



# QCD resummation for SUSY production

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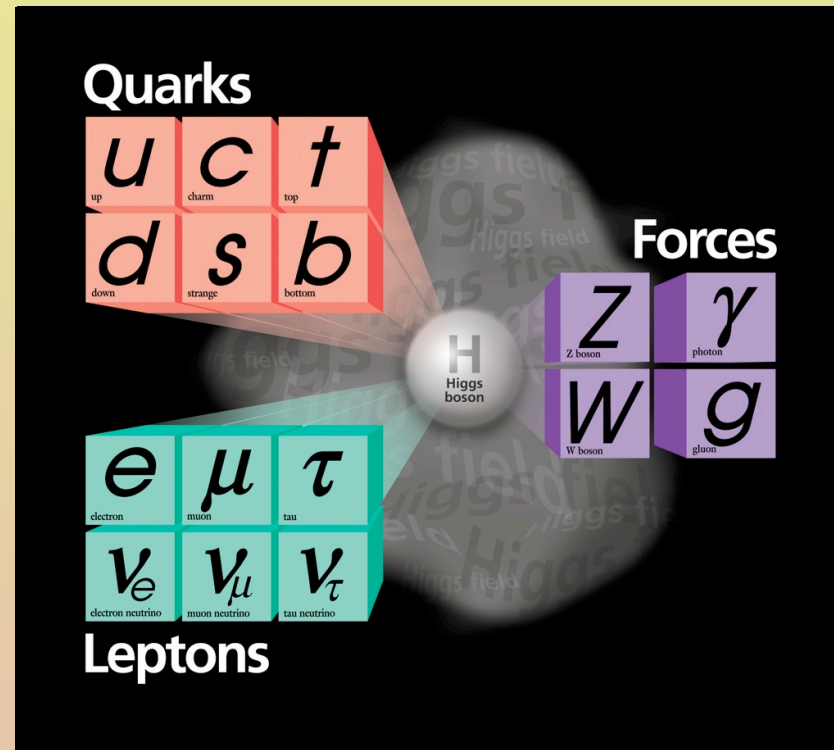
Radboud University Nijmegen

In collaboration with Wim Beenakker, Silja Brensing,  
Michael Krämer, Anna Kulesza and Eric Laenen



# Supersymmetry

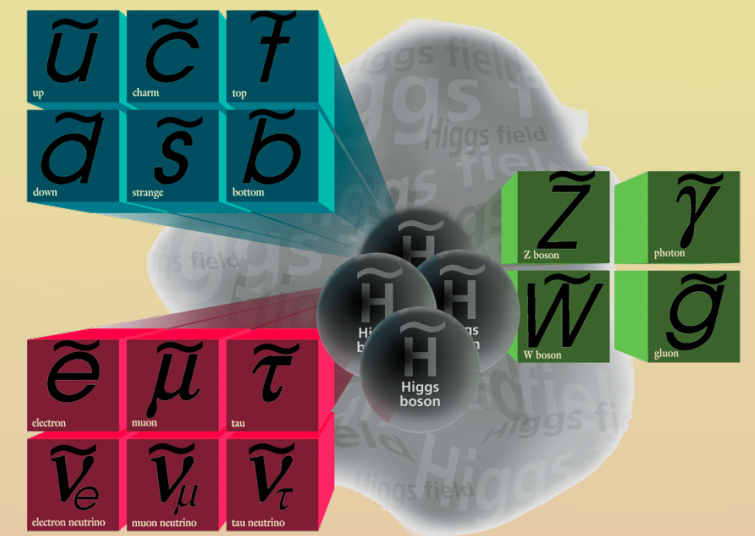
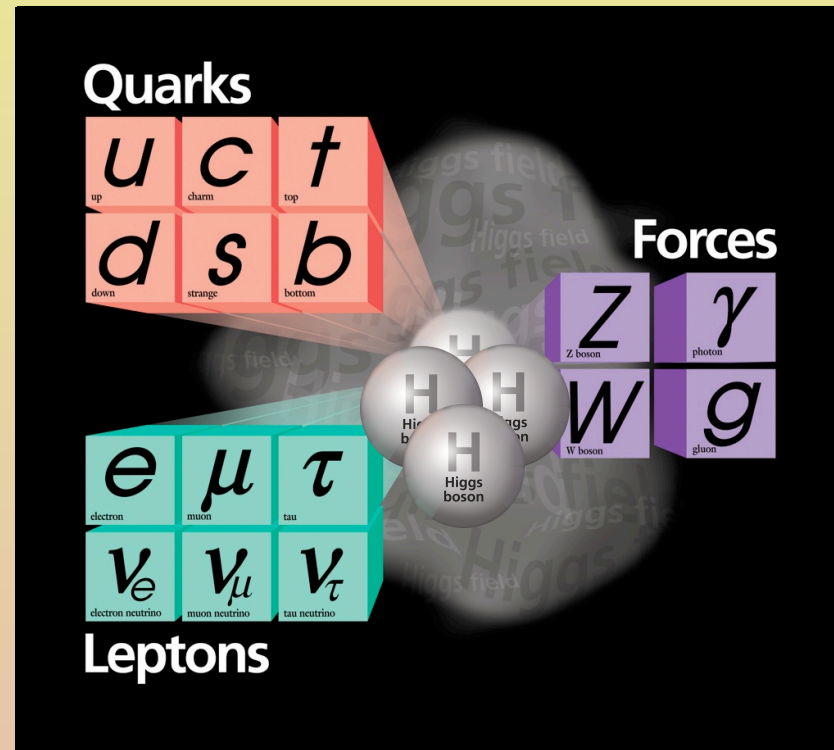
- Hierarchy problem
- Gauge coupling unification
- Dark matter





# Supersymmetry

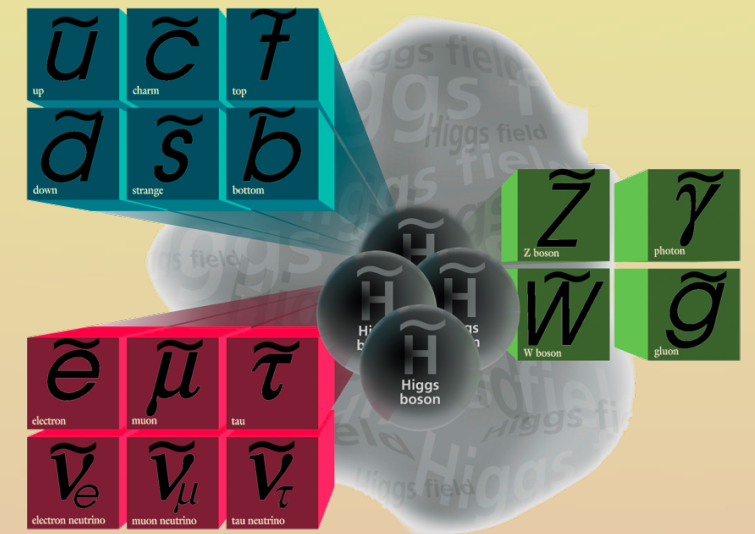
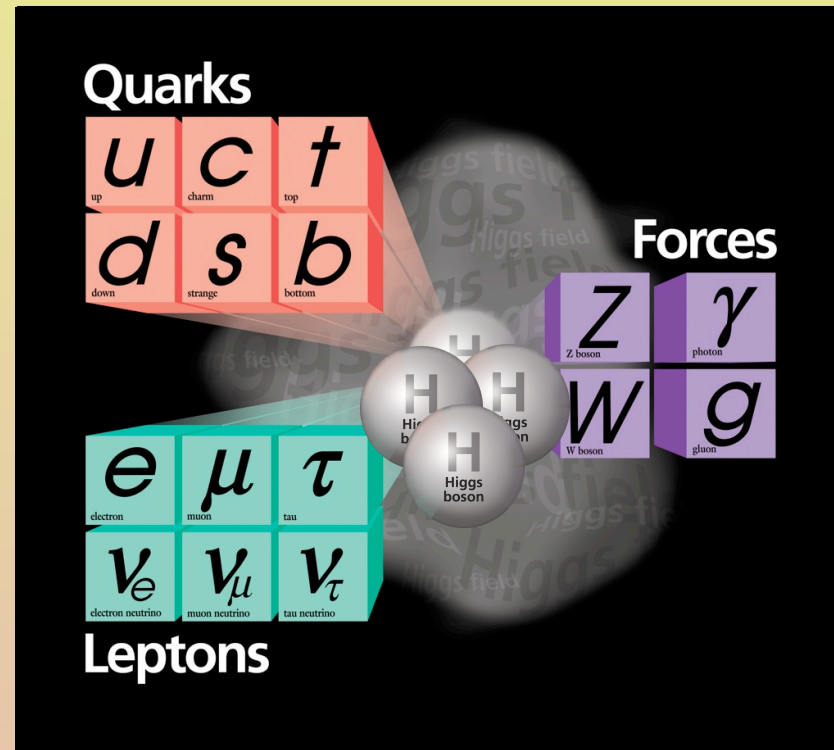
- ✓ Hierarchy problem
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# Supersymmetry

