NC STATE UNIVERSITY

Applied (High Temperature) Superconductivity

Academic Training
Lecture 2

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SCHWARTZ GROUP

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Outline for the week of training

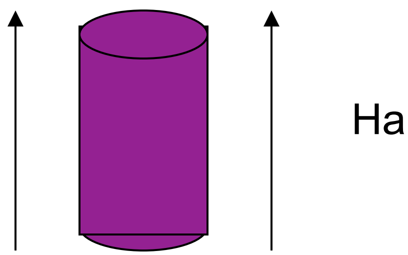
- Lectures 1& 2: Introduction & "Just enough" physics
  - Brief introduction … what is superconductivity and why is it useful?
  - Basic physics of superconductivity and the superconducting state
  - Applications-relevant physics of superconductivity & superconducting state
- Lecture 3: Technical superconductors
  - A brief summary of NbTi and Nb_3Sn
  - HTS conductor options: Bi2212 & YBCO
- Lecture 4: Electromechanical behavior
  - A brief summary of NbTi and Nb_3Sn
  - HTS conductor options: Bi2212 & YBCO
- Lecture 5: Quench behavior and high field magnets
  - What is quench protection?
  - Quench protection in HTS magnets

Thermodynamics of Type I S/C

- Superconducting transition is reversible (thermodynamic system)
- Gorter (1933) predicted the Meissner Effect by showing the S/C transition is reversible
- Thermodynamics describes the transition and the state properties.
- **No assumptions** about "superconducting state"
- **No contribution** to understanding the **transport properties**

Thermodynamics of Magnetic Systems

- Cylinder in parallel magnetic field (M > 0)



- Gibbs Free Energy (with magnetic term) for system with external variables mostly constant

$$G = U - TS + pV - \mu_0 MH$$

- Differentiating with T at fixed p, H:

$$dG = dU - TdS - SdT + pdV - \mu_0 HdM$$

- and applying the First Law of Thermodynamics:

$$dU = TdS - pdV + \mu_0 HdM$$

one finds: $dG = - SdT$

Differential form of Gibbs Energy

- If instead one assumes constant T & p but not constant H,

$$dG = -SdT + Vdp - \mu_0 MdH = -\mu_0 MdH$$

- Now consider a Type I superconductor: perfect diamagnetism $\Longrightarrow$ M = -H

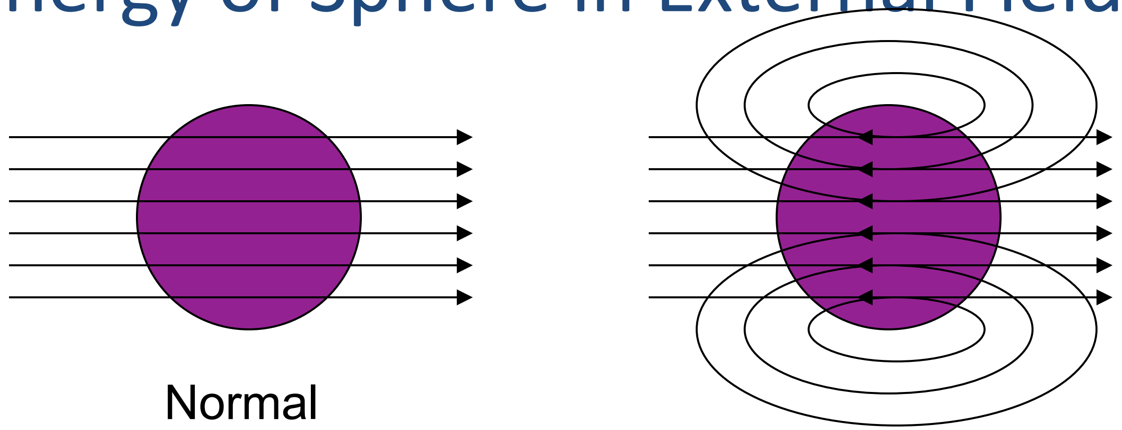
$$dG_s = \mu_0 HdH$$

- For a finite change in applied field ($0 \rightarrow H_a$) at constant T and p, integrate

$$G_s(H_a,T) - G_s(0,T) = 1/2\ \mu_0 H_a^2 \text{ for } H_a < H_c$$

(magnetic energy density)

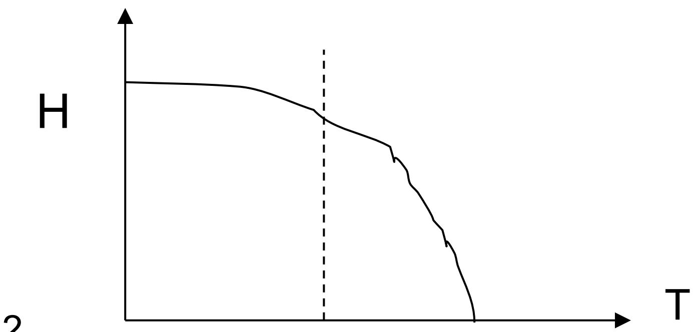
Free Energy of Sphere in External Field



Normal
(non-magnetic)

Superconducting
diamagnetic

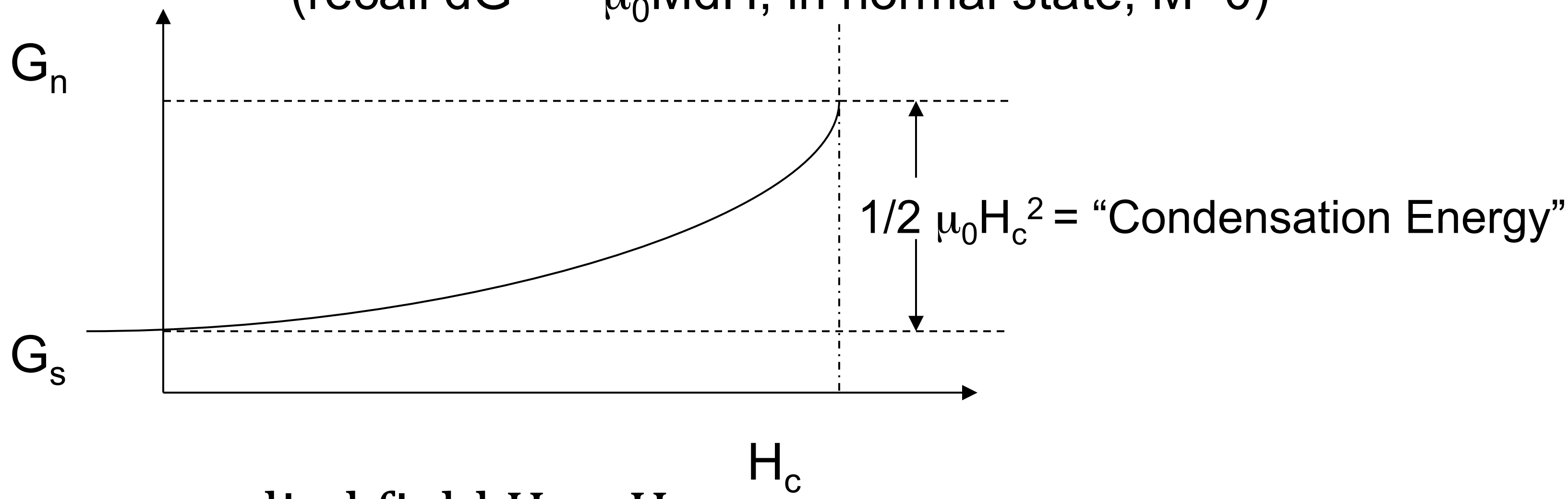
- Increase in energy associated with field needed to oppose applied field and preserve diamagnetic state
- Superconducting transition is reversible ⇨ Gibbs free energy must be continuous across the transition
- At constant T, $H_a \approx H_c$
- $G_s\ (H_c, T) = G_n\ (H_c, T)$



Gibbs free energy difference

$$G_s\,(0,\,T) + 1/2\ \mu_0 H_c^{\,2} = G_n\,(0,T) = G_n(H_a,T)$$

(recall $dG = -\mu_0 M dH$; in normal state, M=0)



For any applied field $H_a < H_c$:

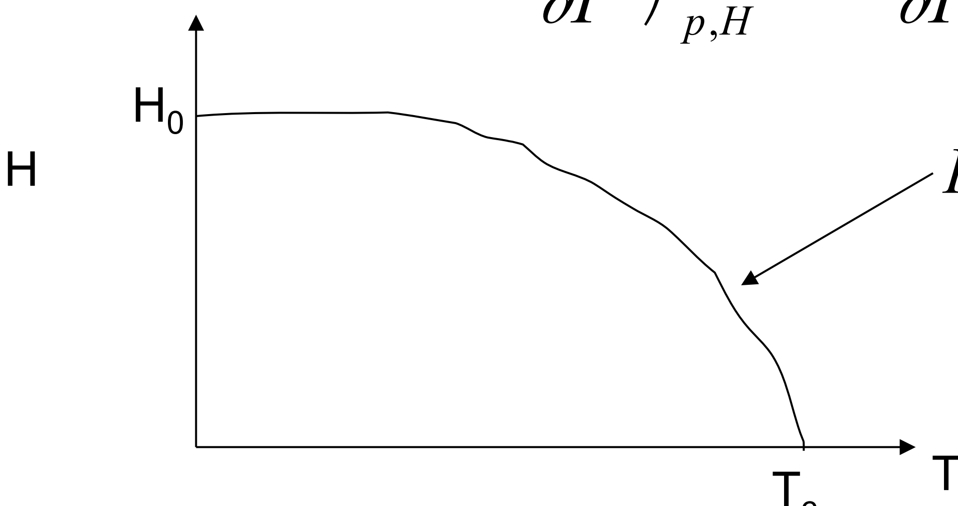
$$G_s(H_a,T) = G_n(H_a,T) - 1/2\ \mu_0\,(H_c^{\,2} - H_a^{\,2})$$

