Parton Distributions and Uncertainties at the LHC

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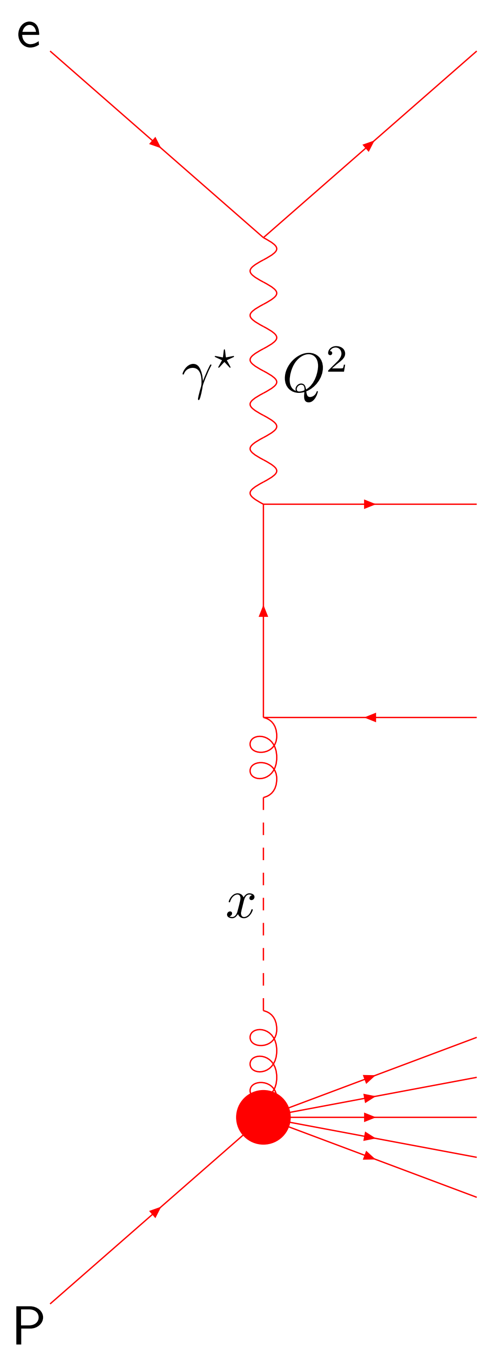
Strong force makes it difficult to perform analytic calculations of scattering processes involving hadronic particles.

The weakening of $\alpha_S(\mu^2)$ at higher scales $\longrightarrow$ the **Factorization Theorem**.

Hadron scattering with an electron factorizes.

Q^2 – Scale of scattering

$x = \frac{Q^2}{2m\nu}$ – Momentum fraction of Parton (ν=energy transfer)



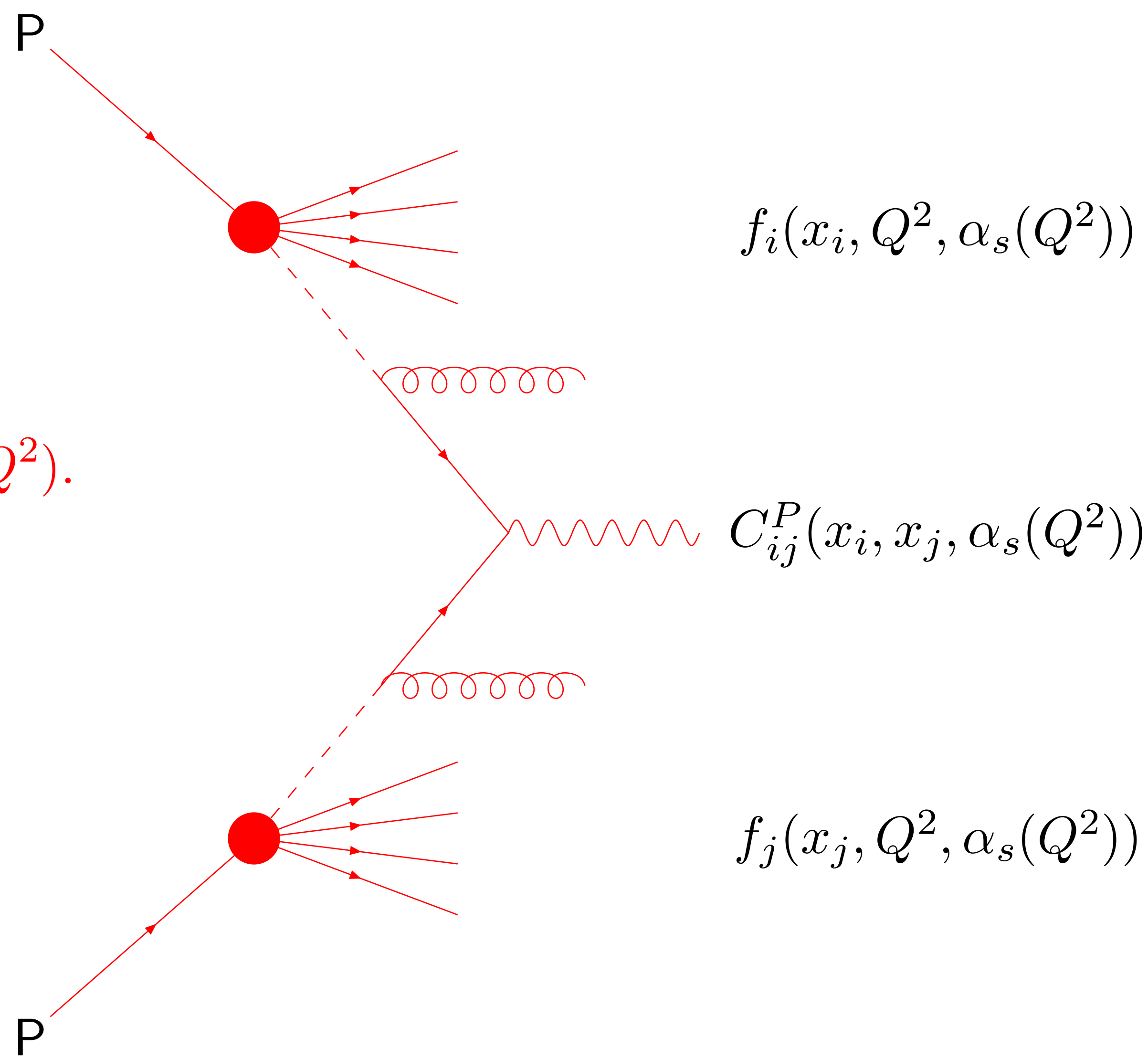
perturbative
calculable
coefficient function
$C_i^P(x, \alpha_s(Q^2))$

nonperturbative
incalculable
parton distribution
$f_i(x, Q^2, \alpha_s(Q^2))$

The coefficient functions $C_i^P(x, \alpha_s(Q^2))$ are process dependent (new physics) but are calculable as a power-series in $\alpha_s(Q^2)$.

$$C_i^P(x, \alpha_s(Q^2)) = \sum_k C_i^{P,k}(x)\alpha_s^k(Q^2).$$

Since the parton distributions $f_i(x, Q^2, \alpha_s(Q^2))$ are process-independent, i.e. **universal**, and evolve in Q^2 perturbatively, once they have been measured at one experiment, one can predict many other scattering processes.



Global fits to determine parton distributions use all available data - largely $ep \to eX$ (Structure Functions), and the most up-to-date QCD calculations to best determine **parton distributions** and their consequences. (Also $\to$ good determination of strong coupling constant.)

Currently use NLO–in–$\alpha_s(Q^2)$, i.e.

$$\begin{aligned} C_i^P(x,\alpha_s(Q^2)) &= \alpha_S^P(Q^2)(C_i^{P,0}(x) + \alpha_S(Q^2)C_i^{P,1}(x)). \\ P_{ij}(x,\alpha_s(Q^2)) &= \alpha_S(Q^2)P_{ij}^0(x) + \alpha_S^2(Q^2)P_{ij}^1(x). \end{aligned}$$

NNLO splitting functions are now known and NNLO coefficient functions are known for most processes in the fit so NNLO partons are also used.

General procedure. Start parton evolution at low scale $Q_0^2 \sim 1\mathrm{GeV}^2$. Input partons parameterised as, e.g.

$$xf(x,Q_0^2) = a_1(1-x)^{a_2}(1+a_3x^{0.5}+a_4x)x^{a_5}.$$

Evolve partons upwards using NLO DGLAP equations. Fit data for scales above $2-5\mathrm{GeV}^2$. Obtain MRST/MSTW, CTEQ, Alekhin, ZEUS, H1 .... distributions. Different levels of **global** in each fit.

In determining partons need to consider that not only are there 6 different combinations of partons, but also wide distribution of x from 0.75 to 0.00003. Need many different types of experiment for full determination.

H1 $F_2^{e^+p}(x,Q^2)$ 1996-97 moderate Q^2 and 1996-97 high Q^2, and $F_2^{e^-p}(x,Q^2)$ 1998-99 high Q^2 small x. ZEUS $F_2^{e^+p}(x,Q^2)$ 1996-97 small x wide range of Q^2. 1999-2000 high Q^2. H1 and ZEUS $F_2^{c,b}(x,Q^2)$.

NMC $F_2^{\mu p}(x,Q^2), F_2^{\mu d}(x,Q^2), (F_2^{\mu n}(x,Q^2)/F_2^{\mu p}(x,Q^2))$, E665 $F_2^{\mu p}(x,Q^2), F_2^{\mu d}(x,Q^2)$ medium x.

BCDMS $F_2^{\mu p}(x,Q^2), F_2^{\mu d}(x,Q^2)$, SLAC $F_2^{\mu p}(x,Q^2), F_2^{\mu d}(x,Q^2)$ large x.

CCFR (NuTeV) $F_2^{\nu(\bar{\nu})p}(x,Q^2), F_3^{\nu(\bar{\nu})p}(x,Q^2)$ large x , singlet, valence.

E605, E866 $pN \to \mu\bar{\mu} + X$ large x sea.

E866 Drell-Yan asymmetry $\bar{u}, \bar{d}$ $\bar{d} - \bar{u}$.

CDF W-asymmetry u/d ratio at high x.

CDF D0 Inclusive jet data high x gluon, HERA jet data medium x gluon.

CCFR (NuTev) Dimuon data constrains strange sea.

Quality of fit.

Determined by χ^2 of fit to data. Various alternatives to calculating this.

Simplest - adding statistical and systematic errors in quadrature. Ignores correlations between data points. Should be improved (but sometimes perhaps not so bad?)

Covariance Matrix. The covariance matrix is constructed as

$$C_{ij} = \delta_{ij}\sigma^2_{i,stat} + \sum_{k=1}^{n} \rho^k_{ij}\sigma_{k,i}\sigma_{k,j},$$

where k runs over each source of correlated systematic error and ρ^k_{ij} are the correlation coefficients. The χ^2 is defined by

$$\chi^2 = \sum_{i=1}^{N}\sum_{j=1}^{N}(D_i - T_i(a))C^{-1}_{ij}(D_j - T_j(a)),$$

where N is the number of data points, D_i is the measurement and $T_i(a)$ is the theoretical prediction depending on parton input parameters a. Unfortunately this relies on inverting large matrices.

Minimisation w.r.t. systematic errors. Here we incorporate the systematic errors into the theory prediction

$$f_i(a,s) = T_i(a) + \sum_{k=1}^{n} s_k \Delta_{ik},$$

where Δ_{ik} is the one-sigma correlated error for point i from source j. In this case the χ^2 is defined by

$$\chi^2 = \sum_{i=1}^{N} \left(\frac{D_i - f_i(a,s)}{\sigma_{i,unc}} \right)^2 + \sum_{k=1}^{n} s_k^2,$$

where the second term constrains the values of s_k. This allows the data to move en $masse$ relative to the theory, but assumes the correlated systematic errors are Gaussian distributed.

Can solve for each of the s_k analytically. Defining

$$B_k = \sum_{i=1}^{N} \frac{\Delta_{ik}(D_i - T_i(a))}{\sigma_{i,unc}^2}, \qquad A_{kl} = \delta_{kl} + \sum_{i=1}^{N} \frac{\Delta_{ik}\Delta_{il}}{\sigma_{i,unc}^2},$$

$$\frac{\partial \chi^2}{\partial s_k} = 0 \quad \rightarrow \quad s_i(a) = \sum_{l=1}^{n} (A^{-1})_{kl} B_l.$$

This leads to the χ^2 definition

$$\chi^2 = \sum_{i=1}^{N} \left(\frac{(D_i - T_i(a))}{\sigma_{i,unc}} \right)^2 - \sum_{k=1}^{n} \sum_{l=1}^{n} B_k (A^{-1})_{kl} B_l.$$

It can be shown that this is identical to the correlation matrix definition of χ^2, but it has the double advantage that smaller matrices need inverting and one sees the shift of data relative to theory explicitly.

However, are Gaussian correlated errors realistic? Is it valid to move data to compensate for the shortcomings of theory?

MRST find that for HERA data increments in χ^2 using this method are similar as for adding in quadrature, and data moves towards theory rather than *vice versa*.

More sophisticated treatment is found to be essential for Tevatron jets where correlated systematic errors dominate. Used by the groups which include this data.

Parton Uncertainties. Number of approaches.

Hessian (Error Matrix) approach first used by H1 and ZEUS, and extended by CTEQ.

$$\chi^2 - \chi^2_{min} \equiv \Delta\chi^2 = \sum_{i,j} H_{ij}(a_i - a_i^{(0)})(a_j - a_j^{(0)})$$

The Hessian matrix H is related to the covariance matrix of the parameters by

$$C_{ij}(a) = \Delta\chi^2 (H^{-1})_{ij}.$$

We can then use the standard formula for linear error propagation.

