Recent new results for $t\bar{t}$ threshold resummation and off-shell single-top production

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M. Beneke, PF, S. Klein, C. Schwinn Nucl. Phys. B855 (2012) 695-741
PF, F. Giannuzzi, P. Mellor, A. Signer Phys. Rev. D83 (2011) 094013
PF, P. Mellor, A. Signer Phys. Rev. D82 (2010) 054028

NNLL threshold resummation of the $t\bar{t}$ production cross section

The top-pair production cross section

Total $t\bar{t}$ cross section measured at Tevatron with $\Delta\sigma/\sigma \pm 7-8\%$... **LHC is catching up quickly!**

- Atlas ($0.7\,\text{fb}^{-1}$): $179 \pm 12 (\Delta\sigma/\sigma \pm 6.6\%)$
- CMS ($36\,\text{pb}^{-1}$): $154 \pm 18 (\Delta\sigma/\sigma \pm 12\%)$

Precise measurements of $\sigma_{t\bar{t}}$ relevant for

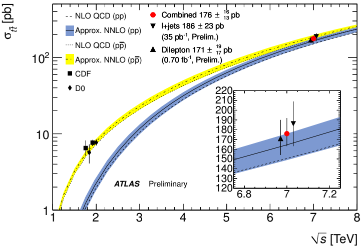
- testing **SM** and **new-physics** models
- constraining **gluon PDF** in the proton
- theoretically clean extraction of the **top quark mass**

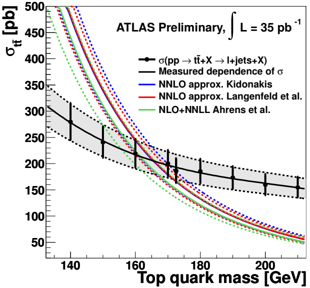
require that theoretical uncertainties are well understood and under control, $\Delta\sigma^{\text{th}} \lesssim \Delta\sigma^{\text{exp}}$:

$$\sigma_{t\bar{t}}^{\text{NLO}} = 162^{+24}_{-26}\,\text{pb}$$

$\Delta\sigma^{\text{NLO}}/\sigma^{\text{NLO}} \pm 15\%$ (scale+PDF+α_s)

$\Rightarrow$ **need better prediction than fixed-o. NLO!**





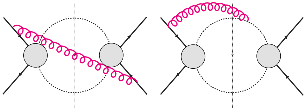
$t\bar{t}$ production theory overview

- **NLO**: QCD corrections (Nason, Dawson, Ellis '88;...) EW corrections (Bernreuther Fücker, Si '06; Kühn, Scharf, Uwer '06) finite-width effects (Denner et al. '10; Bevilacqua et al. '10)
- **Towards the full NNLO result**: several ingredients already known (Czakon '08; Bonciani et al. '09/'10; Körner et al. '08; Anastasiou, Aybat '08;...)
- **LL/NLL resummations**: Laenen et al. '92; Catani et al. '96; Berger, Contopanagos '96; Kidonakis, Smith '96; Bonciani et al. '98; Kidonakis et al. '01
- **NNLL resummation/NNLO approximated results**:
  - IR structure of QCD amplitudes (Neubert, Becher '09) soft-Coulomb factorization (Beneke, PF, Schwinn '10) 2-loop anomalous dimensions (Beneke et al. '09; Czakon et al. '09)
  - NNLO approximations for total cross section (HATHOR , Aliev et al. '11; Beneke et al. '09/'10) and differential cross section in 1PI/PIM kinematics (in Mellin and SCET formalism Kidonakis '09-'11; Ahrens et al. '10/'11;)
  - NNLL resummation for 1PI/PIM cross section in SCET (Ahrens et al. '10/'11), NNLL total cross section in Mellin space (Cacciari et al. '11), combined soft and Coulomb resummation in SCET/NRQCD (Beneke, PF, Klein, Schwinn '11)

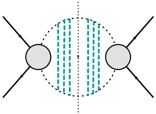
Soft and Coulomb corrections

Total NLO **partonic cross sections** enhanced near **threshold**, $\beta \equiv \sqrt{1 - 4m_t^2/\hat{s}} \to 0$

- **Threshold logarithms**: $\sim \alpha_s^n \ln^m \beta$
  $\Leftrightarrow$ suppression of **soft-gluon emission**



