

Models behind supersymmetry (or how model is build)

Sparticle masses

- If supersymmetry is the symmetry of vacuum, particle and its superpartner should have same mass, as $[Q, H]=0$.
- If the vacuum is not invariant under Q , it is no longer correct.
- In supersymmetric theory it corresponds to non zero value of some field.
- The SUSY breaking leads the mass difference between particle and sparticle masses. They are called "soft" SUSY breaking as they do not induce quadratic divergences

$$\langle 0|\phi|0\rangle = v_i (v \neq 0) \quad [Q, \phi_i] = \delta\phi = \alpha^i T_{jl}^i \phi_l \quad \longrightarrow \quad \langle 0|\delta\phi|0\rangle \neq 0$$

SUSY breaking

$$\delta\psi = -i\sqrt{2}\not{\partial}\phi\bar{\alpha} + \sqrt{2}F\alpha \quad \longrightarrow \quad \langle 0|\delta\psi|0\rangle = -\sqrt{2}\langle 0|F|0\rangle\alpha \neq 0$$

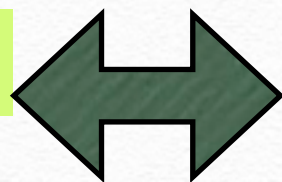
SUSY breaking and Flavor violation

- SUSY breaking in tree level and global supersymmetry
- Therefore SUSY breaking sector must be placed in hidden sector.
The SUSY breaking is introduced to our sector through gravity/ loop effect
- The sector must be “universal” in flavor so that there are no large FCNC

Flavor mixing of quark

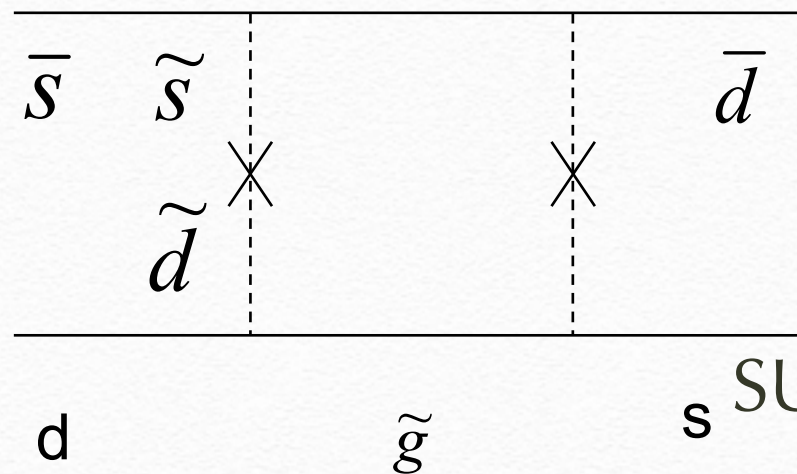
Yukawa effect

$$\left[\frac{10 \text{TeV}}{m_{\tilde{q}, \tilde{g}}} \right]^2 \left[\frac{\Delta m_{\tilde{q}_{12}}^2 / m^2}{0.1} \right]^2 \leq 1$$



Flavor mixing of squark

Yukawa effect + SUSY breaking



SUSY breaking must be flavor universal

SUSY breaking in Hidden sector

SUSY breaking

$$Z = 1 + \langle F_Z \rangle \theta\theta$$

gravity?

You need symmetry
to make scalar mass
universal

radiative
collection?

Higher dimensional Op

$$\frac{1}{M^2} Z \bar{Z} \Phi \bar{\Phi} |_{\theta\theta} = \frac{|\langle F_Z \rangle|^2}{M^2} \phi \phi^*$$

MSSM particles

$$\Phi = \phi + \theta\psi + \theta\theta F$$

squark quark

gaugino mass and squark left right
mixing, higgs mass parameter also arise

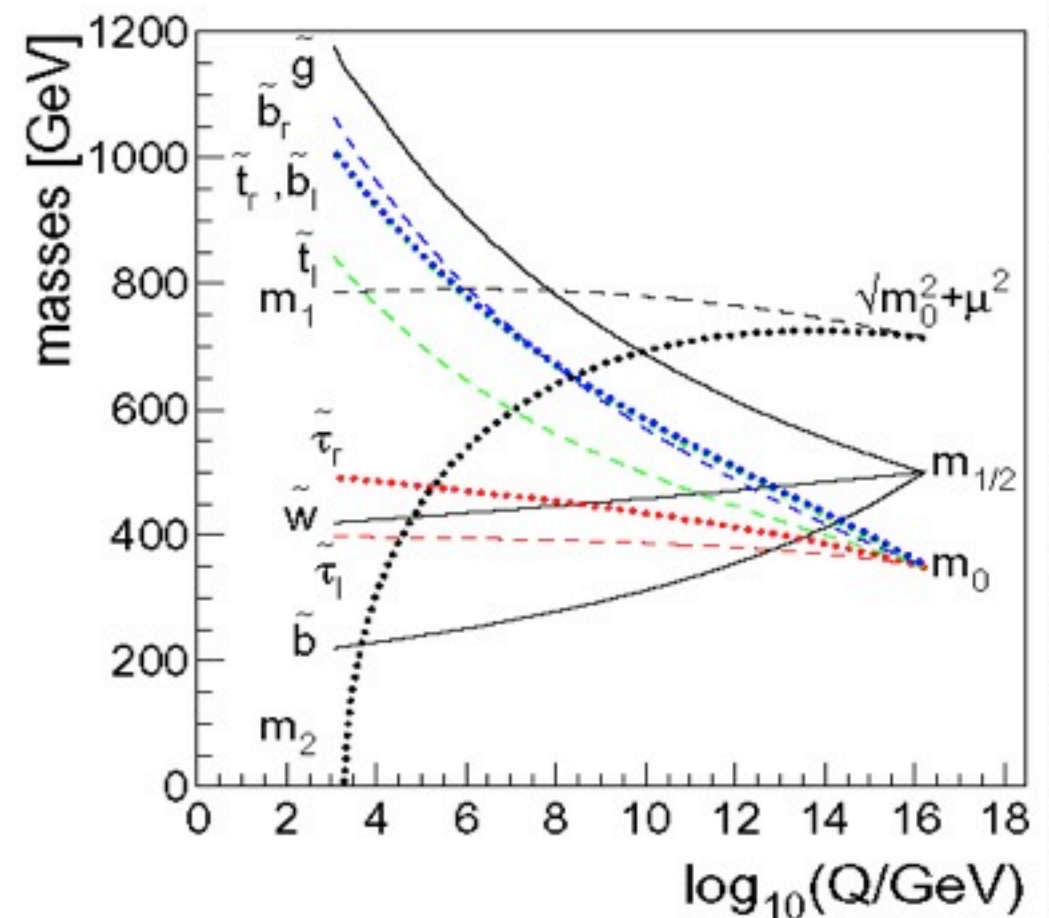
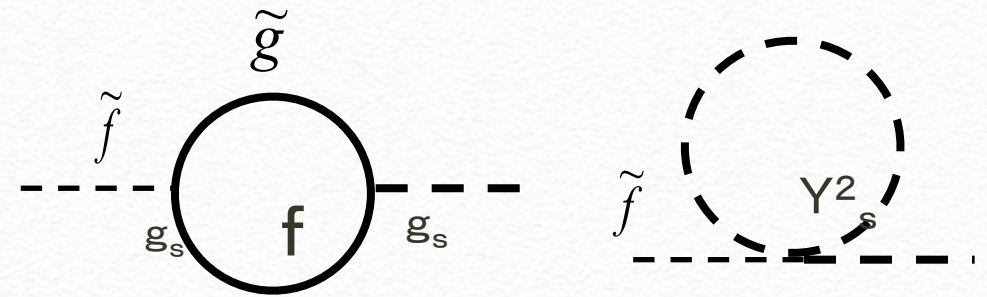
Supergravity model and soft masses

- gravity is the mediator of SUSY breaking
- Prediction at GUT scale \rightarrow (RGE) \rightarrow Low energy mass spectrum
- Strongly interacting particles are heavy and weakly interacting particles are light.
- Yukawa coupling derive particle masses small. (Radiative symmetry breaking)

Example: Unification relation of gaugino mass

$$M_1/\alpha_1 = M_2/\alpha_2 = M_3/\alpha_3$$

$$\rightarrow M_1 : M_2 : M_3 = 0.4 : 0.8 : 2.4$$



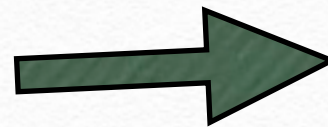
gauge mediation

SUSY breaking sector

$$X = M + \theta\theta F$$

Messenger

$$L = \lambda X \Phi \bar{\Phi} |_{\theta\theta}$$



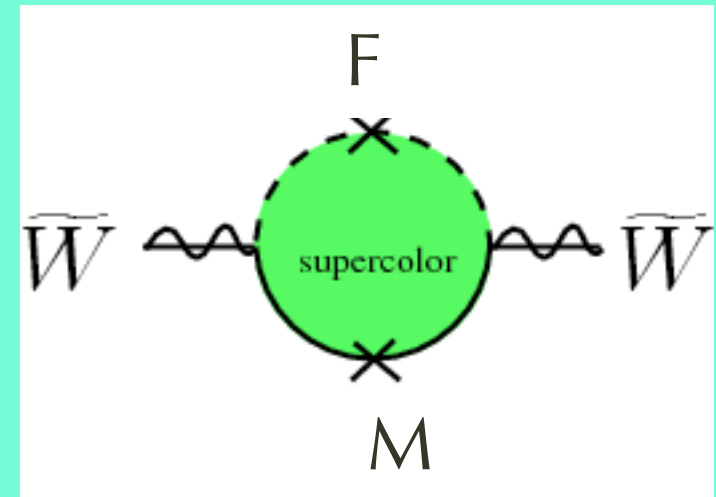
Messenger mass
induced by hidden
sector fields

