NC STATE UNIVERSITY

Applied (High Temperature) Superconductivity

Academic Training
Lecture 1

Justin Schwartz
Department of Materials Science and Engineering
North Carolina State University

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Department of Materials Science and Engineering

Outline for the week of training

- Lectures 1& 2: Introduction & "Just enough" physics
  - Brief introduction … what is superconductivity and why is it useful?
  - Basic physics of superconductivity and the superconducting state
  - Applications-relevant physics of superconductivity & superconducting state
- Lecture 3: Technical superconductors
  - A brief summary of NbTi and Nb_3Sn
  - HTS conductor options: Bi2212 & YBCO
- Lecture 4: Electromechanical behavior
  - A brief summary of NbTi and Nb_3Sn
  - HTS conductor options: Bi2212 & YBCO
- Lecture 5: Quench behavior and high field magnets
  - What is quench protection?
  - Quench protection in HTS magnets

What is superconductivity?

- Let's start with basic, relatively well-known behaviors
- Most of you likely know:
  - Zero resistivity (usually)
    - Persistent magnets are common in MRI/NMR
    - Requires proper electromagnetic conditions
    - Only during DC operation; AC (or any transient) is intrinsically lossy
    - Power systems are mostly AC
  - Perfect diamagnetism (sometimes but not functionally)
    - Makes great demonstrations
    - Perfect diamagnetism not desirable for applications
    - Magnetic behavior very complex and can dominate electrical performance

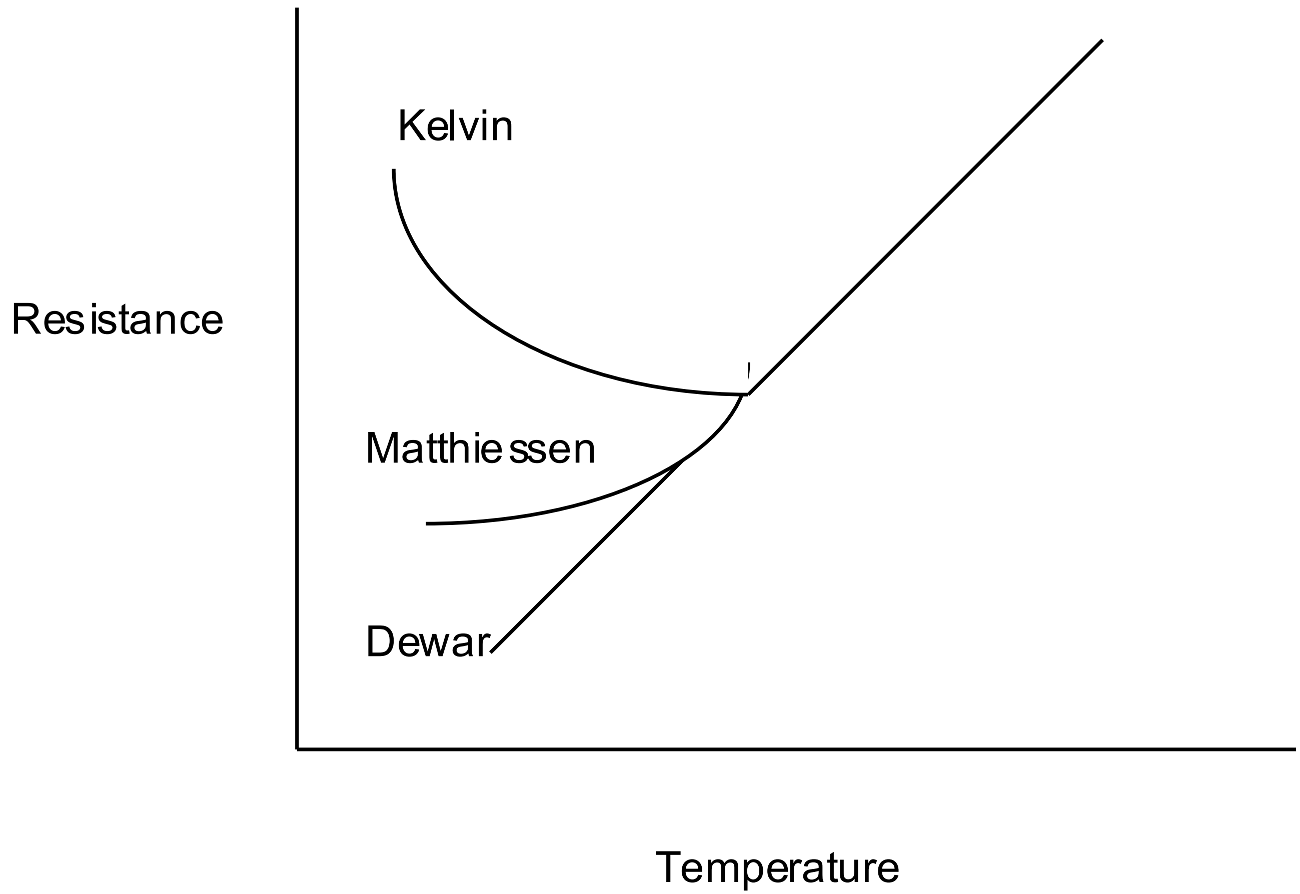
Who is interested in superconductivity?

- Science: high energy physics and nuclear magnetic resonance
- Medicine: magnetic resonance imaging – becoming functionalized to individual body parts
- Energy: we have a 21st Century economy, with 21st Century electrical demands driven by 21st Century loads and 21st Century applications, all supported by a post-WWII power grid with energy still primarily generated by fossil fuel and nuclear power

Superconductivity – an introduction

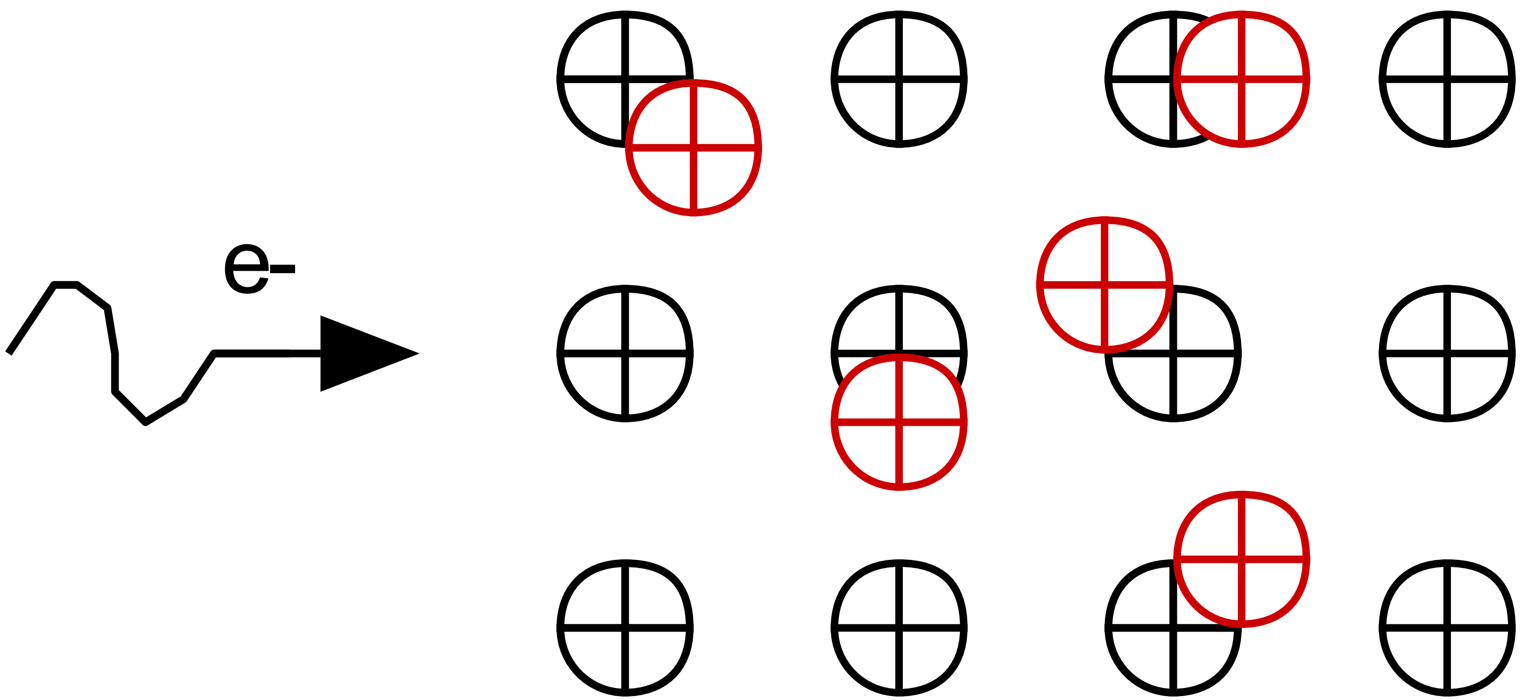
- Discovery of Superconductivity- 1911 H. Kamerlingh Onnes
- Preceeded by liquefaction of Helium- 1908
- Study of the low temperature resistivity of metals- no theory
- Predictions on resistivity

Resistivity of Metals at Low Temperature



Theory of Metallic Resistivity

T=0, coherent, no electron-ion scattering
T > 0, scattering, non-periodic lattice due to lattice vibrations (quantum = phonon)