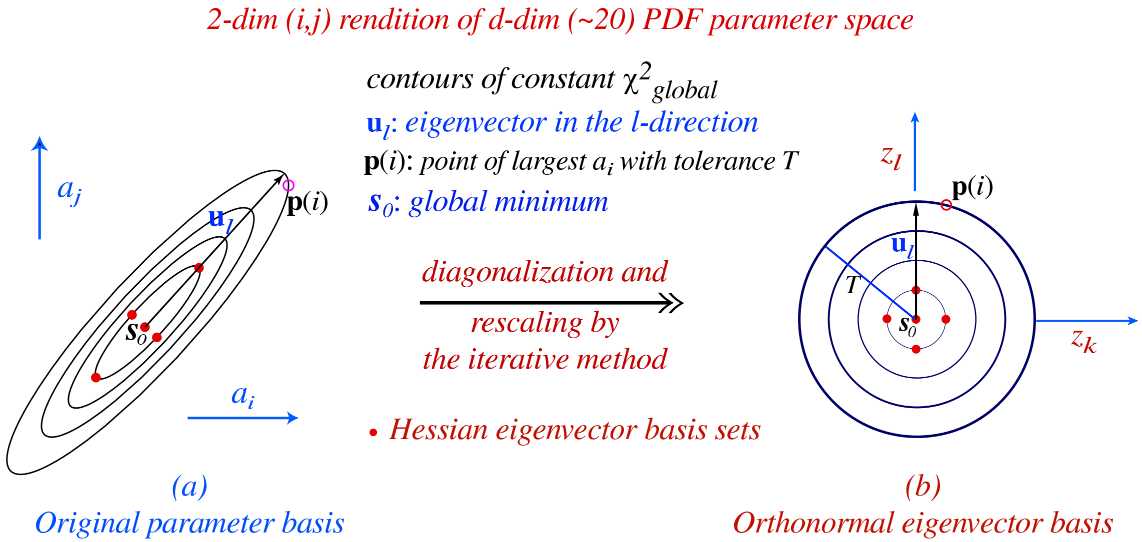
$$(\Delta F)^2 = \Delta\chi^2 \sum_{i,j} \frac{\partial F}{\partial a_i}(H)^{-1}_{ij}\frac{\partial F}{\partial a_j},$$

This is now the standard approach.

Problematic due to extreme variations in $\Delta\chi^2$ in different directions in parameter space.

Solved by finding and rescaling eigenvectors of H leading to diagonal form

$$\Delta\chi^2 = \sum_i z_i^2$$



Implemented by CTEQ, then others. Uncertainty on physical quantity then given by

$$(\Delta F)^2 = \sum_i \left(F(S_i^{(+)}) - F(S_i^{(-)})\right)^2,$$

where $S_i^{(+)}$ and $S_i^{(-)}$ are PDF sets displaced along eigenvector direction by given $\Delta\chi^2$. Art in choosing "correct" $\Delta\chi^2$ given complication of errors in full fit. CTEQ use $\Delta\chi^2 \sim 100$ and MRST/MSTW use $\Delta\chi^2 \sim 50$. Other fits less global.

Offset method. Best fit obtained by minimising

$$\chi^2 = \sum_{i=1}^{N} \left(\frac{(D_i - f_i(a,s))}{\sigma_{i,unc}} \right)^2,$$

i.e. best fit and parameters a_0 obtained from only uncorrelated errors. Forces theory to be close to unshifted data. Quality of fit then estimated by adding in quadrature. Systematic errors on a_i determined by letting each $s_k = \pm 1$ and add deviation in quadrature. Replaced by calculating 2 Hessian matrices

$$M_{ij} = \frac{\partial^2 \chi^2}{\partial a_i \partial a_j} \qquad V_{ij} = \frac{\partial^2 \chi^2}{\partial a_i \partial s_j},$$

and defining covariance matrices

$$C_{stat} = M^{-1} \qquad C_{sys} = M^{-1} V V^T M^{-1}$$

$$C_{tot} = C_{stat} + C_{sys}.$$

Used by early H1 fits (Zomer and Pascaud) and by ZEUS (Botje) (DIS (and E866) data only).

$\rightarrow$ bigger uncertainty for same $\Delta\chi^2$. Not often used now.

Statistical approach used by **Neural Network** group (Del Debbio *et al.*) construct a set of Monte Carlo replicas $\sigma^k(p_i)$ of the original data set $\sigma^{data}(p_i)$. Representation of $P[\sigma(p_i)]$ at points p_i.

Train a neural network for the parton distribution function on each replica, obtaining a representation of the pdfs $q_i^{(net)(k)}$.

The set of neural nets is a representation of the probability density – mean μ_O and deviation σ_O of observable O then given by

$$\mu_O = \frac{1}{N_{rep}} \sum_1^{N_{rep}} O[q_i^{(net)(k)}], \quad \sigma_O^2 = \frac{1}{N_{rep}} \sum_1^{N_{rep}} (O[q_i^{(net)(k)}] - \mu_O)^2.$$

Can incorporate full information about measurements and their error correlations in the distribution of $\sigma^{data}(p_i)$.

This is statistically correct, and does not rely on the approximation of linear propagation of errors in calculating observables, but is more complicated and time intensive.

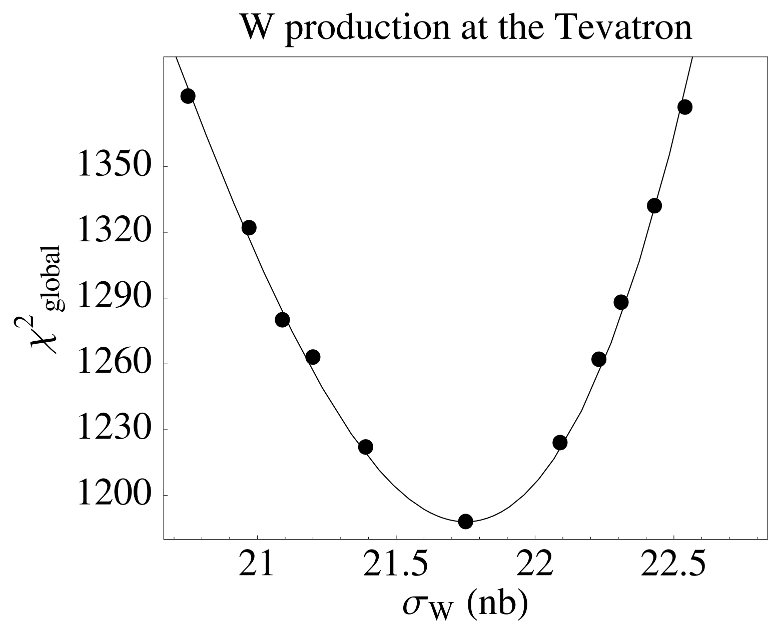
Currently uses only two non-singlet DIS data sets for simplicity.

Attractive but ambitious large-scale project with a lot of work still to be done.

Can also look at uncertainty on a given physical quantity using Lagrange Multiplier method, first suggested by CTEQ and often used by MRST, i.e. perform fit while constraining value of some physical quantity. Minimise

$$\Psi(\lambda, a) = \chi^2_{global}(a) + \lambda F(a)$$

for various values of λ. Gives best fits for particular values of quantity $F(a)$ without relying on Gaussian approx for χ^2. Uncertainty then determined by deciding allowed range of $\Delta\chi^2$. CTEQ use $\Delta\chi^2 \sim 100$ and MRST/MSTW use $\Delta\chi^2 \sim 50$.



More appropriate method if α_S is a free parameter.

Treatment of errors.

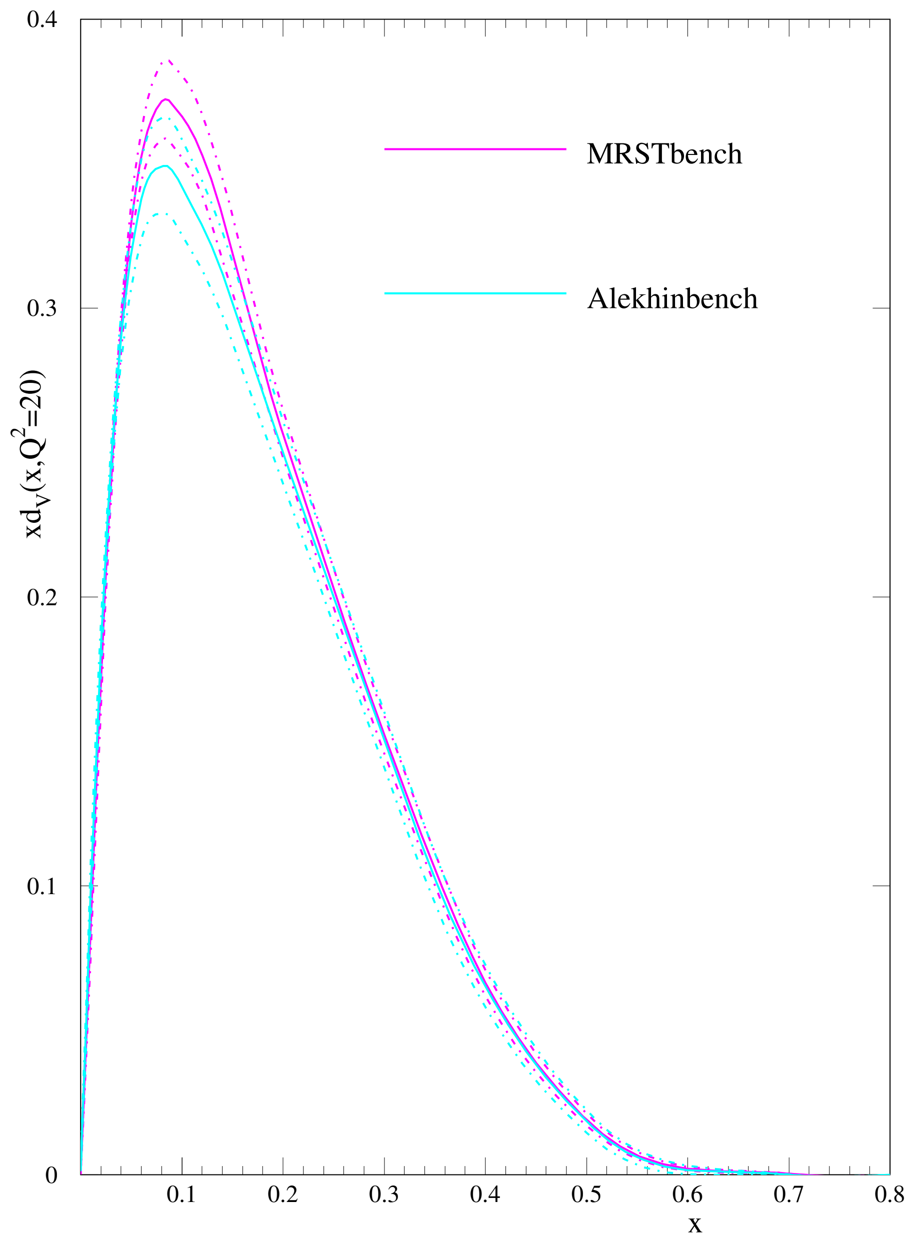
Exercise for $HERA-LHC$ meeting. Fit proton and deuteron structure function data from H1, ZEUS, NMC and BCDMS, for $Q^2 > 9\mathrm{GeV}^2$ using $ZM-VFNS$ and same form of parton inputs at same $Q_0^2 = 1\mathrm{GeV}^2$.

Very conservative fit. Safe partons?

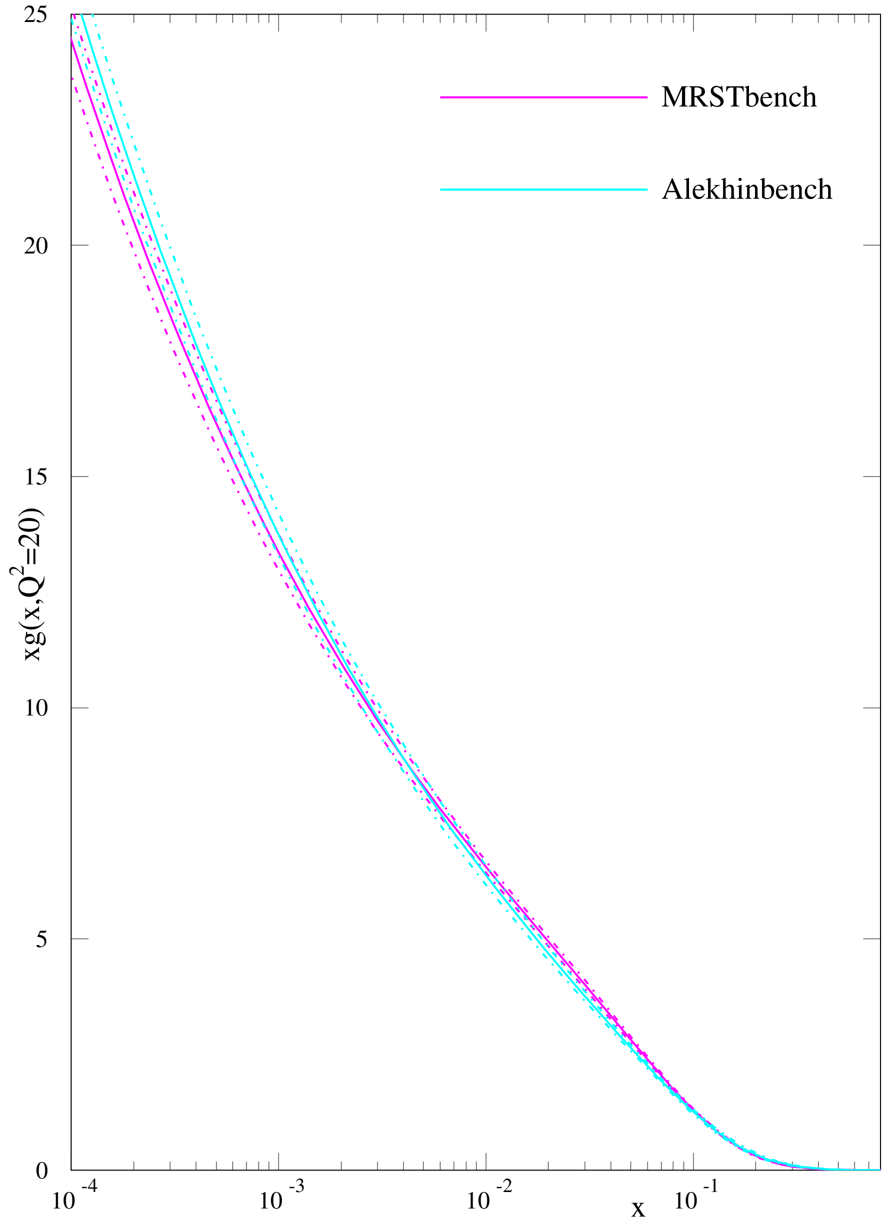
Compare rigorous treatment of all systematic errors (Alekhin) with simple quadratures approach (MRST), both with $\Delta\chi^2 = 1$.

