Subtraction method for parton shower to $2 \to 2$ matrix element matching in $k_\perp$-factorisation

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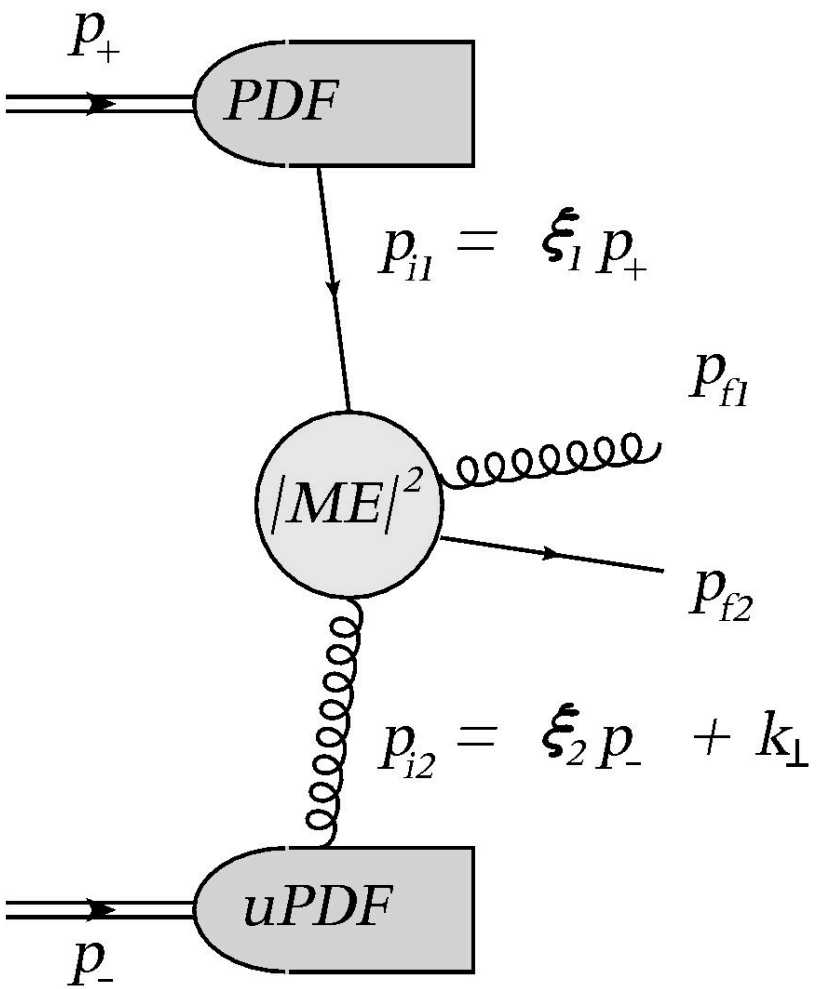
M. D., F. Hautmann, H. Jung, and K. Kutak, JHEP 09, 121 . 0908.0538;
M. D., F. Hautmann, H. Jung, and K. Kutak . 1012.6037

