RWTH AACHEN UNIVERSITY

Baryogenesis & Leptogenesis

Quantum Kinetic (Boltzmann) Equations from the Closed-Time Path (CTP) Formalism (today mostly for Leptogenesis)

- Improved predictions for established models
- New models with interesting signatures & their viability

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Goals

- Explain origin & size of baryon asymmetry of the Universe (BAU) $\frac{n_B}{s} = \begin{cases} (6{,}7 - 9{,}2) \cdot 10^{-11} & (BBN) \\ (8{,}36 - 9{,}32) \cdot 10^{-11} & (CMB) \end{cases}$

- Repeat successful predictions* from Big Bang Nucleosynthesis (BBN) and for baryon acoustic oscillations in the Cosmic Microwave Background (CMB)

- Calculations of BAU, BBN & CMB rely on suitable kinetic equations

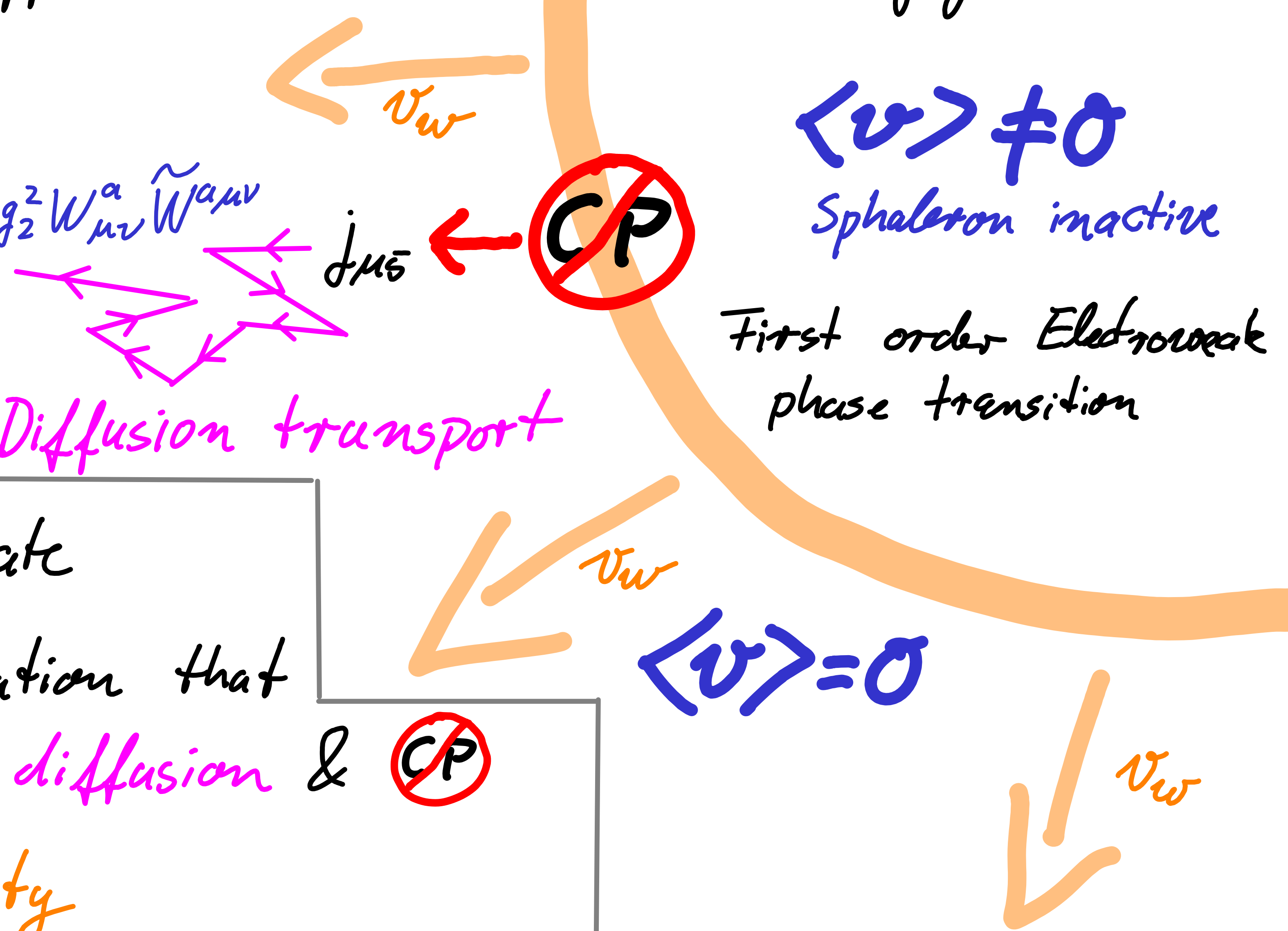
- Theory predictions of better than $\mathcal{O}(1)$ accuracy necessary

*Of course, also like to achieve this for Dark Matter

Challenges & Opportunities: Electroweak Baryogenesis

$$\partial^\mu j_\mu^{(B+L)} = -\frac{2n_f}{32\pi^2} g_2^2 W^a_{\mu\nu}\tilde{W}^{a\mu\nu}$$

$\rightarrow$ Sphaleron



Theory Challenges:

- Spahleron Rate
- Kinetic Equation that encompasses diffusion & $\cancel{CP}$
- Wall velocity

$\Rightarrow$ Difficult to obtain quantitative predictions based on controlled approximations.

Prospects for Observations*

Electroweak Baryogenesis (EWBG):

- Higgs coupled to new light bosons and/or non-renormalisable self interactions in order to achieve 1st order PT
- $\cancel{CP}$ beyond the Standard Model (SM)
- Gravitational waves

Leptogenesis (LG):

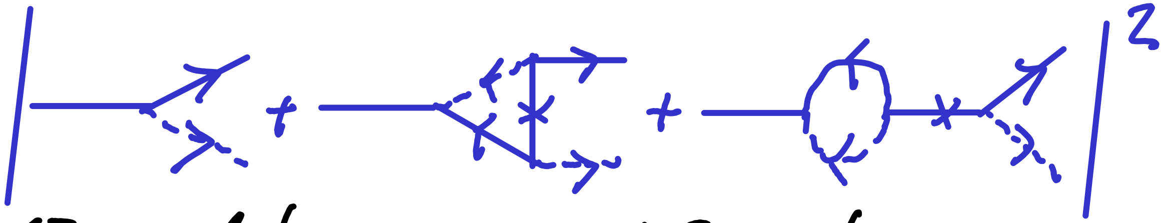
- Neutrino oscillations, neutrino mass scale, $0\nu\beta\beta$
- $\cancel{CP}$ from Majorana phases — experimentally inaccessible

*These comments hold for the most minimal/simplest models. Consideration of loopholes is, of course, worthwile.