- **Coulomb corrections**: $\sim (\alpha_s/\beta)^n$
  $\Leftrightarrow$ **potential interactions** of non-relativistic particles



if **hadronic cross section** is dominated by partonic threshold resummation of soft logs and Coulomb singularities leads to improved predictions and reduced theoretical uncertainties

Counting scheme: $\alpha_s/\beta \sim \alpha_s \ln \beta \sim 1$

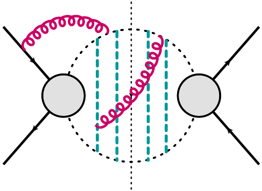
$$\hat{\sigma}_{pp'} \quad \propto \quad \hat{\sigma}^{(0)} \sum_{k=0} \left(\frac{\alpha_s}{\beta}\right)^k \exp\Big[\underbrace{\ln\beta\, g_0(\alpha_s \ln\beta)}_{\text{(LL)}} + \underbrace{g_1(\alpha_s \ln\beta)}_{\text{(NLL)}} + \underbrace{\alpha_s g_2(\alpha_s \ln\beta)}_{\text{(NNLL)}} + \dots\Big]$$

$$\times \left\{ 1\ \text{(LL,NLL)};\ \alpha_s,\ \beta\ \text{(NNLL)};\ \alpha_s^2,\ \alpha_s\beta,\ \beta^2\ \text{(NNNLL)};\ \dots \right\}$$

Factorisation of pair production near threshold at NNLL

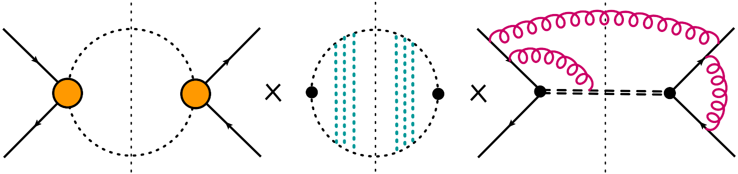
- **non-relativistic H, H' and Coulomb gluons:** $E \sim m_H \beta^2$, $|\vec{p}| \sim m_H \beta$
- **soft gluons:** $q_s \sim m_H \beta^2$

potential and soft modes have the same energy and can "talk" to each other



Effective-theory description of pair production near threshold $\hat{s} \sim (m_H + m_{H'})^2$
[Beneke, PF, Schwinn, '09/'10] ⇒ factorization of hard, Coulomb and soft contributions

$$\hat{\sigma}_{pp'}(\hat{s}, \mu_f) = \sum_i H_i(M, \mu_f) \int d\omega \sum_{R_\alpha} J_{R_\alpha}(E - \frac{\omega}{2})\, W_i^{R_\alpha}(\omega, \mu_f)$$



Soft radiation couples to total colour charge of the pair!

Resummation of soft/hard corrections in momentum space

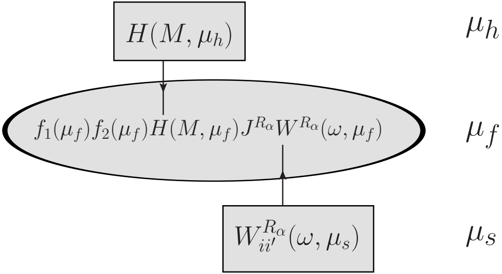
RG evolution equations for the soft function $W_i^{R_\alpha}$ and the hard function $H_i^{R_\alpha}$ follow from IR structure of QCD amplitudes and scale-invariance of the hadronic cross section (generalisation of DY result [Becher, Neubert, Xu '07] to **arbitrary** R_α)

$$\frac{d}{d\ln\mu_f} W_i^{R_\alpha}(\omega,\mu_f) = -2\left[(C_r + C_{r'})\,\Gamma_{\text{cusp}} \ln\left(\frac{\omega}{\mu_f}\right) + 2\gamma_{H,s}^{R_\alpha} + 2\gamma_s^{r} + 2\gamma_s^{r'}\right] W_i^{R_\alpha}(\omega,\mu_f)$$
$$-2\,(C_r + C_{r'})\,\Gamma_{\text{cusp}} \int_0^\omega d\omega' \, \frac{W_i^{R_\alpha}(\omega',\mu_f) - W_i^{R_\alpha}(\omega,\mu_f)}{\omega-\omega'}$$

and similar for hard function $H_i(M,\mu_f)$

Resummation strategy

- solve evolution equation in momentum space
- evolve the function H_i from the hard scale $\mu_h = 2m_t$ to μ_f
- evolve soft function $W_i^{R_\alpha}$ from a soft scale $\mu_s = 2m_t\beta^2$ to μ_f.

