Masse Fermioniche e Simmetrie Discrete

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based on

AF1 = Guido Altarelli and F. F. hep-ph/0504165

AF2 = Guido Altarelli and F. F. hep-ph/0512103

AFL = Guido Altarelli, F.F. and Yin Lin hep-ph/0610165

FHLM = F.F., Claudia Hagedorn, Yin Lin and Luca Merlo hep-ph/0702194

Lepton Mixing Angles

[Fogli, Lisi, Marrone, Palazzo 0608060]
[Schwetz 0606060]

$[2\sigma$ errors (95% C.L.)]

$\sin^2\vartheta_{23} = 0.45\,(1^{+0.35}_{-0.20})$

$\vartheta_{23} = (42.1^{+9.1}_{-5.3})^0 \quad [2\sigma]$

$\sin^2\vartheta_{13} = 0.8^{+2.3}_{-0.8} \times 10^{-2}$

$\sin^2\vartheta_{12} = 0.314\,(1^{+0.18}_{-0.15})$

[Hall, Murayama, Weiner 2000
De Gouvea, Murayama 0301050]

different viewpoints: - angles are all generically large [anarchy]

- angles reflect an underlying order

$\sin^2\vartheta_{23} = \frac{1}{2}$

$\sin^2\vartheta_{13} = 0$

$\sin^2\vartheta_{12} = \frac{1}{3} \qquad \vartheta_{12} = 35.3^0$

$\vartheta_{12} = (34.1\,^{+1.7}_{-1.6})^0 \quad [1\sigma]$

[Harrison, Perkins and Scott (HPS) mixing pattern
or Tri-Bimaximal mixing]

not a bad 1st order approximation!

θ_{12} right within $1\sigma \approx 2^0 \leq 0.04$ rad $\approx \lambda^2$, where λ=0.22
errors on θ_{23} and θ_{13} are still large…

future [< 10 yr] precision/sensitivity on θ_{23} and θ_{13} down to about λ^2
could confirm HPS mixing pattern

$\vartheta_{13} \approx \delta\vartheta_{23} \approx \lambda^2 \approx 0.04 \div 0.05\ \mathrm{rad}\ (2.1^0 \div 2.9^0)$

[Gonzalez-Garcia, Maltoni, Smirnov 0408170]

If future data will confirm HPS down to about λ^2 precision

$$U_{PMNS} = \begin{pmatrix} \frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix} + O(\lambda^2)$$

quite symmetric!
 also called
``Tri-Bimaximal=TB''

$$|\nu_3\rangle = \frac{1}{\sqrt{2}}\left(-|\nu_\mu\rangle + |\nu_\tau\rangle\right)$$

$$|\nu_2\rangle = \frac{1}{\sqrt{3}}\left(|\nu_e\rangle + |\nu_\mu\rangle + |\nu_\tau\rangle\right)$$

reminiscent of

$$\pi^0 = \frac{|uu\rangle - |dd\rangle}{\sqrt{2}} \qquad \eta = \frac{|uu\rangle + |dd\rangle - 2|ss\rangle}{\sqrt{6}} \qquad \eta' = \frac{|uu\rangle + |dd\rangle + |ss\rangle}{\sqrt{3}}$$

theoretical challenge:
- how to derive TB from a model?
 (eventually modified by small, $O(\lambda^2)$, corrections)?

TB mixing from vacuum alignment

choose the basis where charged leptons are diagonal

$$\boxed{m_l \propto \begin{pmatrix} y_e & 0 & 0 \\ 0 & y_\mu & 0 \\ 0 & 0 & y_\tau \end{pmatrix}}$$

minimal symmetry guaranteeing the diagonal pattern

$$Z_3 \text{ group} : G_T = \{1, T, T^2\}$$

$$T = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega^2 & 0 \\ 0 & 0 & \omega \end{pmatrix} \qquad \omega \equiv e^{i\frac{2\pi}{3}}$$

$$T^+ m_l T = m_l$$

in this basis, TB mixing is entirely due to

$$\boxed{m_\nu = \frac{m_3}{2}\begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & -1 & 1 \end{pmatrix} + \frac{m_2}{3}\begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} + \frac{m_1}{3}\begin{pmatrix} 4 & -2 & -2 \\ -2 & 1 & 1 \\ -2 & 1 & 1 \end{pmatrix}}$$

eigenvectors

$$m_3 \leftrightarrow \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ -1 \\ 1 \end{pmatrix} \quad m_2 \leftrightarrow \frac{1}{\sqrt{3}}\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \quad m_1 \leftrightarrow \frac{1}{\sqrt{6}}\begin{pmatrix} 2 \\ -1 \\ -1 \end{pmatrix}$$

minimal symmetry guaranteeing this pattern

$$Z_2 \times Z_2 \text{ group} : G_{SU} = G_S \times G_U ,$$

$$G_S = \{1, S\} \qquad G_U = \{1, U\}$$

$$S = \frac{1}{3}\begin{pmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{pmatrix} \qquad U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

$$S^T m_\nu S = m_\nu$$

$$U^T m_\nu U = m_\nu$$

algorithm to reproduce TB mixing

1. start from a flavour symmetry group $G \supset G_T,\ G_{SU}$

[In practice, G_S is already sufficient]

