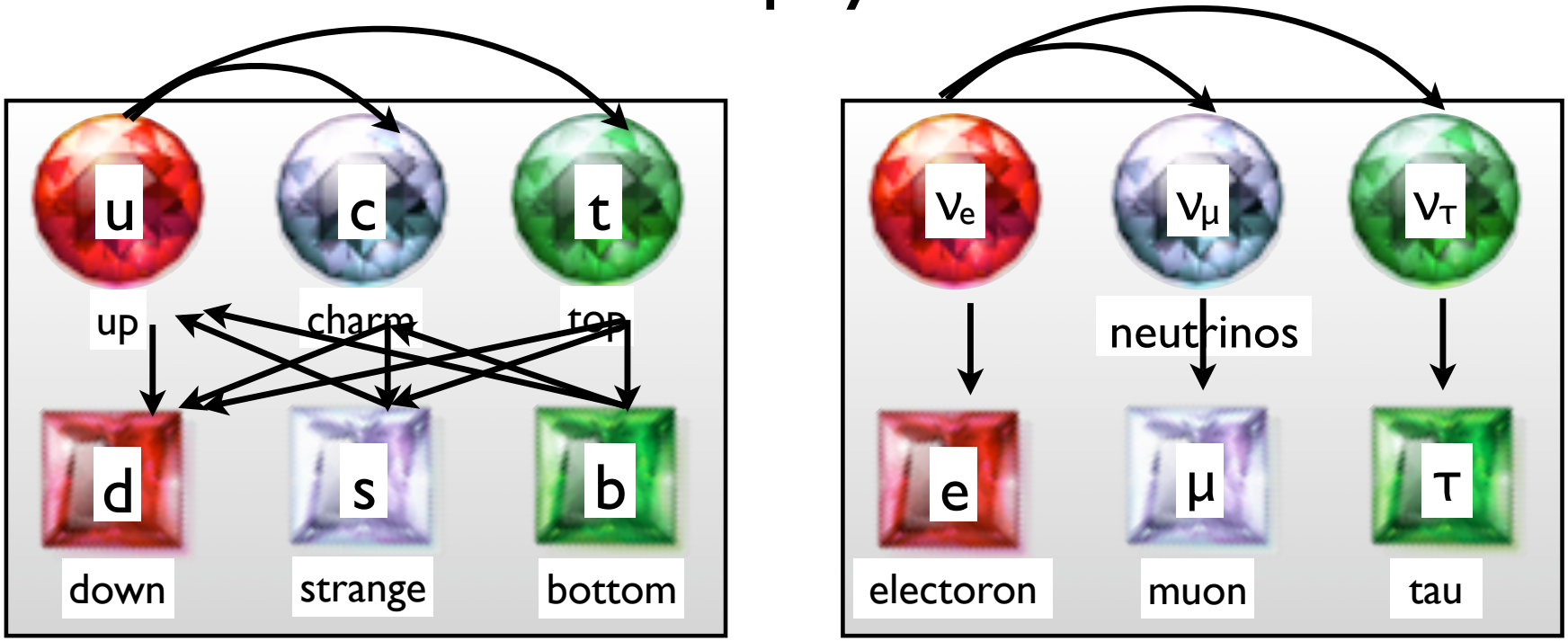
Flavour Physics and CP Violation I

AEPSHEP 2012 at Fukuoka

Emi KOU (LAL/IN2P3)

19/10/2011

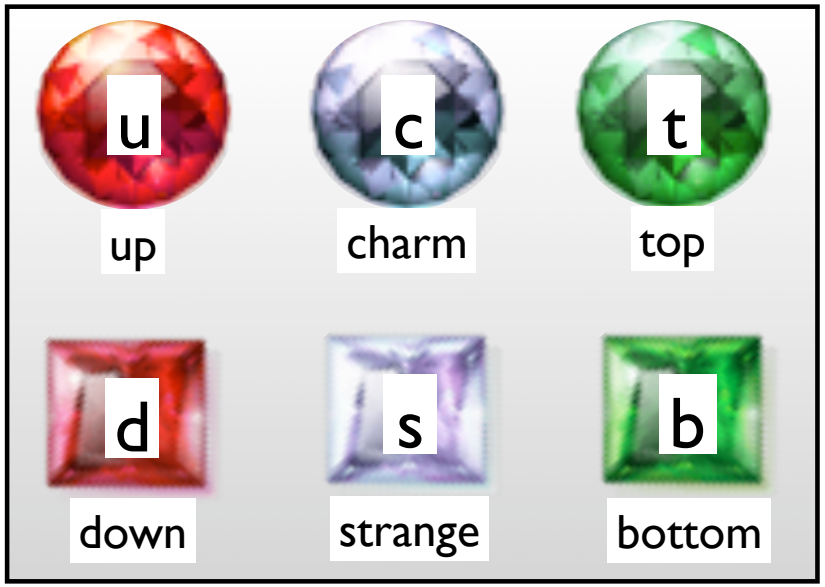
Flavour physics

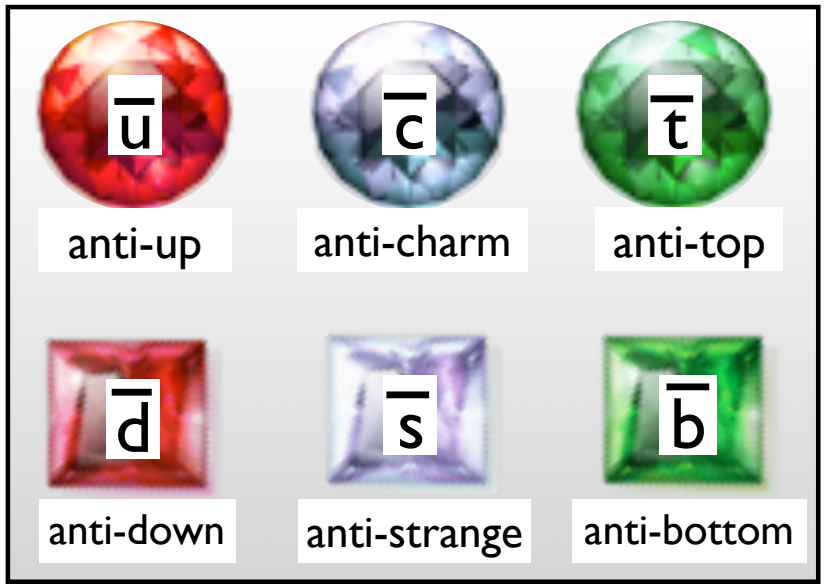


Quark mass

Lepton mass

Matter Anti-Matter Asymmetry (CP violation)

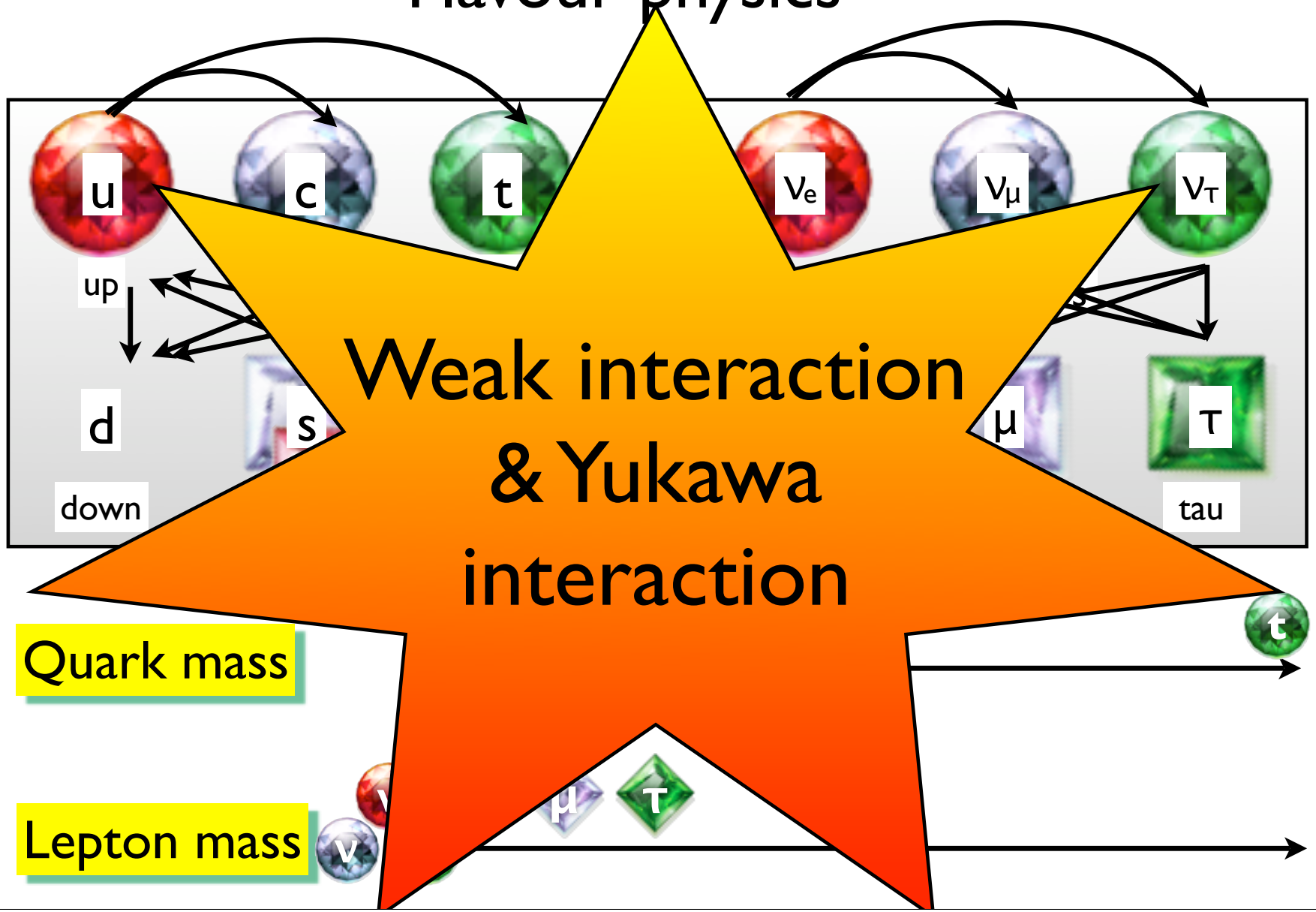




proton, neutron, electron:
Exist in nature

anti-proton, anti-neutron, positron
Exist only for short time

Flavour physics



Weak interaction
& Yukawa
interaction

Plan

- 1st lecture: Introduction to flavour physics
  - ★ Weak interaction processes: historical review
  - ★ Discovery of CP violation in the K system
  - ★ Charged/Neutral processes and GIM mechanism
- 2nd lecture: Describing flavour physics and CP violation within SM
  - ★ Charged/Neutral current and CP violation in SM
  - ★ Measuring CP violating phase in B factories
  - ★ Testing the unitarity of the CKM matrix

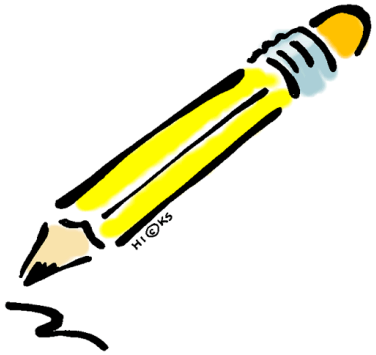
Plan

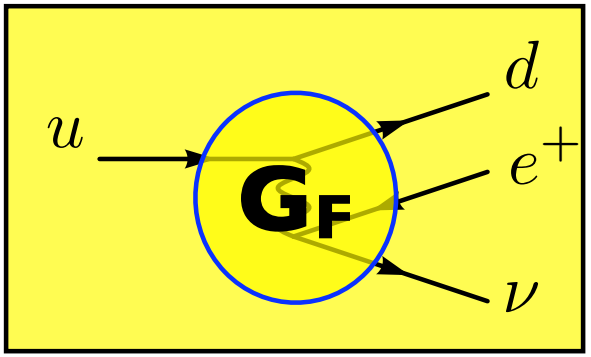
- 3rd lecture: Searching new physics with flavour physics
  - ★ Some examples in the past
  - ★ Some examples in the future

What kind of weak interaction processes do you know?

Discovery of Parity Violation

in beta decays





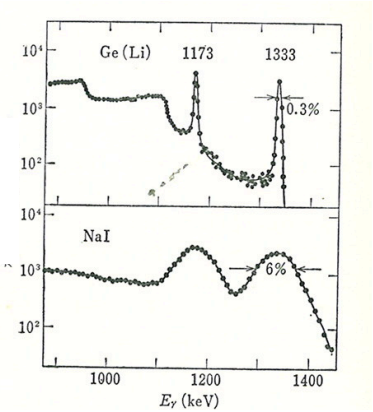
Beta decay: continuous spectrum

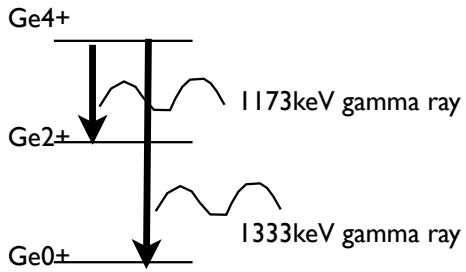
Fermi's theory of beta decay (1933)

Fermi, postulation of neutrino

The spectrum of beta decay is continuous not like gamma ray. This is because in beta decay, zero mass, zero charge neutrinos are emitted together with the electrons/positrons.

γ decay





Gamma ray is typically emitted when the excited state transfer to a lower state.

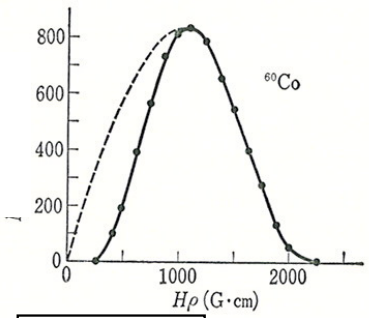
Two body decay

$$Ge^{4+} \rightarrow Ge^{2+}\gamma$$

$$E(Ge^{4+}) = E(Ge^{2+}) + E(\gamma)$$

The emitted energy is fixed for given nucleus.

β decay



Beta ray spectrum:
1. Continuous
2 The maximum energy:
$E_{initial} - E_{final}$

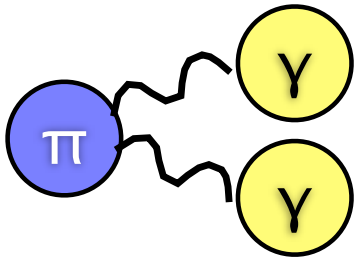
Three body decay

$$H^3 \rightarrow He^3 + e^- + \nu$$

$$E(H^3) = E(He^3) + E(e^-) + E(\nu)$$

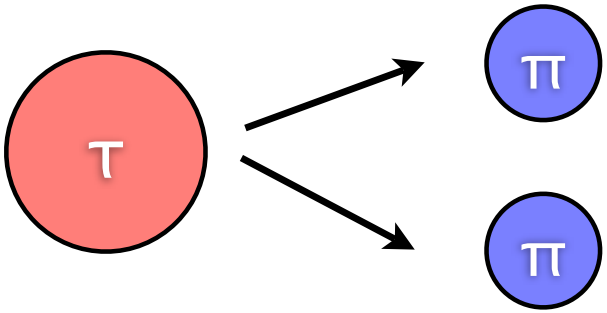
Emitted energy is shared by $e^{\pm}$ and ν. As a result, the spectrum becomes continuous.

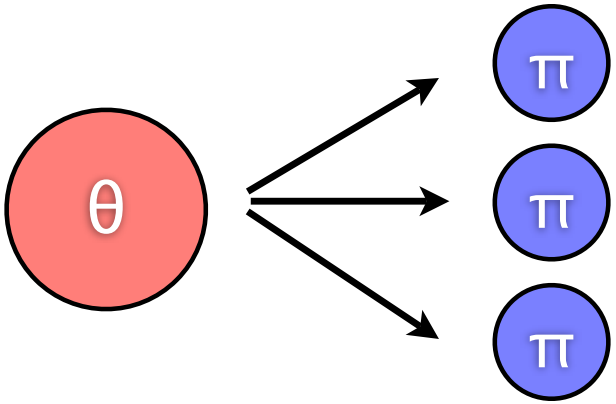
τ-θ puzzle



parity of pion is (-1)

In early '50's, In cosmic ray, two particles which have an identical mass while, decaying into two and three pions were observed.

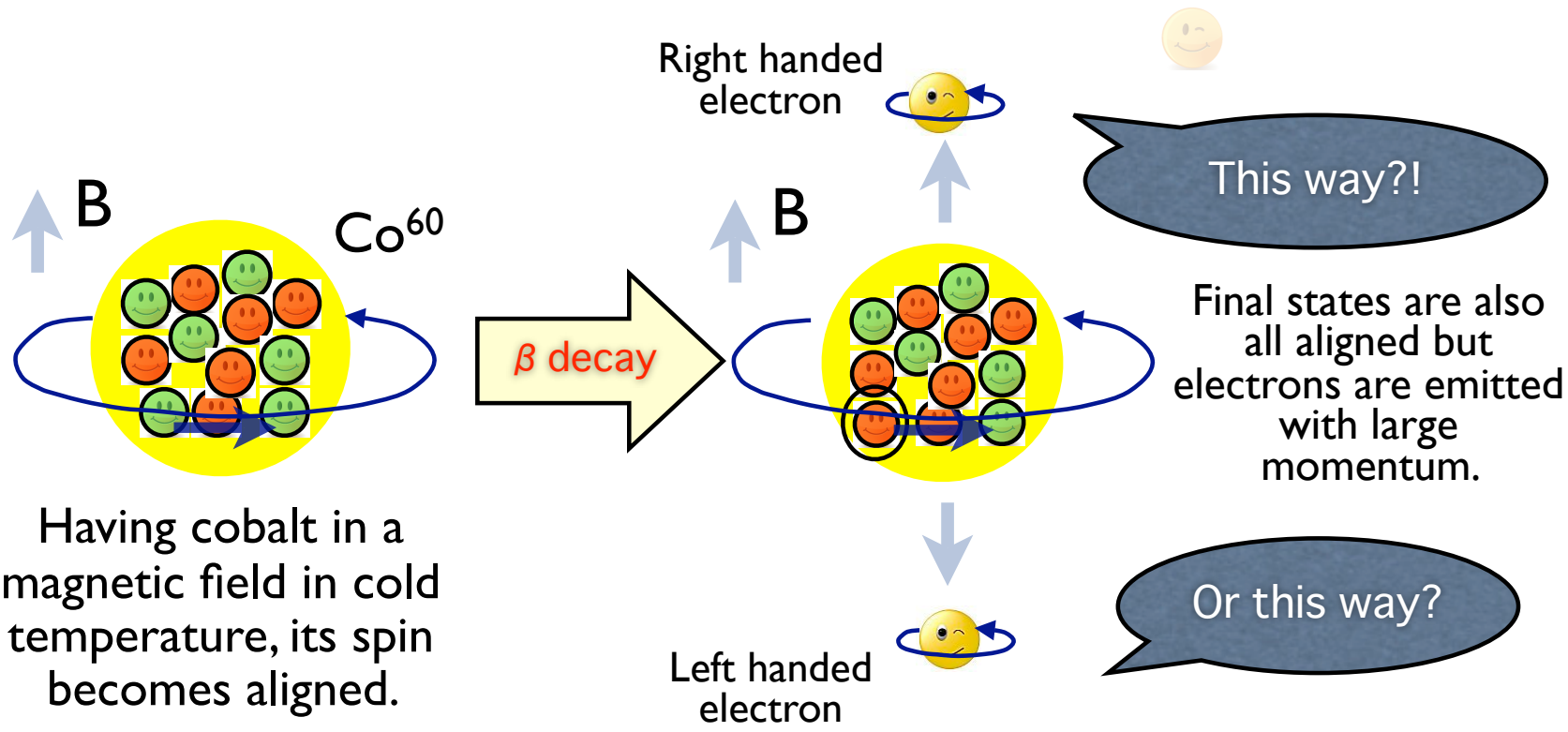




Possibility of broken Parity in weak decay? Lee & Yang '56

Observation of parity violation

$$Co^{60} \rightarrow Ni^{60} + e^{-} + \bar{\nu}$$



Having cobalt in a magnetic field in cold temperature, its spin becomes aligned.

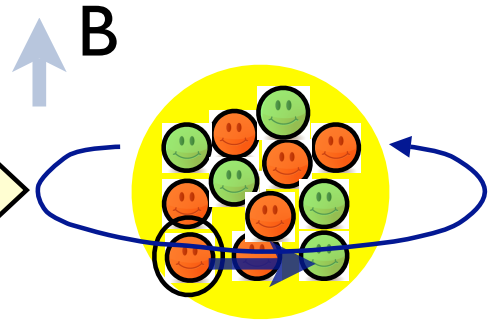
Observation of parity violation

$$Co^{60} \rightarrow Ni^{60} + e^{-} + \bar{\nu}$$

Observation of parity violation by Mme.Wu '56

Wu has tested if there is any preference in the direction of the emitted electron (if Parity is conserved, it should be equal)!

magnetic field in cold temperature, its spin becomes aligned.



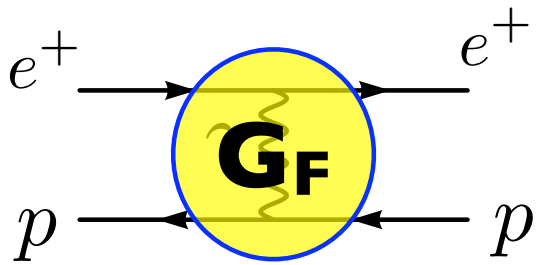
Final states are also all aligned but electrons are emitted with large momentum.

Left handed electron

More in this way!

V-A theory for weak interaction

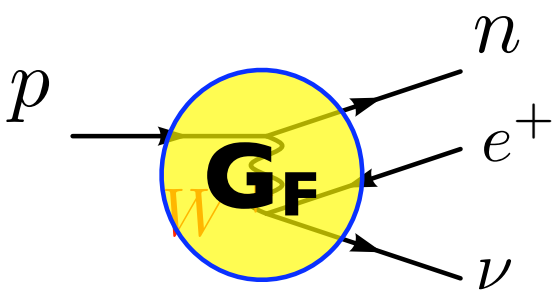
Electro-magnetic interaction



$$\mathcal{M} = (e\overline{u}_p\gamma^\mu u_p)\left(\frac{1}{q^2}\right)(-e\overline{u}_e\gamma_\mu u_e)$$

Vector coupling

Weak interaction



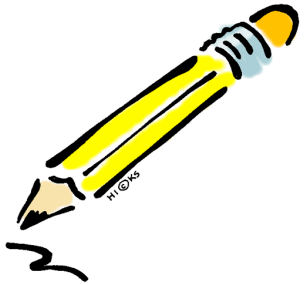
V-A theory: '57

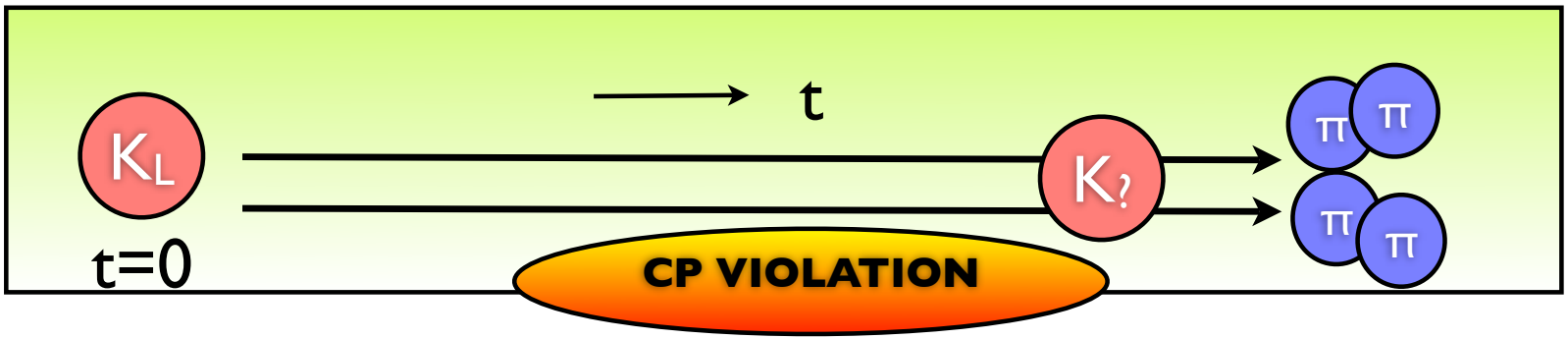
$$\mathcal{M} = \frac{G_F}{\sqrt{2}}(\overline{u}_n\gamma^\mu(1-\gamma^5)u_p)(\overline{u}_{\nu_e}\gamma_\mu(1-\gamma_5)u_e)$$

V-A coupling

Discovery of CP Violation

in K decays





CP transformation in a few words

C: Charge transformation
P: Parity transformation

A few key equations...