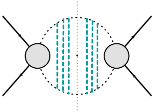


Resummation of Coulomb corrections

Resummation of Coulomb effects well understood from **PNRQCD** and quarkonia physics:

$$J_{R_\alpha}(E) = 2\mathrm{Im}\left[G^{(0)}_{C,R_\alpha}(0,0;E)\Delta_{\mathrm{nc}}(E) + G^{(1)}_{C,R_\alpha}(0,0;E) + \ldots\right]$$

$G^{(0)}_{C,R_\alpha}$ $\Leftrightarrow$



$$= -\frac{m_t^2}{4\pi}\left\{\sqrt{-\frac{E}{m_t}} + \alpha_s(-D_{R_\alpha})\left[\frac{1}{2}\ln\left(-\frac{4\,m_t E}{\mu_C^2}\right) - \frac{1}{2} + \gamma_E + \psi\left(1 - \frac{\alpha_s(-D_{R_\alpha})}{2\sqrt{-E/m_t}}\right)\right]\right\}$$

Includes bound-states below threshold (E<0)

Coulomb scale μ_C: set by typical virtuality of Coulomb gluons $\sqrt{|q^2|} \sim m_t\beta \sim m_t\alpha_s$

$$\Rightarrow \mu_C = \max\{2m_t\beta, C_F m_t \alpha_s(\mu_C)\}$$

$\hookrightarrow$ twice inverse Bohr radius of first bound state

NNLL/NNLO total $t\bar{t}$ cross section

$m_t = 173.3$ GeV, $\mu_F = m_t$, MSTW2008NLO/NNLO

Beneke, PF, Klein, Schwinn, Nucl. Phys. B855 (2012) 695-741

$\sigma_{t\bar{t}}$[pb]	Tevatron	LHC@7	LHC@14
NLO	$6.68^{+0.36+0.51}_{-0.75-0.45}$	$158.1^{+18.5+13.9}_{-21.2-13.1}$	$884^{+107+65}_{-106-58}$
NNLO$_{\text{app}}$	$7.06^{+0.27+0.69}_{-0.34-0.53}$	$161.1^{+12.3+15.2}_{-11.9-14.5}$	891^{+76+64}_{-69-63}
NNLL	$7.22^{+0.31+0.71}_{-0.47-0.55}$	$162.6^{+7.4+15.4}_{-7.5-14.7}$	896^{+40+65}_{-37-64}

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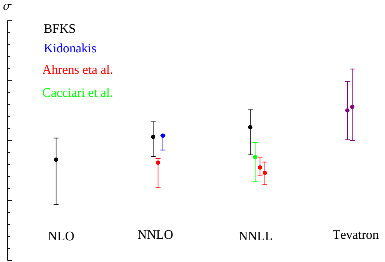
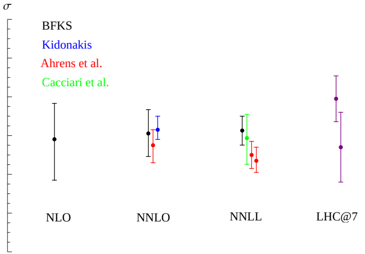
Tevatron

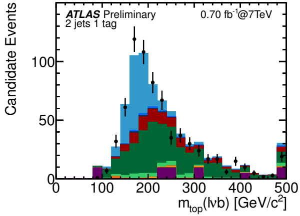


LHC@7TeV



Off-shell and non-factorizable contributions in single-top production

Single top production

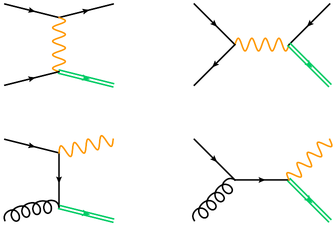


[ATLAS-CONF-2011-101]

Observed two years ago by CDF and D0 at Tevatron

Larger production rate expected at the LHC

$$\sigma^{\text{CMS}}_{t-\text{chan}} = 83.6 \pm 29.8 \pm 3.3\,\text{pb}$$



Single-t prod. proceeds via EW interactions

$\Rightarrow$ information on charged-current interactions of the top quark and on possible anomalous couplings

+ sensitivity to CKM element V_{tb}, bottom PDF, background for Higgs production...