2. arrange symmetry breaking of G:

$$\begin{tikzcd} & G \arrow[dl, "\text{in the charged lepton sector}"'] \arrow[dr, "\text{in the neutrino sector}"] & \\ G_T & & G_{SU} \end{tikzcd}$$

spontaneous symmetry breaking

vacuum alignment problem

$$\langle \varphi_T \rangle, \quad \langle \varphi_{SU} \rangle, \ldots$$

should have specific magnitudes and relative directions in flavour space.

minimal choice for G

[Ma, Rajasekaran 2001; Babu, Ma, Valle 2003; Hirsch, Romao, Skandage, Valle, Villanova de Moral 2003; Ma 0409075]

matrices S and T satisfy

$$S^2 = (ST)^3 = T^3 = 1$$

the group generated by S and T is A_4 and has 12 elements

$$A_4 = \{1, S, T, ST, TS, T^2, ST^2, STS, TST, T^2S, TST^2, T^2ST\}$$

- group of even permutation of four objects
- subgroup of SO(3) leaving a tetrahedron invariant

A_4 representations:

$$\begin{array}{lll} 1 & S=1 & T=1 \\ 1' & S=1 & T=\omega^2 \\ 1'' & S=1 & T=\omega \end{array}$$

$$3 \quad S = \frac{1}{3}\begin{pmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{pmatrix} \qquad T = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega^2 & 0 \\ 0 & 0 & \omega \end{pmatrix}$$

$$\omega \equiv e^{i\frac{2\pi}{3}}$$

a minimal model (lepton sector)

[AF1,AF2]

	l	e^c	μ^c	τ^c	h_u	h_d	φ_T	φ_S	ξ_i
A_4	3	1	1''	1'	1	1	3	3	1

matter fields — Higgses — A_4 breaking sector

SU(2)xU(1)xA_4 invariant Lagrangian:

[Λ Is the cutoff]

$$L = y_e e^c \left(\frac{\varphi_T}{\Lambda} l\right) h_d + y_\mu \mu^c \left(\frac{\varphi_T}{\Lambda} l\right)' h_d + y_\tau \tau^c \left(\frac{\varphi_T}{\Lambda} l\right)'' h_d$$

$$+ x_a \frac{\xi}{\Lambda}(ll)\frac{h_u h_u}{\Lambda} + x_b \left(\frac{\varphi_S}{\Lambda} ll\right)\frac{h_u h_u}{\Lambda} + \ldots$$

higher dimensional operators in $1/\Lambda$ expansion

- some invariant is missing from L: $\left\{\begin{array}{l} \varphi_S \leftrightarrow \varphi_T \\ x(ll)h_u h_u \end{array}\right.$

  [can be forbidden by an additional Z_3]

under appropriate conditions (e,g, SUSY + Z_3) minimization of V leads to

$$\langle \varphi_T \rangle = (v_T, 0, 0)$$ breaks A_4 down to G_T

$$\langle \varphi_S \rangle = (v_S, v_S, v_S)$$ breaks A_4 down to G_S

$$\langle \xi \rangle = u$$

$$[\langle h_{u,d} \rangle = v_{u,d} << v_T, v_S, u] \qquad v_T, v_S, u \leq \Lambda$$

then:

$$m_l = \begin{pmatrix} y_e & 0 & 0 \\ 0 & y_\mu & 0 \\ 0 & 0 & y_\tau \end{pmatrix} v_d \left(\frac{v_T}{\Lambda}\right)$$

charged fermion masses

$$m_f = y_f v_d \left(\frac{v_T}{\Lambda}\right)$$

free parameters as in the SM at this level

$$m_\nu = \begin{pmatrix} a+\frac{2}{3}b & -\frac{b}{3} & -\frac{b}{3} \\ -\frac{b}{3} & \frac{2}{3}b & a-\frac{b}{3} \\ -\frac{b}{3} & a-\frac{b}{3} & \frac{2}{3}b \end{pmatrix} \frac{v_u^2}{\Lambda}$$

$$a \equiv 2x_a \frac{u}{\Lambda}$$

$$b \equiv 2x_b \frac{v_S}{\Lambda}$$

2 complex parameters in ν sector (overall phase unphysical)

[has also an accidental invariance under G_U]

mixing angles entirely from ν sector:

$$U_{PMNS} = \begin{pmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

independent from
|a|, |b|, $\Delta \equiv$arg(a)-arg(b) !!

ν masses: $m_1 = |a+b|\frac{v_u^2}{\Lambda}$ $\quad m_2 = |a|\frac{v_u^2}{\Lambda}$ $\quad m_3 = |a-b|\frac{v_u^2}{\Lambda}$

$m_2 > m_1$ $\Rightarrow$ $-1 \le \cos\Delta < -\left|\frac{b}{2a}\right|$ $\Rightarrow$ ν spectrum always of normal hierarchy type

$$\left|\frac{b}{2a}\right| \approx \begin{cases} 1 & \text{[almost hierarchical spectrum]} \\ 0 & \text{[almost degenerate spectrum]} \end{cases}$$