Which can be used to determine thermodynamic properties
Note that there is ***less*** free energy in the superconducting state than in the normal state

Thermodynamic properties: Entropy

- S = $-\left.\frac{\partial G}{\partial T}\right)_{p,H}$ holding external variables constant
- We are only interested in the change in the properties **across the transition,**

$$\Delta s = s_n - s_s = -\left.\frac{\partial \Delta G}{\partial T}\right)_{p,H} = -\frac{\partial}{\partial T}\left(\frac{1}{2}\mu_0 H_c^2\right) = -\mu_0 H_c \frac{\partial H_c}{\partial T}$$



$$H_c(T) = H_0\left(1 - (T/T_c)^2\right)$$

$$\frac{\partial H_c}{\partial T} < 0$$

for all T

note, S/C state is more ordered

Macroscopic theory

- Theory does not explain the interaction that brings about SC
- Based on phenomenology
  - i.e. no microscopic interactions
- Able to explain certain experimental observations

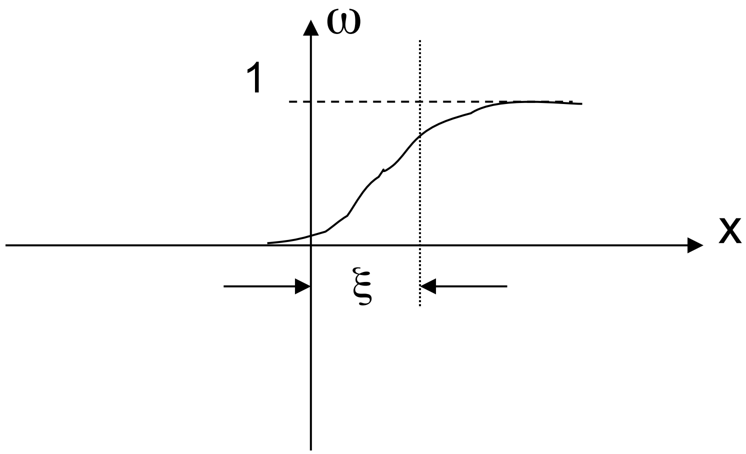
    $\lambda_L \neq \lambda_{exp}$ for many known superconductors
  - existence of Type II superconductivity w/ mixed state
- Based on quantum mechanical theory with SC state being "ordered"

Two fluid model & Ginzburg-Landau Theory (macroscopic theories)

- First proposed by Gorter and Cashmir (1935) as analogue to two-fluid model for liquid helium
- Hypothesis: a superconductor below T_c contains two types of electrons
  - Normal electrons: n_n - number density; J_n (normal e^- current density) = $n_n e v_n$
  - Superconducting electrons: n_s - number density; J_s (SC e^- current density) = $n_s e v_s$
  - Conservation law: $n = n_s + n_n$ & $J = J_s + J_n$
- Order parameter (ω) *defines* the relative fraction in the condensed state, so more in the condensed state means higher order parameter, i.e., more order

Order parameter

- Smoothly varying from ω = 0 @ T_c to ω = 1 @ 0 K
- Spatial variation at SC boundary, conversion from the normal to SC state



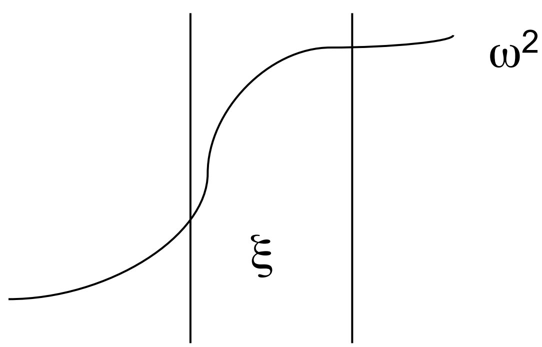
- ξ is the characteristic length for variation of ω
- **"coherence length"**

- Number densities are functions of the order parameter
  - $n_s = f(\omega)$ & $n_n = f(1-\omega)$ - unknown functions

Results from Macroscopic Theories

- Minimize the free energy with respect to the order parameter

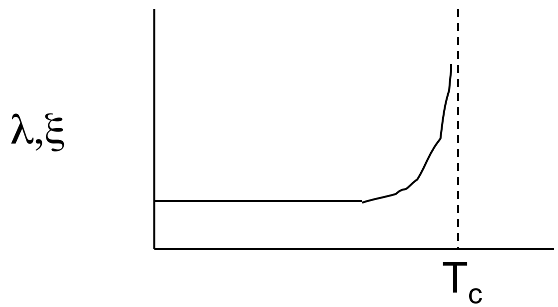
$$\Rightarrow \quad \nabla^2 f = \frac{1}{\xi^2} f \qquad \text{where} \quad \xi^2 = -\frac{\hbar^2}{2m\alpha} \qquad \text{“1st GL equation”}$$



Can’t change the SC state any more rapidly than in length ξ

Ginzburg-Landau Parameter

- Penetration depth (λ) and coherence length (ξ) diverge near T_c



$$\lambda = \lambda_0(1-(T/T_c)^4)^{-1/2}$$

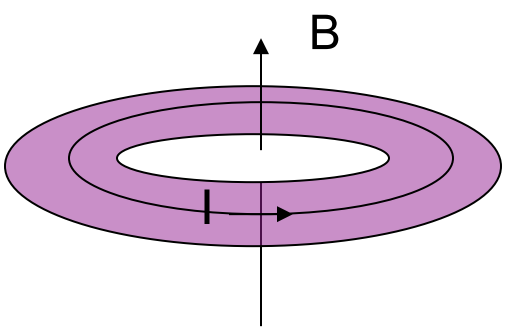
- Ratio of two is ~ constant with respect to temperature (varies from SC to SC)

$$\kappa = \lambda/\xi = \lambda_0/\xi_0 \text{ ... Ginzburg-Landau Parameter}$$

- **κ determines the type of superconductor**
  - Type I for $\kappa < 1/\sqrt{2}$ (small λ, large ξ)
  - Type II for $\kappa > 1/\sqrt{2}$ (large λ, small ξ) ... but what is it?

Microscopic Theory Results … "BCS Theory"

- Carriers in superconducting state are "paired electrons"
- Flux is quantized
- Superconductor with hole in it



$$\Phi = \int B \cdot dA$$

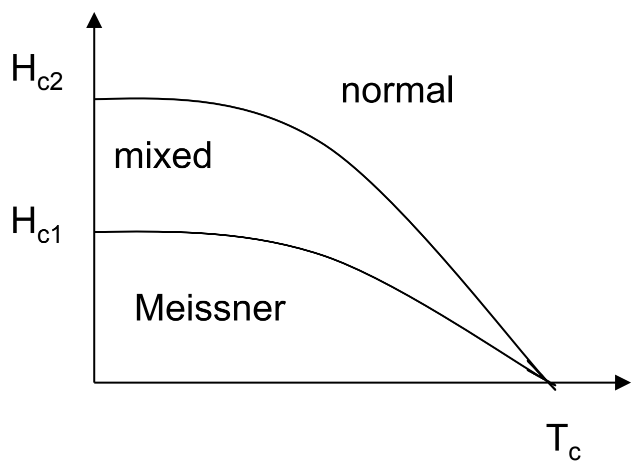
- Flux through hole is quantized in units of h/2e
  - Flux quantum Φ_0= h/2e = 2.07 x 10^{-15} Webers

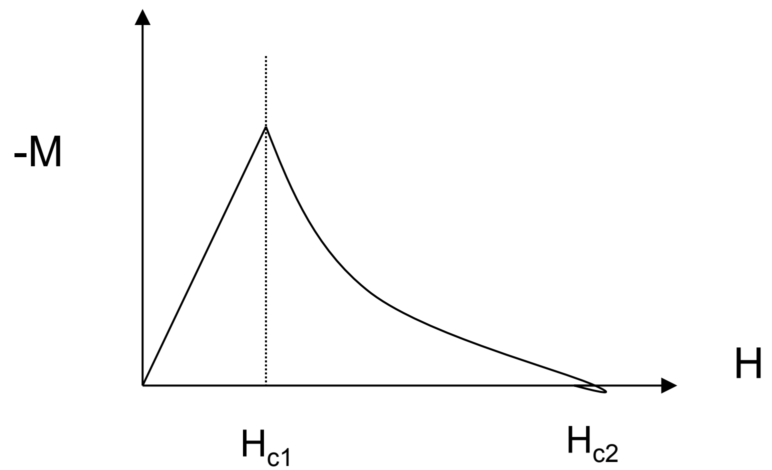
Type II superconductivity

- First observed by DeHaas and Voogd (1930)
  - PbBi has T_c = 8.8 K and H_c = 1.7 T (not fully diamagnetic)
  - Note: PbBi is an ALLOY
- Observed by Bell Labs in Nb_3Sn system (1961)
  - Beginning of applications (magnets)
- All "technical" superconductors are Type II (high H_c)
  - NbTi alloys, Nb_3Sn (and other A-15 compounds), HTS, MgB_2

Mixed state?