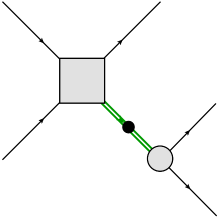
Precise measurements require good theoretical understanding of the production/decay dynamics

Single-top theory overview

- **Stable-top approximation:** NLO QCD corrections for total (Bordes et al. '95; Giele et al. '96; Stelzer et al. '97) and differential cross section (Harris et al.'02; Sullivan '04), EW corrections in SM and MSSM (Beccaria et al.'08; Macorini et al.'10)
- **Narrow-width approximation:** differential NLO QCD corrections (Campbell et al. '04; Cao et al. '05; Heim et al. '10; Schwienhorst et al. '10)
- **Non-factorizable corrections:** t-channel (PF, Mellor, Signer '10) and s-channel production (Pittau '96; PF, Giannuzzi, Mellor, Signer '11)
- **NLO/parton-shower matching**: (MC@NLO; POWHEG)
- **Threshold resummations:** resummation in Mellin-moment space (Kidonakis '07/'10) and SCET (Li, Wang, Zhang, Zhu. '10)
- **4-flavour VS 5-flavour scheme:** (Campbell, Frederix, Maltoni, Tramontano, '09)

The Narrow Width Approximation (NWA)

GENERAL QUESTION: how to treat unstable particles in perturbative calculations?
NWA: consider production and decay of an **on-shell** heavy particle X (top quark, W, Z, ...)

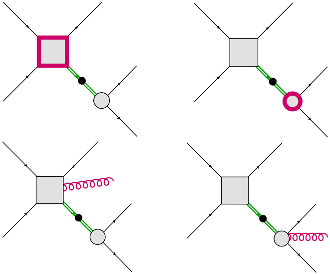


$$p_1 + p_2 \to p_3 + X(M_X^2) \to p_3 + p_4 + p_5$$

- background (non-resonant) diagrams neglected

$$p_1 + p_2 \to Y \to p_3 + p_4 + p_5$$

- off-shell effects and production/decay interferences not included

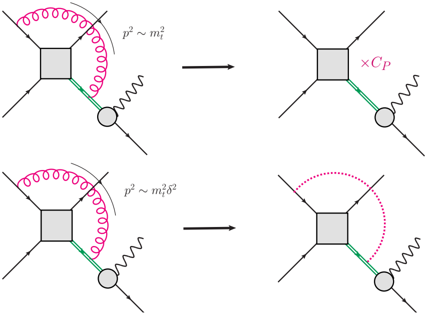


Accuracy of the approximation expected to be Γ_X/M_X for total cross section ($\sim 1\%$ for top quark)

However: expectation might underestimate true error for observables with arbitrary kinematical cuts (cancellation of real and virtual corrections less effective...)

The Effective Theory approach

Consider a resonant unstable particle X (rather than on-shell) and use the small virtuality of X as an expansion parameter, $\delta \equiv (p_X^2 - M_X^2)/M_X^2 \ll 1 \Leftrightarrow$ pole approximation



- **Hard region** ($q^2 \sim M_X^2$)
  $\Rightarrow$ **factorizable corrections**
  ($\Leftrightarrow$ corrections in the NWA)
- **Soft region** ($q^2 \sim M_X^2\delta^2$)
  $\Rightarrow$ **non-factorizable corrections**
  ($\Leftrightarrow$ off-shell effects and production/decay interferences)

Effective-theory expansion resums **finite-width effects**, includes leading **non-factorizable corrections** and preserves **gauge invariance**

+ much simpler than full 1-loop calculation in the Complex Mass Scheme!