$$m_\psi = M$$

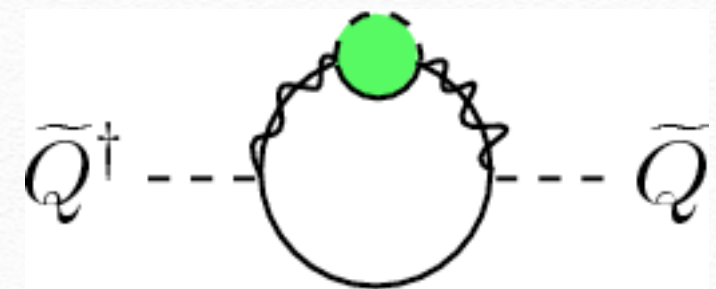
$$m_\phi^2 = M^2 \pm F$$

gauge interaction → Universal

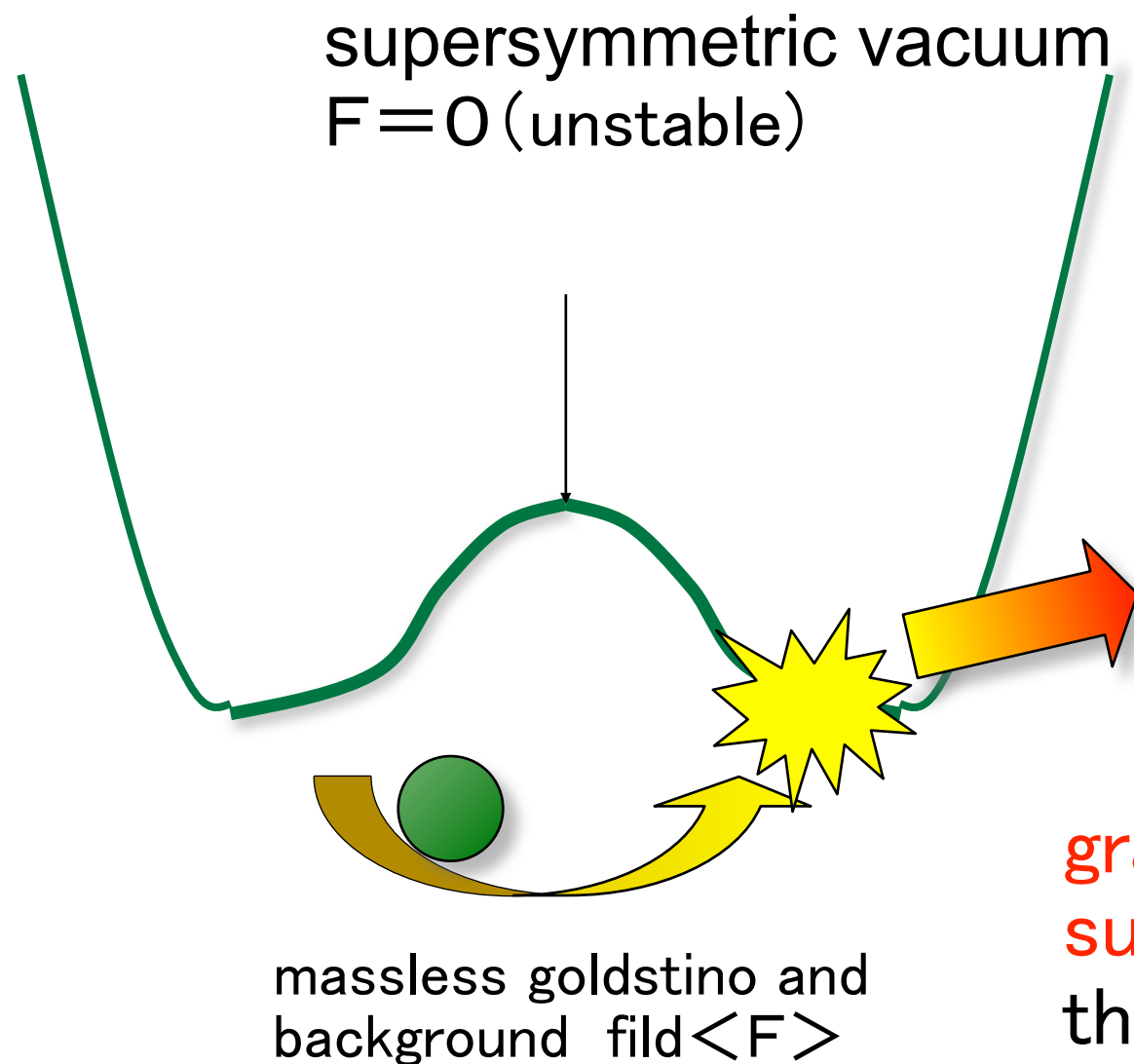
squark mass will be induced by
loop diagram involving gauginos



$$M_{\text{gaugino}} = \frac{\alpha}{4\pi} N_m \frac{F}{M}$$



Supergravity and Super Higgs mechanism



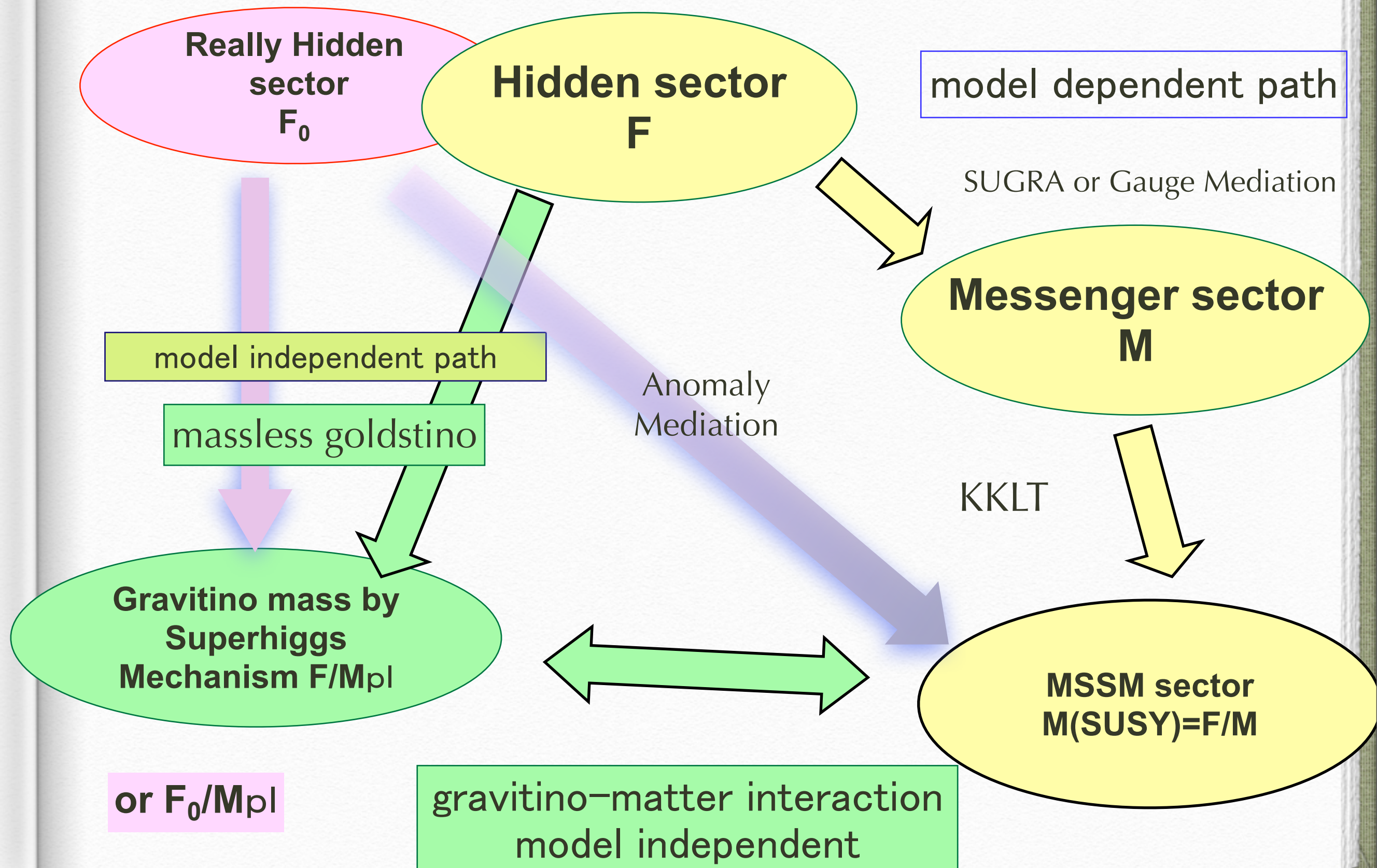
$$\delta L = \frac{1}{M_{pl}} \psi^\mu J_\mu^Q = \frac{1}{M_{pl}} \psi^\mu F \gamma_\mu \tilde{G}$$

$$\Rightarrow \frac{1}{M_{pl}} \psi^\mu \langle F \rangle \gamma_\mu \tilde{G}$$

gravitino-goldstino mixing

gravitino couple with matter though
susy breaking. “gravitino LPS” is
the case that we can study gravity
sector using collider.

Supersymmetry-a picture



gravitino LSP and NLSP

- gravitino is spin 3/2 particle
- non-renormalizable interactions.
- spin 3/2 \rightarrow spin 1 X spin 1/2

$$\partial_\mu \psi^\mu = 0, \quad \gamma_\mu \psi^\mu = 0$$

$$h=-1/2 \quad \psi^0 = -\sqrt{2/3} \frac{|p|}{m} \psi_{-1/2}$$

$$(\psi_1, \psi_2, \psi_3) = \frac{1}{\sqrt{3}} e_2 \psi_{+1/2} - \sqrt{\frac{2}{3}} \frac{E}{m} e_3 \psi_{-1/2}$$

$$L \propto \kappa \left(\bar{\psi}_L \gamma^\mu \gamma^\nu \partial_\nu \phi - \frac{i}{4\sqrt{2}} \lambda^a F_{\mu\rho}^a \gamma^\mu \sigma^{\nu\rho} \right) \psi_\mu$$

- If it is LSP. NLSP decays into gravitino

$$\tilde{\chi}_1^0 \rightarrow \gamma \psi^\mu \quad \tilde{\tau}_1 \rightarrow \tau \psi^\mu$$

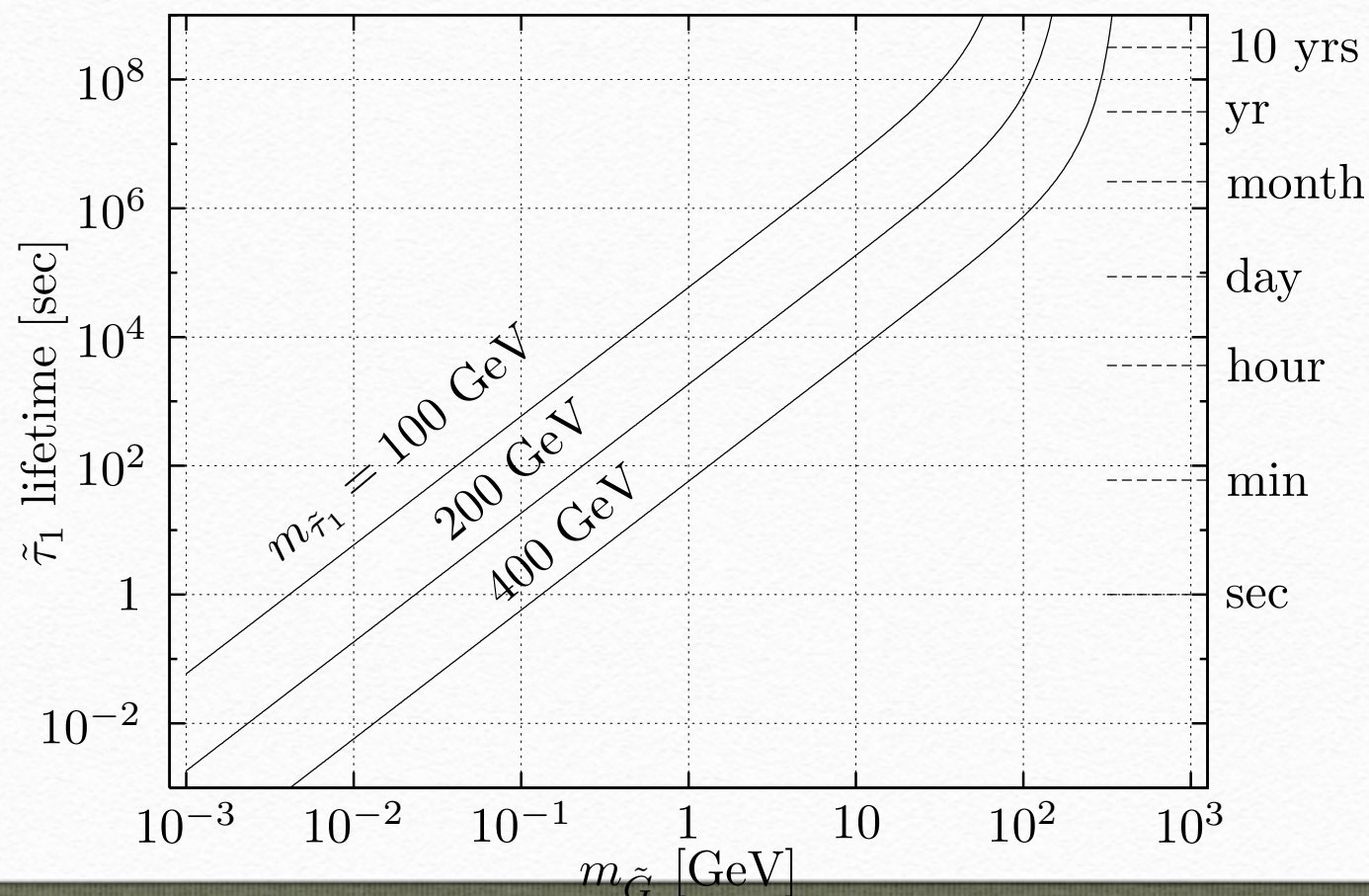
Gravitino mass and NLSP life time

$$\Gamma_{\tilde{\tau}}(\tilde{\tau} \rightarrow \tilde{G}\tau) = \frac{m_{\tilde{\tau}}^5}{48\pi m_{\tilde{G}}^2 M_{\text{pl}}^2} \left(1 - \frac{m_{\tilde{G}}^2 + m_{\tau}^2}{m_{\tilde{\tau}}^2}\right)^4 \left[1 - \frac{4m_{\tilde{G}}^2 m_{\tau}^2}{(m_{\tilde{\tau}}^2 - m_{\tilde{G}}^2 - m_{\tau}^2)^2}\right]^{3/2}$$

$$L = \frac{m_{\tilde{\tau}}^2}{\sqrt{3}m_{3/2}M_{\text{pl}}} (\bar{\chi}\tau_R\tilde{\tau}_R^* + H.c.)$$

NLSP fly and decay

NLSP life time measurement → Hidden sector determination F_0



Life time + Mass measurement →
Planck scale measurement

SUSY breaking scenarios and mass spectrum

- Low energy phenomenology is not the end of the story .
- Hidden sector break supersymmetry. “flavor and CP” problem
 - gravity mediation, gauge mediation, anomaly mediation(string inspired mixed cases) , “geometric separation”
- Problems (why alternatives are searched for)
 - Light higgs boson (hope and/or worry) little hierarchy
 - DM constraints
 - gravitino, string moduli.....