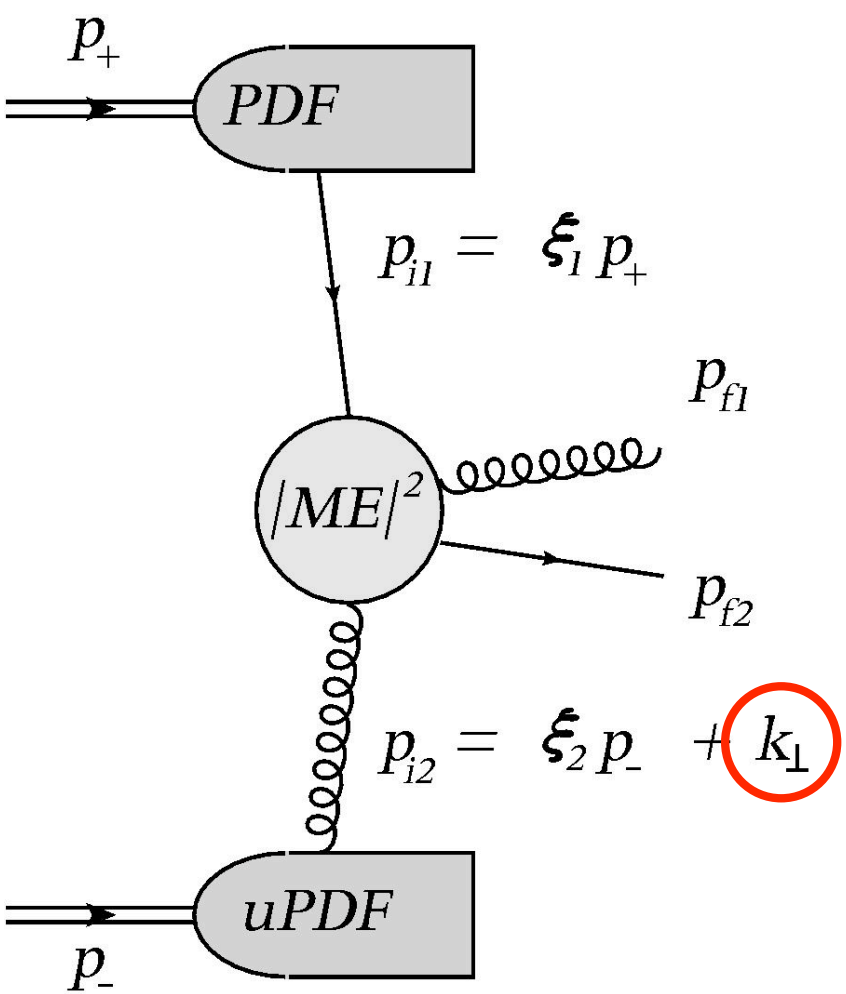
2 → 2 process with uPDF

- 2→2 jet production process - *qg*→qg* in *pp* collisions



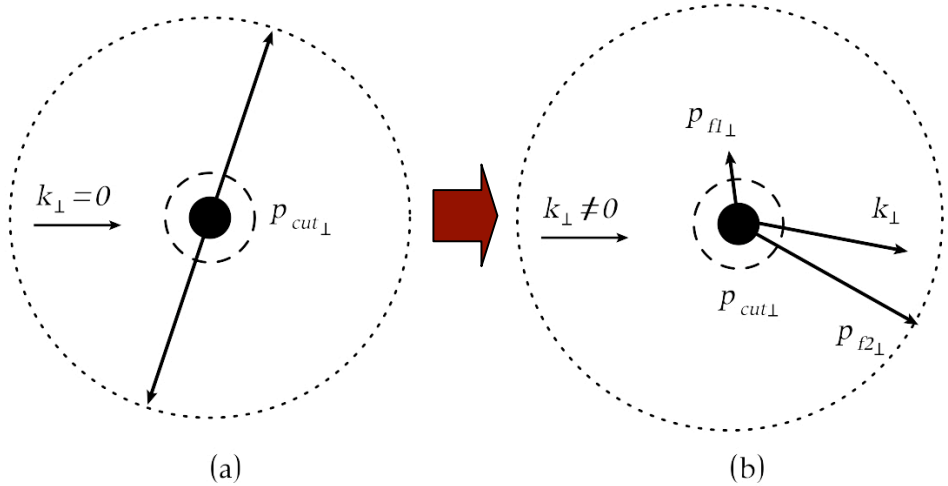
2 → 2 process with uPDF

- 2→2 jet production process - *qg*→qg* in *pp* collisions
- In $k_\perp$-factorisation: extra initial transversal momentum
- Important for resummation of small-x logarithms