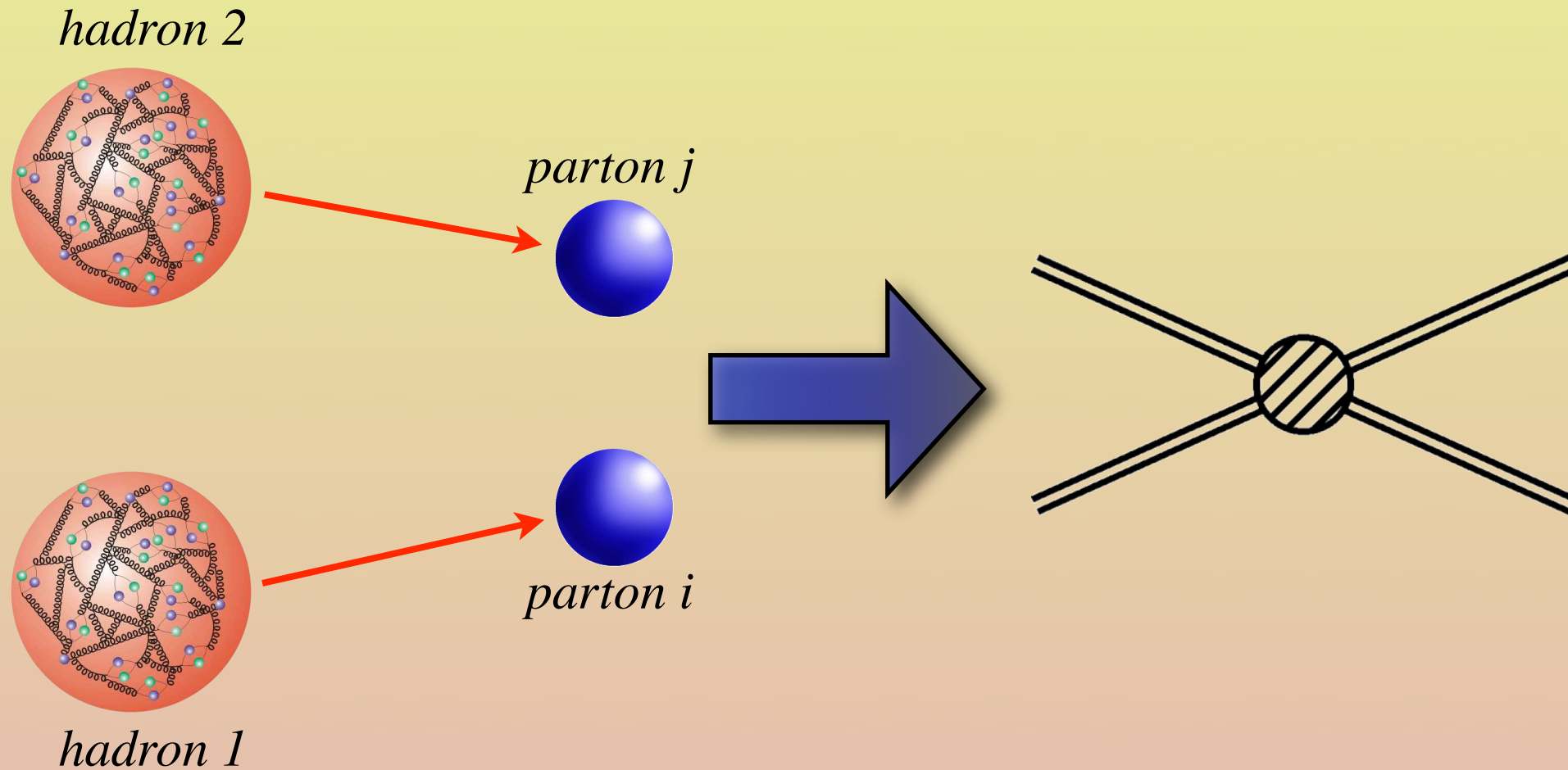
- ✓ Hierarchy problem
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- SUSY particles are heavy