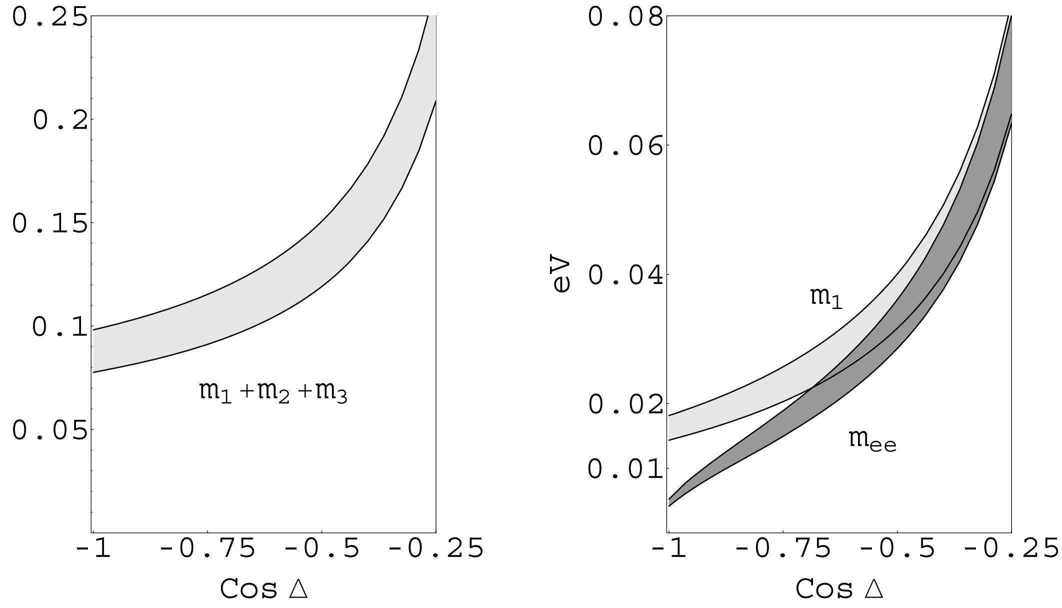
$r \equiv \frac{\Delta m^2_{sol}}{\Delta m^2_{atm}} \approx \frac{1}{35}$ requires a (moderate) tuning

predictions:

$$m_1 \geq 0.017 \ \text{eV}$$

$$\sum_i m_i \geq 0.09 \ \text{eV}$$

$$|m_3|^2 = |m_{ee}|^2 + \frac{10}{9}\Delta m^2_{atm}\left(1 - \frac{\Delta m^2_{sol}}{\Delta m^2_{atm}}\right)$$



range of VEVs:

$$m_\tau = y_\tau v_d \left(\frac{v_T}{\Lambda}\right)$$

$$y_\tau < 4\pi$$

from ν spectrum

$$\frac{v_T}{\Lambda} > 0.002(0.02)$$

$$\tan\beta = 2.5(30)$$

$$\tan\beta = \frac{v_u}{v_d}$$

$$\Lambda = 1.8 \times 10^{15} \left(\frac{v_S}{\Lambda}\right) \sin^2\beta \quad \text{GeV}$$

assuming all VEVs of the same order

$$0.002 < \frac{v_T}{\Lambda} \approx \frac{v_S}{\Lambda} \approx \frac{u}{\Lambda} < 1$$

$$\Lambda < 0.25 \times 10^{15} \quad \text{GeV}$$

sub-leading corrections

arising from higher dimensional operators, depleted by additional powers of $1/\Lambda$.

$\Rightarrow$

they affect m_l, m_ν and they can deform the VEVs.

result

$$U_{PMNS} = \begin{pmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix} + O(\frac{\mathrm{VEV}}{\Lambda})$$

[leading corrections can be even smaller

$$O\left(\frac{VEV}{\Lambda}\right)^2$$

in particular cases]

and similarly for neutrino masses

TB mixing is preserved if corrections are $\leq \lambda^2 \approx 0.04$

given the range $0.002<(\mathrm{VEV}/\Lambda)<1$, corrections can be kept below λ^2

quark masses

simple and good first order approximation:

	q	u^c	c^c	t^c	d^c	s^c	b^c
A_4	3	1	1''	1'	1	1''	1'

- same assignment as in the lepton sector
- compatible with $SU(4)\otimes SU(2)_L \otimes SU(2)_R$ partial unification

quark mass matrices diagonal in the leading order mixing matrix V_{CKM}=1

- unfortunately, corrections induced by higher dimensional operators are negligibly small
- top mass from dim 5 operator

additional sources of A_4 breaking are needed in the quark sector

possible solution within T', the double covering of A_4 [FHLM]

$$S^2 = R \quad R^2 = 1 \quad (ST)^3 = T^3 = 1$$

24 elements

representations: 1 1' 1'' 3 2 2' 2''

	$\begin{pmatrix} u & d \\ c & s \end{pmatrix}$	$\begin{pmatrix} u^c \\ c^c \end{pmatrix}$	$\begin{pmatrix} d^c \\ s^c \end{pmatrix}$	$(t \quad b)$	t^c	b^c	η	ξ''
T'	$2''$	$2''$	$2''$	1	1	1	$2'$	$1''$

[older T' models by
Frampton, Kephard 1994
Aranda, Carone, Lebed 1999, 2000
Carr, Frampton 2007
similar U(2) constructions by
Barbieri, Dvali, Hall 1996
Barbieri, Hall, Raby, Romanino 1997
Barbieri, Hall, Romanino 1997]

- lepton sector as in the A_4 model
- t and b masses at the renormalizable level (τ mass from higher dim operators) at the leading order [including dim 5 operators]

$$m_{u,d} \propto \begin{pmatrix} 0 & 0 & 0 \\ 0 & \times & \times \\ 0 & \times & \times \end{pmatrix}$$