- The "mixed state" has partial flux penetration, two critical fields, and partial diamagnetism

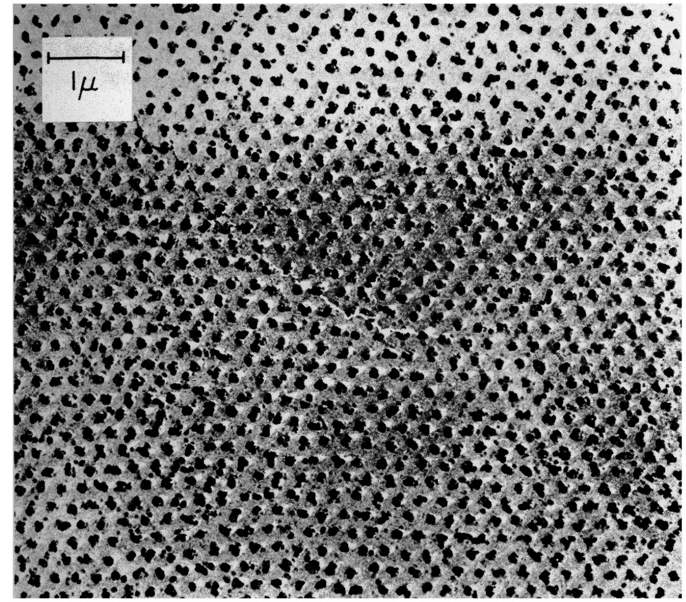


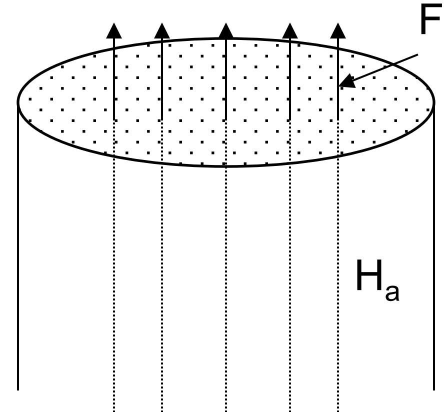


Fields not to typical scale

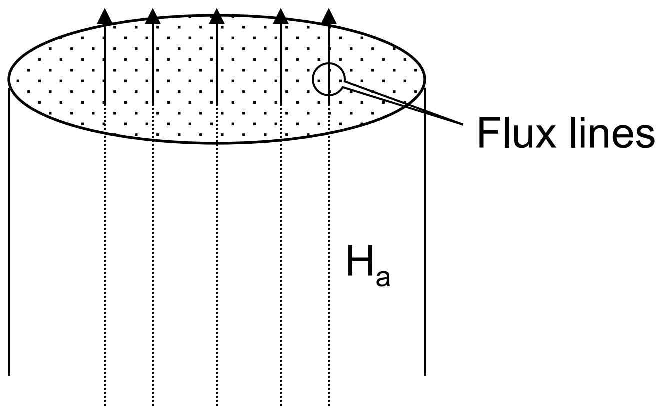
Mixed state in Type II superconductors

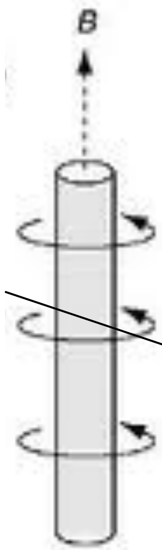
- Experimental observation
  - Magnetic particles on SC
  - Triangular pattern of "normal regions" w/in SC regions (islands within the superconducting background)
  - Number of lines α B_a
  - $\xi < \lambda$
- One individual "island" is called a vortex

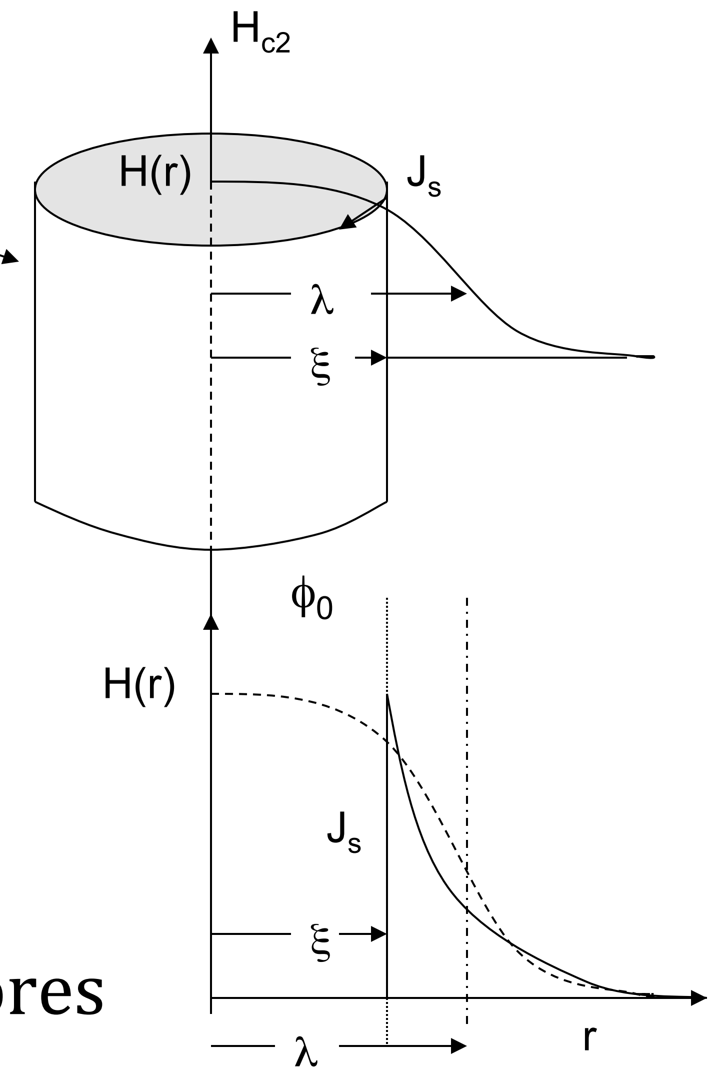




Picture of a Vortex Core







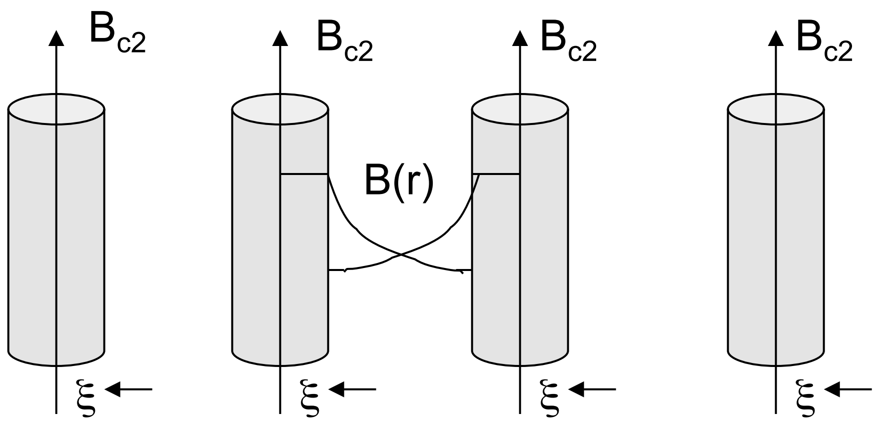
- Within the normal core H = H_{c2}
- Core contains 1 flux quantum

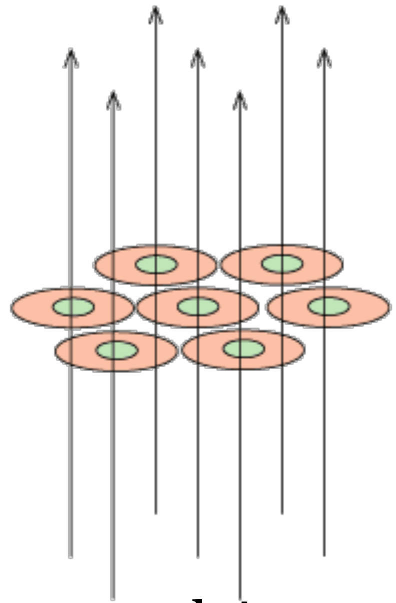
$$\int B \cdot da = \mu_0 H_{c2} \cdot \pi\xi^2 = \phi_0$$