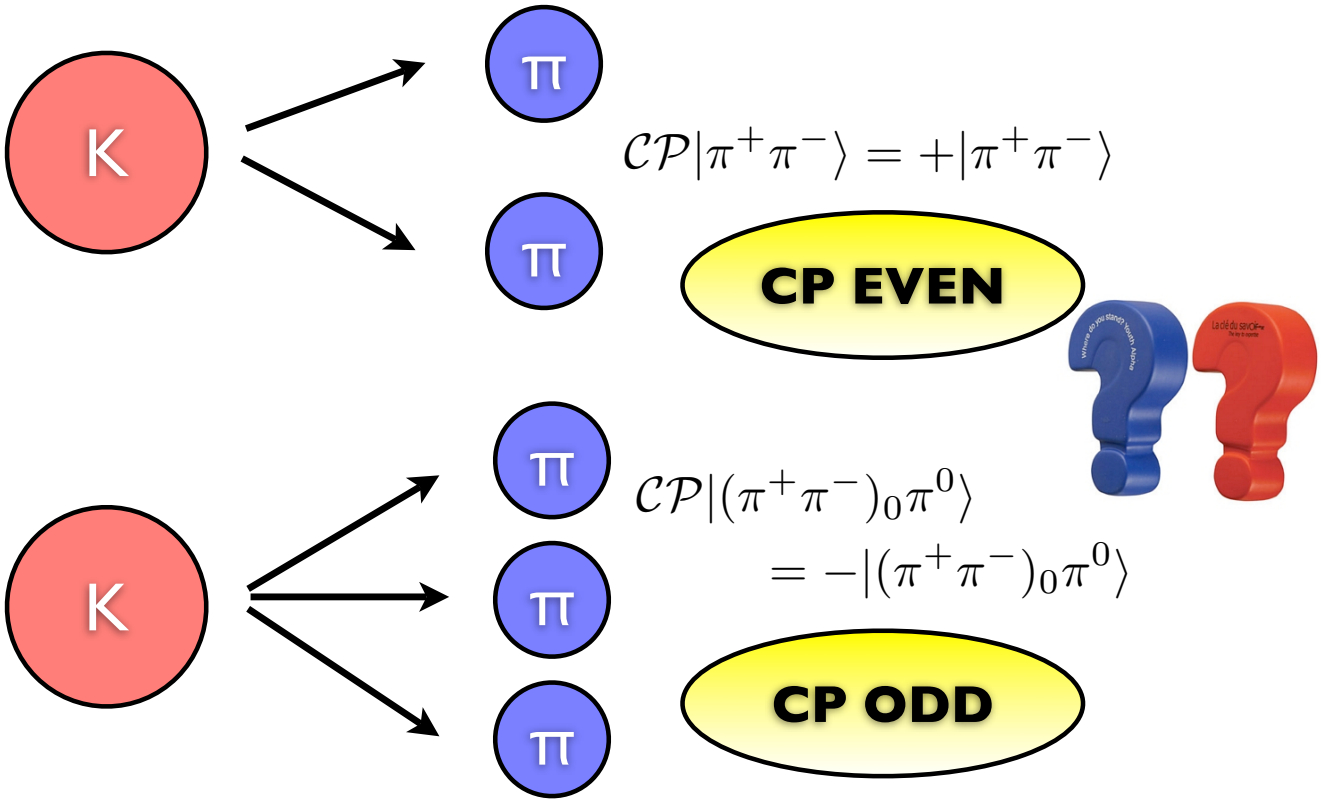
$$\mathcal{CP}|K^0\rangle = |\overline{K}^0\rangle \qquad K^0 = \overline{s}d$$
$$\mathcal{CP}|\overline{K}^0\rangle = |K^0\rangle \qquad \overline{K}^0 = \overline{d}s$$

$$\mathcal{CP}|\pi^0\rangle = -|\pi^0\rangle$$
$$\mathcal{CP}|\pi^+\pi^-\rangle = +|\pi^+\pi^-\rangle$$
$$\mathcal{CP}|(\pi^+\pi^-)_l\pi^0\rangle = (-)^{l+1}|(\pi^+\pi^-)_l\pi^0\rangle$$

$$\pi^0 = u\overline{u} - d\overline{d}$$
$$\pi^+ = u\overline{d},\ \pi^- = d\overline{u}$$

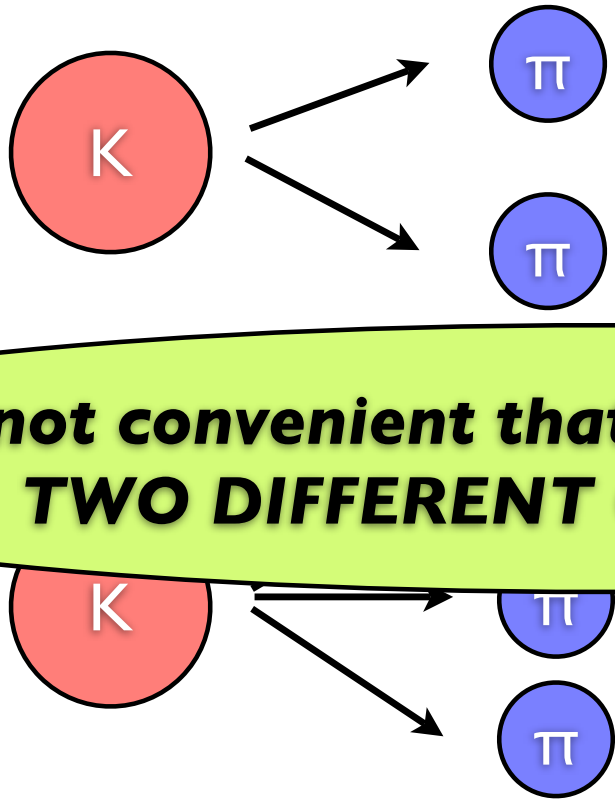
CP "*invariance*" of K system

Two decay channels of K are observed... *(θ-τ puzzle)*



CP "*invariance*" of K system

Two decay channels of K are observed... *(θ-τ puzzle)*



$$\mathcal{CP}|\pi^+\pi^-\rangle = +|\pi^+\pi^-\rangle$$

CP EVEN

CP ODD

It's not convenient that the same K decays to TWO DIFFERENT CP eigen-states!!!

CP "*invariance*" of K system

How can we make two CP (+ and -) states from K^0 and $\overline{K}^0$?

$$\mathcal{CP}|K^0\rangle = |\overline{K}^0\rangle \qquad K^0 = \bar{s}d$$
$$\mathcal{CP}|\overline{K}^0\rangle = |K^0\rangle \qquad \overline{K}^0 = \bar{d}s$$

ANSWER

If the K is a mixed state of K^0 and $\overline{K^0}$ in nature...

$$|K_1\rangle = \frac{1}{\sqrt{2}}(|K^0\rangle + |\overline{K}^0\rangle)$$
$$|K_2\rangle = \frac{1}{\sqrt{2}}(|K^0\rangle - |\overline{K}^0\rangle)$$

$$\mathcal{CP}|K_1\rangle = +\frac{1}{\sqrt{2}}(|K^0\rangle + |\overline{K}^0\rangle) = |K_1\rangle$$

CP EVEN

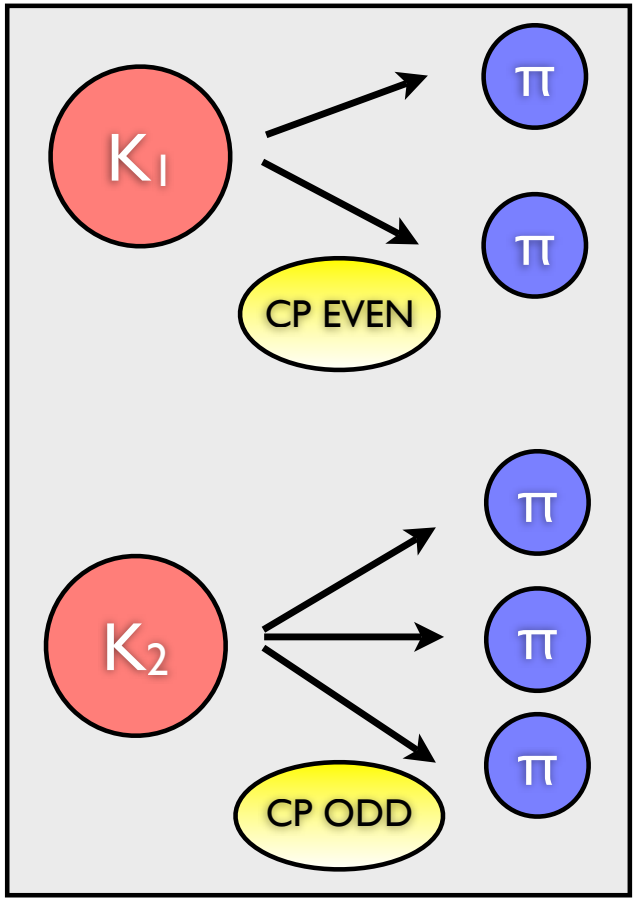
$$\mathcal{CP}|K_2\rangle = -\frac{1}{\sqrt{2}}(|K^0\rangle - |\overline{K}^0\rangle) = -|K_2\rangle$$

CP ODD

CP "*invariance*" of K system

Distinguishing K_1 and K_2

By the decay channel



By the life-time

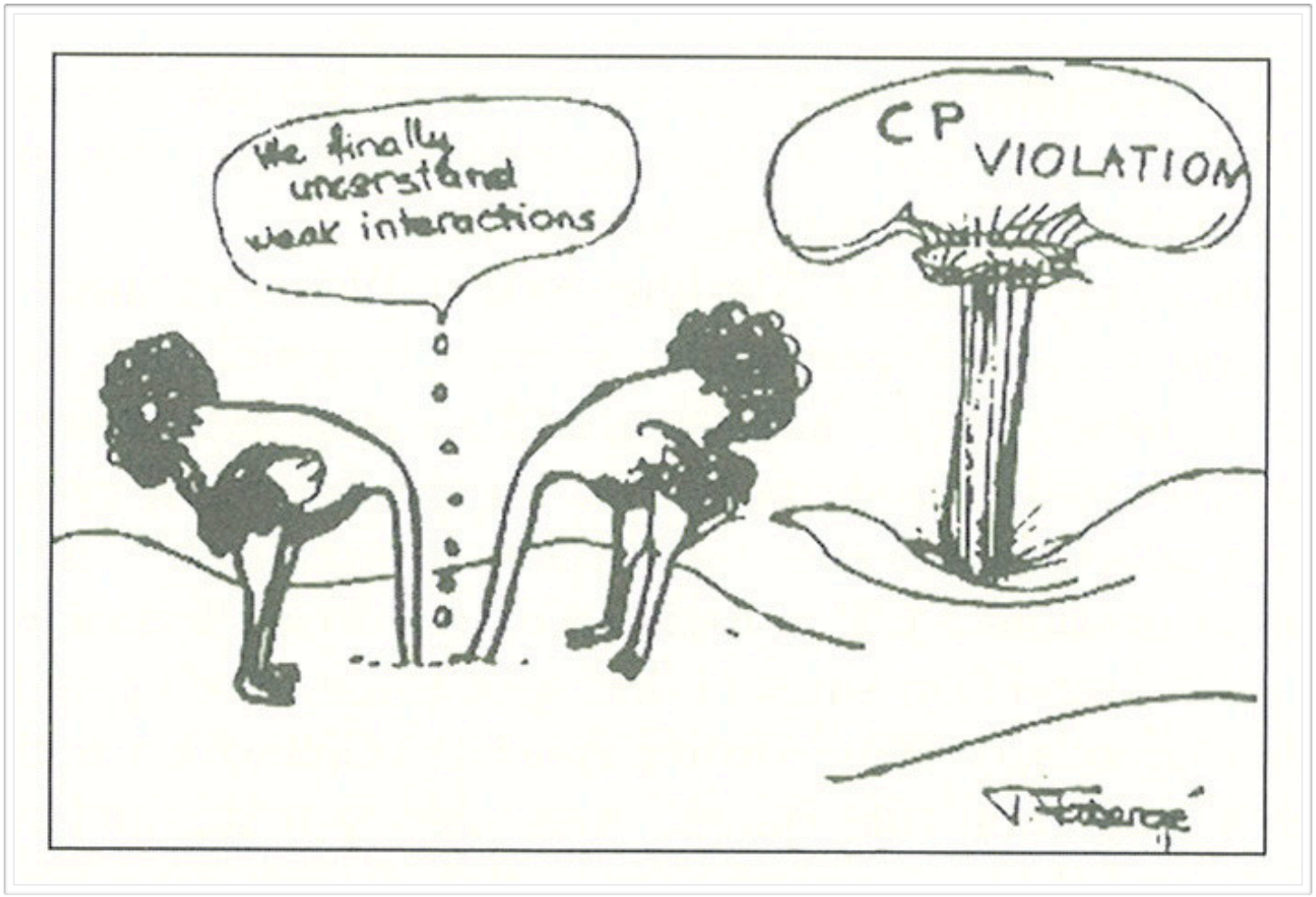
M_K=498MeV
M_π=140 MeV

Phase space for 2π is about 600 larger than for 3π

$$\tau(K_1) \simeq 0.90 \times 10^{-10} s$$
$$\tau(K_2) \simeq 5.1 \times 10^{-8} s$$

Accidental phase space suppression:
short-lived K is K_1 and long-lived one is K_2

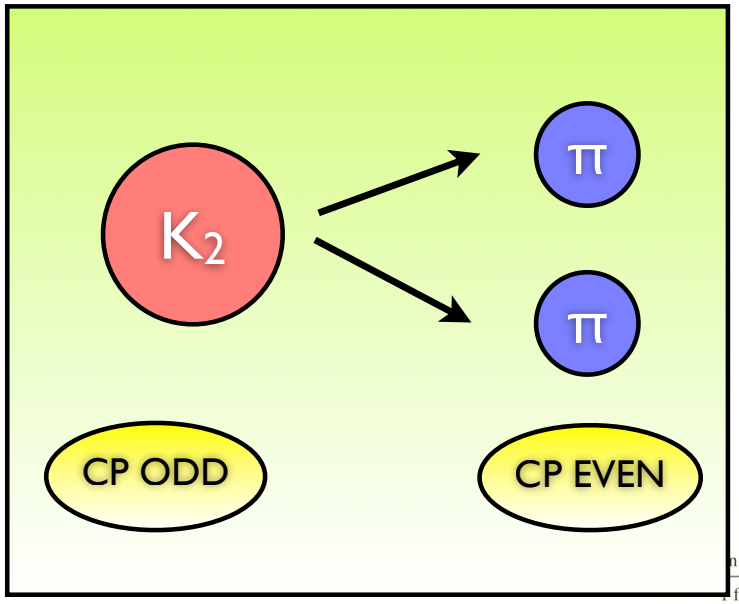
CP non-*invariance* of K system



Cabibbo *(1966)*

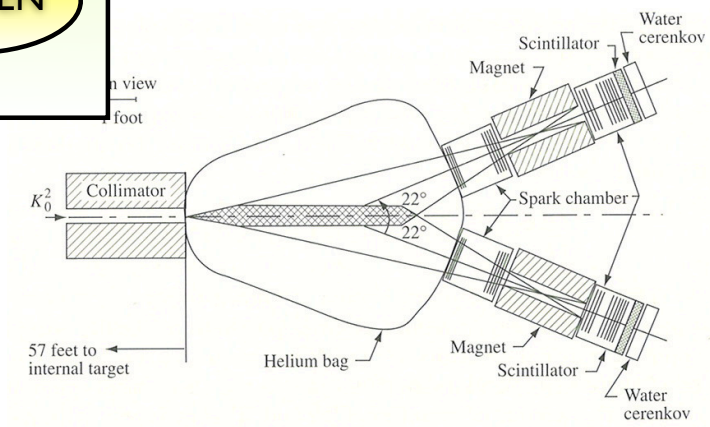
CP non-*invariance* of K system

First observation of the CP violation



Cronin, Fitch Christenson, Turlay
(1964)

Long-lived K_2 decaying to 2π!!!



CP non-*invariance* of K system

We thought...

$$|K_1\rangle = \frac{1}{\sqrt{2}}(|K^0\rangle + |\overline{K}^0\rangle)$$

$$|K_2\rangle = \frac{1}{\sqrt{2}}(|K^0\rangle - |\overline{K}^0\rangle)$$

$$\mathcal{CP}|K_1\rangle = +\frac{1}{\sqrt{2}}(|K^0\rangle + |\overline{K}^0\rangle) = |K_1\rangle$$

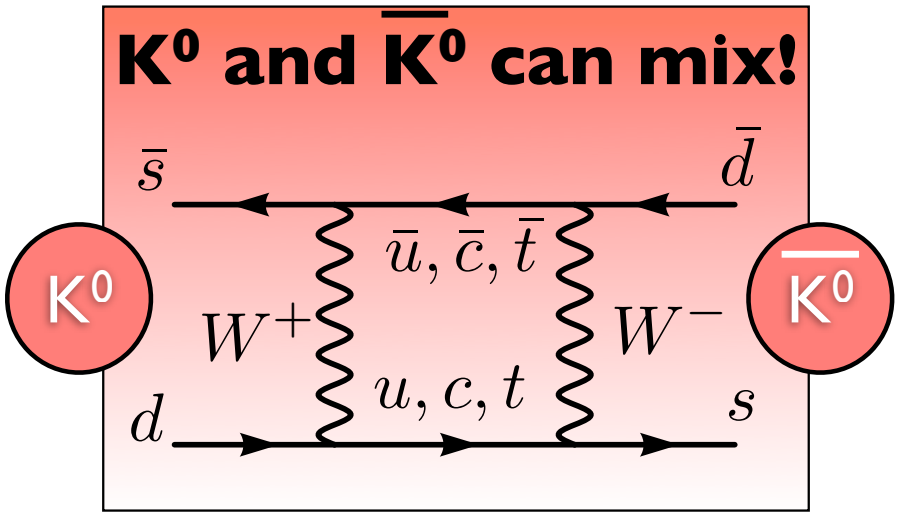
CP EVEN

$$\mathcal{CP}|K_2\rangle = -\frac{1}{\sqrt{2}}(|K^0\rangle - |\overline{K}^0\rangle) = -|K_2\rangle$$

CP ODD

But, actually...

K^0 and $\overline{K}^0$ can mix through box diagram.
Thus, they are not mass eigenstate.



$$|K_S\rangle = \frac{1}{\sqrt{2}}\left(p|K^0\rangle + q|\overline{K}^0\rangle\right)$$

$$|K_L\rangle = \frac{1}{\sqrt{2}}\left(p|K^0\rangle - q|\overline{K}^0\rangle\right)$$

If $q/p \neq 1$,
the mass eigenstate $K_{S/L}$
are not CP eigen state
CP violation!!

CP non-*invariance* of K system

So the mass eigenstate is a mixture of two CP eigenstate!

$$
\begin{aligned}
|K_S\rangle &= \frac{1}{\sqrt{2}}\left(p|K^0\rangle + q|\overline{K}^0\rangle\right) \\
&= \frac{p}{2}\left[(1+\frac{q}{p})|K_1\rangle + (1-\frac{q}{p})|K_2\rangle)\right] \\
|K_L\rangle &= \frac{1}{\sqrt{2}}\left(p|K^0\rangle - q|\overline{K}^0\rangle\right) \\
&= \frac{p}{2}\left[(1-\frac{q}{p})|K_1\rangle + (1+\frac{q}{p})|K_2\rangle)\right]
\end{aligned}
$$

✍ At $p=q=1$, we recover the previous result.

The CP violation comes from $q/p \neq 1$!!!

Mass matrix of the K system

$$\mathcal{H} = \mathbf{M} - \tfrac{i}{2}\mathbf{\Gamma} = \begin{pmatrix} M_{11} - \frac{i}{2}\Gamma_{11} & M_{12} - \frac{i}{2}\Gamma_{12} \\ M_{21} - \frac{i}{2}\Gamma_{21} & M_{22} - \frac{i}{2}\Gamma_{22} \end{pmatrix}$$

Using CPT invariance (M_{11}=M_{22}, Γ_{11}=Γ_{22}) and ***M*** *and* **Γ** *being Hermitian, we find the eigensystem of this matrix:*

$$M_1 + \frac{i}{2}\Gamma_1 \equiv M_{11} - \frac{i}{2}\Gamma_{11} + \frac{q}{p}\left(M_{12} - \frac{i}{2}\Gamma_{12}\right); \quad \begin{pmatrix} p \\ q \end{pmatrix}$$

$$M_2 + \frac{i}{2}\Gamma_2 \equiv M_{11} - \frac{i}{2}\Gamma_{11} - \frac{q}{p}\left(M_{12} - \frac{i}{2}\Gamma_{12}\right); \quad \begin{pmatrix} p \\ -q \end{pmatrix}$$

Thus, the mass eigenstate of K is obtained as:

$$\begin{aligned} |K_1\rangle &= p|K\rangle + q|\overline{K}\rangle \\ |K_2\rangle &= p|K\rangle - q|\overline{K}\rangle \end{aligned} \quad \textit{with} \quad \frac{q}{p} = \pm\sqrt{\frac{M_{12}^* - \frac{i}{2}\Gamma_{12}^*}{M_{12} - \frac{i}{2}\Gamma_{12}}}$$

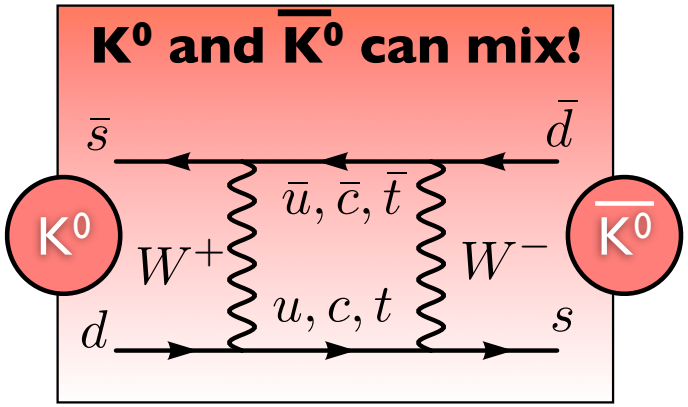
It is clear here that q/p≠1 occurs

when M_{12} and/or Γ_{12} are complex number.