A problem

Transversal momenta cross sections example

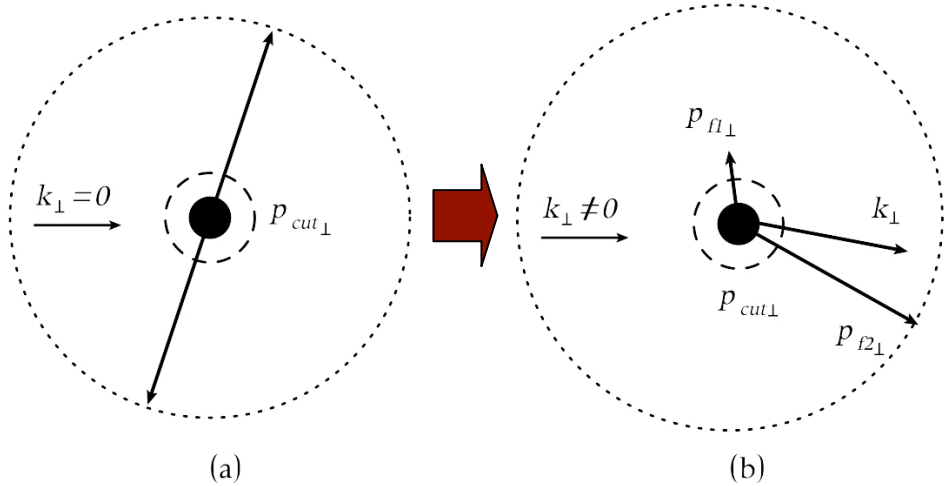


$$\frac{d^2\sigma(\mathbf{p}_f)}{d\mathbf{p}_f^2} = \int_0^1 d\xi_1 \int_0^1 d\xi_2 \, \frac{d^2\hat{\sigma}(\xi_1, \xi_2, \mathbf{p}_f)}{d\mathbf{p}_f^2} H(\xi_1, \xi_2, \mu^2(\mathbf{p}_f))$$

$$\frac{d^2\sigma(\mathbf{p}_{f1})}{d\mathbf{p}_{f1}^2} = \int_0^1 d\xi_1 \int_0^1 d\xi_2 \int_{p_{cut\perp}} d^2\mathbf{p}_{f2} \, \frac{d^2\hat{\sigma}(\xi_1, \xi_2, \mathbf{p}_{f1}, \mathbf{p}_{f2})}{d\mathbf{p}_{f1}^2 d\mathbf{p}_{f2}^2}$$
$$\times \tilde{H}(\xi_1, \xi_2, \mathbf{p}_{f1} + \mathbf{p}_{f2}, \mu^2(\mathbf{p}_{f1}, \mathbf{p}_{f2}))$$

A problem

Transversal momenta cross sections example



$$\frac{d^2\sigma(\mathbf{p}_f)}{d\mathbf{p}_f^2} = \int_0^1 d\xi_1 \int_0^1 d\xi_2 \, \frac{d^2\hat{\sigma}(\xi_1, \xi_2, \mathbf{p}_f)}{d\mathbf{p}_f^2} H(\xi_1, \xi_2, \mu^2(\mathbf{p}_f))$$

$$\boxed{\frac{d^2\sigma(\mathbf{p}_{f1})}{d\mathbf{p}_{f1}^2} = \int_0^1 d\xi_1 \int_0^1 d\xi_2 \int_{\boxed{p_{cut\perp}}} d^2\mathbf{p}_{f2} \, \frac{d^2\hat{\sigma}(\xi_1, \xi_2, \mathbf{p}_{f1}, \mathbf{p}_{f2})}{d\mathbf{p}_{f1}^2 d\mathbf{p}_{f2}^2} \times \tilde{H}(\xi_1, \xi_2, \mathbf{p}_{f1} + \mathbf{p}_{f2}, \mu^2(\mathbf{p}_{f1}, \mathbf{p}_{f2}))}$$

Cross section function of cutoff $p_{cut\perp}$

A solution

- Remove the reason for the cut-off = regulate the cross section
  - If cut-off necessary: reduction of the previous cut-off depence
- Remove the cut-off