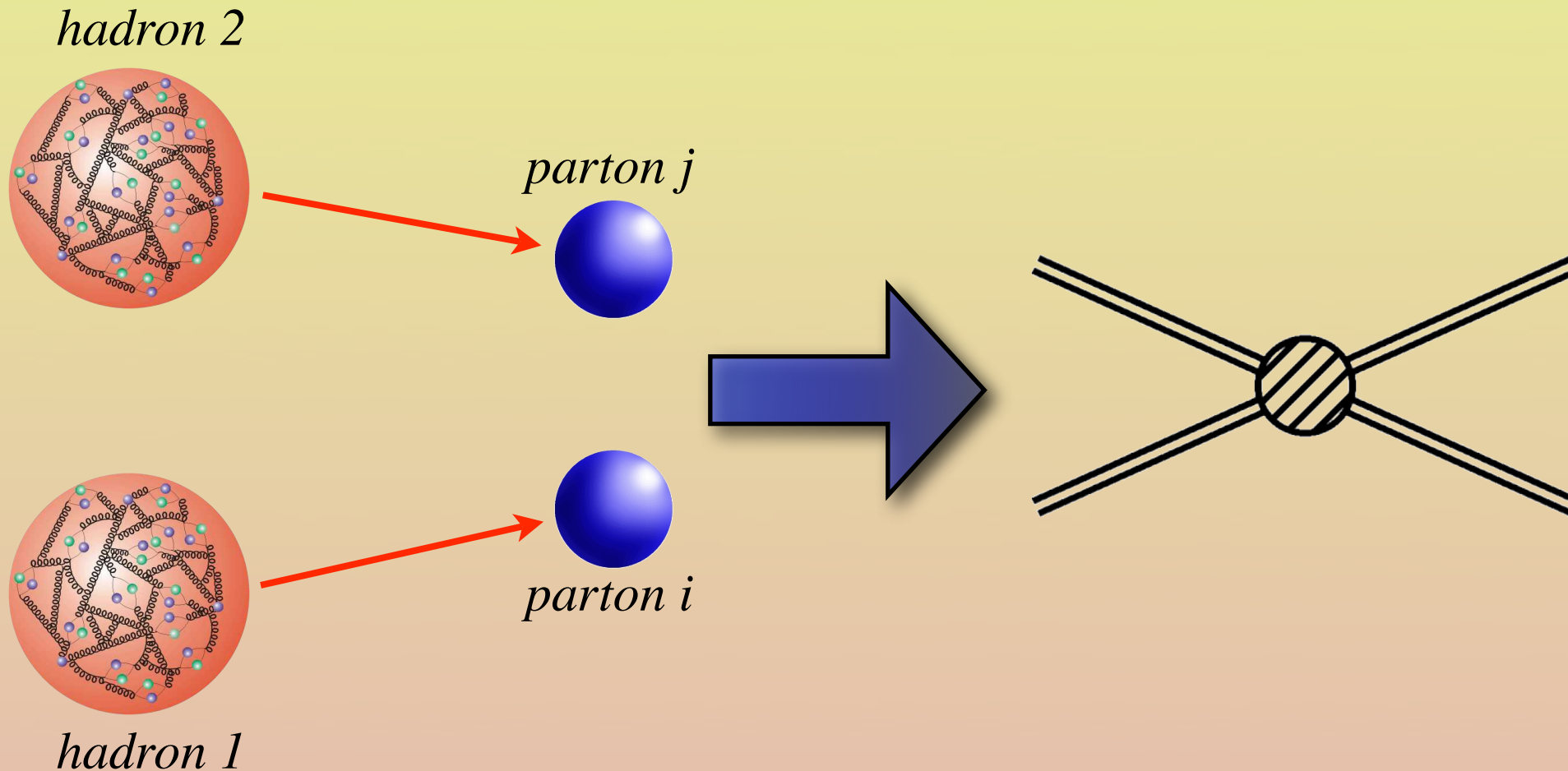


# Colliding Protons





# Colliding Protons

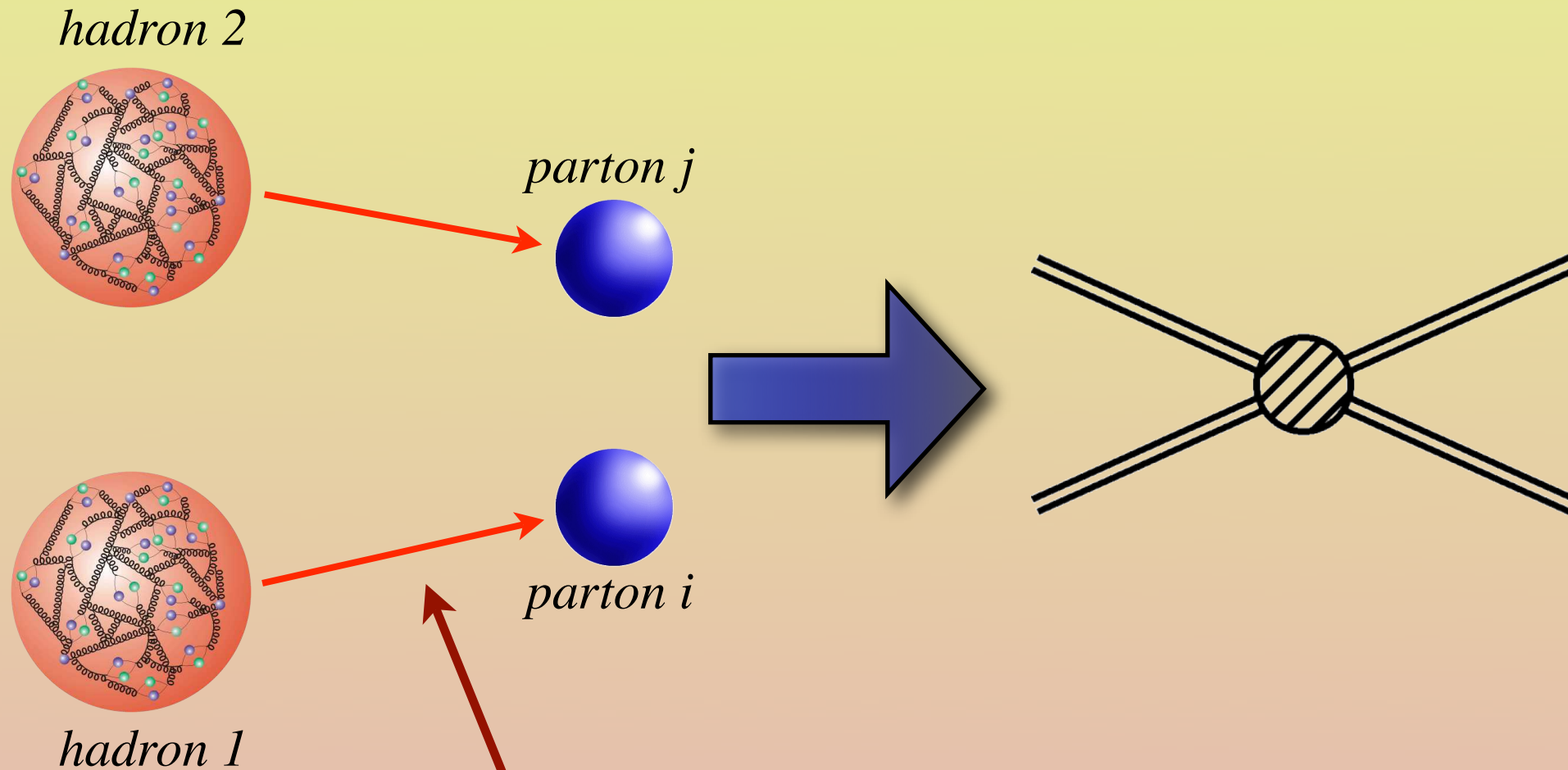


$$\sigma_{h_1 h_2 \rightarrow kl}(\{m^2\}) = \iint \sum_{i,j} f_{i/h_1}(x_1, \mu) f_{j/h_2}(x_2, \mu) \sigma_{ij \rightarrow kl}(\{m^2\}, \mu^2) dx_1 dx_2$$





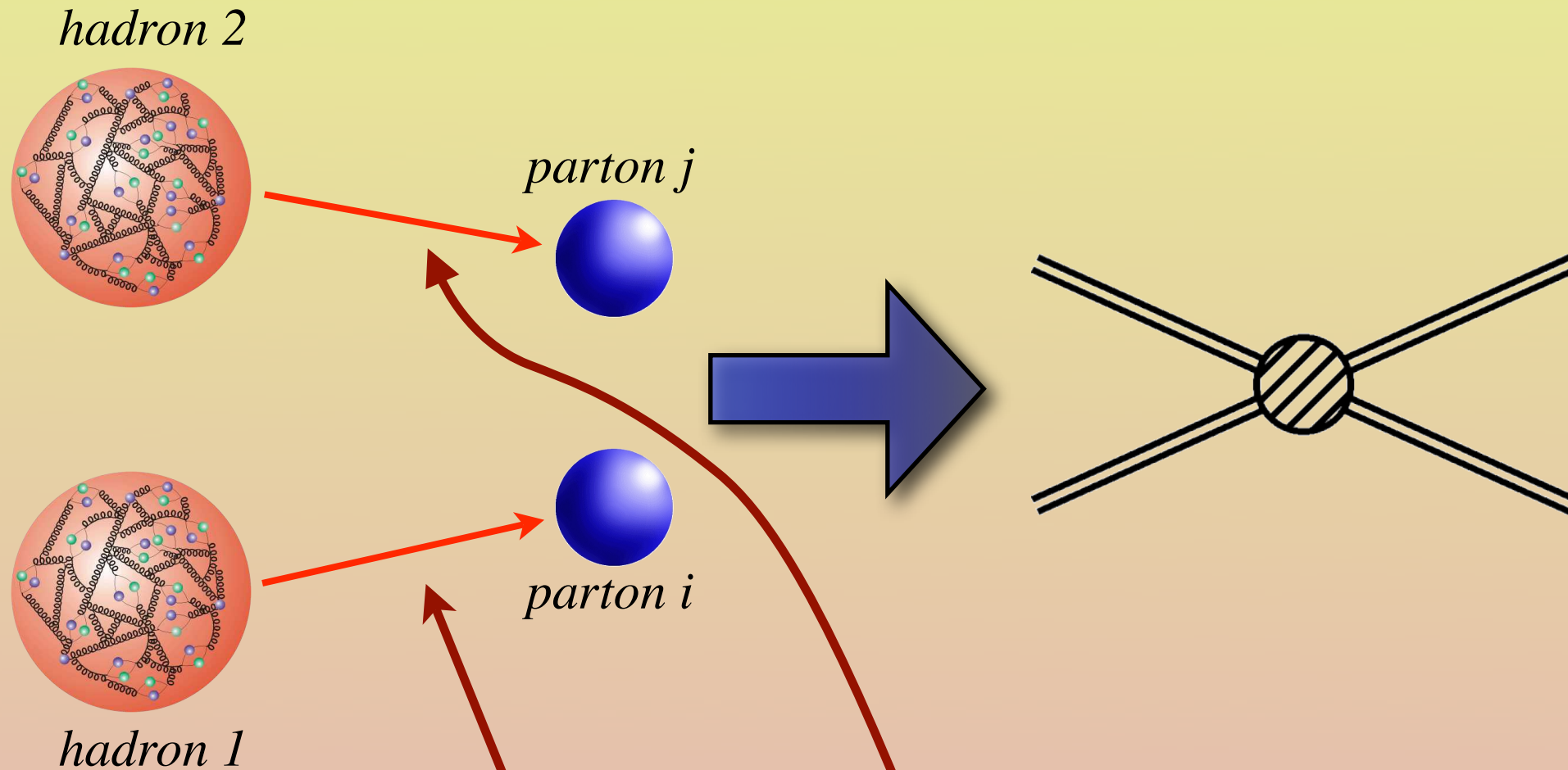
# Colliding Protons



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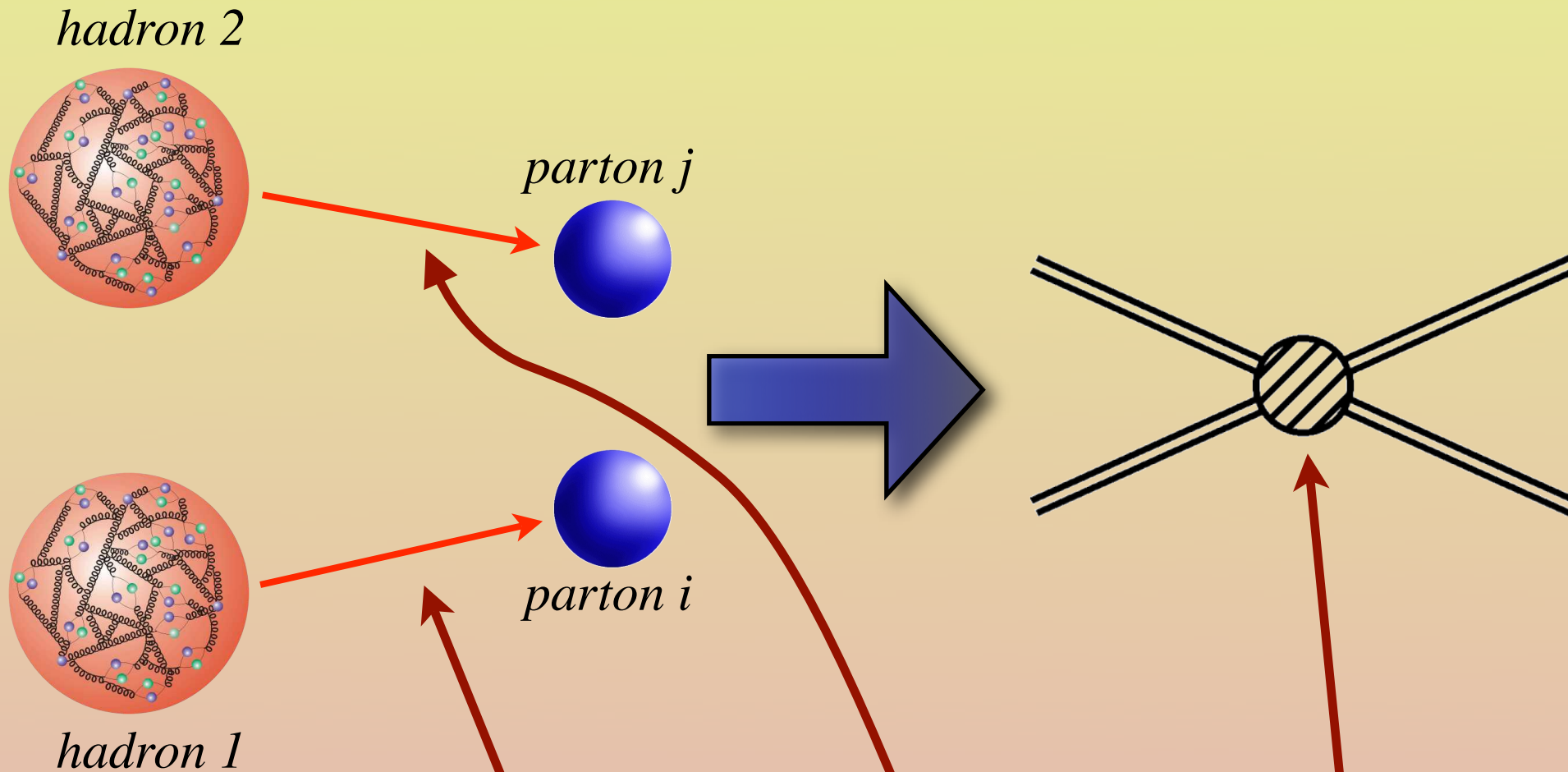
# Colliding Protons