$\longrightarrow$ $H_{c2} \propto 1/\xi^2$

Each core has magnetic moment; cores are mutually repulsive

Representation of the mixed state



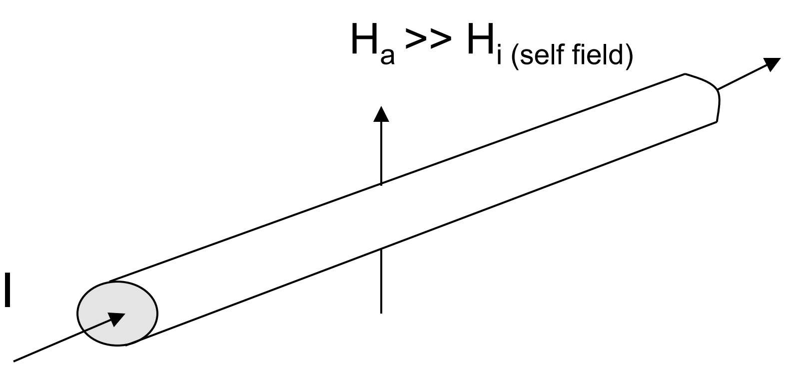


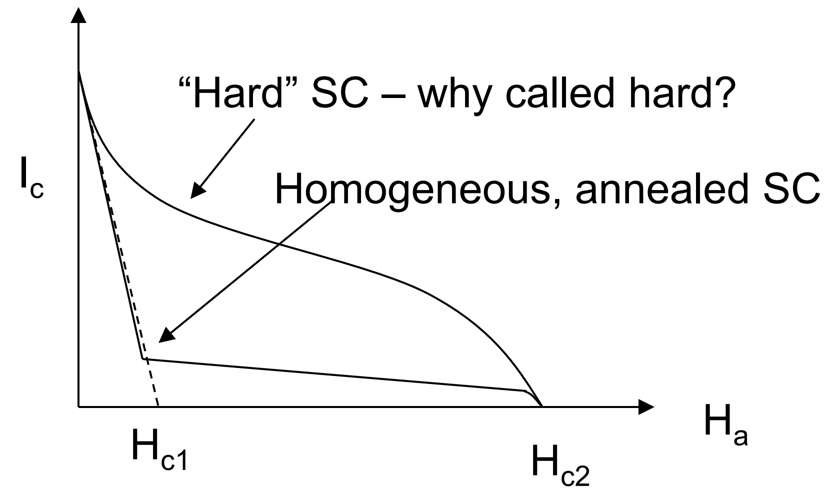
- Magnetic flux overlaps between normal cores → net repulsion (or JxB)
- Increasing field → increasing number of cores/area
- Minimum energy configuration is a triangular array (force balance on each core)
- Cores are created at the surface and migrate inward with increasing field
- H_{c2} occurs when cores overlap, i.e., no room for more vortices!

What does this have to do with magnets?

- Magnets rely on transport current, not circulating shielding currents
- But transport current generates magnetic field which results in vortices
- So transport and shielding currents must co-exist

Transport Current in Type II Superconductors

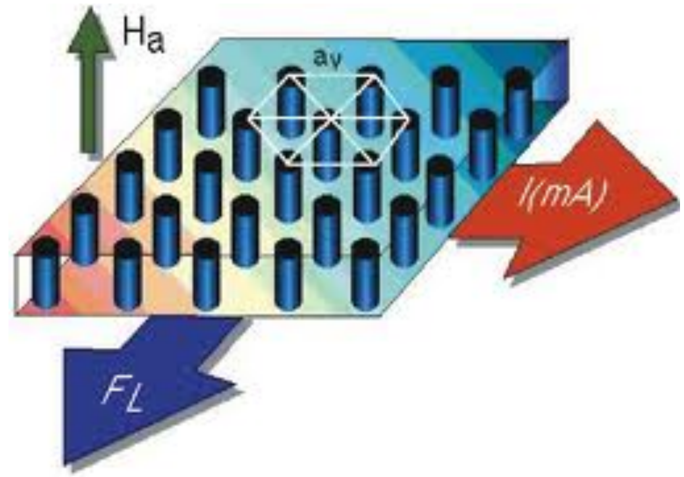




Defects & impurities **improve** I_c in mixed state

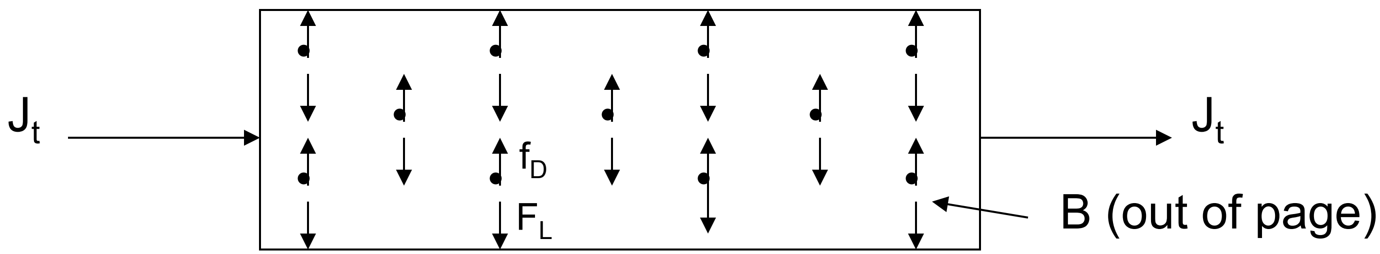
- I_c (or J_c) is NOT a phase transition like T_c and H_c (and I_c in Type-I superconductors) → *not* thermodynamically driven
- I_c is related to interaction between vortices and current; dominated by microstructural impurities and defects
- In HTS, the anisotropic crystal structure and small coherence length results in additional limits/issues

Current-Flux Line Interaction



- Consider Type II SC in the mixed state
  - N = # of vortex lines/area (function of H_a)
  - J_t = transport current density (I_t/A_{sc})
- We show I flowing throughout the superconductor (approximation for Type II)
- J_s(screening current density) is >> J_t so J_t is a small perturbation on J_s
- Lorentz force/volume on vortex lines: $F_L = J_t x$ B which is $\perp$ to J and B; pushes vortices to the edge
- The fundamental laws of physics do not change!
  - F_L acts on vortices and they can and will move

Flux Line Motion



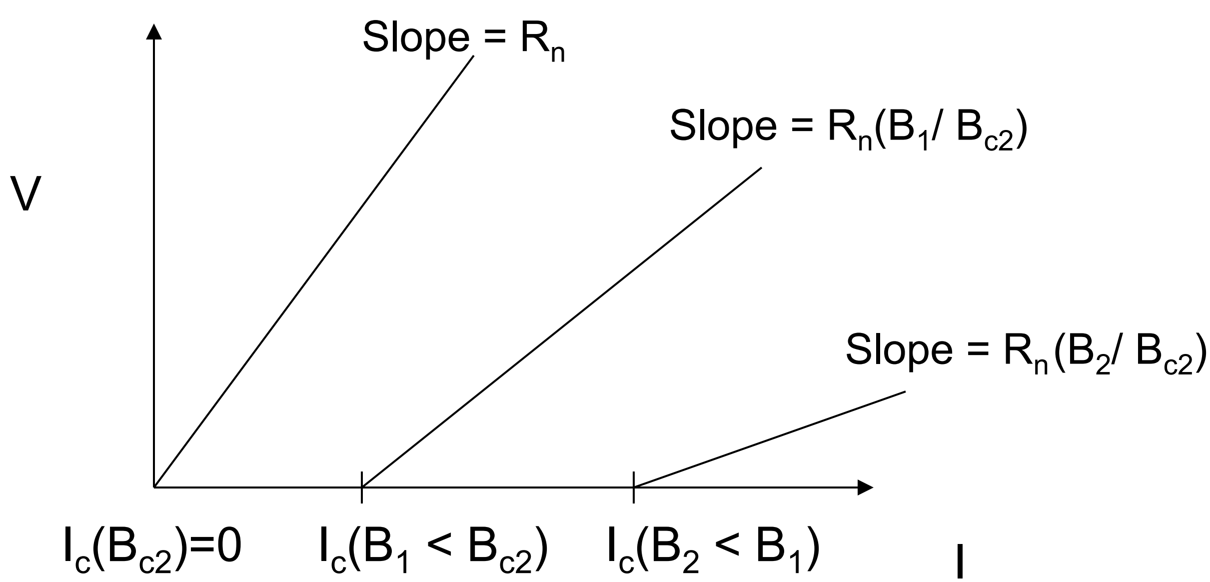
- Lorentz force drives vortex lines through bulk (v_{perp})
- Array moves as whole
  - Assuming J_t & B ~ uniform
  - Due to mutual interactions between vortices
- This motion is called FLUX FLOW
- Vortices are created/annihilated at boundaries
- v_{perp} does not →infinity; must be a drag force too
- Motion in homogeneous Type II SC is like viscous flow with effective viscosity $\eta = F_L/v_{perp}$
- Average flux density $B = N\phi_0$ (flux line contains one quantum)
- Total force on individual flux line: $F_L = J_t\phi_0 = \eta v_{perp}$ which can give a value for v_{perp}

Implications of Flux Flow

- From a static reference from in the superconductor, flux flow is a time-varying magnetic field
- Recall: $$\vec{\nabla} x \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$
- So time-varying magnetic field results in a non-zero, spatially-varying electric field (i.e., a voltage)
- Thus, a transport current results in a voltage and the superconductor does NOT have lossless current flow!!