Rich Field!

Higgs boson discovery in SUSY parameter constraint

tree level

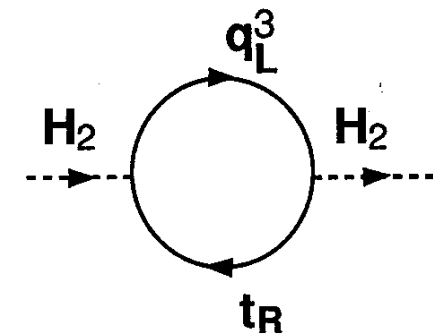
$$V_0 = m_1^2 \phi_1^2 + m_2^2 \phi_2^2 + 2m_3^2 \phi_1 \phi_2 \\ + \frac{1}{8} (g^2 + g'^2) (\phi_2^2 - \phi_1^2)^2.$$

$$m_{h,H}^2 = \frac{1}{2} [m_A^2 + m_Z^2 \\ \mp \sqrt{(m_A^2 + m_Z^2)^2 - 4m_A^2 m_Z^2 \cos^2 2\beta}],$$

Supersymmetric relation

effect of scalar top decoupling

$$\left[\frac{1}{2} \frac{\partial^2 V_1}{\partial \phi^2} \right]_{\phi=v} = \left[\frac{1}{2} \frac{\partial^2 V_0}{\partial \phi^2} + \frac{3g^2}{16\pi^2} \frac{m_t^4}{m_W^2} \log \frac{m_{\tilde{t}}^4}{m_t^4} \right]_{\phi=v},$$



threshold corrections

$$\delta\lambda = \frac{6}{(4\pi)^2} \left(\frac{m_A^2}{m_{\text{SUSY}}^2} - \frac{1}{12} \frac{m_A^4}{m_{\text{SUSY}}^4} \right) h_t^4.$$

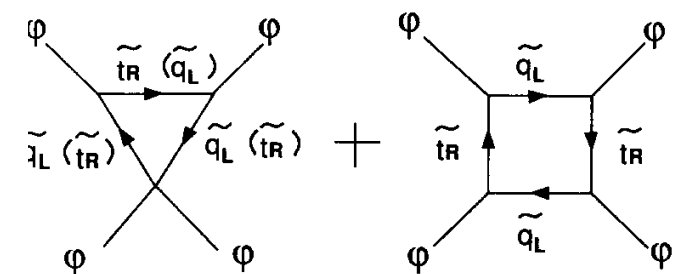
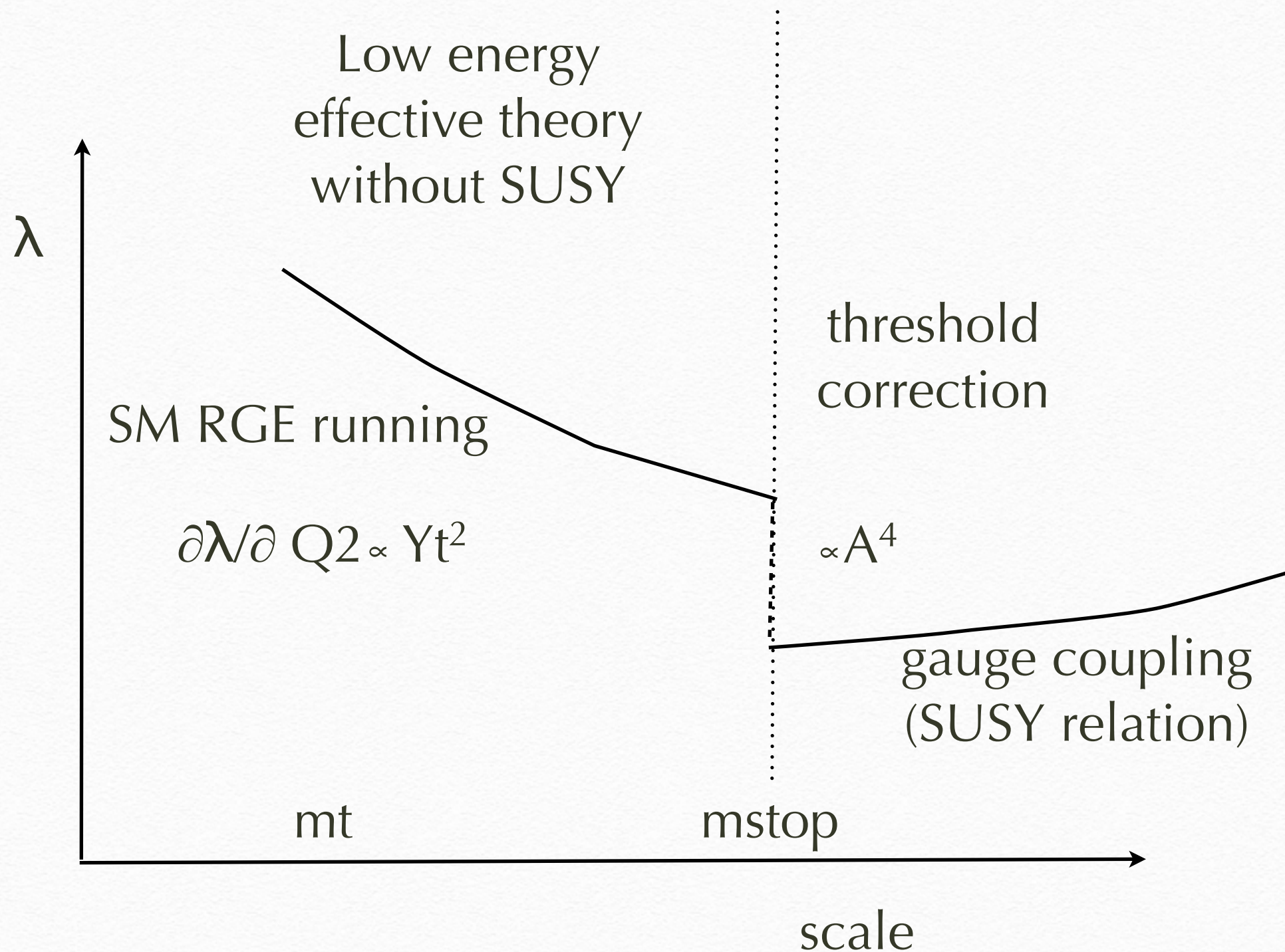


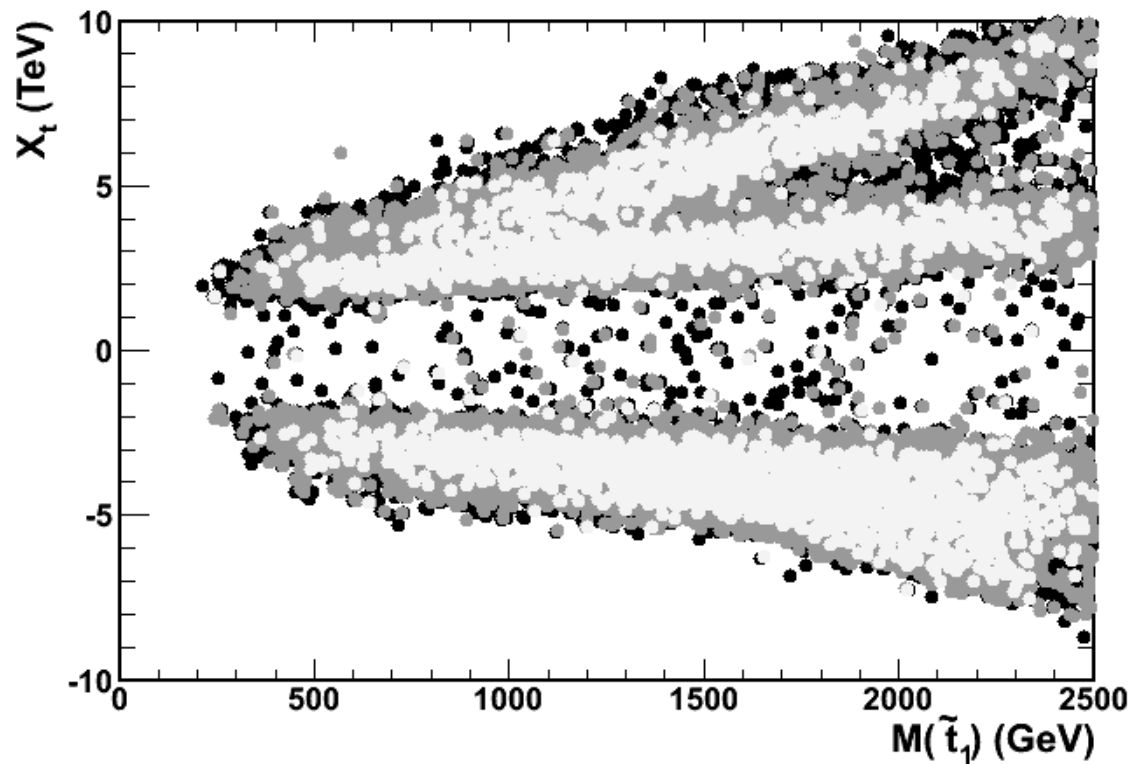
Fig. 2. Feynman diagrams which give a finite, extra $|\phi|^4$ term.