$\longrightarrow$ some difference in central values (other possible reasons) and similar errors.

Fairly consistent.



Same conclusion for all partons, e.g. gluon.



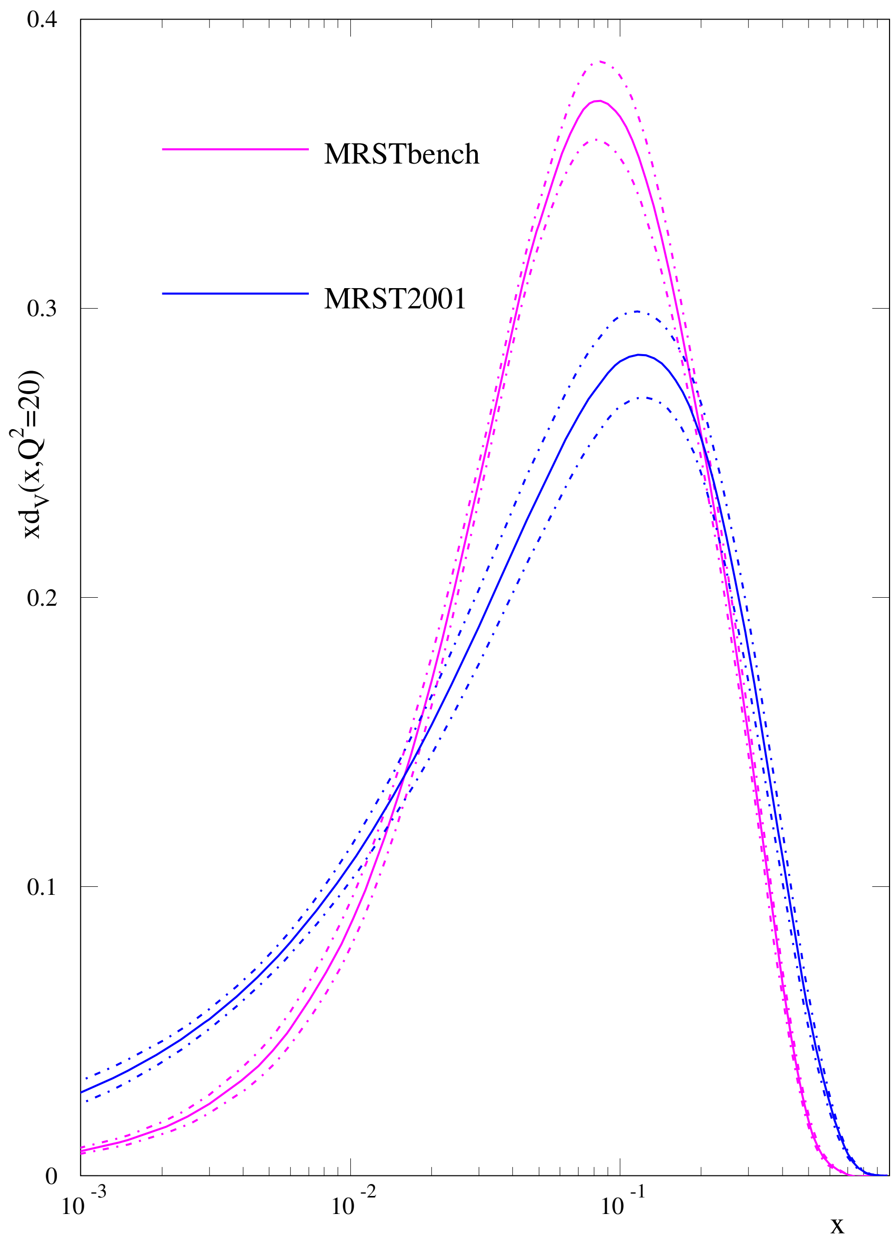
However, how do partons from very conservative, structure function only data compare to global partons?

Compare to MRST01 partons with uncertainty from $\Delta\chi^2 = 50$.

Enormous difference in central values.

Errors similar.

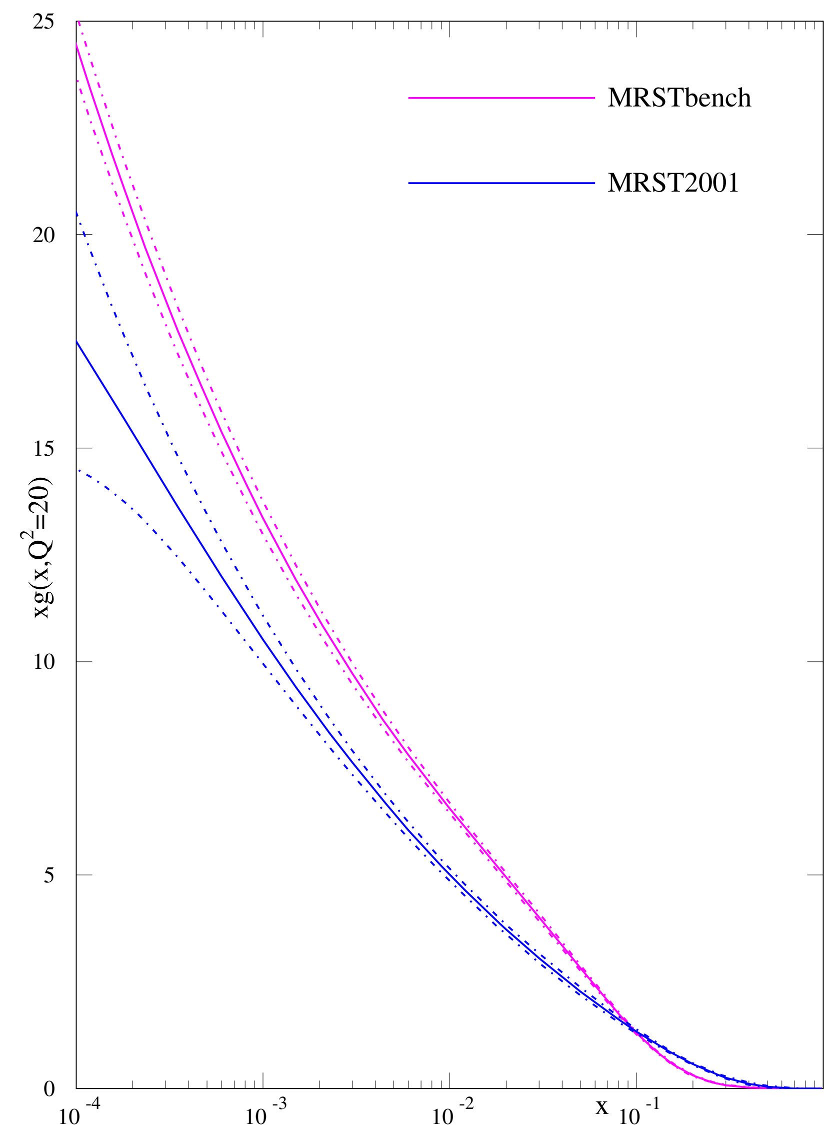
Similar deviations from global partons seen when using partons using non-singlet distributions only Blümlein, Böttcher and Guffanti.



Same conclusion for gluon.

Notice difference in range of small-x error band (more later).

Moreover $\alpha_S(M_Z^2) = 0.1110 \pm 0.0015$ compared to $\alpha_S(M_Z^2) = 0.119 \pm 0.002$.

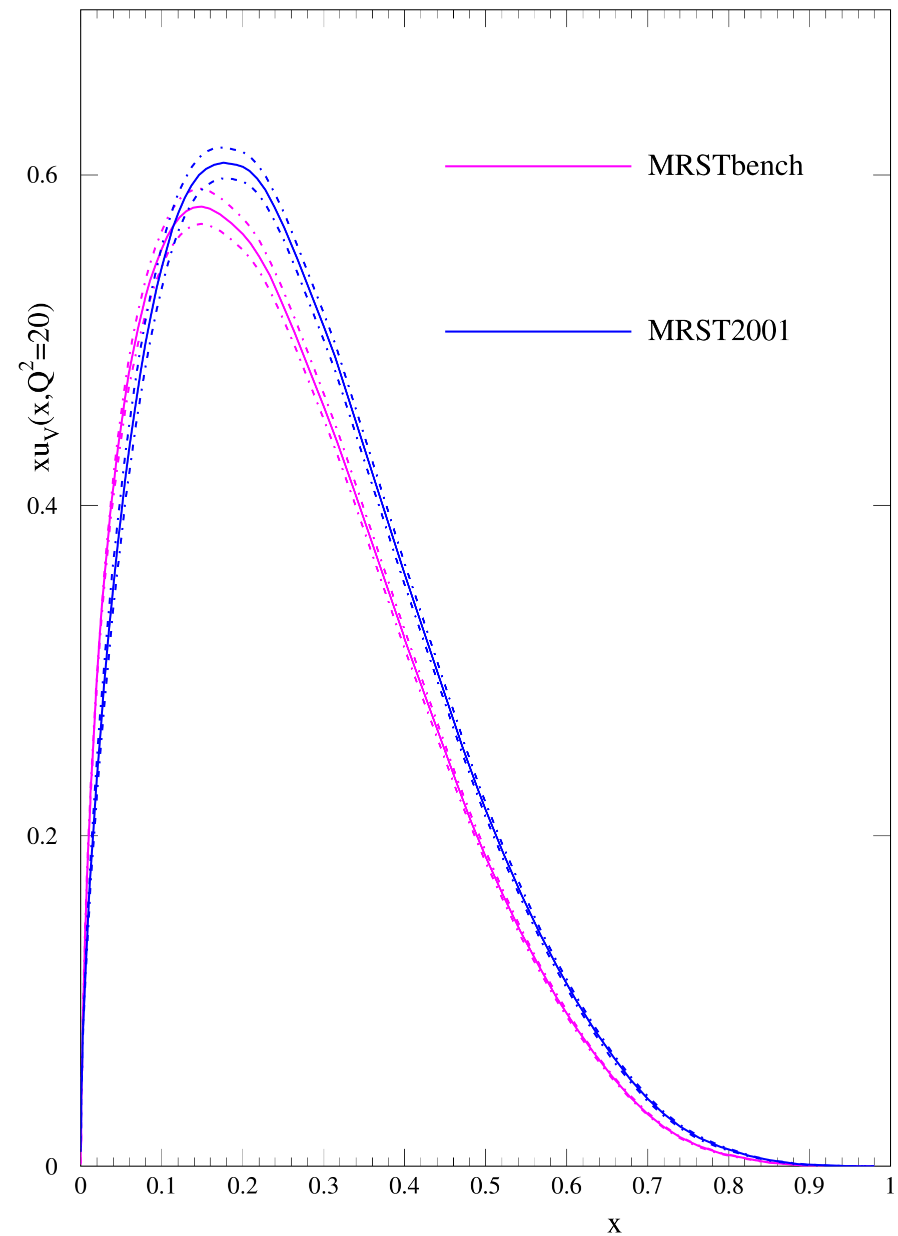


Even for u_V quark.

This, if anything should have been good in benchmark fit.

Sea large at high x – valence quarks too low.

Not consistent with Drell-Yan data.



Something is seriously wrong in one of these analyses.

Central values different by many σ.

Errors similar from $\Delta\chi^2 = 50$ compared to $\Delta\chi^2 = 1$ with only approx. twice the data.