Flux Flow Resistivity

- Flux flow electric field: $E_{ff} = v_{perp}\, B = J_t \phi_0 B/\eta$
- Implies non-zero E_{ff} for all nonzero J_t
- Effective resistivity: $\rho_{ff} = E_{ff}/J_t = \phi_0 B/\eta$
- At B_{c2}, $\rho_{ff} = \rho_n$ (normal state resistivity) therefore $\rho_n = \phi_0 B_{c2}/\eta$ and $\rho_{ff} = \rho_n(B/B_{c2})$



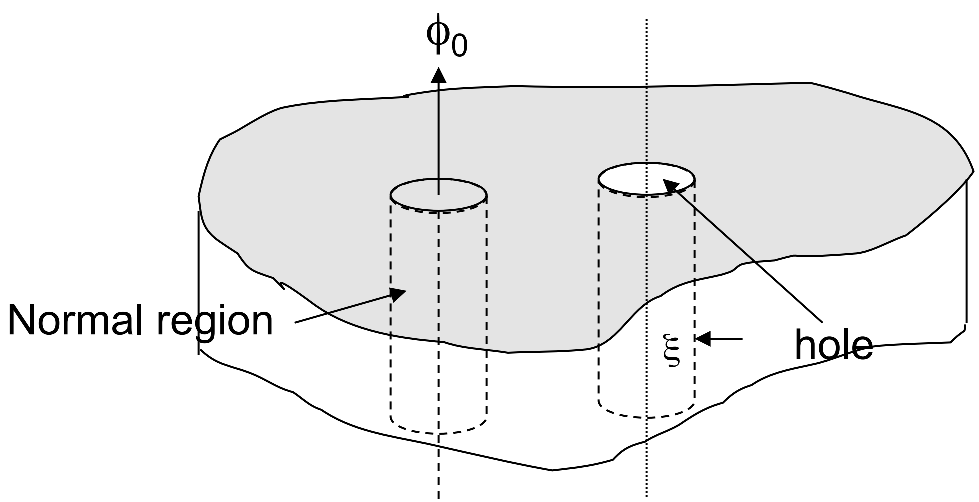
Power dissipation
$P = J_t^2\rho = J_t^2\rho\,(B/B_{c2})$

Flux Pinning

- Flux flow resistance can lead to large power dissipation since normal state resistance of superconductor is high
- To reduce FF resistance, we need a method to prevent lines from moving: "pinning sites" to hold lines in place
- Flux line size is ~ ξ (nm scale) so sites are small
- Types of pinning sites:
  - Grain boundaries
  - Crystal imperfections
  - Non (or weak) superconducting impurities
  - Irradiation damage tracks
  - Intrinsic crystal structure
- "Hard" superconductors have many imperfections & impurities which lead to good pinning
- Detailed microstructure can be optimized for pinning and
- Recall that ξ decreases as H_{c2} and T_c increase so defects must be smaller for HTS than for LTS

Flux Pinning Theory

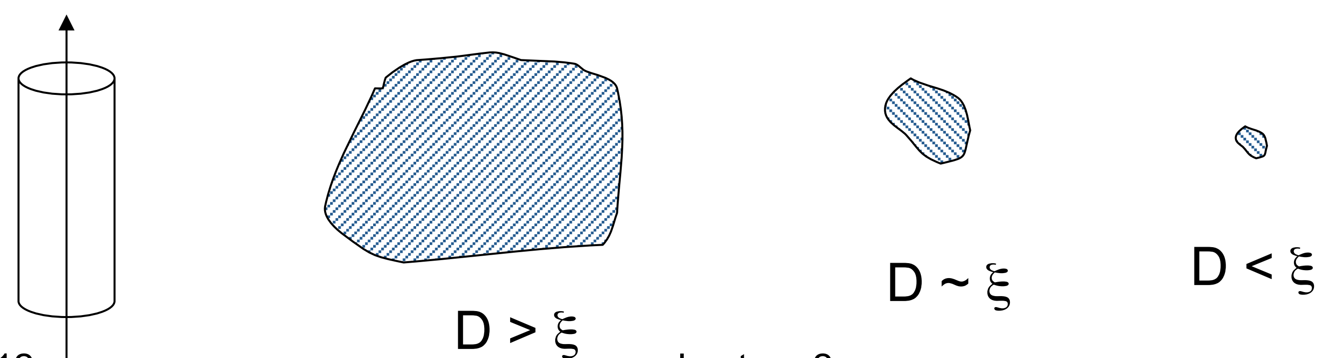
- Ideal pinning site is a normal region or hole of radius ξ
- If normal region exists (defect) then the superconductor doesn't need to create one to accommodate vortices – defect is, in general, energetically favorable location for vortex



Energy/length = $1/2\mu_0 H_c^2 \pi \xi^2$
$\sim 10^{-11}$ J/m

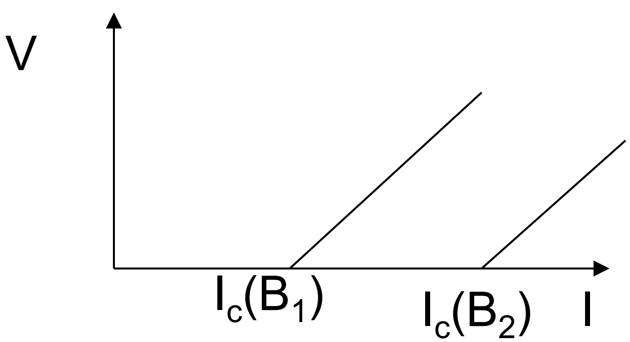
Flux Pinning Theory

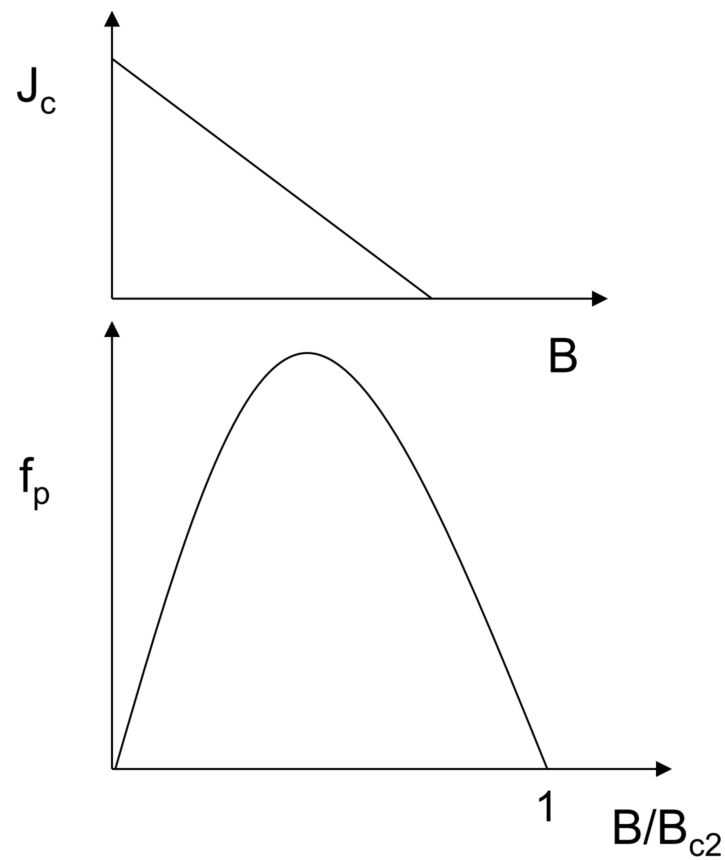
- “Artificial pinning centers” approximate ideal pinning sites while normal precipitates only pin portion of flux line
- Pinning centers that are too large can still pin vortex, but they also “waste space” and thus reduce critical current by reducing the area available for current flow → optimization required
- If pinning center is even larger relative to ξ, then J_c -> 0
- E.g., in Nb_3Sn, grain boundaries are pinning centers. In HTS, grain boundaries can be limit current flow and smaller pinning sites are required (why?)