33>>22,23,32

$$\langle \eta \rangle = \begin{pmatrix} v_1 \\ 0 \end{pmatrix}$$

$$m_t, m_b > m_c, m_s \neq 0$$

$$V_{cb}$$

- masses and mixing angles of 1st generation from higher-order effects
- despite the large number of parameters two relations are predicted

$$\sqrt{\frac{m_d}{m_s}} = |V_{us}| + O(\lambda^2)$$

$0.213 \div 0.243 \qquad 0.2257 \pm 0.0021$

$$\sqrt{\frac{m_d}{m_s}} = \left|\frac{V_{td}}{V_{ts}}\right| + O(\lambda^2)$$

$0.208^{+0.008}_{-0.006}$

- vacuum alignment explicitly solved
- lepton sector not spoiled by the corrections coming from the quark sector

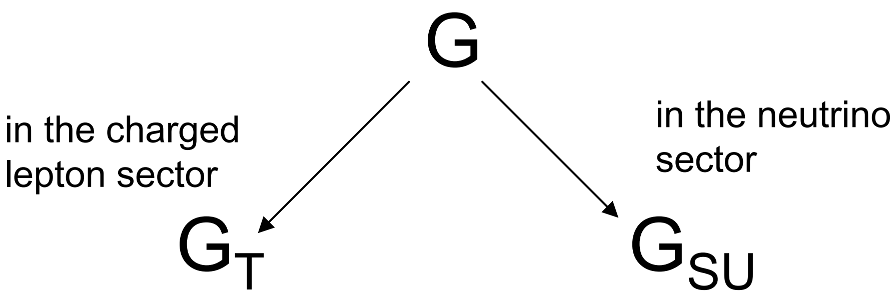
conclusion

mixing in the lepton sector is well described by the TB pattern

$\vartheta_{23} = 45^0$ $\quad$ $\vartheta_{13} = 0$ $\quad$ $\sin^2 \vartheta_{12} = \frac{1}{3}$

errors on θ_{23} and θ_{13} are still large and future data are needed to confirm TB at the λ^2 level

TB mixing can arise from



natural vacuum alignment
preserved by high-order effects

Here: an existence proof based on the discrete group A_4 [T']

vacuum alignment, stability and extension to quarks are non-trivial

- neutrino spectrum is of normal hierarchy type

$$|m_3|^2 = |m_{ee}|^2 + (10/9)\Delta m^2_{atm}\left(1 - \Delta m^2_{sol} / \Delta m^2_{atm}\right)$$

$$m_1 > 0.017 \text{ eV} \qquad \sum_i m_i > 0.09 \text{ eV}$$

$$\sqrt{\frac{m_d}{m_s}} = |V_{us}| + O(\lambda^2) \qquad \sqrt{\frac{m_d}{m_s}} = \left|\frac{V_{td}}{V_{ts}}\right| + O(\lambda^2)$$

relation to the modular group

modular group PSL(2,Z): linear fractional transformation

complex variable

$$z \rightarrow \frac{a\,z+b}{c\,z+d} \qquad a,b,c,d \in Z \qquad ad-bc=1$$

discrete, infinite group generated by two elements

$$\underbrace{z \rightarrow -\frac{1}{z}}_{S} \qquad \underbrace{z \rightarrow z+1}_{T}$$

obeying

$$S^2 = (ST)^3 = 1$$

the modular group is present everywhere in string theory

[any relation to string theory approaches to fermion masses?]

Ibanez; Hamidi, Vafa; Dixon, Friedan, Martinec, Shenker; Casas, Munoz; Cremades, Ibanez, Marchesano; Abel, Owen

A_4 is a finite subgroup of the modular group and

$$A_4 = \frac{PSL(2,Z)}{H} \quad \Rightarrow \quad \text{representations of } A_4 \text{ are representations of } PSL(2,Z)$$

infinite discrete normal subgroup of PSL(2,Z)

A_4 as a leftover of Poincare symmetry in D>4

[AFL]

D dimensional Poincare symmetry $\Longleftrightarrow$ usually broken by compactification down to 4 dimensions

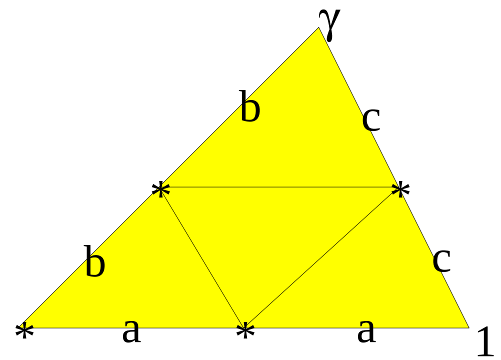
a discrete subgroup of the (D-4) euclidean group can survive in specific geometries