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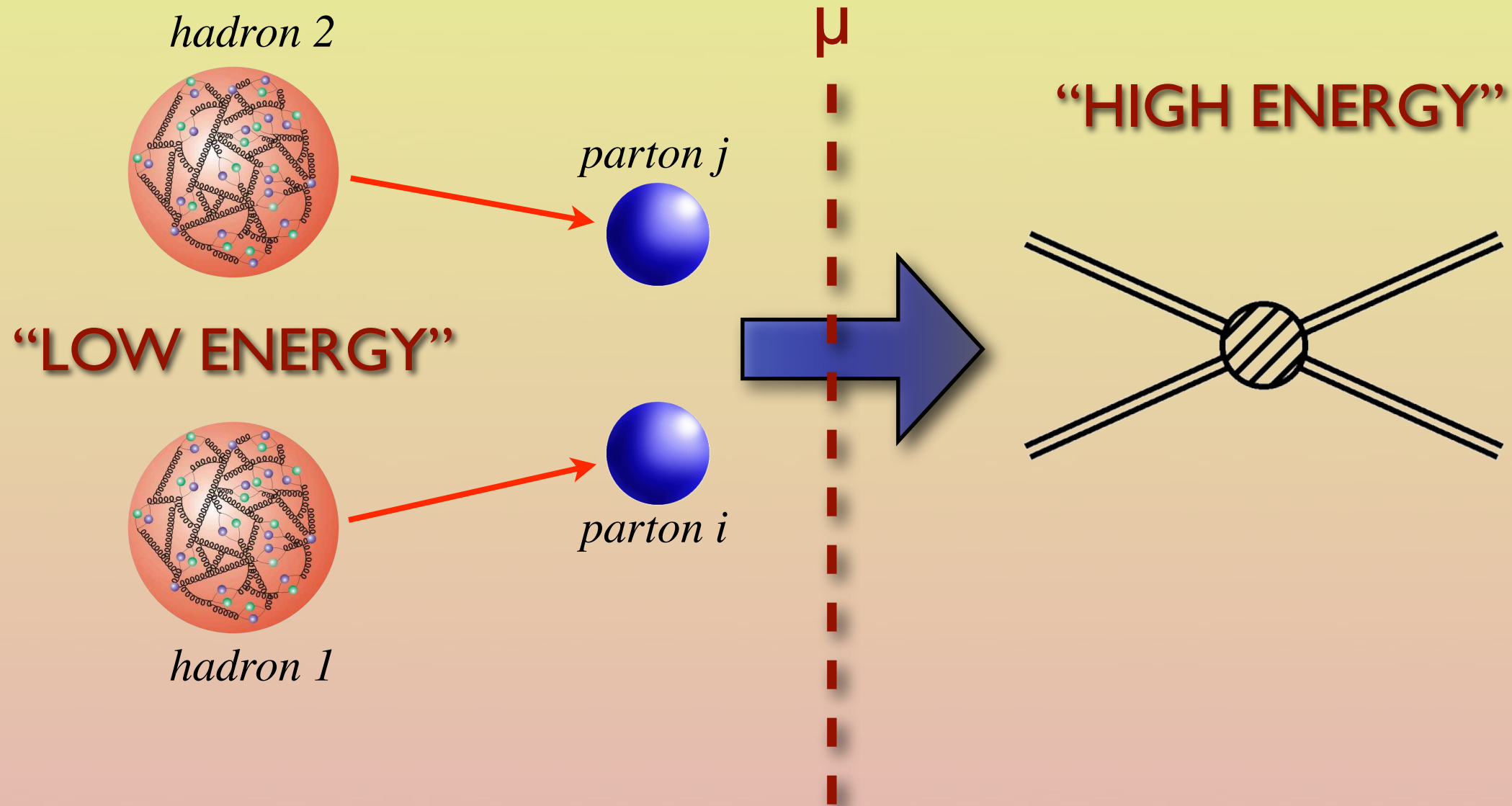
# Colliding Protons



$$\sigma_{h_1 h_2 \rightarrow kl}(\{m^2\}) = \iint \sum_{i,j} f_{i/h_1}(x_1, \mu) f_{j/h_2}(x_2, \mu) \sigma_{ij \rightarrow kl}(\{m^2\}, \mu^2) dx_1 dx_2$$



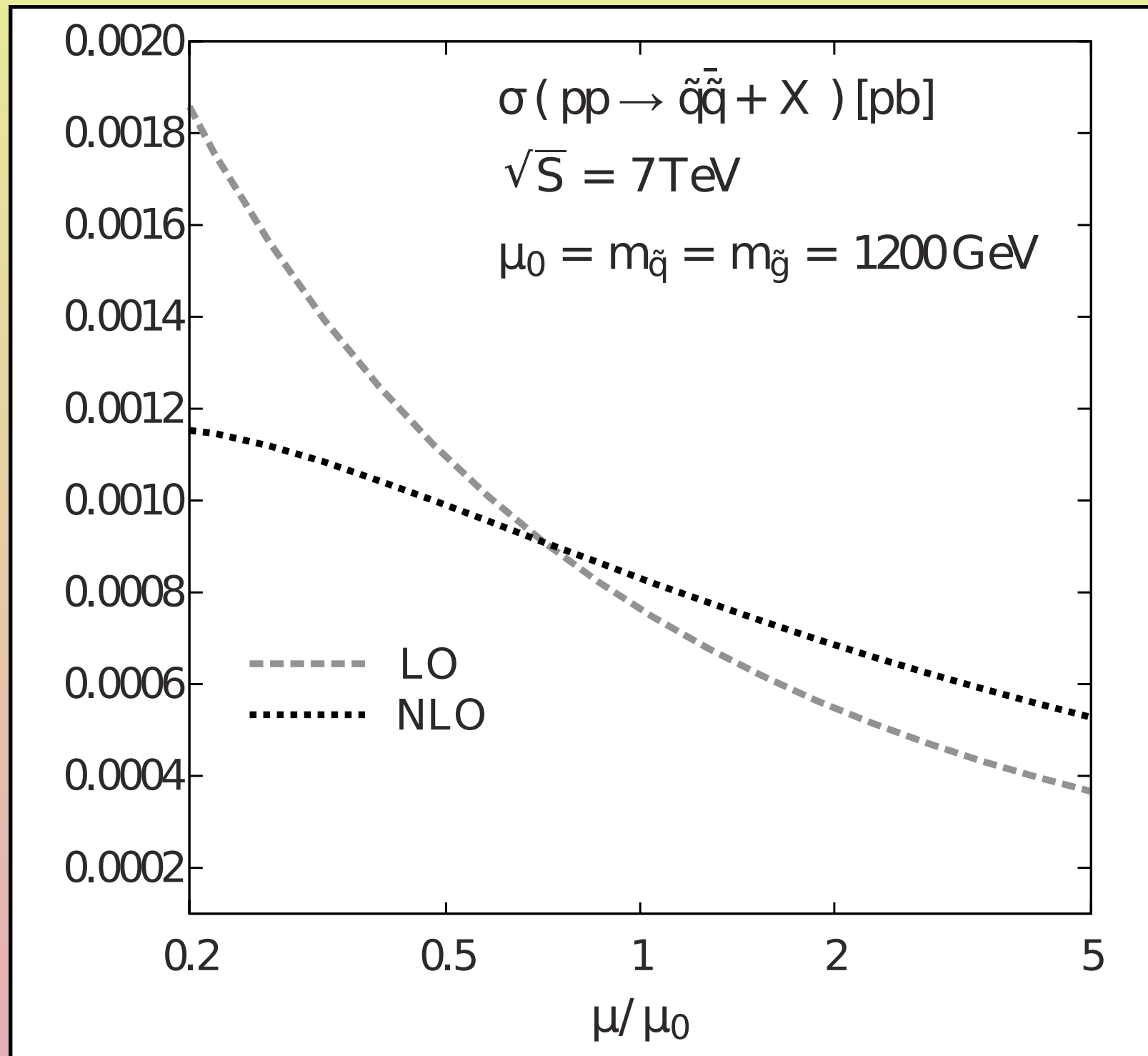
# Colliding Protons



$$\sigma_{h_1 h_2 \rightarrow kl}(\{m^2\}) = \iint \sum_{i,j} f_{i/h_1}(x_1, \mu) f_{j/h_2}(x_2, \mu) \sigma_{ij \rightarrow kl}(\{m^2\}, \mu^2) dx_1 dx_2$$



