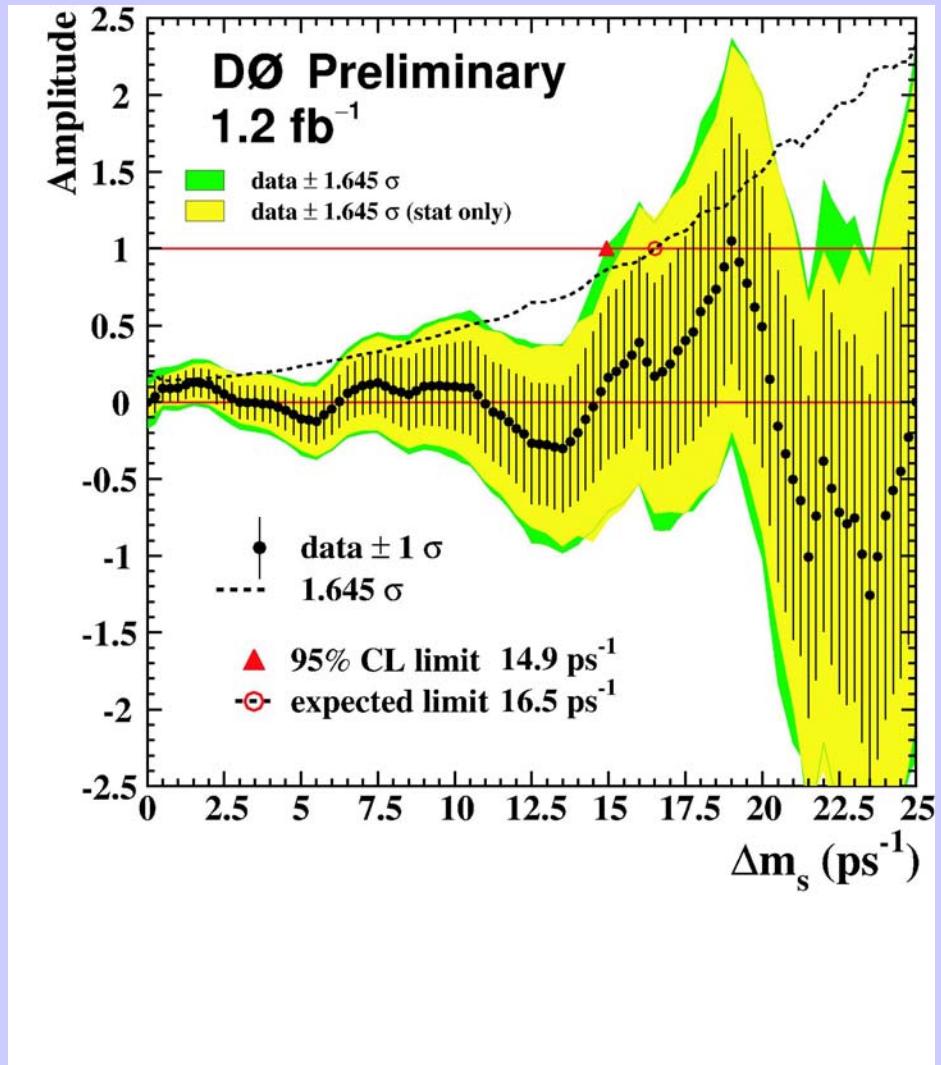
# Bs mixing – D0

D0 results:  
( $1.2 \text{ fb}^{-1}$ )

$\text{Bs} \rightarrow \text{Ds e X}$   
( $\text{Ds} \rightarrow \phi\pi$ )

$\text{Bs} \rightarrow \text{Ds } \mu \text{ X}$   
( $\text{Ds} \rightarrow \phi\pi, \text{K}^*\text{K}, \text{KsK}$ )

$\Delta m_s > 14.9 \text{ ps}^{-1}$  @ 95% CL  
(expected:  $16.5 \text{ ps}^{-1}$ )



# Flavour charge asymmetries - I

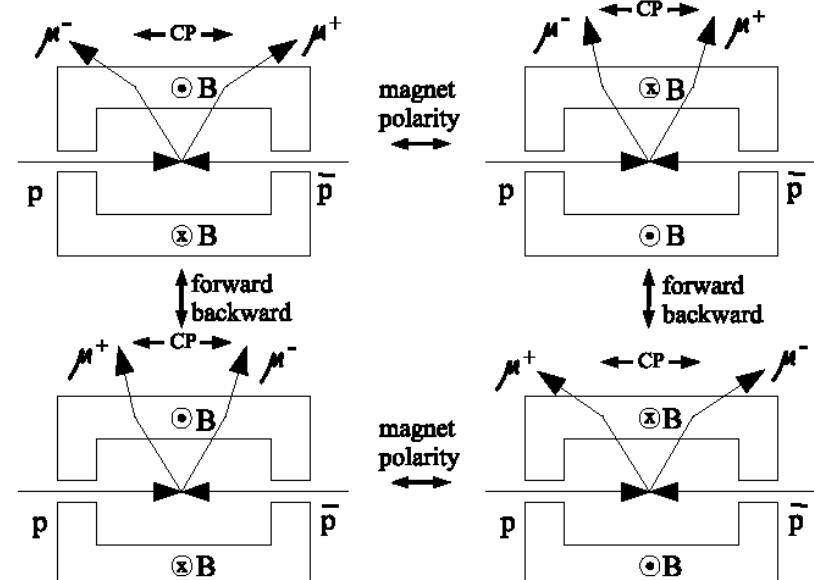
Very sensible to new physics. In the  $B_s$  system we have two measures:

$$D0: p\bar{p} \rightarrow b\bar{b} \rightarrow \mu^\pm \mu^\pm \quad (\mathcal{L} \approx 1.0 \text{fb}^{-1})$$

[Phys. Rev. D 74, 092001 (2006)]

$$A_{SL}^{\mu\mu} = \frac{N(b\bar{b} \rightarrow \mu^+ \mu^+ X) - N(b\bar{b} \rightarrow \mu^- \mu^- X)}{N(b\bar{b} \rightarrow \mu^+ \mu^+ X) + N(b\bar{b} \rightarrow \mu^- \mu^- X)}$$

$$= \frac{1}{4} \left( A_{SL}^d + \frac{f_s}{f_d} \frac{Z_s(\Gamma_s, \Delta\Gamma_s, \Delta m_s)}{Z_d(\Gamma_d, \Delta\Gamma_d, \Delta m_d)} A_{SL}^s \right)$$



$$D0: B_s \rightarrow D_s \mu^\pm X \quad (\mathcal{L} \approx 1.3 \text{fb}^{-1})$$

[hep-ex/0701007 (2007)]

$$A_{SL}^{s,unt} = \frac{N(D_s^- \mu^+) - N(D_s^+ \mu^-)}{N(D_s^- \mu^+) + N(D_s^+ \mu^-)}$$

$$= \frac{1}{2} A_{SL}^s = \frac{1}{2} \frac{\Delta\Gamma_s}{\Delta m_s} \tan\phi_s$$

## Similar methods

## Some highlights:

- Both detector and physics contribute to charge asymmetry.
- Flip B-field @ 0.1%: reduce det. syst.
- Divide sample in 8 sub-samples (B polarity,  $\mu$  charge,  $\eta$  sign)
- Extract all asymmetries (6) from data

# Flavour charge asymmetries - II

$$n_q^{\alpha\beta} = \frac{1}{4} N \epsilon^\beta (1 + qA)(1 + q\gamma A_{fb})(1 + \gamma A_{\text{det}}) \cdot (1 + q\beta\gamma A_{ro})(1 + q\beta A_{q\beta})(1 + \beta\gamma A_{\beta\gamma})$$

q= charge of  $\mu$   
 $\beta$ = polarity of B  
 $\gamma$ = sign of  $\eta$

|                                       | D <sub>s</sub> $\mu$ | $\mu\mu$             |
|---------------------------------------|----------------------|----------------------|
| A                                     | 0.0102 $\pm$ 0.0081  | -0.0005 $\pm$ 0.0013 |
| A <sub>fb</sub>                       | -0.0046 $\pm$ 0.0081 | 0.0004 $\pm$ 0.0005  |
| A <sub>det</sub>                      | -0.0051 $\pm$ 0.0081 | -0.0176 $\pm$ 0.0005 |
| A <sub>ro</sub>                       | -0.0352 $\pm$ 0.0081 | -0.0275 $\pm$ 0.0005 |
| A <sub><math>\beta\gamma</math></sub> | -0.0097 $\pm$ 0.0081 | -0.0008 $\pm$ 0.0005 |
| A <sub>q<math>\beta</math></sub>      | 0.0030 $\pm$ 0.0081  | 0.0064 $\pm$ 0.0005  |

From the raw A asymmetry  
 correct for bg contamination

$$A_{SL}^{\mu\mu} = -0.0092 \pm 0.0044(\text{stat.}) \pm 0.0032(\text{syst.}) \rightarrow \text{inclusive } \mu\mu \text{ measure}$$

$$A_{SL}^s = 0.0245 \pm 0.0193(\text{stat.}) \pm 0.0035(\text{syst.}) \rightarrow D_s \mu \text{ measure}$$

# Flavour charge asymmetries - III



$A_{SL}^{\mu\mu}$  contains effects from both  $B_s$  and  $B_d$ :

$$A_{SL}^{\mu\mu} = \frac{1}{4} \left( A_{SL}^d + \frac{f_s}{f_d} \frac{Z_s(\Gamma_s, \Delta\Gamma_s, \Delta m_s)}{Z_d(\Gamma_d, \Delta\Gamma_d, \Delta m_d)} A_{SL}^s \right)$$

using measured values of  $\Gamma_{s,d}$ ,  $\Delta\Gamma_{s,d}$ ,  $\Delta m_{s,d}$ , we have for  $A_{SL}^s$

$$A_{SL}^s = -0.0064 \pm 0.0101$$

[D0 Collab., hep-ex/0702030]