Pinning force density f_p

- Superconductor will have no flux flow resistance if $f_p > f_L$
- Pinning force density can be measured by J_c vs. B
- Dependent on type of superconductor, defect, temperature, …

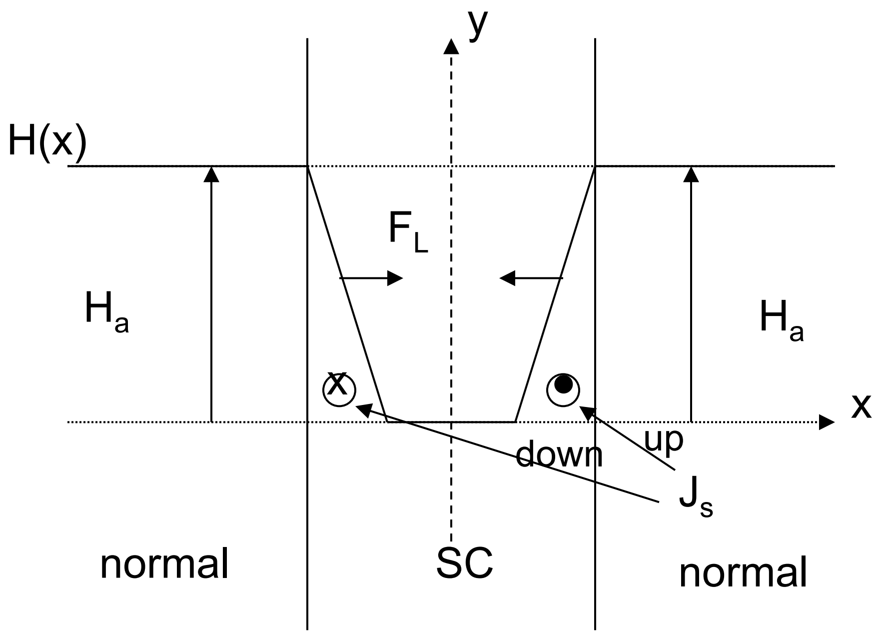




Critical State Model for Type II SC

- Where does current really flow in a Type II superconductor?
- Originally proposed by C. Bean (1963)
- Consider a Type II SC in mixed state w/ many pinning sites so flux lines are tightly held in place.
  - This is a 'strong pinning' model
- Externally applied (increasing) magnetic field, $H_a > H_{c1}$
- 2D slab geometry (most wires are round)

Critical State Model for Type II SC



- Flux lines move under influence of Lorentz force

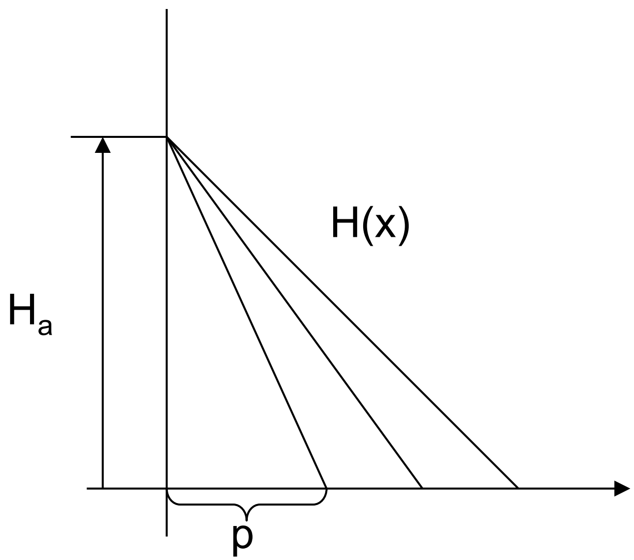
$$F_L = J_s \times B = \mu_0 J_s H$$

- Ampere's Law: $\nabla \times H = J$ for given geometry: $J_c = dH/dx$
- Screening currents try to keep B = 0 inside SC

Critical State Model (continued)

- Flux will move into SC as long as $F_L > F_p$
- "Critical State": $F_L = F_p$ and $F_L = \mu_0 H\ (dH/dx) = d/dx(1/2\mu_0 H^2)$

J_c = slope of H(x) within superconductor



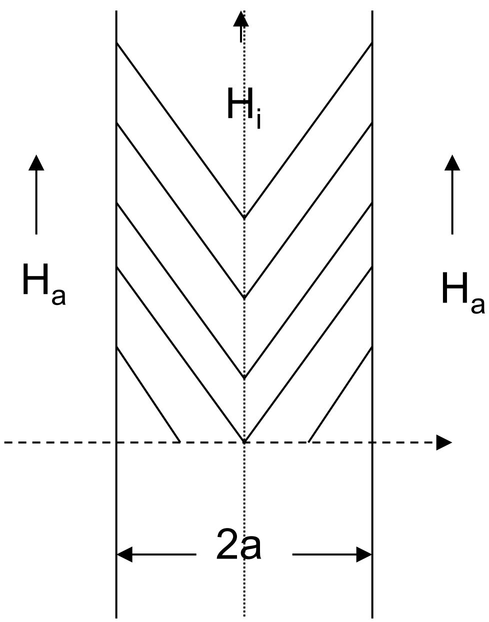
- "Penetration depth": $p = H_a/J_c$ distance flux penetrates into Type II SC
- NOT the same as $\boldsymbol{\lambda}$
- Typical values: J_c = 2000 A/mm^2 @ 5 T then p = $B/\mu_0 J_c$ ~ 2 mm
- Note: SC filaments in composite conductors are typically d ~ 10 mm

Note difference as compared to Type I SC:

1. p >> λ (screening due to flux penetration & pinning)
2. B(x) varies linearly (not exponential)

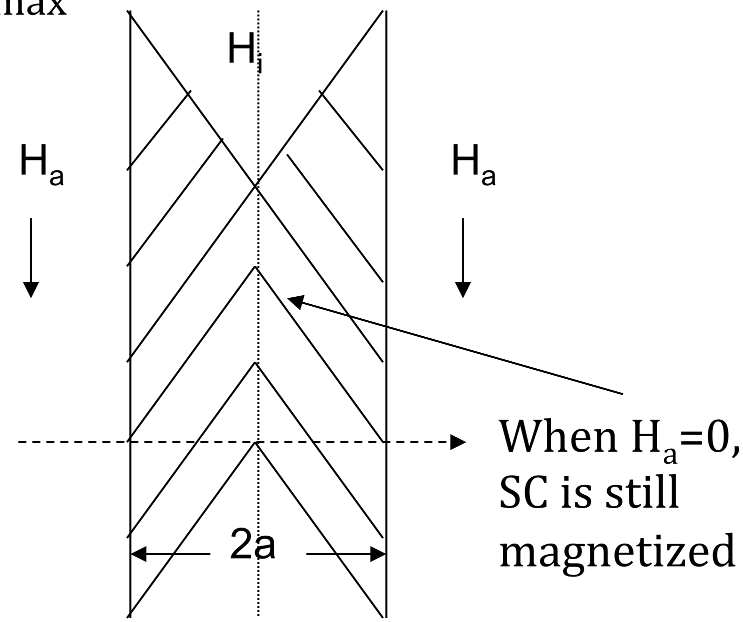
Field Variation - Critical State Model

- Increasing H_a from 0 "unmagnetized" sample



- Penetration field, $H_p = J_c a$
- Screening current density = J_c
- H_i increases, but lags behind H_a

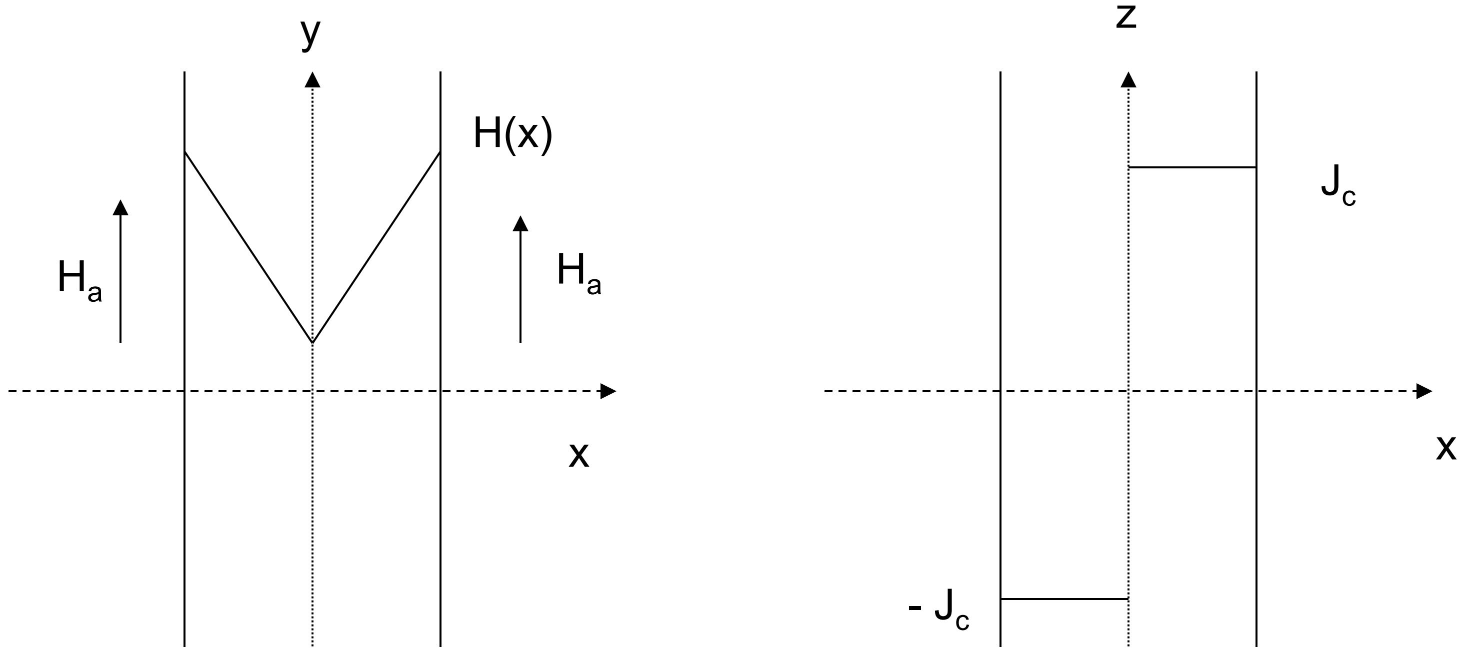
- Subsequently decreasing from H_{max} to 0