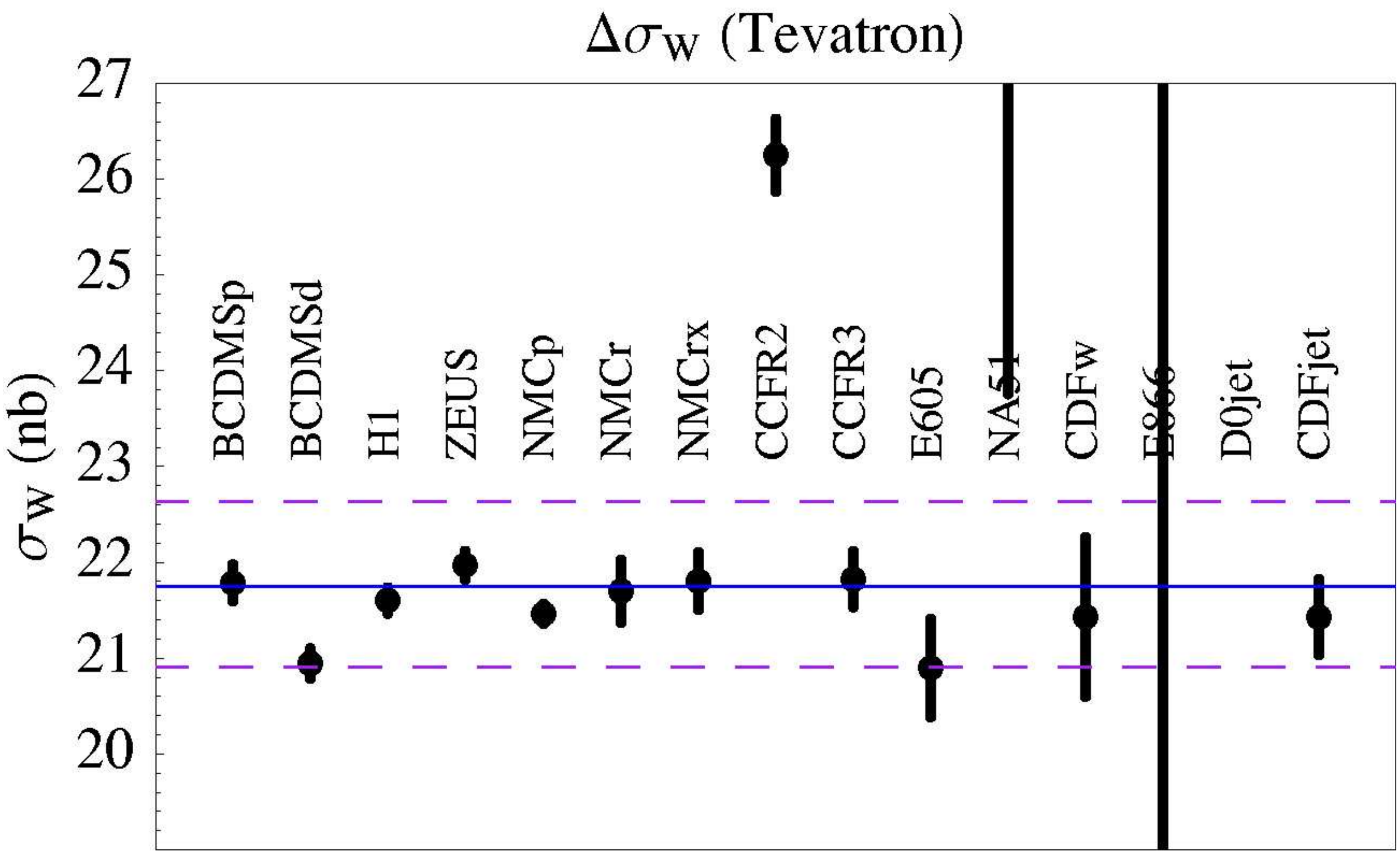
Very confident benchmark fit is wrong. Fails when compared to pretty much all data not in the fit. Is all this data unreliable?

Also, rigorously defined error analysis of this data is wrong. It does not produce true uncertainty by some way.

Partons should be constrained by all possible reliable data. Benchmark fit extreme, but not so extreme. Some partons listed are similar in data used, but many input implicit constraints from elsewhere.

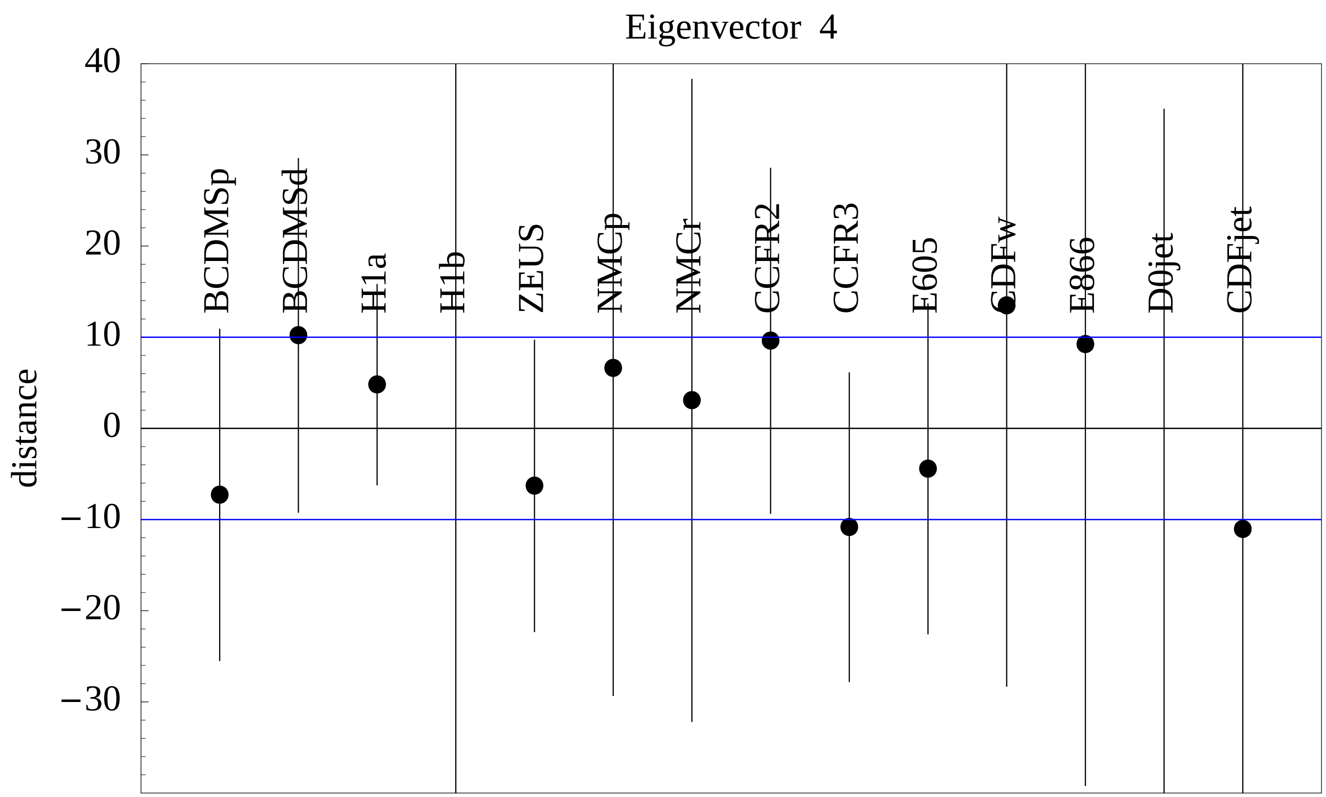
In global fit $\Delta\chi^2 = 1$ is not reliable due to strict incompatibility of different data sets, limitations in theoretical understanding, flexibility in parameterisations $etc.$.

Further illustration of inappropriateness of $\Delta\chi^2 = 1$ given by best value of σ_W and uncertainty using $\Delta\chi^2 = 1$ for individual data sets obtained by CTEQ using Lagrange Multiplier technique.



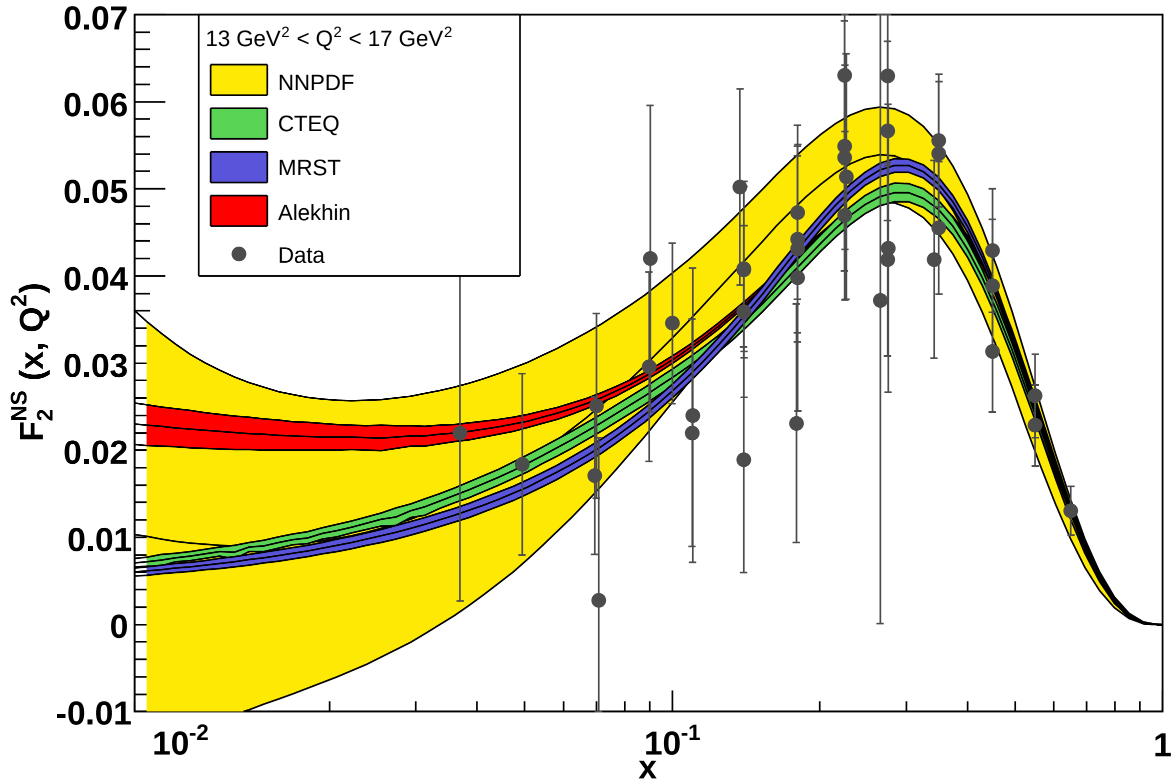
Alternative reasoning, allow $\Delta\chi^2$ to take a value such that every data set remains roughly within its 90% confidence limit.

These limits shown for CTEQ6 eigenvector 4 as function of $T = \sqrt{\Delta\chi^2}$. Some sets somewhat outside for $T = 10$



Using similar sort of reasoning MRST/MSTW use $\Delta\chi^2 \sim 50$ for 90% confidence level.

Much larger uncertainty from neural network approach with fit to a small number of non-singlet data sets on $F_2^p(x,Q^2) - F_2^n(x,Q^2)$ (similar to parton combination $u(x,Q^2) - d(x,Q^2) + \bar{d}(x,Q^2) - \bar{u}(x,Q^2)$).

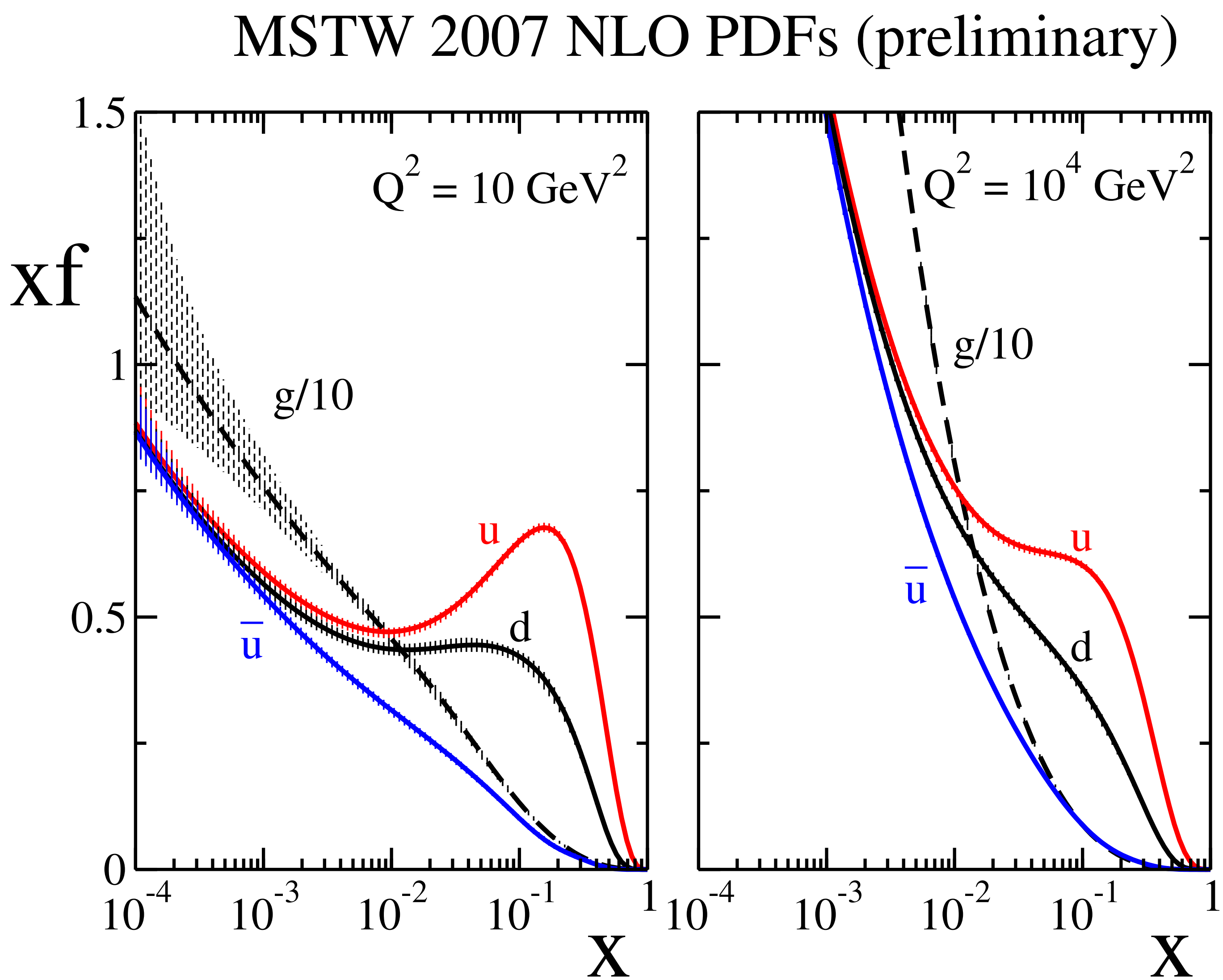


Not comparable with other parton uncertainties since they have (for MRST and CTEQ) many extra constraints. But certainly consistent.