Merits of an Improved Kinetic Theory of Leptogenesis

- Well motivated, plausible scenario
- Achieve improved quantitative predictions in many parametric regimes: So far, accuracy better than order one only for unflavoured leptogenesis in strong washout.
- Development of methods of relevance for (EWBG) within a simpler setting
- Incorporate unitary evolution (important for $\cancel{CP}$), flavour coherence of active leptons (flavoured LG) & sterile neutrinos (resonant LG) into kinetic theory

Standard Approach to Leptogenesis & Unitarity



CP-violating squared S-matrix elements (quantum)

$L[f] = C[f]$

Boltzmann equation (classical)

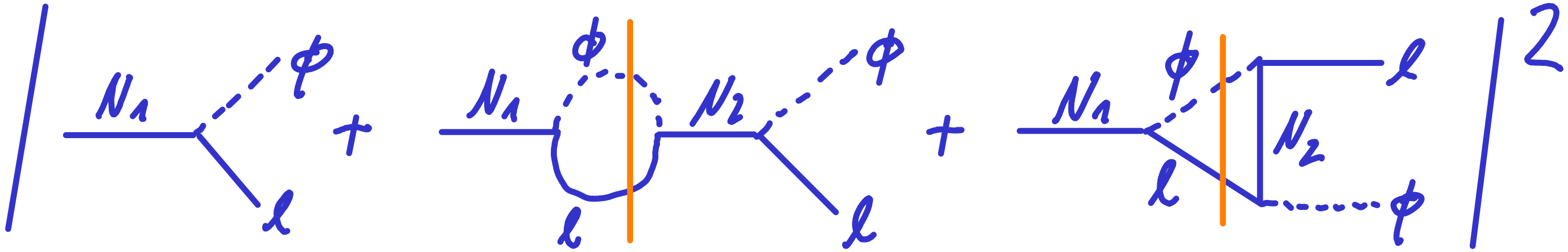
Lepton Asymmetry

Fukugita & Yanagida (1986)

Asymmetry in Boltzmann Approach

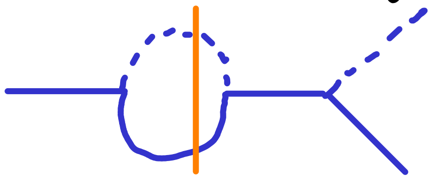
Fukugita & Yanagida (1986)
Covi, Roulet, Vissani (1996)
Buchmüller & Plümacher (1996)

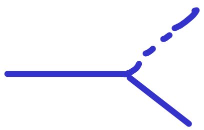
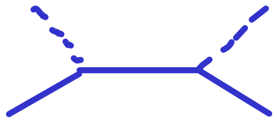
- Interference of tree & loop amplitudes ⟶ CP violation



(*)

- CP violating contributions from discontinuities
  ⟶ loop momenta where cut particles are on shell
  (Cutkosky rules)

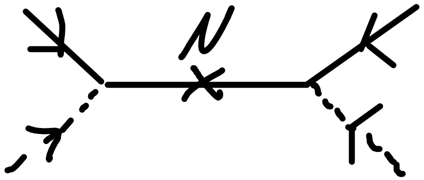
- Is  an extra process or is it already

  accounted for by  and  ?

- Including (*) only ⟶ CP asymmetry generated
  even in equilibrium

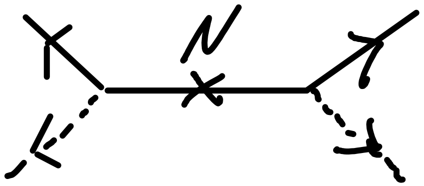
What the Vacuum Matrix Elements Can Do for Us:

$2 \longleftrightarrow 2$



$$|\mathcal{M}_{\ell\phi\to\bar{\ell}\phi^*}|^2$$

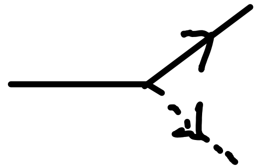
$$=$$



$$|\mathcal{M}_{\bar{\ell}\phi^*\to\ell\phi}|^2$$

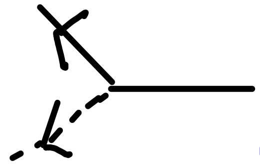
No asymmetry ever generated

$1 \longleftrightarrow 2$

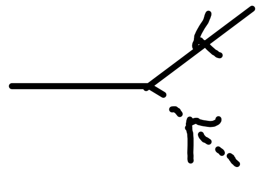


$$|\mathcal{M}_{N\to\ell\phi}|^2 \sim 1+\varepsilon$$

$$\overset{CPT}{\equiv}$$

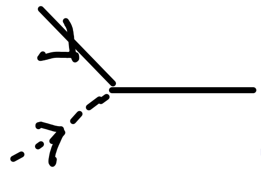


$$|\mathcal{M}_{\bar{\ell}\phi^*\to N}|^2 \sim 1+\varepsilon$$



$$|\mathcal{M}_{N\to\bar{\ell}\phi^*}|^2 \sim 1-\varepsilon$$

$$\overset{CPT}{\equiv}$$



$$|\mathcal{M}_{\ell\phi\to N}|^2 \sim 1-\varepsilon$$

Naive multiplication* suggests asymmetry even in equilibrium:

$$\Gamma_{\bar{\ell}\phi^*\to\ell\phi} \sim 1+2\varepsilon$$

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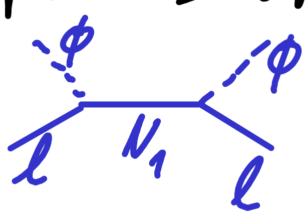
*Do not try this at home: The unstable N cannot constitute elements of a unitary S-matrix.