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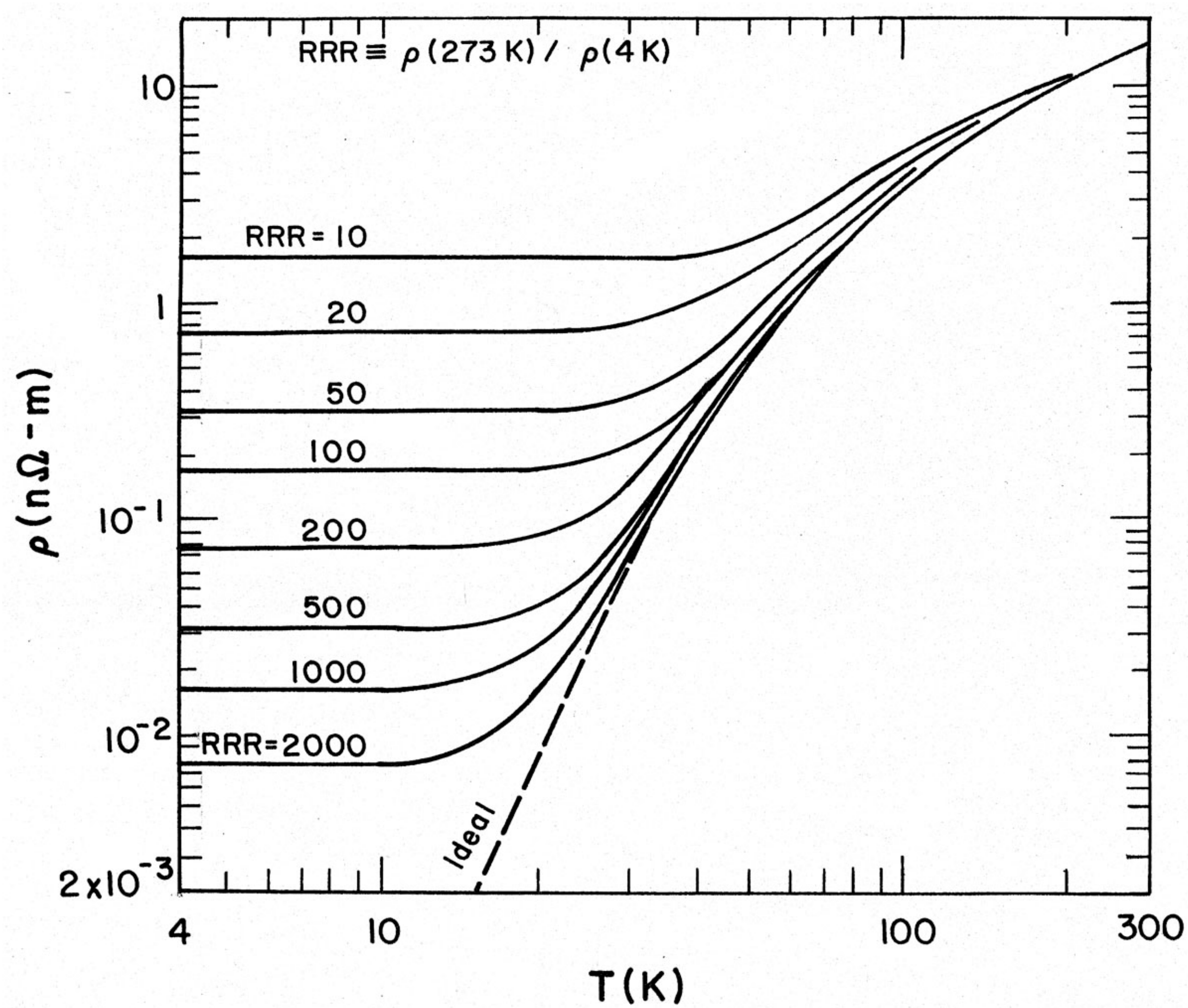
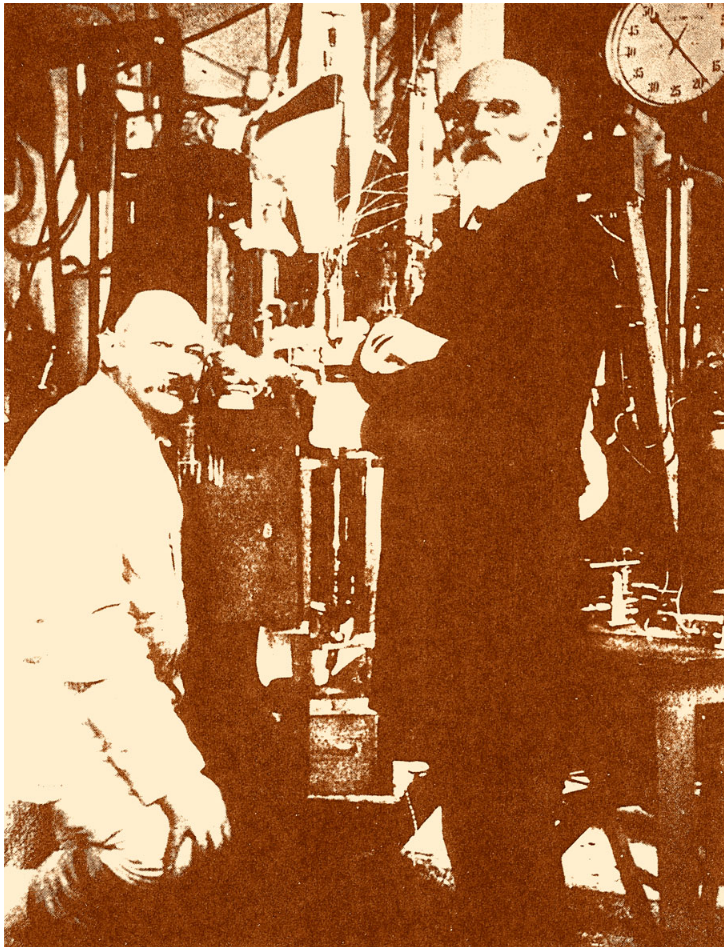
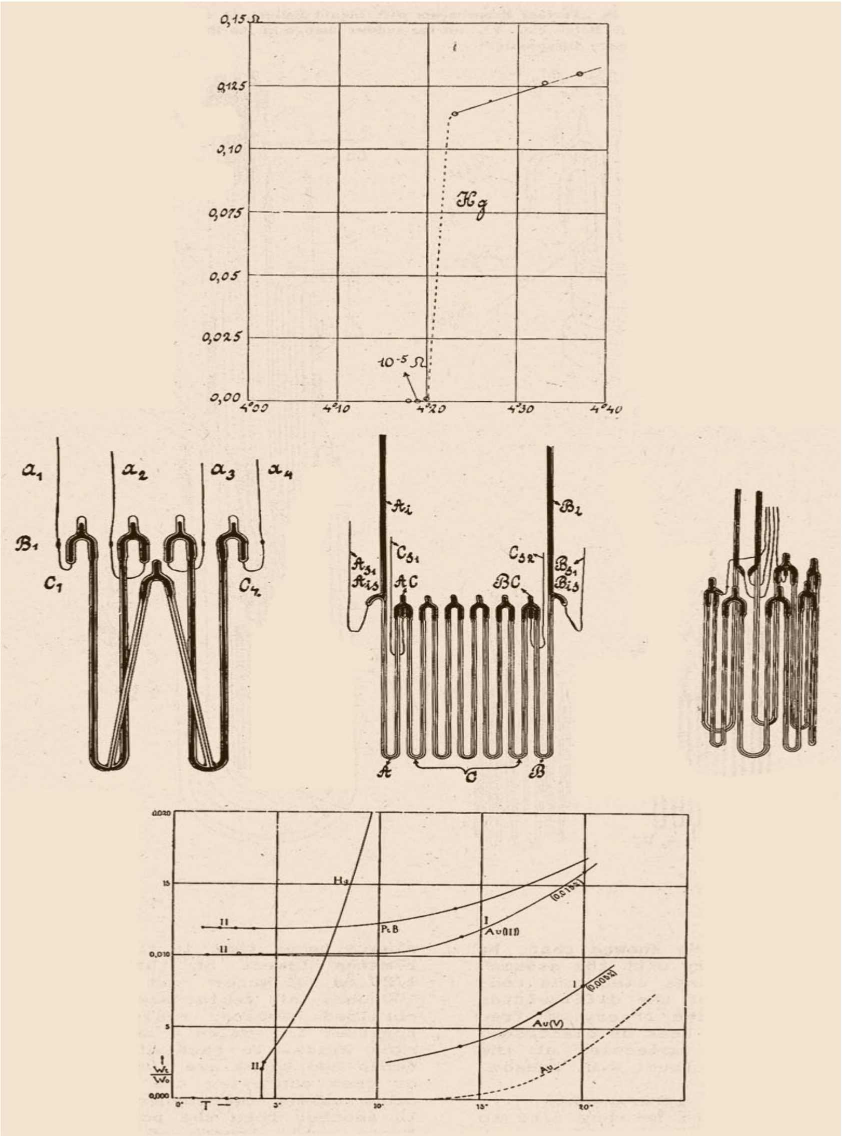
Resistivity of Metals – For example, Cu