While the  $B_s$ -semileptonic measure gives:

$$A_{SL}^s = 0.0245 \pm 0.0193(stat.) \pm 0.0035(syst.)$$

The two measures are nearly independent {  
    ( $<1\%$  of  $B_s$  in  $\mu\mu$  sample)  
    ( $\approx 10\%$  of  $\mu\mu$  in  $B_s$  sample)}

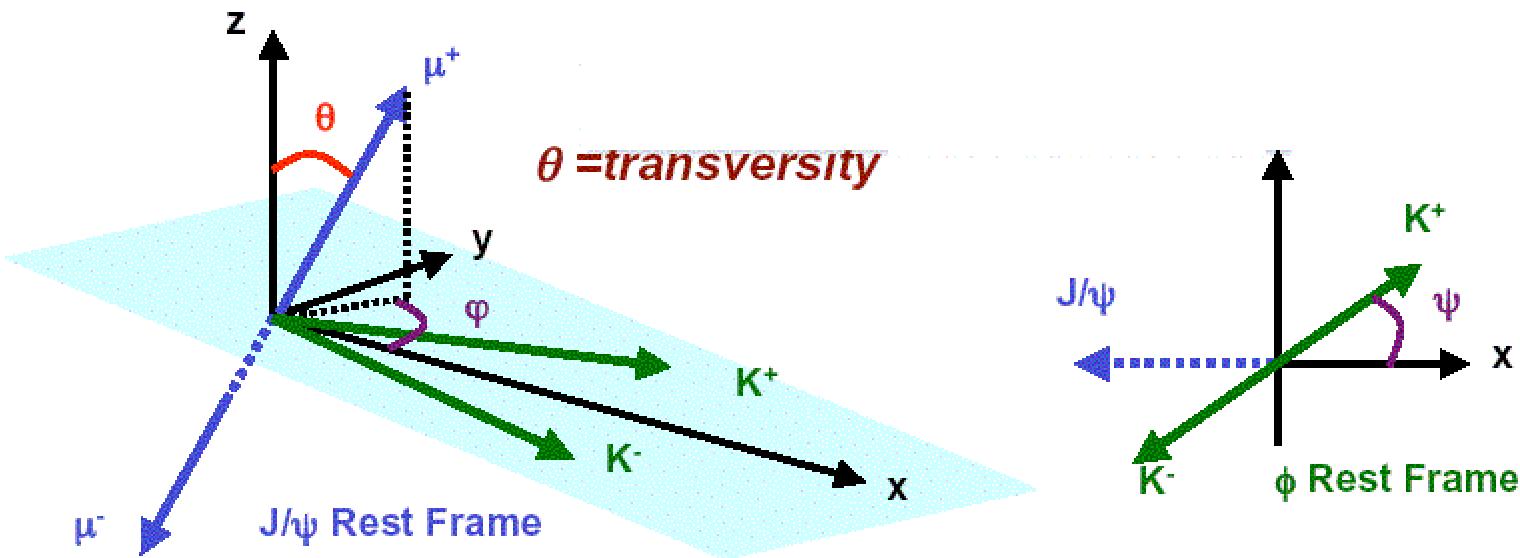
$$\Rightarrow A_{SL}^s = 0.0001 \pm 0.0090$$

To be compared to the SM prediction:

$$A_{SL}^{s,SM} = (2.06 \pm 0.57) \cdot 10^{-5}$$

[Lenz, Nierste; hep-ph/0612167]

# Transversity



$$\begin{aligned}
 & \frac{d^3 \Gamma}{dcos\theta \, d\phi \, dcos\psi} \propto \frac{9}{16\pi} \left[ 2|A_0(0)|^2 e^{-\Gamma_L t} \cos^2\psi (1 - \sin^2\theta \cos^2\phi) \right. \\
 & + \sin^2\psi \left\{ |A_{||}(0)|^2 e^{-\Gamma_L t} (1 - \sin^2\theta \sin^2\phi) + |A_{\perp}(0)|^2 e^{-\Gamma_R t} \sin^2\theta \right\} \\
 & + \frac{1}{\sqrt{2}} \sin 2\psi \left\{ |A_0(0)||A_{\perp}(0)| \cos(\delta_2 - \delta_1) e^{-\Gamma_L t} \sin^2\theta \sin 2\phi \right\} \\
 & + \left\{ \frac{1}{\sqrt{2}} |A_0(0)||A_{\perp}(0)| \cos \delta_2 \sin 2\psi \sin 2\theta \cos \phi \right\} \frac{1}{2} (e^{-\Gamma_R t} + e^{-\Gamma_L t}) \delta\phi \\
 & \left. - \left\{ \frac{1}{\sqrt{2}} |A_{||}(0)||A_{\perp}(0)| \cos \delta_1 \sin^2\psi \sin 2\theta \sin \phi \right\} \frac{1}{2} (e^{-\Gamma_R t} + e^{-\Gamma_L t}) \delta\phi \right] H(\cos\psi) F(\phi) G(\cos\theta)
 \end{aligned}$$