CPT, Unitarity & RIS

- Generation of CP asymmetry in equilibrium ↯ CPT theorem ↯
- N_1 unstable, not an asymptotic state of a unitary S-matrix
  $\longrightarrow$ Multiplication of matrix elements implicit in Boltzmann equations leads to non-unitary evolution
- Usual fix: subtract Real Intermediate States (RIS) from



Kolb & Wolfram (1980)

Purpose of the Closed-Time-Path (CTP) Formalism

- Scattering theory: S-matrix elements between free asymptotic in- and out-states
- S-matrix elements are obtained from time-ordered Green functions (LSZ reduction formula)
- Here, we are interested in the expectation value of an operator without time-ordering:

  $\langle \bar{\psi}_\ell \gamma^0 \psi_\ell \rangle$ lepton charge density

  $\longrightarrow$ Calculate this using the CTP formalism

Functional Approach (in-out)

- In-out generating functional for time ordered expectation values:

$$Z[J] = \mathcal{N}^{-1} \langle vac_{out} | vac_{in} \rangle_J = \int \mathcal{D}\phi \, e^{i\int d^4x \left(\mathcal{L} + J(x)\phi(x)\right)}$$

$$\langle T[\phi(x)\phi(y)] \rangle = -\frac{\delta^2}{\delta J(x)\,\delta J(y)} \log Z[J] \Big|_{J=0}$$

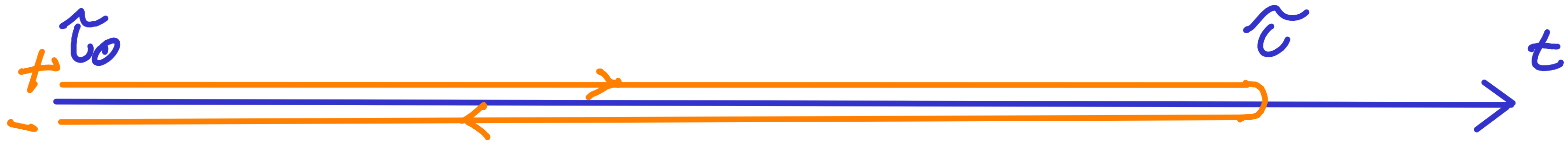
The Closed-Time-Path

- **In-In** generating functional: $\phi_{in}(\vec{x}) = \phi(\vec{x}, \tau_0)$

$$Z[J_+, J_-] = \int \mathcal{D}\psi\, \mathcal{D}\phi^-_{in}\, \mathcal{D}\phi^+_{in} \langle \phi^-_{in} | \psi, \tau \rangle_{J_-} \langle \psi, \tau | \phi^+_{in} \rangle_{J_+} \langle \phi^-_{in} | \varrho | \phi^+_{in} \rangle$$

$$= \int \mathcal{D}\phi^+ \mathcal{D}\phi^-\, e^{i \int d^4x \left\{ \mathcal{L}[\phi^+] + J_+ \phi^+ - \mathcal{L}[\phi^-] - J_- \phi^- \right\}} \langle \phi^-_{in} | \varrho | \phi^+_{in} \rangle$$

The Closed Time Path:



- Path ordered Green functions:

$$i\Delta^{ab}_{\phi}(u,v) = - \frac{\delta^2}{\delta J_a(u)\, \delta J_b(v)} \log Z[J_+, J_-] \Big|_{J_\pm = 0}$$

$$= i \langle \mathcal{C}[\phi^a(u) \phi^b(v)] \rangle$$

Path Ordered Green Functions

$$i\Delta_\phi^{<}(u,v) = i\Delta_\phi^{+-}(u,v) = \langle \phi(v)\,\phi(u)\rangle$$

$$i\Delta_\phi^{>}(u,v) = i\Delta_\phi^{-+}(u,v) = \langle \phi(u)\,\phi(v)\rangle$$

$$i\Delta_\phi^{T}(u,v) = i\Delta_\phi^{++}(u,v) = \langle T[\phi(u)\,\phi(v)]\rangle$$

$$i\Delta_\phi^{\bar{T}}(u,v) = i\Delta_\phi^{--}(u,v) = \langle \bar{T}[\phi(u)\,\phi(v)]\rangle$$

Free propagators in Minkowski-space:

(anti-) particle distributions

$$i\Delta_\phi^{<}(p) = 2\pi\,\delta(p^2-m_\phi^2)\left[\vartheta(p_0)\,f_\phi(\vec{p}) + \vartheta(-p_0)\left(1+\bar{f}_\phi(-\vec{p})\right)\right]$$

$$i\Delta_\phi^{>}(p) = 2\pi\,\delta(p^2-m_\phi^2)\left[\vartheta(p_0)\left(1+f_\phi(\vec{p})\right) + \vartheta(-p_0)\,\bar{f}_\phi(-\vec{p})\right]$$

$$i\Delta_\phi^{T}(p) = \frac{i}{p^2-m_\phi^2+i\varepsilon} + 2\pi\,\delta(p^2-m_\phi^2)\left[\vartheta(p_0)\,f_\phi(\vec{p}) + \vartheta(-p_0)\,\bar{f}_\phi(-\vec{p})\right]$$

$$i\Delta_\phi^{\bar{T}}(p) = \frac{i}{p^2-m_\phi^2-i\varepsilon} + 2\pi\,\delta(p^2-m_\phi^2)\left[\vartheta(p_0)\,f_\phi(\vec{p}) + \vartheta(-p_0)\,\bar{f}_\phi(-\vec{p})\right]$$

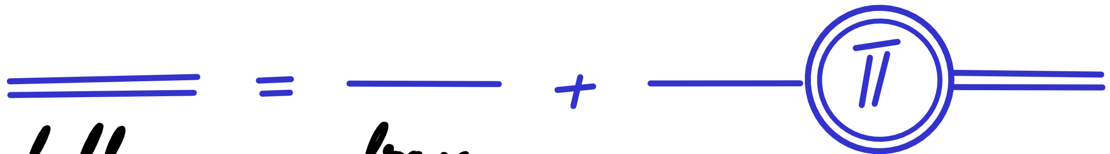
Feynman Rules

- Vertices either $+$ or $-$, factor -1 for each $-$ vertex
- Connect vertices $a = \pm$ and $b = \pm$ with $i\Delta^{ab}$