Resistivity increases with impurities



Discovery of Superconductivity - 1911

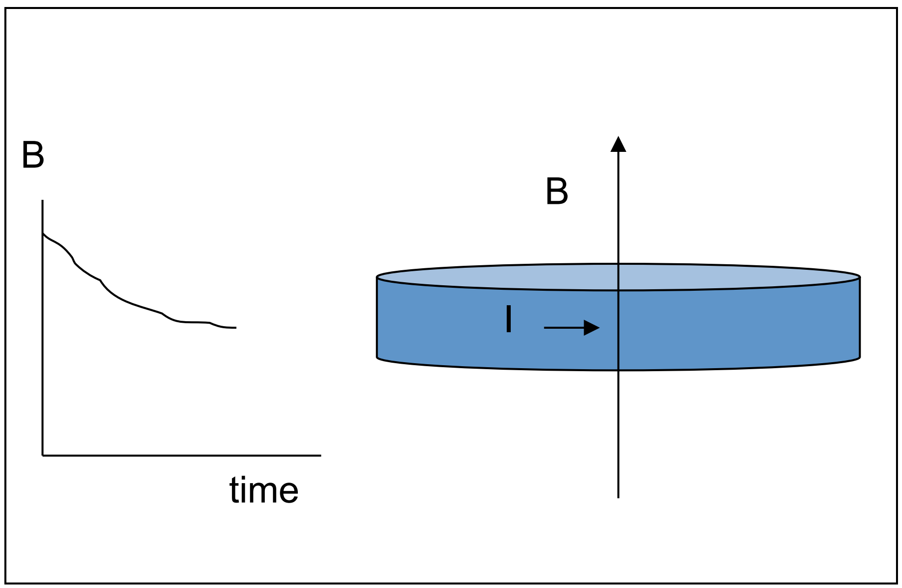


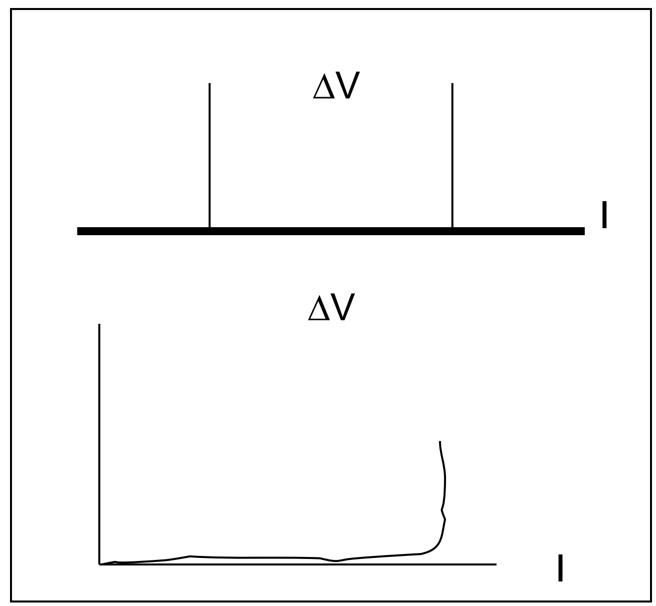


by K. Onnes in pure Hg

Superconductors

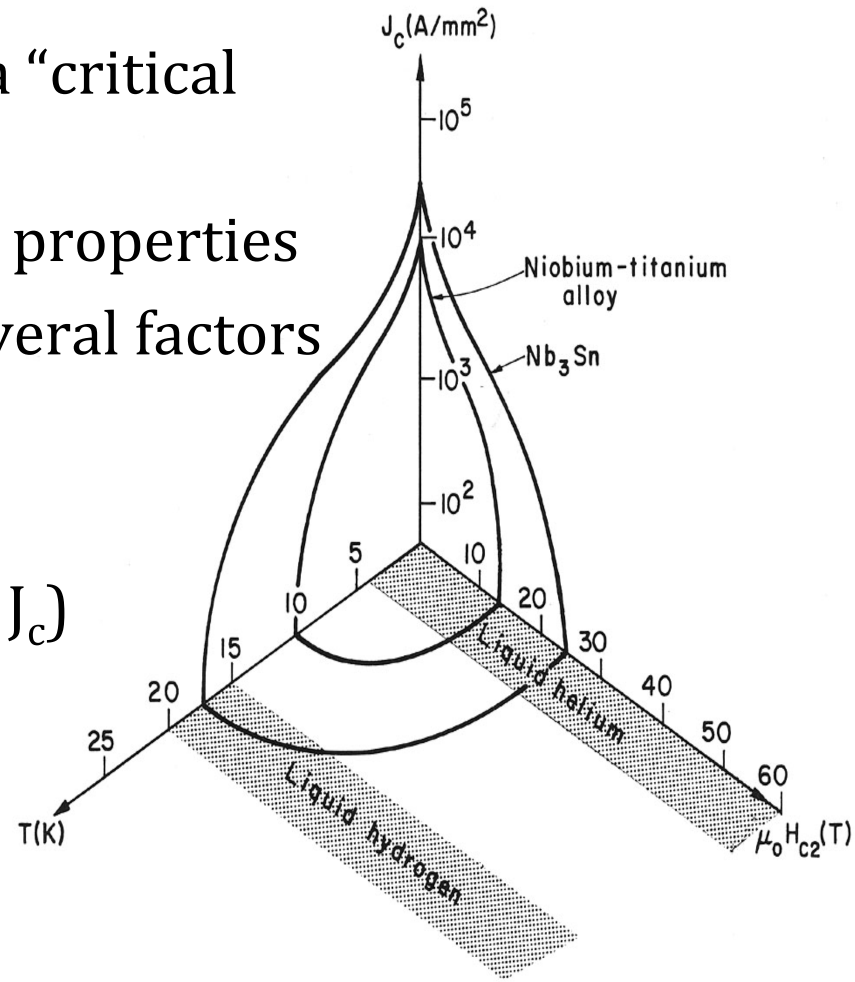
- Many materials (metals, alloys, oxides, polymers)
- Resistive transition (how measured originally)
- Persistent current method





Superconductor Critical Surface

- Superconductivity occurs within a "critical surface" in T_c, B_c & J_c space
- Note the tradeoff between critical properties
- Critical parameters depend on several factors
  - Elements
  - Crystal structure
  - Microstructure (mostly affects J_c)



Critical Temperature (T_c)

- Property that wins Nobel prizes
- Values
  - Lowest Rh (T_c = 325 mK)
  - Highest element Nb (T_c = 9.2 K)
  - Highest metallic alloys Nb_3Ge (T_c = 23 K); MgB_2 (T_c = 39 K)
  - Highest oxide $HgBa_2Ca_2Cu_3O_{10}$ (T_c = 135 K)
    - Under pressure … 160 K
- The search continues!

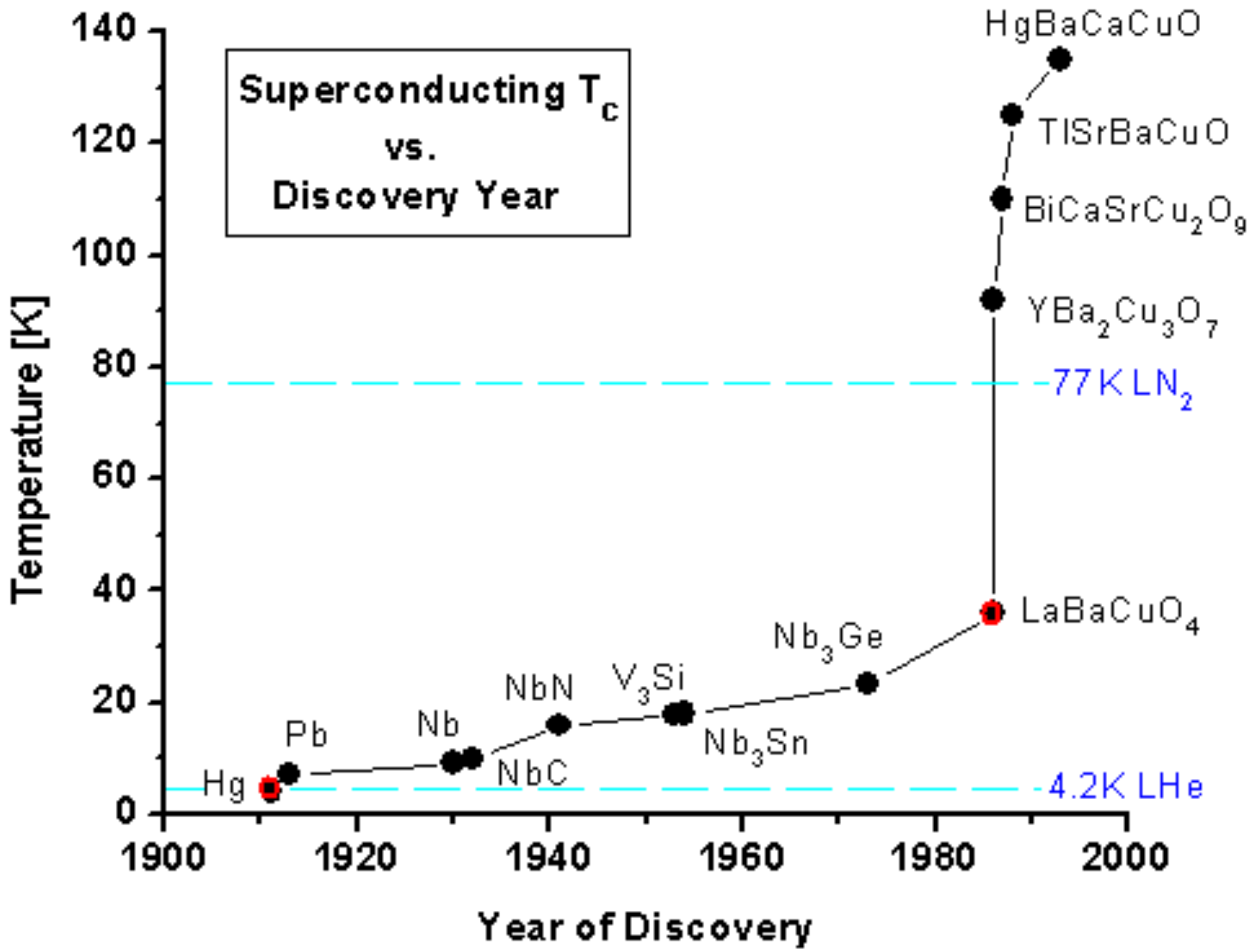
Critical temperatures and magnetic fields of "Type I" superconductors: all values are very low!