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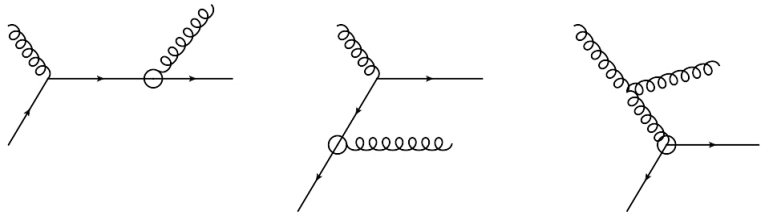
Regularisation

$$d\sigma^{exact}_{qg^* \to qg}$$ ⟷ |leading order exact ME|^2 × PhSp

Includes collinear singularities +
finite terms

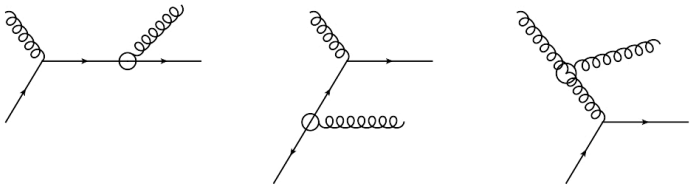
$$d\sigma^{0}_{qg^* \to qg}$$ ⟷ $\Sigma_i|\alpha_S{}^1$ parton *i* emission$|^2$ × PhSp$_i$

Includes collinear singularities



$$d\sigma^{shower}_{qg^* \to qg}$$ ⟷ $\Sigma_i|$LL parton *i* emission$|^2$ × PhSp$_i$

Includes collinear singularities +
resummed loop corrections - regular



Regularisation

- First we remove the singular terms

$$d\sigma^{finite}_{qg^* \to qg} = d\sigma^{exact}_{qg^* \to qg} - d\sigma^{0}_{qg^* \to qg}$$

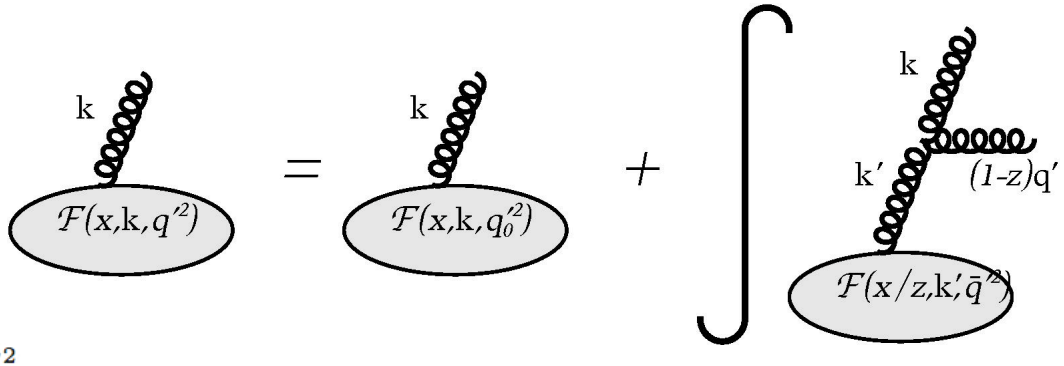
- We add the finite terms to the finite shower

$$d\sigma^{corrected}_{qg^* \to qg} = d\sigma^{shower}_{qg^* \to qg} + d\sigma^{finite}_{qg^* \to qg}$$

- Using Monte Carlo implementation

CCFM equation

- The CCFM equation
  - Includes BFKL kernel
  - Coherence effects
  - Angular ordering
- The equation



$$\mathcal{F}(x,\mathbf{k},\mathbf{q}'^2) = \mathcal{F}(x,\mathbf{k},\mathbf{q}_0'^2) + \int_{\mathbf{q}_0'^2}^{\mathbf{q}'^2} \frac{d^2\bar{\mathbf{q}}'}{\bar{\mathbf{q}}'^2} \frac{N_C\alpha_S}{\pi}$$

$$\int_x^{1-\frac{Q_0}{|\mathbf{q}'|}} \frac{dz}{z} \mathcal{F}(x/z,\mathbf{k}',\bar{\mathbf{q}}'^2) \left(\frac{\Delta_{NS}(\mathbf{k}'^2,(z\bar{\mathbf{q}}')^2)}{z} + \frac{1}{1-z} \right) \Delta_S(\mathbf{q}_0'^2,(z\bar{\mathbf{q}}')^2)$$