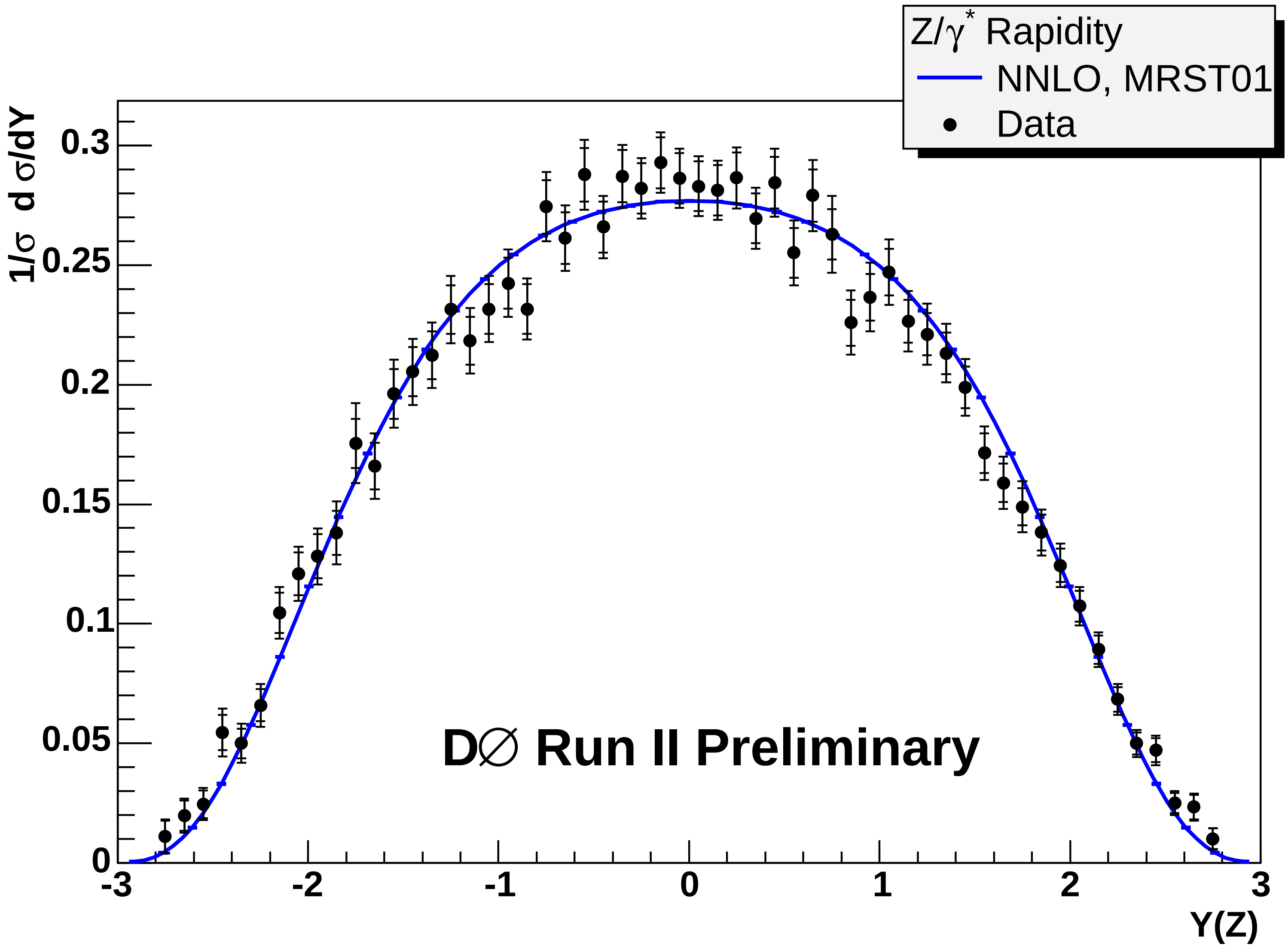
Get effective $\Delta\chi^2 = 1.7$ for 1σ, even with only two data sets.

Overall global fitting procedure with $\Delta\chi^2 > 1$ is generally successful and is part of a large-scale, ongoing project. Results in partons of the form shown.

All LHC cross-sections rely on our understanding of these partons.



Excellent predictive power – comparison of MRST prediction for Z rapidity distribution with preliminary data.



Interplay of LHC and pdfs/QCD

Make predictions for all processes, both SM and BSM, as accurately as possible given current experimental input and theoretical accuracy.

Check against well-understood processes, e.g. central rapidity W, Z production (luminosity monitor), lowish-E_T jets, ...

Compare with predictions with more uncertainty and lower confidence, e.g. high-E_T jets, high rapidity bosons or heavy quarks .....

Improve uncertainty on parton distributions by improved constraints, and check understanding of theoretical uncertainties, and determine where NNLO, electroweak corrections, resummations $etc.$ needed.

Make improved predictions for both background and signals with improved partons and surrounding theory.

Spot new physics from deviations in these predictions. As a nice by-product improve our understanding of the strong sector of the Standard Model considerably.

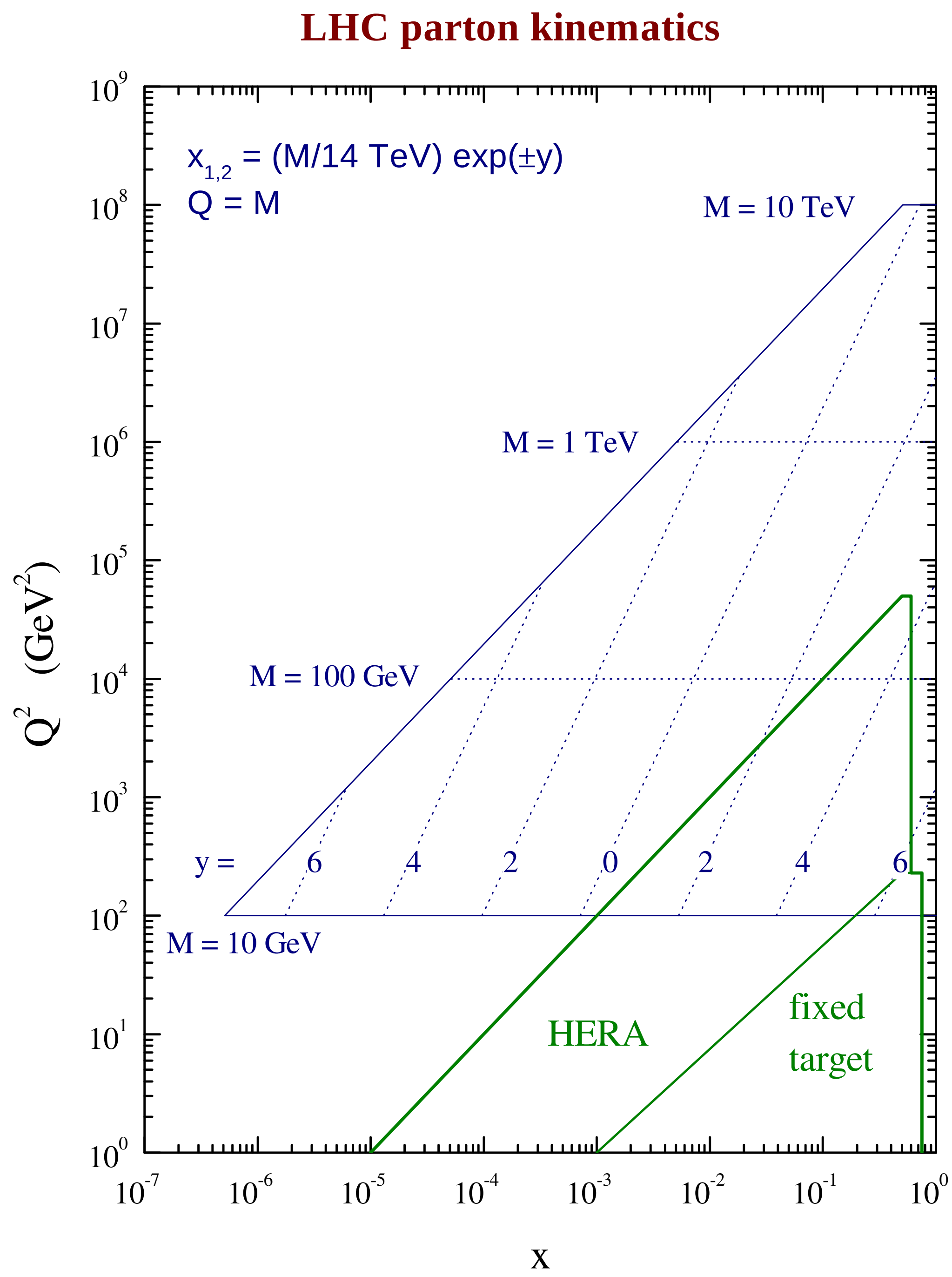
Remainder of talk describes this process in more detail.

LHC **Physics**

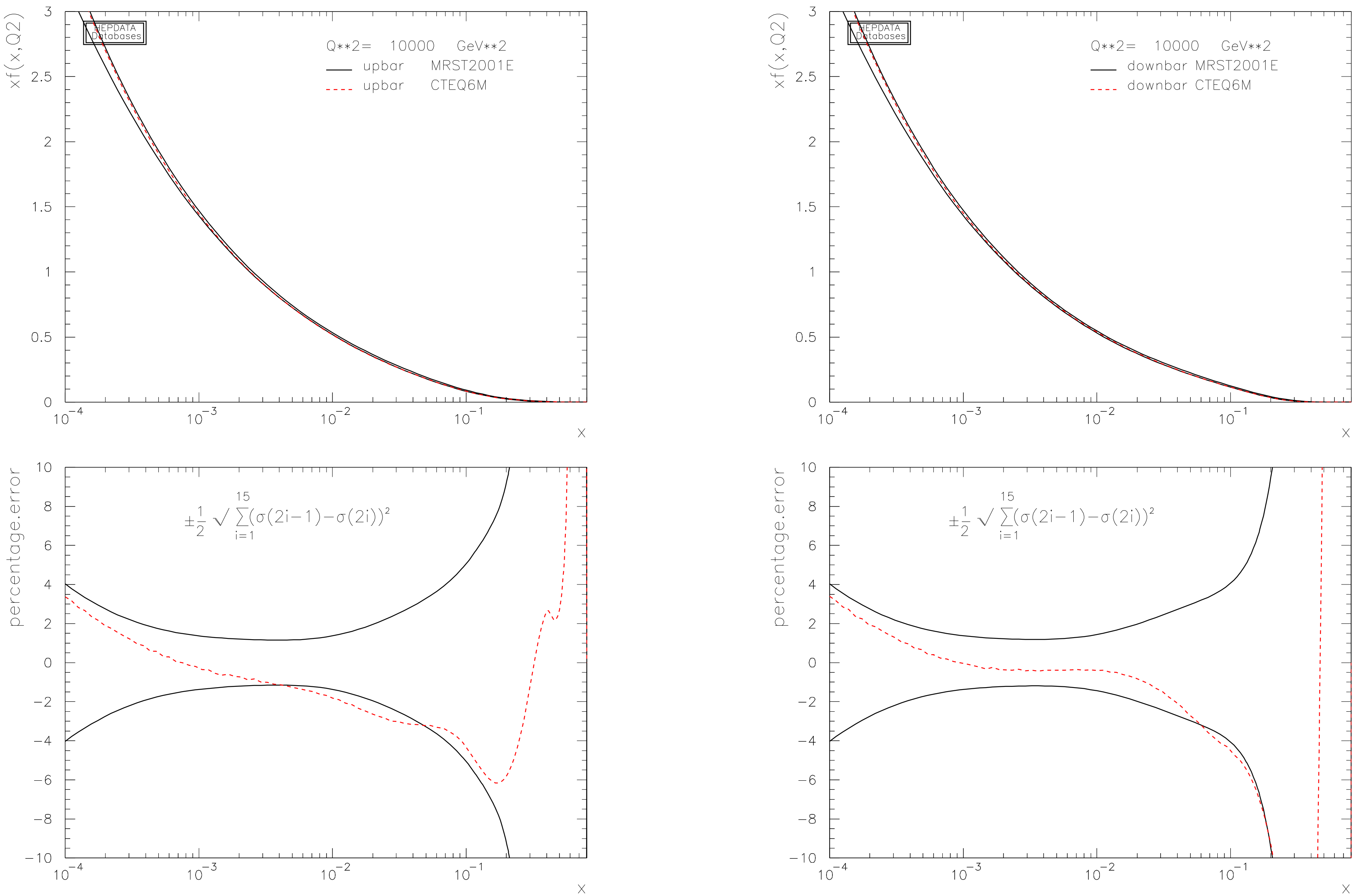
The kinematic range for particle production at the LHC is shown.

Smallish $x \sim 0.001 - 0.01$ parton distributions therefore vital for understanding the standard production processes at the LHC.

However, even smaller (and higher) x required when one moves away from zero rapidity, e.g. when calculating total cross-section.



Uncertainty on MRST $\bar{u}$ and $\bar{d}$ distributions, along with CTEQ6. Central rapidity $x = 0.006$ is ideal for MRST uncertainty in W, Z (Higgs?) at the LHC.



Current best (MRST) estimate

$$\delta\sigma^{\rm NLO}_{W,Z}(\text{expt pdf}) = \pm 2\%$$

but note that there is a greater theoretical uncertainty in the NLO prediction, mainly due to possible problems at small x in the global fit to DIS data.

