$\overline{B} \to X_s\gamma$ at NNLO in the SM

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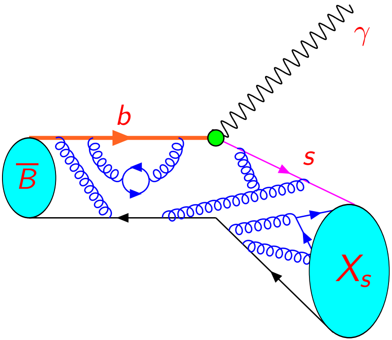
Outline

1. Introduction: generalities of the $b \to s\,\gamma$ process
2. $\mathcal{B}(\overline{B}^0 \to X_s\,\gamma)$ at NNLO
3. Calculation of the NNLO photon energy spectrum
   - The contribution of $\mathcal{O}_7 - \mathcal{O}_7$

Inclusive $\overline{B}^0 \to X_s\,\gamma$ Decay



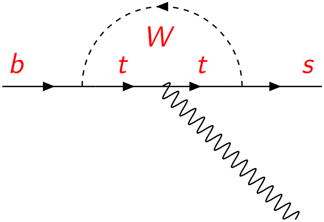
Well approximated by the partonic process

$$\Gamma(\overline{B} \to X_s\,\gamma) = \Gamma(b \to s\,\gamma) + \Delta^{\text{non pert.}}$$

FCNC process at the parton level

The $b \to s\gamma$ Process

In the SM in is loop suppressed, mediated by "Penguin" diagrams



In principle, particularly sensitive to new physics beyond the Standard Model

Effective Theory

Suitable framework for the resummation of large $\alpha_s(m_b)\ln(m_b/M) \longrightarrow$ effective low-energy theory with 5 quarks

$$\begin{aligned}\mathcal{L} &= \mathcal{L}_{\text{QED}\otimes\text{QCD}}\,(\text{u,d,c,s,b}) \\ &+ \sum_{i=1}^{8} \frac{4G_F}{\sqrt{2}} V_{\text{ts}}^* V_{\text{tb}} C_i(\mu, M_{\text{heavy}}) \mathcal{O}_i(\mu)\end{aligned}$$

- The Wilson coefficients C_i describe the short distance physics
- The matrix elements of the effective operators $\mathcal{O}_i$ describe the long distance dynamics
- In the case of B decays, the factorization scale $\mu \ll M_W, m_t$

Calculational Steps

1. Matching: Evaluating $C_i(\mu_0)$ at the renormalization scale $\mu_0 \sim m_t, M_W$ by requiring equality of the SM and effective theory Green's functions at the leading order in (external momenta)/(m_t, M_W)
2. Mixing: Calculating the operator mixing under renormalization, deriving the effective theory RG equations and evolving $C_i(\mu_0)$ from μ_0 down to $\mu_b \sim m_b$

$$\mu \frac{d}{d\mu} C_i(\mu) = \gamma_{ji} C_j(\mu)$$

3. Matrix elements: Evaluating the $b \to X_s^{\text{parton}} \gamma$ amplitudes at $\mu_b \sim m_b$

$$\langle s\,\gamma | \mathcal{O}_i(\mu_b) | b \rangle$$

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$\mathcal{B}(\overline{B}^0 \to X_s \gamma)$: Experimental Status

Experimental value $(E_\gamma > 1.6\,\mathrm{GeV})$

$$\mathcal{B}\left(\overline{B} \to X_s \gamma\right) = (3.55 \pm 0.24^{+0.09}_{-0.10} \pm 0.03) \times 10^{-4}$$

Heavy Flavor Averaging Group ('06)

1. first error combined statistical and systematic
2. second error theory input on the shape function/extrapolation
3. third error $b \to d\gamma$ contamination

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Combined experimental error $\longrightarrow$ same size as the expected NNLO QCD corrections to $\Gamma(b \to X_s\,\gamma)$, larger than the non-perturbative corrections