Integrated cross section ($m_t = 172\,\text{GeV}$, $\mu_R = \mu_F = m_t/2$)

$pp \to J_b J_l e^+ \not{E}_T + X$	$pp \to J_b J_{\bar{b}} e^+ \not{E}_T + X$
$p_T(J_b) > 20$ GeV	$p_T(J_b) > 20$ GeV
$p_T(\text{hardest } J_l) > 20$ GeV	$p_T(J_{\bar{b}}) > 30$ GeV
$p_T(\text{extra } J_{\bar{b}}) < 15$ GeV	$p_T(\text{extra } J_l) < 15$ GeV
$\not{E}_T + p_T(e) > 60$ GeV	$\not{E}_T + p_T(e) > 60$ GeV

LHC@7TeV

$pp \to J_b J_l e^+ \not{E}_T + X$		**Eff. Theory**	**NWA**
	LO[pb]	$3.460^{+0.278}_{-0.403}$	3.505
	NLO[pb]	$1.609^{+0.303}_{-0.240}$	1.642
$pp \to J_b J_{\bar{b}} e^+ \not{E}_T + X$		**Eff. Theory**	**NWA**
	LO[pb]	$0.1654^{+0.0001}_{-0.0010}$	0.1677
	NLO[pb]	$0.1618^{+0.0021}_{-0.0005}$	0.1635

Differences between effective-theory calculation and NWA $\sim 2\%$

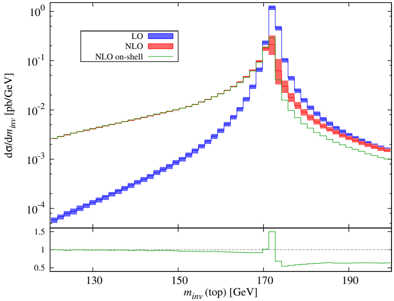
$\Rightarrow$ consistent with expectation $\sim \Gamma_t/m_t$...

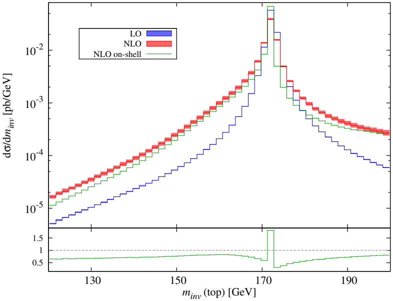
Similar effects found in $t\bar{t}$ production [Bevilacqua et al. '10, Denner et al. '10]

Invariant-mass distribution: m_{inv}

LHC@7TeV

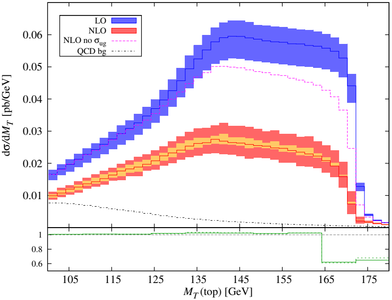
$pp \to J_b J_l e^+ \not{E}_T + X$ $\qquad$ $pp \to J_b J_{\bar{b}} e^+ \not{E}_T + X$

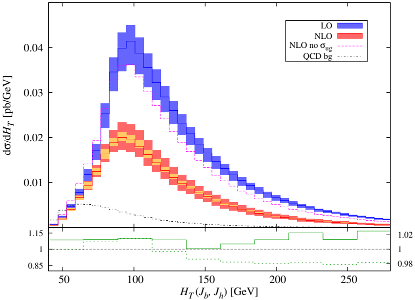


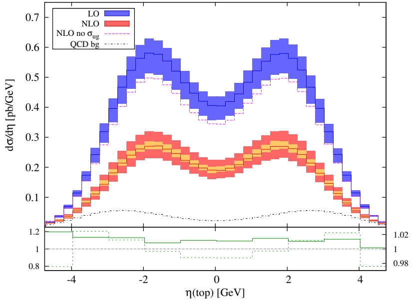


- large off-shell effects (up to 50%) close to the peak
- non-factorizable corrections change sign around the peak
  $\Leftrightarrow$ **explains small effect on the total cross section**

$pp \to J_b J_l e^+ \not{E}_T + X$







- off-shell and non-factorizable effects **generally small** ($\sim 2\%$) due to averaging effect over m_{inv}
- **sizeable corrections** (up to 40%) close to kinematics edges, e.g. $M_T \sim m_t$ (depends strongly on observable)