This is because the large rapidity W and Z total cross-sections sample very small x

$\sigma(W^+)/\sigma(W^-)$ is **gold-plated**

$$R_\pm = \frac{\sigma(W^+)}{\sigma(W^-)} \simeq \frac{u(x_1)\bar{d}(x_2)}{d(x_1)\bar{u}(x_2)} \simeq \frac{u(x_1)}{d(x_1)}$$

since sea is u, d symmetric at small x, and using MRST2001E

$$\delta R_\pm(\text{expt. pdf}) = \pm 1.4\%$$

Assuming all other uncertainties cancel, this is probably the most accurate SM cross-section test at LHC.

However, not quite as simple as this.

Other Sources of Uncertainty

It is vital to consider theoretical uncertainties. These include ....

- Underlying assumptions in procedure, e.g. parameterisations.

- QED and Weak (comparable to NNLO ?) ($\alpha_s^3 \sim \alpha$). Sometime enhancements – large E_T.

- higher orders (NNLO)

- large x ($\alpha_s^n \ln^{2n-1}(1-x)$)

- low Q^2 (higher twist)

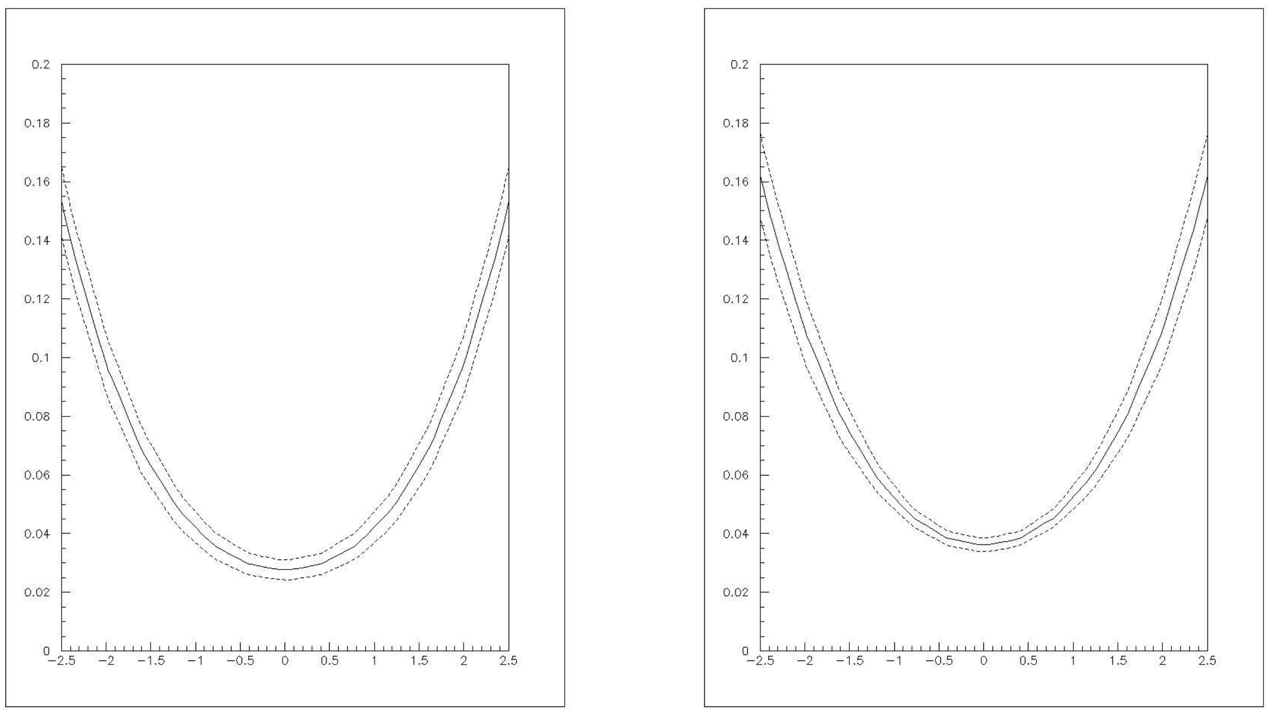
- small x ($\alpha_s^n \ln^{n-1}(1/x)$)

Lead to differences in current partons, and to corrections in predicted cross-sections.

Example of issue – "safe" predictions from different groups differ by more than their quoted uncertainties, e.g. study by ZEUS/ATLAS parton analysis group (Cooper-Sarkar *et al*) of

$$\frac{(\sigma(W^+) - \sigma(W^-))}{(\sigma(W^+) + \sigma(W^-))}$$

Left – MRST Right – CTEQ



At $y = 0$ MRST give 0.026 ± 0.005 while CTEQ give 0.036 ± 0.004.

Different ideas about quark decomposition at lowish x , i.e. $y = 0$ corresponds to $x = 0.006$ – i.e. separation of valence and sea quarks.

$$\frac{(\sigma(W^+) - \sigma(W^-))}{(\sigma(W^+) + \sigma(W^-))} \approx \frac{u(x)\bar{d}(x) - d(x)\bar{u}(x)}{u(x)\bar{d}(x) + d(x)\bar{u}(x)}$$

At this x to a good approximation $\bar{u}(x) = \bar{d}(x)$ so

$$\frac{(\sigma(W^+) - \sigma(W^-))}{(\sigma(W^+) + \sigma(W^-))} \approx \frac{u(x) - d(x) - (\bar{u}(x) - \bar{d}(x))}{u(x) + d(x)} = \frac{u_V(x) - d_V(x)}{u(x) + d(x)}$$

Total quark distributions well-constrained but valence quarks obtained only by extrapolation into this region (and number sum rule).

Best prediction actually for $x_1 > 0.01$, i.e for rapidity $y \approx 1.5 - 2.5$, not at centre.

Something for LHCb to look for.

So different approaches to fits generally lead to similar uncertainty for measured quantities, but can lead to different central values.

Many can be as important as experimental errors on data used (or more so).

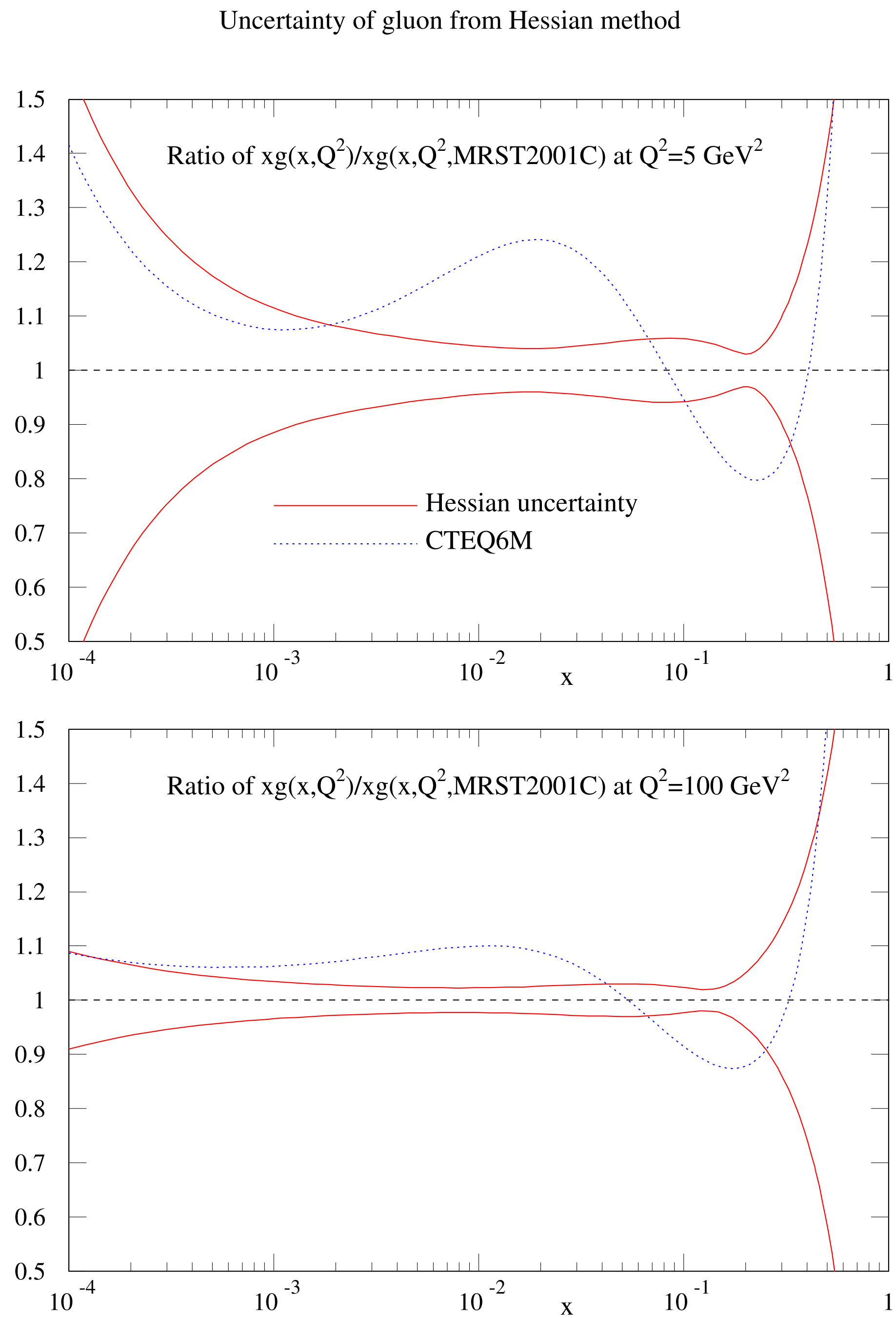
Example W total cross-section at LHC at NLO

CTEQ6.5 partons $\sigma = 202 \pm 9\ nb$.

MRST2004 partons $\sigma = 190 \pm 5\ nb$.

Due to different gluons and hence different evolution.

Determined by prejudice about input gluon.



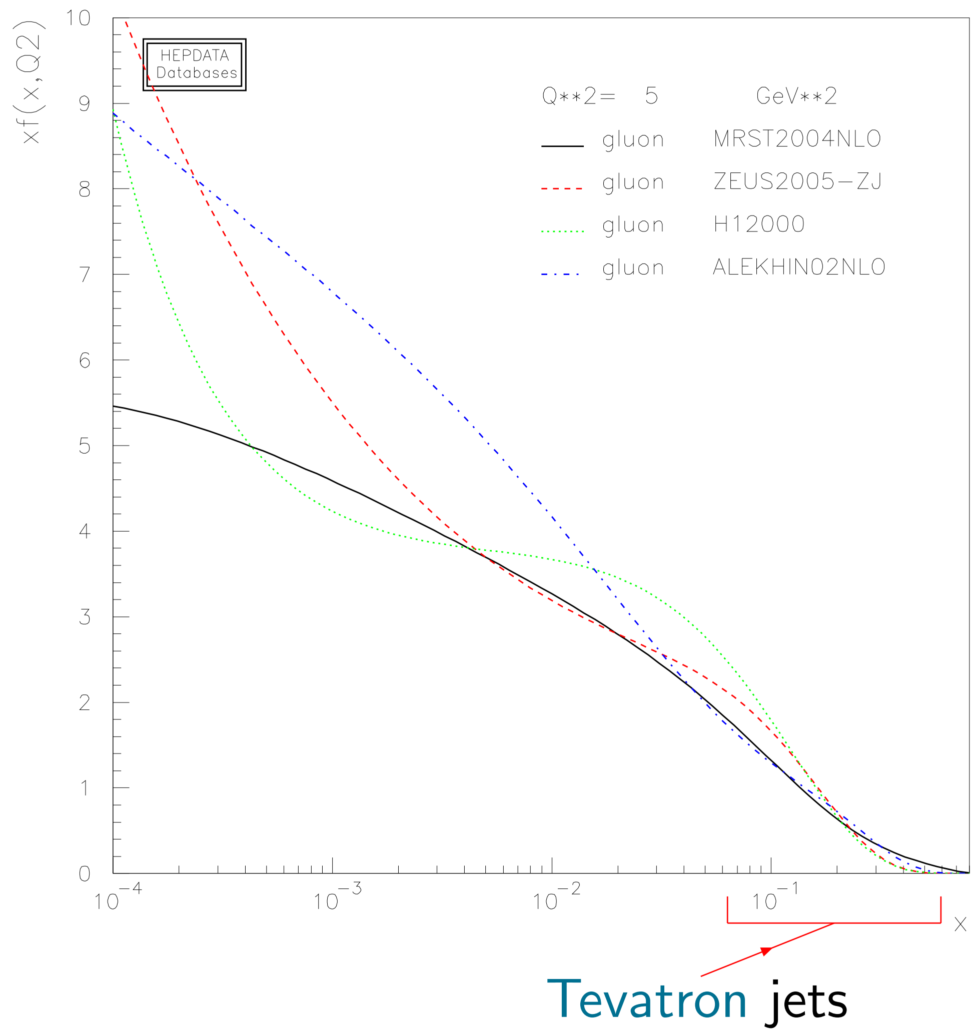
Gluon still very uncertain at low x and Q^2.

All partons fit to same small-x HERA data.

Very wide variety in gluon distributions.

Different approaches to fits.