# Scale dependence





# Resummation

$LO$	1						
$NLO$	$\alpha_s L^2$	$\alpha_s L$	$\alpha_s$				
$NNLO$	$\alpha_s^2 L^4$	$\alpha_s^2 L^3$	$\alpha_s^2 L^2$	$\alpha_s^2 L$	$\alpha_s^2$		
$N^3 LO$	$\alpha_s^3 L^6$	$\alpha_s^3 L^5$	$\alpha_s^3 L^4$	$\alpha_s^3 L^3$	$\alpha_s^3 L^2$	$\alpha_s^3 L$	$\alpha_s^3$
$N^4 LO$	...						

$$L = \log(8\beta^2) \quad \beta = \sqrt{1 - \rho} \quad \rho = \frac{4m^2}{s}$$



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$LO$	1						
$NLO$	$\alpha_s L^2$	$\alpha_s L$	$\alpha_s$				
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$N^4 LO$	...						

$$\tilde{f}(N) = \int_0^1 d\rho \rho^{N-1} f(\rho)$$

$$L = \log(8\beta^2) \quad \beta = \sqrt{1-\rho} \quad \rho = \frac{4m^2}{s}$$



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$N^4 LO$	...						

$$\tilde{f}(N) = \int_0^1 d\rho \rho^{N-1} f(\rho) \quad L \longrightarrow \log(N)$$

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$$\tilde{f}(N) = \int_0^1 d\rho \rho^{N-1} f(\rho) \quad L \rightarrow \log(N)$$

$$\tilde{\sigma}^{\text{resum}} = \tilde{\sigma}^{\text{thr}} e^{LP_1(\alpha_s L)} e^{P_2(\alpha_s L)} e^{\alpha_s P_3(\alpha_s L)}$$



# Resummation

$LO$	1							
$NLO$	$\alpha_s L^2$	$\alpha_s L$	$\alpha_s$					
$NNLO$	$\alpha_s^2 L^4$	$\alpha_s^2 L^3$	$\alpha_s^2 L^2$	$\alpha_s^2 L$	$\alpha_s^2$			
$N^3 LO$	$\alpha_s^3 L^6$	$\alpha_s^3 L^5$	$\alpha_s^3 L^4$	$\alpha_s^3 L^3$	$\alpha_s^3 L^2$	$\alpha_s^3 L$	$\alpha_s^3$	
$N^4 LO$	...							

$$\tilde{f}(N) = \int_0^1 d\rho \rho^{N-1} f(\rho) \quad L \rightarrow \log(N)$$

$$\tilde{\sigma}^{\text{resum}} = \tilde{\sigma}^{\text{thr}} \underset{\substack{\uparrow \\ LO}}{\text{LL}} e^{LP_1(\alpha_s L)} e^{P_2(\alpha_s L)} e^{\alpha_s P_3(\alpha_s L)}$$



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$N^4 LO$	...						

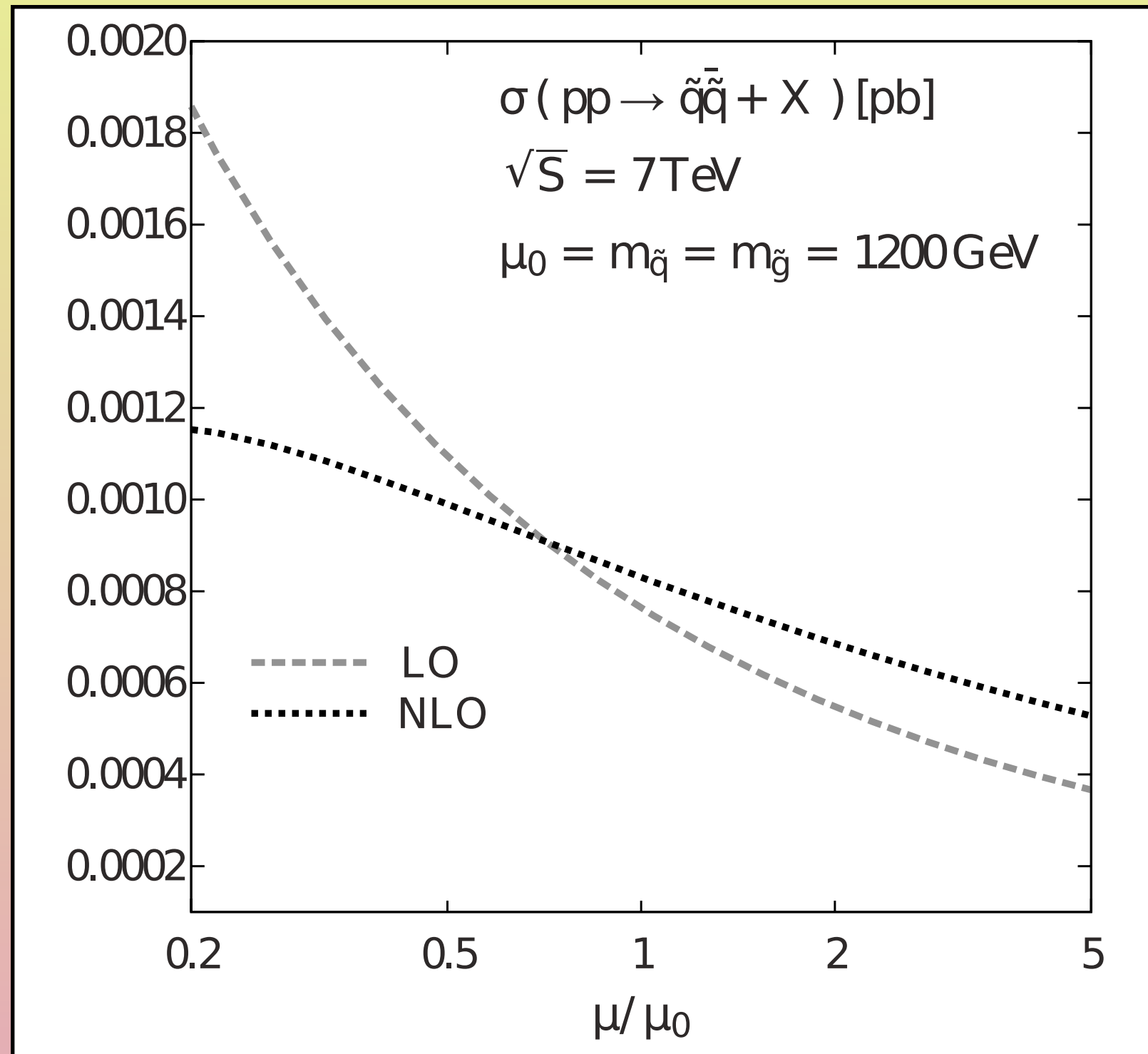
$$\tilde{f}(N) = \int_0^1 d\rho \rho^{N-1} f(\rho) \quad L \rightarrow \log(N)$$