two contributions



Higgs mass vs SUSY

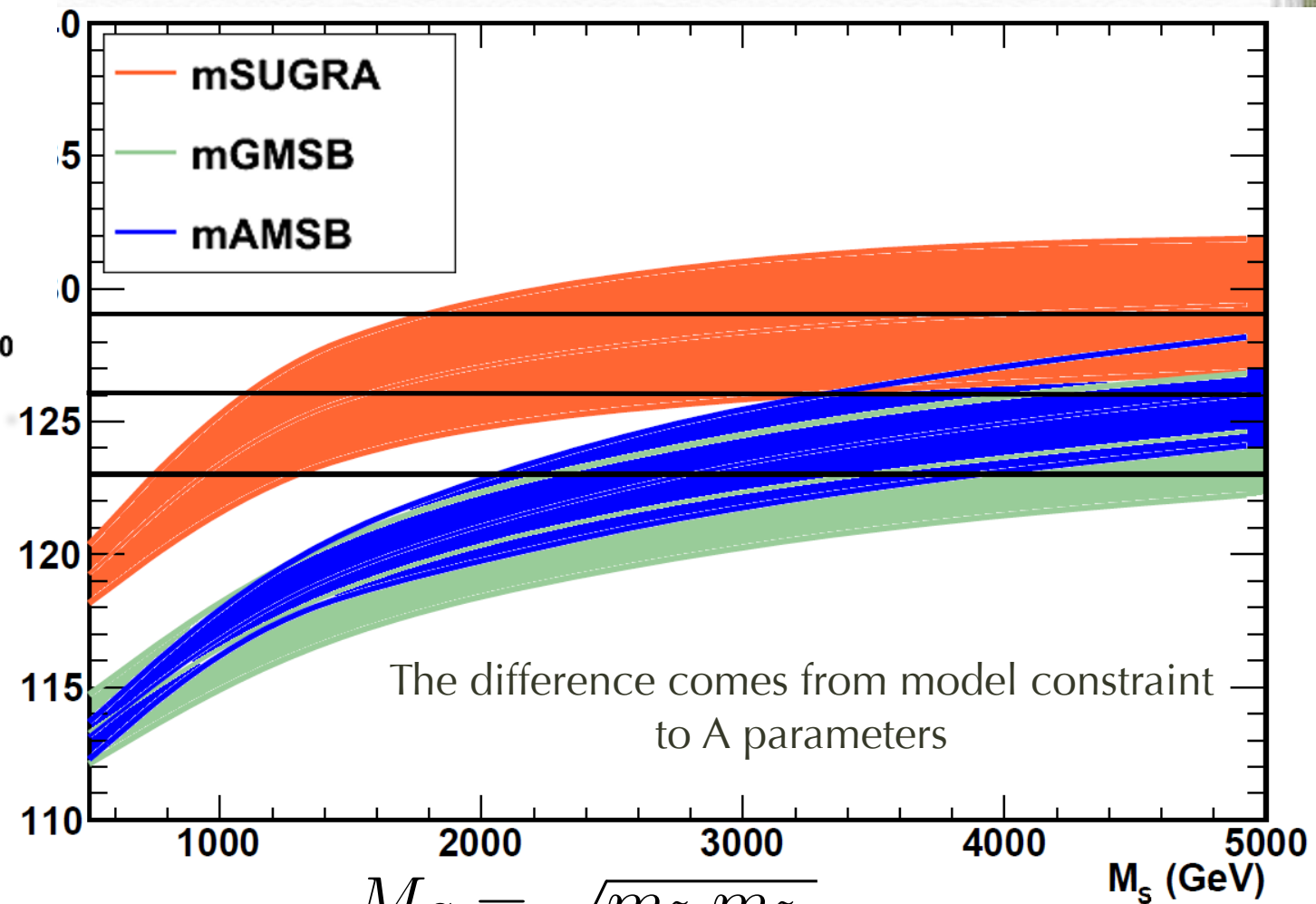
off diagonal part



large SUSY scale required
otherwise if you
assume symmetry
breaking mechanism

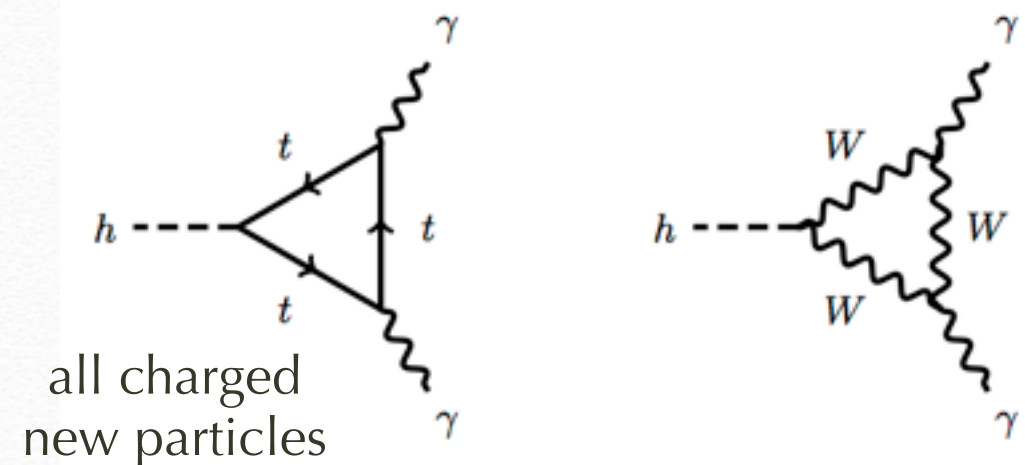
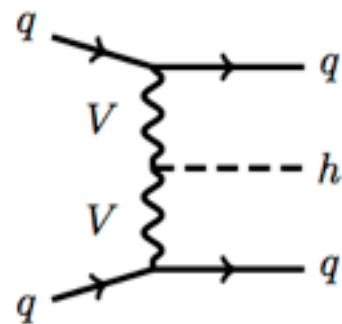
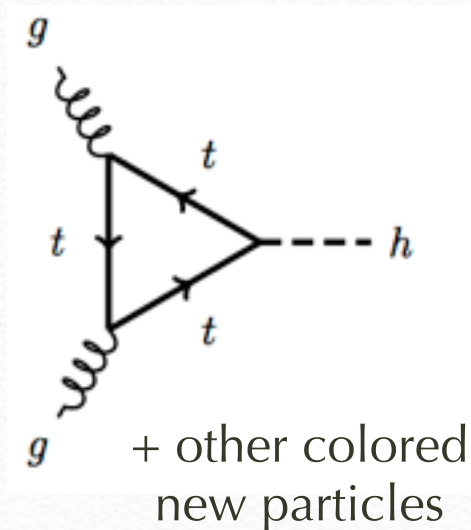
Tension is extremely
strong for gauge mediation

large stop mixing required for
light stop mass in model
independent approach



$$M_S = \sqrt{m_{\tilde{t}_1} m_{\tilde{t}_2}}$$

possible deviations?



production of
Higgs boson

very light stau or
third generation loop
may change Higgs
branches

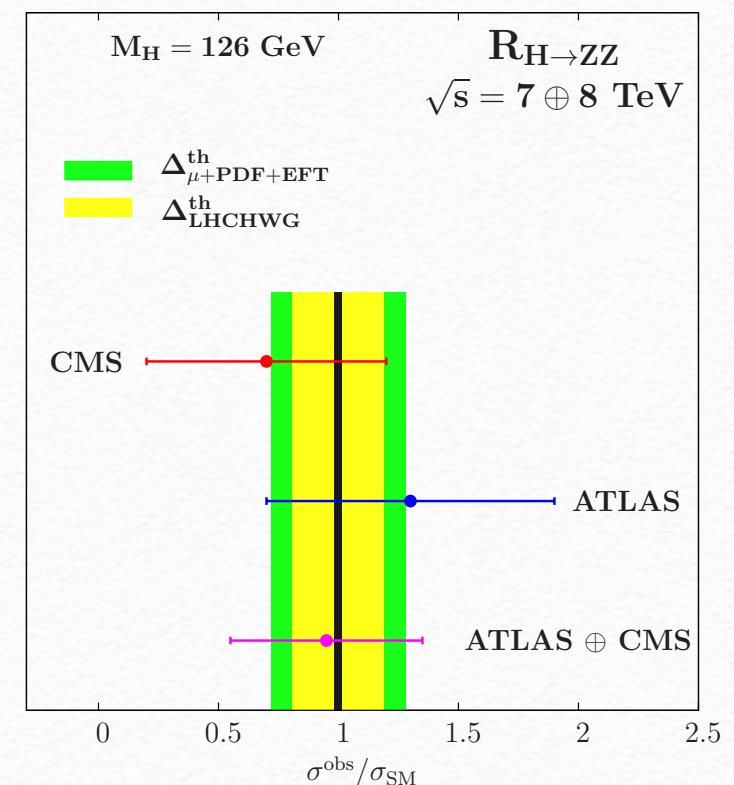
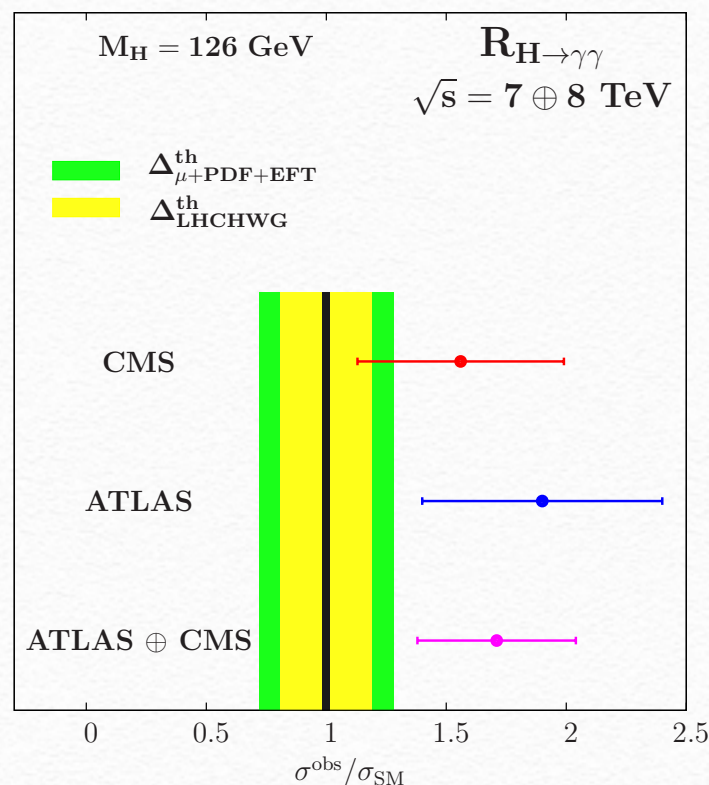
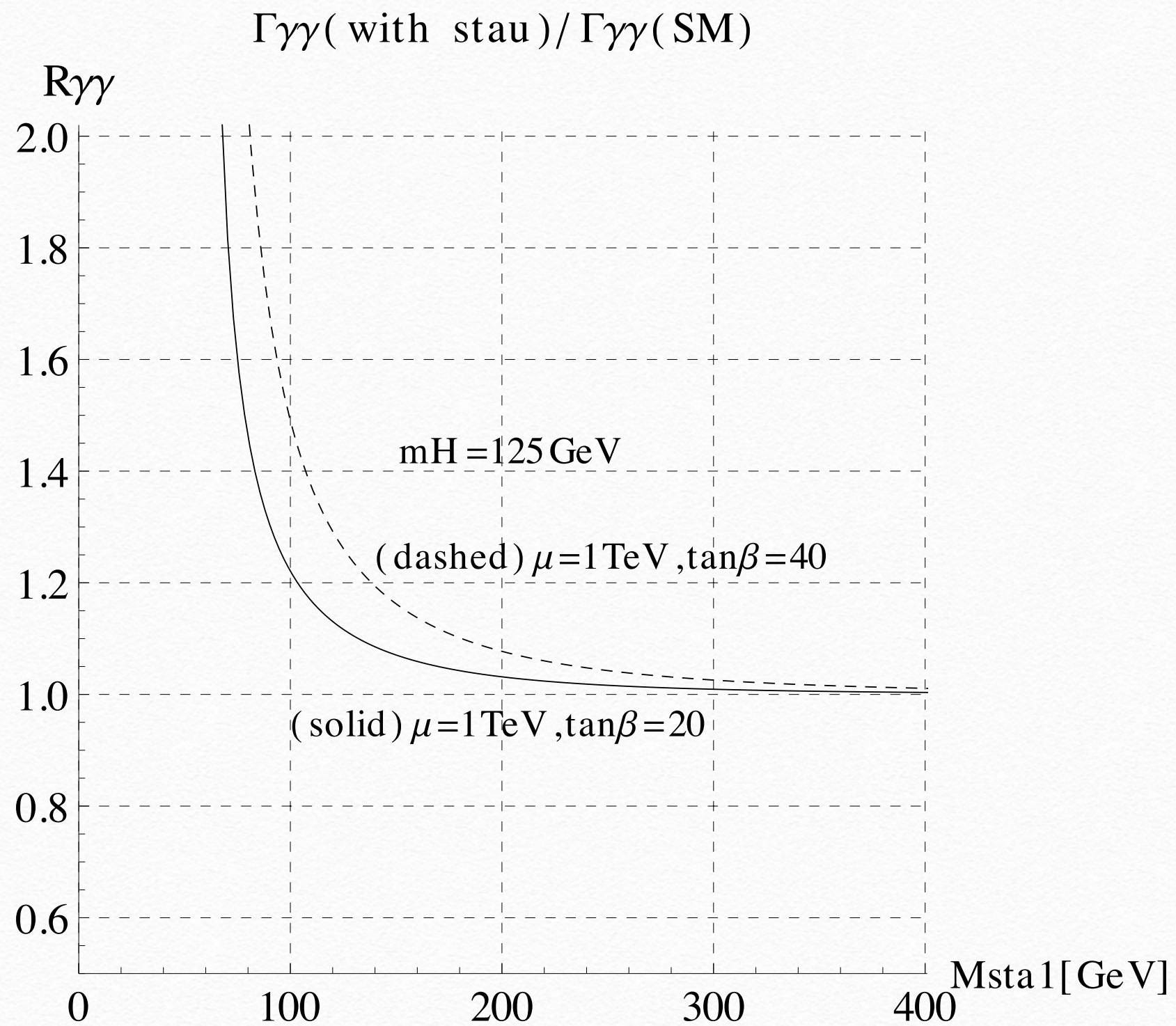


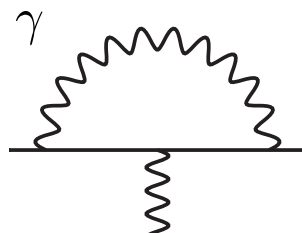
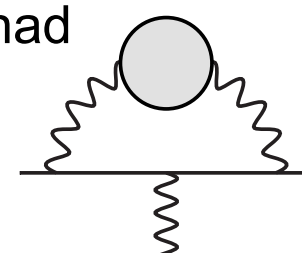
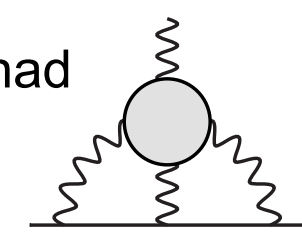
Figure 2: The value of R_{XX} for the $H \rightarrow \gamma\gamma$ and ZZ final states given by the ATLAS and CMS collaborations, as well as their combination, compared to the theoretical uncertainty bands.