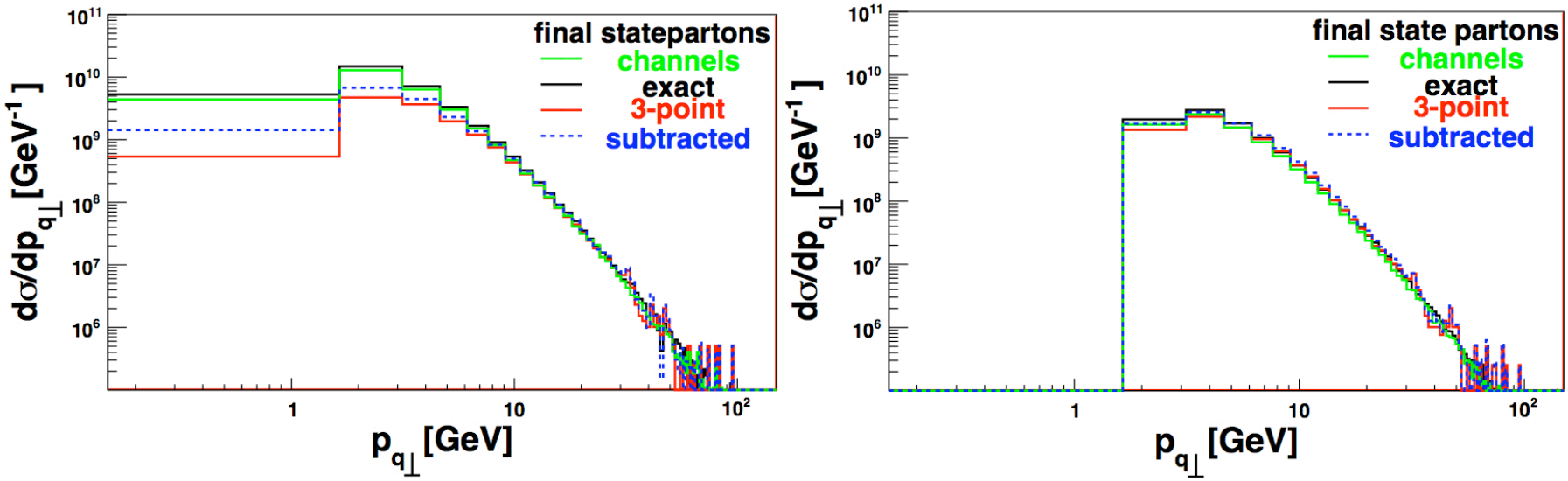
M. Ciafaloni, Nucl. Phys. B296, 49 (1988);
S. Catani, F. Fiorani, and G. Marchesini, Phys. Lett. B234, 339 (1990);
S. Catani, F. Fiorani, and G. Marchesini, Nucl. Phys. B336, 18 (1990);
G. Marchesini, Nucl. Phys. B445, 49 (1995)

CASCADE

- Monte Carlo generator **CASCADE** (version 2.2.0, H. Jung et al.; Eur.Phys.J.C70:1237-1249,2010) - implementation of the CCFM equation
  - Backward evolution algorithm for initial state parton showers for
    - Exact kinematics in each step of the parton shower
    - No difference between parton shower evolved uPDF and CCFM evolved uPDF
    - Gluon chains
    - Valence quarks/Non-singlet uPDFs from one-loop CCFM equation
  - Final state parton showers by Pythia algorithm
  - Hadronisation of partons by the Lund String Model
  - Gluon uPDFs obtained from fits to HERA data