Material	T_c (K)	$\mu_0 H_0$ (mT)
Aluminum	1.2	9.9
Cadmium	0.52	3.0
Gallium	1.1	5.1
Indium	3.4	27.6
Iridium	0.11	1.6
Lanthanum α	4.8	
β	4.9	
Lead	7.2	80.3
Lutecium	0.1	35.0
Mercury α	4.2	41.3
β	4.0	34.0
Molybdenum	0.9	
Osmium	0.7	~6.3
Rhenium	1.7	20.1
Rhodium	0.0003	4.9
Ruthenium	0.5	6.6
Tantalum	4.5	83.0
Thalium	2.4	17.1
Thorium	1.4	16.2
Tin	3.7	30.6
Titanium	0.4	
Tungsten	0.016	0.12
Uranium α	0.6	
β	1.8	
Zinc	0.9	5.3
Zirconium	0.8	4.7

Trends in Critical Temperature

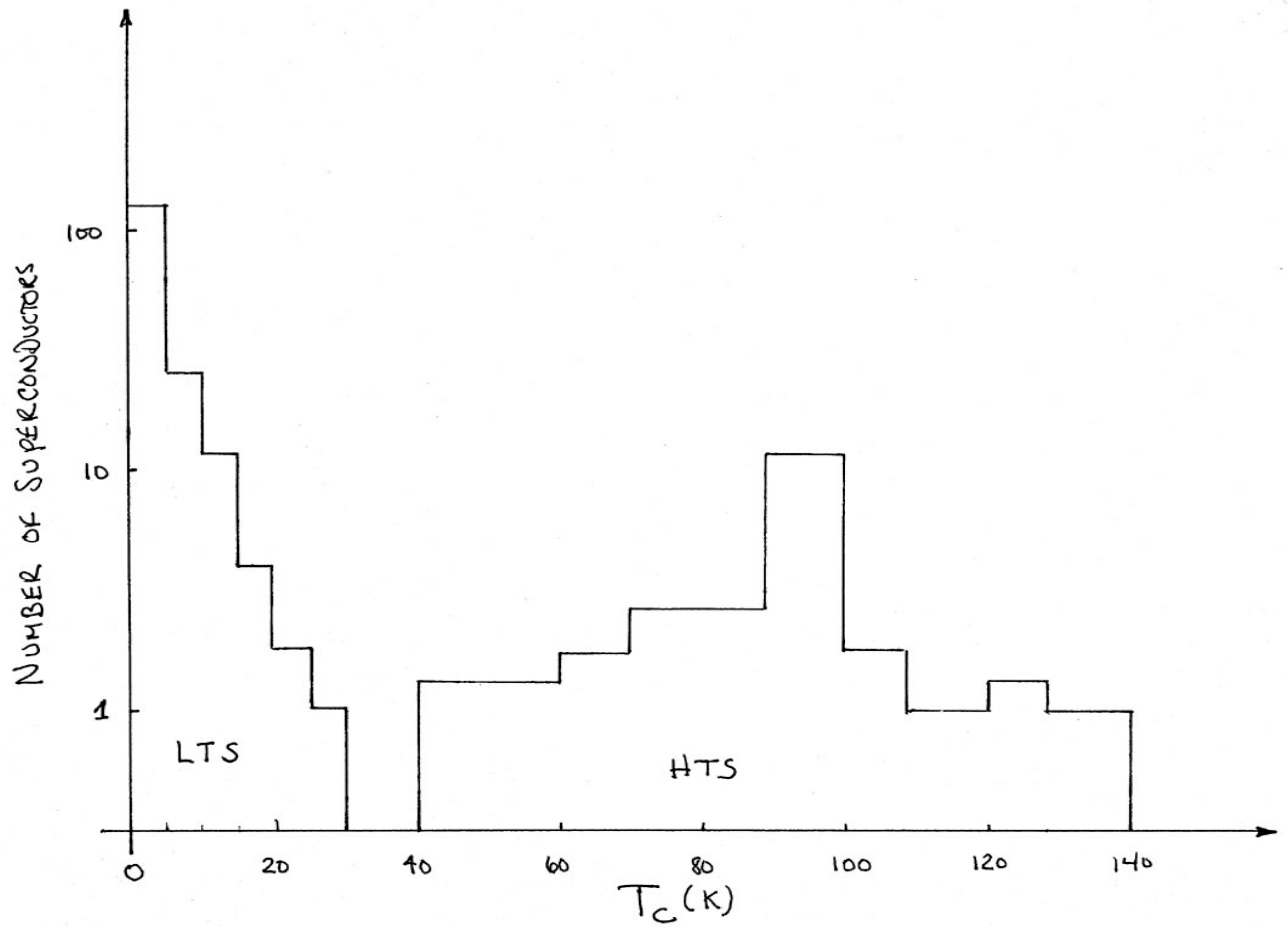
- T_c increases w/ # of constituents
  - T_c (Nb) < $T_c(Nb_3Sn)$ < $T_c(YBa_2Cu_3O_y)$<$T_c(HgBa_2Ca_2Cu_3O_y)$
- T_c is maximum with odd Z compounds
  - (Mattias' Rule)
- T_c function of crystal structure
  - α - U: T_c = 0.68 K; β - U: T_c = 1.8 K
  - A-15 compounds, HTS
  - structural anisotropy seems to result in higher T_c
- for a given stoichiometry, T_c depends on isotopic mass