Summary

- **Threshold resummation for $t\bar{t}$ production**
  - ⇒ simultaneous resummation of **soft logarithms** and **Coulomb singularities** in momentum space
  - ⇒ **NNLL** corrections $\sim 13\%$ at Tevatron and $\sim 9\%$ at LHC
  - ⇒ significant **reduction of theoretical uncertainty** compared to NLO result
  - ⇒ **good agreement with experiments** and different theoretical predictions (though some tension remains at Tevatron...)
- **Off-shell and non-factorizable effects in single top production**
  - ⇒ general formalism based on **effective theories** to include off-shell effects
  - ⇒ off-shell and production/decay interferences generally small
    ⇒ **NWA works well in most situations!**
  - ⇒ non-factorizable effects can be sizeable ($\sim 40\%$) close to **kinematical edges**

Backup slides

$t\bar{t}$ production at NNLL/NNLO

All ingredients for NNLL resummation of $t\bar{t}$ cross section are known:

- 1-loop colour-separated hard functions $H_i^{(1)}$ [Czakon, Mitov '09]
- 2-**loop soft anomalous dimension** [Beneke, PF, Schwinn '09; Czakon, Mitov, Sterman '09]

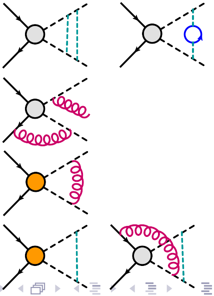
$$\gamma_{H,s}^{R_\alpha,(1)} = C_{R_\alpha}\left[-C_A\left(\frac{98}{9}-\frac{2\pi^2}{3}+4\zeta_3\right)+\frac{20}{9}n_f\right]$$

- NLO Coulomb and non-Coulomb potentials [Beneke, Signer, Smirnov '99]

Can be used to construct approx. NNLO containing all terms singular in β

[Beneke, PF, Czakon, Mitov, Schwinn '09; HATHOR Aliev et al. '10]

$$\begin{aligned}\hat{\sigma}^{\text{NNLO}}_{\text{approx.}} = &\frac{k_{\text{LO}}^2}{\beta^2}+\frac{1}{\beta}\left[k_{\text{NLO},1}\ln\beta+k_{\text{NLO},0}\right]+k_{\text{n-C}}\ln\beta\\ &+c_{S,4}^{(2)}\ln^4\beta+c_{S,3}^{(2)}\ln^3\beta+c_{S,2}^{(2)}\ln^2\beta+c_{S,1}^{(2)}\ln\beta\\ &+H^{(1)}\left[c_{S,2}^{(1)}\ln^2\beta+c_{S,1}^{(1)}\ln\beta\right]\\ &+\frac{k_{\text{LO}}}{\beta}\left[c_{S,2}^{(1)}\ln^2\beta+c_{S,1}^{(1)}\ln\beta+c_{S,0}^{(1)}+H^{(1)}\right]\end{aligned}$$

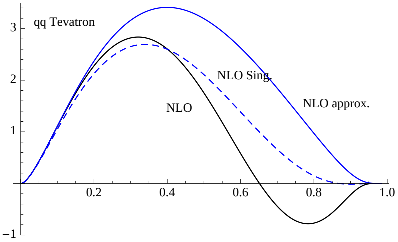


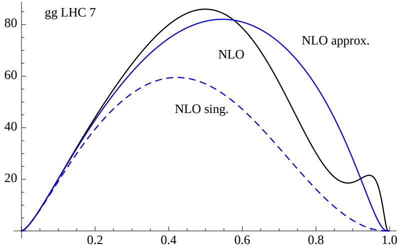
Contribution of threshold-enhanced terms

$\sqrt{s} \gg 2m_t \Rightarrow$ **How good is the threshold approximation?**
can study the approximation at the NLO level...

Plot $8\beta m_t^2/(s(1-\beta^2)^2)\mathcal{L}_{gg}(\beta)\hat{\sigma}_{tt}(\beta)$:

- **NLO**: exact NLO result
- **NLO sing.**: only singular terms in β
- **NLO approx.**: singular terms + O(1) term ($\Leftrightarrow H_i^{(1)}$)





NLO sing. is good approximation only up to $\beta \sim 0.3$
However: expect NNLO approximation to be better (more singular terms at $O(\alpha_s^2)$...)