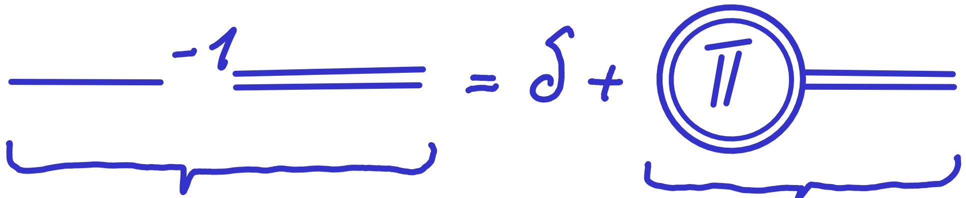
Schwinger-Dyson Equations

- $i\Delta^{ab} = i\Delta^{(0)ab} + cd\, i\Delta^{(0)ac} \odot \Pi^{cd} \odot \Delta^{db}$

$$A(x,w) \odot B(w,y) = \int d^4w\, A(x,w)\, B(w,y)$$



full propagator = bare propagator + self energy: 1PI, full propagators



"flow term" = δ + "collision term"

- Kinetic equations in terms of Green functions — no reference to asymptotic states $\longrightarrow$ by construction, no unitarity problem

Wigner Transformation

- Wigner transform:

$$A(k,x) = \int d^4\tau \, e^{ik\tau} A\left(x+\frac{\tau}{2}, x-\frac{\tau}{2}\right)$$

x → average coordinate — macroscopic evolution

k → relative coordinate — microscopic (quantum) properties

- For the convolutions, can show that:

$$\int d^4\tau \, e^{ik\tau} \int d^4w \, A\left(x+\frac{\tau}{2}, w\right) B\left(w, x-\frac{\tau}{2}\right) = e^{-i\Diamond}\{A(k,x)\}\{B(k,x)\}$$

where $\Diamond\{\cdot\}\{\cdot\} = \frac{1}{2}\left(\partial_x^{(1)} \cdot \partial_k^{(2)} - \partial_k^{(1)} \cdot \partial_x^{(2)}\right)$

Gradient Expansion

- For slowly evolving system, expand in powers of
  $\partial_x \cdot \partial_k \sim H/T \sim T/m_{Pl} \ll 1$
  ∂_k → typical momentum scale, i.e. T
  ∂_x → typical time scale, i.e. Hubble time H^{-1}
  $\vec{\nabla}_x = 0$ for spatially homogeneous system

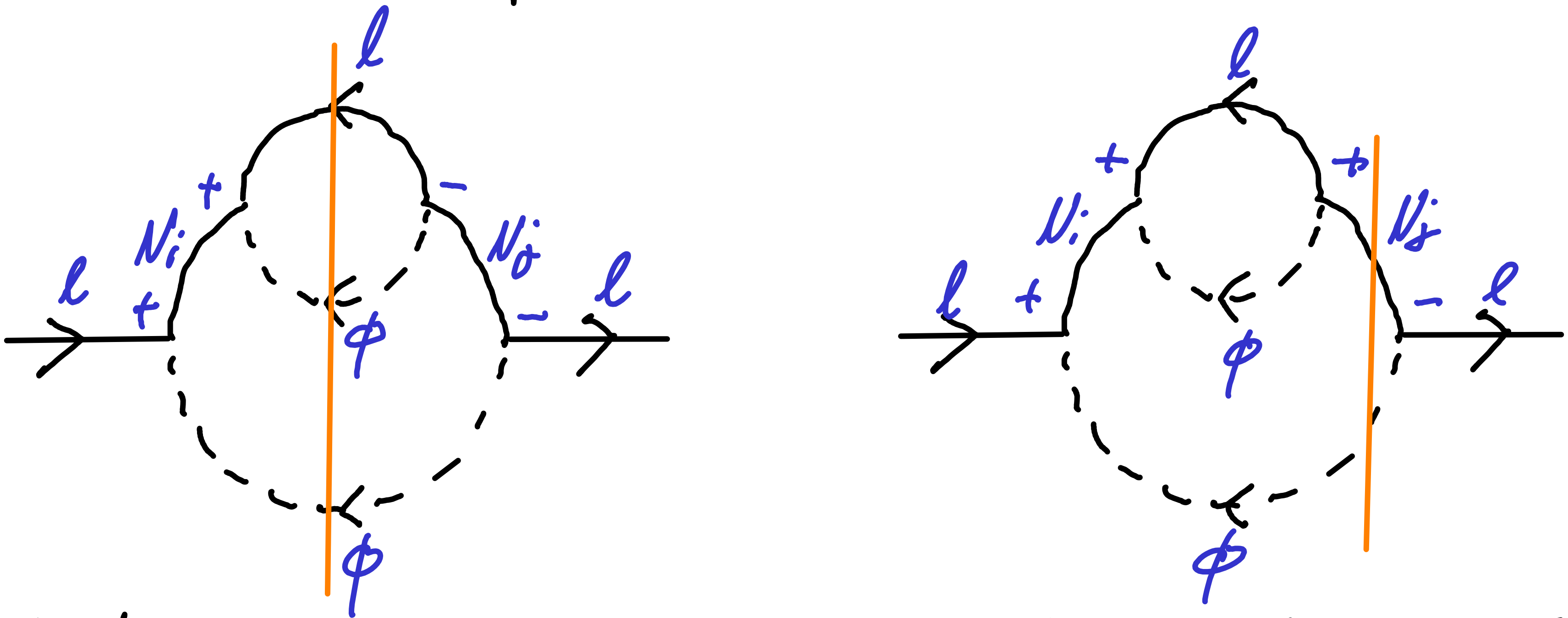
- Leptogenesis most efficient when
  $\Gamma = Y^2 \frac{1}{16\pi} M \sim H$ and $M \sim T \Rightarrow Y^2 \frac{1}{16\pi} \sim H/T \ll 1$

⟹ Expand in $\partial_x \cdot \partial_k$ and Y^2 $(\sim T/m_{Pl})$

M, Y: Mass & Yukawa coupling of the singlet Majorana neutrino

- For $M = m_{GUT}$ or below, numerically excellent expansion parameter ⟶ controlled approximation scheme

Wave-function Contribution



Interference between two s-channel scatterings.

Interference between loop and tree-level decays.

When neglecting quantum-statistical corrections (i.e. $(1-f_{\ell,N}) \longrightarrow 1$ and $(1+f_\phi) \longrightarrow 1$), and cutting as indicated, we recover the usual RIS-subtraction.