Record T_c versus time



Ref: http://hoffman.physics.harvard.edu/materials/SCintro.php

Number of Superconductors vs T_c

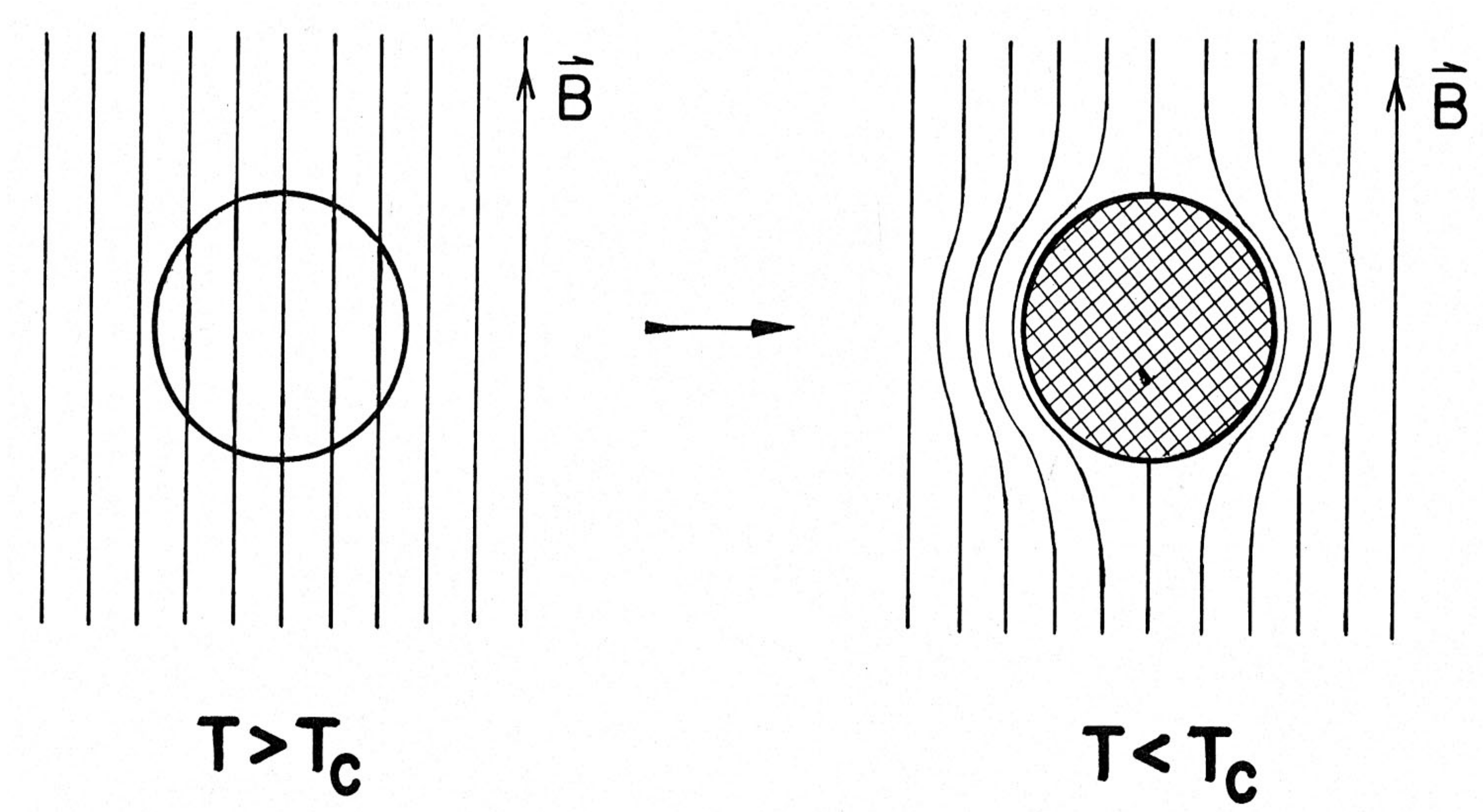


About ½ of all elements and 100s of alloys are superconductors

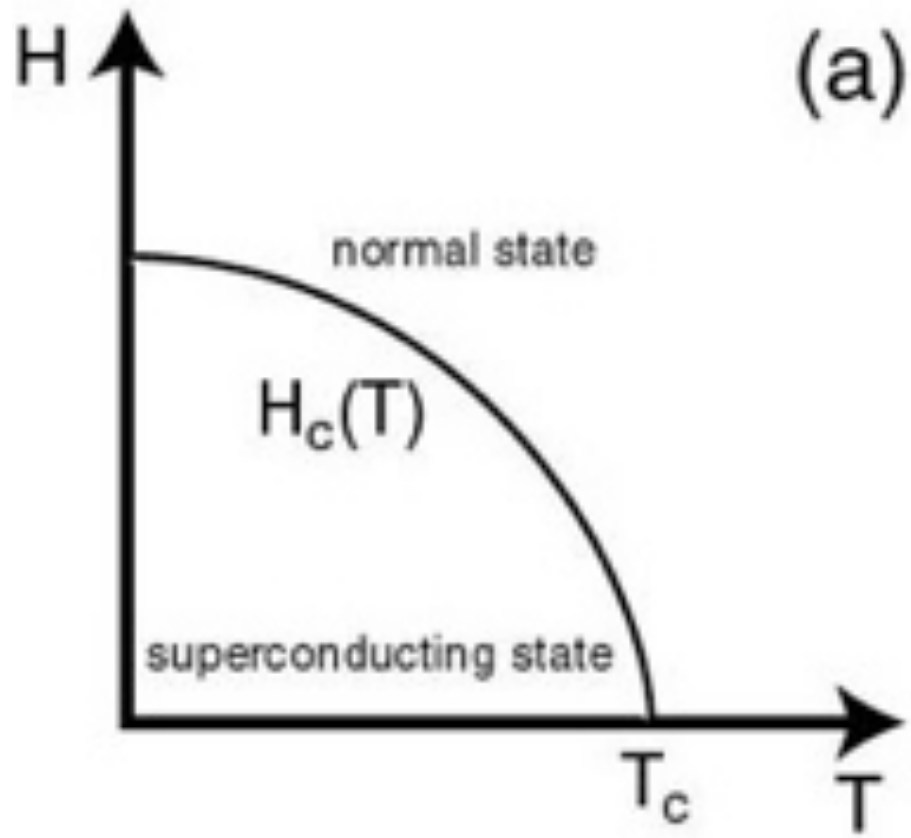
Critical Field (H_c) or Flux Density (B_c)

- As important as T_c for applications
- Two classes: Type I and Type II
- Values at T=0 K
  - W, B_c = 0.12 mT
  - Pb, B_c = 80.3 mT (highest Type I
  - Nb_3Sn, B_{c2} ~ 25 T (Type II)
  - $Bi_2Sr_2Ca_{n-1}Cu_nO_{2n+4}$, B_{c2} ~ 100 T

Perfect diamagnetism (Type I)