CP non-*invariance* of K system

So the mass eigenstate is a mixture of two CP eigenstate!

$$
\begin{aligned}
|K_S\rangle &= \frac{1}{\sqrt{2}}\left(p|K^0\rangle + q|\overline{K}^0\rangle\right) \\
&= \frac{p}{2}\left[(1+\frac{q}{p})|K_1\rangle + (1-\frac{q}{p})|K_2\rangle)\right] \\
|K_L\rangle &= \frac{1}{\sqrt{2}}\left(p|K^0\rangle - q|\overline{K}^0\rangle\right) \\
&= \frac{p}{2}\left[(1-\frac{q}{p})|K_1\rangle + (1+\frac{q}{p})|K_2\rangle)\right]
\end{aligned}
$$



- *The box diagram induces an oscillation.*
- *But the CP violation can occur only if there is a complex number in the weak interaction*

BARYON ASYMMETRY

~ When did the anti-matter disappeared?!~

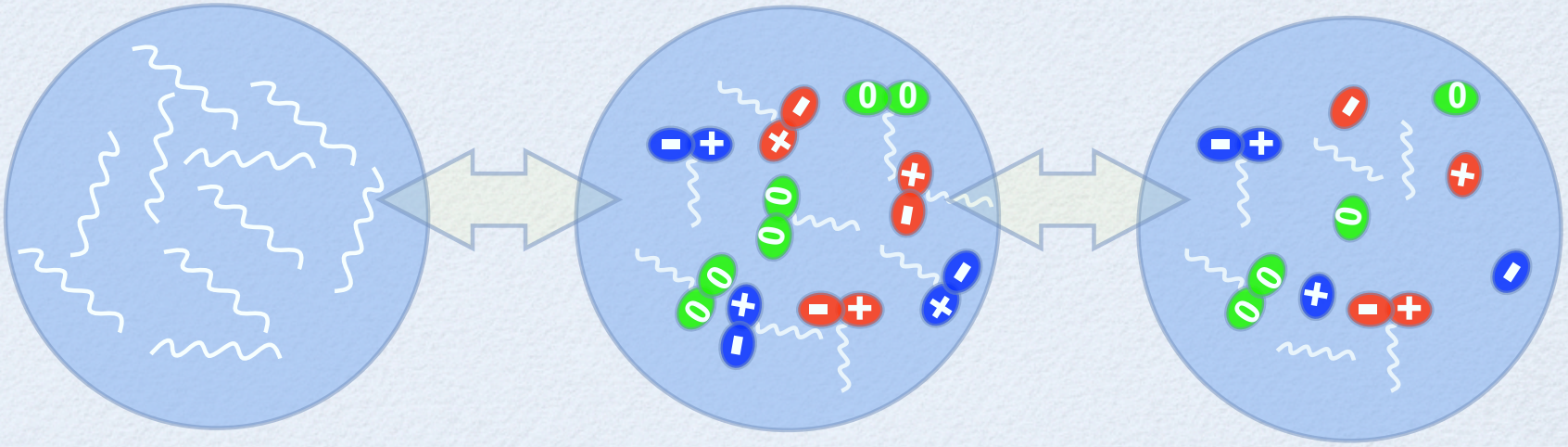
Our universe was born 13.7 billion years ago

Extremely high temperature (4000K=10^{19} GeV)

As the universe expanded, the temperature rapidly decreased.

During the first few seconds...

At the beginning, it was in the thermal equilibrium.



High energy photon | Pair creation | Pair annihilation

proton / anti-proton | positron / electron | neutron / anti-neutron

BARYON ASYMMETRY

~ When did the anti-matter disappeared?!~

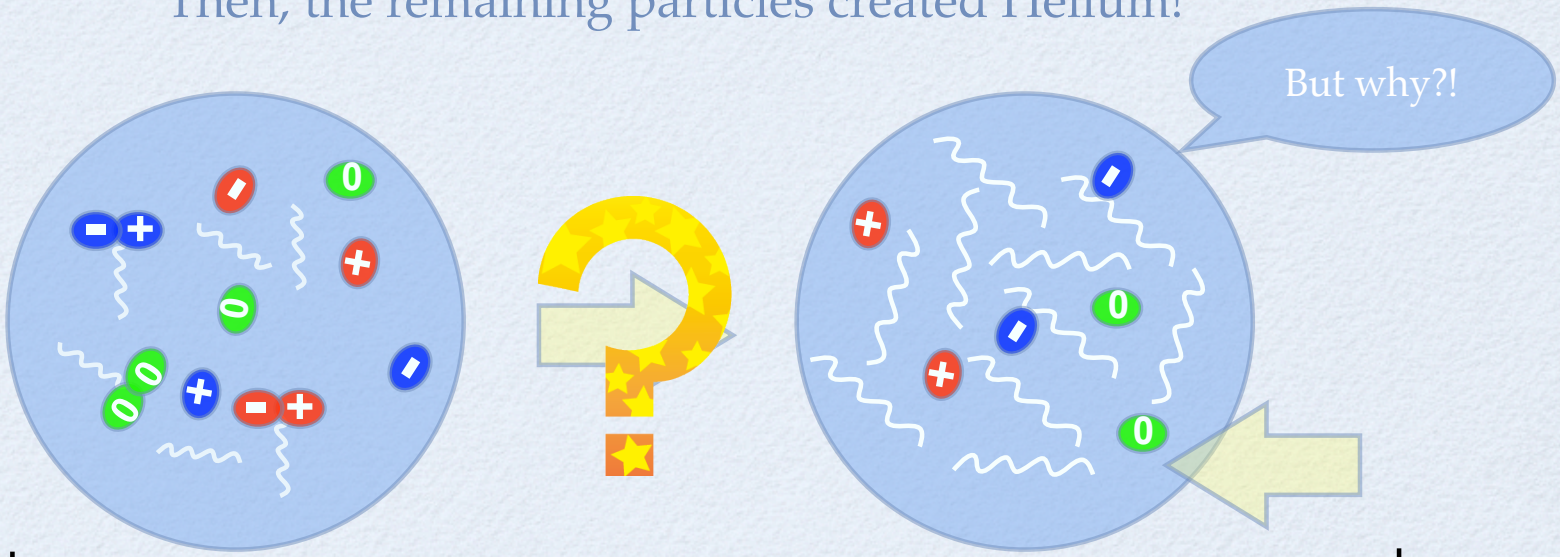
Our universe was born 13.7 billion years ago

Extremely high temperature (4000K=10^{19} GeV)

As the universe expanded, the temperature rapidly decreased.

In reality, a tiny number of particles remained for some reason!

Then, the remaining particles created Helium!



This question is still a mystery in particle physics...

anti-proton electron anti-neutron

BARYON ASYMMETRY

~ When did the anti-matter disappeared?!~

Our universe was born 13.7 billion years ago

Extremely high temperature (4000K=10^{19} GeV)

As the universe expanded, the temperature rapidly decreased.

In reality, a tiny number of particles remained for some reason!

Then, the remaining particles created Helium!

But why?!

So far, what we know is

When the temperature is about 10^{15} GeV, number of particle exceeded to number of anti-particle. In order to this to happen, the following three condition has to be satisfied:

1, Baryon number violating reaction to occur

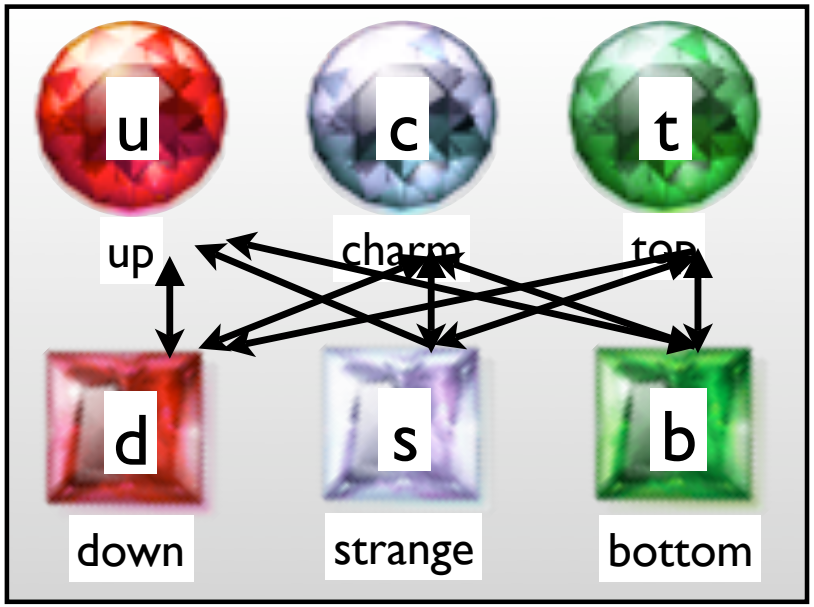
2, C and CP violation to occur

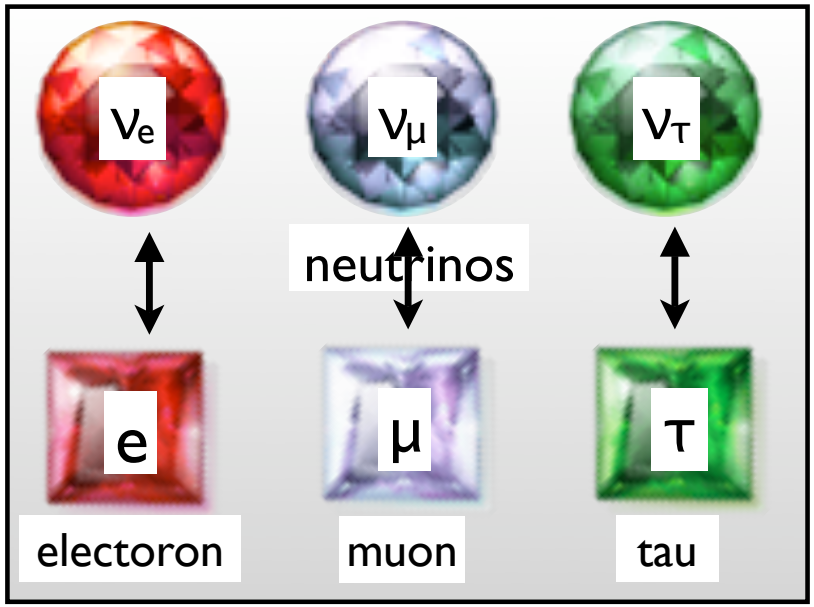
3, Non-equilibrium state to occur

Sakharov's three conditions

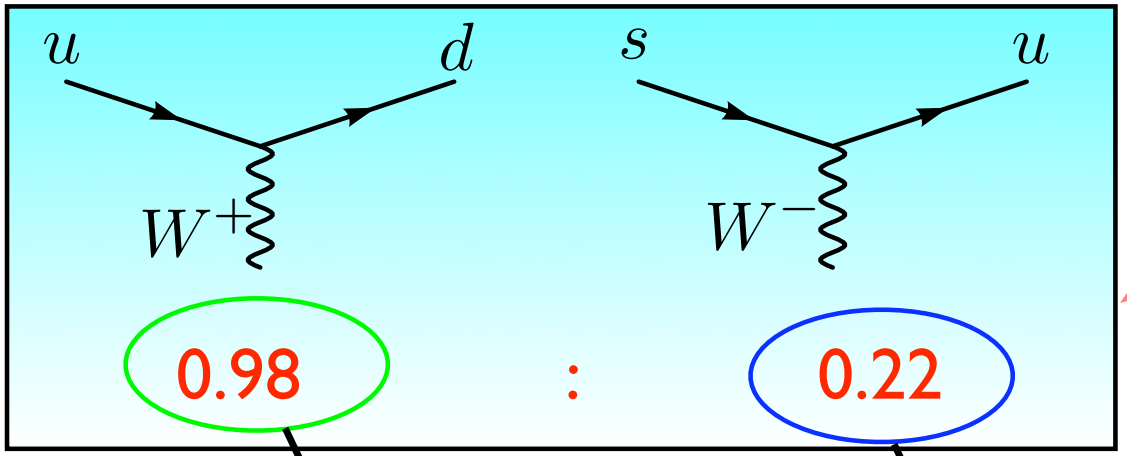
This question is still a mystery in particle physics...

Charged Current





Different strength for different flavours?!



Strange-Up coupling is even smaller than Up-Down!!!

Physical Down-Strange quarks are defined as

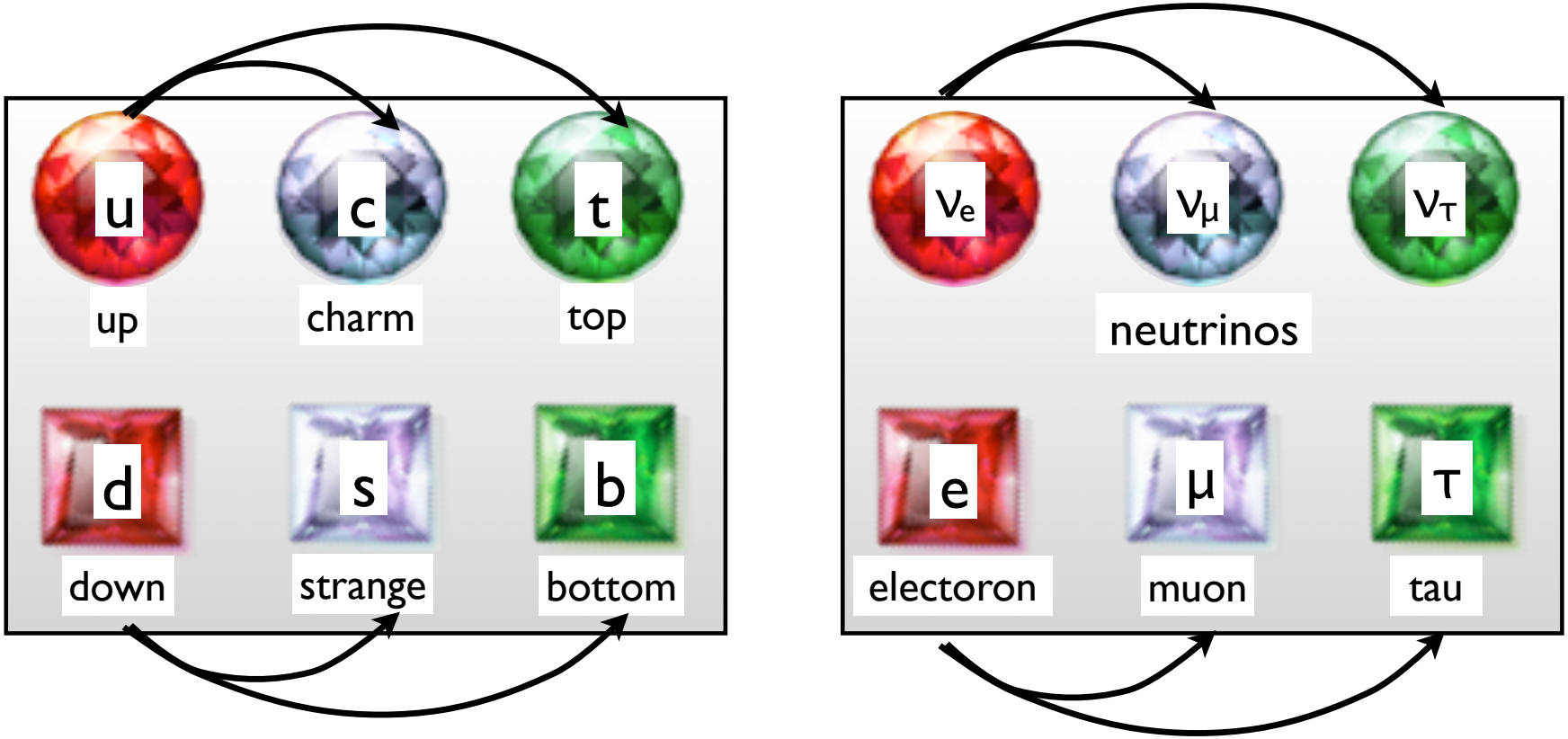
$$\begin{pmatrix} d' \\ s' \end{pmatrix} \longrightarrow \begin{pmatrix} \cos\theta_c & \sin\theta_c \\ -\sin\theta_c & \cos\theta_c \end{pmatrix} \begin{pmatrix} d \\ s \end{pmatrix}$$

$$J_\mu^{\text{hadron}} = \cos\theta_c \bar{u}\gamma^\mu(1-\gamma_5)d + \sin\theta_c \bar{u}\gamma^\mu(1-\gamma_5)s$$

$\sin\theta_c = 0.22, \cos\theta_c = 0.98$

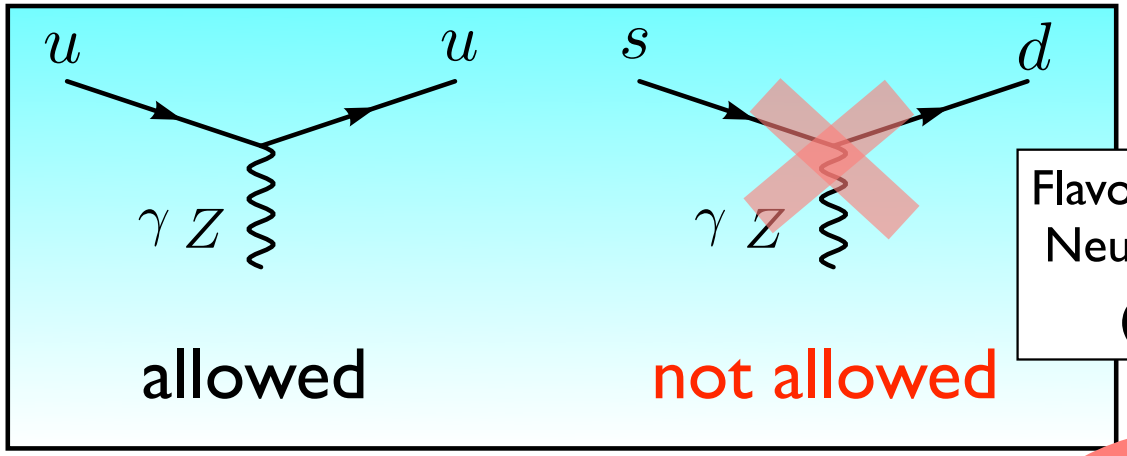
Cabibbo angle '63

Neutral Current



* Actually forbidden...

Forbidding the FCNC
~ GIM mechanism ~



Flavour Changing Neutral Current (FCNC)

Prediction of charm quark!

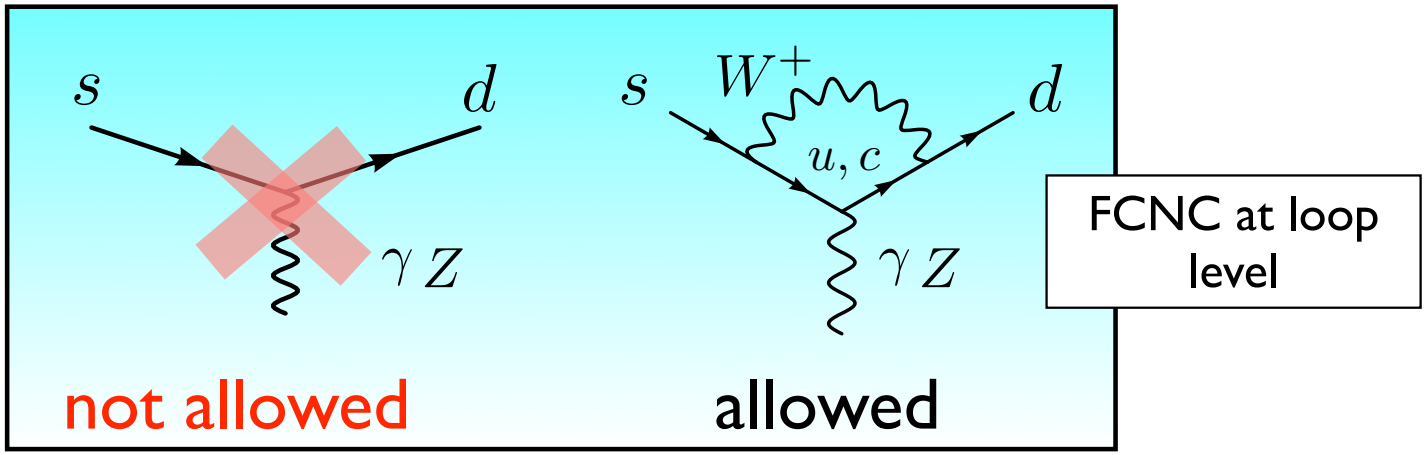
If SU(2) doublet is

$$\begin{pmatrix} \nu_e \\ e \end{pmatrix}, \quad \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}, \quad \begin{pmatrix} u \\ d\cos\theta_c + s\sin\theta_c \end{pmatrix}, \quad \begin{pmatrix} c \\ -d\cos\theta_c + s\sin\theta_c \end{pmatrix}$$

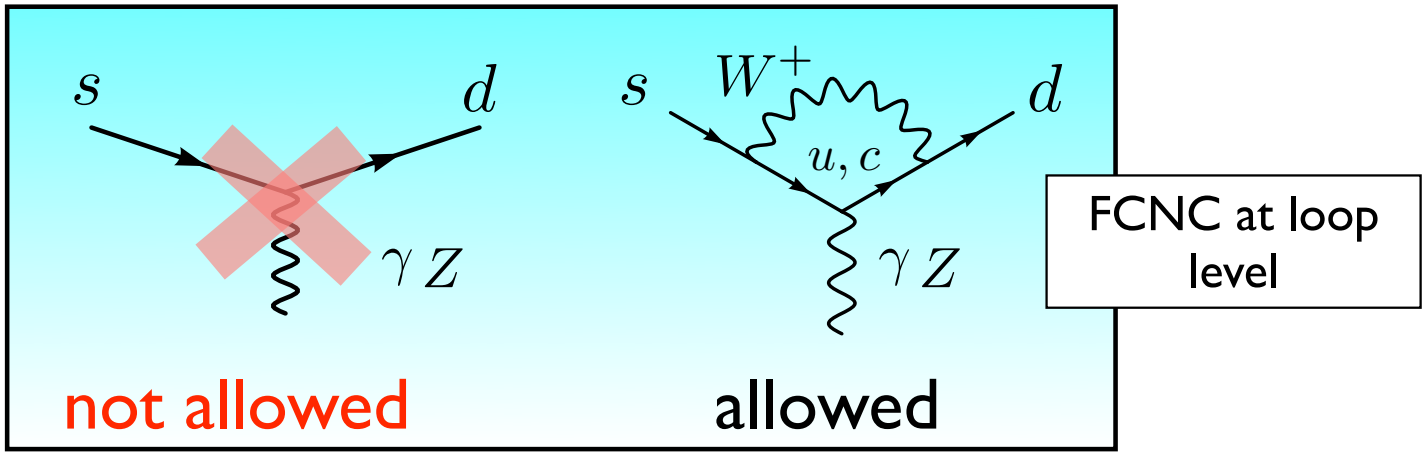
The cross-term would cancel out!