Example: D=6

2 dimensions compactified on T^2/Z_2

$$z \to z+1$$
$$z \to z+\gamma$$
$$z \to -z$$

four fixed points



$$\text{if} \quad \gamma = e^{i\frac{\pi}{3}}$$

regular tetrahedron invariant under

$$S: \quad z \to z + \frac{1}{2}$$
$$T: \quad z \to \gamma^2 z$$

$$S^2 = T^3 = (ST)^3 = 1$$

$\sin^2\theta_{23}$

$\delta(\sin^2\theta_{23})$ reduced by future LBL experiments from $\nu_\mu \rightarrow \nu_\mu$ disappearance channel

$$\vartheta_{23} \approx \frac{\pi}{4}$$

$$P_{\mu\mu} \approx 1 - \sin^2 2\vartheta_{23} \sin^2\left(\frac{\Delta m^2_{31} L}{4E}\right)$$

$$\delta\vartheta_{23} \approx \frac{\sqrt{\delta P_{\mu\mu}}}{2}$$

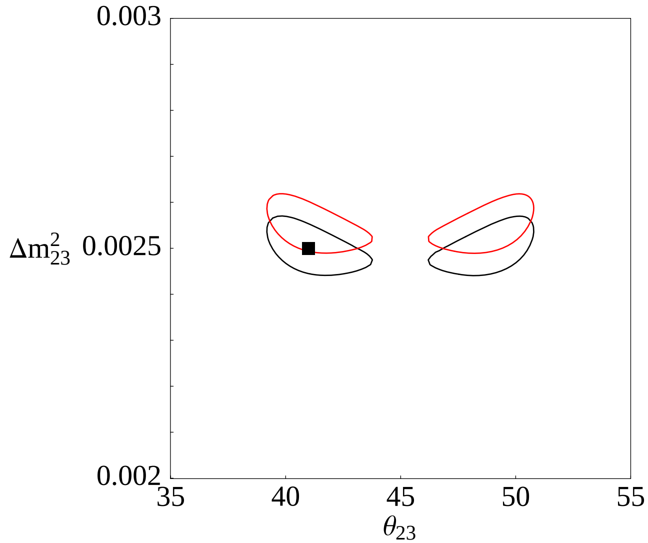
i.e. a small uncertainty on $P_{\mu\mu}$ leads to a large uncertainty on θ_{23}

- no substantial improvements from conventional beams
- superbeams (e.g. T2K in 5 yr of run)

$$\delta P_{\mu\mu} \approx 0.01$$

$$\delta\vartheta_{23} \approx 0.05\,\mathrm{rad} \leftrightarrow 2.9^0$$

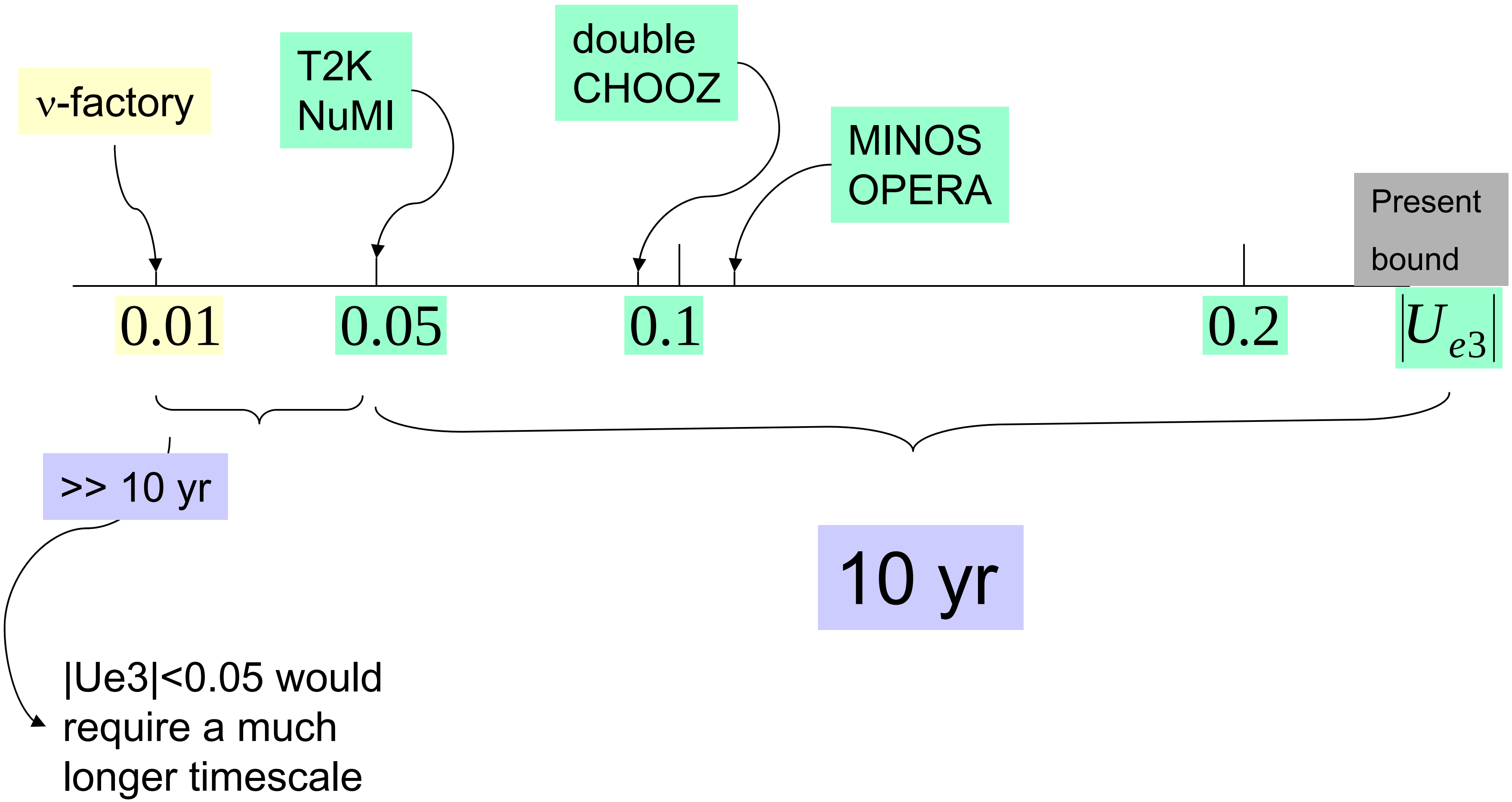
improvement by about a factor 2



T2K-1
90% CL
black = normal hierarchy
red = inverted hierarchy
true value 41^0
[courtesy by Enrique Fernandez]