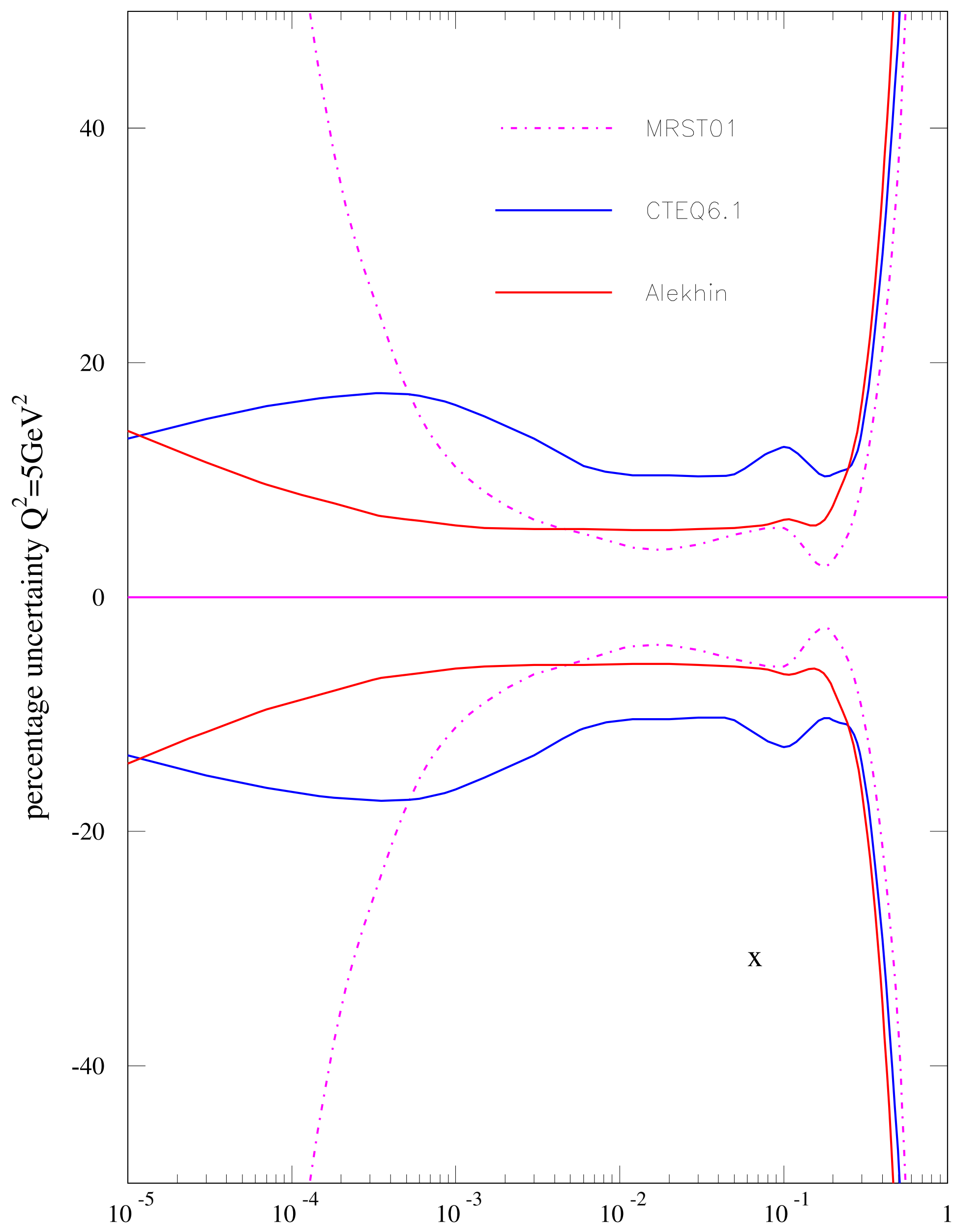
Affects uncertainty as well as central values.



MRST uncertainty blows up for very small x, whereas Alekhin (and ZEUS and H1) gets slowly bigger, and CTEQ saturates (or even decreases).

Related to input forms and scales.

($Neck$ in MRST gluon cured in MSTW).



MRST (MSTW) parameterise at $Q_0^2 = 1\mathrm{GeV}^2$ but allow negative and positive small x contributions. Very flexible. Represent true uncertainty at low x?

Alekhin and ZEUS gluons input at higher scale – behave like $x^{-\lambda}$ at small x. Uncertainty due to uncertainty in one parameter.

CTEQ gluons input at $Q_0^2 = 1.69\mathrm{GeV}^2$. Behave like x^{λ} at small x where λ large and positive. Input gluon valence-like.

Requires fine tuning. Evolving backwards from steep gluon at higher scale valence-like gluon only exists for very narrow range of Q^2 (if at all).

Small x input gluon tiny – very small absolute error. At higher Q^2 all uncertainty due to evolution driven by higher x, well-determined gluon. Very small x gluon no more uncertain than at $x = 0.01 - 0.001$.

Also, sea quarks usually use some assumption for fixed strange quark proportion.

Now fitting to strange from NUTEV dimuon data (along with G. Watt).

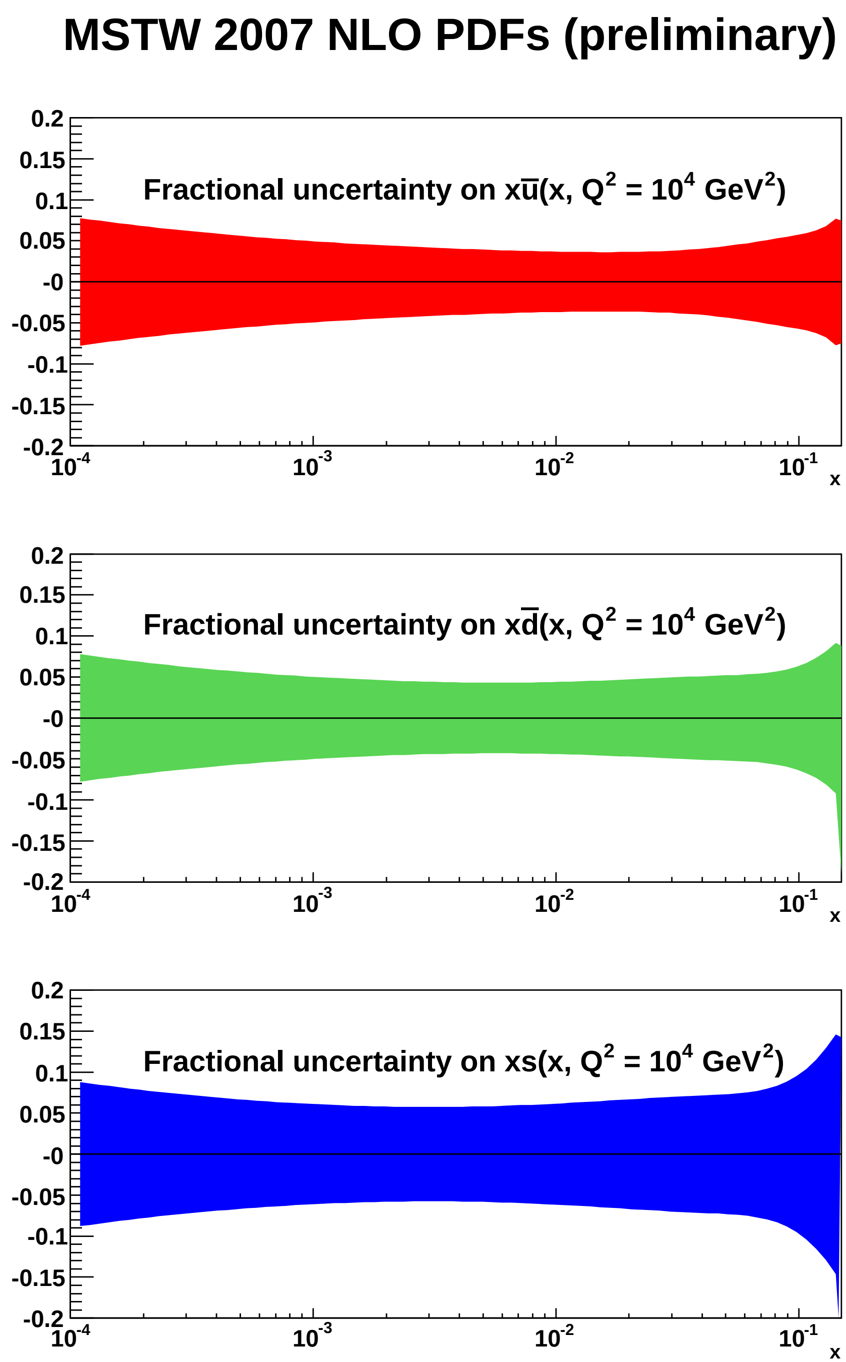
$\nu + s \rightarrow \mu^- + c(\mu^+)$, $\bar{\nu} + \bar{s} \rightarrow \mu^+ + \bar{c}(\mu^-)$.

Allows genuine *larger* uncertainty on $s(x)$– feeds into that on $\bar{u}$ and $\bar{d}$ quarks.

The size of the uncertainty on the small x antiquarks roughly doubles – $\sim 1.5\% \rightarrow \sim 3\%$.

Strong (anti)-correlations. Total cross-sections fairly stable. Ratios less so.

Important to check with early data to improve separation of valence and sea quarks, and of flavours.

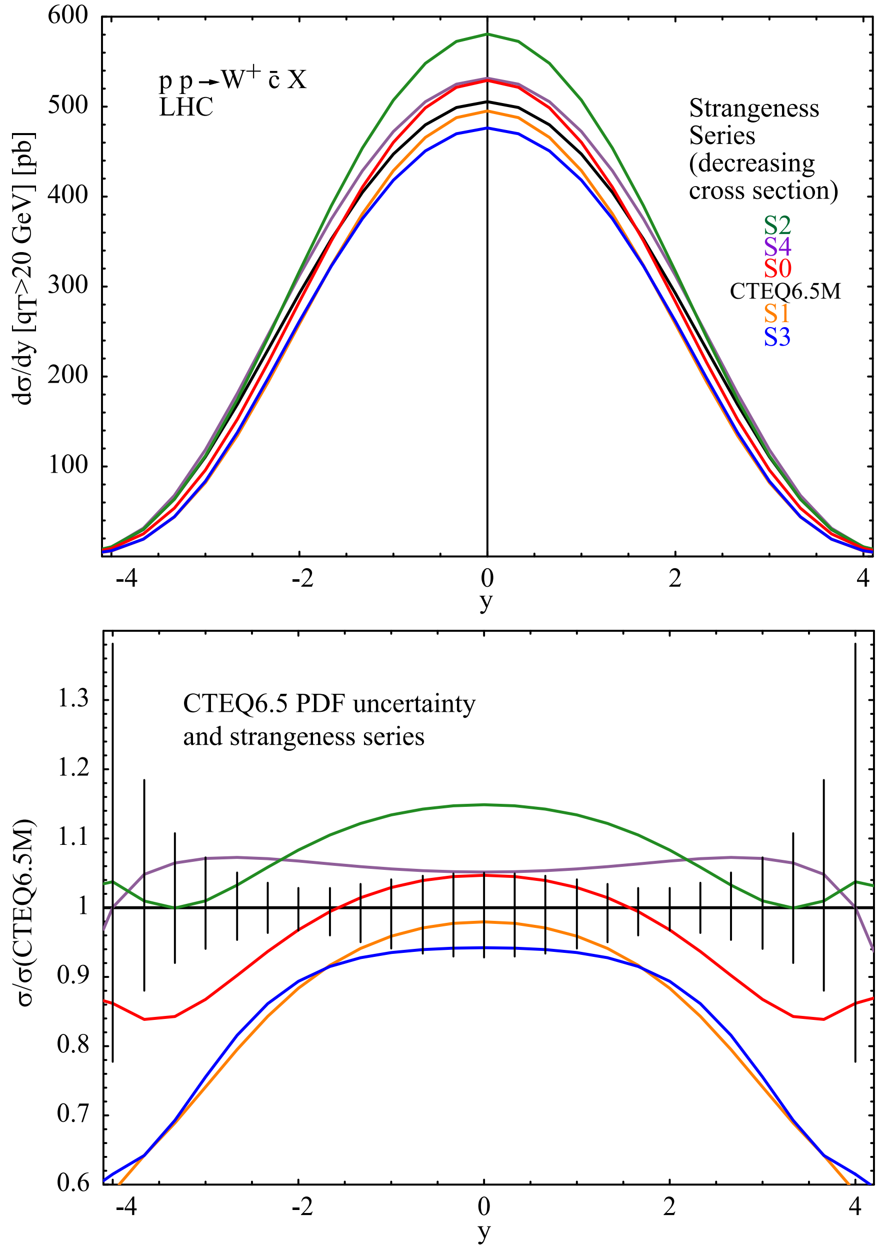


CTEQ look at special sets with fits to dimuon data and possible (generous) variations.

Band represents uncertainty of default CTEQ6.5 set.

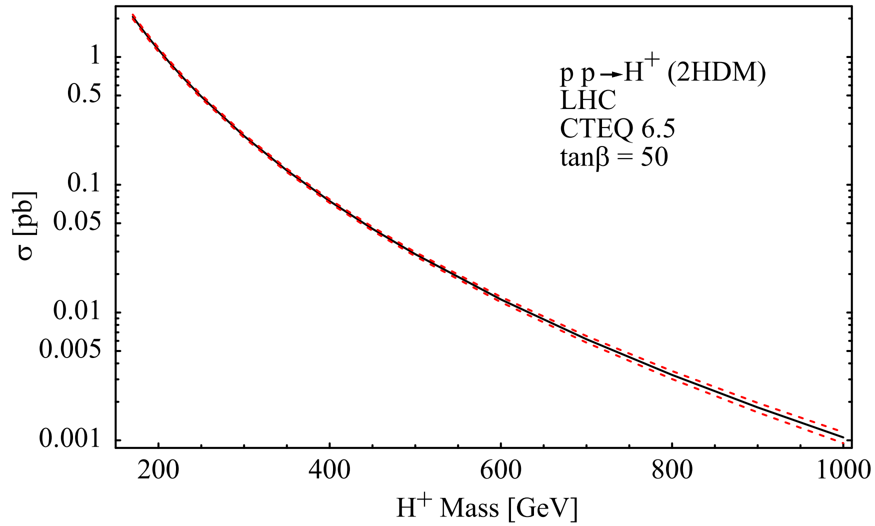
Not yet part of default sets. Best fit CTEQS0 differs from CTEQ6.5.