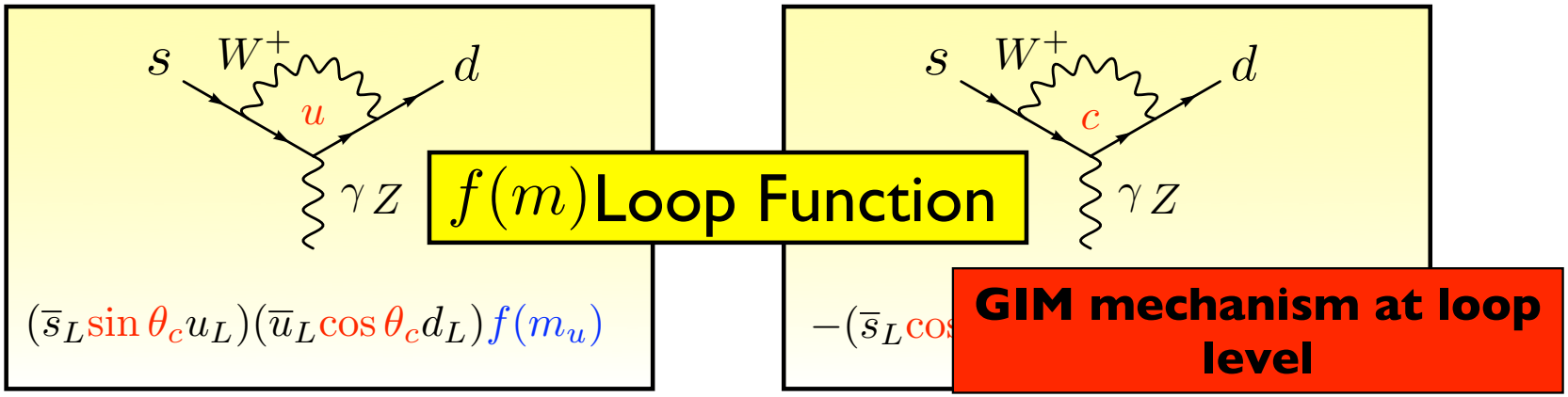
Glashow, Illiopolous, Maiani
(GIM mechanism '70)

Forbidding the FCNC at loop level
~ GIM mechanism 2~



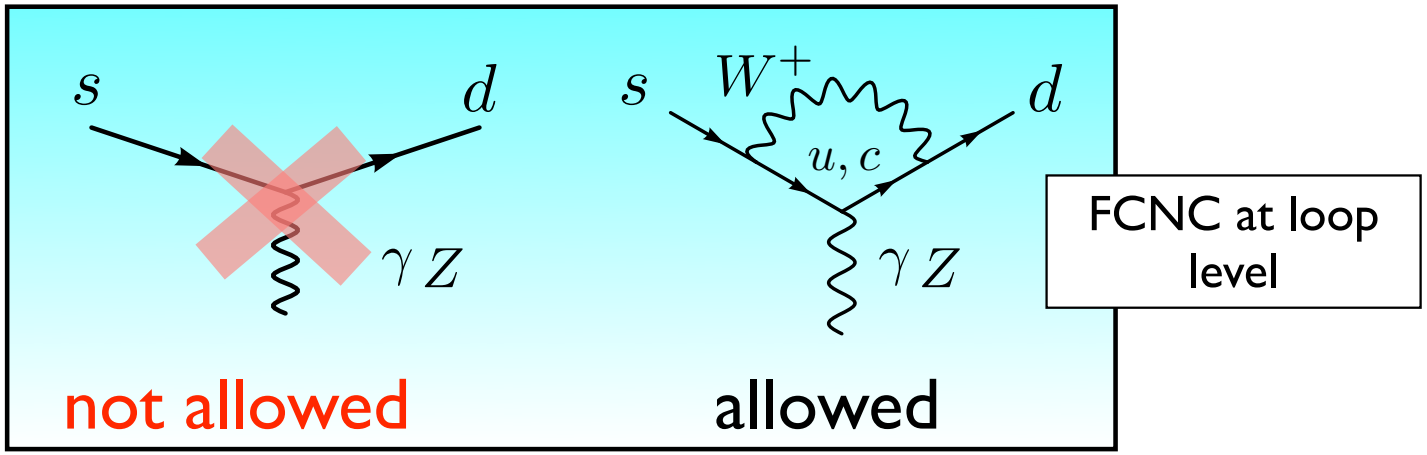
Forbidding the FCNC at loop level
~ GIM mechanism 2~

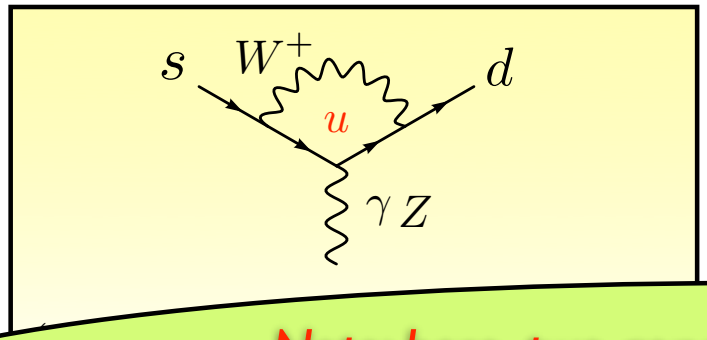


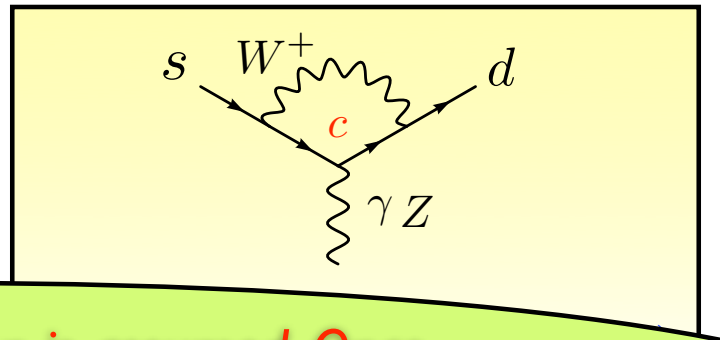


At the limit of mu=mc, two diagrams exactly cancel!!!

Forbidding the FCNC at loop level
~ GIM mechanism 2~

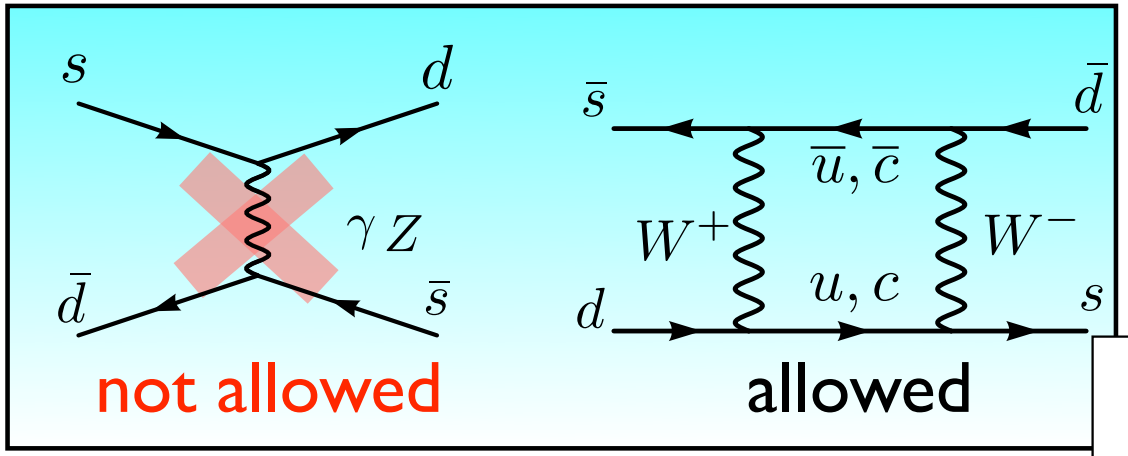




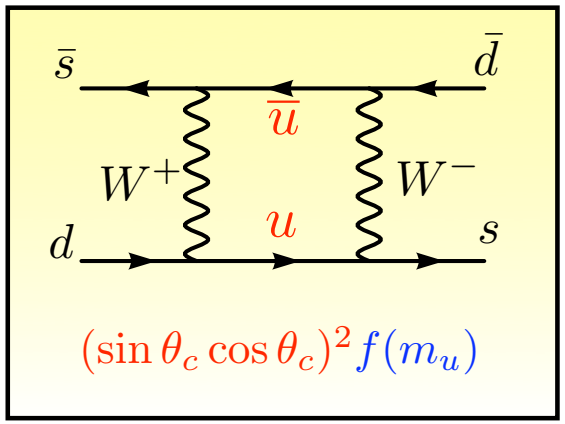


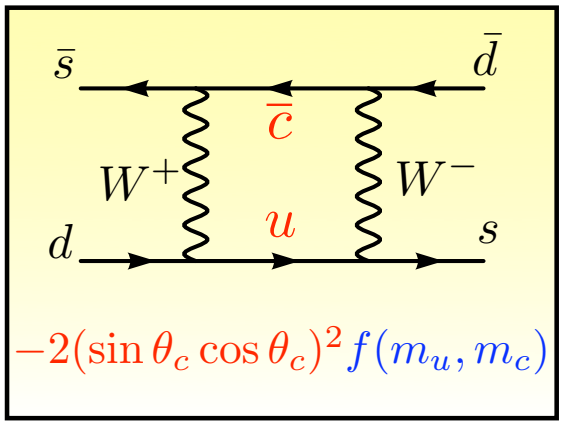
Note: here, two-generation is assumed. Once the top quark is introduced, the FCNC at loop level becomes significantly large! (we'll see soon...)

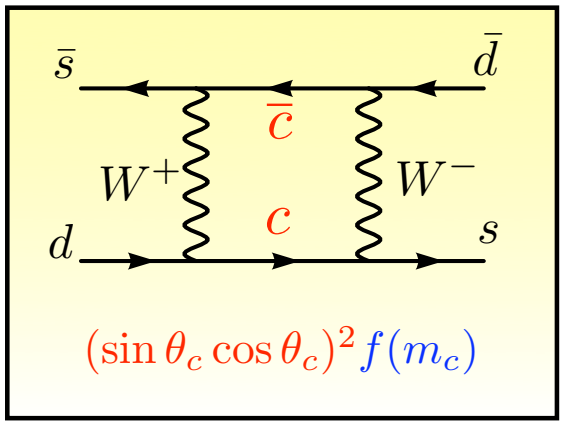
Forbidding the FCNC at loop level
~ GIM mechanism 2~



FCNC at loop level

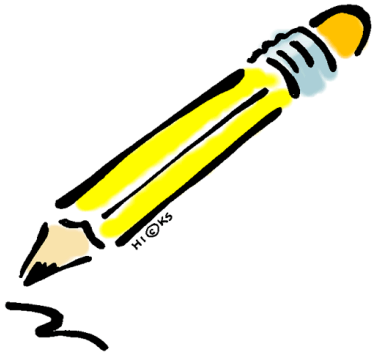


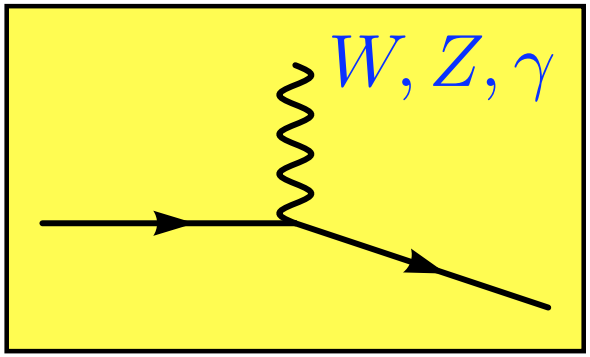


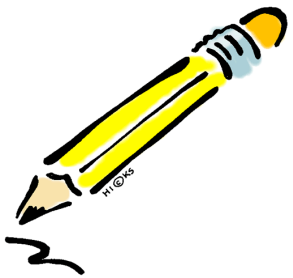


Again, at the limit of mu=mc, three diagrams exactly cancel.

Theoretical description of
Electroweak Interaction of SM







Electroweak interaction

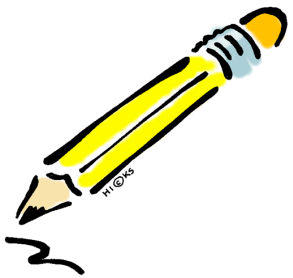
$$G_F = \frac{g^2}{4\sqrt{2}m_W^2}$$

$$\begin{aligned}
\mathcal{L} &= \sum_i \Big[\overline{E_{iL}}(i\partial\!\!\!/)E_{iL} + \overline{l_{iR}}(i\partial\!\!\!/)l_{iR} + \overline{Q_{iL}}(i\partial\!\!\!/)Q_{iL} + \overline{u_{iR}}(i\partial\!\!\!/)u_{iR} + \overline{d_{iR}}(i\partial\!\!\!/)d_{iR} \\
&\quad + g(W_\mu^+ J_W^{\mu+} + W_\mu^- J_W^{\mu-} + Z_\mu^0 J_Z^\mu) + eA_\mu J_{\mathrm{EM}}^\mu\Big] \\
\mathcal{L}_Y &= \sum_{ij} Y_{ij}^u \overline{Q_{iL}}\begin{pmatrix} \phi^0 \\ \phi^- \end{pmatrix} u_{jR} + \sum_{ij} Y_{ij}^d \overline{Q_{iL}}\begin{pmatrix} -\phi^{-\dagger} \\ \phi^{0\dagger} \end{pmatrix} d_{jR} + \sum_{ij} m_{ij}^l \overline{L_{iL}}\begin{pmatrix} -\phi^{-\dagger} \\ \phi^{0\dagger} \end{pmatrix} l_{jR} + h.c.
\end{aligned}$$

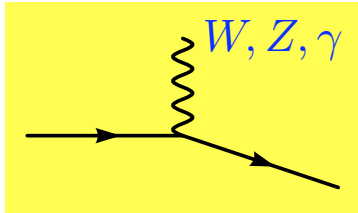
$$q_L = \frac{(1-\gamma_5)}{2}q, \quad q_R = \frac{(1+\gamma_5)}{2}q, \quad , l_L = \frac{(1-\gamma_5)}{2}l, \quad l_R = \frac{(1+\gamma_5)}{2}l$$

$$E_{1L} = \begin{pmatrix} \nu_e \\ e^- \end{pmatrix}_L, E_{2L} = \begin{pmatrix} \nu_\mu \\ \mu^- \end{pmatrix}_L, E_{3L} = \begin{pmatrix} \nu_\tau \\ \tau^- \end{pmatrix}_L, Q_{1L} = \begin{pmatrix} u \\ d \end{pmatrix}_L, Q_{2L} = \begin{pmatrix} v \\ d \end{pmatrix}_L, Q_{3L} = \begin{pmatrix} t \\ b \end{pmatrix}_L$$

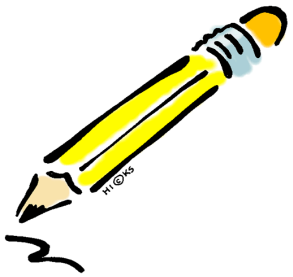
$$u_{1R} = u_R,\ u_{2R} = c_R,\ u_{3R} = t_R,\ d_{1R} = d_R,\ d_{2R} = s_R,\ d_{3R} = b_R,\ l_{1R} = e_R,\ l_{2R} = \mu_R,\ l_{3R} = \tau_R$$



Charged and Neutral Currents



$$
\begin{aligned}
J_W^{\mu+} &= \frac{1}{\sqrt{2}}(\overline{\nu_{iL}}\gamma^\mu L_{iL} + \overline{U_{iL}}\gamma^\mu D_{iL}) \\
J_W^{\mu-} &= \frac{1}{\sqrt{2}}(\overline{L_{iL}}\gamma^\mu \nu_{iL} + \overline{D_{iL}}\gamma^\mu U_{iL}) \\
J_Z^\mu &= \frac{1}{\sqrt{\cos\theta_w}}\Big[\overline{\nu_{iL}}\gamma^\mu(\frac{1}{2})\nu_{iL} + \overline{L_{iL}}\gamma^\mu(-\frac{1}{2}+\sin^2\theta_w)L_{iL} + \overline{l_{iR}}\gamma^\mu(\sin^2\theta_w)l_{iR} \\
&\quad +\overline{U_{iL}}\gamma^\mu(\frac{1}{2}-\frac{2}{3}\sin^2\theta_w)U_{iL} + \overline{u_{iR}}\gamma^\mu(-\frac{2}{3}\sin^2\theta_w)u_{iR} \\
&\quad +\overline{D_{iL}}\gamma^\mu(-\frac{1}{2}+\frac{1}{3}\sin^2\theta_w)D_{iL} + \overline{d_{iR}}\gamma^\mu(\frac{1}{3}\sin^2\theta_w)d_{iR}\,\Big] \\
J_{\mathrm{EM}}^\mu &= \overline{l_i}\gamma^\mu(-1)l_i + \overline{u_i}\gamma^\mu(+\frac{2}{3})u_i + \overline{d_i}\gamma^\mu(-\frac{1}{3})d_i
\end{aligned}
$$



Yukawa Interaction (quark)

$$\mathcal{L}_Y = \sum_{ij} Y^u_{ij} \overline{Q_{iL}} \begin{pmatrix} \phi^0 \\ \phi^- \end{pmatrix} u_{jR} + \sum_{ij} Y^d_{ij} \overline{Q_{iL}} \begin{pmatrix} -\phi^{-\dagger} \\ \phi^{0\dagger} \end{pmatrix} d_{jR} + h.c.$$

$Y_{u,d}$ Yukawa coupling (generic 3x3 matrix)

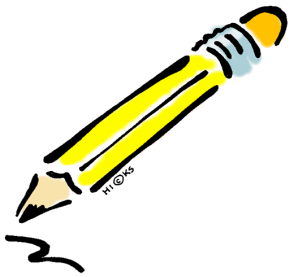
Spontaneous Symmetry Breaking

$$\langle \phi^0 \rangle_{\rm VAC} = v$$

$$m^u_{ij} = Y^u_{ij} \langle \phi^0 \rangle_{\rm VAC}, \quad m^d_{ij} = Y^d_{ij} \langle \phi^0 \rangle^*_{\rm VAC}$$

$$\mathcal{L}_Y = \sum_{ij} m^u_{ij} \overline{U_{iL}} u_{jR} + \sum_{ij} m^d_{ij} \overline{D_{i_L}} d_{jR} + h.c.$$

$$(\overline{u}_L, \overline{c}_L, \overline{t}_L) \begin{pmatrix} \cdots & \cdots & \cdots \\ \cdots & m^u_{ij} & \cdots \\ \cdots & \cdots & \cdots \end{pmatrix} \begin{pmatrix} u_R \\ c_R \\ t_R \end{pmatrix} \quad (\overline{d}_L, \overline{s}_L, \overline{b}_L) \begin{pmatrix} \cdots & \cdots & \cdots \\ \cdots & m^d_{ij} & \cdots \\ \cdots & \cdots & \cdots \end{pmatrix} \begin{pmatrix} d_R \\ s_R \\ b_R \end{pmatrix}$$



Yukawa Interaction (quark)

Mass eigen-basis

$K^{U/D}_{L/R}$: 3x3 matrix, "^" is for 3 vector

$$\hat{u}_L = K^u_L \hat{u}'_L, \quad \hat{u}_R = K^u_R \hat{u}'_R \qquad \hat{d}_L = K^d_L \hat{d}'_L, \quad \hat{d}_R = K^d_R \hat{d}'_R$$

$$m^{u\prime}_{\rm diag} = K^{u\dagger}_L m^u K^u_R, \quad m^{d\prime}_{\rm diag} = K^{d\dagger}_L m^d K^d_R$$

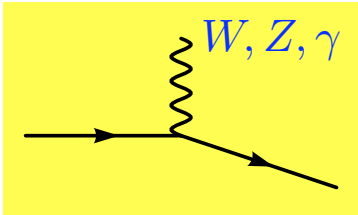
ex. down-type

$$(\overline{d}_L, \overline{s}_L, \overline{b}_L) \begin{pmatrix} \cdots & \cdots & \cdots \\ \cdots & m^d_{ij} & \cdots \\ \cdots & \cdots & \cdots \end{pmatrix} \begin{pmatrix} d_R \\ s_R \\ b_R \end{pmatrix}$$

Inserting the unit matrix

$$= \underbrace{(\overline{d}_L, \overline{s}_L, \overline{b}_L) K^d_L}_{\overline{\hat{d}}'_L} \underbrace{(K^d_L)^\dagger \begin{pmatrix} \cdots & \cdots & \cdots \\ \cdots & m^d_{ij} & \cdots \\ \cdots & \cdots & \cdots \end{pmatrix} K^d_R}_{\begin{pmatrix} m_d & 0 & 0 \\ 0 & m_s & 0 \\ 0 & 0 & m_b \end{pmatrix}} \underbrace{(K^d_R)^\dagger \begin{pmatrix} d_R \\ s_R \\ b_R \end{pmatrix}}_{\hat{d}'_R}$$

Charged and Neutral Currents on the mass basis



$$
\begin{aligned}
J_W^{\mu+} &= \frac{1}{\sqrt{2}}(\hat{\bar{u}}_L \gamma^\mu \hat{d}_L) \\
J_W^{\mu-} &= \frac{1}{\sqrt{2}}(\hat{\bar{d}}_L \gamma^\mu \hat{u}_L) \\
J_3^{\mu} &= \frac{1}{2}\hat{\bar{u}}_L \gamma^\mu \hat{u}_L - \frac{1}{2}\hat{\bar{d}}_L \gamma^\mu \hat{d}_L
\end{aligned}
$$

$$
\hat{u}_L = K_L^u \hat{u}'_L, \quad \hat{u}_R = K_R^u \hat{u}'_R
$$
$$
\hat{d}_L = K_L^d \hat{d}'_L, \quad \hat{d}_R = K_R^d \hat{d}'_R
$$

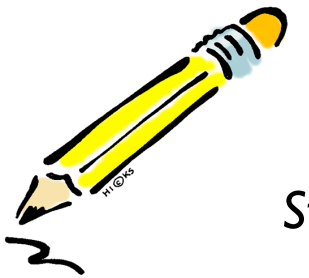
mass basis

$$
\begin{aligned}
(K_L^u)^\dagger K_L^d &\equiv V_{\rm CKM} \\
(K_L^d)^\dagger K_L^u &\equiv V_{\rm CKM}^\dagger
\end{aligned}
$$

$$
\begin{aligned}
J_W^{\mu+} &= \frac{1}{\sqrt{2}}(\hat{\bar{u}}'_L (K_L^u)^\dagger \gamma^\mu K_L^d \hat{d}'_L) \\
J_W^{\mu-} &= \frac{1}{\sqrt{2}}(\hat{\bar{d}}'_L (K_L^d)^\dagger \gamma^\mu K_L^u \hat{u}'_L) \\
J_3^{\mu} &= \frac{1}{2}\hat{\bar{u}}'_L (K_L^u)^\dagger \gamma^\mu K_L^u \hat{u}'_L - \frac{1}{2}\hat{\bar{d}}'_L (K_L^d)^\dagger \gamma^\mu K_L^d \hat{d}'_L
\end{aligned}
$$

$$
\begin{aligned}
(K_L^u)^\dagger K_L^u &= \mathbf{1} \\
(K_L^d)^\dagger K_L^d &= \mathbf{1}
\end{aligned}
$$

No tree FCNC in SM!

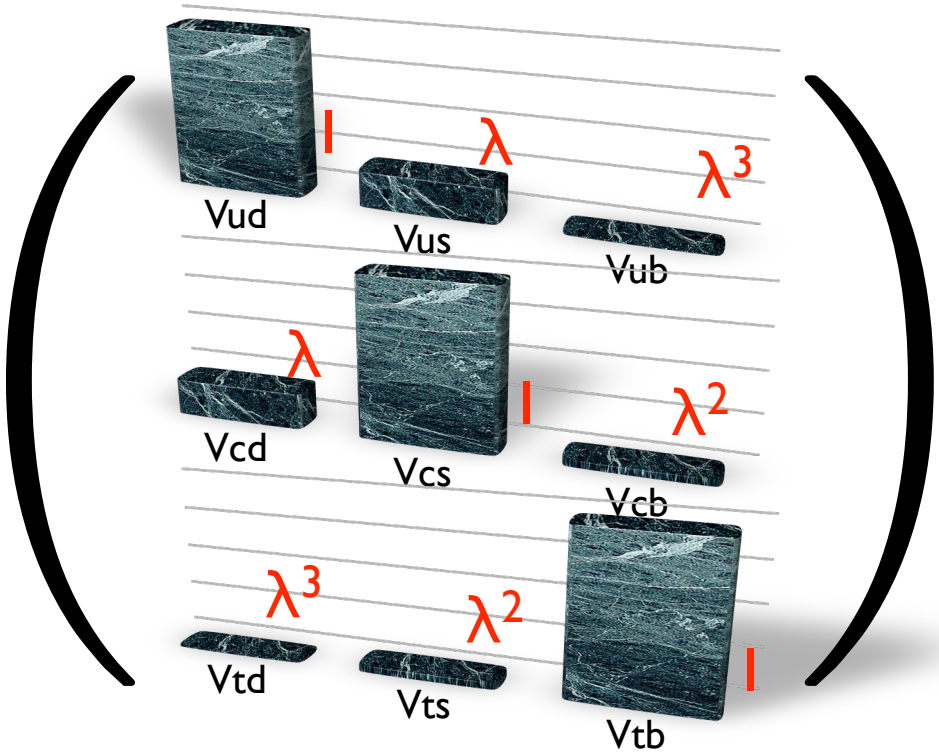


CKM matrix elements

Strength of the weak couplings between different flavours

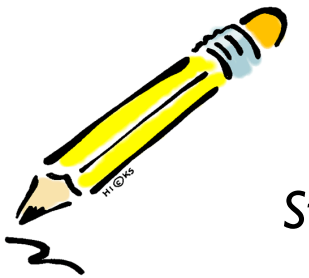
$$J_W^{\mu+} = (\overline{u_L}, \overline{c_L}, \overline{t_L}) \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d_L \\ s_L \\ b_L \end{pmatrix}$$

It turned out that the CKM matrix is close to diagonal!



It contains many information.
Ex) the life time difference of K, D, B:

$\tau_D \ll \tau_K$, $\tau_D < \tau_B$



CKM matrix elements

Strength of the weak couplings between different flavours

$$J_W^{\mu+} = (\overline{u_L}, \overline{c_L}, \overline{t_L}) \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d_L \\ s_L \\ b_L \end{pmatrix}$$

It provides a source of CP violation!

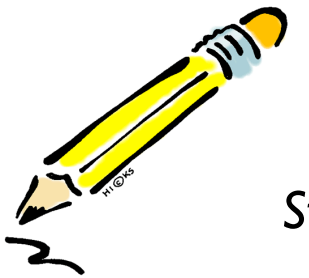
It took nearly 10 years to find the solution for this complex coupling since discovery of CP violation...

