Bayesian Implications of 2011 LHC and Xenon$_{100}$ Searches for the Constrained MSSM

BayesFIT group

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Outline

- Supersymmetry defined by CMSSM
- Bayesian approach
- Impact on SUSY from:
  - Various experimental measurements
  - LHC (CMS α_T 1.1/fb)
  - Dark Matter ($Xenon_{100}$) searches
- BestFIT results:
  - CMSSM's parameters constraints including the latest limits

SUSY w.r.t. experimental data

SUSY can not be experimentally ruled out

BUT can (potentially) contribute to various measurable quantities:

- signals of **supersymmetric and Higgs particles**

  direct collider searches: still only lower bounds on masses

- **Indirect (e.g. loop) contributions**
  - electroweak observables: $\sin\theta_W, m_Z, \Gamma_Z, ...$
  - flavor processes: $b \to s\gamma, \mathrm{BR}(\bar{B}_s \to \mu^+\mu^-), \mathrm{BR}(\overline{B}_u \to \tau\nu)$

    NEW: $\mathrm{BR}(\bar{B}_s \to \mu^+\mu^-) < 1.5 \times 10^{-8}$ (LHCb)
  - anomalous magnetic moment of the muon $(g-2)_\mu$
- **neutralino WIMP** as dark matter to give correct relic abundance $\Omega_\chi h^2$

Goal: compare data with theory

- Rigid step-function application of limits/ranges
- Input from experimental measurements and limits

SM (nuisance) parameter	Mean μ	Error σ (expt)
M_t	172.9 GeV	1.1 GeV
$m_b(m_b)^{\overline{MS}}$	4.19 GeV	0.12 GeV
α_s	0.1184	0.0006
$1/\alpha_{\rm em}(M_Z)$	127.916	0.015

SM: $\mathrm{BR}(\bar{\mathrm{B}} \to \mathrm{X_s}\gamma) \times 10^4 =$
3.15 ± 0.23
(Misiak & Steinhauser, Sept 06)

Derived observable	Mean μ	Errors σ (expt)	τ (th)
M_W	80.399 GeV	23 MeV	15 MeV
$\sin^2\theta_{\rm eff}$	0.231160	13×10^{-5}	15×10^{-5}
$\delta(g-2)_\mu \times 10^{10}$	30.5	8.6	1
$\mathrm{BR}(\bar{\mathrm{B}} \to \mathrm{X_s}\gamma) \times 10^4$	3.6	0.23	0.21
ΔM_{B_s}	17.77	0.12	2.4
$\mathrm{BR}(\overline{B}_u \to \tau\nu) \times 10^4$	1.66	0.66	0.38
$\Omega_\chi h^2$	0.1120	0.0056	10%

take w/o error: $M_Z = 91.1876(21)$ GeV, $G_F = 1.16637(1)\times 10^{-5}$ GeV^{-2}

Derived observable	upper/lower limit	Constraints $\xi_{\rm lim}$	τ (theor.)
$\mathrm{BR}(\mathrm{B_s} \to \mu^+\mu^-)$	UL	1.5×10^{-8}	14%
m_h	LL	114.4 GeV (91.0 GeV)	3 GeV
$\zeta_h^2 \equiv g^2_{ZZh}/g^2_{ZZH_{\rm SM}}$	UL	$f(m_h)$	3%
m_χ	LL	50 GeV	5%
$m_{\chi_1^\pm}$	LL	103.5 GeV (92.4 GeV)	5%
$m_{\tilde{e}_R}$	LL	100 GeV (73 GeV)	5%
$m_{\tilde{\mu}_R}$	LL	95 GeV (73 GeV)	5%
$m_{\tilde{\tau}_1}$	LL	87 GeV (73 GeV)	5%
$m_{\tilde{\nu}}$	LL	94 GeV (43 GeV)	5%
$m_{\tilde{t}_1}$	LL	95 GeV (65 GeV)	5%
$m_{\tilde{b}_1}$	LL	95 GeV (59 GeV)	5%
$m_{\tilde{q}}$	LL	318 GeV	5%
$m_{\tilde{g}}$	LL	233 GeV	5%
$(\sigma_p^{\rm SI}$	UL	WIMP mass dependent	$\sim 100\%)$

We use Bayesian (vs Frequentist) approach

Frequentist: "probability is the number of times the event occurs over the total number of trials, in the limit of an infinite series of equiprobable repetitions"

Bayesian: "probability is a measure of the degree of belief about a proposition"

- theory defined by a number of free parameters
- its contributions to observables can be confronted with diverse data
- statistical question: Which ranges of the model's parameters (if any) fit the data well/so-so/poorly,... ?
- draw probability maps in the model's parameter space
- compare different models...

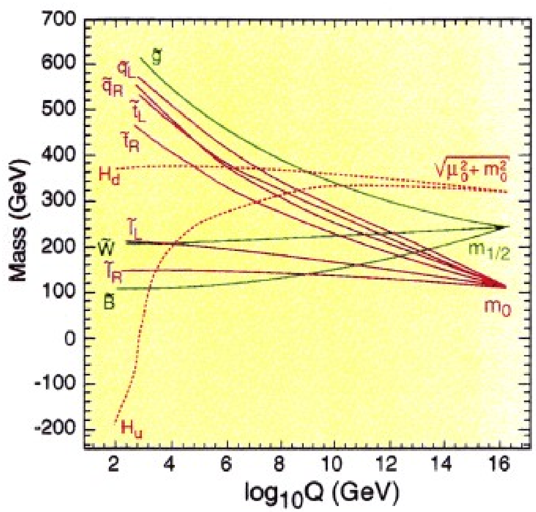
Constrained MSSM

Kane, Kolda, LR, Wells (1993)

..."benchmark framework" for the LHC

(...e.g., mSUGRA)

CMSSM



At $M_{\rm GUT} \simeq 2 \times 10^{16}\,{\rm GeV}$:

- gauginos $M_1 = M_2 = m_{\tilde{g}} = m_{1/2}$
- scalars $m_{\tilde{q}_i}^2 = m_{\tilde{l}_i}^2 = m_{H_b}^2 = m_{H_t}^2 = m_0^2$
- 3–linear soft terms $A_b = A_t = A_0$
- radiative EWSB $\mu^2 = \frac{m_{H_b}^2 - m_{H_t}^2 \tan^2\beta}{\tan^2\beta - 1} - \frac{m_Z^2}{2}$
- five independent parameters: $m_{1/2},\ m_0,\ A_0,\ \tan\beta,\ {\rm sgn}(\mu)$
- well developed machinery to compute masses and couplings
- neutralino χ mostly bino

some useful mass relations:

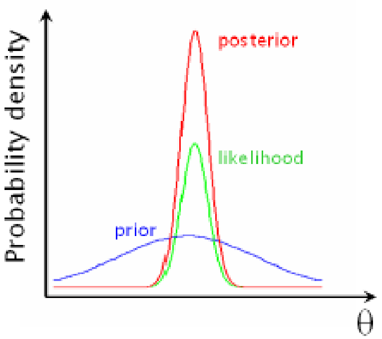
- bino: $m_\chi \simeq 0.4 m_{1/2}$
- gluino $\tilde{g}$: $m_{\tilde{g}} \simeq 2.7 m_{1/2}$
- supersymmetric tau (stau) $\tilde{\tau}_1$: $m_{\tilde{\tau}_1} \simeq \sqrt{0.15 m_{1/2}^2 + m_0^2}$

Baysian analysis of the CMSSM

Apply to the CMSSM: recent development, led by 2 groups

- $m = (\theta, \psi)$ – model's all relevant parameters
- CMSSM parameters $\theta = m_{1/2}, m_0, A_0, \tan\beta$
- relevant SM param's $\psi = M_t, m_b(m_b)^{\overline{MS}}, \alpha_s^{\overline{MS}}, \alpha_{\rm em}(M_Z)^{\overline{MS}}$
- $\xi = (\xi_1, \xi_2, \ldots, \xi_m)$: set of derived variables (observables): $\xi(m)$
- d: data $(\Omega_{\rm CDM}h^2, b \to s\gamma, m_h,$ etc)
- Bayes' theorem: posterior pdf

$$p(\theta, \psi | d) = \frac{p(d|\xi)\pi(\theta,\psi)}{p(d)}$$



$$\text{posterior} = \frac{\text{likelihood} \times \text{prior}}{\text{normalization factor}}$$

- $p(d|\xi) = \mathcal{L}$: likelihood
- $\pi(\theta, \psi)$: prior pdf
- $p(d)$: evidence (normalization factor)
- usually marginalize over SM (nuisance) parameters $\psi \Rightarrow p(\theta|d)$

SuperBayeS for CMSSM scans

SuperBayeS program used to perform 4 scans of the CMSSM's parameters:

- For a comparison with subsequent scans, a scan **with** a likelihood from **non-LHC experiments** and **without** a likelihood from **Xenon100**
- To validate our method, a scan **with** a likelihood that we derived from the **CMS aT 1.1/fb** analysis, and **with** a likelihood from the experiments that constrain the **SM's nuisance parameters**
- Scan **with non-LHC, CMS aT 1.1/fb** and **without Xenon100**
- Scan **with all in**

Parameter	Description	Prior Range	Scale
CMSSM			
m_0	Universal scalar mass	100, 2000	Log
$m_{1/2}$	Universal gaugino mass	100, 1000	Log
A_0	Universal trilinear coupling	−2000, 2000	Linear
$\tan\beta$	Ratio of Higgs vevs	3, 62	Linear
$\mathrm{sgn}\,\mu$	Sign of Higgs parameter	+1	Fixed
Nuisance			
m_t^{POLE}	Top quark pole mass	163.7, 178.1	Linear
$m_b(m_b)^{\overline{MS}}$	Bottom quark mass	3.92, 4.48	Linear
$\alpha_s(M_Z)^{\overline{MS}}$	Strong coupling	0.1096, 0.1256	Linear
$1/\alpha_{\mathrm{em}}(M_Z)^{\overline{MS}}$	Reciprocal of electromagnetic coupling	127.846, 127.99	Linear