$\sin\theta_{13}$

a similar sensitivity is expected on θ_{13} ($U_{e3}=\sin\theta_{13}$)



[Donini, Meloni, Rigolin 0506100]

many models predicts a large but not necessarily maximal θ_{23}

an example: abelian flavour symmetry group $U(1)_F$

$$F(l) = (\times, 0, 0) \qquad [\times \neq 0]$$

$$F(e^c) = (\times, \times, 0)$$

$$m_e = \begin{pmatrix} \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & O(1) & O(1) \end{pmatrix} v_d \qquad m_\nu = \begin{pmatrix} \cdot & \cdot & \cdot \\ \cdot & O(1) & O(1) \\ \cdot & O(1) & O(1) \end{pmatrix} \frac{v_u^2}{\Lambda}$$

$$\Downarrow$$

$\vartheta_{23} \approx O(1)$ maximal only by a fine-tuning!

similarly for all other abelian charge assignements

$$F(l) = (1, -1, -1) \qquad m_\nu = \begin{pmatrix} \cdot & O(1) & O(1) \\ O(1) & \cdot & \cdot \\ O(1) & \cdot & \cdot \end{pmatrix} \frac{v_u^2}{\Lambda} \qquad \vartheta_{23} \approx O(1) + \text{charged lepton contribution}$$

no help from the see-saw mechanism within abelian symmetries…

θ_{23} maximal by RGE effects?

[Ellis, Lola 1999
Casas, Espinoza, Ibarra, Navarro 1999-2003
Broncano, Gavela, Jenkins 0406019]

running effects important only for quasi-degenerate neutrinos

2 flavour case

boundary conditions at Λ>> e.w. scale $\quad m_2, m_3, \vartheta_{23}$

at $Q < \Lambda$

$$\vartheta_{23}(Q) \approx \frac{\pi}{4} \quad \Leftrightarrow \quad \varepsilon \approx -\frac{\delta m}{m}\cos 2\vartheta_{23}$$

$$\varepsilon \approx \frac{1}{16\pi^2} y_\tau^2 \log\frac{\Lambda}{Q}$$

[possible only if $\quad \delta m \equiv m_2 - m_3 << m_2 + m_3 \approx 2m$]

gives the scale Q at which $\theta_{23}(Q)$ becomes maximal $\Rightarrow$ $\boxed{m_2, m_3, \vartheta_{23} \text{ fine tuned to obtain Q at the e.w. scale}}$

a similar conclusion also for the 3 flavour case:

$$\sin^2 2\vartheta_{12} = \frac{\sin^2\vartheta_{13}\sin^2 2\vartheta_{23}}{(\sin^2\vartheta_{23}\cos^2\vartheta_{13} + \sin^2\vartheta_{13})^2}$$

infrared stable fixed point

[Chankowski, Pokorski 2002]

if $\vartheta_{23} = \frac{\pi}{4}$

$$\sin^2 2\vartheta_{12} = \frac{4\sin^2\vartheta_{13}}{(1+\sin^2\vartheta_{13})^2} < 0.2 \ \ \text{(Chooz)}$$

wrong!

patterns of symmetry breaking

A_4 triplet $\qquad \varphi = (\varphi_1, \varphi_2, \varphi_3)$

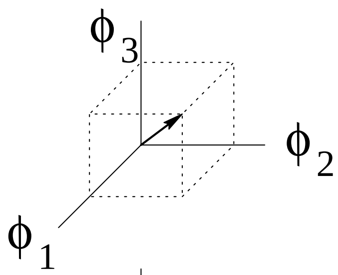
$$\langle \varphi \rangle \propto (1,1,1) \qquad A_4 \to G_S$$

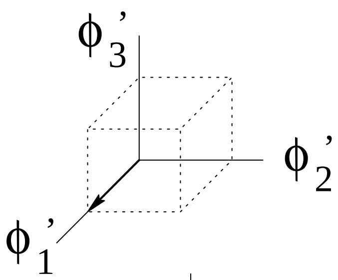
$$\langle \varphi \rangle \propto (1,0,0) \qquad A_4 \to G_T$$

as we have seen

G_S is the low-energy symmetry in the ν sector

G_T is the low-energy symmetry in the charged lepton sector





low-energy parameters

ν masses

[3 light active ν]