Results

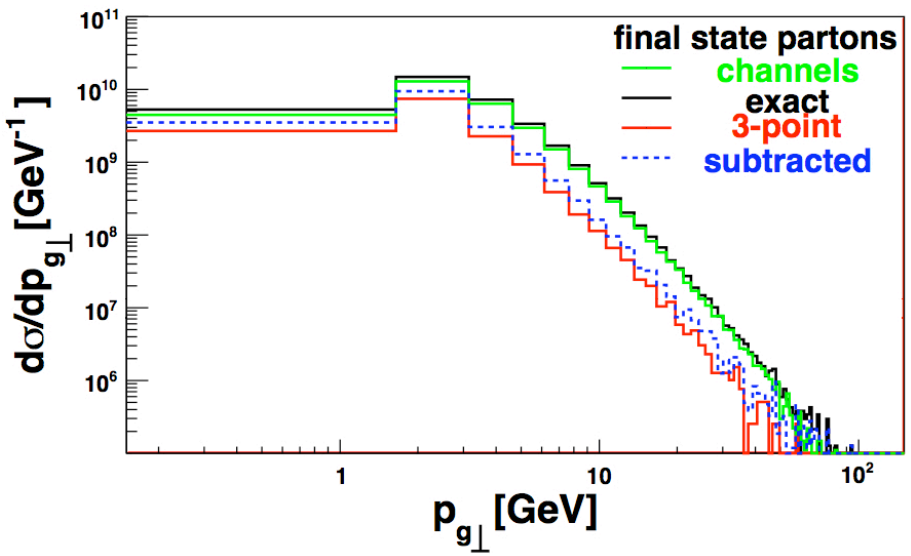
- Transversal momentum of the quark for cut-off *1* and *2 GeV*