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LL
NLL

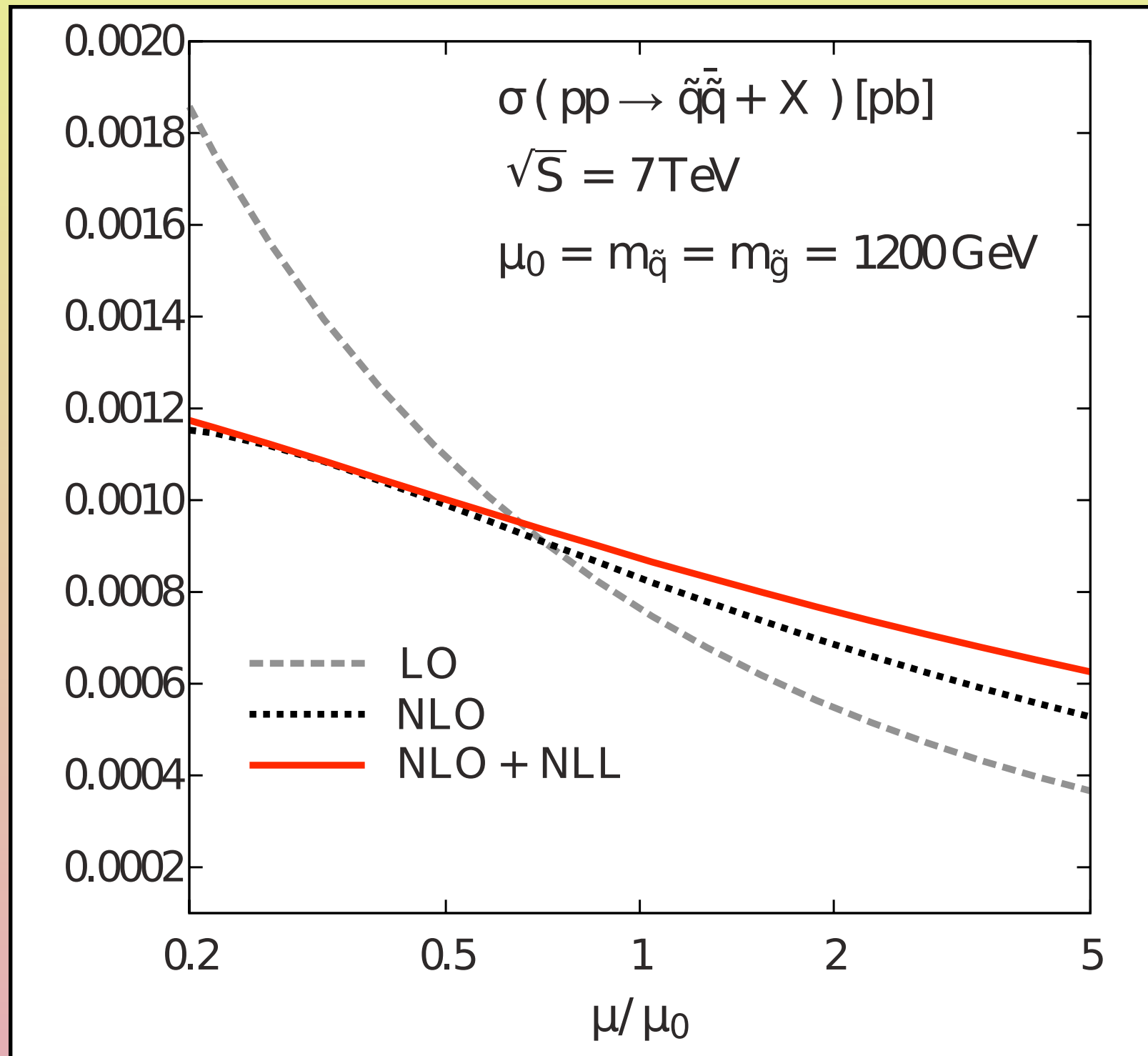


# NLL: scale dependence



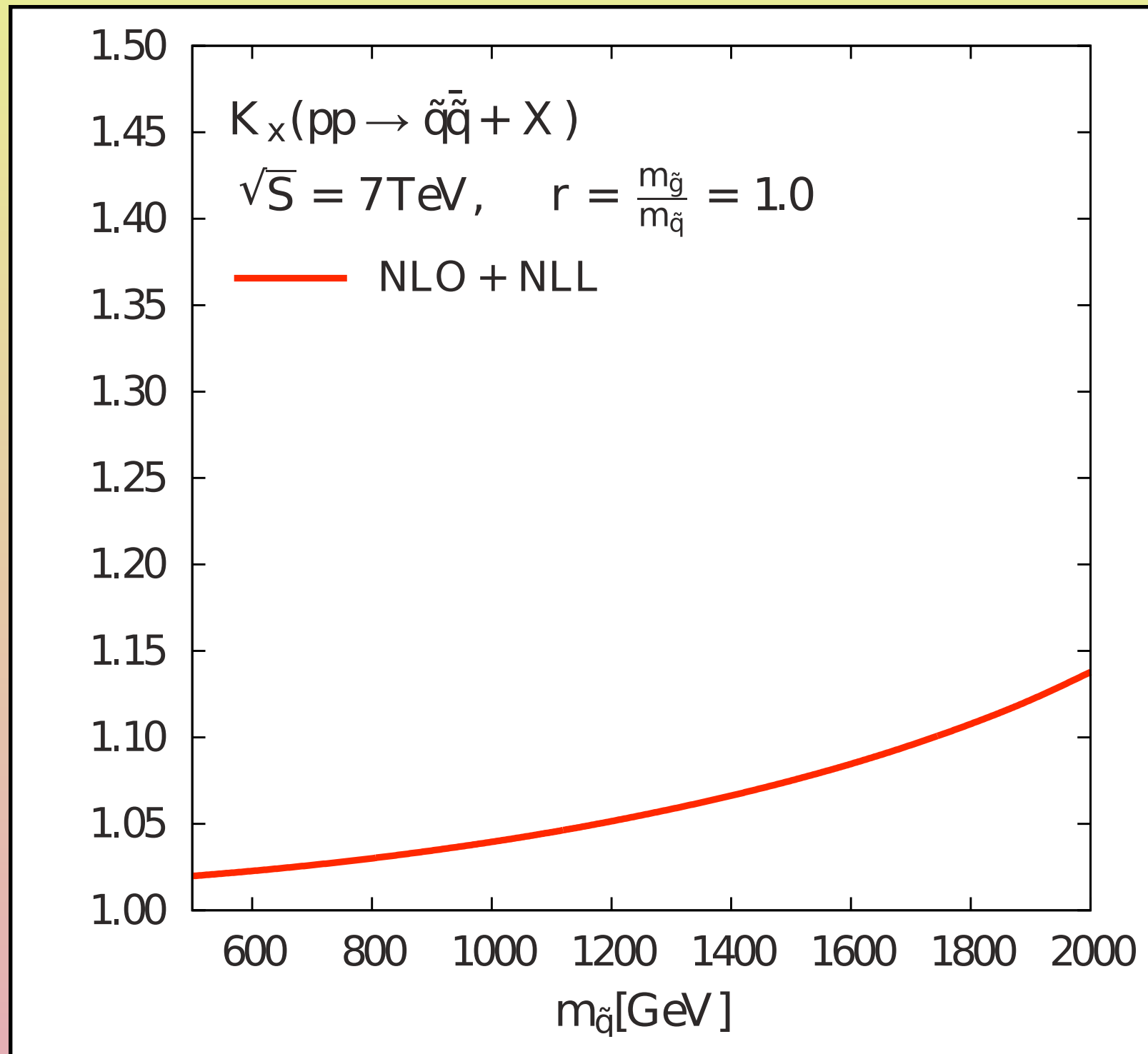


# NLL: scale dependence





# NLL: K-factor







# NLLfast

- Grid and fast interpolation code for NLL resummation in SUSY-QCD
  - MSTW 2008 NLO PDFs
  - Masses from 500 GeV to 2000 GeV
  - LO, NLO (PROSPINO) and NLL cross sections
  - Uncertainty from scale, PDF and  $\alpha_s$
- <http://web.physik.rwth-aachen.de/service/wiki/bin/view/Main/SquarksandGluginos>



# Going beyond

$LO$	1						$L = \log(N)$
$NLO$	$\alpha_s L^2$	$\alpha_s L$	$\alpha_s$				
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$N^3 LO$	$\alpha_s^3 L^6$	$\alpha_s^3 L^5$	$\alpha_s^3 L^4$	$\alpha_s^3 L^3$	$\alpha_s^3 L^2$	$\alpha_s^3 L$	$\alpha_s^3$
$N^4 LO$	...						

$$\tilde{\sigma}^{\text{resum}} = \tilde{\sigma}^{\text{thr}} \underset{\substack{\uparrow \\ LO}}{\text{LO}} e^{LLP_1(\alpha_s L)} e^{NLLP_2(\alpha_s L)} e^{\alpha_s P_3(\alpha_s L)}$$



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$N^4 LO$	$\dots$					$\alpha_s^3$

$$\tilde{\sigma}^{\text{resum}} = \tilde{\sigma}^{\text{thr}} e^{LP_1(\alpha_s L)} e^{P_2(\alpha_s L)} e^{\alpha_s P_3(\alpha_s L)}$$

↑

NLO

LL

NLL

NNLL



# NNLL resummation

$$\sigma^{\text{NLO,thr}} = C^{\text{NNLL}} \sigma^{\text{LO,thr}}$$



# NNLL resummation

$$\sigma^{\text{NLO,thr}} = C^{\text{NNLL}} \sigma^{\text{LO,thr}}$$

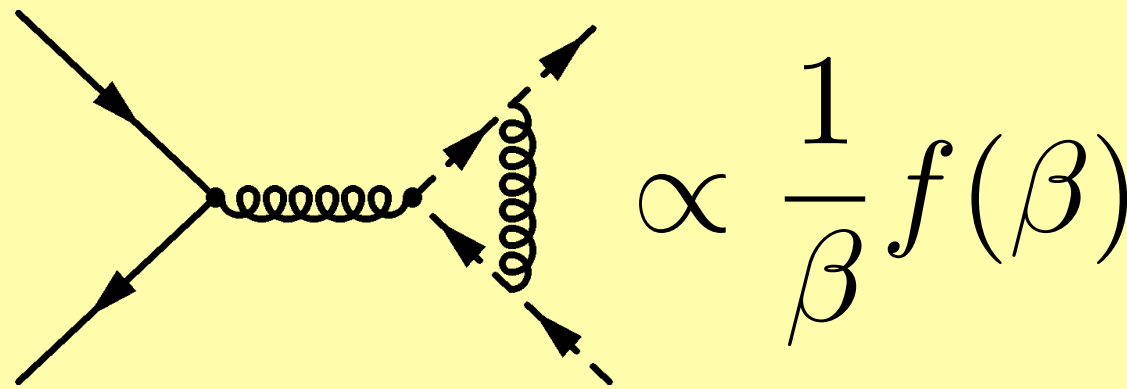
$$C^{\text{NNLL}} = \left( 1 + \frac{\alpha_s}{\pi} \mathcal{C}^{\text{Coul,(1)}}(N, \{m^2\}, \mu^2) \right) \left( 1 + \frac{\alpha_s}{\pi} \mathcal{C}^{(1)}(\{m^2\}, \mu^2) \right)$$