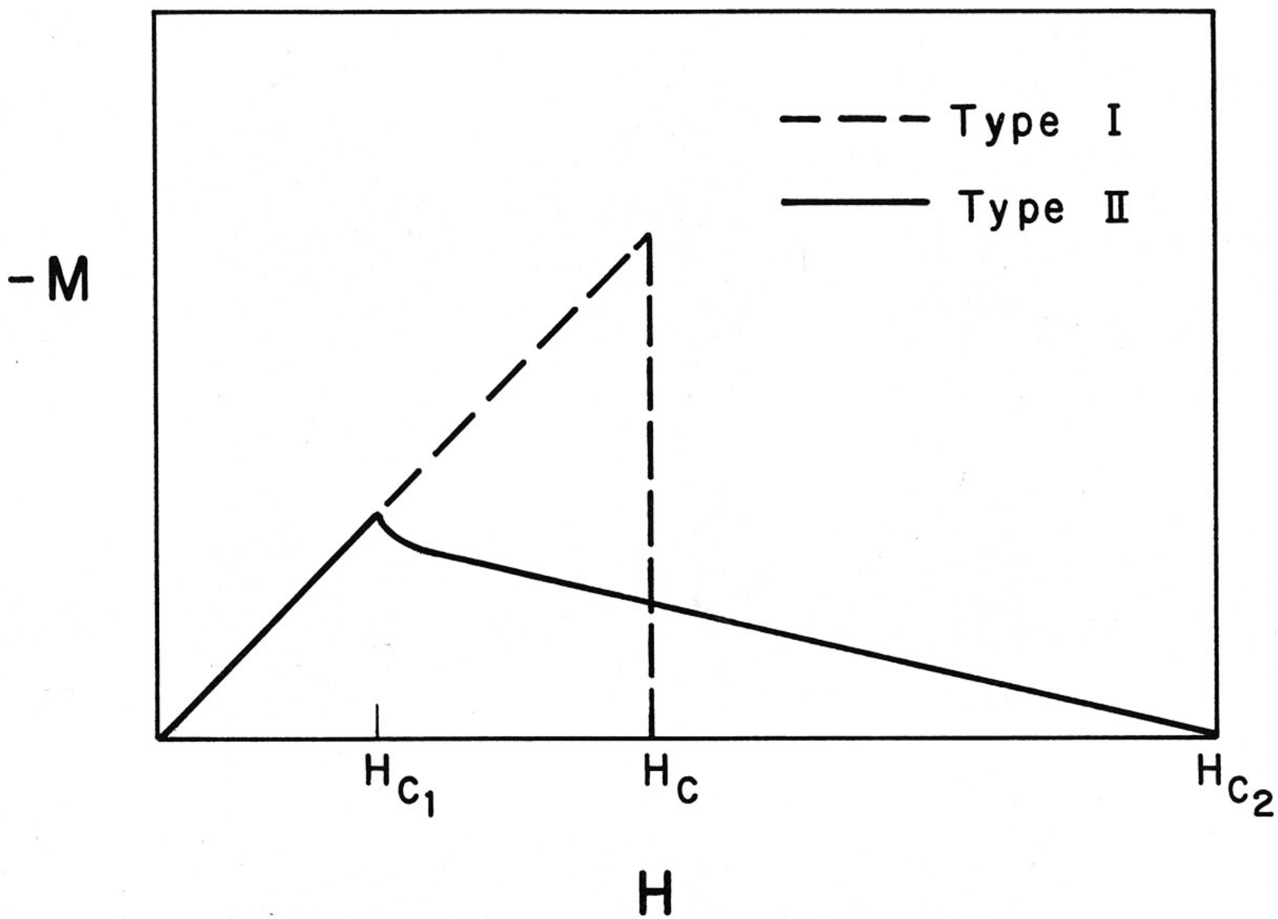
Type I Superconductors $H_c(T)$



Type II Superconductors

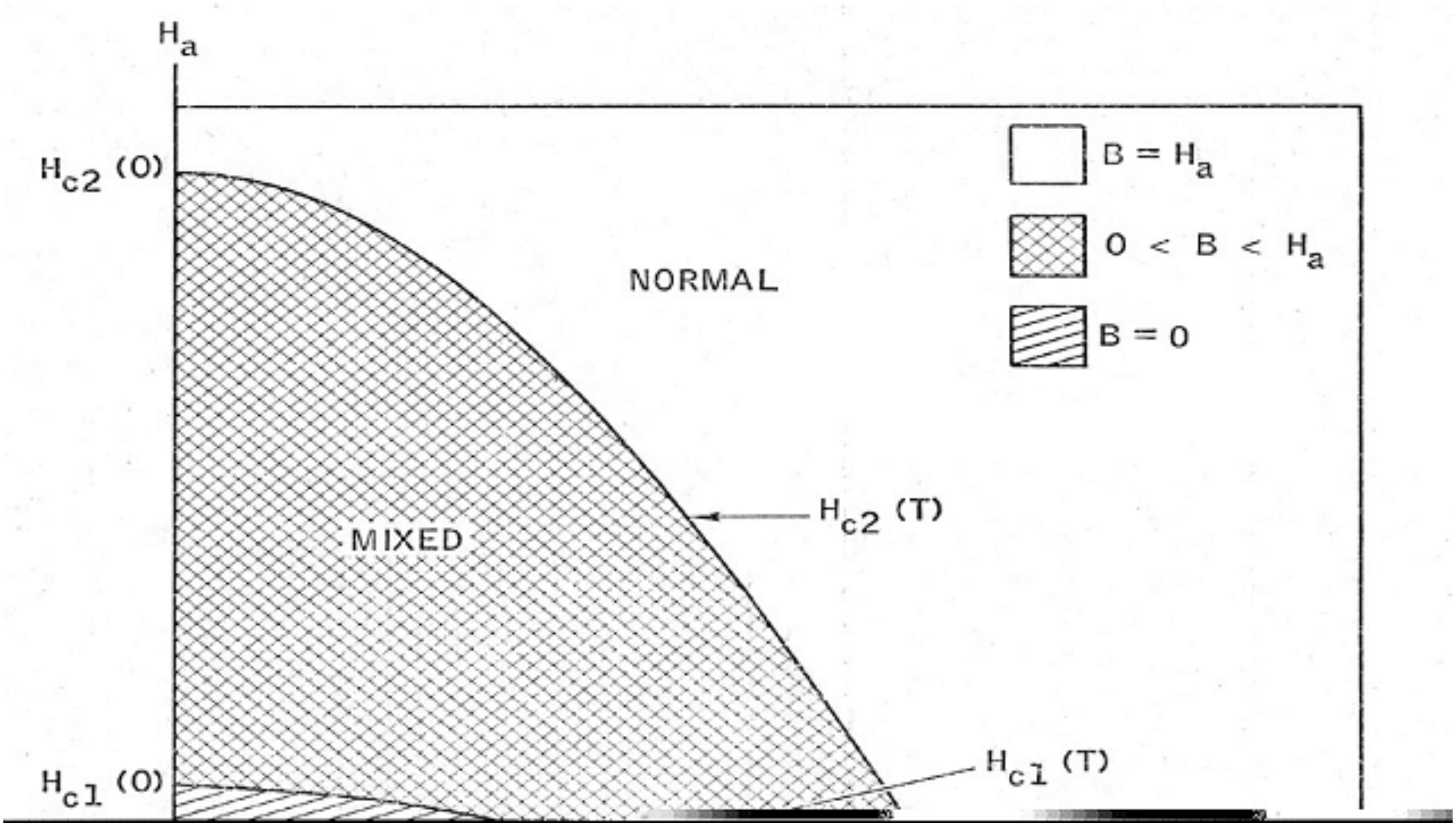
- First discovered in 1960s
  - PbBi alloy not a perfect diamagnet above H_{c1}
- $H_{c2} > H > H_{c1}$ → "mixed state"
  - Magnetic flux within superconductor: $|M| < |H|$
- New class of superconductors for applications

Type I versus Type II



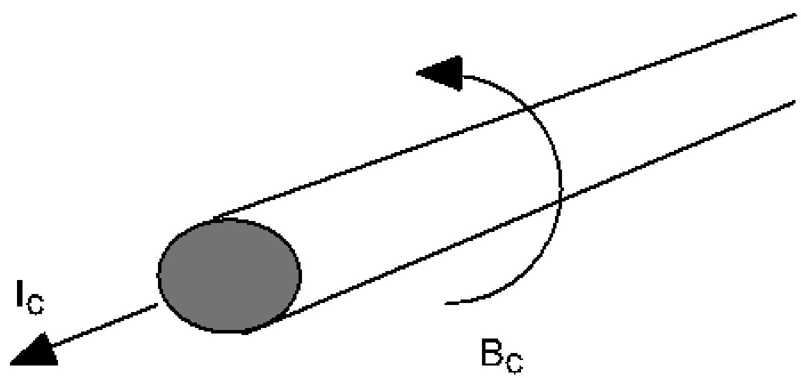
Note: H_{c2} typically >> H_c, H_{c1}

Type II Superconductors H_c vs T



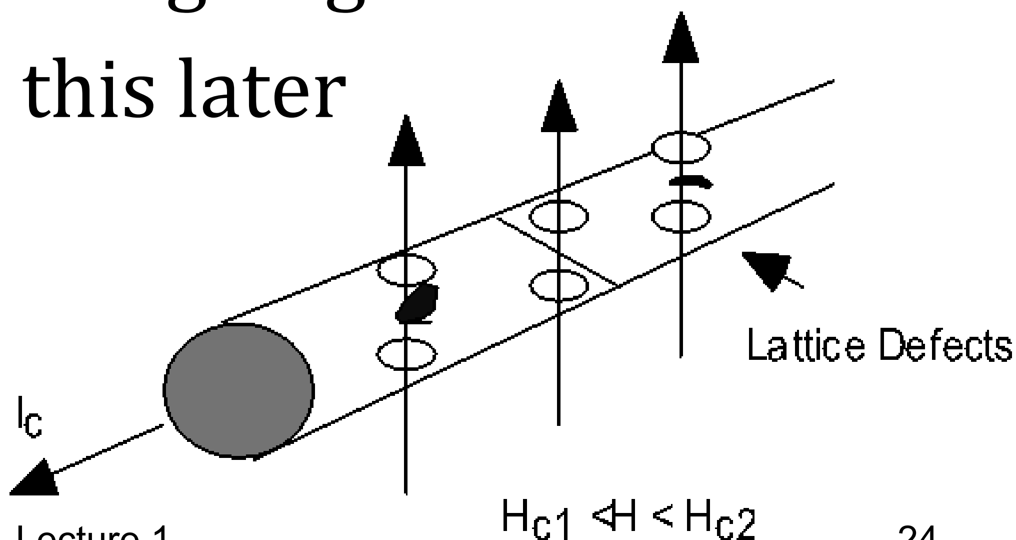
Type I: Critical Current $I_c(B, T)$

- Determined by B_c at the surface of the wire
- $B_c = \mu_o I_c / 2\pi r$
- Limits (eliminates) applications



Type II: Critical Current $I_c(B, T)$

- Depends on microstructure and thus is strongly influenced by processing
- Pinning of flux lines and connectivity between grains
- Main focus of much on-going research
- We'll come back to this later



Review of Basic Electromagnetism
Maxwell's Equations

- Differential Form
- Integral Form

- Gauss' Law: $\vec{\nabla} \bullet \vec{D} = \rho$ $\int \vec{D} \bullet d\vec{s} = Q$
- No Magnetic Monopoles

$$\vec{\nabla} \bullet \vec{B} = 0 \qquad \int \vec{B} \bullet d\vec{s} = 0$$

Review of Basic Electromagnetism
Maxwell's Equations

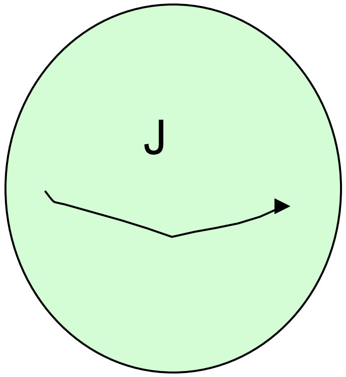
- Differential Form Integral Form
- Ampere's Law: $\vec{\nabla} x \vec{H} = \vec{j} + \frac{\partial \vec{D}}{\partial t}$ $\oint \vec{H} \bullet d\vec{l} = I$
- Faraday's Law: $\vec{\nabla} x \vec{E} = -\frac{\partial \vec{B}}{\partial t}$ $\oint \vec{E} \bullet d\vec{l} = -\frac{\partial \phi}{\partial t}$

Perfect Conductivity Model

- Early (<1930) model for the superconducting state
  - Obeys Maxwell's Equations
  - Ohm's Law (E = ρJ, ρ = 0)
  - PC requires E = 0, but J > 0
- Application of Faraday's Law for E = 0