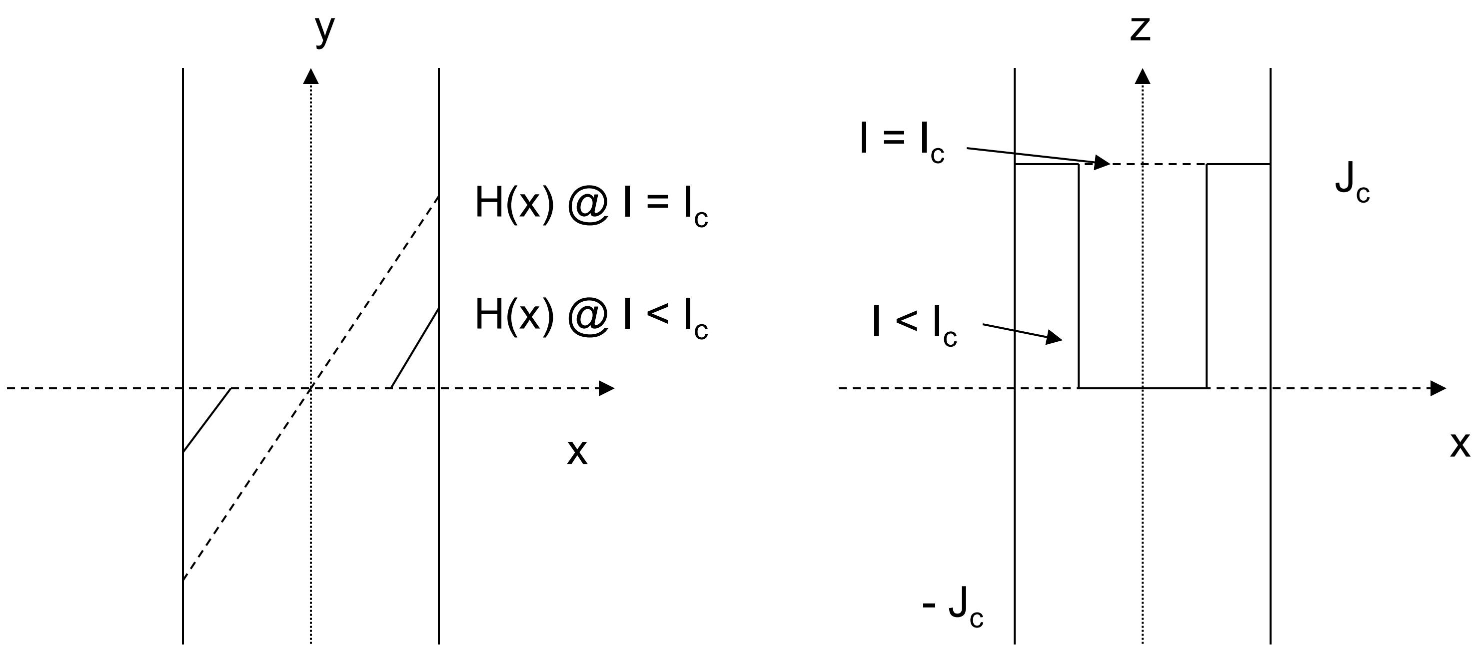
- Flux moves out from surface
- Like superposition of negative field
- $H_i > H_a$ because trapped flux by screening current
- $H_i = 0$ for $H_a = -J_c a$
- This is a lossy process

Transport Current in Hard SC

- External field with screening currents ($J_t = 0$)
  - Amperes Law: $\nabla x H = J \Rightarrow J_s = \frac{\partial H_y}{\partial x}\hat{z}$
  - Net current: $\langle J \rangle = \frac{1}{2a}\int_{-a}^{a} J \cdot dx = 0$

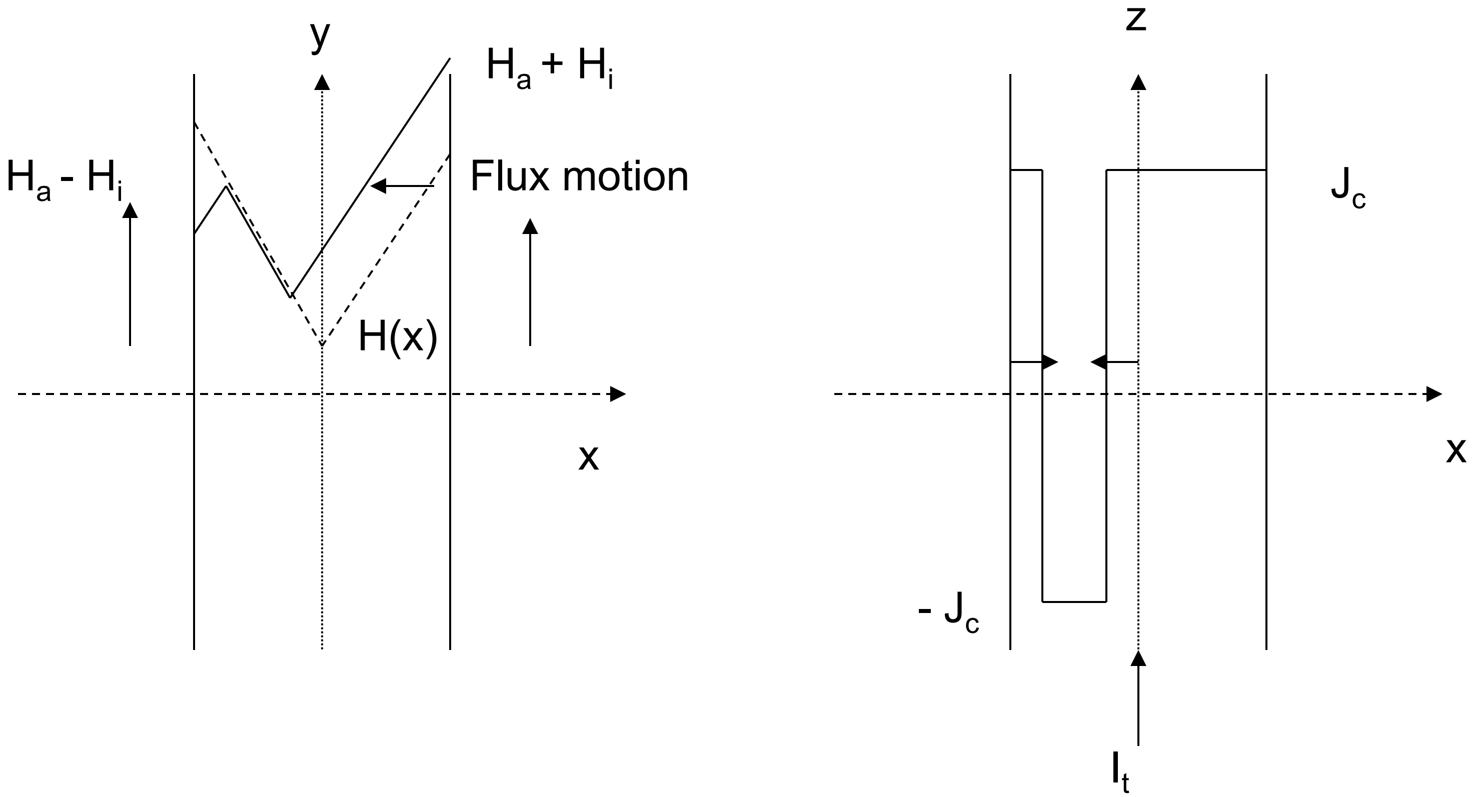


Transport Current w/ Self Field



- Critical current corresponds to full penetration of H(x)
- Use "Right Hand Rule" to determine direction of self field

Now combine external field & transport current



- $J = J_c$ (or $-J_c$) everywhere w/in superconductor
- Flux motion so that filament screens H_i
- Normally, $H_a >> H_i$ so the effect is small

Thermally Activated Flux Creep

- Two cases considered so far:
  - fully mobile flux flow: $f_L \sim \eta v_{perp}$
  - fully pinned flux up to $f_p > f_L$
- Transition between two cases is not abrupt
- When $f_L \sim$ but $< f_p$ some flux motion can occur due to thermal activation ($T > 0$ K)
- Technical importance of the problem
  - controls sharpness of sc to normal transition
  - important for "persistent current" magnets (magnets that are powered to some field and then disconnected from the power supply)
  - High temperature SC are affected by this process due to expanded temperature range and anisotropic flux pinning