Method

In frequentist statistics

- K-dim CMSSM's parameter space is in the best agreement with exprerimentt for min X^2,where

$$F\left(\Delta\chi^2, k\right) = 1 - \epsilon,$$

- The best-fit point ↔ a parameter point with min X^2 → confidence intervals are constructed from the best point

Method

In bayesian statistics

- question of posterior – **what is the probability of the CMSSM and its particular model parameters, given the experimental data?** We invert the likelihood using Bayes' theorem to find the posterior:

$$p(\theta,\phi|d) = \frac{\mathcal{L}(\theta,\phi)\pi(\theta,\phi)}{p(d)}$$

- 2D region of the CMSSM's parameter space that is in best agreement with the experiments (wrt posterior – the **credible region**) is the smallest region that contains a given fraction of the posterior, that is, the smallest region:

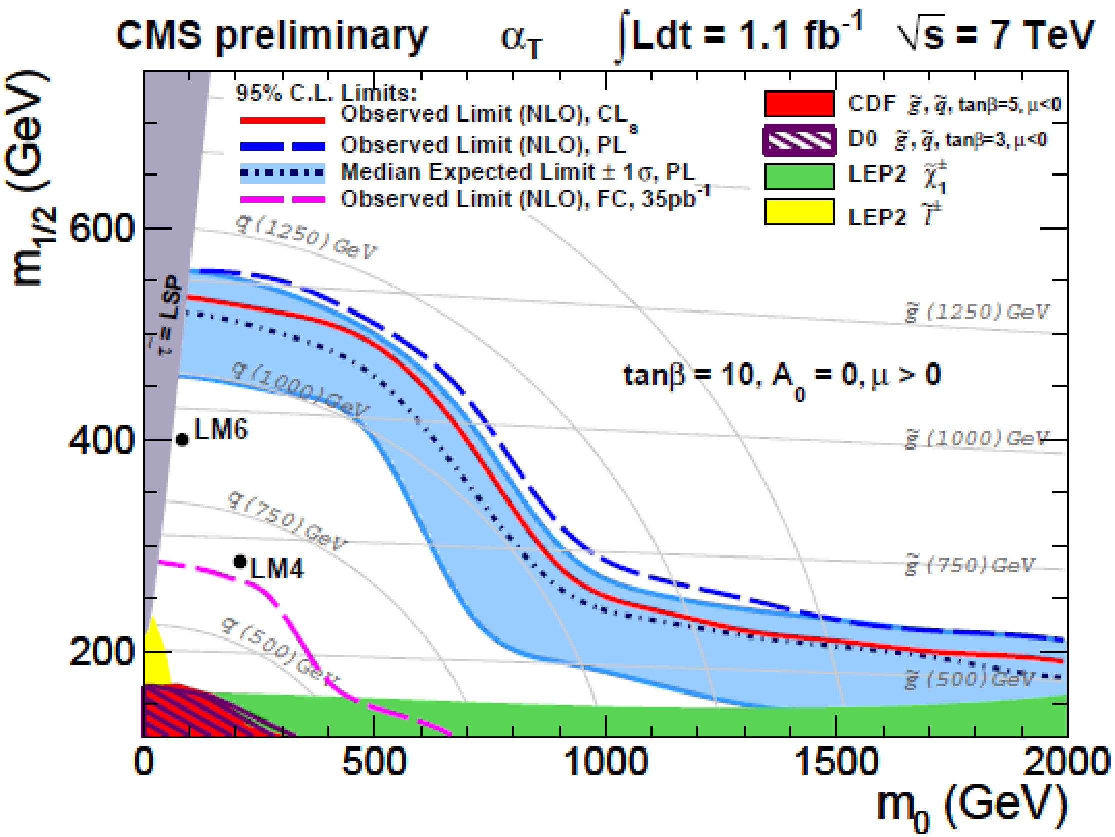
$$\int_R p(m_0, m_{1/2}|d)\,\mathrm{d}m_0\mathrm{d}m_{1/2} = 1-\epsilon$$

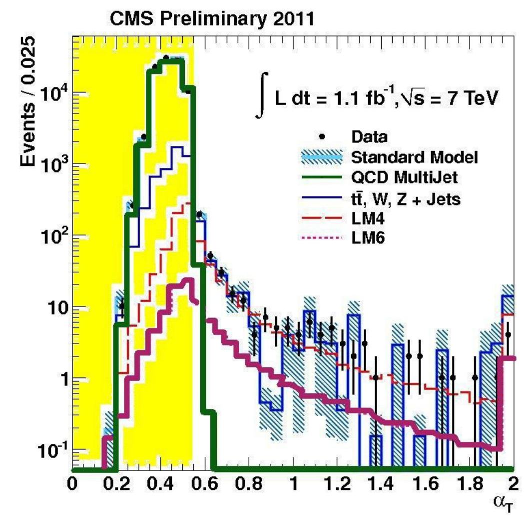
- **We consider:**

68 %	95%
$\epsilon = 0.32$ for 1σ	$\epsilon = 0.05$ for 2σ
$\Delta\chi^2 = 2.30$ for 1σ	$\Delta\chi^2 = 5.99$ for 2σ

Strongest limits from data - LHC

- Searches in all-hadronic states at LHC
- 1.1/fb CMS limits for the aT variable

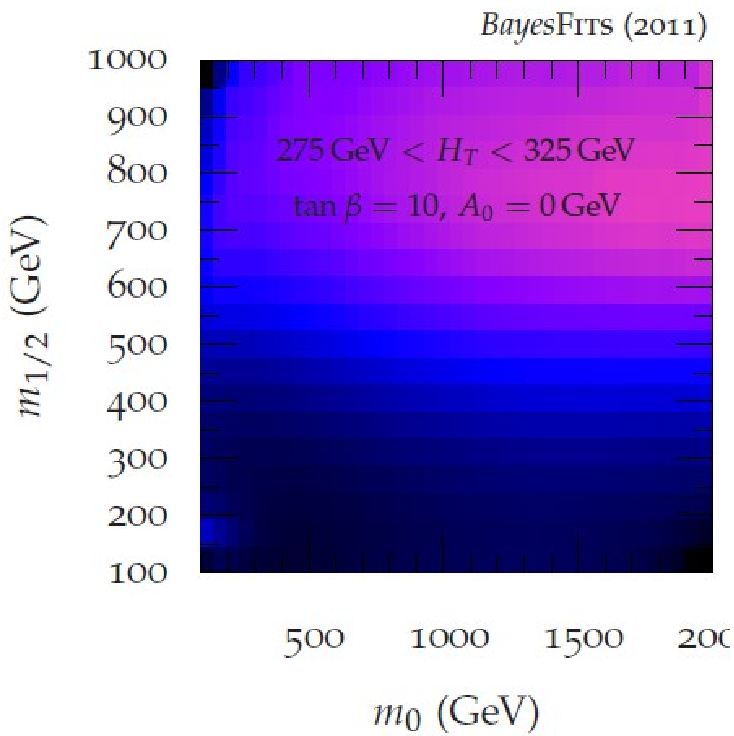


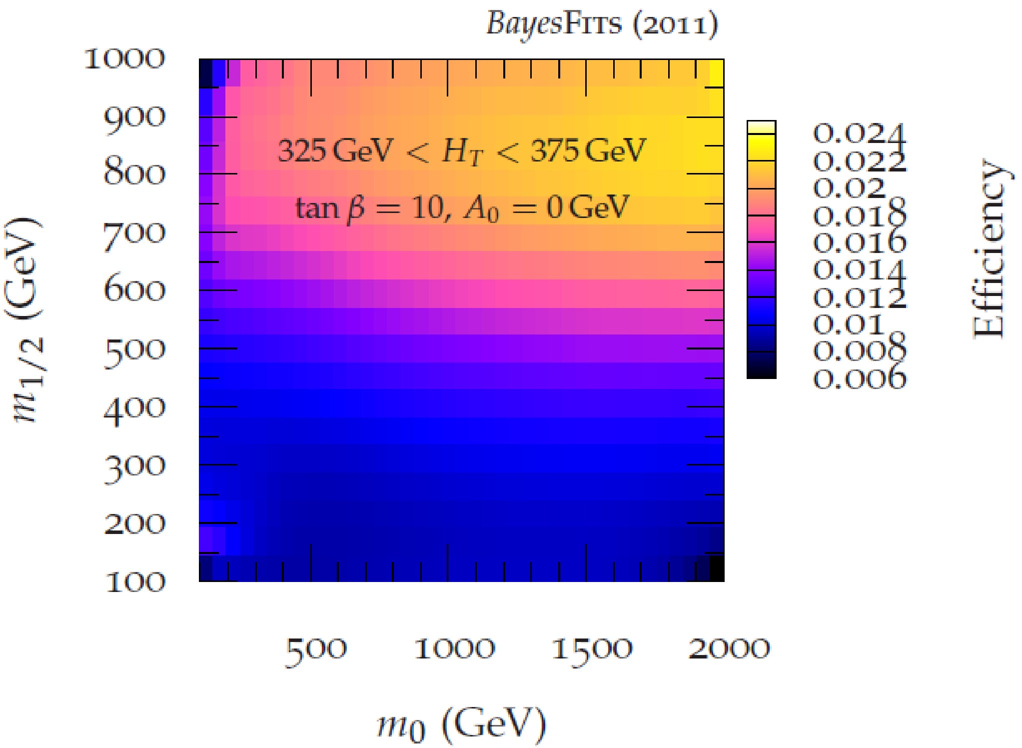


$$\alpha_T = \frac{E_T^{jet_2}}{M_T}$$

Efficiency MC map simuation

- Efficiency of the event selection for aT analysis at the MC level, 10k events per CMSSM points
- Used SoftSUSY, SUSY-HIT, Pythia → Xsec at LO