Result for Wave-Function Contribution

$$\int \frac{d^3k}{(2\pi)^3} \epsilon_\ell^{wf}(\vec{k}) = 4\,\mathrm{Im}\left[Y_1^2 Y_2^{*2}\right] \frac{M_1 M_2}{M_1^2 - M_2^2}$$

$$* \int \frac{d^3k'}{(2\pi)^3\, 2\sqrt{\vec{k}'^2 + M_1^2}}\, \delta f_N(\vec{k}') \frac{\Sigma_{N\mu}(\vec{k}')\,\Sigma_N^\mu(\vec{k}')}{g_w}$$

where we have the thermal decay rate

$$\Sigma_N^\mu(\vec{k}) = g_w \int \frac{d^3p}{(2\pi)^3\, 2|\vec{p}|} \frac{d^3q}{(2\pi)^3\, 2|\vec{q}|} (2\pi)^4 \delta^4(k-p-q)$$

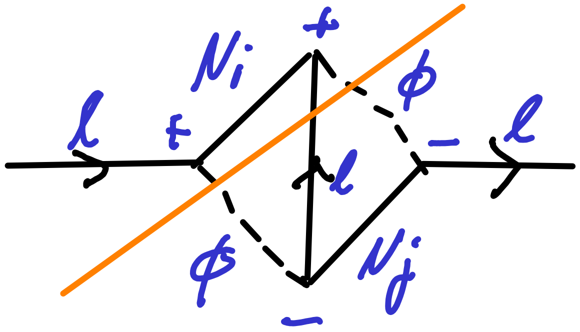
$$* \, p^\mu \left[1 - f_\ell(\vec{p}) + f_\phi(\vec{q})\right]$$

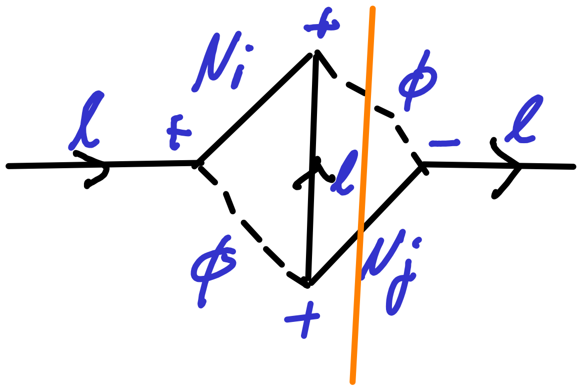
consistent derivation of quantum-statistical corrections

External phase-space & loop integral enter into the CP asymmetry at the same level. → cf. Cutkosky's rules

$$\Sigma_N^\mu(\vec{k}) \xrightarrow[M_N \gg T]{} g_w \frac{k^\mu}{16\pi}$$ recover standard approximation

Vertex Contribution





Interference between s- and t-channel scatterings

Interference between loop and tree-level decays.

$$\int \frac{d^3p'}{(2\pi)^3} \epsilon^{V}(\vec{p}') = 4\,\mathrm{Im}\left[Y_1^2 Y_2^{*2}\right] \int \frac{d^3k}{(2\pi)^3\, 2\sqrt{\vec{k}^2 + M_1^2}}\, \delta f_{N_1}(\vec{k})\, V(k)$$

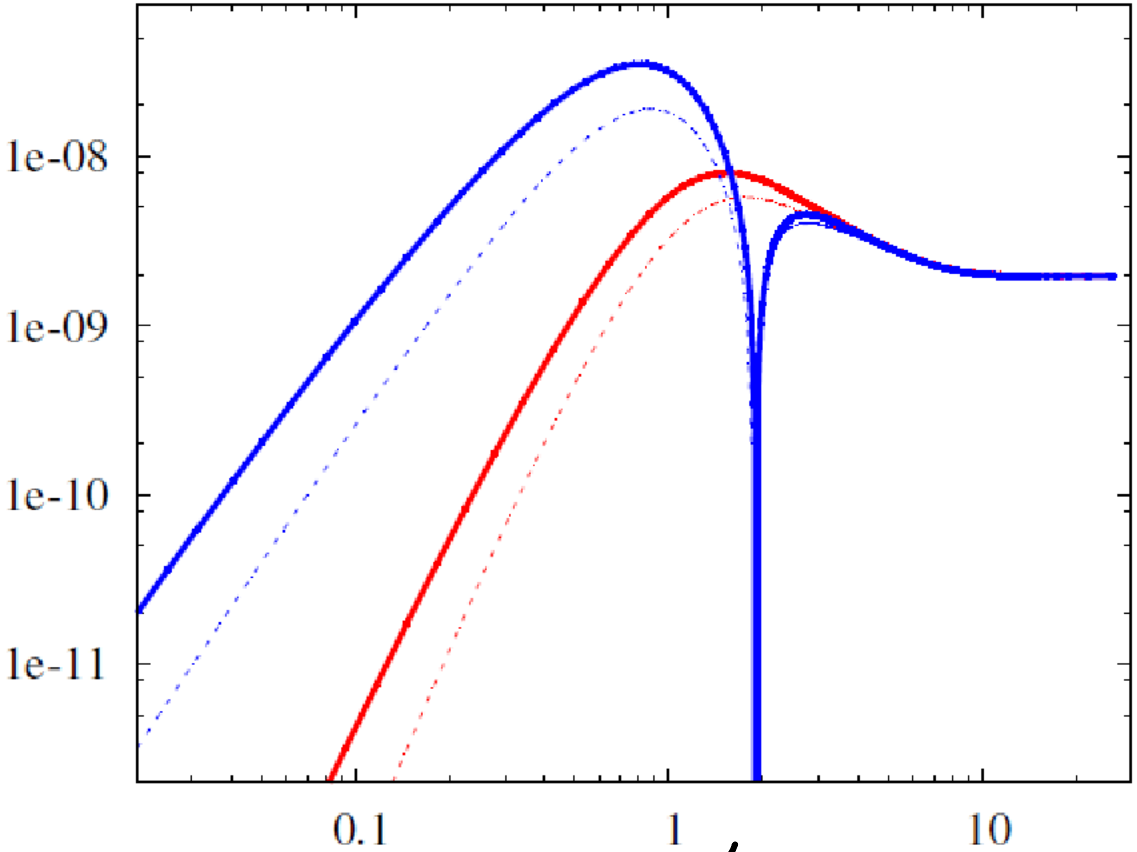
$$V(k) = \int \frac{d^3p'}{(2\pi)^3 2|\vec{p}'|} \frac{d^3p''}{(2\pi)^3 2|\vec{p}''|} (2\pi)^4 \delta^4(k-p'-p'') p'^{\mu}\, \Gamma_\mu(k,p'') \left[1 - f_\ell(\vec{p}') + f_\phi(\vec{p}'')\right]$$