$\mathcal{B}(\overline{B}^0 \to X_s\gamma)$: General Structure

the partonic decay $b \to s\gamma$ is usually normalized to the semileptonic decay rate in order to get rid of uncertainties from CKM and m_b^5

courtesy of U. Haisch

$$\mathcal{B}(\overline{B}^0 \to X_s\gamma)_{\mathrm{SM}}^{E_\gamma > 1.6\,\mathrm{GeV}} = \mathcal{B}(\overline{B}^0 \to X_c e\overline{\nu}) \left[\frac{\Gamma(b \to s\gamma)}{\Gamma(b \to ce\overline{\nu})}\right]_{\mathrm{LO}}$$

$$\left\{1 + \mathcal{O}(\alpha_s) + \mathcal{O}(\alpha) + \mathcal{O}(\alpha_s^2) + \mathcal{O}\left(\frac{\Lambda^2}{m_b^2}\right) + \mathcal{O}\left(\frac{\Lambda^2}{m_c^2}\right) + \mathcal{O}\left(\alpha_s \frac{\Lambda}{m_b}\right)\right\}$$

- NLO QCD 25%
- NLO EW 4%
- NNLO QCD 7%

→ Perturbative

- LO QCD + NLO m_b 1%
- LO QCD + NLO m_b 3%
- NLO QCD + LO m_b 5%

→ Non-Perturbative

Estimate of $\mathcal{B}(\overline{B}^0 \to X_s \gamma)$ at NNLO

The calculation of the NNLO QCD corrections is a big enterprise:

Bieri, Greub, Steinhauser ('03), Misiak, Steinhauser ('04), Gorbahn, Haisch ('04), Gorbahn, Haisch, Misiak ('05), Melnikov, Mitov ('05), Blokland, Czarnecki, Misiak, Slusarczyk, Tkachov ('05), Asatrian, Hovhannisyan, Poghosyan, Ewerth, Greub, Hurth ('06), Asatrian, Ewerth, AF, Gambino, Greub ('06), Czakon, Haisch, Misiak ('06)

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NNLO ESTIMATE ($E_\gamma > 1.6\,\mathrm{GeV}$)

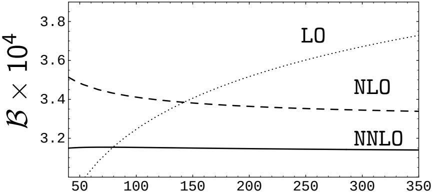
$$\mathcal{B}\left(\overline{B} \to X_s\,\gamma\right) = (3.15 \pm 0.23) \times 10^{-4}$$

Misiak *et al.*

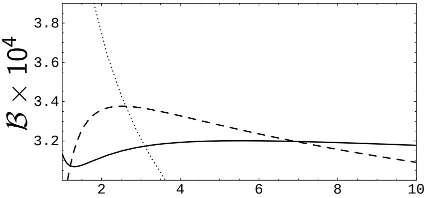
The result is lower than the NLO results and it is about 1σ lower than the experimental average

Uncertainties: non-perturbative (5%), parametric (3 %), higher order (3%), m_c interpolation ambiguity (3 %)

Scale Dependence



matching scale μ_0 [GeV]



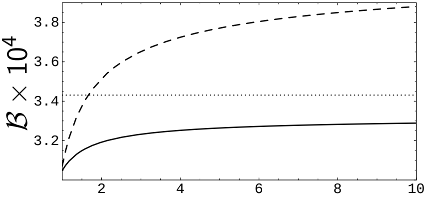
low-energy scale μ_b [GeV]

Clear reduction of scale dependence

Charm mass dependence remains visible

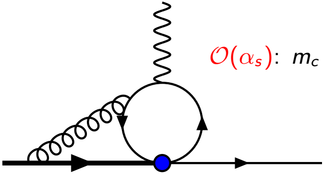
Central values:

$\mu_b = 2.5$ GeV $\mu_c = 1.5$ GeV $\mu_0 = m_t$

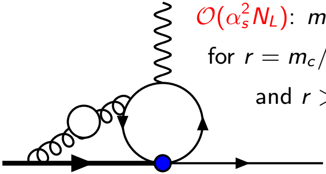


charm mass renormalization scale μ_c [GeV]

m_c Dependence



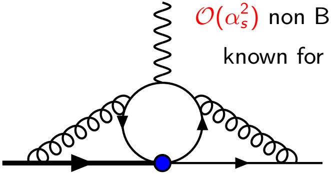
$\mathcal{O}(\alpha_s)$: m_c dep known



$\mathcal{O}(\alpha_s^2 N_L)$: m_c dep known

for $r = m_c/m_b \ll 1/2$

and $r \gg 1/2$



$\mathcal{O}(\alpha_s^2)$ non BLM: m_c dep

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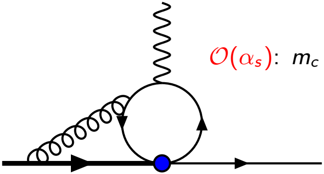
Misiak & Steinhauser '06

m_c DEPENDENCE



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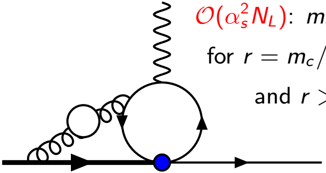

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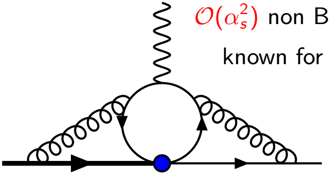

$\mathcal{O}(\alpha_s^2)$ non BLM: m_c dep

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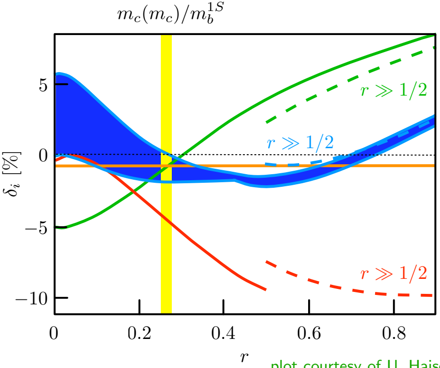
Misiak & Steinhauser '06

BLM corrections generally dominant. Since small-large m_c interpolation works very well at NLO and for NNLO-BLM use it as a first step

Three ansatzt for matching NON-BLM

NNLO terms at $m_c = 0$ are employed and

averaged: 3% uncertianty on the BR



plot courtesy of U. Haisch

Photon Energy Spectrum-I

In the $b \to s\,\gamma$ decay at LO the photon energy is fixed

$$E_\gamma = \frac{m_b}{2} \qquad \text{(in the b rest frame)}$$

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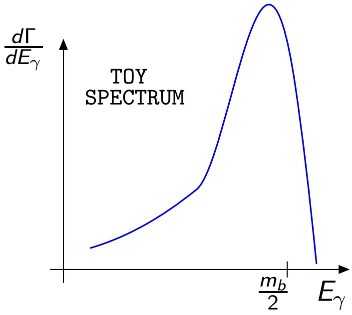

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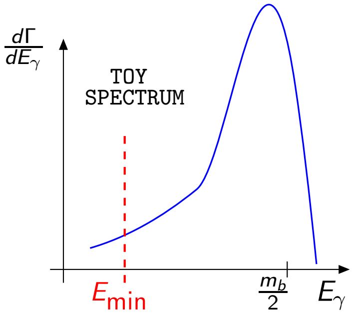

Experimental lower cut on the photon energy

Photon Energy Spectrum-II

The photon energy spectrum originates from

- perturbative gluon / quark pair bremsstrahlung
  $b \to s\,\gamma + n \times g + m \times q\overline{q}$
- non-perturbative motion of the b quark in the B meson (Fermi motion)

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$$\mathcal{O}_7\mathcal{O}_7 \Longrightarrow \begin{cases} \texttt{Melnikov, Mitov'05} \\ \texttt{Asatrian, Ewerth, Greub+} \\ \texttt{AF, Gambino'06} \end{cases}$$

$$\mathcal{O}_7\mathcal{O}_8 \Longrightarrow \begin{cases} \texttt{Greub}\ \textit{et al.} \\ \texttt{AF, Gambino, Mitov, Ossola} \qquad \textit{in progress} \end{cases}$$