Simulation of aT likelihood

consider CMS limit from α_T at 1 fb^{-1}

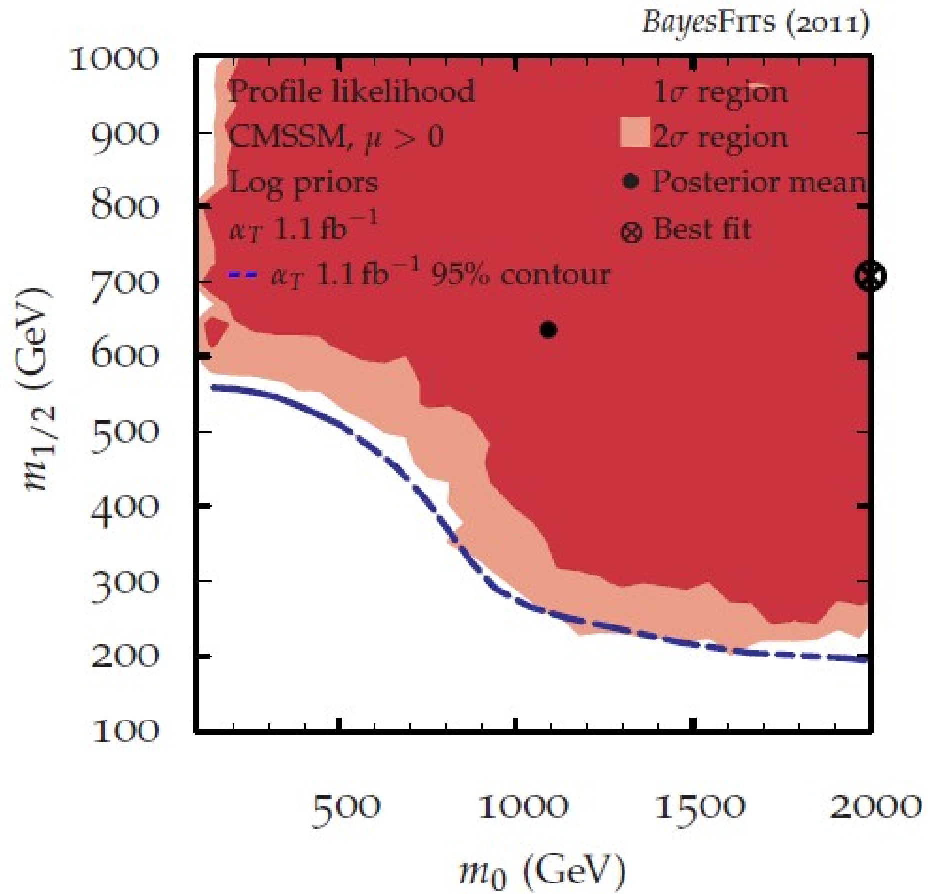
- o – observed nr of events
- b – expected nr of events from SM (bgnd)
- s – expected nr of events from SUSY (for a given parameter point)

$$s = \epsilon \cdot \sigma \cdot \int L$$

- ϵ – simulated detector efficiency
- σ – cross section
- $\int L$ – integrated luminosity

Likelihood

$$\mathcal{L} = \frac{e^{-s+b}\,(s+b)^o}{o!}$$

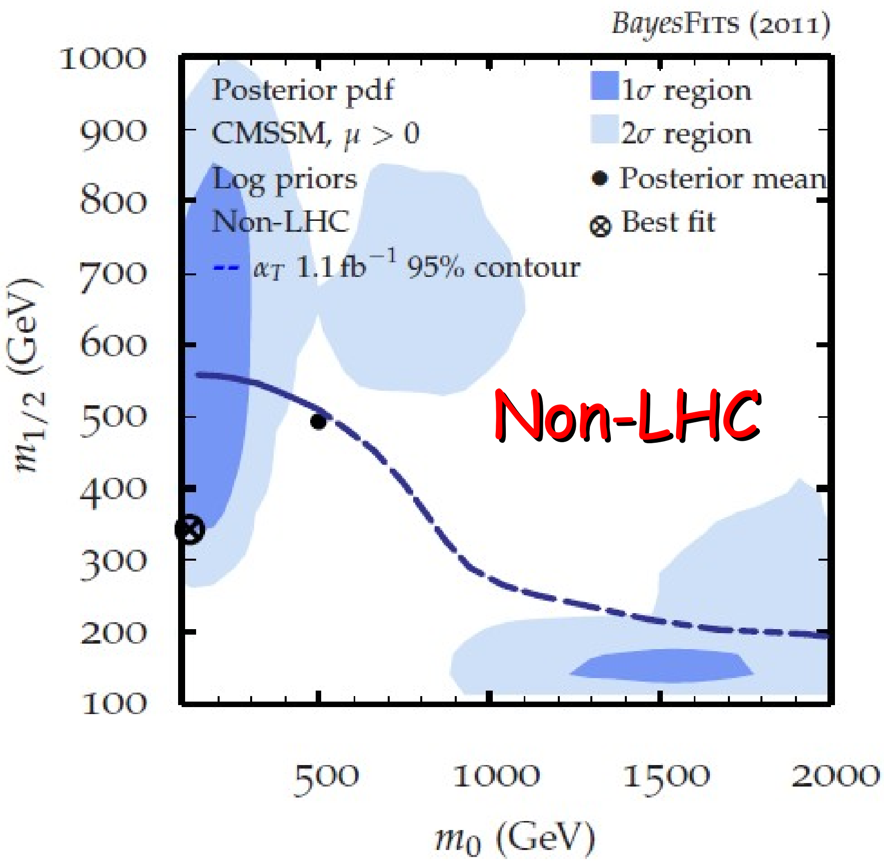


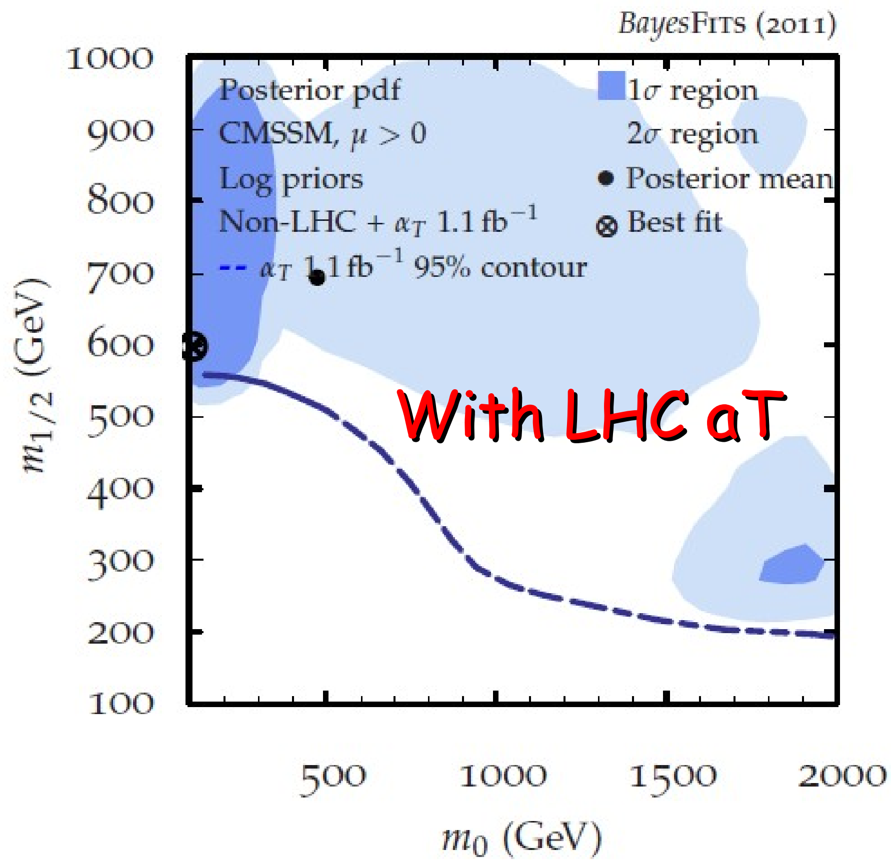
H_T Bin (GeV)	275 – 325	325 – 375	375 – 475	475 – 575	575 – 675	675 – 775	775 – 875	875 – ∞
$p_T^{leading}$ (GeV)	73	87	100	100	100	100	100	100
p_T^{second} (GeV)	73	87	100	100	100	100	100	100
p_T^{others} (GeV)	37	43	50	50	50	50	50	50
Observed Data								
$\alpha_T > 0.55$	782	321	196	62	21	6	3	1

- **Excellent agreement** between our 95% contour and the official CMS 95% contour