$$m_1, m_2, m_3$$

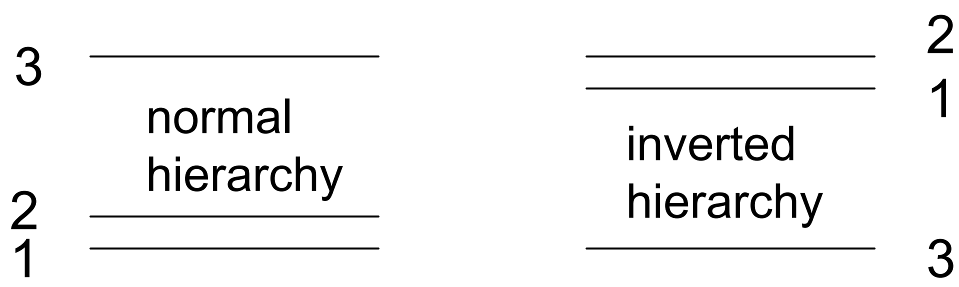
order $m_1 < m_2$

$$\Delta m^2_{21} < \left|\Delta m^2_{32}\right|, \left|\Delta m^2_{31}\right| \qquad [\Delta m^2_{ij} \equiv m^2_i - m^2_j]$$

i.e. 1 and 2 are, by definition, the closest levels

two possibilities:

$$\left.\begin{array}{l} \Delta m^2_{21} = 7.9\,(1 \pm 0.09) \times 10^{-5} \quad \mathrm{GeV}^2 \\ \left|\Delta m^2_{31}\right| = 2.4\,(1^{+0.21}_{-0.26}) \times 10^{-3} \quad \mathrm{GeV}^2 \end{array}\right\} \text{ at } 2\,\sigma$$



Mixing matrix (analogous to V_{CKM})

$$U_{PMNS} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{i\delta} \\ -s_{12}c_{23} - c_{12}s_{13}s_{23}e^{-i\delta} & c_{12}c_{23} - s_{12}s_{13}s_{23}e^{-i\delta} & c_{13}s_{23} \\ -c_{12}s_{13}c_{23} + s_{12}s_{23}e^{-i\delta} & -s_{12}s_{13}c_{23} - c_{12}s_{23}e^{-i\delta} & c_{13}c_{23} \end{pmatrix} \times \underbrace{\begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\alpha} & 0 \\ 0 & 0 & e^{i\beta} \end{pmatrix}}$$

$$c_{12} \equiv \cos\vartheta_{12}, \ldots$$

- only if ν are Majorana
- drops in oscillations

plan of the seminar

- lepton mixing angles and Tri-Bimaximal (TB) mixing
- maximal atmospheric mixing angle
- TB mixing from vacuum alignment, minimal model based on A_4
- extension to the quark sector: from A_4 to T'
- microscopic origin of A_4

based on

AF1 = Guido Altarelli and F. F. hep-ph/0504165

AF2 = Guido Altarelli and F. F. hep-ph/0512103

AFL = Guido Altarelli, F.F. and Yin Lin hep-ph/0610165

FHLM = F.F., Claudia Hagedorn, Yin Lin and Luca Merlo hep-ph/0702194 within abelian flavour symmetries

many models predicts a large but not necessarily maximal θ_{23}

$$\vartheta_{23} = \frac{\pi}{4}$$ only by a fine-tuning

$$\vartheta_{23} = \frac{\pi}{4}$$ is not an infrared stable fixed point of RGE evolution

initial conditions at high energy should be fine-tuned in order to achive maximal atmospheric mixing angle at low energy

[Ellis, Lola 1999
Casas, Espinoza, Ibarra, Navarro 1999-2003
Broncano, Gavela, Jenkins 0406019
Chankowski, Pokorski 2002]

further requirements for the vacuum alignment

(1) alignment should be **natural**

no ad-hoc relations: desired VEVs from most general V
in a finite region of parameter space

(2) alignment **not spoiled by sub-leading terms**

$$\vartheta_{13} = 0 + a_1 \frac{\langle\varphi\rangle}{\Lambda} + a_2 \frac{\langle\varphi\rangle^2}{\Lambda^2} + \ldots$$

$$\vartheta_{23} = \underbrace{\frac{\pi}{4}}_{\text{leading order}} + \underbrace{b_1 \frac{\langle\varphi\rangle}{\Lambda} + b_2 \frac{\langle\varphi\rangle^2}{\Lambda^2} + \ldots}_{}$$

leading order

from higher-dimensional operators compatible with gauge and flavour symmetries

often $\frac{\langle\varphi\rangle}{\Lambda} \approx \lambda$

then $a_1 = b_1 = 0$ needed

(3) alignment **compatible with mass hierarchies**

$$\frac{m_e}{m_\tau}, \quad \frac{m_\mu}{m_\tau}$$

should vanish in the limit of exact symmetry

(1) natural vacuum alignment

$$\langle\varphi_T\rangle = (v_T, 0, 0)$$
$$\langle\varphi_S\rangle = (v_S, v_S, v_S)$$
$$\langle\xi\rangle = u$$

$$v_T \approx v_S \approx u$$

it is not a local minimum of the most general renormalizable scalar potential V depending on φ_S, φ_T, ξ and invariant under A_4

a simple solution in 1 extra dimension ≡ ED

[AF1]

[Altarelli, F. 0504165]