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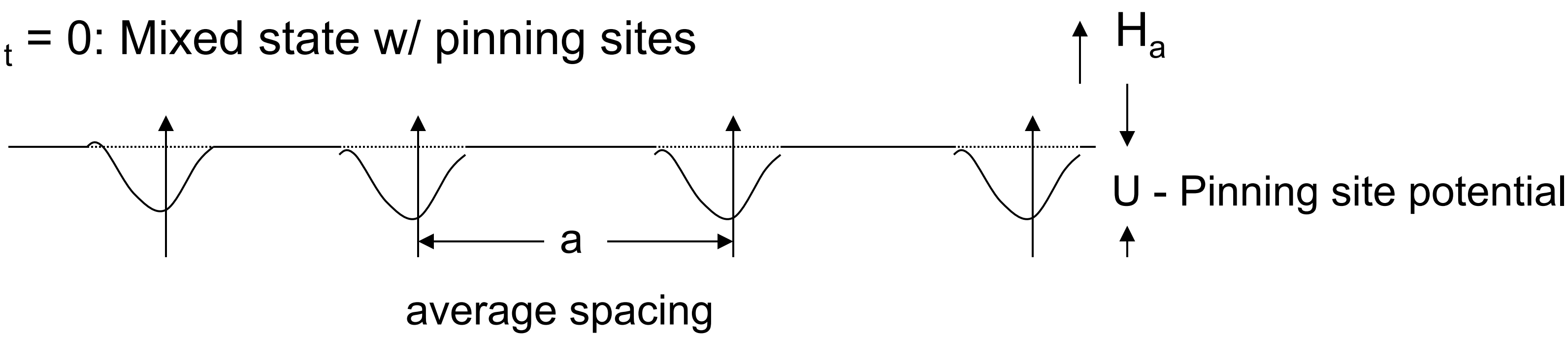
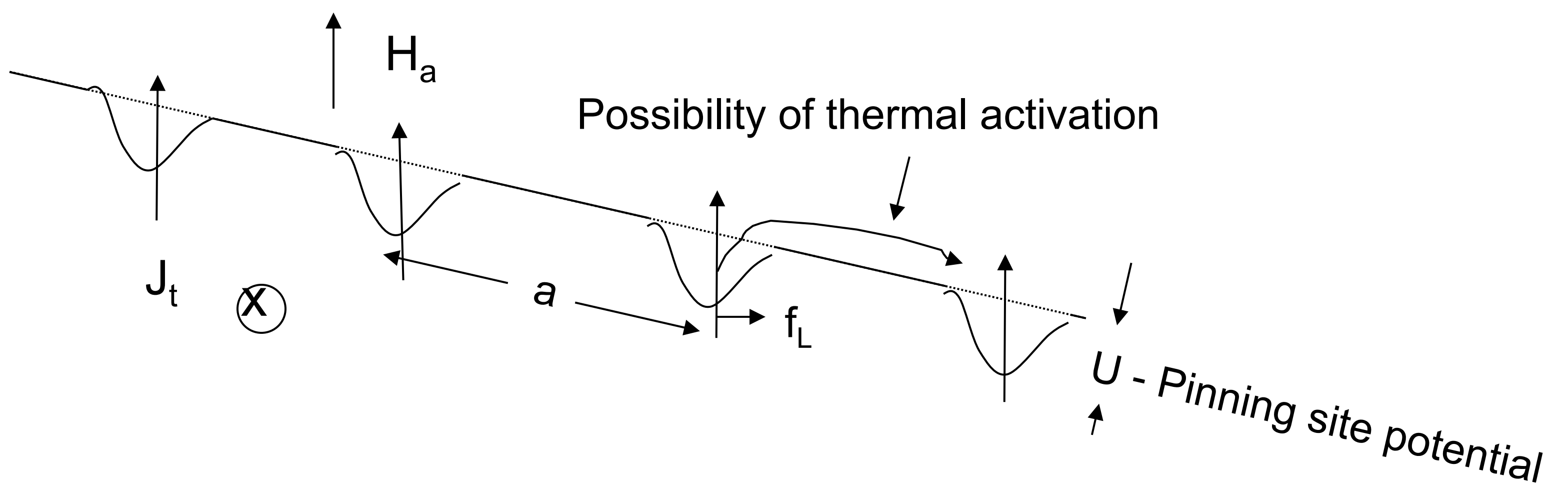
Distribution of pinning sites

$J_t = 0$: Mixed state w/ pinning sites



$J_t > 0$: Lorentz force on flux lines



Flux motion is determined by whether $f_L \cdot a$ > or < U

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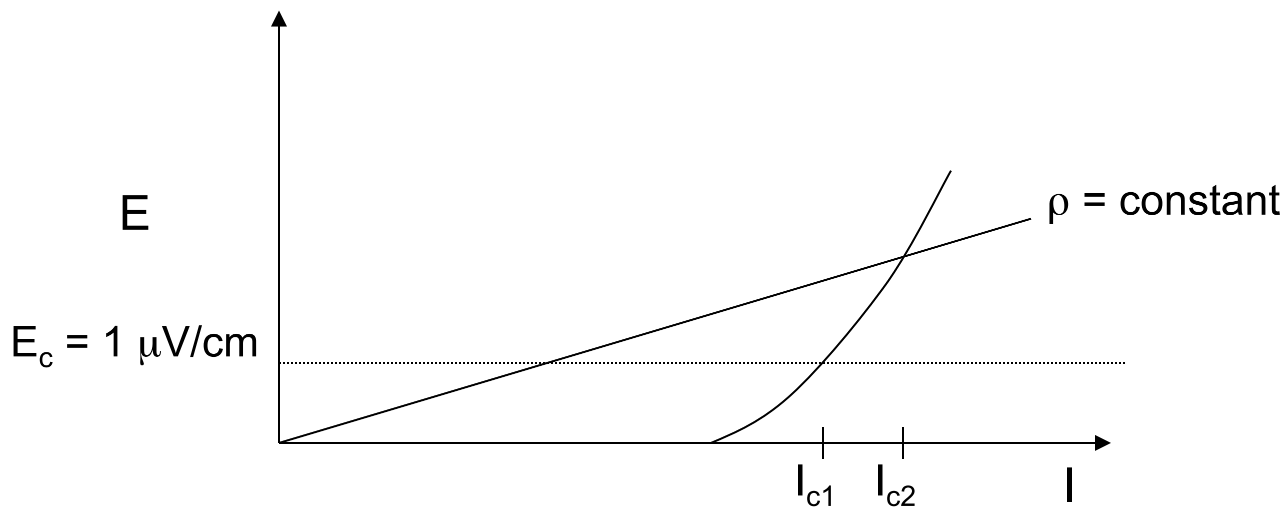
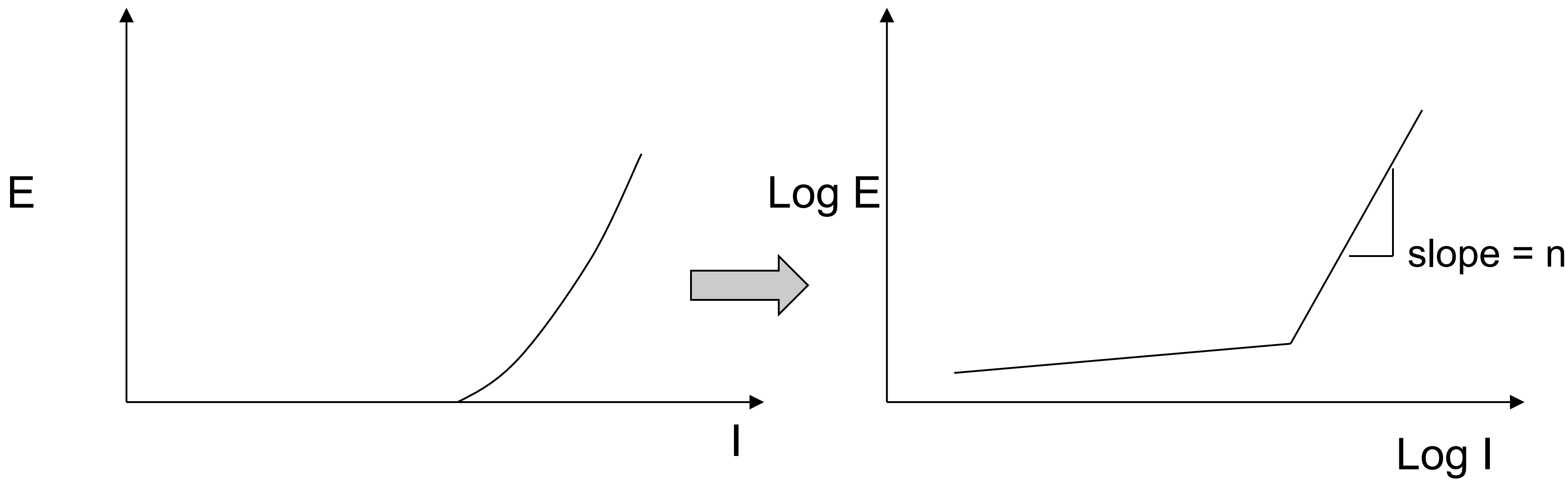
Definition of J_c

- Electric field criterion: e.g. 1 μV/cm, 0.1 μV/cm
- Resistivity criterion: $\rho_{eff} = 10^{-n}$, n ~ 11 to 13



- Best measurements require long samples, high sensitivity voltmeter and low ripple power supply

Fitting the Transition



- Above transition $E \sim (I/I_c)^n$
- "n-value" is the empirical goodness parameter
  - High n-value > 20 indicates uniform SC properties
  - Low n-value < 10 non-uniform structure & thermal activation

Summary of Behavior of Hard SC

- Flux Line Lattice formed in mixed state
- Flux pinning is required for Type II SC to carry much current in mixed state.
- Critical State is a balance between pinning force and Lorentz force
- Critical State Model allows calculation of J_c and magnetic hysteresis from magnetization measurement
- Thermal activation at relatively high temperature controls flux leakage and electric field near I_c
- Engineering magnetic flux pinning in practical Type II superconductors has been key to successful magnets