Impact on CMSSM paramters

- Significant impact on previously favored regions





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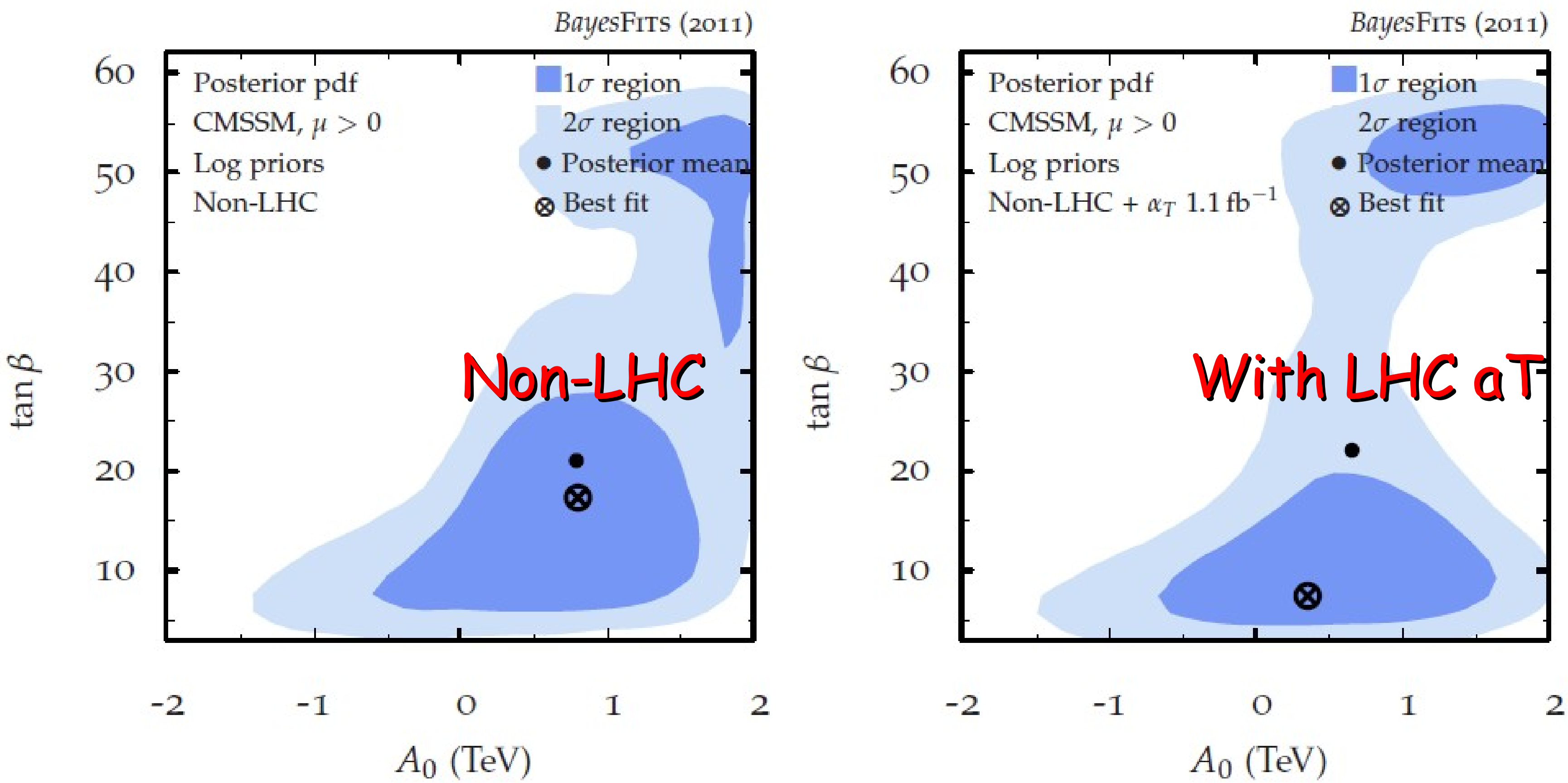
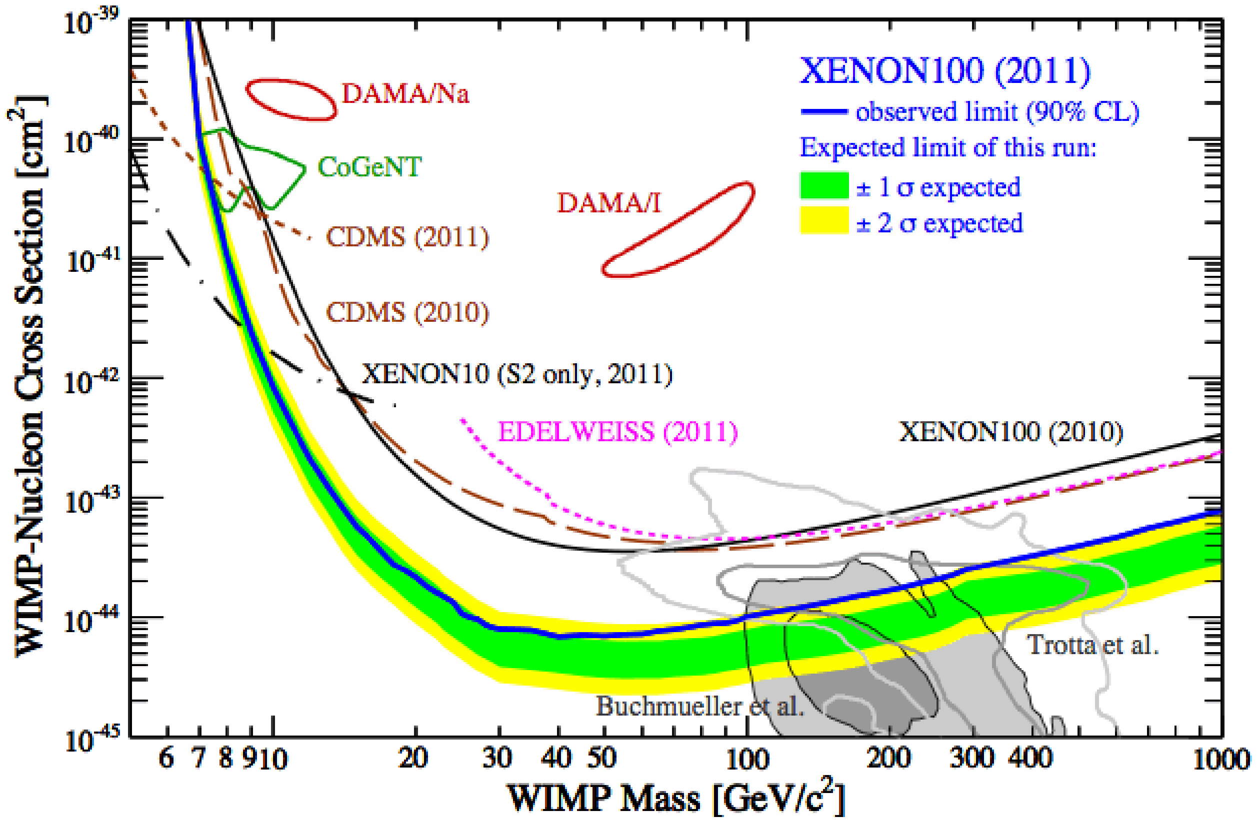