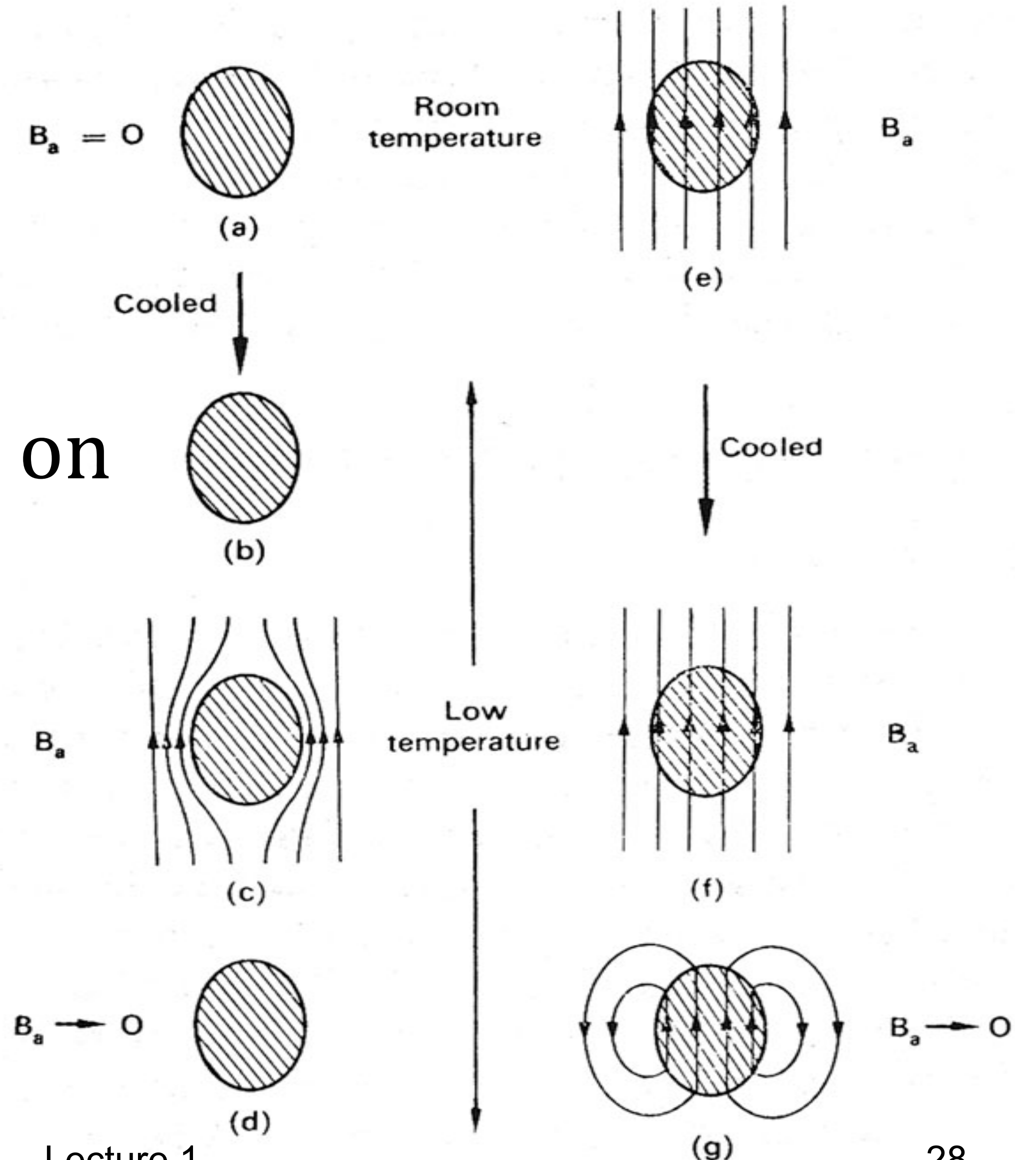
$$\vec{\nabla} x \vec{E} = 0 = -\frac{\partial \vec{B}}{\partial t}$$

  - B = constant and independent of H! How?



What a "perfect conductor" would do

Final "state" depends on sequence

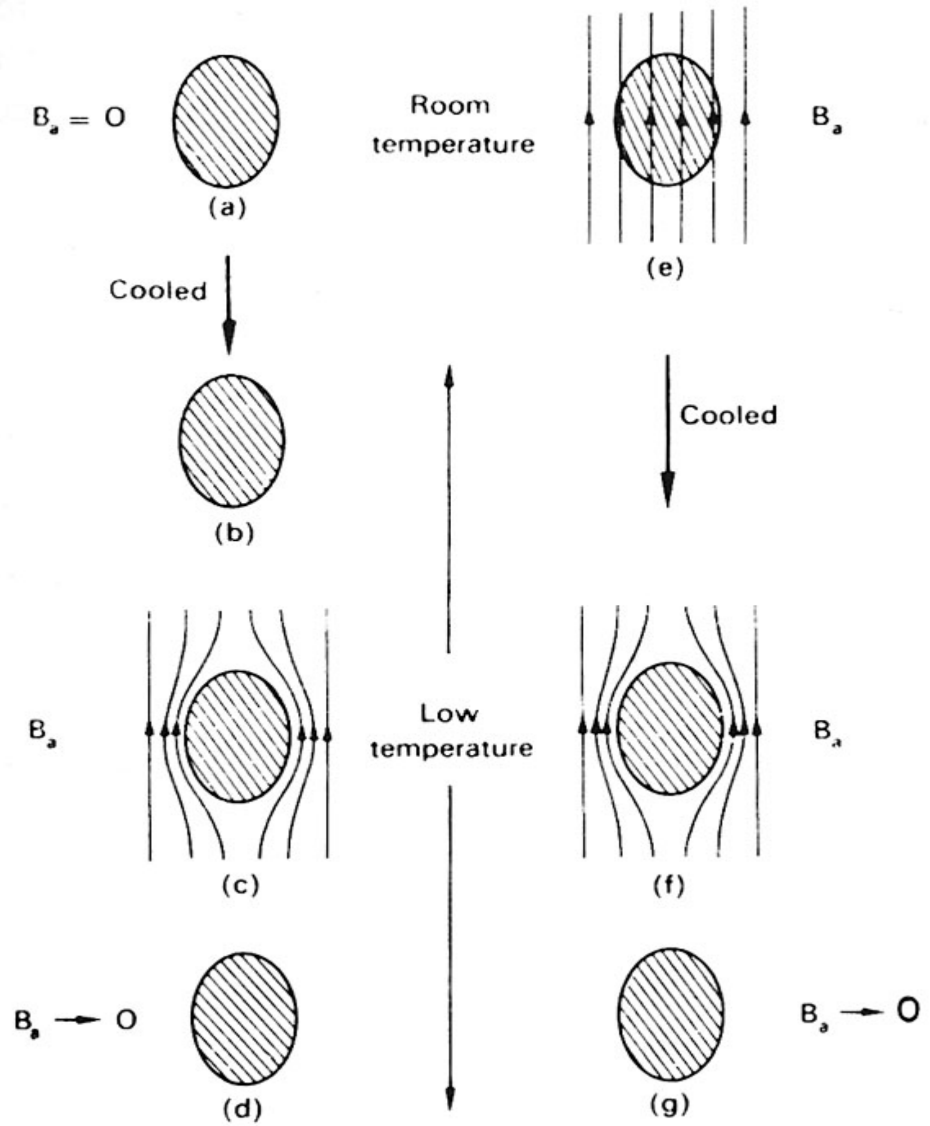


What a superconductor actually does

Meissner Effect

(Meissner & Oshenfield, 1933)

Path-independent



Two-Fluid Model

- Consider a Two-Fluid Model for superconducting electrons
  - n = total number of electrons/volume
  - n_s = number of superconducting electrons/volume
  - n_n = number of normal electrons/volume
  - $n = n_s + n_n$
  - $n_s \sim (1-(T/T_c)^4)$ → so at T_c, n_s → 0
- Then the current density of the superconducting electrons is $$J_s = n_s e v_s$$

Supercurrent density

Start with: $J_s = n_s e v_s$ take d/dt → $\dot{J}_s = n_s e \dot{v}_s$

Use **F**=e**E** = md$\mathbf{v_s}$/dt, take ∇x of both sides and use Faraday's Law ($\vec{\nabla} x \vec{E} = -\dot{\vec{B}}$) to get

$$\vec{\nabla} x \dot{\vec{J}}_s = -\frac{n_s e^2}{m_e} \dot{\vec{B}}$$ First Perfect Conductor Eq.

- Screening current will result from dB/dt

  ρ = 0, so screening current will not decay

Flux Density within Superconductor

- Ampere's Law: $\vec{\nabla} x \vec{B} = \mu_0 \vec{J}_s$
- Take ∇x and d/dt of both sides and substitute using the 1st P.C. equation,

$$\nabla x \nabla x \dot{\vec{B}} = -\frac{\mu_0 n_s e^2}{m_e} \dot{\vec{B}}$$

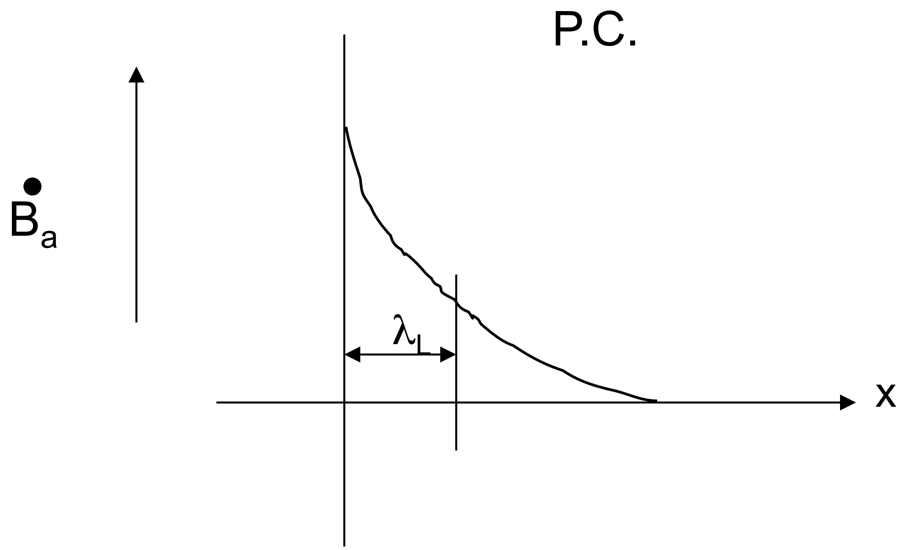
- Calculus identity, $\nabla x \nabla x B = \nabla(\nabla \bullet B) - \nabla^2 B = -\nabla^2 B$