Estimate of theoretical uncertainties

Resummation uncertainties

- β_{cut}: vary β_{cut} by $\pm 20\%$ and take width of envelope of the 8 curves
- $\mu_s = k_s m_t \beta^2$: choose default value as $k_s = 2$ and vary between $k_s = 1$ and $k_s = 4$
- power-suppressed corrections: consider difference between $E = m_t\beta^2$ and $E = \sqrt{s} - 2m_t$

Scale uncertainties

- for NLO and NNLO vary $m_t/2 < \mu_F, \mu_R < 2m_t$ with $1/2 < \mu_R/\mu_F < 2$
- for NNLL vary μ_C in the interval $[\tilde{\mu}_C/2, 2\tilde{\mu}_C]$.
  μ_F, μ_H are varied simultaneously with the constraint $1 < \mu_H/\mu_F < 4$

$O(\alpha_s^2)$ constant

- choose $C^{(2)}_{pp'} = 0$ as default in $\text{NNLO}_{\text{approx}}$
- vary by $\pm {C^{(1)}_{pp'}}^2$, where $C^{(1)}_{pp'}$ is the $\mathcal{O}(\alpha_s)$ constant for the partonic channel pp'

PDF+α_s uncertainty

- MSTW2008 with 90% CL sets
- $\alpha_s(M_Z) = 0.1171 \pm 0.0034$

Real corrections

How to implement expansion of real corrections in δ? No obvious way...
$\Rightarrow$ **be pragmatic and use full matrix element for real corrections**

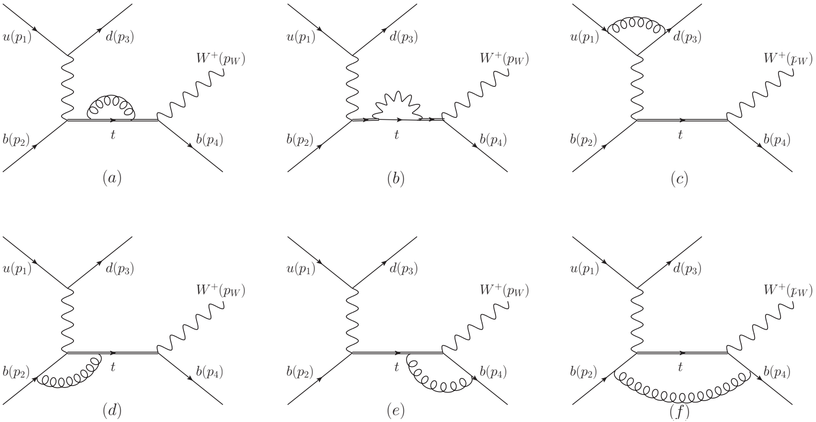
Cancellation of IR singularities requires some attention...

$$
\begin{aligned}
\sigma^{\mathrm{NLO}} &= \int d\Phi_n d\sigma_V + \int d\Phi_{n+1} d\sigma_R \\
&= \int d\Phi_n \left(d\sigma_V + \int d\Phi_1 d\sigma_{\mathrm{subt}} \right) + \int d\Phi_{n+1} (d\sigma_R - d\sigma_{\mathrm{subt}}) \\
&\sim \int d\Phi_n \underbrace{\left(d\sigma_V^{\mathrm{exp}} + \int d\Phi_1 d\sigma_{\mathrm{subt}}^{\mathrm{exp}} \right)}_{\text{expand in}\delta} + \int d\Phi_{n+1} (d\sigma_R - d\sigma_{\mathrm{subt}})
\end{aligned}
$$

- expansion of $d\sigma_{\mathrm{subt}}$ guarantees exact cancellation of IR singularities in virtual corrections and subtraction term
- $\int d\Phi_1 d\sigma_{\mathrm{subt}}^{\mathrm{exp}}$ has Born kinematics $\Rightarrow$ clear what expansion parameter is
- $d\sigma_R - d\sigma_{\mathrm{subt}}$ still contains the full gauge-invariant set of Feynman diagrams

NLO matrix element

Leading tree contribution $M^{\text{tree}} \sim \delta \Rightarrow$ compute all corrections of order $\delta^{3/2}$ (**NLO** approx.)
Arise from subset of one-loop resonant diagrams



Note: **before expansion in δ the subset of diagrams is gauge-dependent!**