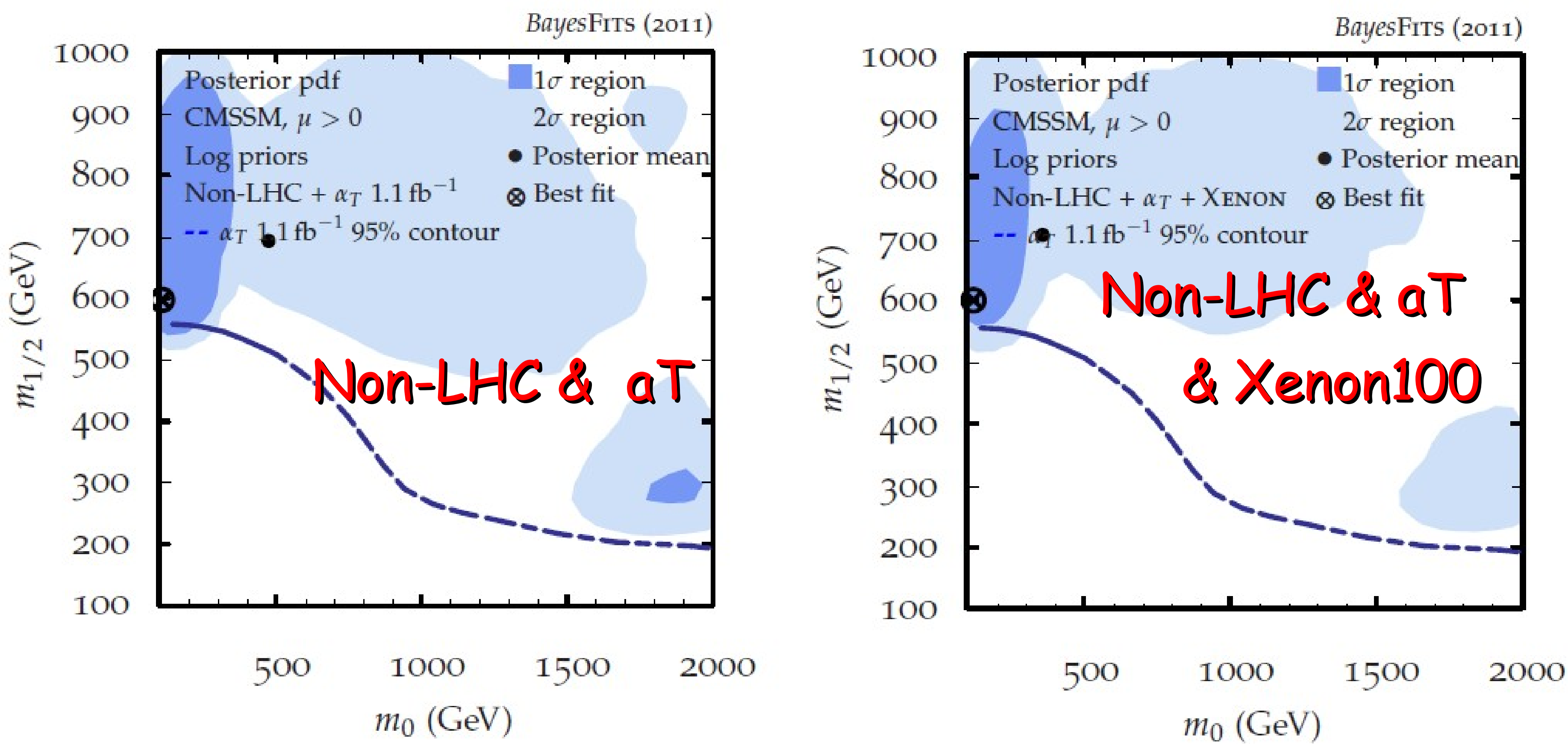
Xenon 100 searches for Dark Matter

- Excluded spin-independent (SI) scattering cross sections below 10^{-8} pb



CMSSM and Xenon100

- Weak effect → slightly squeezing parameters

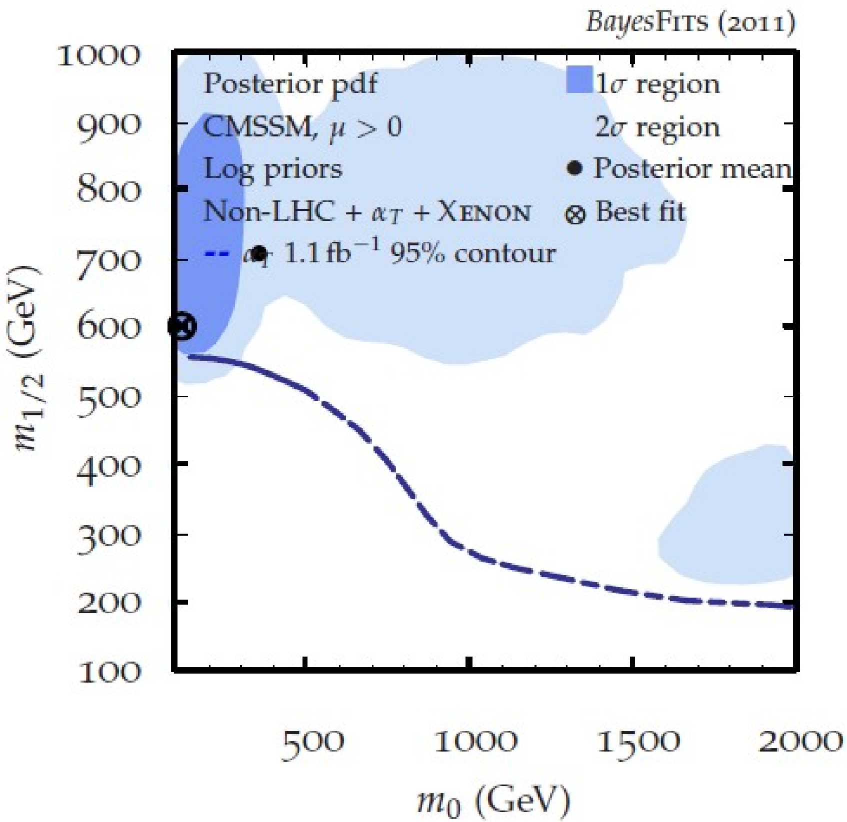


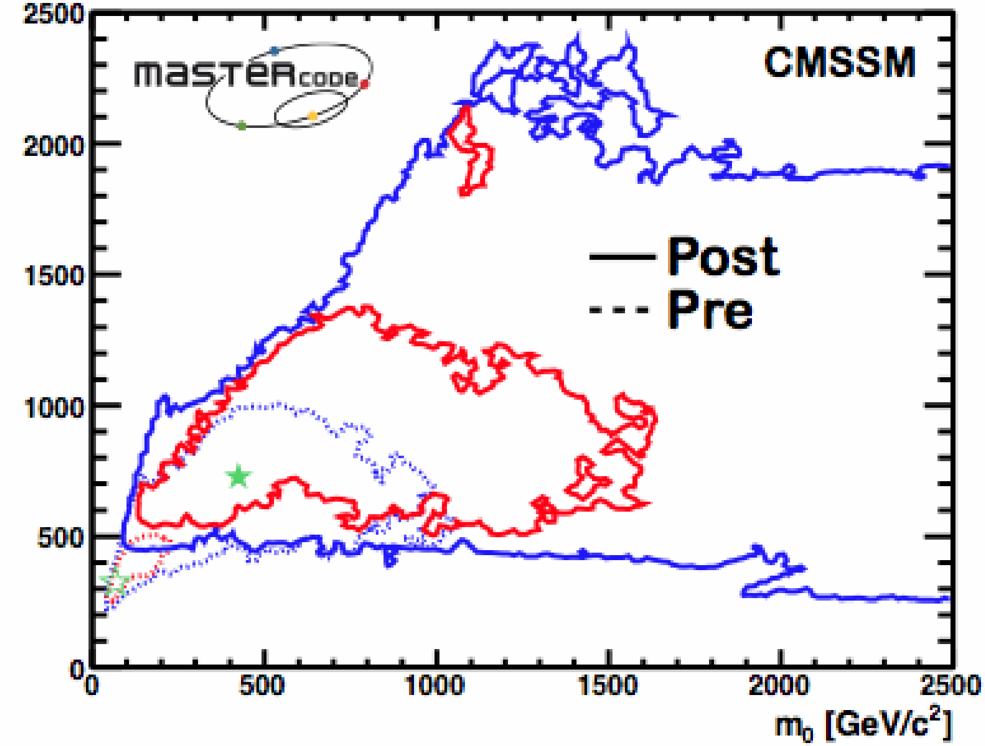
Best-point results

Constraints	m_0	$m_{1/2}$	A_0	$\tan\beta$	χ^2	d.o.f.	p-value
Non-LHC	122 (116, 1391)	343 (142, 702)	806 (236, 1514)	17 (13, 22)	16.53	16	42%
Non-LHC + α_T	111 (129, 1113)	597 (591, 827)	360 (18, 1357)	7 (9, 23)	22.34	24	56%
Non-LHC + α_T + XENON100	122 (127, 741)	600 (608, 820)	677 (82, 1283)	11 (9, 16)	22.21	25	61%

Mass (GeV)	68% Credible Region	95% Credible Region	68% Credible Region	95% Credible Region
	Non-LHC		CMS α_T 1.1/fb analysis	
m_h	(112, 117)	(110, 118)	(115, 120)	(111, 122)
m_χ	(56, 291)	(53, 356)	(169, 358)	(109, 412)
$m_{\chi_1^\pm}$	(110, 554)	(104, 676)	(322, 682)	(203, 782)
$m_{\tilde{q}}$	(326, 808)	(254, 1172)	(708, 1300)	(528, 1615)
$m_{\tilde{g}}$	(403, 1576)	(384, 1885)	(1019, 1906)	(710, 2100)
	CMS α_T 1.1/fb analysis + Non-LHC		CMS α_T 1.1/fb analysis + Non-LHC + XENON100	
m_h	(114, 118)	(112, 119)	(114, 118)	(112, 119)
m_χ	(243, 304)	(99, 393)	(250, 343)	(128, 390)
$m_{\chi_1^\pm}$	(461, 657)	(143, 744)	(475, 651)	(181, 738)
$m_{\tilde{q}}$	(440, 999)	(398, 1383)	(434, 761)	(398, 1302)
$m_{\tilde{g}}$	(1349, 1840)	(727, 2065)	(1380, 1825)	(879, 2043)

Bayesian vs χ^2





- reasonable agreement in the $m_{1/2}$ > m0 region
- disagreement about the best-fit point
- some disagreement about large m0 region

Conclusions

- **LHC 2011 data**: a big bite at the simplest unified SUSY model
- **Xenon100 2011**: complementary search, some additional impact
- **CMSSM**:
  still wide ranges of sparticle and Higgs masses to explore

- much more room in less constrained SUSY models

Main references

- SuperBayeS package, www.superbayes.org
- CMS Collaboration, CMS SUS-11-003
- BayesFIT, “Bayesian Implications of 2011 LHC and Xenon100 Searches for the Constrained MSSM”, forthcoming