Thermal vertex function:

$$\Gamma_\mu(k,p'') = \int \frac{d^3k'}{(2\pi)^3 2|\vec{k}'|} \frac{d^3k''}{(2\pi)^3 2|\vec{k}''|} (2\pi)^4 \delta^4(k-k'-k'') k'_\mu \frac{M_1 M_2}{(k'-p'')^2 - M_2^2} \left[1 - f_\ell(\vec{k}') + f_\phi(\vec{k}'')\right]$$

Statistical Corrections: Strong Washout $(M \gg T)$ @ time of leptogenesis

$$|Y_\ell| = \frac{n_\ell - \bar{n}_\ell}{s}$$



$z = M_1/T$

asymmetry first washed out & eventually freezes in in non-relativistic regime $\longrightarrow$ no quantum-statistical corrections

red: thermal initial f_{N_1} blue: zero initial f_{N_1}

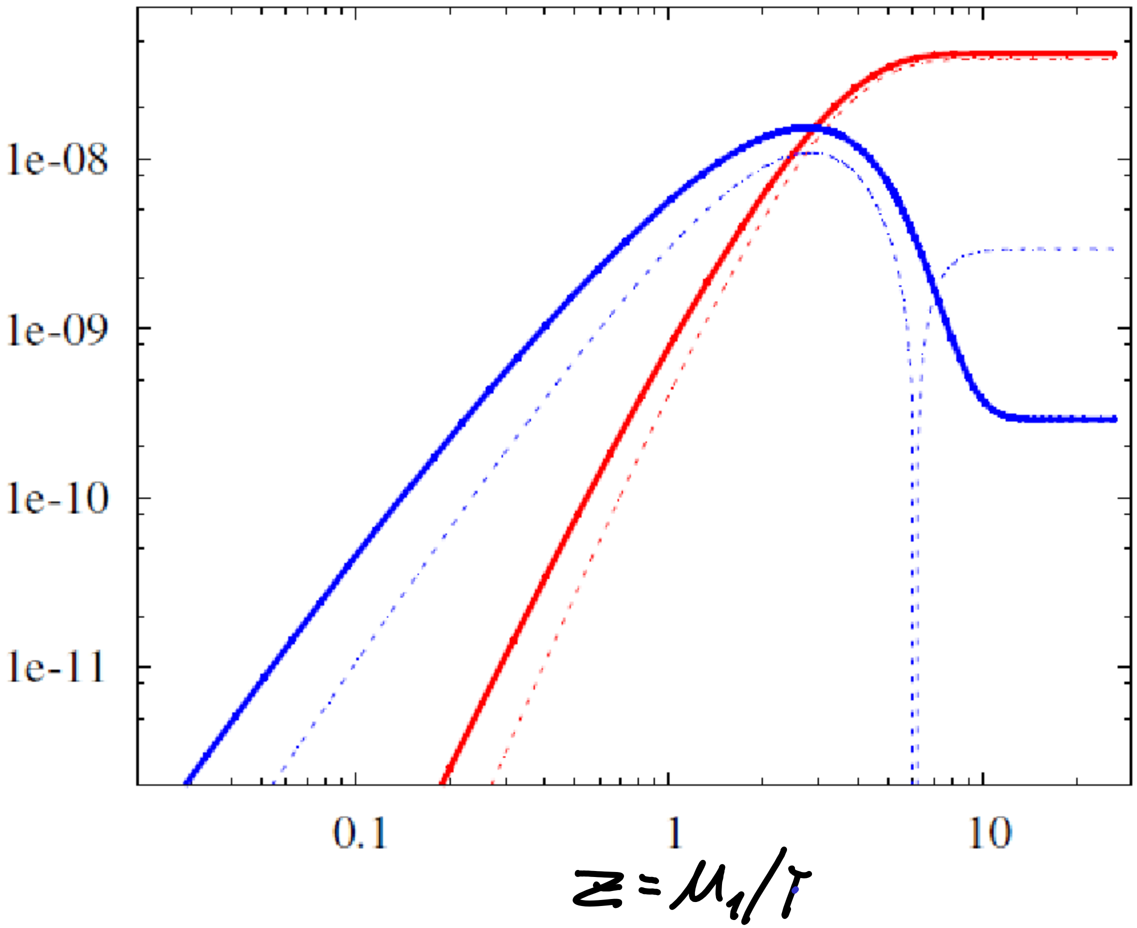
solid: full solution dashed: no thermal corrections in loops

$M_1 = 10^{13}\,\mathrm{GeV}$ $M_2 = 10^{15}\,\mathrm{GeV}$ $Y_1 = 5 * 10^{-2}$ $Y_2 = 10^{-1}$ $\mathrm{Im}[Y_1 Y_2^*] = |Y_1 Y_2|$

Statistical Corrections: Weak Washout

$(M \ll T)$
@ time of leptogenesis

$$|Y_\ell| = \frac{n_\ell - \bar{n}_\ell}{s}$$



sign change for vanishing initial conditions

⚠ thermal corrections lead to $O(1)$ effects for $z \lesssim 0.5$

red: thermal initial f_{N_1} blue: zero initial f_{N_1}

solid: full solution dashed: no thermal corrections in loops

$M_1 = 10^{13}\,\mathrm{GeV}$ $M_2 = 10^{15}\,\mathrm{GeV}$ $Y_1 = 1 * 10^{-2}$ $Y_2 = 10^{-1}$ $\mathrm{Im}[Y_1 Y_2^*] = |Y_1 Y_2|$

Flavoured Leptogenesis

Abada, Davidson, Josse-Michaux, Losada, Riotto (2006)
Nardi, Nir, Roulet, Racker (2006)