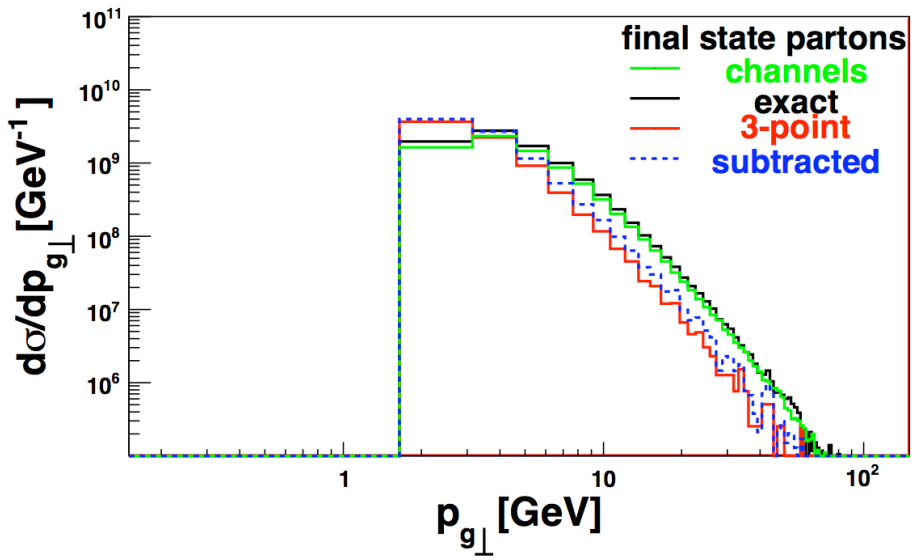


- Good agreement of different cross sections

Results

- Transversal momentum of the gluon for cut-off *1* and *2 GeV*

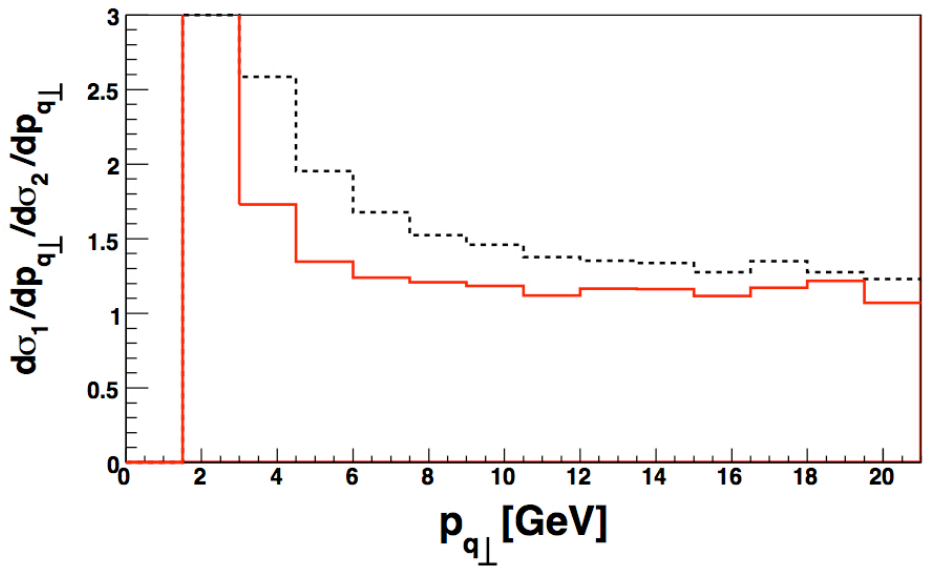


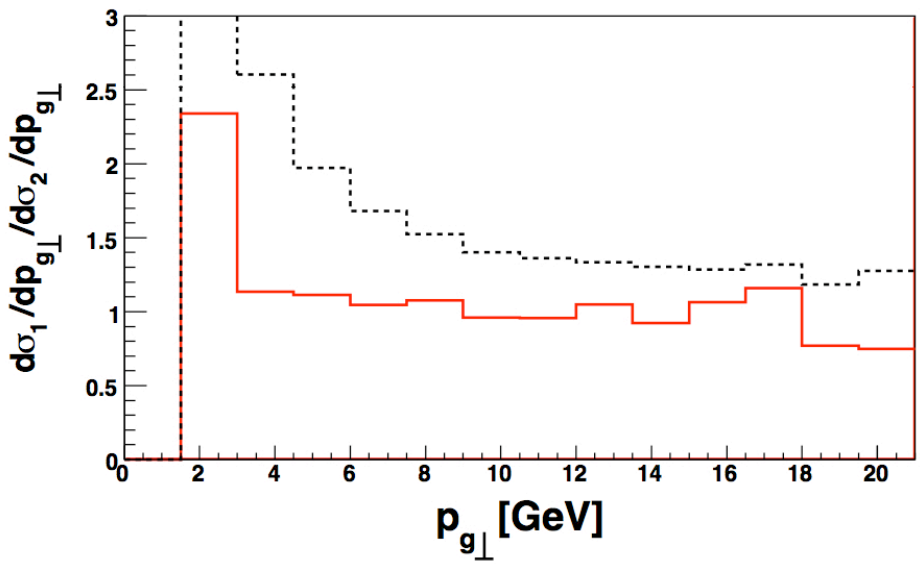


- Disagreement in the magnitude of the cross sections

Results

- Ratios of the cross sections with cut-offs *1* and *2 GeV*





- black dashed - the leading order ME
- red - the result of the subtraction method

Summary and Outlook

- We identified an important problem of cut-off dependence in kt-factorisation 2→2 process
- We suggest a solution to the problem by regularisation by ME to PS matching
- We make a <u>successful</u> “proof of concept” calculation
- Extension to phenomenology necessary
- Event by event implementation needed to remove the cut-off

Back-Up Slides

Other point of view

$$d\sigma^{virtual}_{qg^* \to qg} = d\sigma^{shower}_{qg^* \to qg} - d\sigma^{0}_{qg^* \to qg}$$

$$d\sigma^{corrected}_{qg^* \to qg} = d\sigma^{exact}_{qg^* \to qg} + d\sigma^{virtual}_{qg^* \to qg}$$

Phenomenology results

- Only the 2→2 matrix element