Parameter counting of the unitary matrix to go to diagonalize the Yukawa coupling

Unitarity condition $UU^\dagger = 1 \longrightarrow 2n^2 - n^2 = n^2$

↓

Phase convention $n^2 - (2n-1) = (n-1)^2$



CKM matrix elements

Strength of the weak couplings between different flavours

$$J_W^{\mu+} = (\overline{u_L}, \overline{c_L}, \overline{t_L}) \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d_L \\ s_L \\ b_L \end{pmatrix}$$

For two generation, only 1 rotation remains while for three generation, 3 rotations plus 1 phase remains (prediction of the 3rd generation).

Kobayashi, Maskawa
(1973)

Unitarity condition $UU^\dagger = 1 \longrightarrow 2n^2 - n^2 = n^2$

↓

Phase convention $n^2 - (2n-1) = (n-1)^2$

3 mixings and 1 phase

phase. The rotation is defined as follows:

$$\omega(\theta_{12},0) = \begin{pmatrix} \cos\theta_{12} & \sin\theta_{12} & 0 \\ -\sin\theta_{12} & \cos\theta_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{1}$$

$$\omega(\theta_{13},\delta_1) = \begin{pmatrix} \cos\theta_{13} & 0 & \sin\theta_{13}e^{-i\delta_1} \\ 0 & 1 & 0 \\ -\sin\theta_{13}se^{-i\delta_1} & 0 & \cos\theta_{13} \end{pmatrix} \tag{2}$$

$$\omega(\theta_{23},0) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta_{23} & \sin\theta_{23} \\ 0 & -\sin\theta_{23} & \cos\theta_{23} \end{pmatrix} \tag{3}$$

Then, the standard CKM matrix is obtained by choosing to multiply these matrices in the following order:

$$V_{\mathrm{CKM}}^{3\times3} = \omega(\theta_{23},0)\omega(\theta_{13},\delta_1)\omega(\theta_{12},0). \tag{4}$$

$$V = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix},$$

We need experimental verifications that all 9 complex elements can be explained by the 4 input parameters.

A new parameterization

phase. The rotation is defined as follows:

$$\omega(\theta_{12}, 0) = \begin{pmatrix} \cos\theta_{12} & \sin\theta_{12} & 0 \\ -\sin\theta_{12} & \cos\theta_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{1}$$

We re-parametrize in terms of λ, A, ρ and η:

$$\sin\theta_{12} = \lambda, \sin\theta_{13} = A(\rho - i\eta)\lambda^3, \sin\theta_{23} = A\lambda^2$$

Realizing the hierarchy in the matrix, we expand in terms of λ~=0.22:

$$\sin\theta_{12} = \mathcal{O}(\lambda), \sin\theta_{23} = \mathcal{O}(\lambda^2), \sin\theta_{13} = \mathcal{O}(\lambda^3)$$

Then,
order:

$$V = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix},$$

We need experimental verifications that all 9 complex elements can be explained by the 4 input parameters.

Wolfenstein's parameterization

$$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

Expansion in order λ^3

$$= \begin{pmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-\rho - i\eta) & -A\lambda^2 & 1 \end{pmatrix} + \mathcal{O}(\lambda^4)$$

$$= \begin{pmatrix} 1-\lambda^2/2-\lambda^4/8 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\lambda^2/2+(-1/8-A^2/2)\lambda^4 & A\lambda^2 \\ A\lambda^3(1-\rho - i\eta) & -A\lambda^2 + A\lambda^4(1/2-\rho - i\eta) & 1 \end{pmatrix}$$

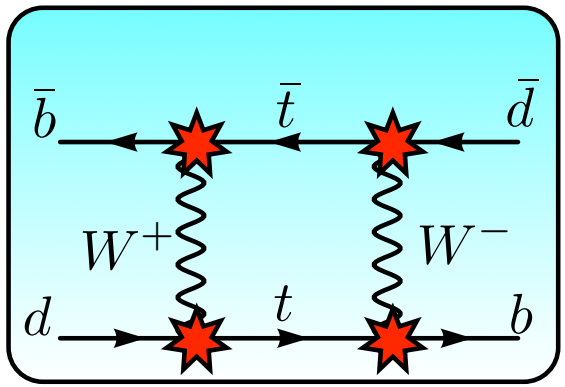
$$+ \quad \mathcal{O}(\lambda^5)$$

Expansion in order λ^4

Computing q/p for B system

In the B system, we have $M_{12} >> \Gamma_{12}$, thus

$$\frac{q}{p} = \sqrt{\frac{M_{12}^* - \frac{i}{2}\Gamma_{12}^*}{M_{12} - \frac{i}{2}\Gamma_{12}}} \simeq \sqrt{\frac{M_{12}^*}{M_{12}}} \equiv e^{i\phi}$$



$$M_{12} = \frac{G_F^2 m_W^2}{16\pi^2}(V_{tb}V_{td}^*)^2 S_0\left(\frac{m_t^2}{m_W^2}\right) \times \eta_{\rm QCD} \frac{\langle B^0|(\bar{d}b)_{\rm V-A}(\bar{d}b)_{\rm V_A}|\overline{B}^0\rangle}{m_B}$$

Loop function dominant=top quark

Strong interaction part

$$V_{td} = A\lambda^3(1-\rho-i\eta)$$
$$V_{tb} = 1$$

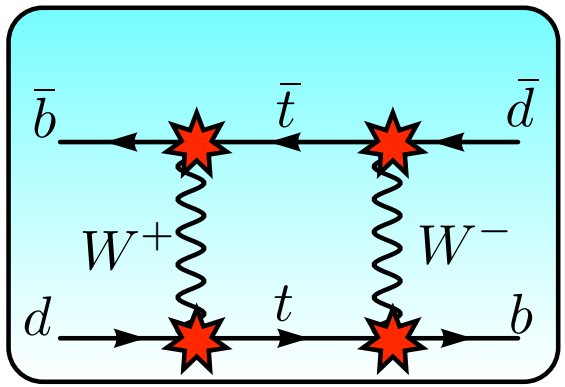
$$\Rightarrow \boxed{\frac{q}{p} = e^{-2i\ arg(V_{tb}^* V_{td})}}$$

DONE!

Computing q/p for B system

In the B system, we have $M_{12} >> \Gamma_{12}$, thus

$$\frac{q}{p} = \sqrt{\frac{M_{12}^* - \frac{i}{2}\Gamma_{12}^*}{M_{12} - \frac{i}{2}\Gamma_{12}}} \simeq \sqrt{\frac{M_{12}^*}{M_{12}}} =$$



$$M_{12} = \frac{G_F^2 m_W^2}{16\pi^2}($$

$$\times \eta_{\rm QCD} \frac{\langle B^0|(\bar{d}$$

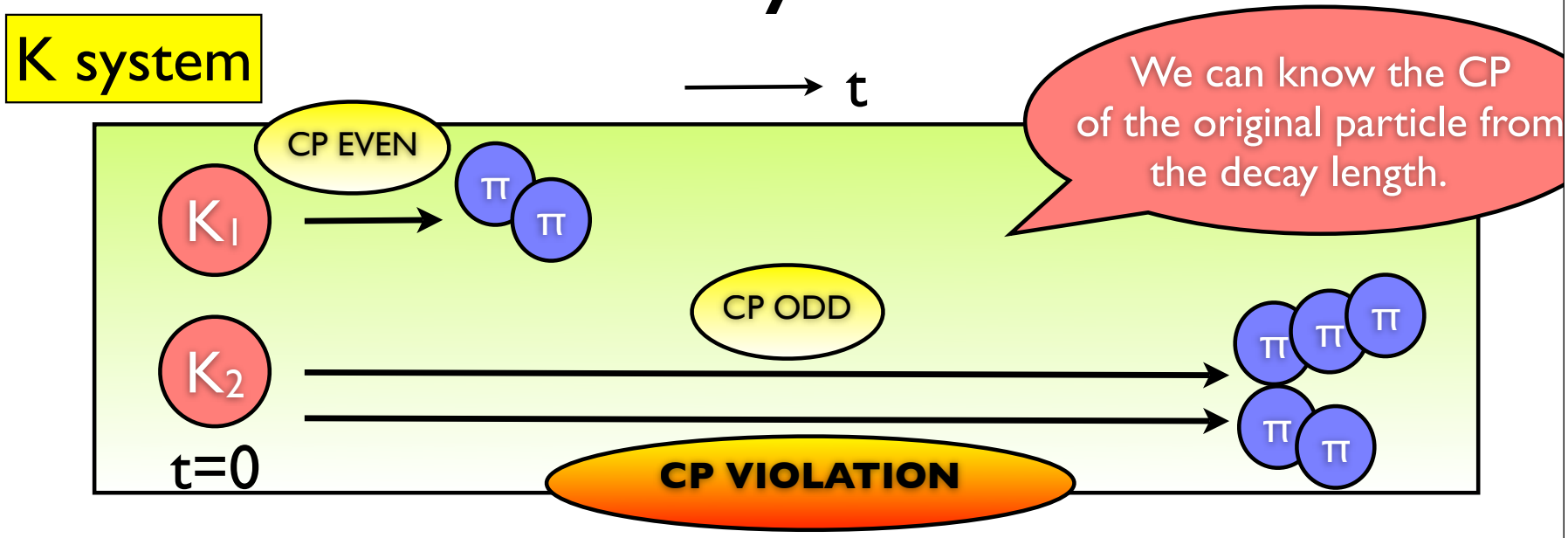
$$V_{td} = A\lambda^3(1-\rho-i\eta)$$
$$V_{tb} = 1$$

$\Rightarrow$ $\frac{q}{p} = e^{-2i\ arg(V_{td}^* V_{tb})}$

If the Kobayashi-Maskawa ansatz is correct, the CP violation in B system should be very large!!!

B factories!

CP violation in K system vs B system



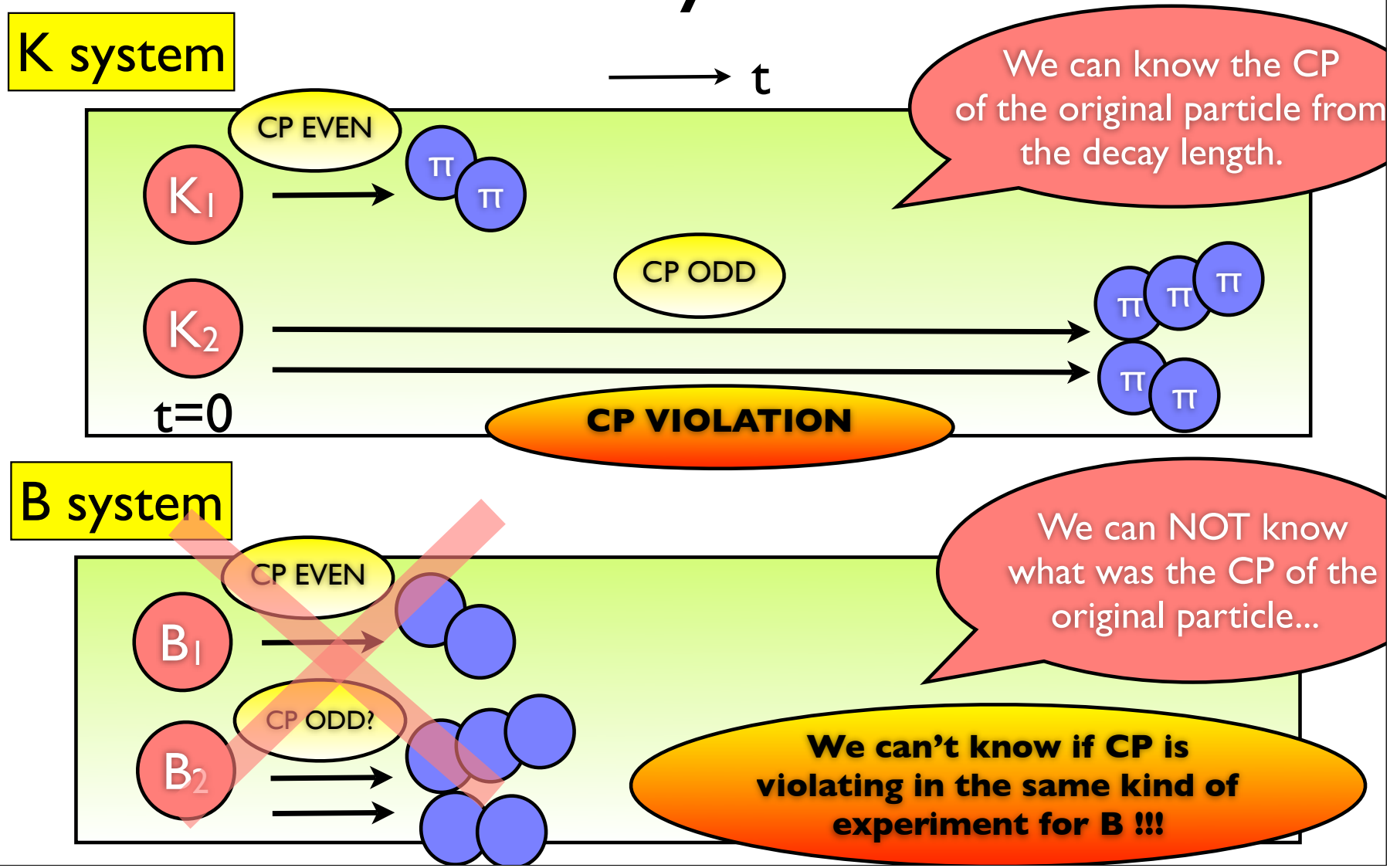
$$\mathcal{CP}|K_1\rangle = +\frac{1}{\sqrt{2}}(|K^0\rangle + |\overline{K}^0\rangle) = |K_1\rangle \quad \boxed{\textbf{CP EVEN}}$$

$$\mathcal{CP}|K_2\rangle = -\frac{1}{\sqrt{2}}(|K^0\rangle - |\overline{K}^0\rangle) = -|K_2\rangle \quad \boxed{\textbf{CP ODD}}$$

$$|K_S\rangle = \frac{1}{\sqrt{2}}\left(p|K^0\rangle + q|\overline{K}^0\rangle\right) = \frac{p}{2}\left[(1+\frac{q}{p})|K_1\rangle + (1-\frac{q}{p})|K_2\rangle)\right]$$

$$|K_L\rangle = \frac{1}{\sqrt{2}}\left(p|K^0\rangle - q|\overline{K}^0\rangle\right) = \frac{p}{2}\left[(1-\frac{q}{p})|K_1\rangle + (1+\frac{q}{p})|K_2\rangle)\right]$$

CP violation in K system vs B system



Time evolution formula

Let us describe the time evolution, in terms of the Hilbert space:

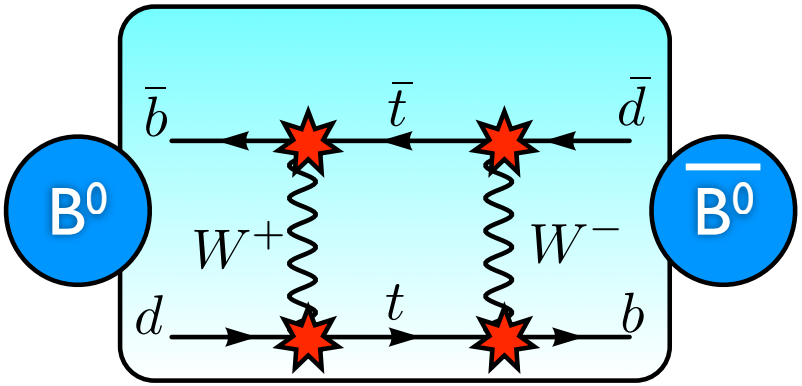
$$|\Psi(t)\rangle = a(t)|B\rangle + b(t)|\overline{B}\rangle$$

The time dependence can be described by the Schroedinger equation:

$$i\hbar\frac{\partial}{\partial t}\Psi(t) = \mathcal{H}\Psi(t); \qquad \Psi(t) = \begin{pmatrix} a(t) \\ b(t) \end{pmatrix}$$

where the Hamiltonian is given as

$$\mathcal{H} = \mathbf{M} - \tfrac{i}{2}\mathbf{\Gamma} = \begin{pmatrix} M_{11} - \frac{i}{2}\Gamma_{11} & M_{12} - \frac{i}{2}\Gamma_{12} \\ M_{21} - \frac{i}{2}\Gamma_{21} & M_{22} - \frac{i}{2}\Gamma_{22} \end{pmatrix}$$



Time evolution formula

$$\mathcal{H} = \mathbf{M} - \tfrac{i}{2}\mathbf{\Gamma} = \begin{pmatrix} M_{11} - \frac{i}{2}\Gamma_{11} & M_{12} - \frac{i}{2}\Gamma_{12} \\ M_{21} - \frac{i}{2}\Gamma_{21} & M_{22} - \frac{i}{2}\Gamma_{22} \end{pmatrix}$$

Using CPT invariance (M_{11}=M_{22}, Γ_{11}=Γ_{22}) and ***M*** *and* **Γ** *being Hermitian, we find the eigensystem of this matrix:*

$$M_1 + \frac{i}{2}\Gamma_1 \equiv M_{11} - \frac{i}{2}\Gamma_{11} + \frac{q}{p}\left(M_{12} - \frac{i}{2}\Gamma_{12}\right); \quad \begin{pmatrix} p \\ q \end{pmatrix}$$

$$M_2 + \frac{i}{2}\Gamma_2 \equiv M_{11} - \frac{i}{2}\Gamma_{11} - \frac{q}{p}\left(M_{12} - \frac{i}{2}\Gamma_{12}\right); \quad \begin{pmatrix} p \\ -q \end{pmatrix}$$

Thus, the mass eigenstate of K is obtained as:

$$\begin{aligned} |B_1\rangle &= p|B\rangle + q|\overline{B}\rangle \\ |B_2\rangle &= p|B\rangle - q|\overline{B}\rangle \end{aligned} \quad \textit{with} \quad \frac{q}{p} = \pm\sqrt{\frac{M_{12}^* - \frac{i}{2}\Gamma_{12}^*}{M_{12} - \frac{i}{2}\Gamma_{12}}}$$

Time evolution formula

Now, we obtain the time evolution of the B states:

These states were B or Bbar at t=0.

$$|B(t)\rangle = f_+(t)|B\rangle + \frac{q}{p} f_-(t)|\overline{B}\rangle$$

$$|\overline{B}(t)\rangle = f_+(t)|\overline{B}\rangle + \frac{p}{q} f_-(t)|B\rangle$$

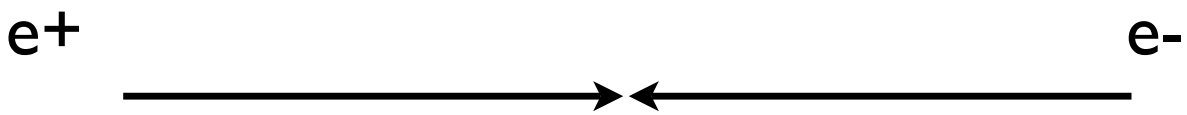
where $$f_\pm = \frac{1}{2} e^{-iM_1 t} e^{-\frac{1}{2}\Gamma_1 t} \left[1 \pm e^{-i\Delta M t} e^{\frac{1}{2}\Delta\Gamma t}\right]$$

with $$\Delta M \equiv M_2 - M_1, \quad \Delta\Gamma \equiv \Gamma_1 - \Gamma_2$$ *+ sign for q/p*

If p/q≠ 1, B and Bbar states behave differently.

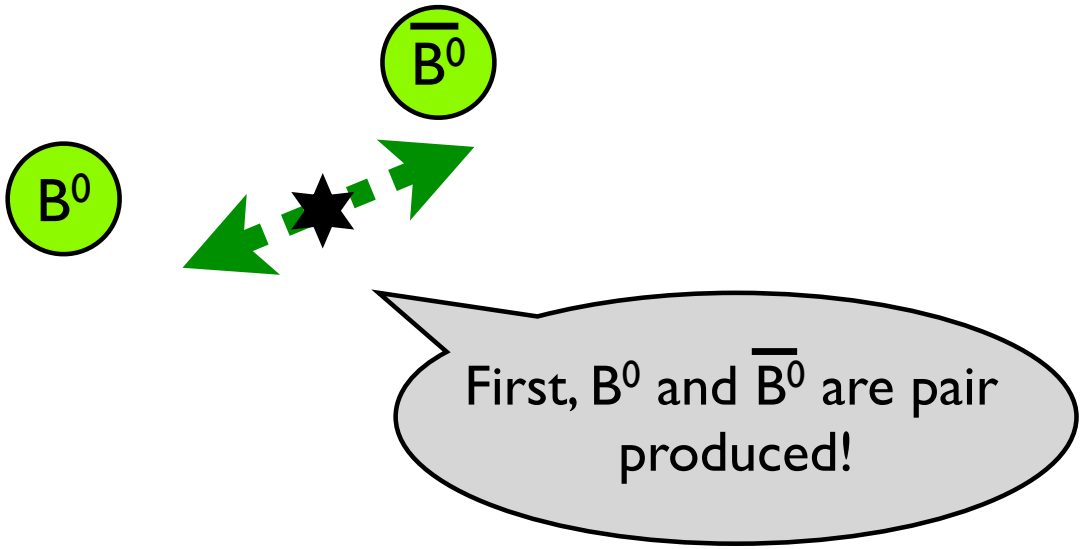
Flavour specific mixing CP violation

(CP Violation in oscillation)



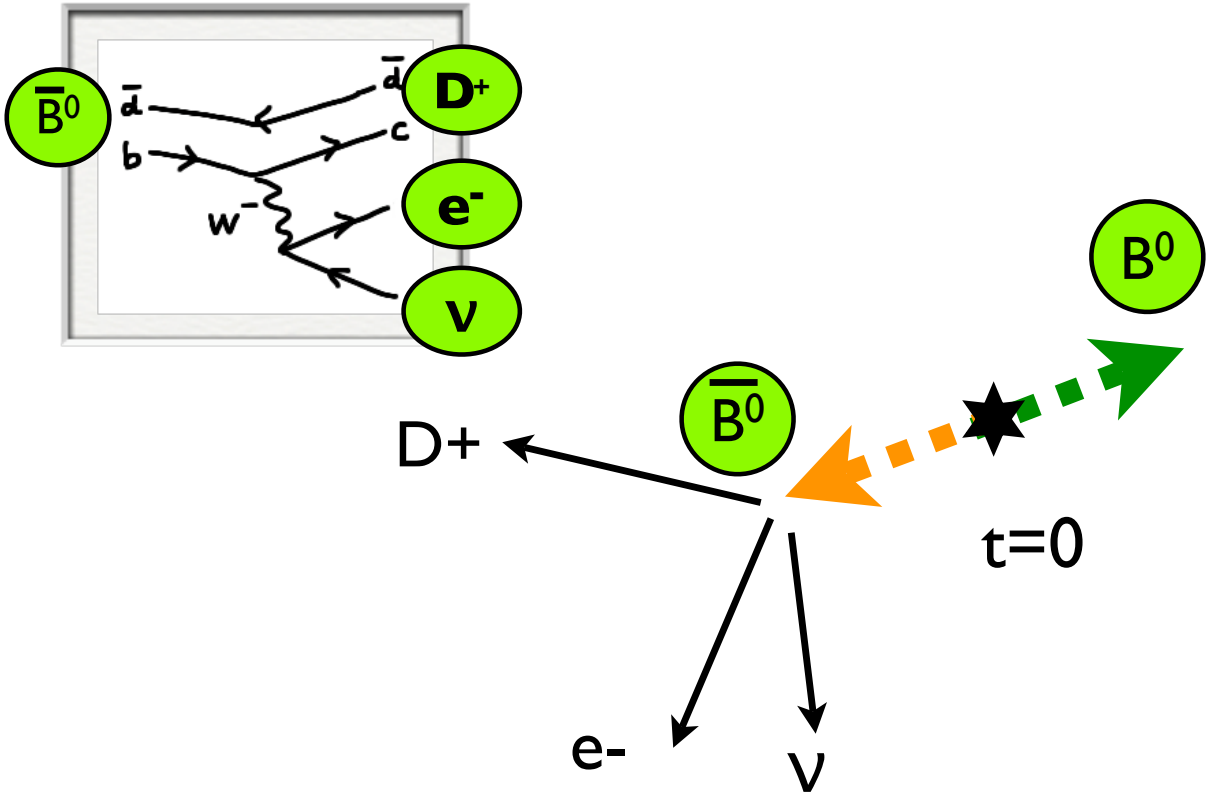
Flavour specific mixing CP violation

(CP Violation in oscillation)