From Amplitudes to Cutkosky Rules

$$z = \frac{2E_\gamma}{m_b}$$

$$\frac{dG_{77}(z)}{dz} = \delta(1-z) + \frac{\alpha_s(m_b)}{\pi} C_F H^{NLO}(z) + \left(\frac{\alpha_s(m_b)}{\pi}\right)^2 C_F H^{NNLO}(z) + O(\alpha_s^3)$$

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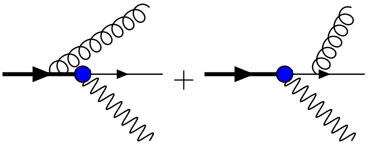
$H^{NLO}(z) \rightarrow$ 

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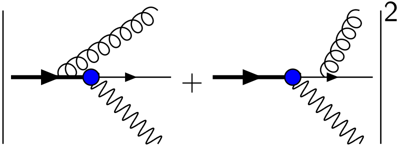
$H^{NLO}(z) \to$ 

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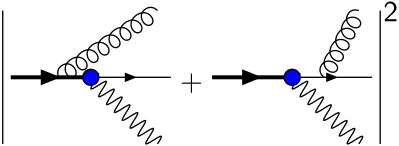

$\delta\left(z - \frac{2E_\gamma}{m_b}\right)$

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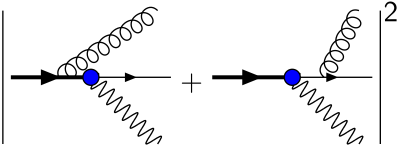
$H^{NLO}(z) \rightarrow \int$ phase space



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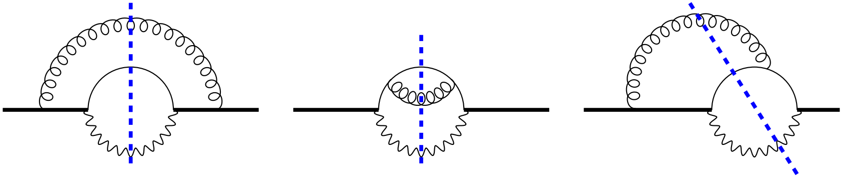
$H^{NLO}(z) \rightarrow$ sum over the Im parts of two-loop self energy diagrams using the Cutkosky rules



REMEMBER: multiply the integrand by $\delta(z - 2E_\gamma/m_b)$

NNLO Cuts:

$$\frac{dG_{77}(z)}{dz} = \delta(1-z) + \frac{\alpha_s(m_b)}{\pi} C_F H^{NLO}(z) + \left(\frac{\alpha_s(m_b)}{\pi}\right)^2 C_F H^{NNLO}(z) + O(\alpha_s^3)$$

non trivial color structure

$$H^{NNLO}(z) = C_F H^{(2,\text{a})}(z) + C_A H^{(2,\text{na})}(z) + T_R N_L H^{(2,\text{NL})}(z)$$

NNLO Cuts: closed fermion loop graphs

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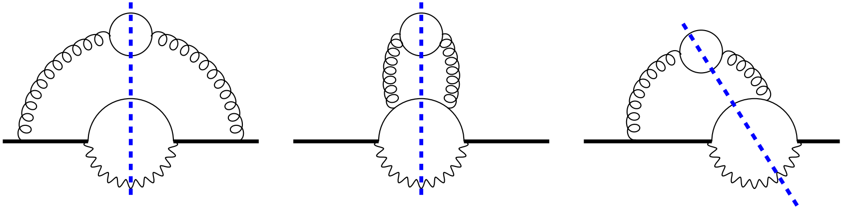