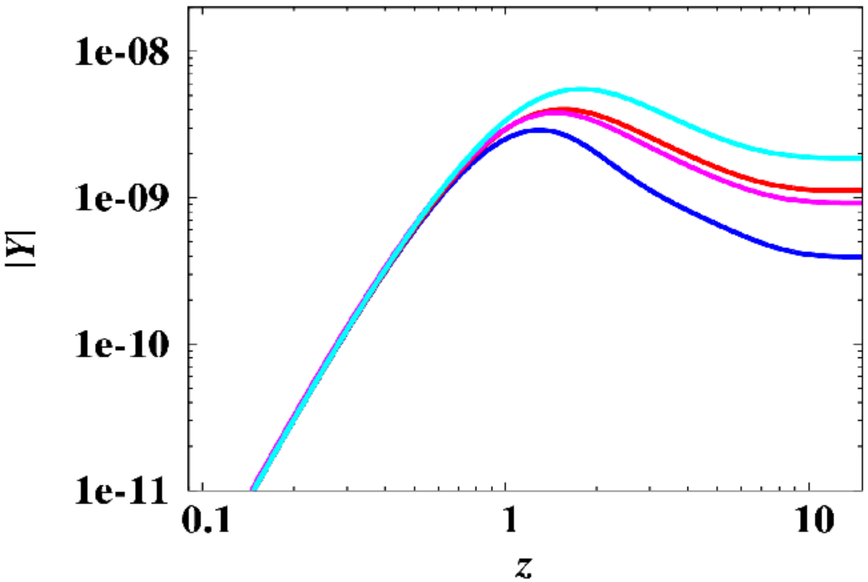
- "Leptogenesis basis" in which lepton asymmetry is produced generally different from lepton flavour basis where $h_{\tau a}$ as in $h_{\tau a} \bar{R}_\tau^\dagger \phi^\dagger l_a$ is diagonal
- For $T \lesssim 10^{12}$ GeV (10^9 GeV, 10^4 GeV) $h_{\tau\tau}$ ($h_{\mu\mu}$, h_{ee}) is in equilibrium (interactions faster than expansion H)

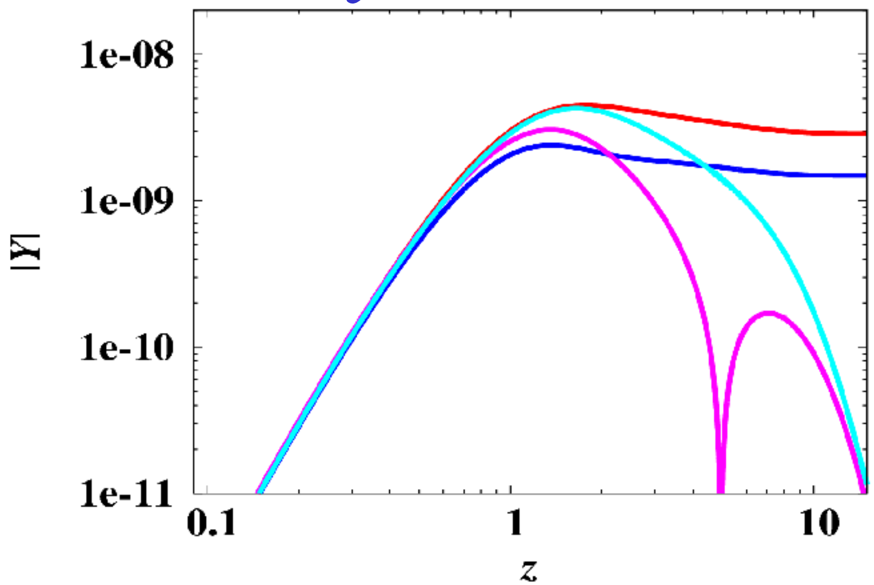
$\Rightarrow$ Lepton charge densities projected on flavour basis (decoherence of flavour off-diagonal correlations)
$\longrightarrow$ suppression of washout (because of "hidden" asymmetry)

- So far: either fully flavoured or unflavoured description; intermediate regime in heuristic Boltzmann/density matrix approach

Suppression of the off-Diagonals

$h_\tau = 0$ $\qquad\qquad$ $h_\tau = 7 * 10^{-3}$





In flavour basis: $\begin{pmatrix} Y_{\ell 11} & Y_{\ell 12} \\ Y_{\ell 21} & Y_{\ell 22} \end{pmatrix}$ lepton number to entropy ratio

$$Y = \begin{pmatrix} 1.4 * 10^{-2} & 1 * 10^{-2} \\ i * 10^{-1} & 10^{-1} \end{pmatrix} \Big\} \text{ r.h. neutrino}$$

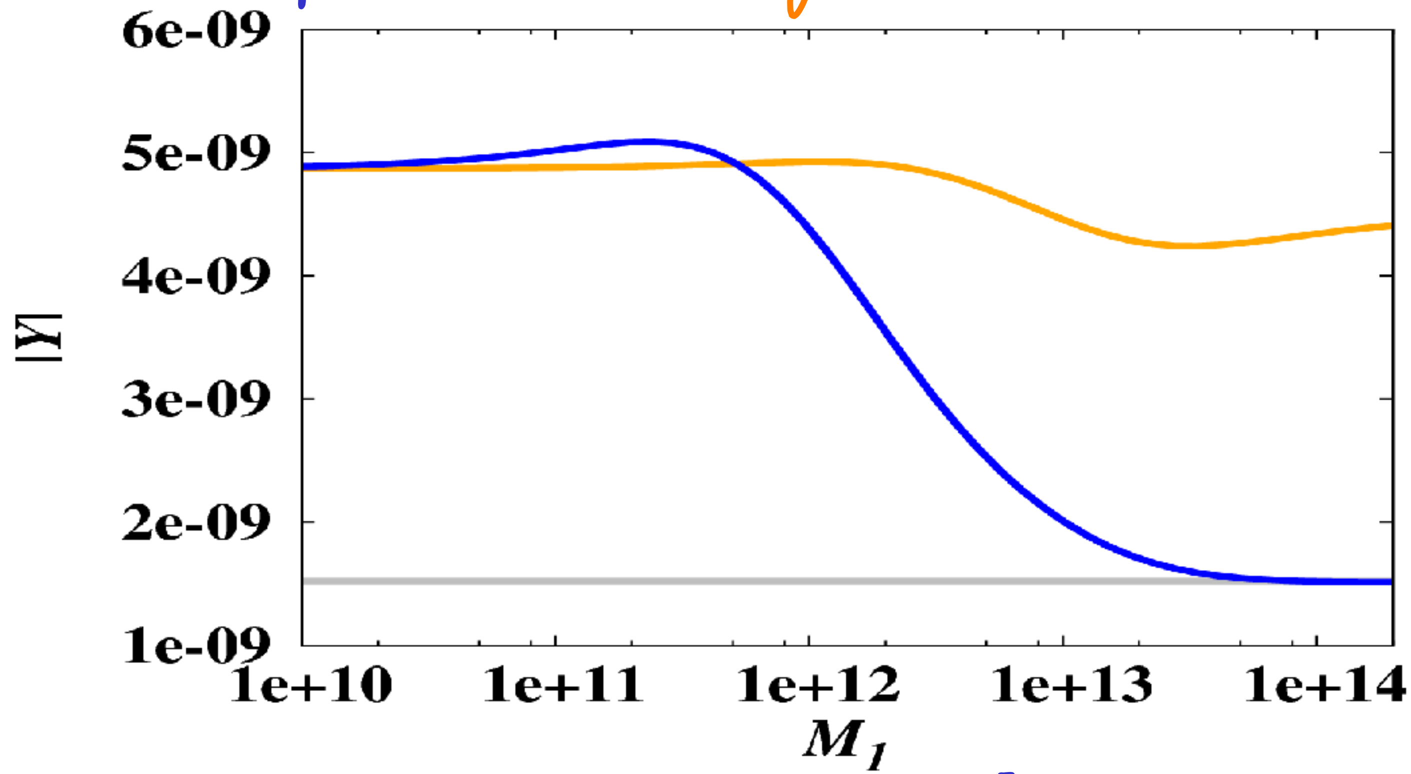
$l_\tau \equiv l_1 \qquad l_{\mu, e} \equiv l_2$