Flavour specific mixing CP violation

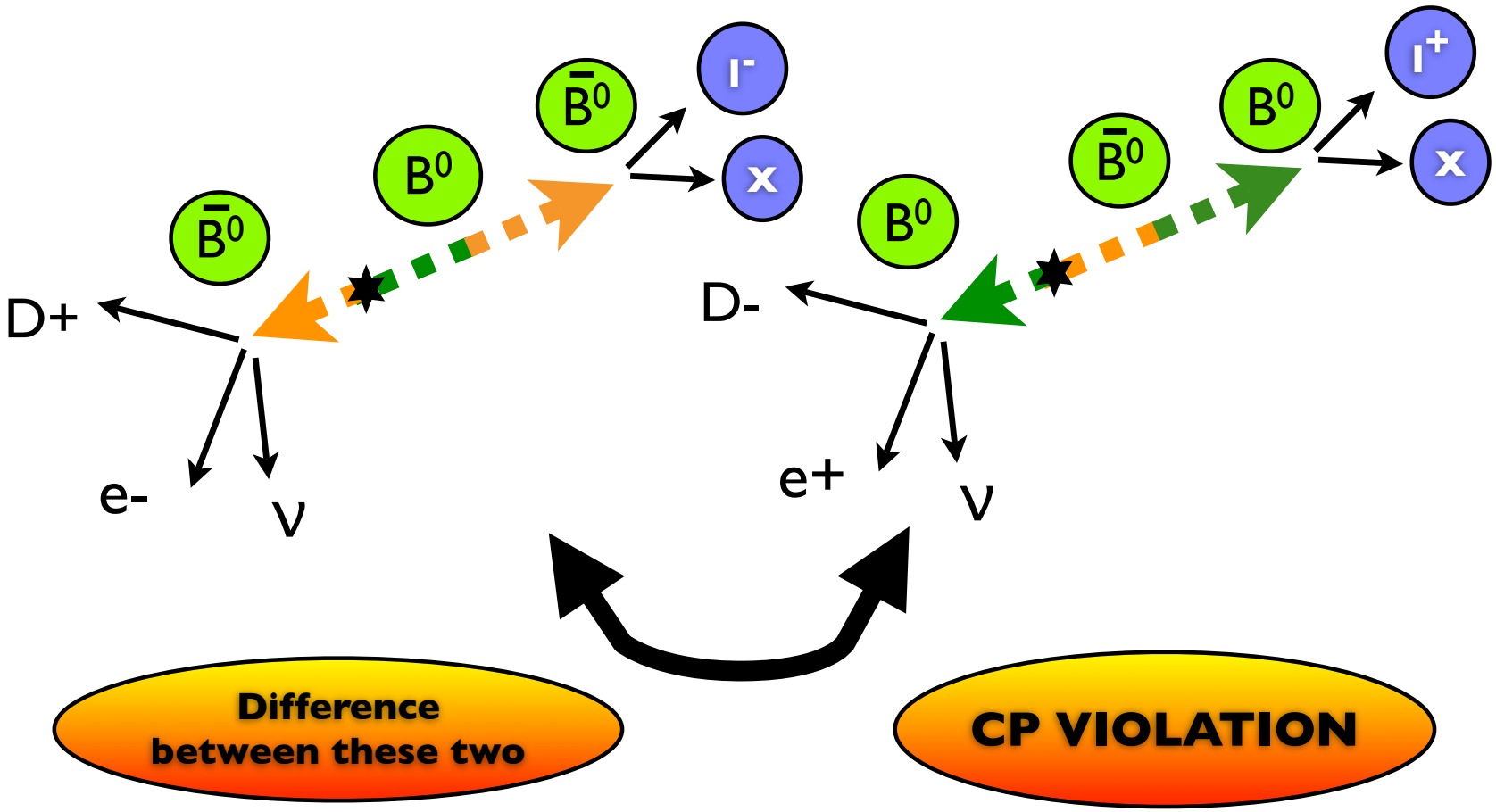
(CP Violation in oscillation)



If one of them decays semi-leptonically, we can tell if it was B^0 or $\overline{B}^0$ on one side at given time, which allows us to tell about the other side.

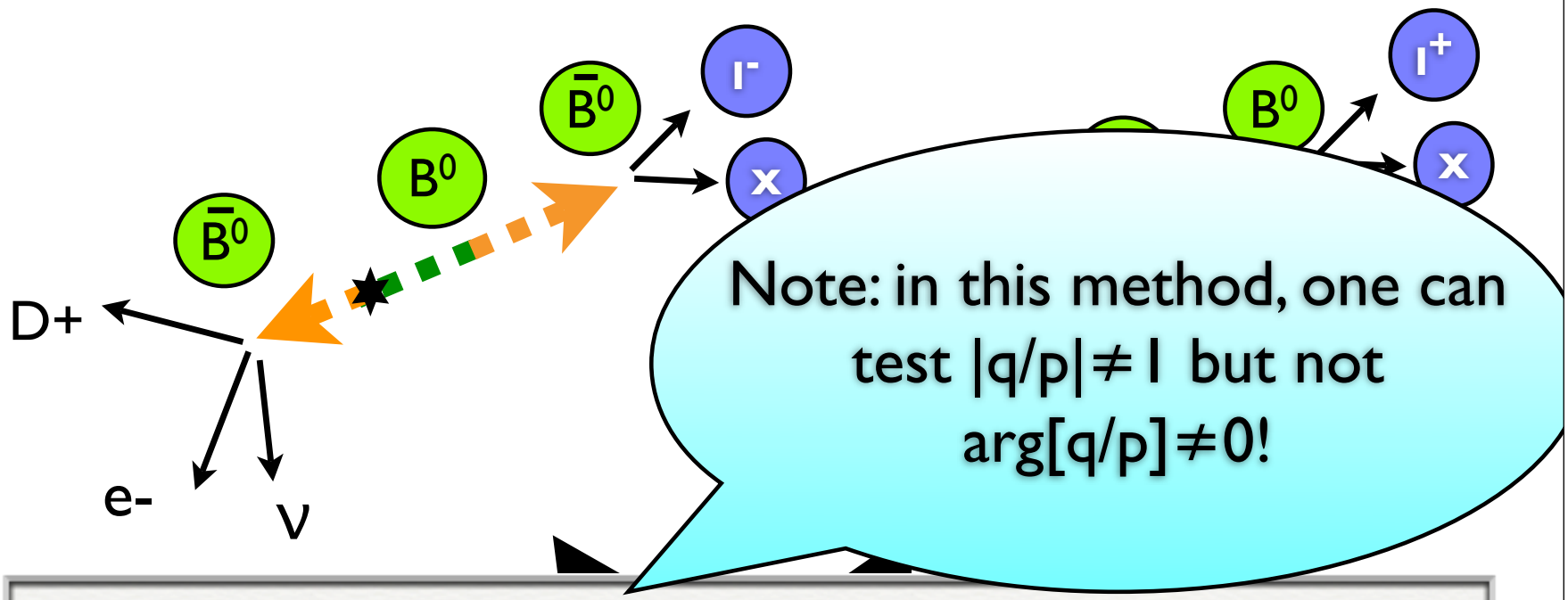
Flavour specific mixing CP violation

(CP Violation in oscillation)



Flavour specific mixing CP violation

(CP Violation in oscillation)

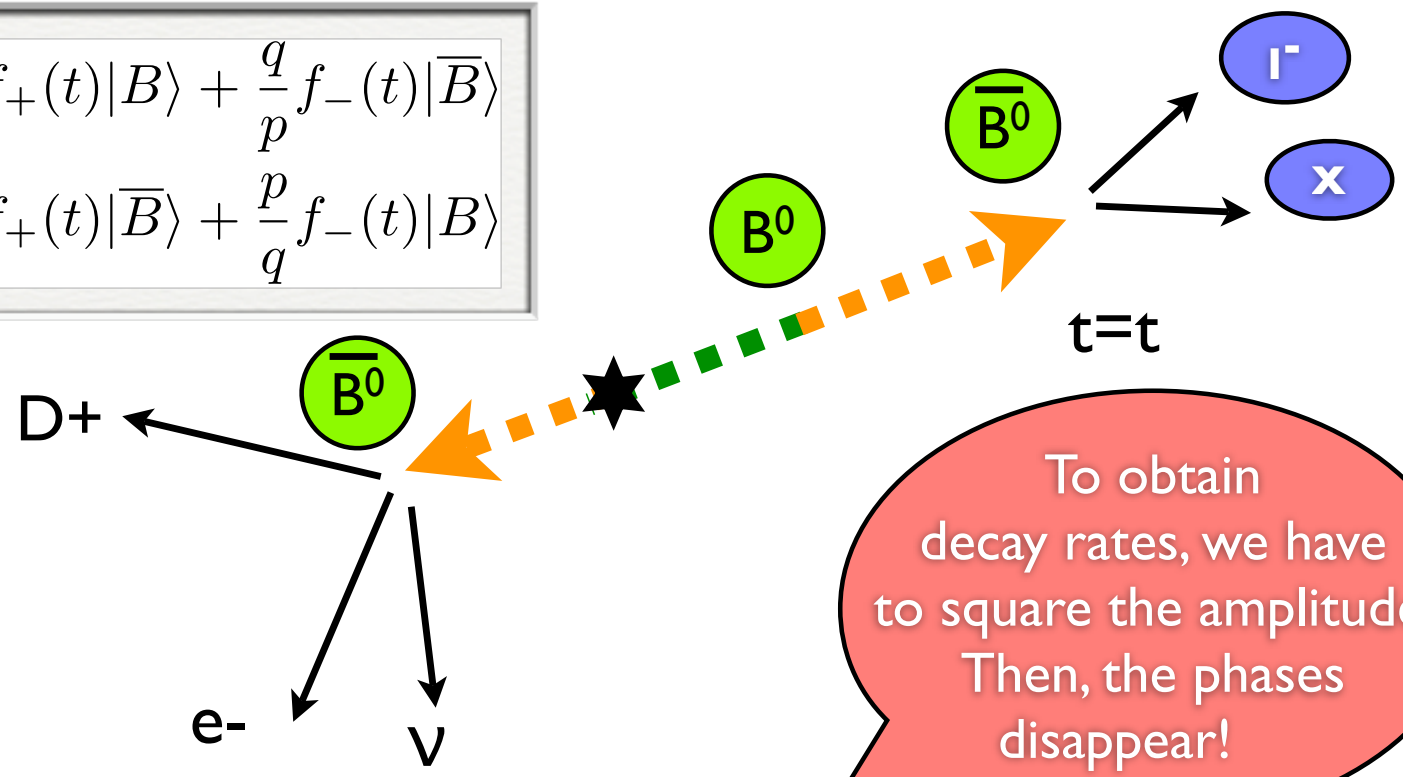


$$\mathcal{A} = \frac{\Gamma(\overline{B}^0 B^0 \to X l^+ l^+) - \Gamma(\overline{B}^0 B^0 \to X l^- l^-)}{\Gamma(\overline{B}^0 B^0 \to X l^+ l^+) + \Gamma(\overline{B}^0 B^0 \to X l^- l^-)} = \frac{|p/q|^4 - 1}{|p/q|^4 + 1}$$

Flavour specific mixing CP violation

(CP Violation in oscillation)

$$
\begin{aligned}
|B(t)\rangle &= f_+(t)|B\rangle + \frac{q}{p} f_-(t)|\overline{B}\rangle \\
|\overline{B}(t)\rangle &= f_+(t)|\overline{B}\rangle + \frac{p}{q} f_-(t)|B\rangle
\end{aligned}
$$



To obtain decay rates, we have to square the amplitude. Then, the phases disappear!

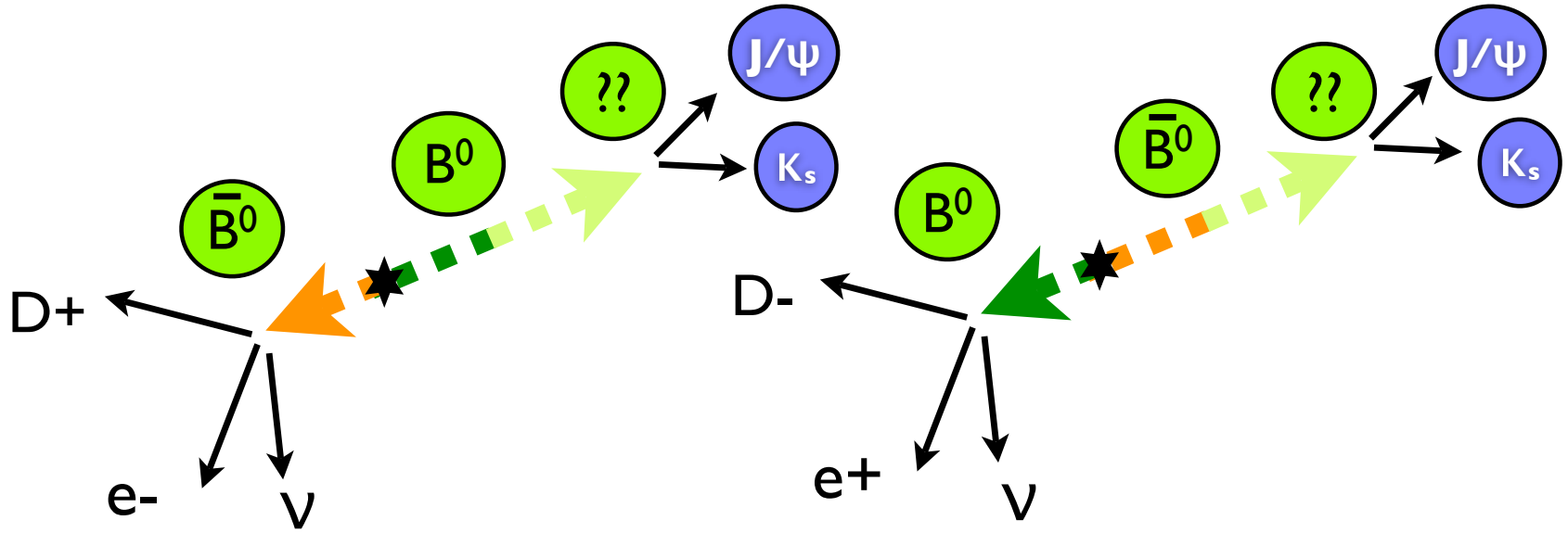
$$\langle l^- X|\mathcal{H}^{\Delta B=1}|B(t)\rangle = f_+(t)\langle l^- X|\mathcal{H}^{\Delta B=1}|B\rangle + \frac{q}{p} f_-(t)\langle l^- X|\mathcal{H}^{\Delta B=1}|\overline{B}\rangle$$

Flavour Non-specific mixing CPV

(CP Violation in oscillation)

Choose a final state which could come both B and Bbar!

ex: $J/\psi K_s$ final state



$$|B(t)\rangle = f_+(t)|B\rangle + \frac{q}{p} f_-(t)|\overline{B}\rangle$$

$$|\overline{B}(t)\rangle = f_+(t)|\overline{B}\rangle + \frac{p}{q} f_-(t)|B\rangle$$

Flavour Non-specific mixing CPV

(CP Violation in oscillation)

Choose a final state which could come both B and Bbar!

ex: J/ψK_s final state

$$
\begin{aligned}
|B(t)\rangle &= f_+(t)|B\rangle + \frac{q}{p} f_-(t)|\overline{B}\rangle \\
|\overline{B}(t)\rangle &= f_+(t)|\overline{B}\rangle + \frac{p}{q} f_-(t)|B\rangle
\end{aligned}
$$

$$
\begin{aligned}
\langle J/\psi K_s|\mathcal{H}^{\Delta B=1}|B(t)\rangle &= f_+(t)\langle J/\psi K_s|\mathcal{H}^{\Delta B=1}|B\rangle + \frac{q}{p} f_-(t)\langle J/\psi K_s|\mathcal{H}^{\Delta B=1}|\overline{B}\rangle \\
\langle J/\psi K_s|\mathcal{H}^{\Delta B=1}|\overline{B}(t)\rangle &= f_+(t)\langle J/\psi K_s|\mathcal{H}^{\Delta B=1}|\overline{B}\rangle + \frac{p}{q} f_-(t)\langle J/\psi K_s|\mathcal{H}^{\Delta B=1}|B\rangle
\end{aligned}
$$

We assume...

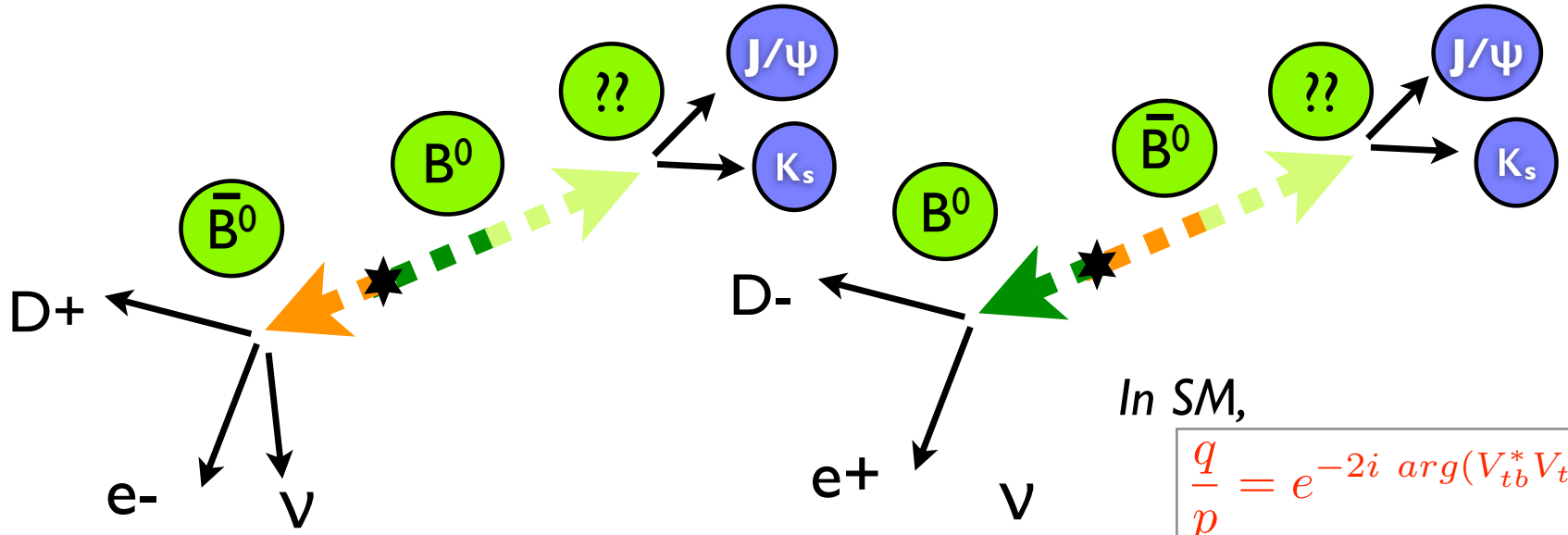
$$\langle J/\psi K_s|\mathcal{H}^{\Delta B=1}|\overline{B}\rangle = \langle J/\psi K_s|\mathcal{H}^{\Delta B=1}|B\rangle \qquad \Gamma_{12} << M_{12}$$

Flavour Non-specific mixing CPV

(CP Violation in oscillation)

Choose a final state which could come both B and Bbar!

ex: $J/\psi K_s$ final state



In SM,

$$\frac{q}{p} = e^{-2i\ arg(V_{tb}^* V_{td})}$$

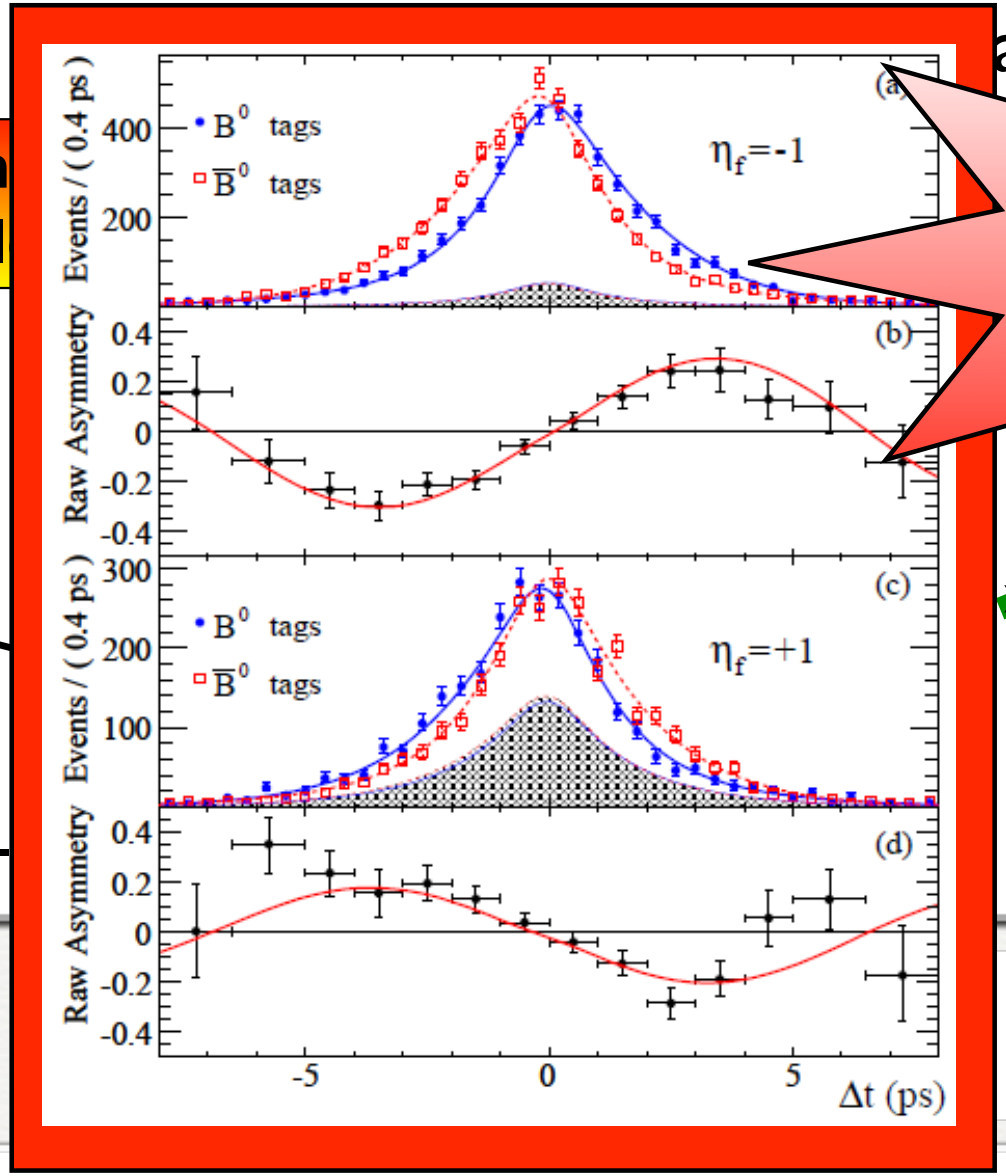
$$\mathcal{A} = \frac{\Gamma(\overline{B}^0 \to J/\psi K_s) - \Gamma(B^0 \to J/\psi K_s)}{\Gamma(\overline{B}^0 \to J/\psi K_s) - \Gamma(B^0 \to J/\psi K_s)} = \sin(\Delta M t) Im\left(\frac{q}{p}\right)$$

Flavour Non-specific mixing CPV

ation)

Ch
coul

state



B factory measurments!

D+

e-

In SM,

$$\frac{q}{p} = e^{-2i\ arg(V_{tb}^{*}V_{td})}$$

$$= \sin(\Delta M t) Im\left(\frac{q}{p}\right)$$

Test of Unitarity of CKM

$$V_{CKM}^{\dagger} V_{CKM} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$V_{CKM} V_{CKM}^{\dagger} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Test of Unitarity

$$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

$$= \begin{pmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix} + \mathcal{O}(\lambda^4)$$

Unitarity: 9 complex numbers can be replaced by the 4 real number parameters

⇩

We must test *at which extent* this is satisfied!