Similar graphs with closed gluon/ghost loop contribute to $H^{(2,\mathrm{a})}$

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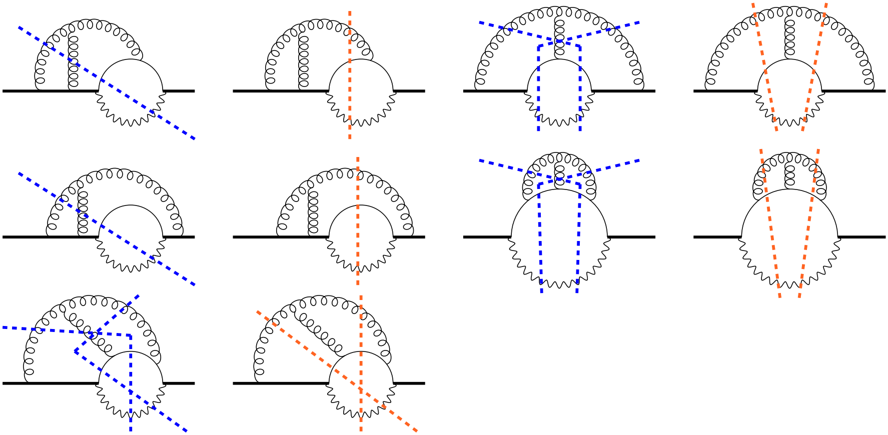

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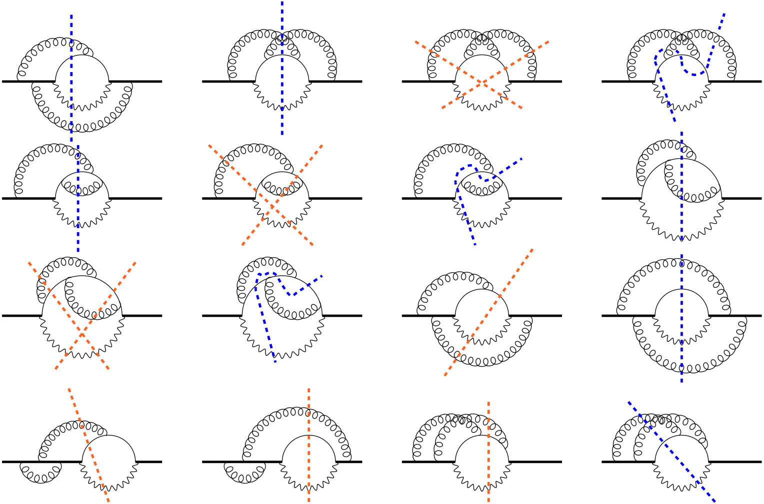

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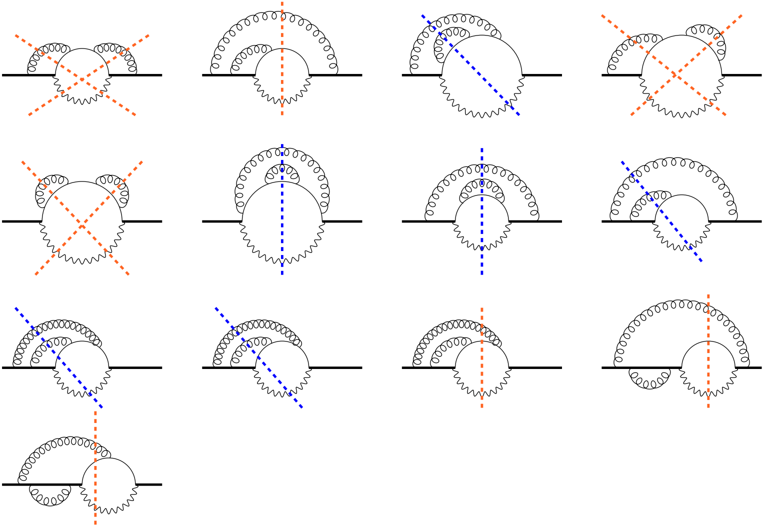

What to do With the Integrals?