long standing deviation from SM

muon g-2

Standard Model Prediction

Exp (E821)		116 592 089	(63)	[10 ⁻¹¹]	
QED (α^5)		116 584 718.962	(0.08)		
EW (W/Z/H _{SM} , NLO)		153.2	(1.8)		
Hadronic (leading)	[HLMNT]	6 949.1	(43)*		
	[DHMZ]	6 923	(42)		
Hadronic (α higher)		-98.4	(0.7)		
Hadronic (LbL)	[RdRV]	105	(26)*		
	[NJN]	116	(39)		

$$a_{\mu}^{\text{exp}} - a_{\mu}^{\text{SM}} = (26.1 \pm 8.0) \cdot 10^{-10} > 3\sigma \text{ deviation}$$

muon g-2 in SUSY

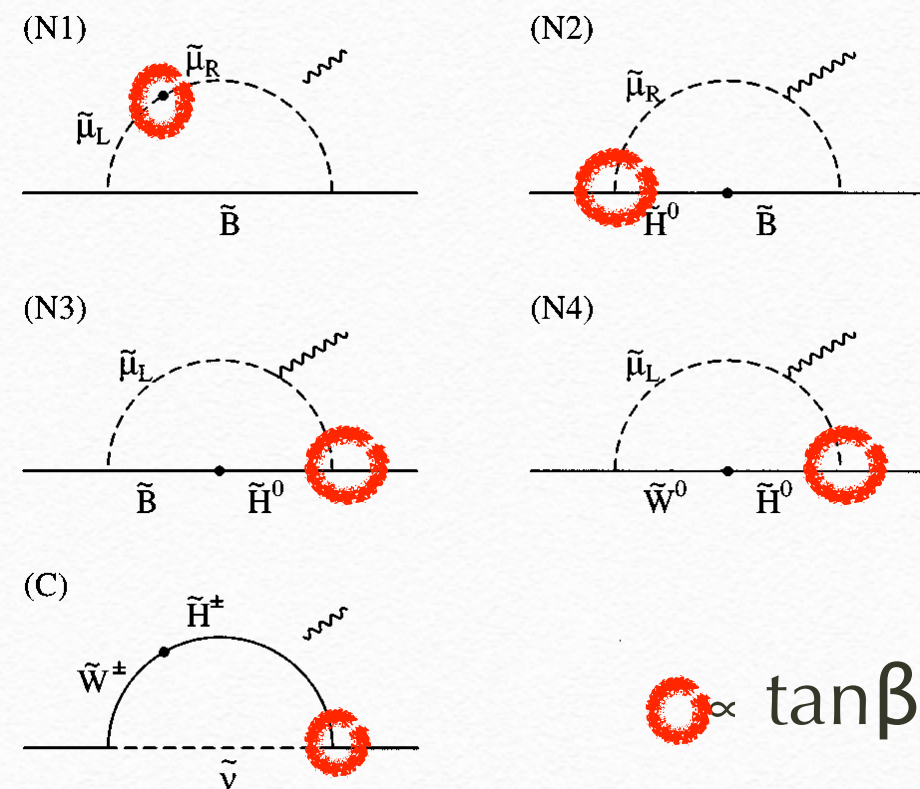
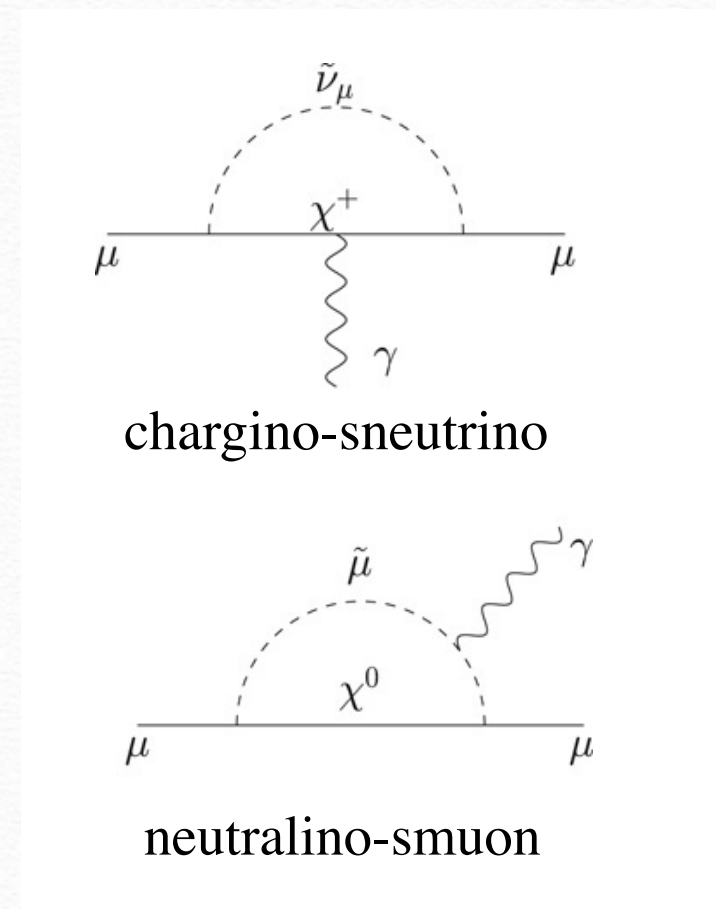


FIG. 2. Feynman diagrams which give rise to the muon MDM in the mass insertion method.

$$\Delta a_{\mu}^{\chi^0 \tilde{\mu}} \simeq \Delta a_{\mu}^{N1} + \Delta a_{\mu}^{N2} + \Delta a_{\mu}^{N3} + \Delta a_{\mu}^{N4}$$

$$= \frac{1}{192\pi^2} \frac{m_{\mu}^2}{m_{\text{SUSY}}^2} (g_1^2 - g_2^2) \tan\beta,$$

$$\Delta a_{\mu}^{\chi^{\pm} \tilde{\nu}} \simeq \Delta a_{\mu}^C = \frac{1}{32\pi^2} \frac{m_{\mu}^2}{m_{\text{SUSY}}^2} g_2^2 \tan\beta.$$

Note that we cannot take
very large $\tan\beta$ without worrying about B constraints...

Higgs mass and $g-2$

Endo, Hamaguchi, Iwamoto, Nakayama Yokozaki

$$A_0 (= A_u = A_d = A_e)$$

only A_u is truned on..

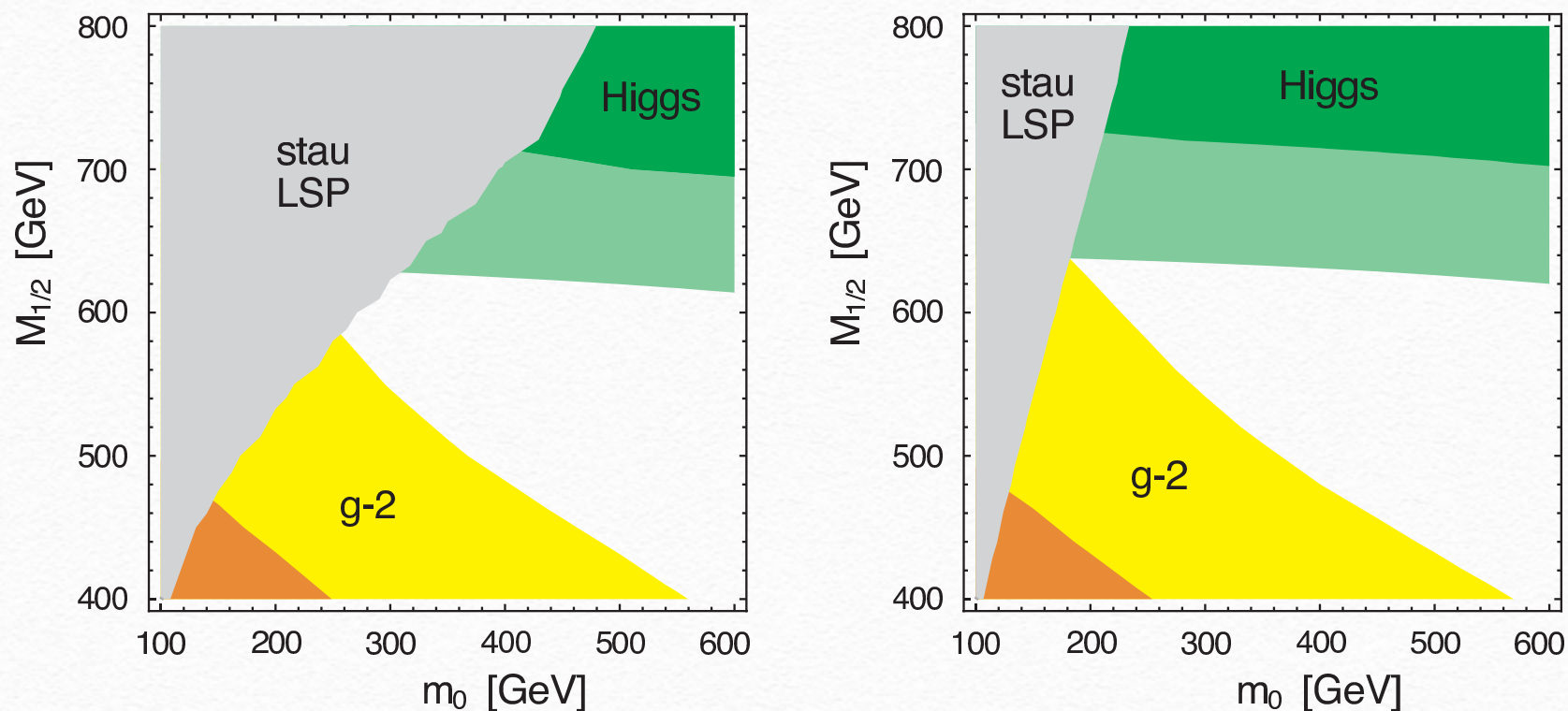


Figure 3: Contours of the Higgs mass and the muon $g - 2$ are shown. The Higgs mass are maximized by choosing A_0 and A_u appropriately under the $\text{Br}(\bar{B} \rightarrow X_s \gamma)$ constraint in the CMSSM models (left) and the extension (right), respectively (“ m_h -max scenario”). In the dark green region, the Higgs mass is 124–126 GeV, and it becomes larger than 124 GeV in the light green region once the uncertainties are included. In the orange (yellow) regions, the muon $g - 2$ is explained at the 1σ (2σ) level. The LSP is the (lighter) stau in the upper-left shaded region, while the lightest neutralino in the rest.

changing higgs mass relation by introducing additional particles

- Singlet in Higgs potential (no μ parameter)

$$W = \lambda \hat{S} \hat{H}_u \hat{H}_d + \frac{1}{3} \kappa \hat{S}^3 + h_t \hat{Q} \hat{H}_u \hat{T}_R^c - h_b \hat{Q} \hat{H}_d \hat{B}_R^c .$$

$$-\mathcal{L}_{\text{soft}} = m_{H_u}^2 |H_u|^2 + m_{H_d}^2 |H_d|^2 + m_S^2 |S|^2 + m_Q^2 |Q|^2 + m_T^2 |T_R^2| + m_B^2 |B_R^2| \\ + (\lambda A_\lambda H_u H_d S + \frac{1}{3} \kappa A_\kappa S^3 + h_t A_t Q H_u T_R^c - h_b A_b Q H_d B_R^c + \text{h.c.}) .$$