Unitarity triangles

$$\underbrace{V_{ud}V_{us}^*}_{\mathcal{O}(\lambda)} + \underbrace{V_{cd}V_{cs}^*}_{\mathcal{O}(\lambda)} + \underbrace{V_{td}V_{ts}^*}_{\mathcal{O}(\lambda^5)} = 0$$

ds

$$\underbrace{V_{ud}V_{cd}^*}_{\mathcal{O}(\lambda)} + \underbrace{V_{us}V_{cs}^*}_{\mathcal{O}(\lambda)} + \underbrace{V_{ub}V_{cb}^*}_{\mathcal{O}(\lambda^5)} = 0$$

uc

$$\underbrace{V_{us}V_{ub}^*}_{\mathcal{O}(\lambda^4)} + \underbrace{V_{cs}V_{cb}^*}_{\mathcal{O}(\lambda^2)} + \underbrace{V_{ts}V_{tb}^*}_{\mathcal{O}(\lambda^2)} = 0$$

sb

$$\underbrace{V_{td}V_{cd}^*}_{\mathcal{O}(\lambda^4)} + \underbrace{V_{ts}V_{cs}^*}_{\mathcal{O}(\lambda^2)} + \underbrace{V_{tb}V_{cb}^*}_{\mathcal{O}(\lambda^2)} = 0$$

ct

$$\underbrace{V_{ud}^*V_{ub}}_{\mathcal{O}(\lambda^3)} + \underbrace{V_{cd}^*V_{sb}}_{\mathcal{O}(\lambda^3)} + \underbrace{V_{td}^*V_{tb}}_{\mathcal{O}(\lambda^3)} =$$

db

$$\underbrace{V_{ud}V_{ub}^*}_{\mathcal{O}(\lambda^3)} + \underbrace{V_{cd}V_{cb}^*}_{\mathcal{O}(\lambda^3)} + \underbrace{V_{td}V_{tb}^*}_{\mathcal{O}(\lambda^3)}$$

ut

Unitarity triangles

$$\underbrace{V_{ud}V_{us}^*}_{\mathcal{O}(\lambda)} + \underbrace{V_{cd}V_{cs}^*}_{\mathcal{O}} + \underbrace{V_{td}V_{ts}^*}_{} = 0$$

$$\underbrace{V_{ud}V_{cd}^*}_{} + \underbrace{V_{us}V_{cs}^*}_{} + \underbrace{V_{ub}V_{cb}^*}_{\mathcal{O}(\lambda^5)} = 0$$

uc

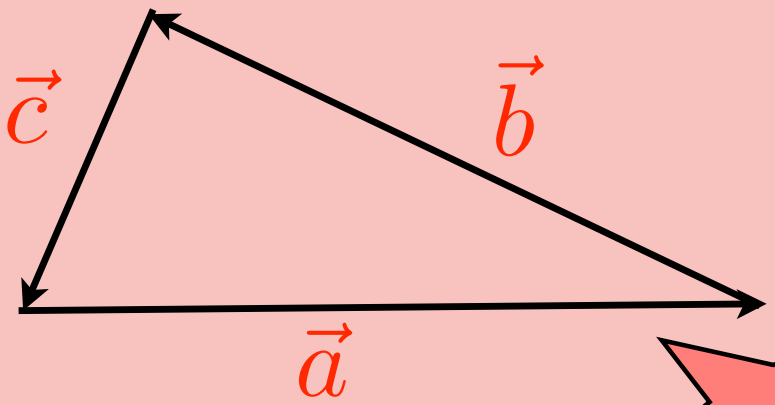
$$\underbrace{V_{us}V_{ub}^*}_{\mathcal{O}(\lambda^4)} + \underbrace{V_{c}}_{\mathcal{O}}$$

$$\underbrace{{}_{tb}V_{cb}^*}_{\mathcal{O}(\lambda^2)} = 0$$

ct

$$\underbrace{V_{ud}^*V_{ub}}_{\mathcal{O}(\lambda^3)} + \underbrace{V_{c}}_{\mathcal{O}(\lambda^3)} \quad \underbrace{\phantom{V_{td}V_{ts}^*}}_{\mathcal{O}(\lambda^3)} \quad \underbrace{\phantom{V_{ud}V_{cd}^*}}_{\mathcal{O}(\lambda^3)}$$

$$\underbrace{V_{1i}^*V_{1k}}_{\vec{a}} + \underbrace{V_{2i}^*V_{1k}}_{\vec{b}} + \underbrace{V_{3i}V_{1k}}_{\vec{c}} = 0$$



Unitarity Triangle

Unitarity triangles

$$\underbrace{V_{ud}V_{us}^*}_{\mathcal{O}(\lambda)} + \underbrace{V_{cd}V_{cs}^*}_{\mathcal{O}(\lambda)} + \underbrace{V_{td}V_{ts}^*}_{\mathcal{O}(\lambda^5)} = 0$$

ds

K physics

$$\underbrace{V_{ud}V_{cd}^*}_{\mathcal{O}(\lambda)} + \underbrace{V_{us}V_{cs}^*}_{\mathcal{O}(\lambda)} + \underbrace{V_{ub}V_{cb}^*}_{\mathcal{O}(\lambda^5)} = 0$$

uc

D physics

$$\underbrace{V_{us}V_{ub}^*}_{\mathcal{O}(\lambda^4)} + \underbrace{V_{cs}V_{cb}^*}_{\mathcal{O}(\lambda^2)} + \underbrace{V_{ts}V_{tb}^*}_{\mathcal{O}(\lambda^2)} = 0$$

sb

B_s physics

$$\underbrace{V_{td}V_{cd}^*}_{\mathcal{O}(\lambda^4)} + \underbrace{V_{ts}V_{cs}^*}_{\mathcal{O}(\lambda^2)} + \underbrace{V_{tb}V_{cb}^*}_{\mathcal{O}(\lambda^2)} = 0$$

ct

The Unitarity Triangle!

db

B_d physics

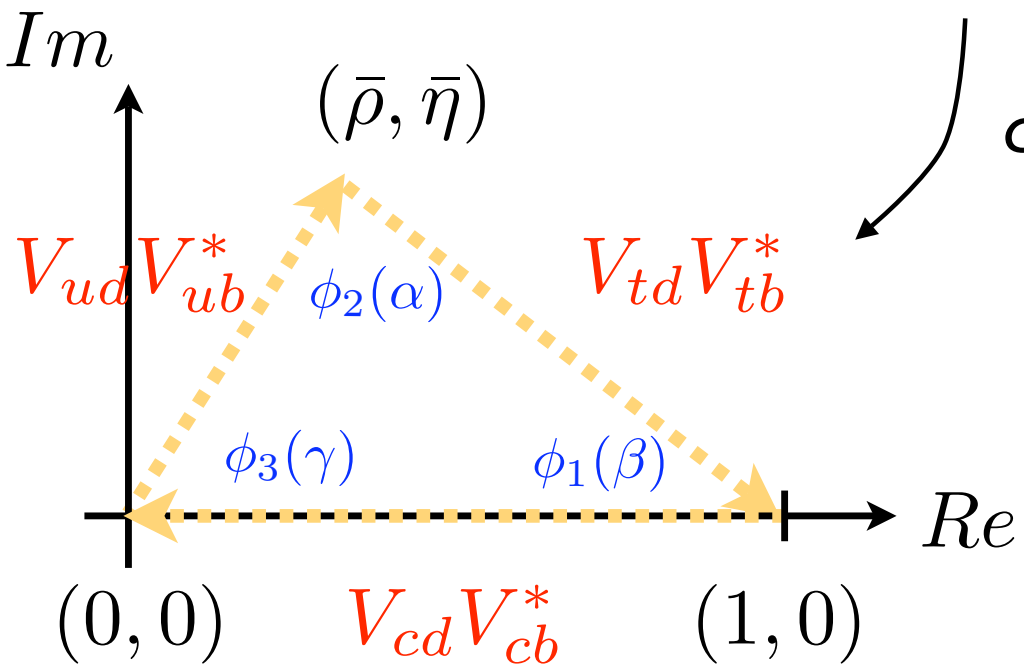
$$\underbrace{V_{td}V_{ud}^*}_{\mathcal{O}(\lambda^3)} + \underbrace{V_{ts}V_{us}^*}_{\mathcal{O}(\lambda^3)} + \underbrace{V_{tb}V_{ub}^*}_{\mathcal{O}(\lambda^3)} = 0$$

ut

The Unitarity Triangle

$$\underbrace{V_{ud}V_{ub}^*}_{A\lambda^3(\rho+i\eta)} + \underbrace{V_{cd}V_{cb}^*}_{-A\lambda^3} + \underbrace{V_{td}V_{tb}^*}_{A\lambda^3(1-\rho-i\eta)} = 0$$

divide by $A\lambda^3$



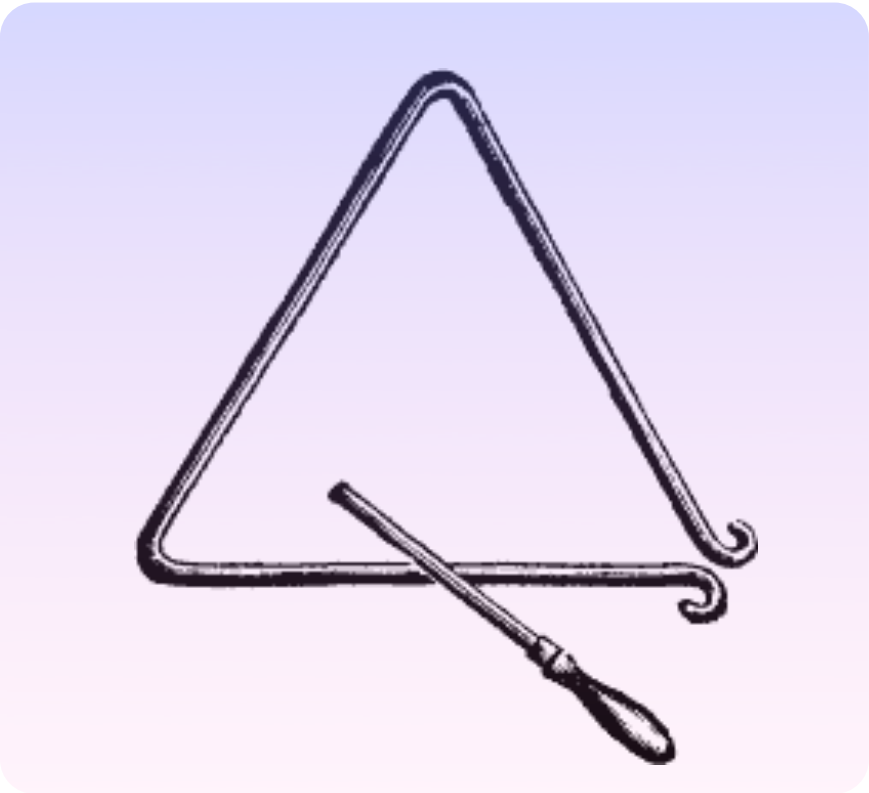
$$arg\left(\frac{V_{tb}^* V_{td}}{V_{cb}^* V_{cs}}\right) \equiv -\phi_1$$

$$arg\left(\frac{V_{tb}^* V_{td}}{V_{ub}^* V_{ud}}\right) \equiv -\phi_2$$

$$arg\left(\frac{V_{cb}^* V_{cs}}{V_{ub}^* V_{ud}}\right) \equiv -\phi_3$$

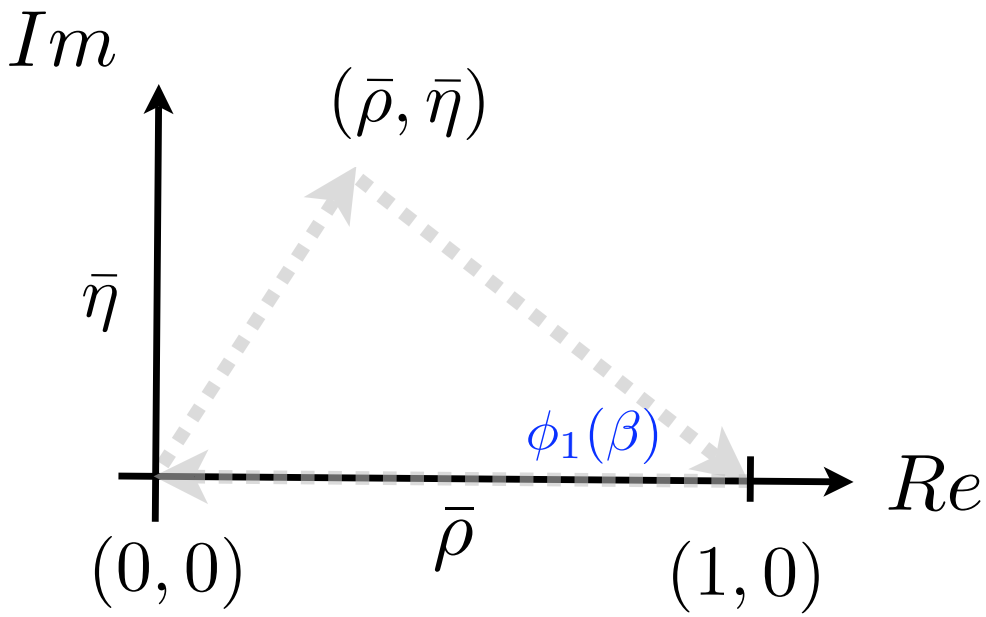
B_d physics

Determination of the CKM matrix



Determination of the CKM matrix: sin2Φ₁ (β) (phase)

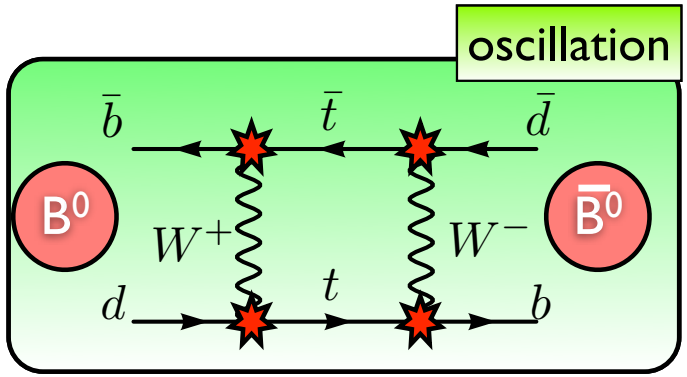
$$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

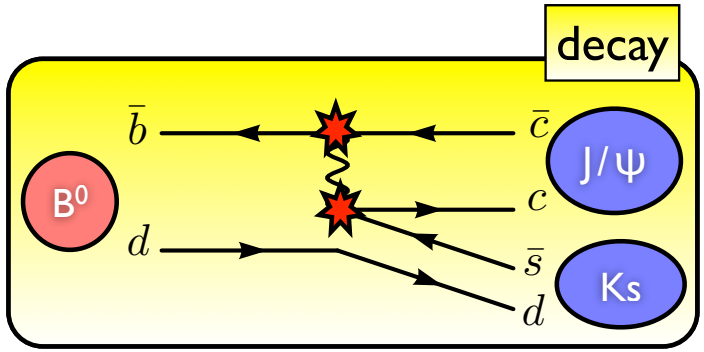


CP asymmetry in B→J/ψK$_S$

$$A_{J/\psi K_S}(t) = \frac{\Gamma(\overline{B}^0(t)\to J/\psi K_S)-\Gamma(B^0(t)\to J/\psi K_S)}{\Gamma(\overline{B}^0(t)\to J/\psi K_S)+\Gamma(B^0(t)\to J/\psi K_S)} = S_{J/\psi K_s} \sin \Delta M_B \; t$$

$$\begin{aligned} S_{J/\psi K_s} &= Im \left[\underbrace{\frac{M_{12}}{M_{12}^*}}_{oscill.} \underbrace{\frac{A(\overline{B} \to J/\psi K_S)}{A(B \to J/\psi K_S)}}_{decay} \right] \\ &= Im \left[\underbrace{\frac{V_{tb}V_{td}^*}{V_{tb}^*V_{td}}}_{oscill.} \underbrace{\frac{V_{cb}V_{cs}^*}{V_{cb}^*V_{cs}}}_{decay} \right] \\ &= \sin 2\phi_1 \end{aligned}$$



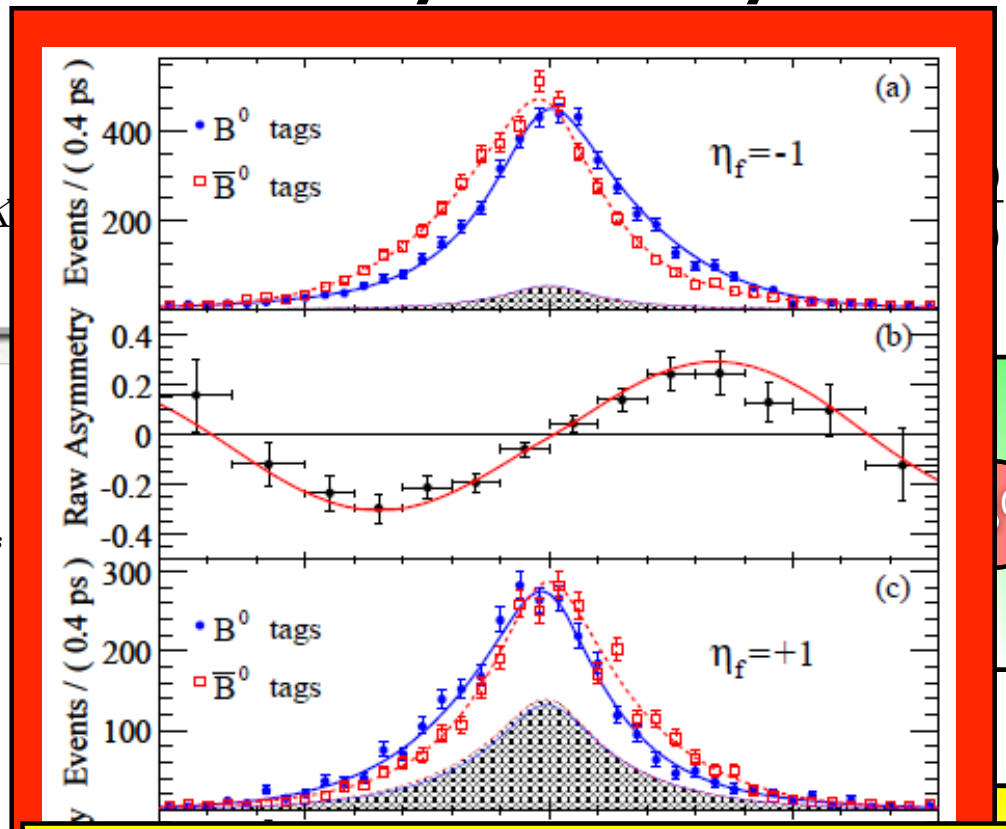


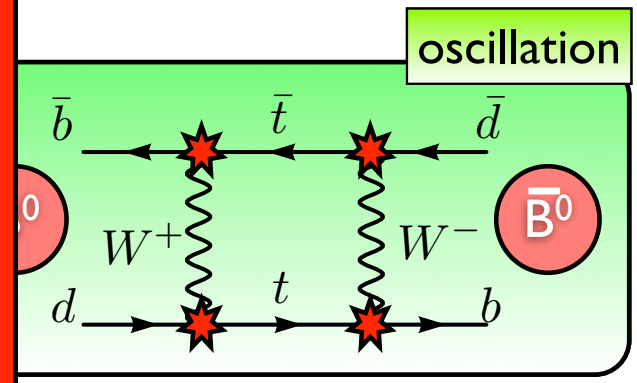
CP asymmetry in B→J/ψK$_S$

$A_{J/\psi K}$

$S_{J/\psi K_s}$

$= S_{J/\psi K_s} \sin \Delta M_B \; t$

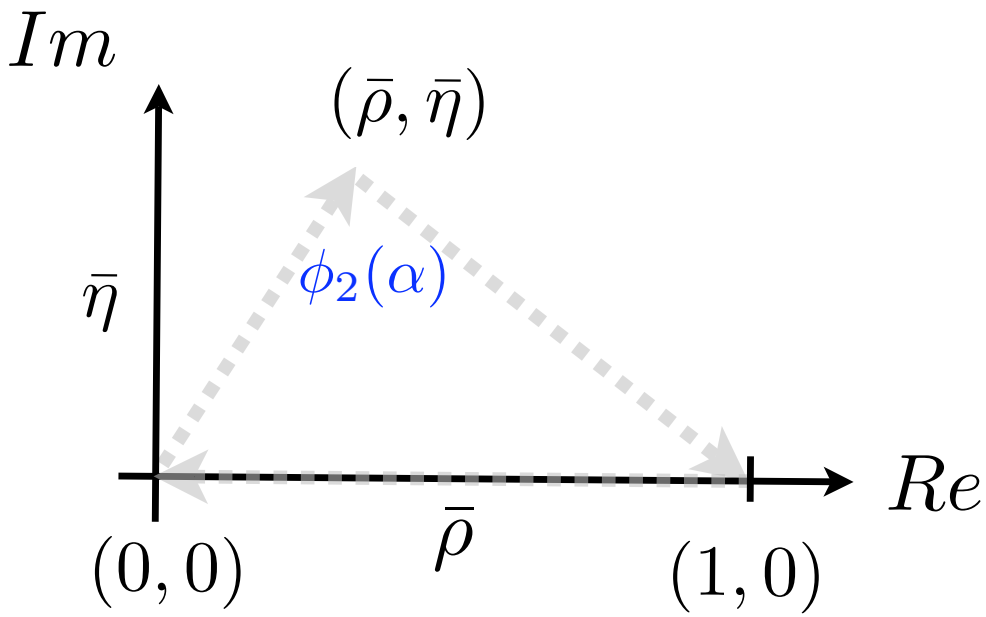




Including various b->cc$_{bar}$s the measurements, we find Φ_I=(21.1±0.9) °

Determination of the CKM matrix: sin2Φ$_2$ (α) (phase)

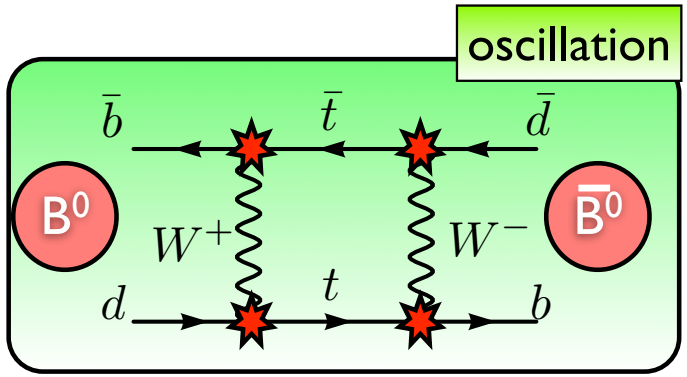
$$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

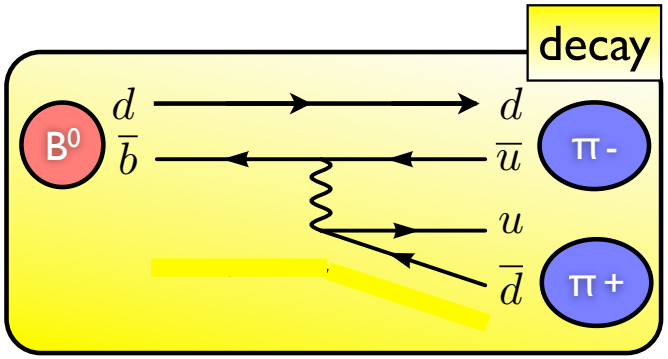


CP asymmetry in B→π⁺π⁻

$$A_{\pi^+\pi^-}(t) = \frac{\Gamma(\overline{B}^0(t) \to \pi^+\pi^-) - \Gamma(B^0(t) \to \pi^+\pi^-)}{\Gamma(\overline{B}^0(t) \to \pi^+\pi^-) + \Gamma(B^0(t) \to \pi^+\pi^-)} = S_{\pi^+\pi^-} \sin \Delta M_B \; t$$

$$\begin{aligned} S_{\pi^+\pi^-} &= Im \left[\underbrace{\frac{M_{12}}{M_{12}^*}}_{oscill.} \underbrace{\frac{A(\overline{B} \to \pi^+\pi^-)}{A(B \to \pi^+\pi^-)}}_{decay} \right] \\ &= Im \left[\underbrace{\frac{V_{tb}V_{td}^*}{V_{tb}^*V_{td}}}_{oscill.} \underbrace{\frac{V_{ub}V_{ud}^*}{V_{ub}^*V_{ud}}}_{decay} \right] \\ &= \sin 2\phi_2(\alpha) \end{aligned}$$



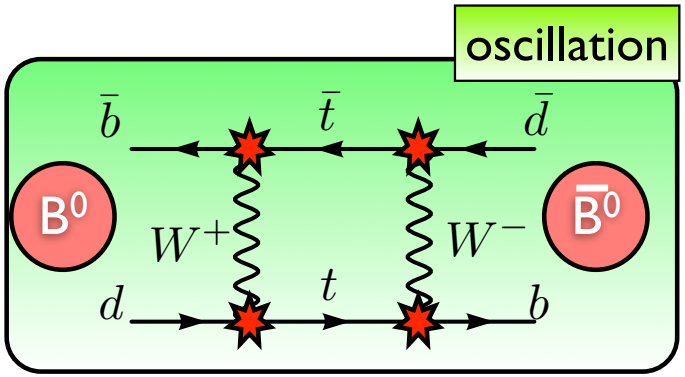


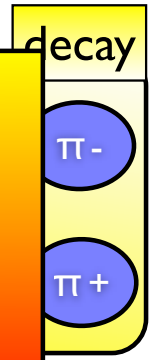
*Assuming tree-dominant...