We need to calculate the imaginary parts of three-loop self energy diagrams

For each given cut, after summing over the spin of the b-quark, we are left with with linear combinations of integrals of the form

$$\int \mathfrak{D}^D k_1 \mathfrak{D}^D k_2 \mathfrak{D}^D k_3 \frac{S_1^{n_1} \cdots S_q^{n_q}}{\mathcal{D}_1^{m_1} \cdots \mathcal{D}_t^{m_t}}$$

$S \rightarrow$ scalar products $k_i \cdot k_j$ or $k_i \cdot p$

$\mathcal{D} \rightarrow$ propagators $[\sum k \ (+p)]^2 \ (+m_b^2)$

Luckily, just a "small" number of these diagrams are independent: the MIs

Use the technical tools usually employed in the calculation of multi-loop Feynman diagrams to select a set of MI and to evaluate them

$\Longrightarrow$ IBPs + differential equation method

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Dirac Deltas as Propagators

The cut propagators (phase space Dirac delta s) can be written as difference of propagators

$$\delta\left(q^2+m^2\right)=\frac{1}{2\pi i}\left(\frac{1}{q^2+m^2-i\delta}-\frac{1}{q^2+m^2+i\delta}\right)$$

All the integrals in which one of the cut propagators is simplified or raised to a negative power are zero, since the $\pm i\delta$ prescription becomes irrelevant: the reduction procedure is simplified

Anastasiou Melnikov ('02)

The same can be done for the Dirac delta enforcing the kinematic constrain

$$\delta\left(E_\gamma+\frac{p_b\cdot p_\gamma}{m_b}\right)=2m_b\delta\left((p_b+p_\gamma)^2+(1+z)m_b^2\right)$$

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Dirac Deltas as Propagators

The cut propagators (phase space Dirac delta s) can be written as difference of propagators

$$\delta\left(q^2+m^2\right)=\frac{1}{2\pi i}\left(\frac{1}{q^2+m^2-i\delta}-\frac{1}{q^2+m^2+i\delta}\right)$$

All the integrals in which one of the cut propagators is simplified or raised to a negative power are zero, since the $\pm i\delta$ prescription becomes irrelevant: the reduction procedure is simplified

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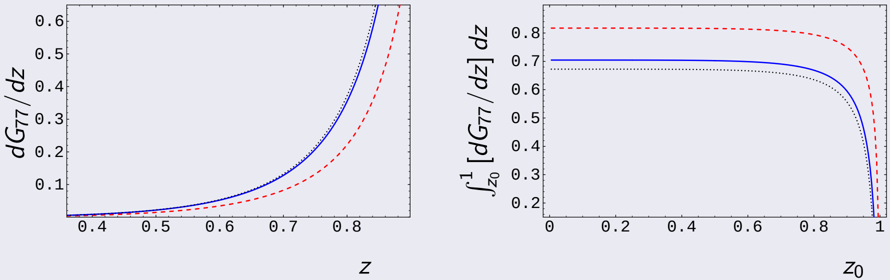
The same can be done for the Dirac delta enforcing the kinematic constrain

$$\delta\left(E_\gamma+\frac{p_b\cdot p_\gamma}{m_b}\right)=2m_b\delta\left((p_b+p_\gamma)^2+(1+z)m_b^2\right)$$

Comparison of NLO and NNLO

$\mathcal{O}_7 - \mathcal{O}_7$ contribution to the spectrum



The BLM corrections ($\sim \alpha_s^2 \beta_0$) provide the dominant part of the NNLO corrections. They are obtained by naive non-abelianization:

$$\alpha_s^2 N_L \longrightarrow -\frac{3}{2}\alpha_s^2 \beta_0$$

red line NLO, dotted line NNLO-BLM, blue line full NNLO

Summary & Conclusions

- A NNLO calculation of $\mathcal{B}(\overline{B}^0 \to X_s \gamma)$ is needed in order to match the expected 5% experimental error at the end of B factories.
- With the NNLO results recently obtained it was possible to obtain a NNLO estimate for the branching ratio. The theory uncertainty is now at the level of the experimental one.
- The estimate suffers a 3% uncertainty due to the fact that we do not know the full m_c dependence at NNLO.
- The theory uncertainty is at the moment dominated by non-perturbative effects.
- The calculation of the spectrum is part of this NNLO program, the dominant $\mathcal{O}_7 - \mathcal{O}_7$ contribution has been calculated independently by two groups, using multi-loop Feynman diagrams calculation techniques. The calculation of the contribution of other operators (ex $\mathcal{O}_7 - \mathcal{O}_8$) is under way.