- additional matter that couple to Higgs boson

$$W = Y' H_u Q' U' + M' (Q' \bar{Q}' + U' \bar{U}')$$

$$\Delta m_h \simeq \frac{3v^2}{4\pi^2} Y'^4 \ln \frac{m_S^2}{m_F^2} + \dots$$

$m_{S(F)}$: vector scalar(fermion) mass

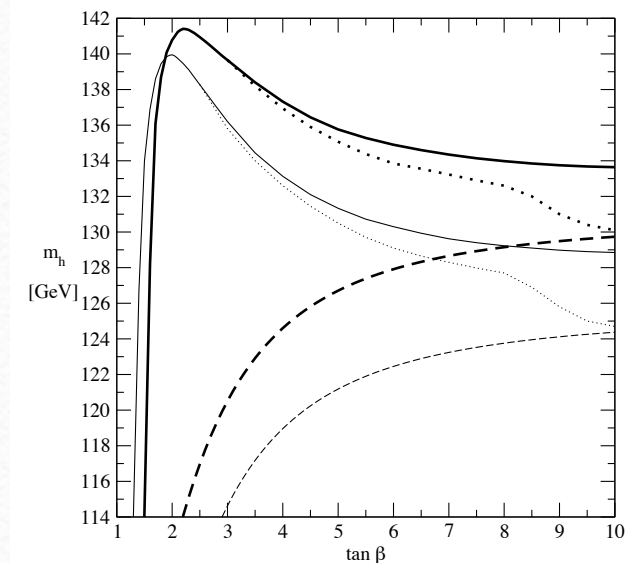
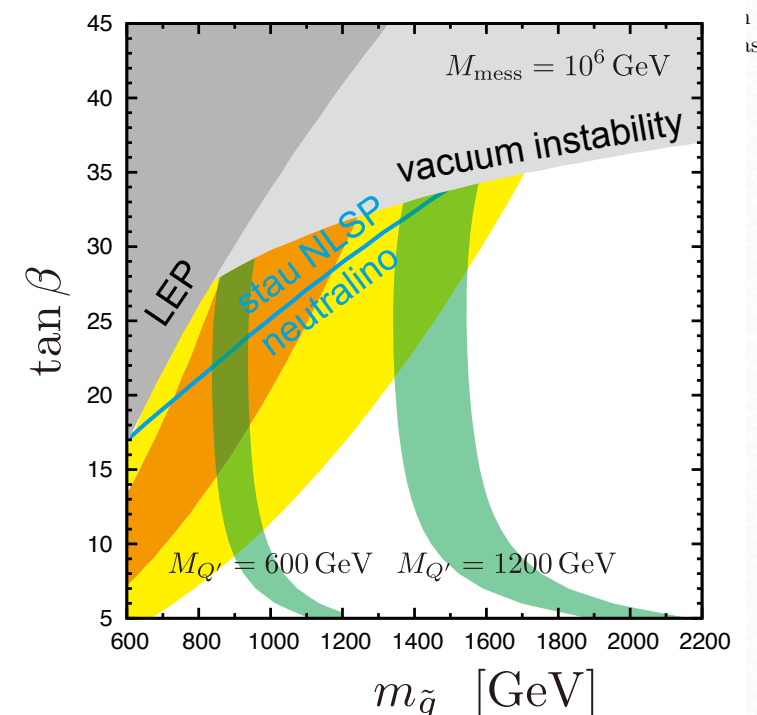


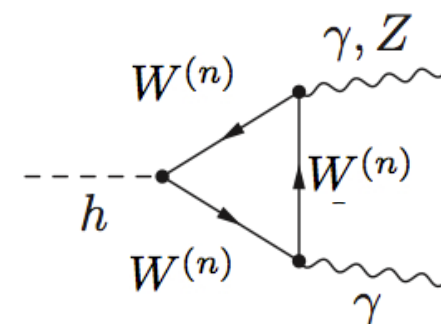
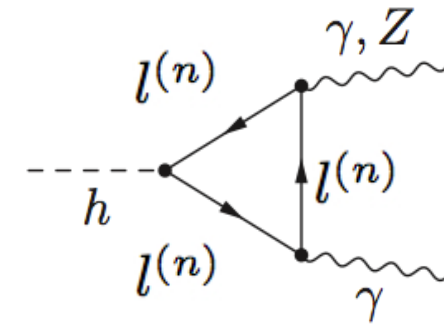
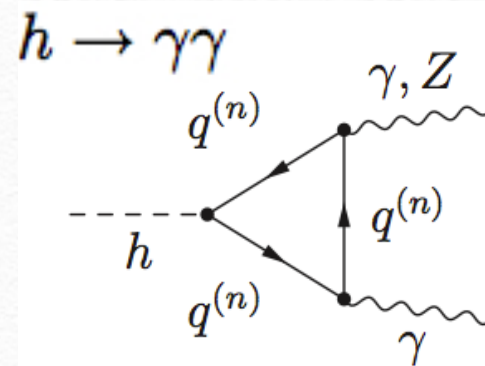
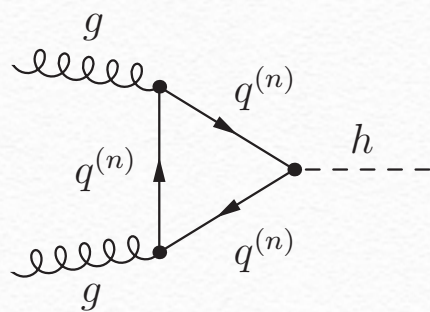
Figure 1: Upper bound on the lightest Higgs mass in the NMSSM for $m_{top} = 178$ GeV (thick full line: m_A arbitrary, thick dotted line: $m_A = 1$ TeV) and $m_{top} = 171.4$ GeV (thin full line: $m_A = 1$ TeV, thin dotted line: m_A arbitrary, thin dashed line: $m_A = 1$ TeV).



M (with
dashed
lines are

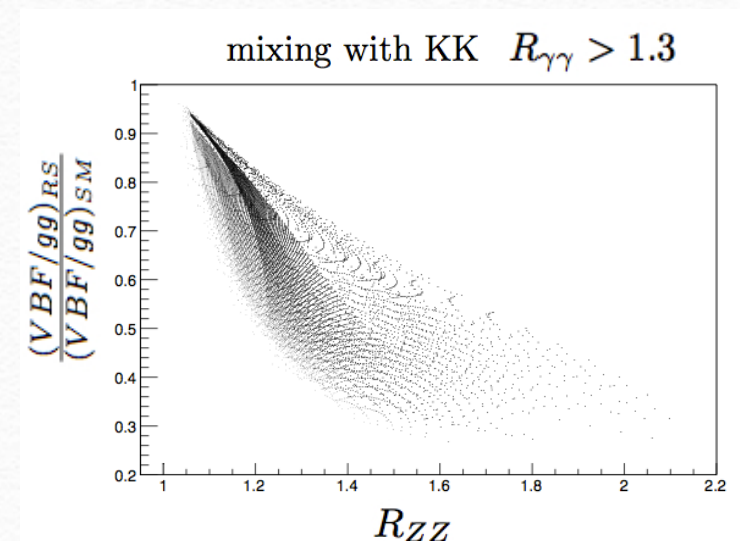
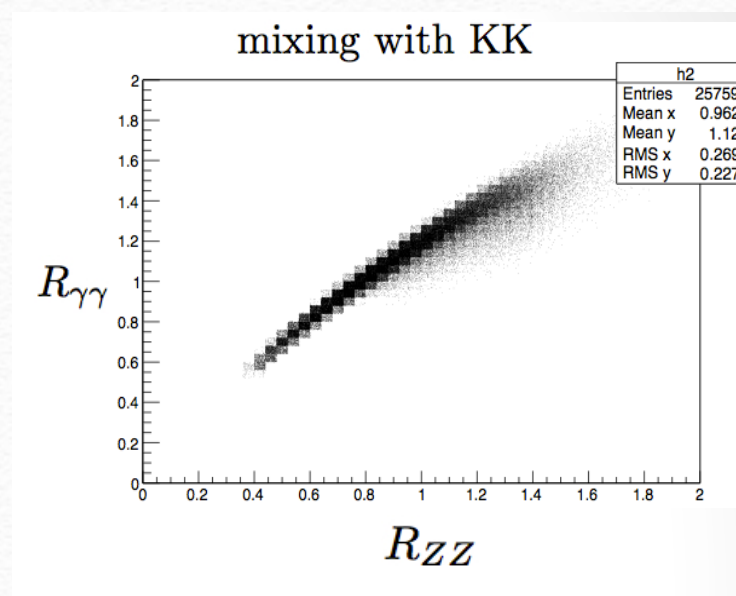
Other new physics?

- RS Higgs sector Large radiative corrections due to large overlap of KK mode to the visible brane.



In addition there are mixing between radion and higgs boson

$$\mathcal{L}_\xi = \sqrt{g_{\text{ind}}}\xi R(g_{\text{ind}})H^\dagger H$$



DM candidate in SUSY

- neutralino LSP
 - a neutralino is a mixture of gauginos and Higgsinos
 - $\Omega(h^2) \sim 0.1 \Leftrightarrow$ light slepton, Higgs exchange, or gaugino-higgsino mixing, light annihilation.
- gravitino LSP
 - no prediction on the density.
 - direct detection is not possible
- sneutrino essentially excluded

But in general, it is good to have
a DM candidate in the model

Why dark matter is in the Universe

- Metric (homogeneous and isotropic)
Robertson-Walker metric

$$ds^2 = -c^2 dt^2 + R(t)^2 \left(\frac{dr^2}{1 - kr^2} + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2 \right)$$

- Universe must be in thermal eq. in early Universe

- particle density

$$\rho = \frac{\pi^2}{30} g_B T^4 + \frac{7}{8} \frac{\pi^2}{30} g_F T^4$$

- It is adiabatic expansion (most of the time)

$$T \propto R^{-1} \quad \begin{array}{l} \rho(\text{rad}) \propto \frac{1}{R^4} \\ \rho(\text{NR}) \propto \frac{1}{R^3} \end{array}$$

- Expansion rate (Einstein equation)

$$H = \frac{\dot{R}}{R} \quad H^2 = \frac{8\pi G}{3} \rho \quad \longrightarrow \quad H \propto T^2$$

Decoupling of stable particle in early universe

Boltzmann equation of the number density of dark matter

number density reduces
as Universe expands $\propto R^{-3}$

dark matter pair annihilate
to reduce the number

$$\frac{dn}{dt} + 3Hn = -\langle\sigma v\rangle(n^2 - \cancel{n_{EQ}^2})$$

$O(T^5)$ $O(T^6)$

If in thermal eq,
n reduces as
 T^3 or $\exp(-m/T)$

dark matter density is
equal to that in thermal eq.
if interaction is large enough

For sufficiently low temperature, the right hand side of the equation does not contribute, and number density does not reduce any more. We call this thermal relic density

thermal relic density of cold dark matter

decoupling temperature
(LHS~RHS)

$$H \sim \sigma v n$$

If $T(\text{dec}) < m$, the number density drops quickly and $T(\text{dec})$ does not depend on cross section so much
note that

$$n \sim T^{3/2} \exp(-m/T)$$

$$n \sim 1/H$$

Then the number density is roughly proportional to $1/\sigma$

The number density can constrain model parameter
or if one measures model parameter very well, one
can check big bang assumption

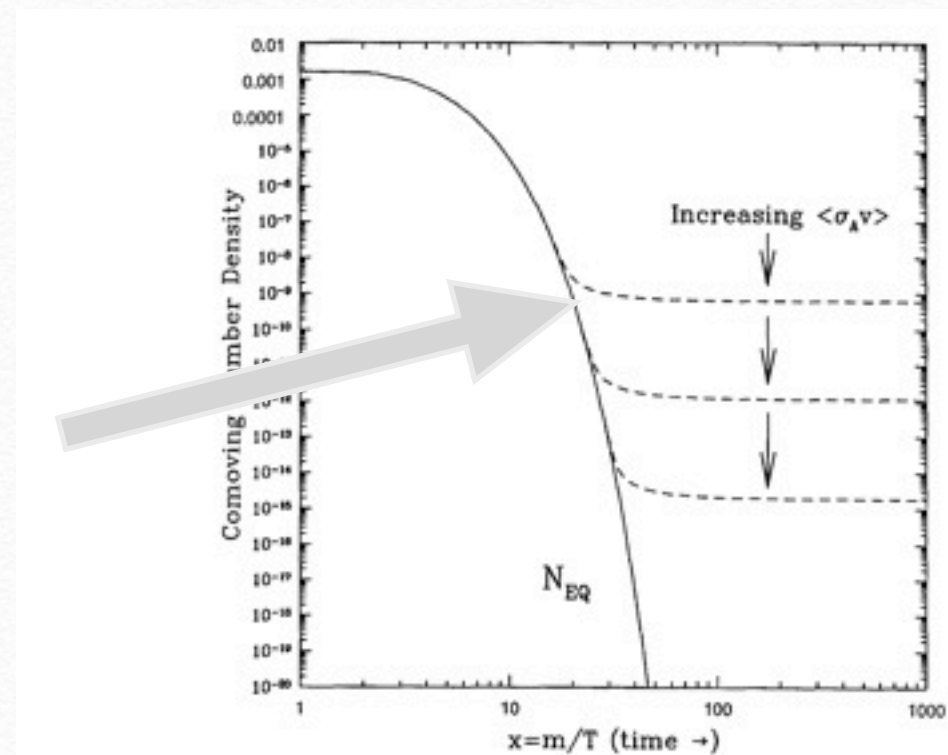


Fig. 4. Comoving number density of a WIMP in the early Universe. The dashed curves are the actual abundance, and the solid curve is the equilibrium abundance. From [31].