$\langle\varphi_T\rangle = (v_T, 0, 0)$

local minimum of V_0

e^c, μ^c, τ^c

$l, h_{u,d}$

0 y L

$\langle\varphi_S\rangle = (v_S, v_S, v_S)$

$\langle\xi\rangle = u$

local minimum of V_L

ν masses arise from local operators at y=L

$$\frac{(\varphi_S ll) h_u h_u}{\Lambda^2} \qquad \frac{\xi(ll) h_u h_u}{\Lambda^2}$$

this explains also the absence of the terms with $\varphi_S \leftrightarrow \varphi_T$

charged lepton masses from non-local operators

bulk fermion Y=-1

$$\left.\begin{array}{l} \dfrac{(f^c \varphi_T F)\delta(y)}{\sqrt{\Lambda}} \\ -MFF^c \\ \dfrac{(F^c l) h_d}{\sqrt{\Lambda}}\delta(y-L) \end{array}\right\} \quad E << M$$

$$\frac{(f^c \varphi_T l) h_d}{\Lambda} e^{-ML}$$

a 4D supersymmetric solution ≡ SUSY

[AF2]

L is identified with the superpotential w_{lepton} in the lepton sector

w_{lepton} is invariant under $A_4 \times Z_3 \times U(1)_R$

	l	e^c	μ^c	τ^c	$h_{u,d}$	φ_T	φ_S	ξ	$\tilde{\xi}$	φ_0^T	φ_0^S	ξ_0
A_4	3	1	1''	1'	1	3	3	1	1	3	3	1
Z_3	ω	ω^2	ω^2	ω^2	1	1	ω	ω	ω	1	ω	ω
$U(1)_R$	1	1	1	1	0	0	0	0	0	2	2	2

matter fields (l, e^c, μ^c, τ^c) — Higgses ($h_{u,d}$) — A_4 breaking sector (φ_T, φ_S, ξ, $\tilde{\xi}$) — ``driving fields'' (φ_0^T, φ_0^S, ξ_0)

absence of $\varphi_S \leftrightarrow \varphi_T$ $x(ll)$ automatic

$$w = w_{lepton} + w_d + \ldots$$

$$w_d = M(\varphi_0^T \varphi_T) + g(\varphi_0^T \varphi_T \varphi_T) + g_1(\varphi_0^S \varphi_S \varphi_S) + g_2 \tilde{\xi}(\varphi_0^S \varphi_S) + g_3 \xi_0(\varphi_S \varphi_S) + g_4 \xi_0 \xi^2 + g_5 \xi_0 \xi \tilde{\xi} + g_6 \xi_0 \tilde{\xi}^2$$

minimum of the scalar potential at:

$$\begin{aligned} \langle \varphi_T \rangle &= (v_T, 0, 0) \\ \langle \varphi_S \rangle &= (v_S, v_S, v_S) \\ \langle \xi \rangle &= u \\ \langle \tilde{\xi} \rangle &= 0 \end{aligned}$$

$$v_T = -\frac{3M}{2g} \qquad v_S^2 = -\frac{g_4}{3g_3} u^2$$

u undetermined

(3) alignment and mass hierarchies

$$m_l = \begin{pmatrix} y_e & 0 & 0 \\ 0 & y_\mu & 0 \\ 0 & 0 & y_\tau \end{pmatrix} v_d \left(\frac{v_T}{\Lambda} \right)$$

charged fermion masses are already diagonal

$$m_e << m_\mu << m_\tau$$

easily reproduced by U(1) flavour symmetry

$$Q(e^c) = 4 \quad Q(\mu^c) = 2 \quad Q(\tau^c) = 0$$
$$Q(l) = 0$$
$$Q(\vartheta) = -1 \qquad \langle \vartheta \rangle \neq 0$$

compatible with A_4

$$\Downarrow$$

$$y_e \approx \frac{\langle \vartheta \rangle^4}{\Lambda^4} \qquad y_\mu \approx \frac{\langle \vartheta \rangle^2}{\Lambda^2} \qquad y_\tau \approx 1$$

these models have a see-saw realization

by including right-handed neutrinos $\nu^c \approx 3$

$$m_\nu^D \propto 1$$

$$M \propto \begin{pmatrix} a+\frac{2}{3}b & -\frac{b}{3} & -\frac{b}{3} \\ -\frac{b}{3} & \frac{2}{3}b & a-\frac{b}{3} \\ -\frac{b}{3} & a-\frac{b}{3} & \frac{2}{3}b \end{pmatrix}$$

as m_ν before

$$m_\nu = -\left(m_\nu^D\right) M^{-1} m_\nu^D \propto M^{-1}$$

inverse of what was before

- mixing matrix is the same

- eigenvalues are the inverse and now also the case of inverted hierarchy is allowed

θ_{23} maximal from non-abelian flavour symmetries ?

an obstruction: $\vartheta_{23} = 45^0$ can never arise in the limit of an exact realistic symmetry

charged lepton mass matrix:

$$m_l = m_l^0 + \delta m_l^0$$

symmetric limit

symmetry breaking effects: vanishing when flavour symmetry F is exact

realistic symmetry:

(1) $\left|\delta m_l^0\right| < \left|m_l^0\right|$

(2) m_l^0 has rank ≤1

$$m_l^0 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & m_\tau \end{pmatrix}$$

ϑ_{12}^e undetermined

$$U_{PMNS} = U_e^+ U_\nu$$

[omitting phases]

$$\tan\vartheta_{23}^0 = \tan\vartheta_{23}^\nu \cos\vartheta_{12}^e + \left(\frac{\tan\vartheta_{13}^\nu}{\cos\vartheta_{23}^\nu}\right)\sin\vartheta_{12}^e$$

undetermined

$\vartheta_{23} = 45^0$

determined entirely by the pattern of breaking effects
(different, in general, for ν and e sectors)