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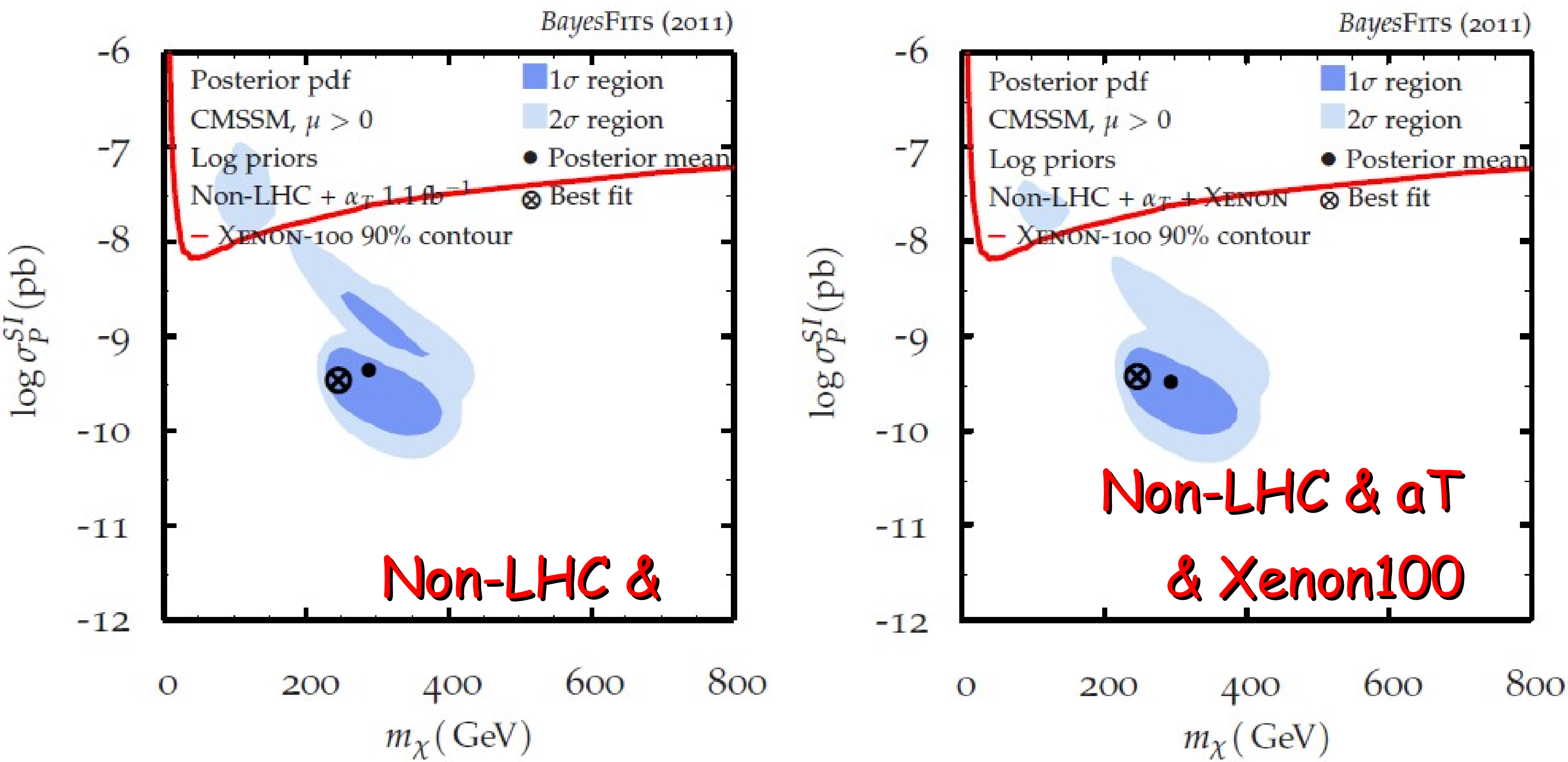
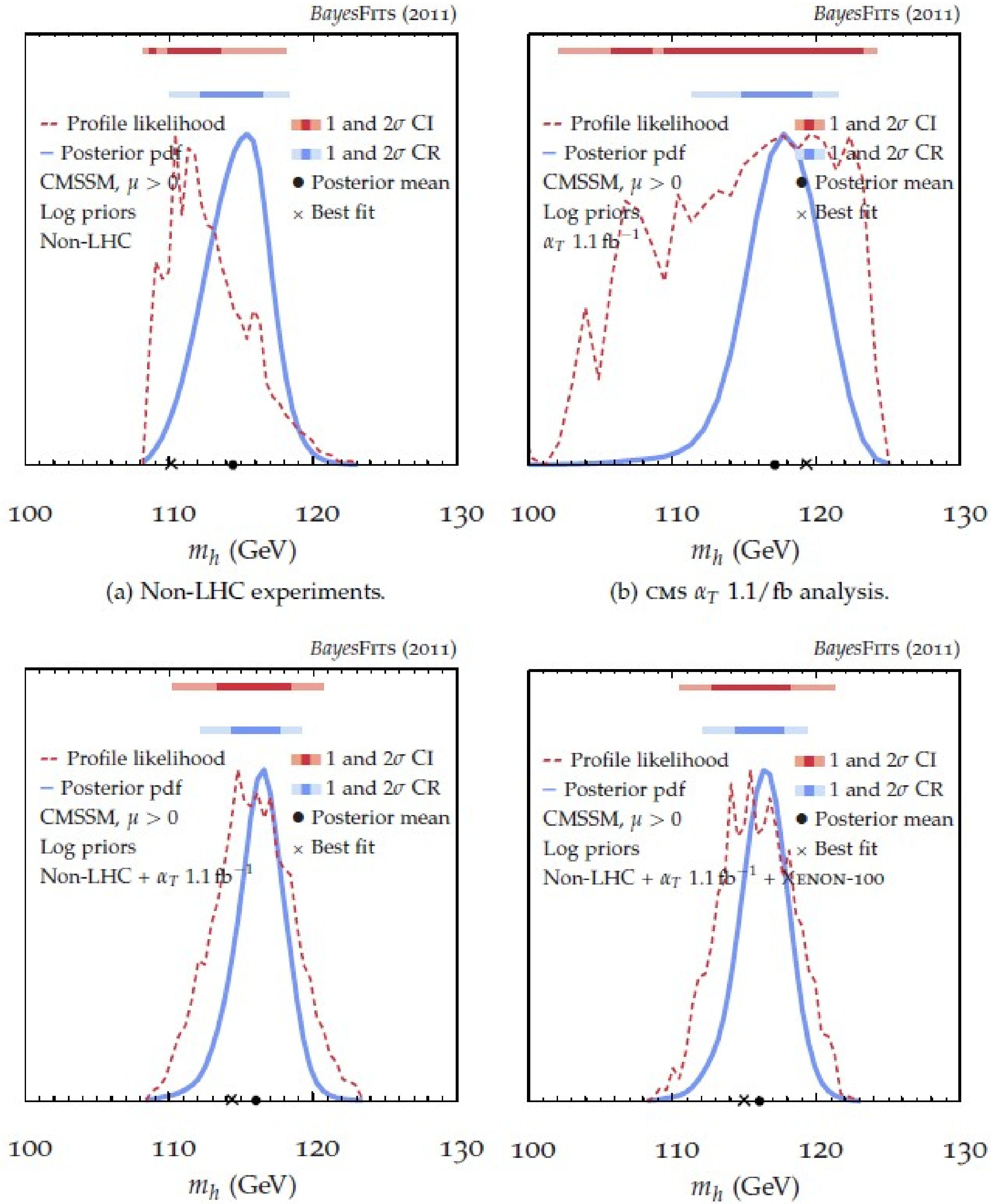
Backup

Xenon100

- Slightly squeezing parameters




BayesFITS (2011)
Profile likelihood
Posterior pdf
CMSSM, $\mu > 0$
Log priors
Non-LHC
1 and 2σ CI
1 and 2σ CR
Posterior mean
Best fit
100
110
120
130
m_h (GeV)
(a) Non-LHC experiments.
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(b) CMS α_T 1.1/fb analysis.
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m_h (GeV)
BayesFITS (2011)
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Posterior pdf
CMSSM, $\mu > 0$
Log priors
Non-LHC + α_T 1.1 fb^{-1} + XENON-100
1 and 2σ CI
1 and 2σ CR
Posterior mean
Best fit
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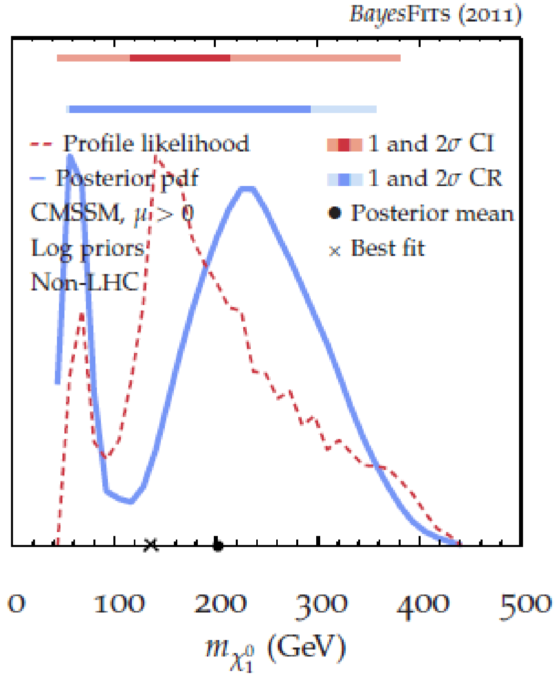
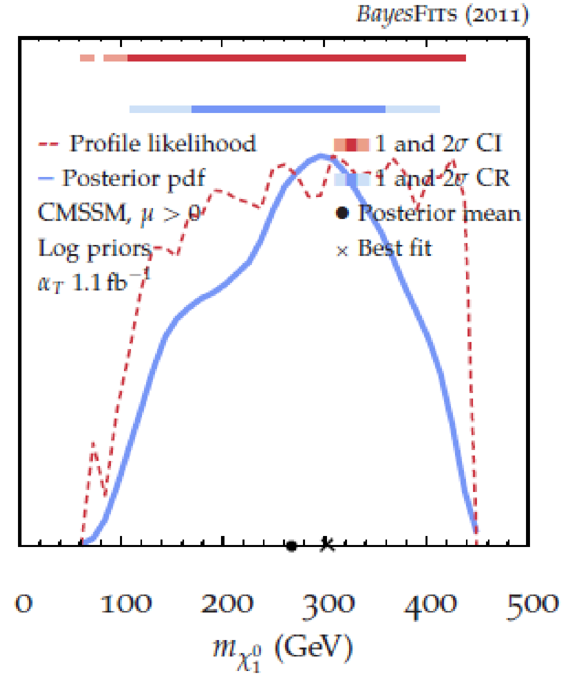
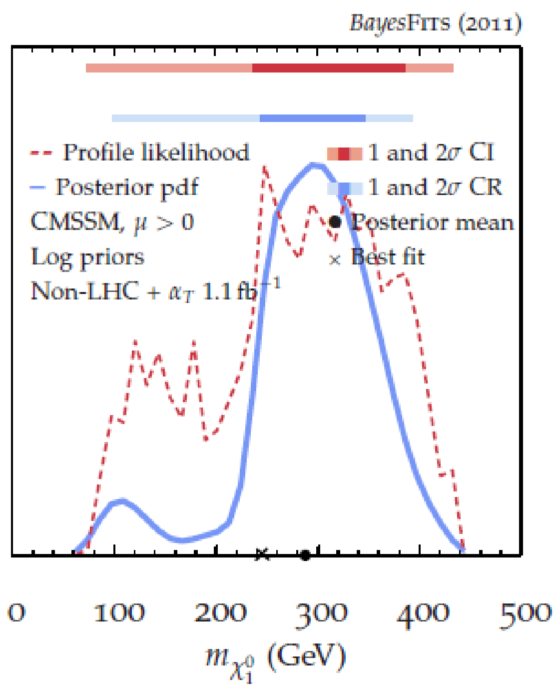
110
120
130
m_h (GeV)