The nature of the Lightest Neutralino

Neutralino is a mixture of gaugino and higgsino. The higgsino-gaugino mixing comes from Higgs vacuum expectation value

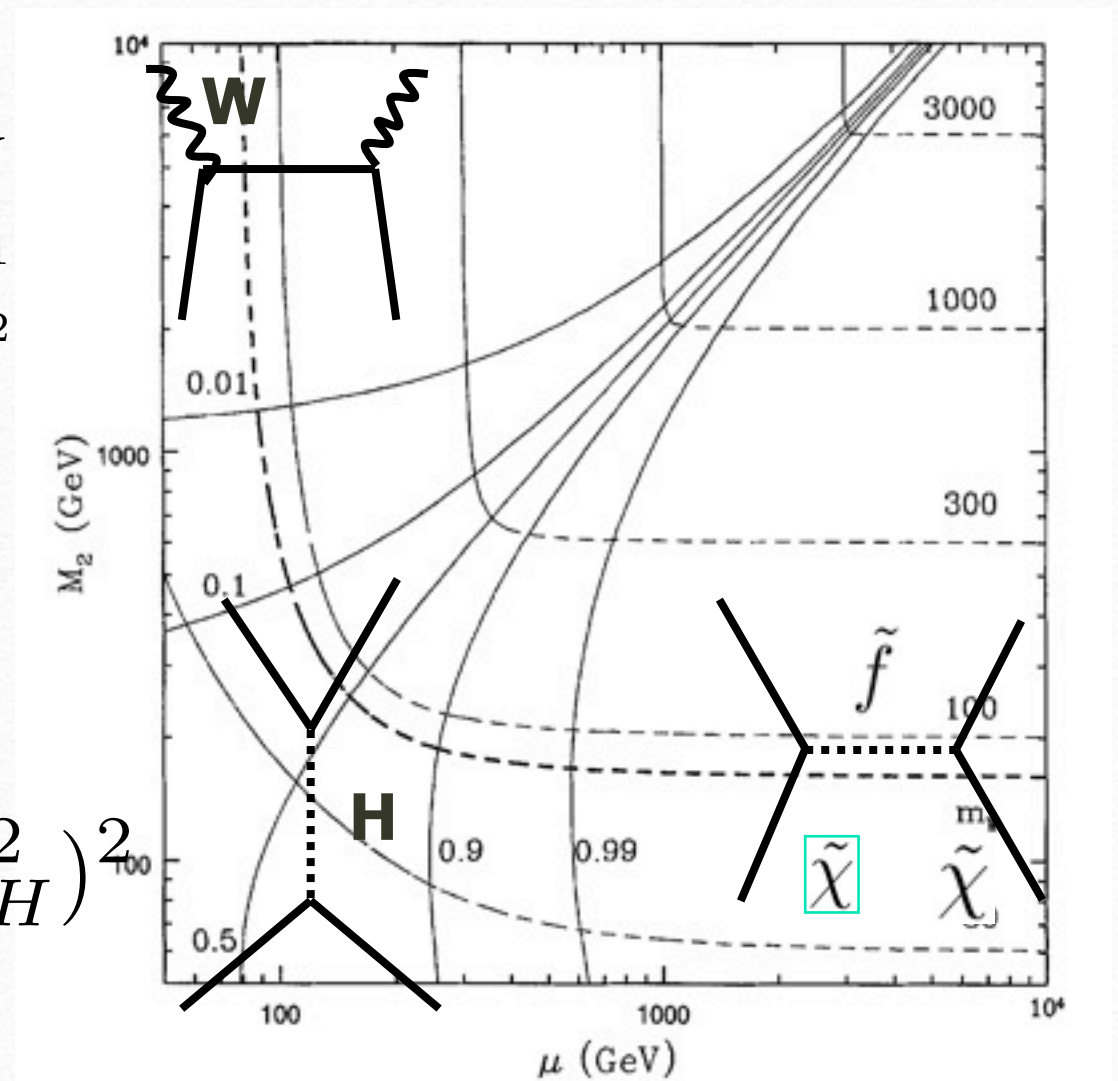
Neutralino mass matrix

$$M = \begin{pmatrix} M_1 & 0 & -m_Z s_W c_\beta & m_Z s_W s_\beta \\ 0 & M_2 & m_Z c_W c_\beta & -m_Z c_W s_\beta \\ & & 0 & -\mu \\ & & -\mu & 0 \end{pmatrix} \begin{pmatrix} \tilde{B} \\ \tilde{W} \\ \tilde{H}_1 \\ \tilde{H}_2 \end{pmatrix}$$

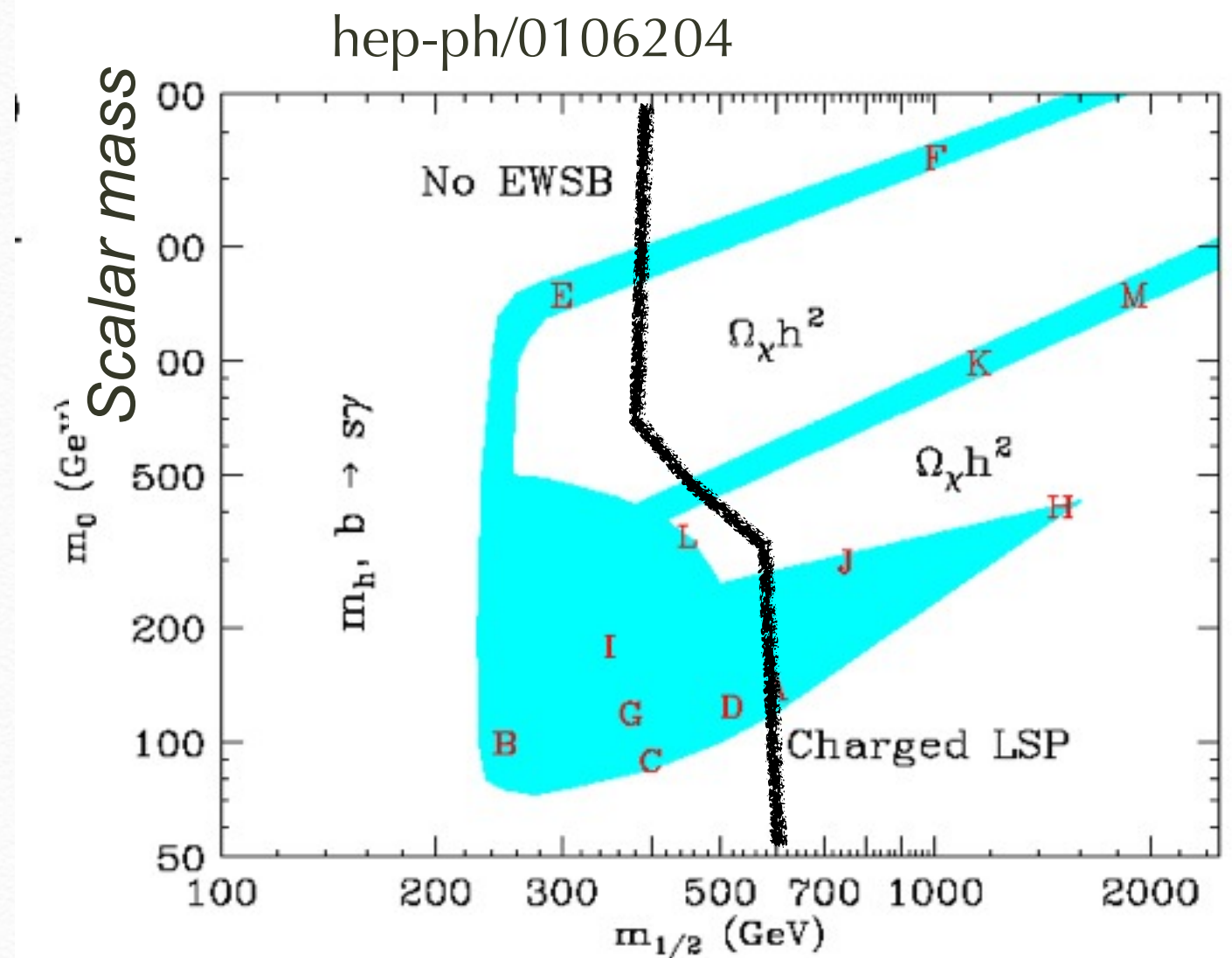
$$M_1 \ll \mu \quad \sigma v \propto m_{\tilde{\chi}}^2 / m_{\tilde{l}}^4$$

$$M_1 \gg \mu \quad \sigma v \propto 1 / m_{\tilde{\chi}}^2$$

$$M_1 \sim \mu \quad \sigma v \propto m_{\tilde{\chi}}^2 / (4m_{\tilde{\chi}}^2 - m_H^2)^2$$



DM density constraint is important in
“MSUGRA”



Have we excluded “bulk regions??

Gaugino mass

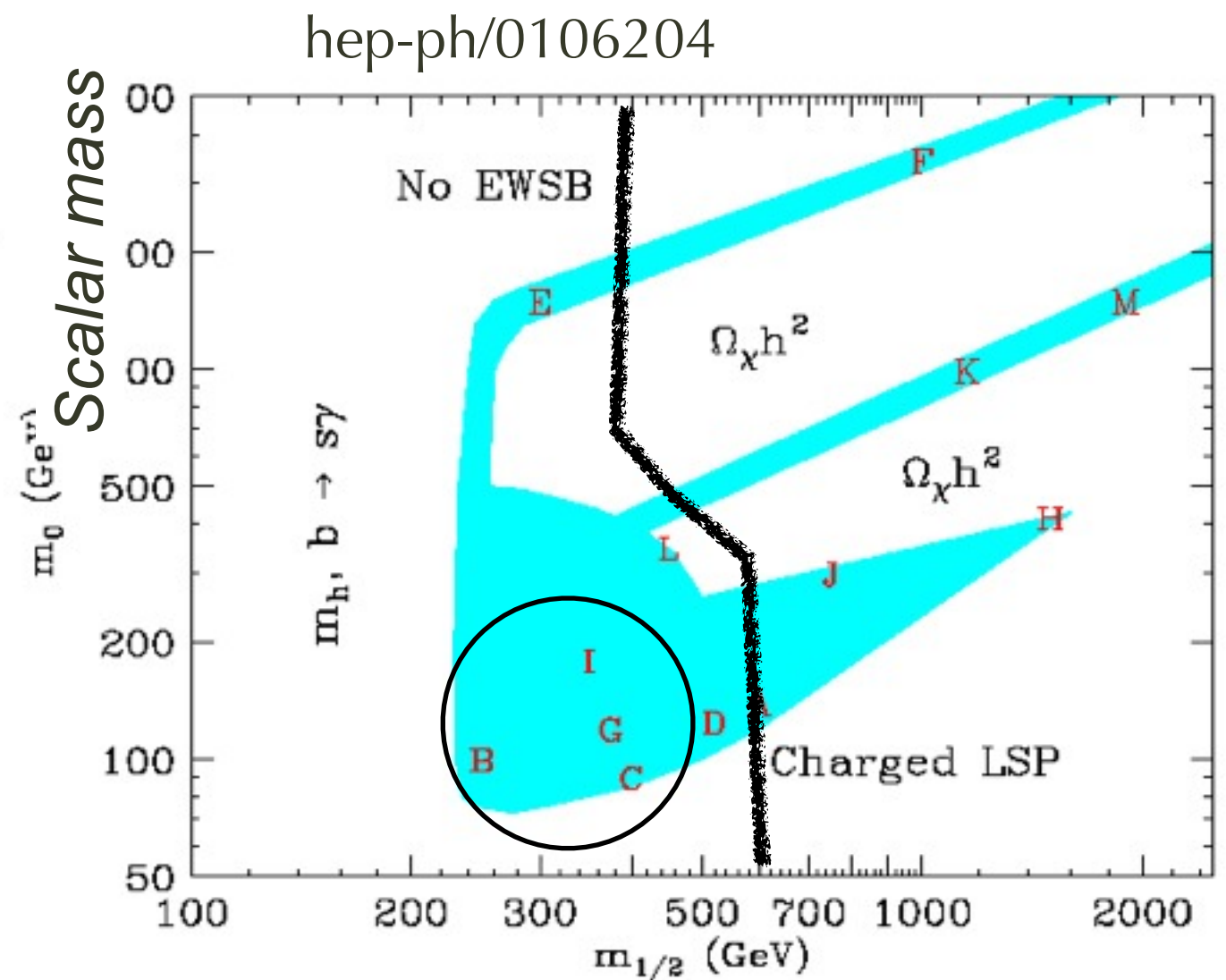
DM density constraint is important in “MSUGRA”

1) bulk region : LSP Bino like.