CP asymmetry in $B \to \pi^+\pi^-$

$$A_{\pi^+\pi^-}(t) = \frac{\Gamma(\overline{B}^0(t) \to \pi^+\pi^-) - \Gamma(B^0(t) \to \pi^+\pi^-)}{\Gamma(\overline{B}^0(t) \to \pi^+\pi^-) + \Gamma(B^0(t) \to \pi^+\pi^-)} = S_{\pi^+\pi^-} \sin \Delta M_B \, t$$

Penguin pollution prevents a precise measurement of Φ_2 (α)





Including the measurements with

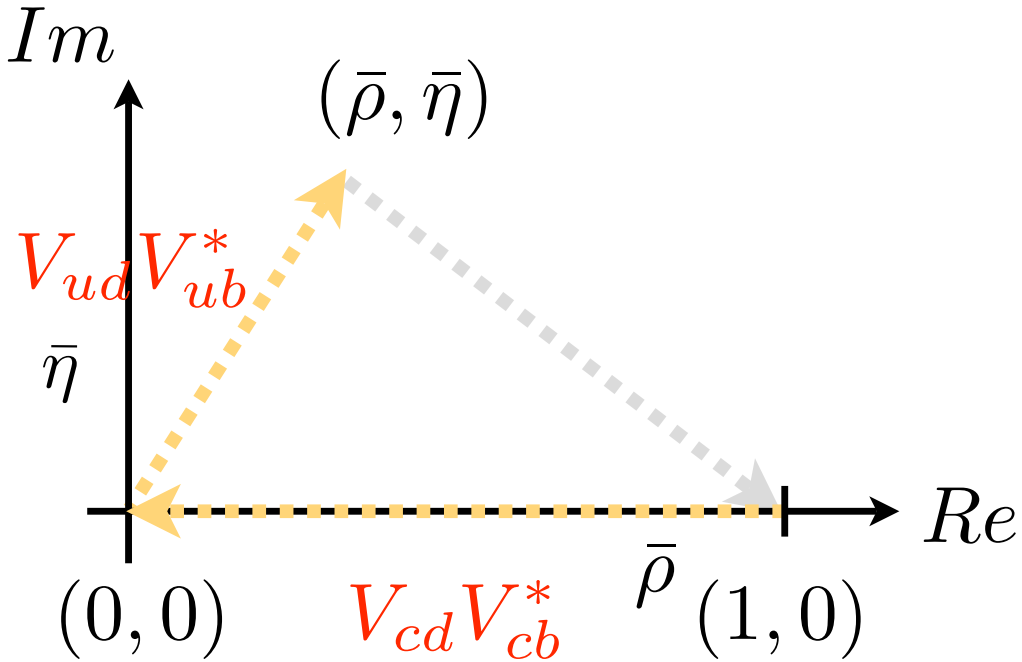
$\pi\pi$, $\rho\rho$, $\pi\rho$,

$\alpha = (89.0^{+4.4}_{-4.2})$ °

* Assuming tree-dominant...

Determination of the CKM matrix: $|V_{ub}|$ and $|V_{cb}|$

$$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

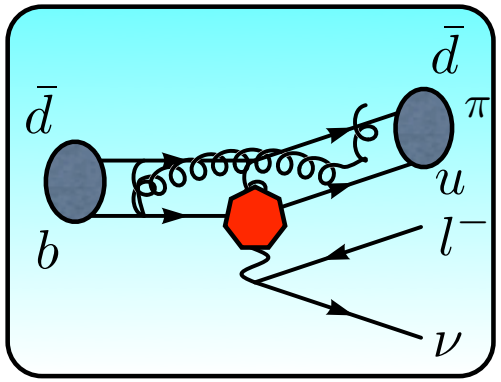


Determination of the CKM matrix: $|V_{ub}|$

$$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

** Similar in $|V_{cb}|$*

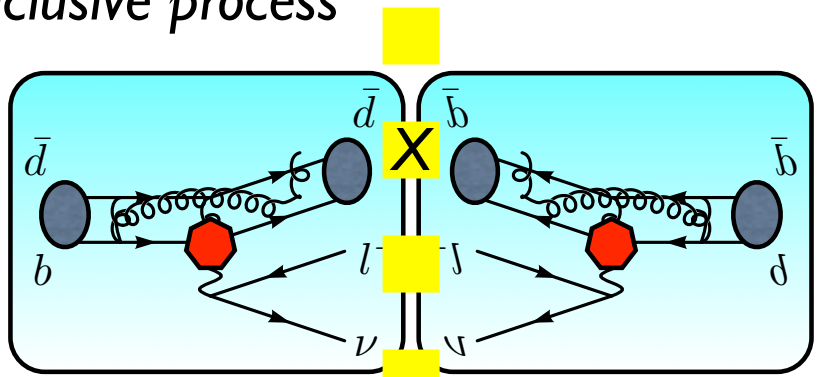
Exclusive process



$$\mathcal{A}(B \to \pi l \nu) \propto |V_{ub}| F^{B\to\pi}(q^2)$$

$|V_{ub}|=(3.89\pm0.44)10^{-3}$

Inclusive process



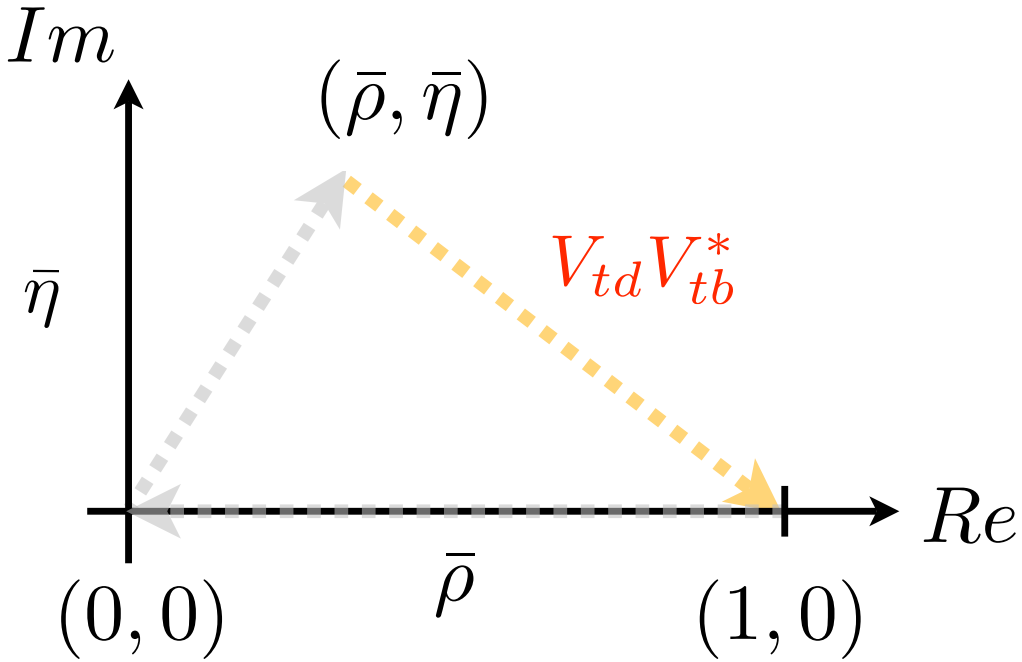
Optical theorem

$$\sum |\mathcal{A}(B \to X_u l \nu)|^2 \propto |V_{ub}|^2 f(q^2, \mu_\pi, ...)$$

$|V_{ub}|=(4.27\pm0.38)10^{-3}$

Determination of the CKM matrix: $|V_{td}V_{tb}^*|$

$$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$



Determination of the CKM matrix: $|V_{td}V_{tb}^*|$

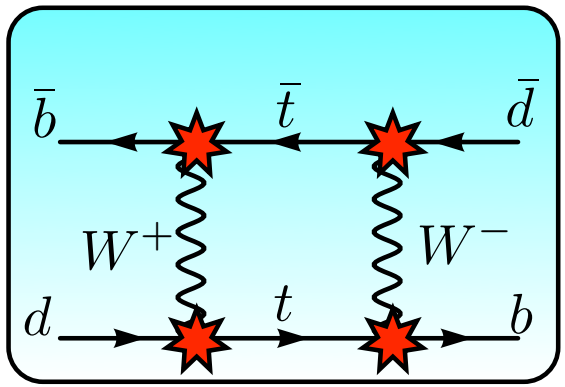
$$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

Perturbative QCD correction

Loop function dominant=top quark

$$\Delta M_d (\propto |M_{12}|) = \frac{G_F^2 m_W^2}{16\pi^2} |V_{tb}V_{td}^*|^2 S_0\left(\frac{m_t^2}{m_W^2}\right) \eta_{\rm QCD} \times \frac{1}{m_B} \langle B^0 | (\bar{d}b)_{V-A} (\bar{d}b)_{V-A} | \overline{B}^0 \rangle$$

$$\equiv \frac{8}{3} B_B f_B^2 m_B^2$$



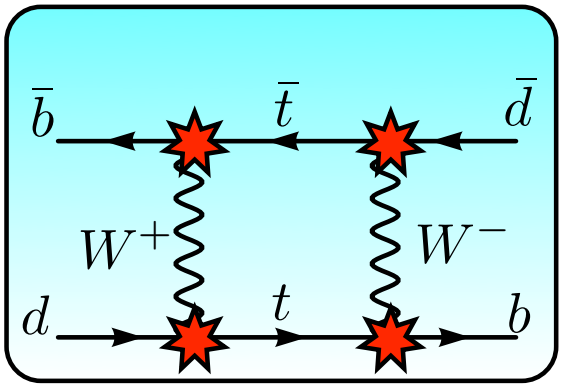
$\Delta M_d \Leftrightarrow |V_{td}V_{tb}^*|$

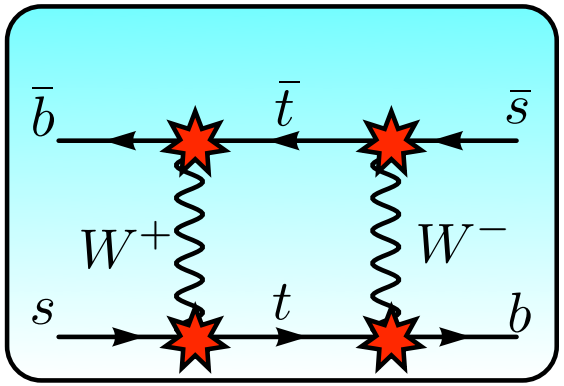
Main source of the hadronic uncertainties in determining |Vtb|: Lattice QCD computation very important!

Determination of the CKM matrix: $|V_{td}V_{tb}^*|$ and $|V_{ts}V_{tb}^*|$

$$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

Some hadronic uncertainties cancel in the ratio between ΔM_d and ΔM_s

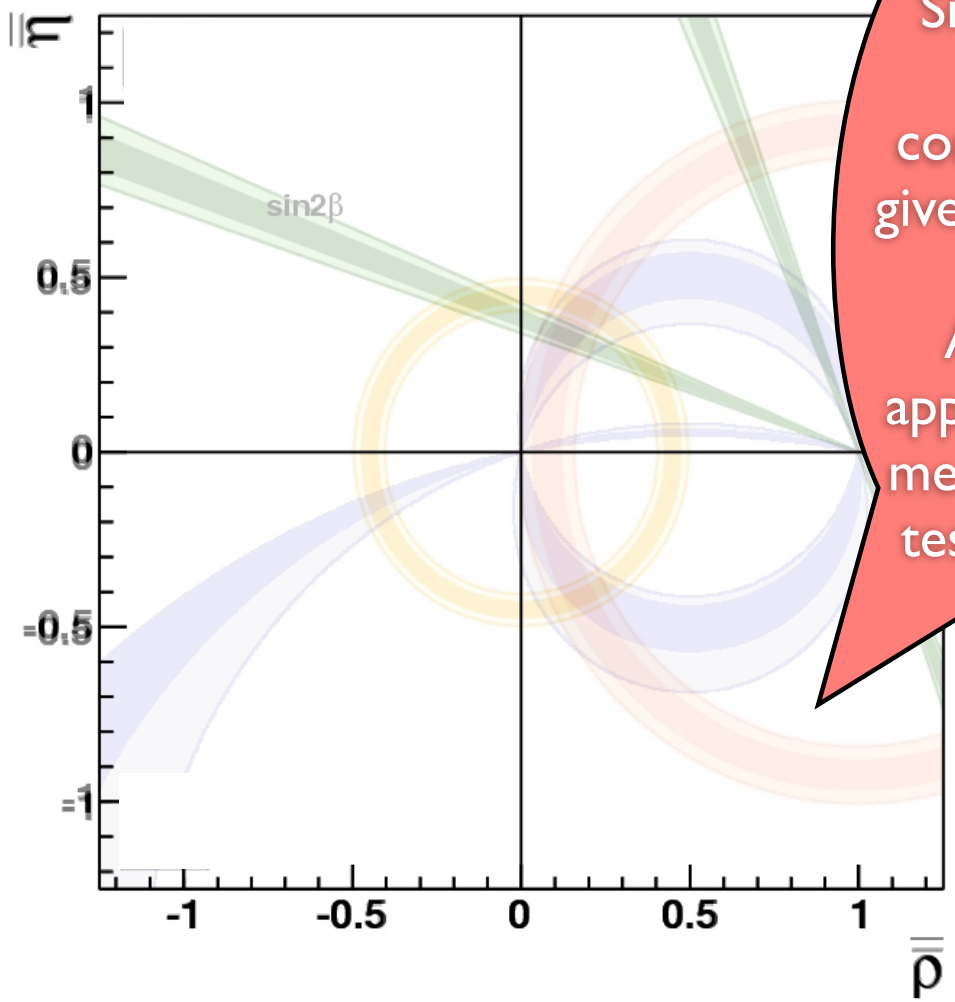




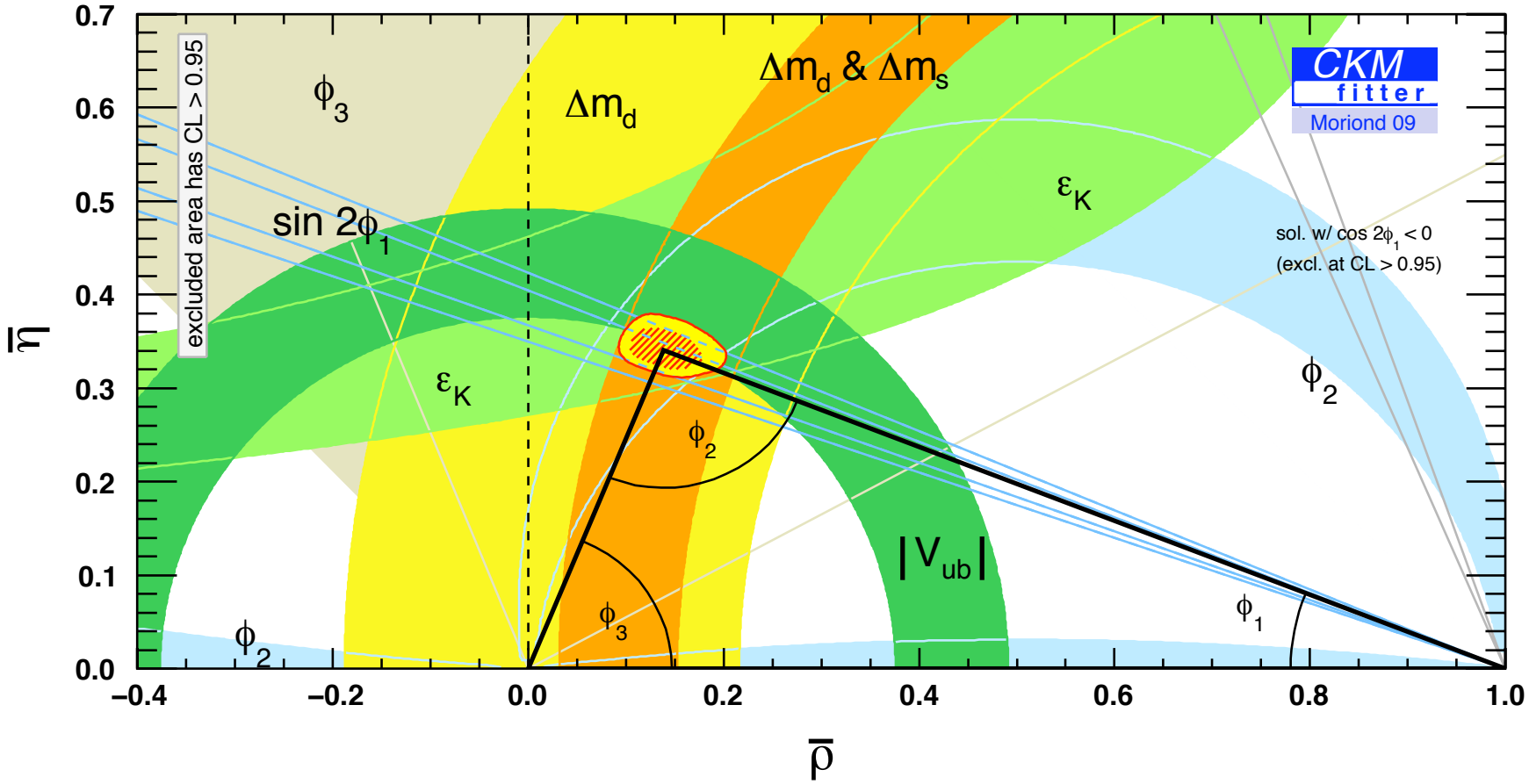
$\Delta M_d \Rightarrow |V_{td}V_{tb}^*|$

$\Delta M_s \Rightarrow |V_{td}V_{tb}^*|$

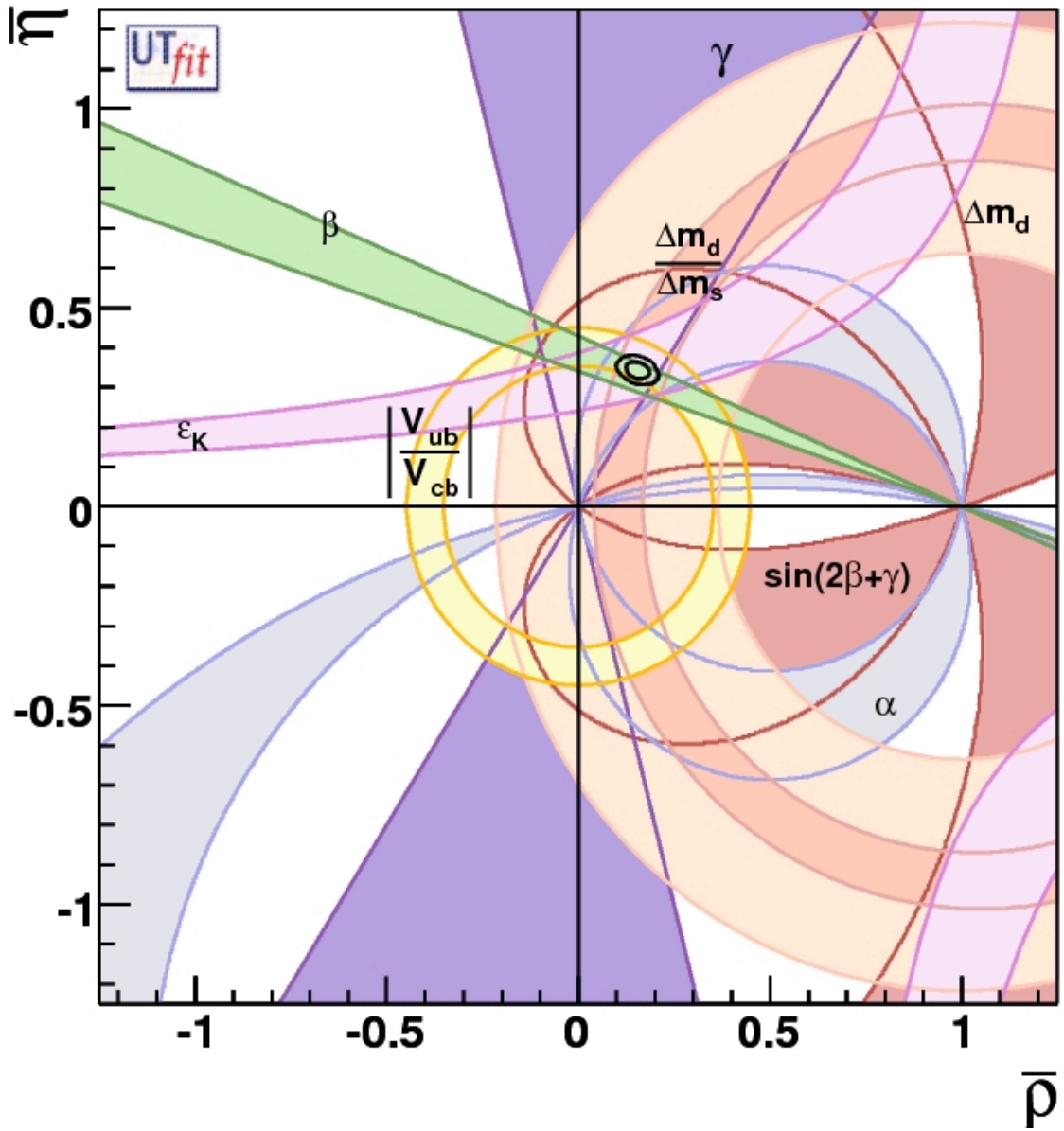
Combining the constra



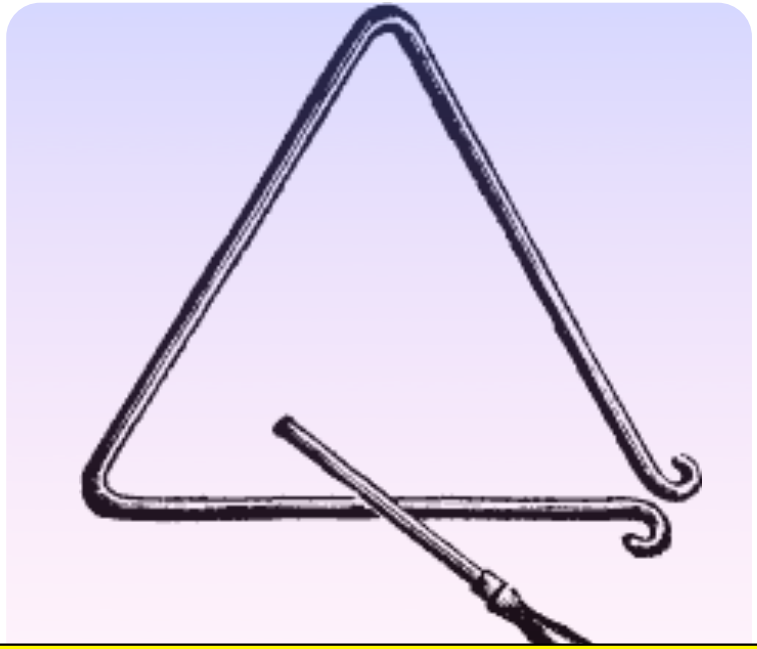
Warning:
Simply overlapping the different constraints does not give a right confidence level.
An applicaton of appropriate statistical methods is crucial to test the unitarity!!



http://ckmfitter.in2p3.fr/



http://www.utfit.org/UTfit/



We can say that the main part of the CP violation comes from the complex phase in the CKM matrix. However, there is still a possibility that the unitarity is not exact for a certin extent. Our challenges for more precise experiemental data as well as improvements in the theoretical predcitions continue!!!