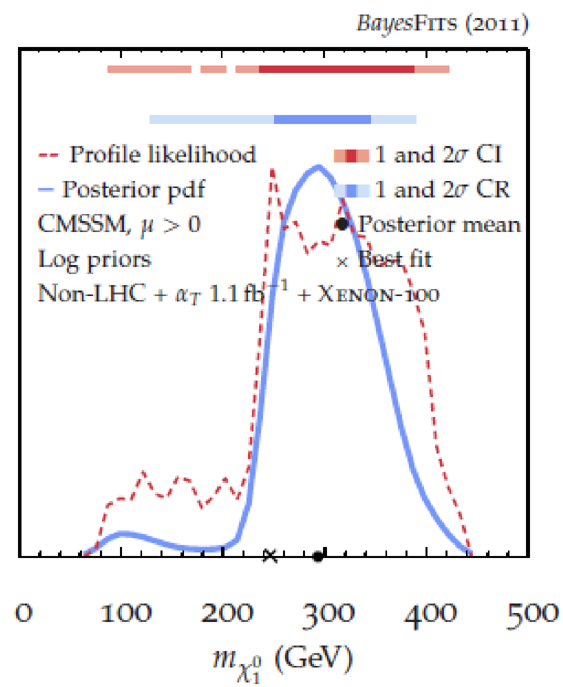


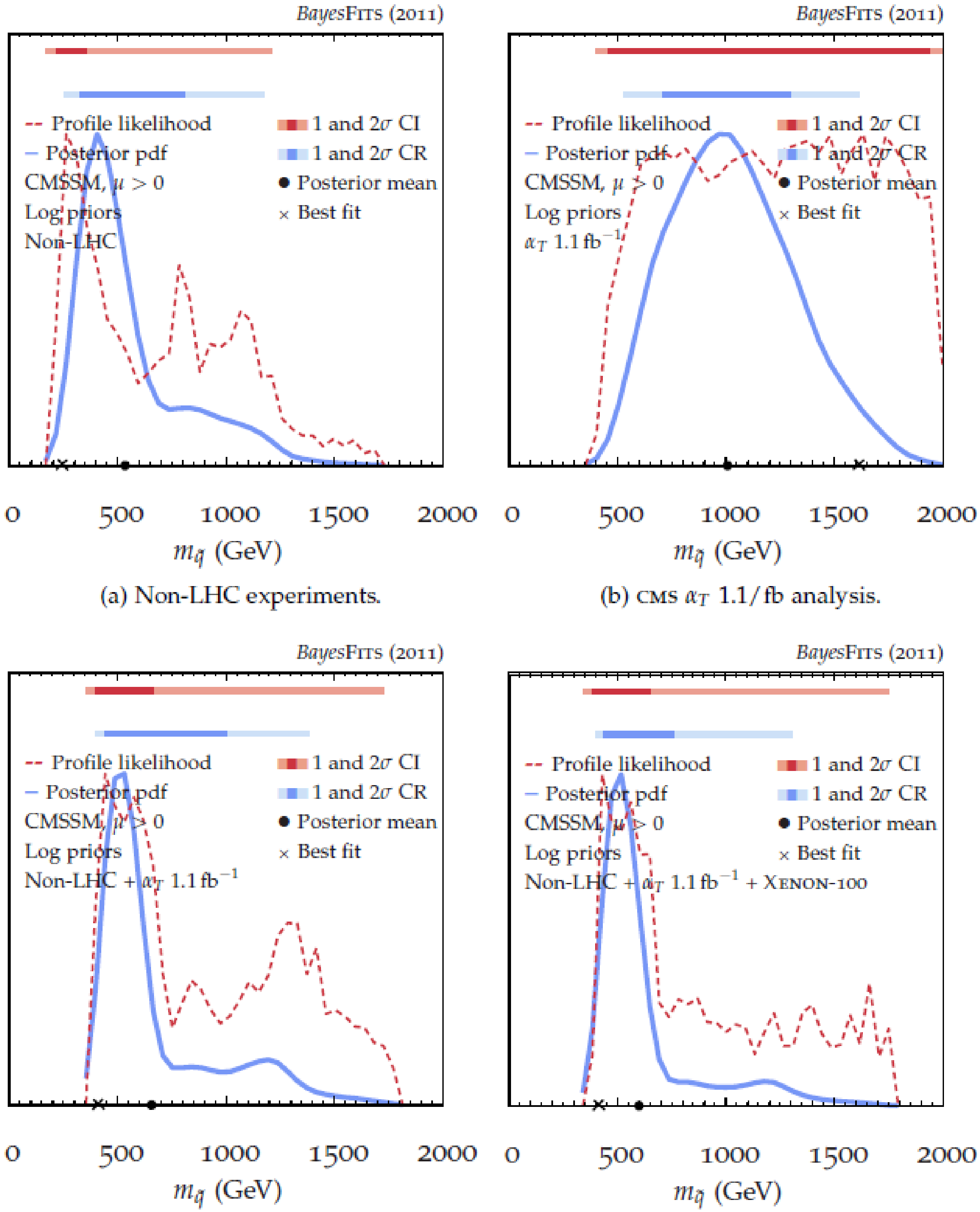
(a) Non-LHC experiments.



(b) CMS α_T 1.1/fb analysis.

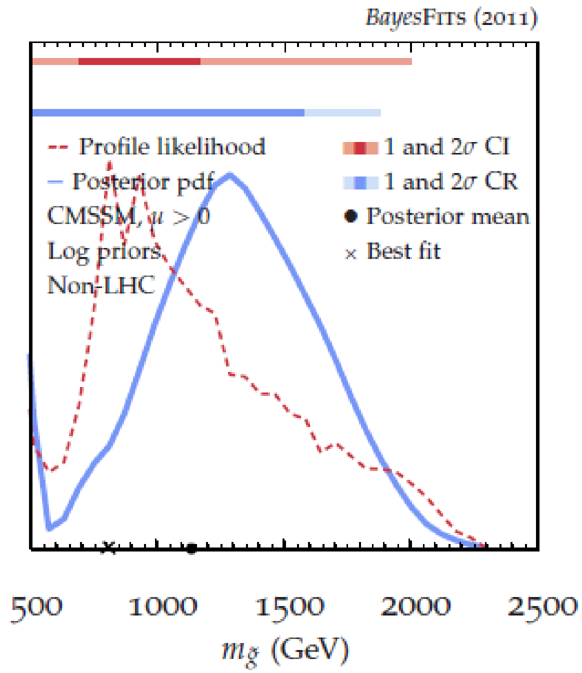






(a) Non-LHC experiments.

(b) CMS α_T 1.1/fb analysis.



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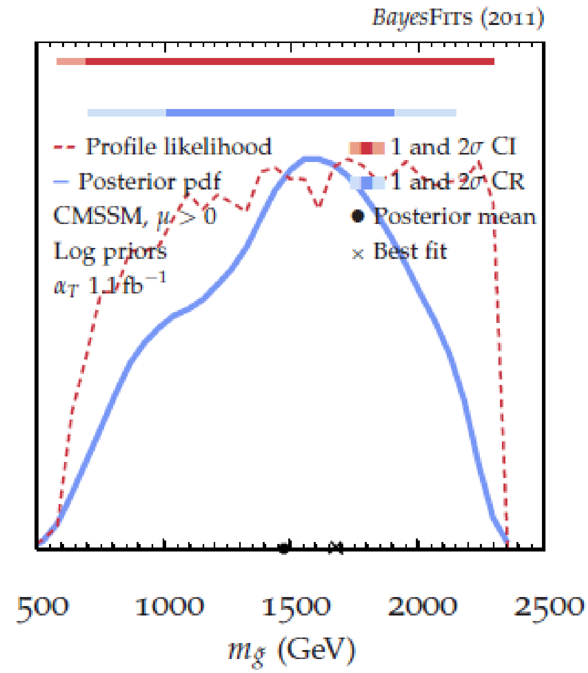
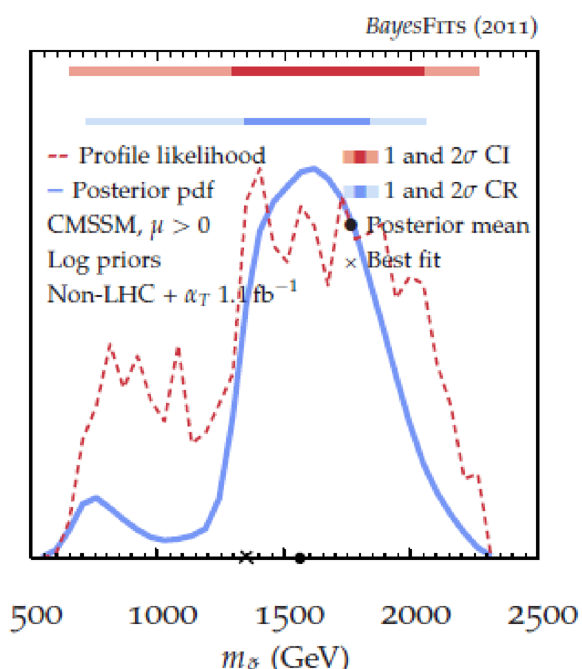
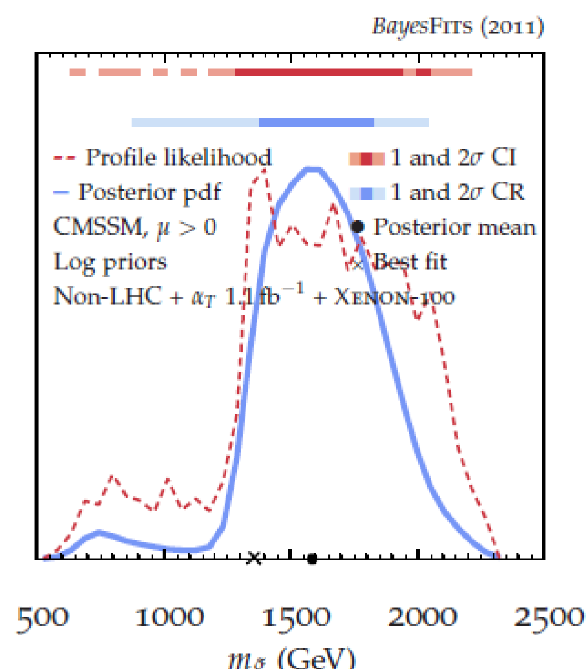


(b) CMS α_T 1.1/fb analysis.