→ Slepton exchange

$$\Omega h^2 \propto m_{\tilde{l}}^4 / m_{\tilde{\chi}}^2$$

too large mass density



Have we excluded “bulk regions??

Gaugino mass

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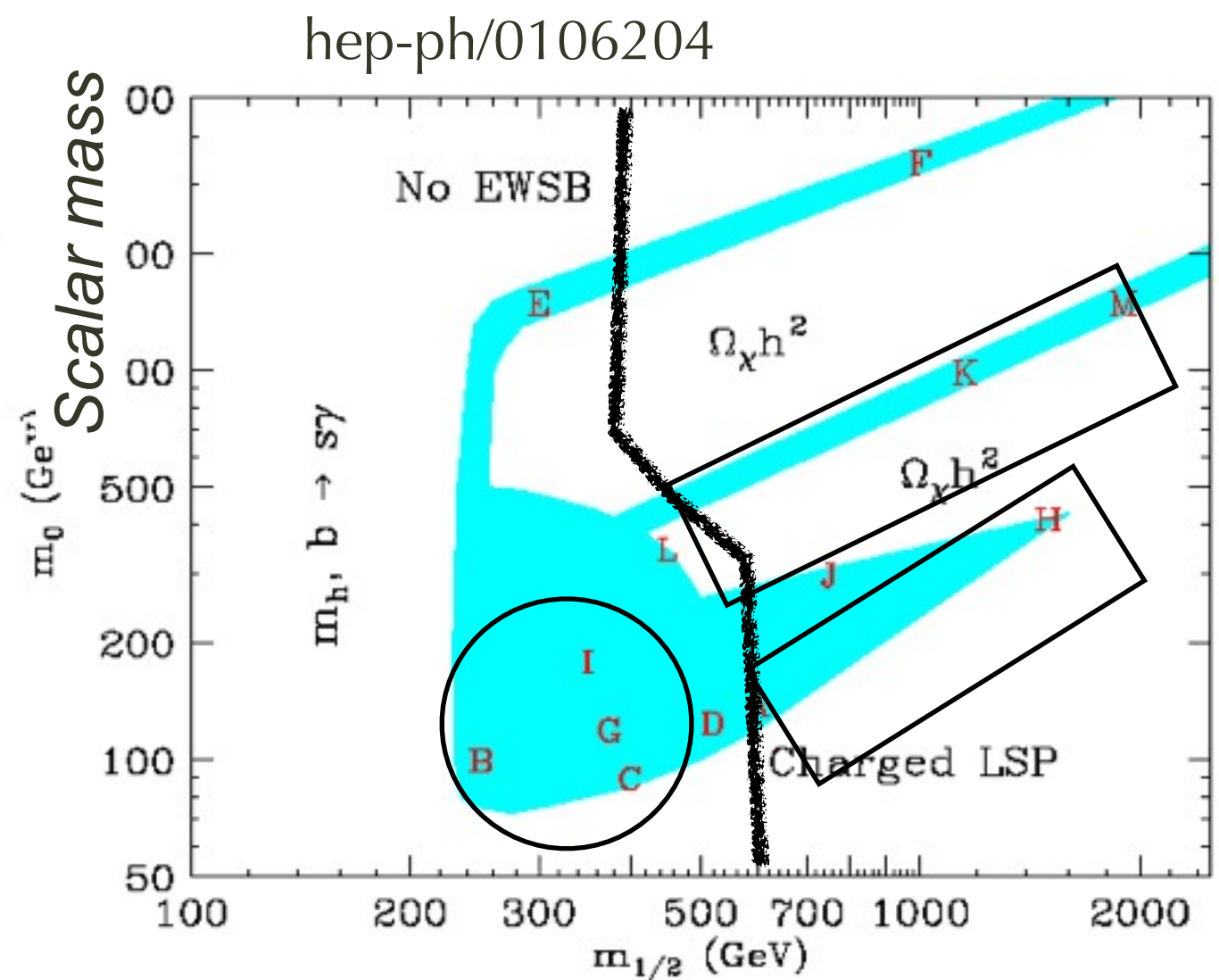
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2) Higgs pole effect $m_H = 2m_\chi$

3) coannihilation



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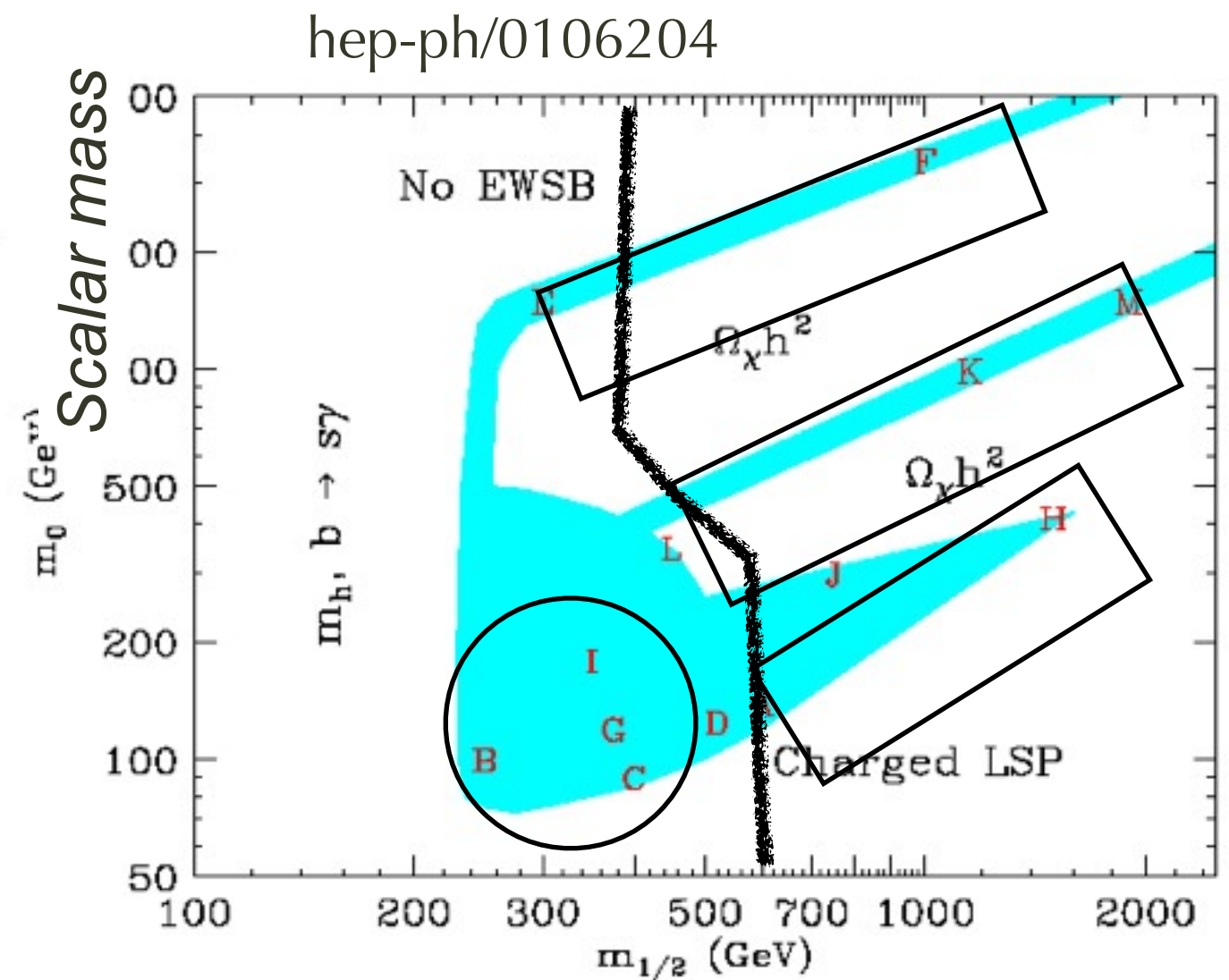
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2) Higgs pole effect $m_H = 2m_\chi$

3) coannihilation

4) focus point region:
higgsino-gaugino mixing



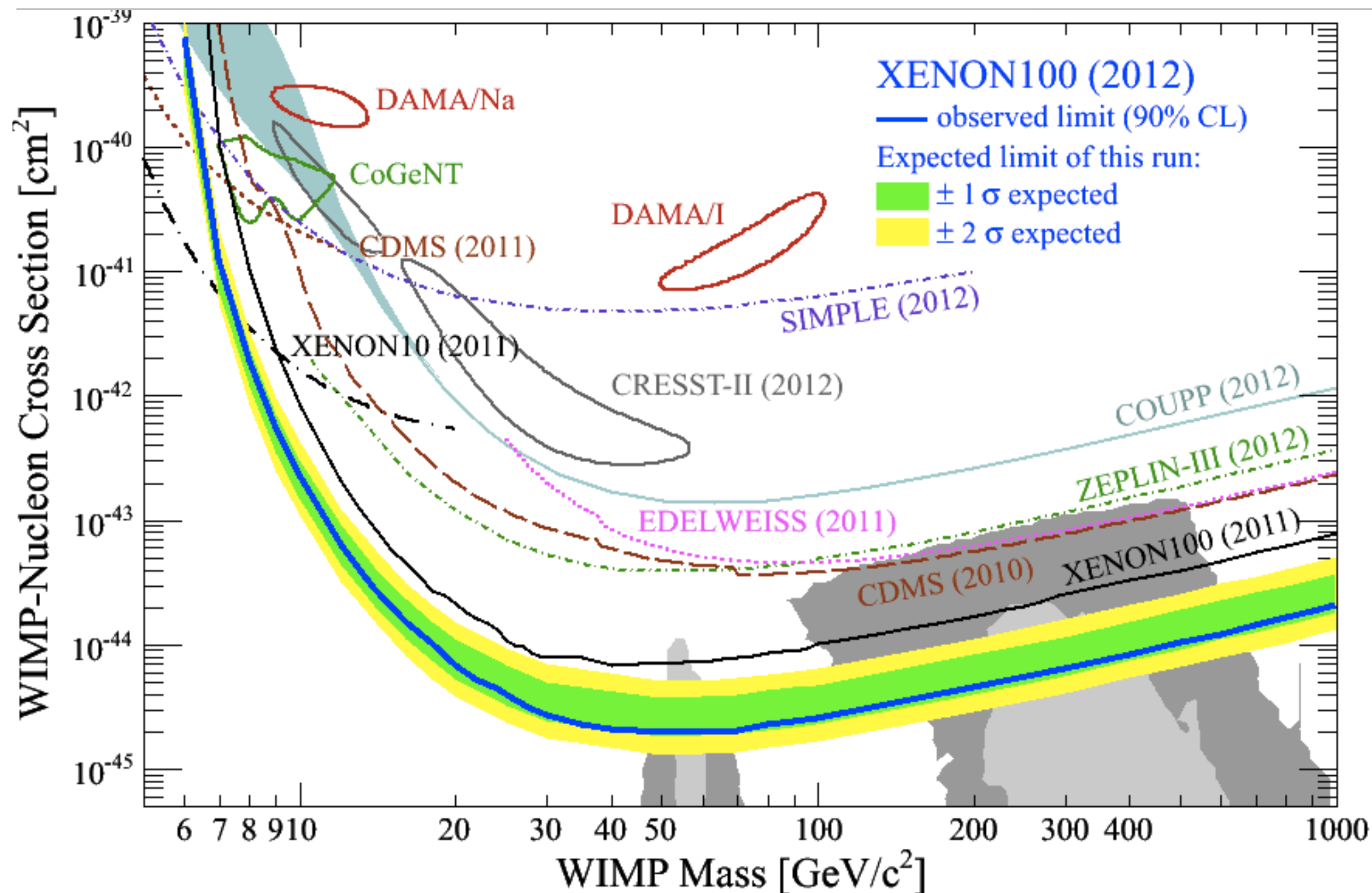
Have we excluded “bulk regions??

Gaugino mass

Time for serious thought about BSM and dark matter

- LSP may be light even if light squark and gluino are excluded (**lifting GUT relations**) . $g-2$ still pointing to light
- the LSP maybe higgsino even if scalar masses are small. (**lifting GUT relation of higgs mass**)
- any particle can degenerate with LSP...
- **More direct and model independent information needed.**
 - Direct bounds on chargino and neutralino/no tau excess/are we too much relying on GUT relation?

Direct search will be serious constraint this year



Upper Limit (90% C.L.) is $2 \times 10^{-45} \text{ cm}^2$ for $55 \text{ GeV}/c^2$ WIMP