# NNLL resummation

$$\sigma^{\text{NLO,thr}} = C^{\text{NNLL}} \sigma^{\text{LO,thr}}$$

$$C^{\text{NNLL}} = \left( 1 + \frac{\alpha_s}{\pi} \mathcal{C}^{\text{Coul},(1)}(N, \{m^2\}, \mu^2) \right) \left( 1 + \frac{\alpha_s}{\pi} \mathcal{C}^{(1)}(\{m^2\}, \mu^2) \right)$$



**Leading Coulomb  
corrections  
(bound state effects)**

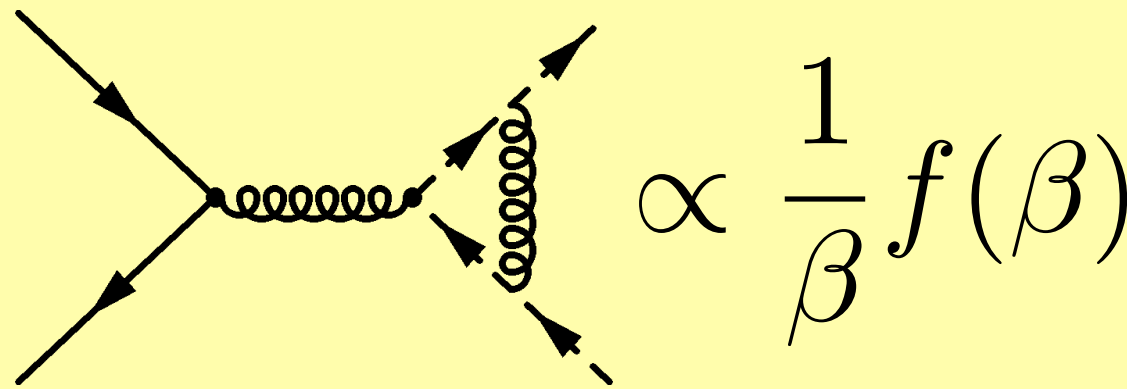




# NNLL resummation

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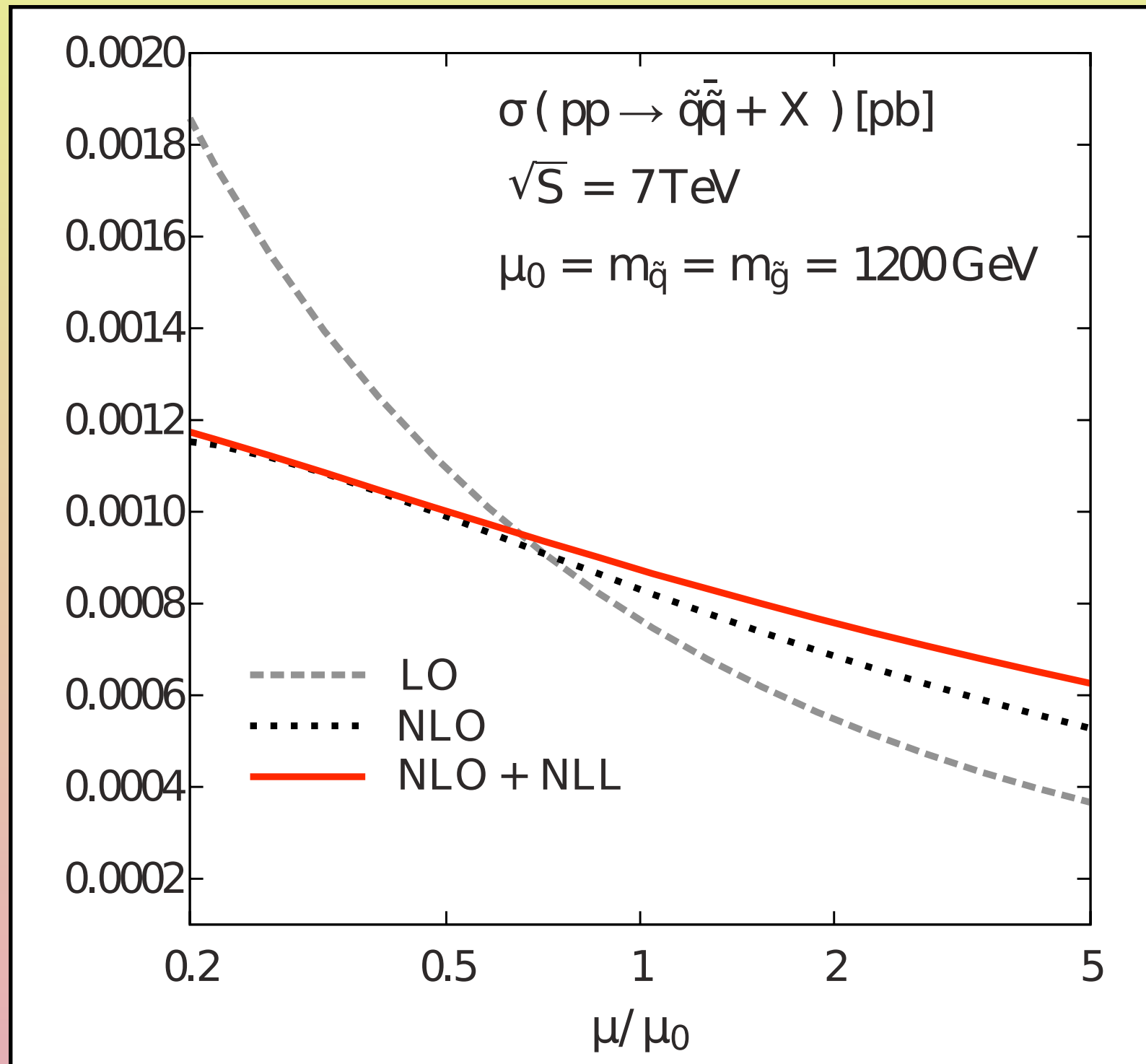


Leading Coulomb  
corrections  
(bound state effects)