$M_1 = 10^{12}$ GeV

$M_2 = 10^{14}$ GeV

Full Result Interpolates Between Flavoured/Unflavoured Limits

full result / fully flavoured / unflavoured



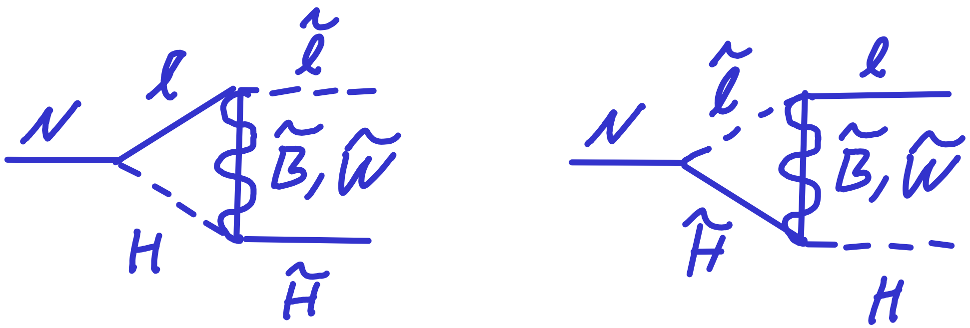
$h_\tau = 7 * 10^{-3}$

$$\left.\begin{array}{l} M_1 \longrightarrow \alpha M_1 \\ Y_{11} \longrightarrow \alpha Y_{11} \\ Y_{12} \longrightarrow \alpha Y_{12} \end{array}\right\} \text{fixed } Y_\ell \text{ in the unflavoured limit}$$

$$Y = \begin{pmatrix} 1.4 * 10^{-2} & 1 * 10^{-2} \\ i * 10^{-1} & 10^{-1} \end{pmatrix} \Big\} \text{r.h. neutrino}$$

$l_\tau \equiv l_1 \qquad l_{\mu,e} \equiv l_2$

A Model with Observable $\cancel{CP}$: Soft Leptogenesis

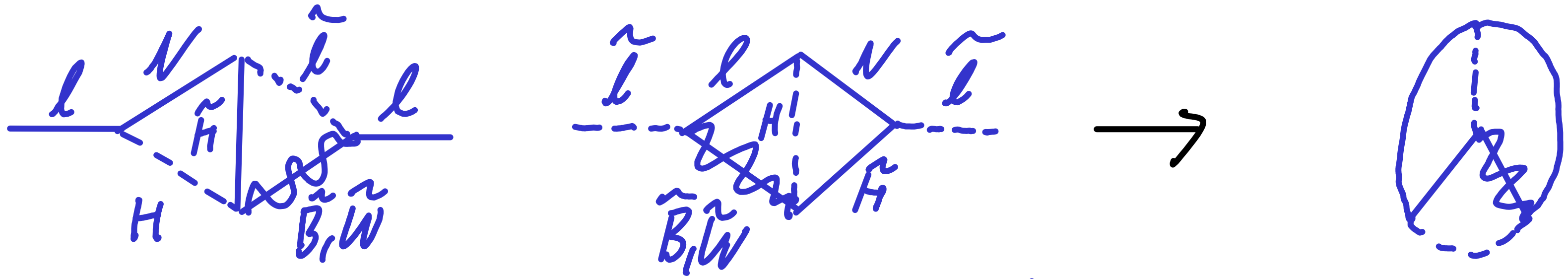


- Relies on $\cancel{CP}$ - phase ϑ_μ (when gaugino phase is chosen zero)
  $\longrightarrow$ permanent EDMs
- In vacuum however: $\Gamma_{N\to\tilde{\ell}\tilde{H}} - \Gamma_{N\to\tilde{\ell}^*\bar{\tilde{H}}} + \Gamma_{N\to\ell H} - \Gamma_{N\to\bar{\ell}H^*} \equiv 0$
- It is argued that the cancellation disappears when taking the thermal phase space of the final states into account (Fermi blocking & Bose enhancement)

Fong & Gonzales-Garcia (2009)

A Model with Observable $\cancel{CP}$: Soft Leptogenesis

- Phase space of the internal propagators must also be taken into account $\longrightarrow$ Automatically within CTP approach



No difference between external states & cut particles

➤ Cancellation persists also for finite T. BG, in preparation

Applies as well to other proposals Hall, March-Russell, West (2010) Kayser & Segre (2011)

- Use CTP methods to explore viable models of Leptogenesis connected with experimentally accessible $\cancel{CP}$

Summary

- Improved kinetic equations needed for EWBG & LG for most portions of parameter space
- Use CTP formalism & consider LG first, for simplicity
  Systematic approach to unitarity & $\cancel{CP}$, coherent particle mixing @ finite temperature
- $\mathcal{O}(1)$ improvements for LG in weak washout, resonant or flavour (de)coherent regime
- Search for possibilities of Baryo-/Leptogenesis from out-of-equilibrium decays with experimentally accessible signatures