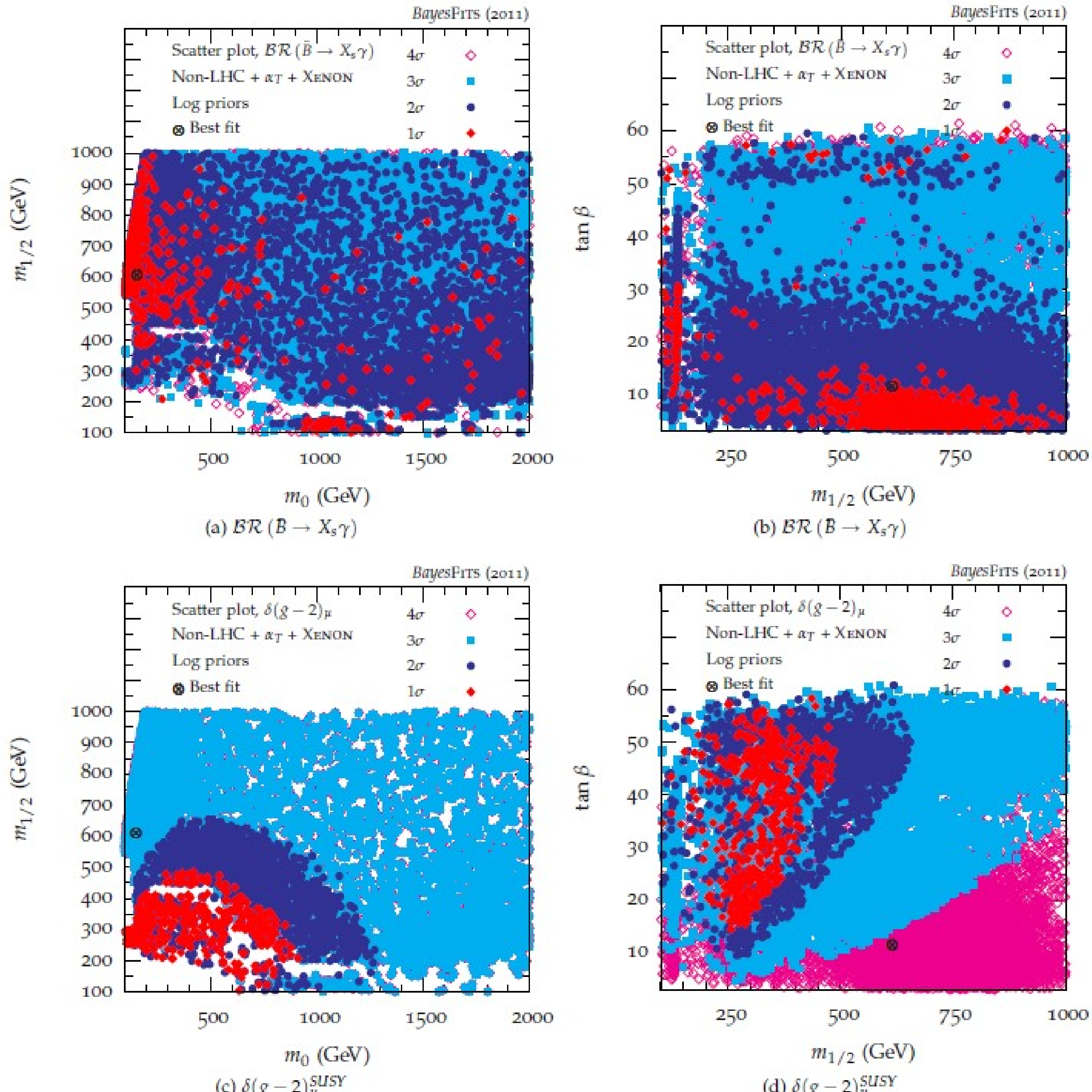
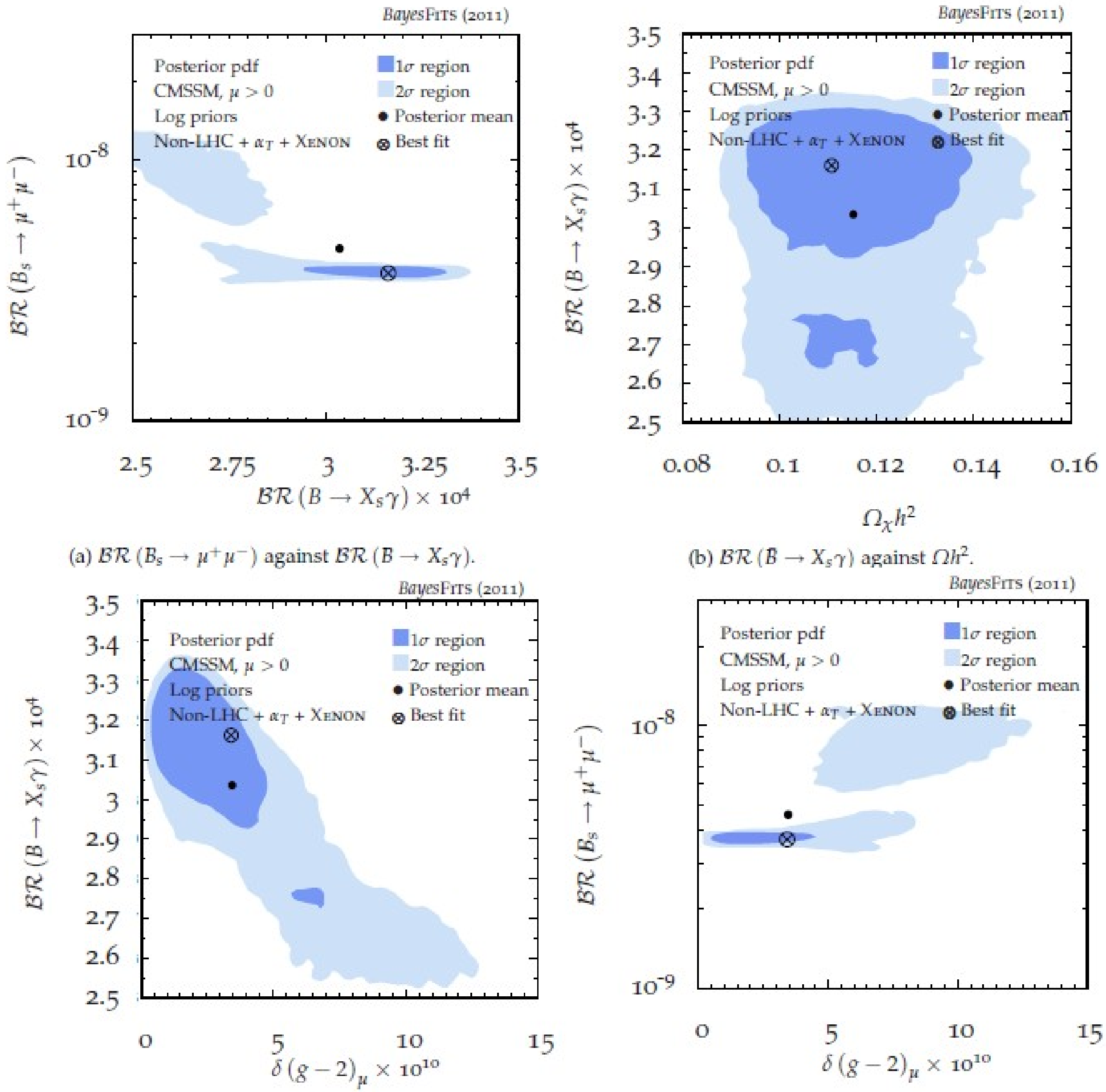


Figure 14: Scatter plots of points from a scan with a likelihood from α_T, XENON100 and non-LHC experiments, coloured by the values of $\mathcal{BR}(\bar{B} \to X_s\gamma)$ (top row) and $\delta(g-2)_\mu^{SUSY}$ (bottom row). The colours show the discrepancy between the CMSSM's predicted value and the measured value, measured in experimental errors.



(a) $\mathcal{BR}(B_s \to \mu^+\mu^-)$ against $\mathcal{BR}(\bar{B} \to X_s\gamma)$.

(b) $\mathcal{BR}(\bar{B} \to X_s\gamma)$ against Ωh^2.

(c) $\mathcal{BR}(\bar{B} \to X_s\gamma)$ against $\delta(g-2)_\mu^{SUSY}$.

(d) $\mathcal{BR}(B_s \to \mu^+\mu^-)$ against $\delta(g-2)_\mu^{SUSY}$.

Figure 15: Marginalised posterior pdf for combinations of the experimental observables $\mathcal{BR}(\bar{B} \to X_s\gamma)$, $\delta(g-2)_\mu^{SUSY}$, $\mathcal{BR}(B_s \to \mu^+\mu^-)$ and Ωh^2, from a scan with a likelihood from α_T, XENON100 and non-LHC experiments.