Other NLO  
contributions

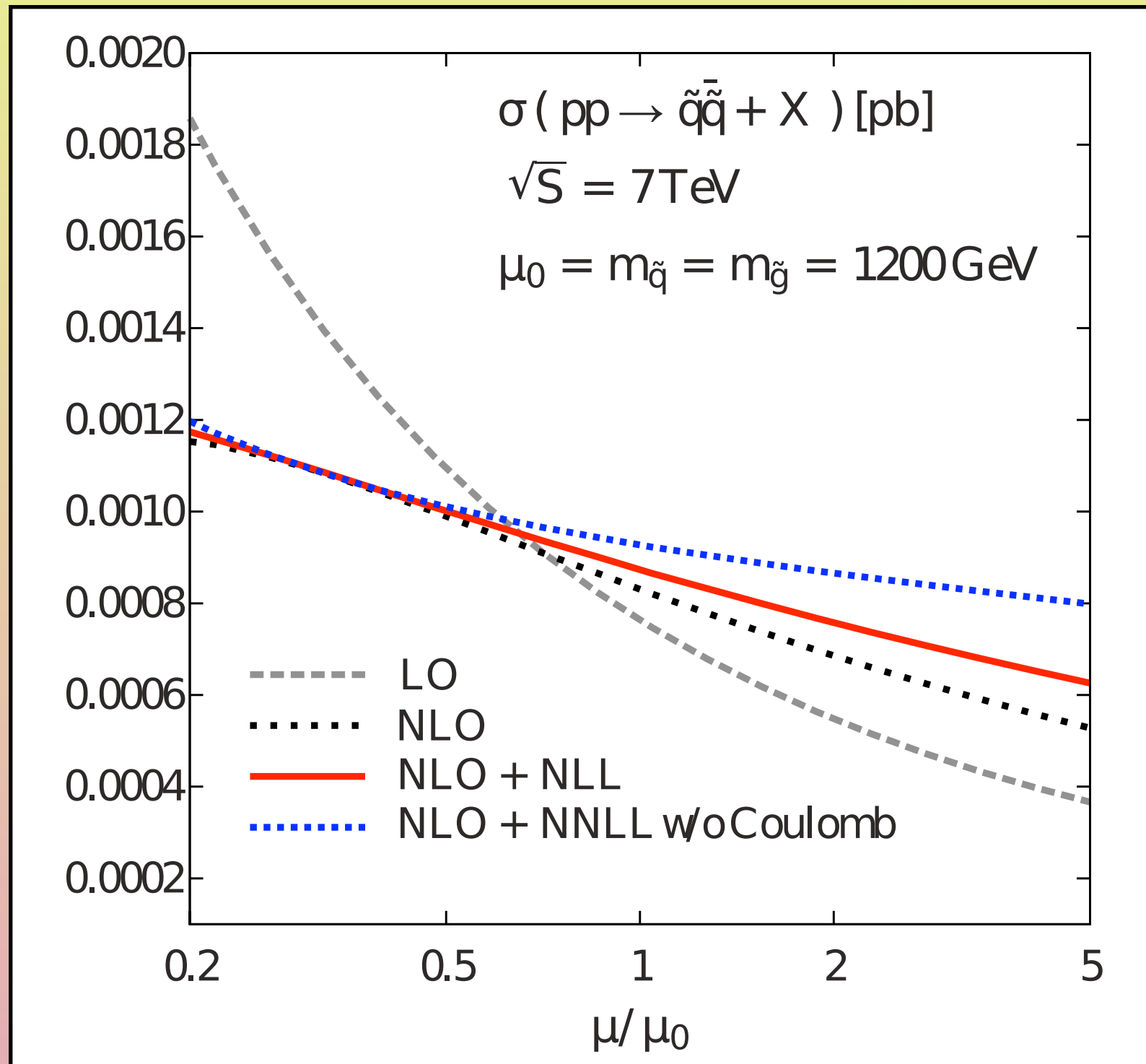


# NNLL: scale dependence



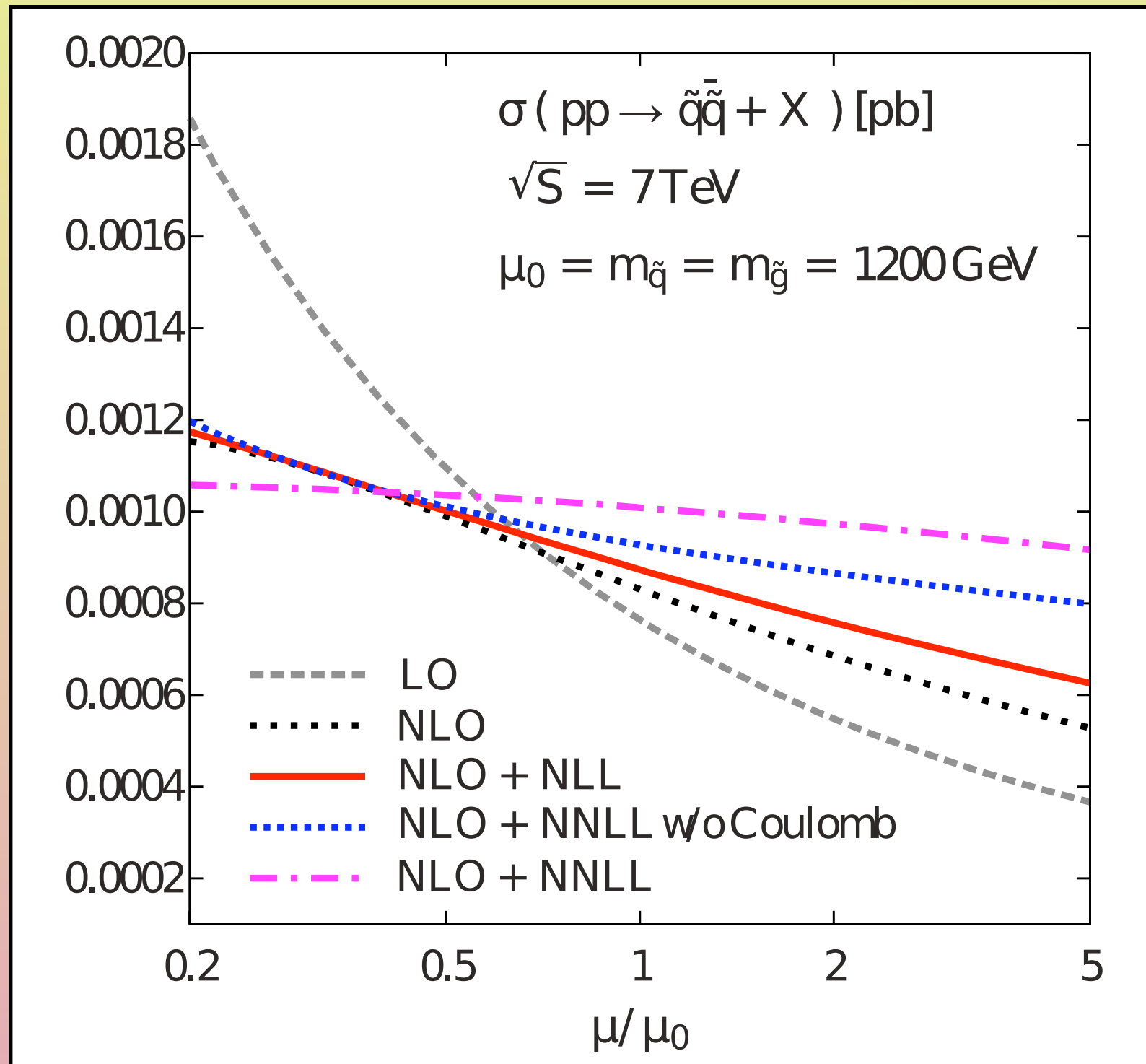


# NNLL: scale dependence



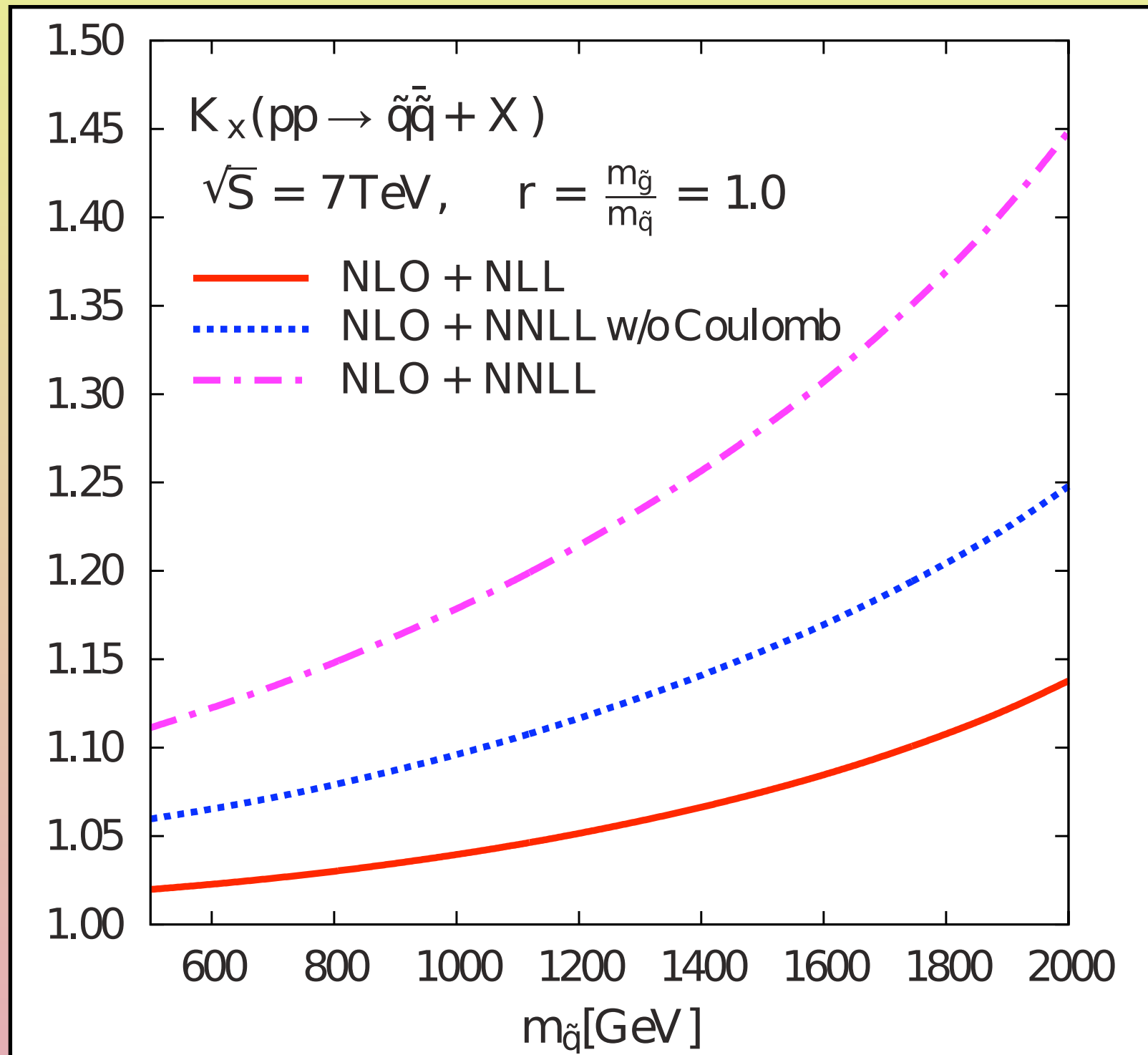


# NNLL: scale dependence





# NNLL: K-factor





# Conclusion

QCD resummation for SUSY:

- Reduces scale dependence
- Increases cross section at central scale
- NLLfast: code for NLL-resummed SUSY-QCD





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QCD resummation for SUSY:

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Still to do:

- Include anything experiments need in NLLfast
- Include other processes in NNLL calculation