Since $\vec{\nabla} \bullet \vec{B} = 0$

- 2nd P.C Eq:

$$\nabla^2 \dot{\vec{B}} = \frac{\mu_0 n_s e^2}{m} \dot{\vec{B}} = \frac{1}{\lambda_L^2} \dot{\vec{B}}$$

Semi-infinite Perfect Conductor Solution



$$\dot{B} = \dot{B}_a e^{-x/\lambda_L}$$

$$\lambda_L = \left[\frac{m_e}{\mu_0 n_s e^2}\right]^{1/2}$$

One-dimensional solution to the 2nd PC equation

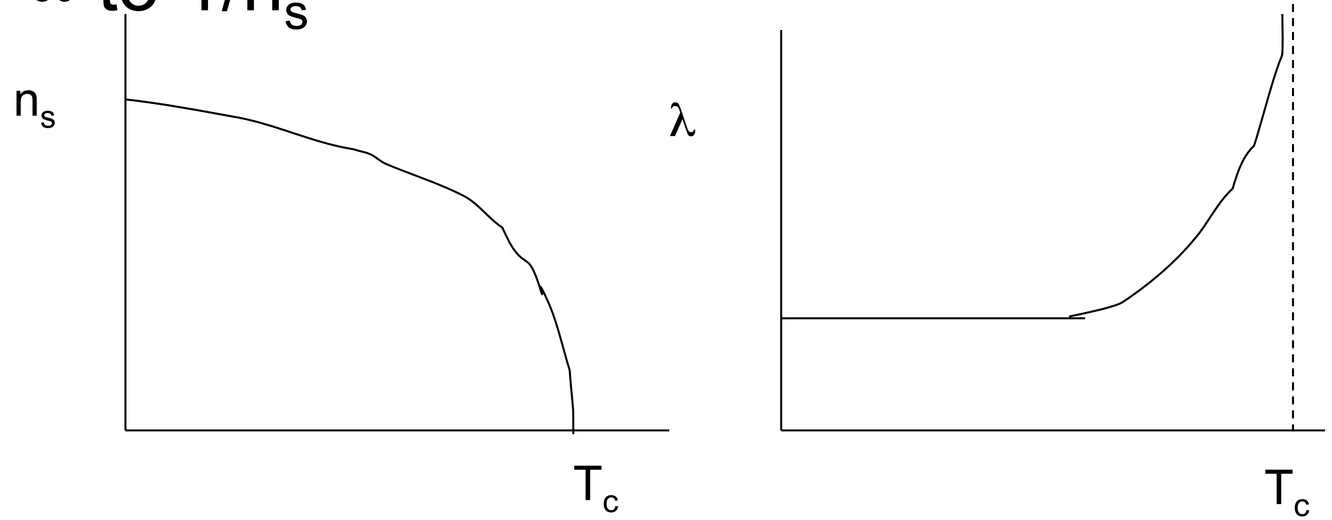
Current and magnetic field are in a surface layer of characteristic length scale called the PENETRATION DEPTH λ_L

Thin, but non-zero, why?? → if zero, J must be infinite

On the order of 10-100 nm for pure metals

Temperature Dependence of λ

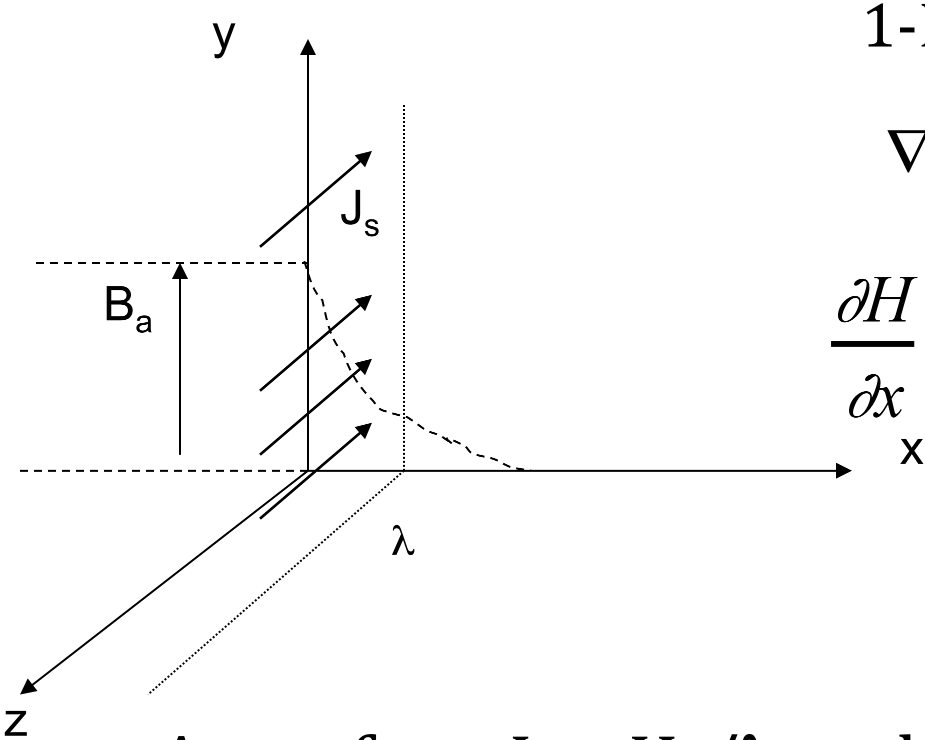
λ is characteristic of the superconducting state ∝ to $1/n_s^{1/2}$



$$\frac{n_s}{n} \approx 1 - \left(\frac{T}{T_c}\right)^4$$

Surface Current Density

- Current flows on surface



1-D Maxwell's Equation

$$\nabla x H = \frac{\partial H}{\partial x}\hat{z} = J_s \hat{z}$$

$$\frac{\partial H}{\partial x} = \frac{1}{\mu_o}\frac{\partial B}{\partial x} = -\frac{B_a}{\mu_0 \lambda_L} e^{-x/\lambda_L}\hat{z}$$

At surface, $J_s = H_a/\lambda_L$, which is max when $H_a = H_c$

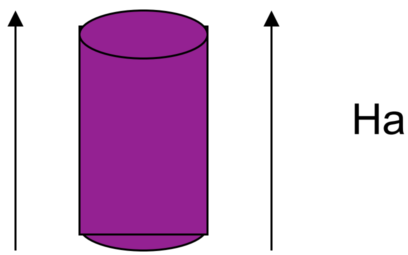
Note that J_s is constant

Thermodynamics of Type I S/C

- Superconducting transition is reversible
  (thermodynamic system)
- Gorter (1933) predicted the Meissner Effect by showing the S/C transition is reversible
- Thermodynamics describes the transition and the state properties.
- **No assumptions** about "superconducting state"
- **No contribution** to understanding the **transport properties**

Thermodynamics of Magnetic Systems

- Cylinder in parallel magnetic field (M > 0)



- Gibbs Free Energy (with magnetic term) for system with external variables mostly constant

$$G = U - TS + pV - \mu_0 MH$$

- Differentiating with T at fixed p, H:

$$dG = dU - TdS - SdT + pdV - \mu_0 HdM$$

- and applying the First Law of Thermodynamics:

$$dU = TdS - pdV + \mu_0 HdM$$

one finds: $dG = - SdT$

Differential form of Gibbs Energy

- If instead one assumes constant T & p but not constant H,

$$dG = -SdT + Vdp - \mu_0 MdH = -\mu_0 MdH$$

- Now consider a Type I superconductor: perfect diamagnetism ⟹ M = -H

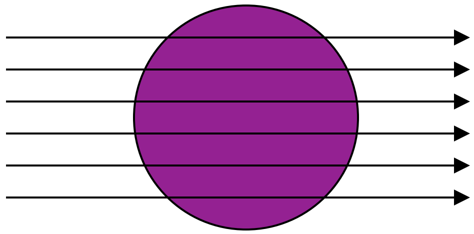
$$dG_s = \mu_0 HdH$$

- For a finite change in applied field ($0 \rightarrow H_a$) at constant T and p, integrate

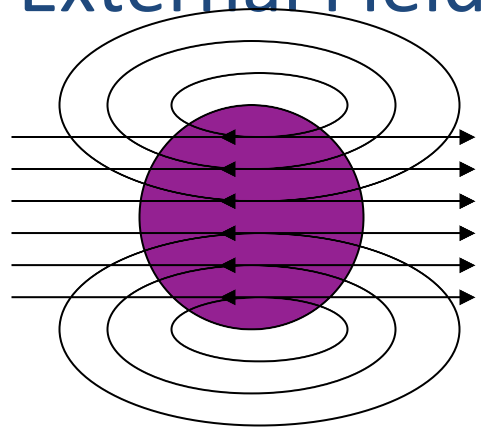
$$G_s(H_a,T) - G_s(0,T) = 1/2\ \mu_0 H_a^2 \text{ for } H_a < H_c$$

(magnetic energy density)

Free Energy of Sphere in External Field



Normal
(non-magnetic)



Superconducting
diamagnetic

- Increase in energy associated with field needed to oppose applied field and preserve diamagnetic state
- Superconducting transition is reversible ➡ Gibbs free energy must be continuous across the transition
- At constant T, $H_a \approx H_c$
- $G_s\ (H_c, T) = G_n\ (H_c, T)$