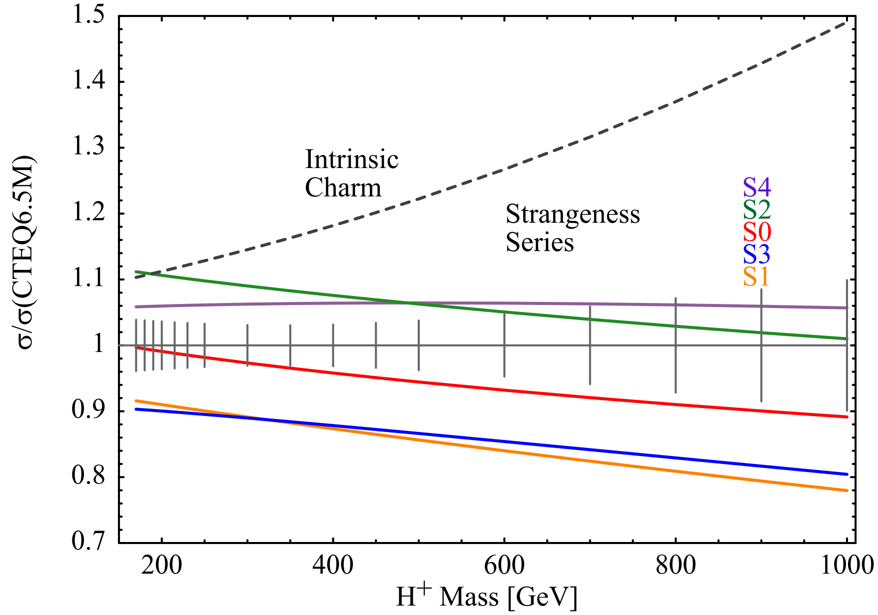
Look at implications for strange sensitive final state, i.e. $W + c$ at LHC.



Also examine uncertainty of predictions for BSM physics, e.g. $+\bar{s} \to H^{+}$.

Again allowed sets give wider range of predictions than default uncertainty.

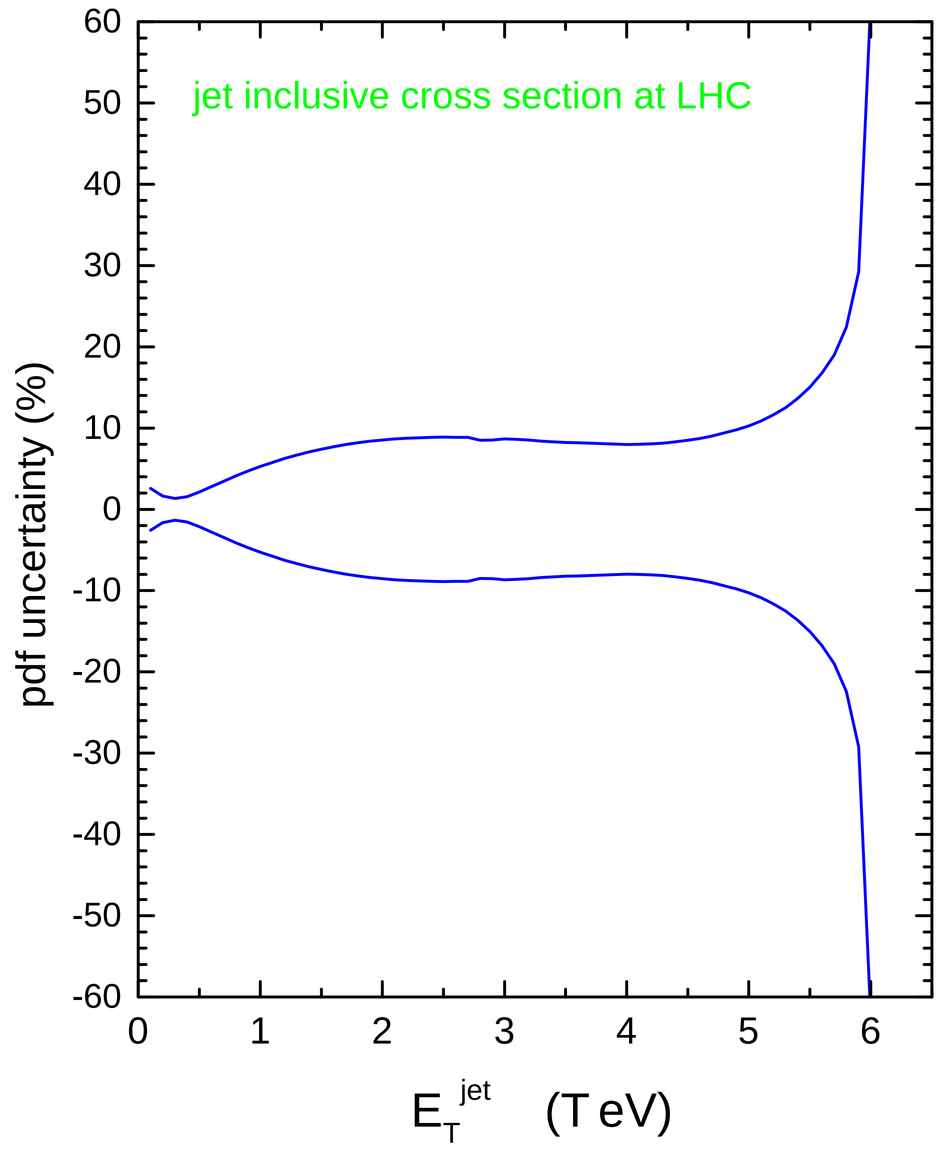




High-E_T Jets

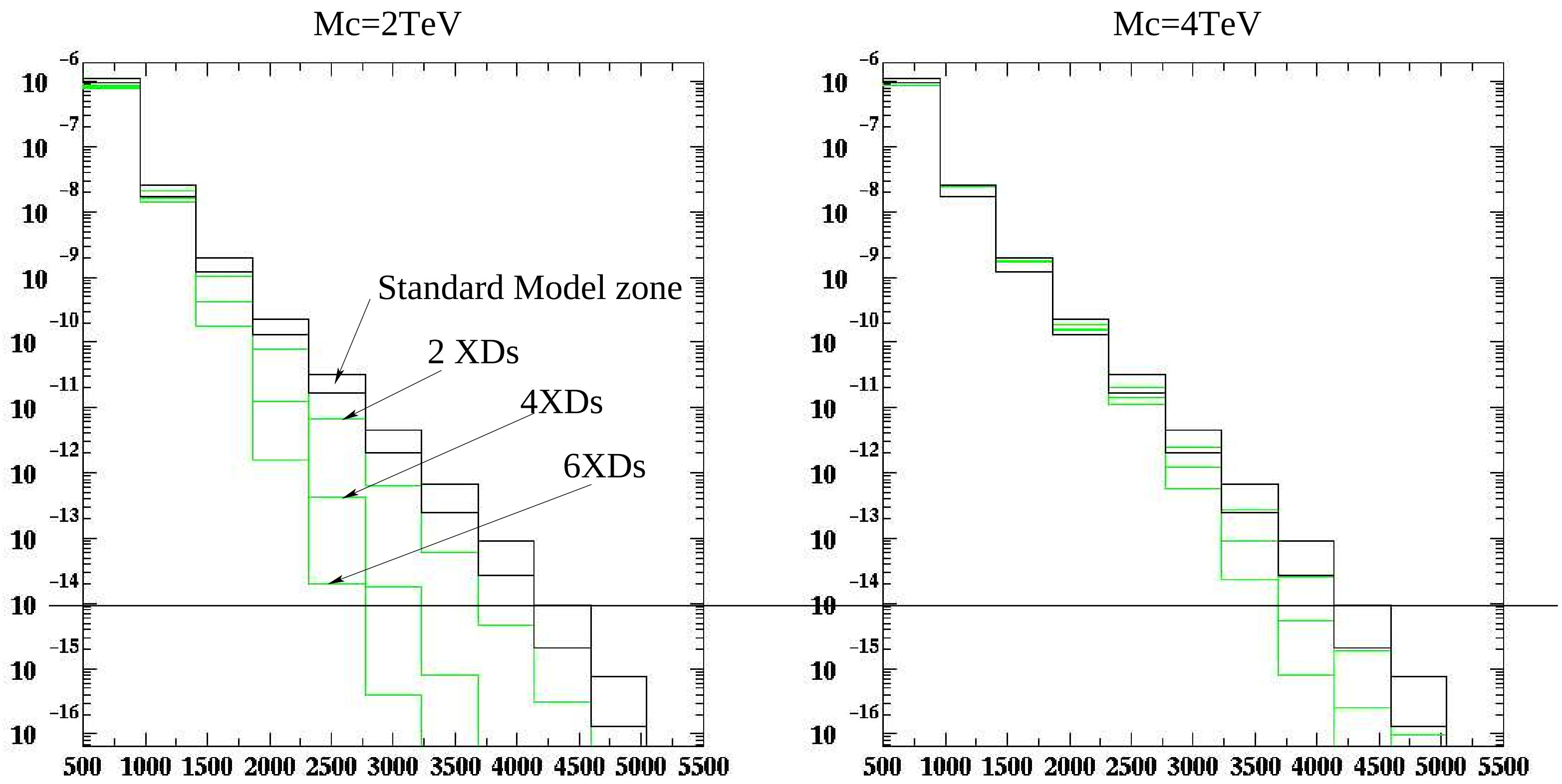
The error on predictions for very high-E_T jets at the LHC is dominated by the parton uncertainties.

Sensitive to relatively poorly known high-x gluon.



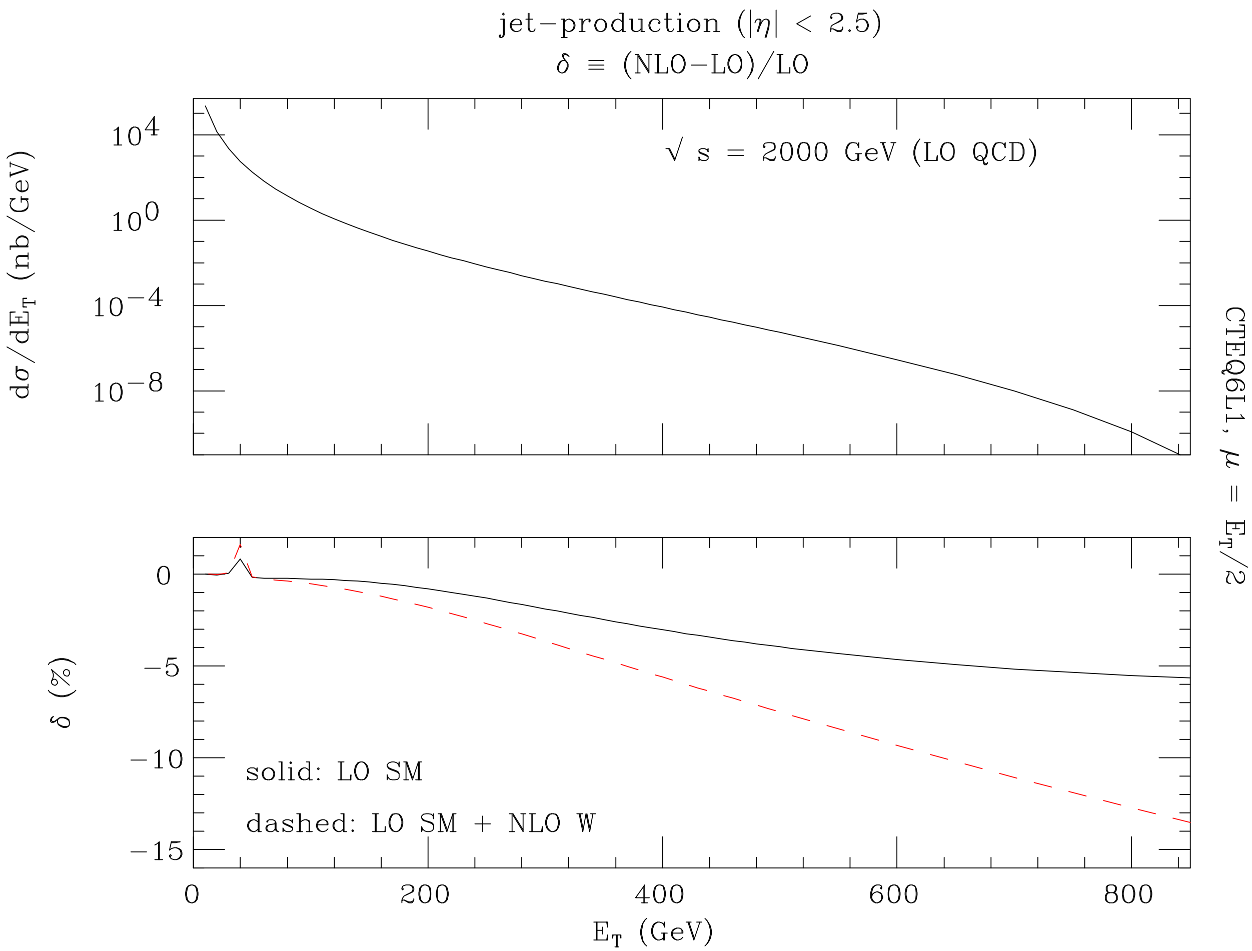
Comparison of variations in dijet production from large extra dimensions (alters running of $\alpha_S(Q^2)$) with given compactification scale and from uncertainties in $g(x, Q^2)$ (Ferrag).

Limit on M_C changes from $5\text{TeV} \rightarrow 2\text{TeV}$. Depends on particular parton set and uncertainties.



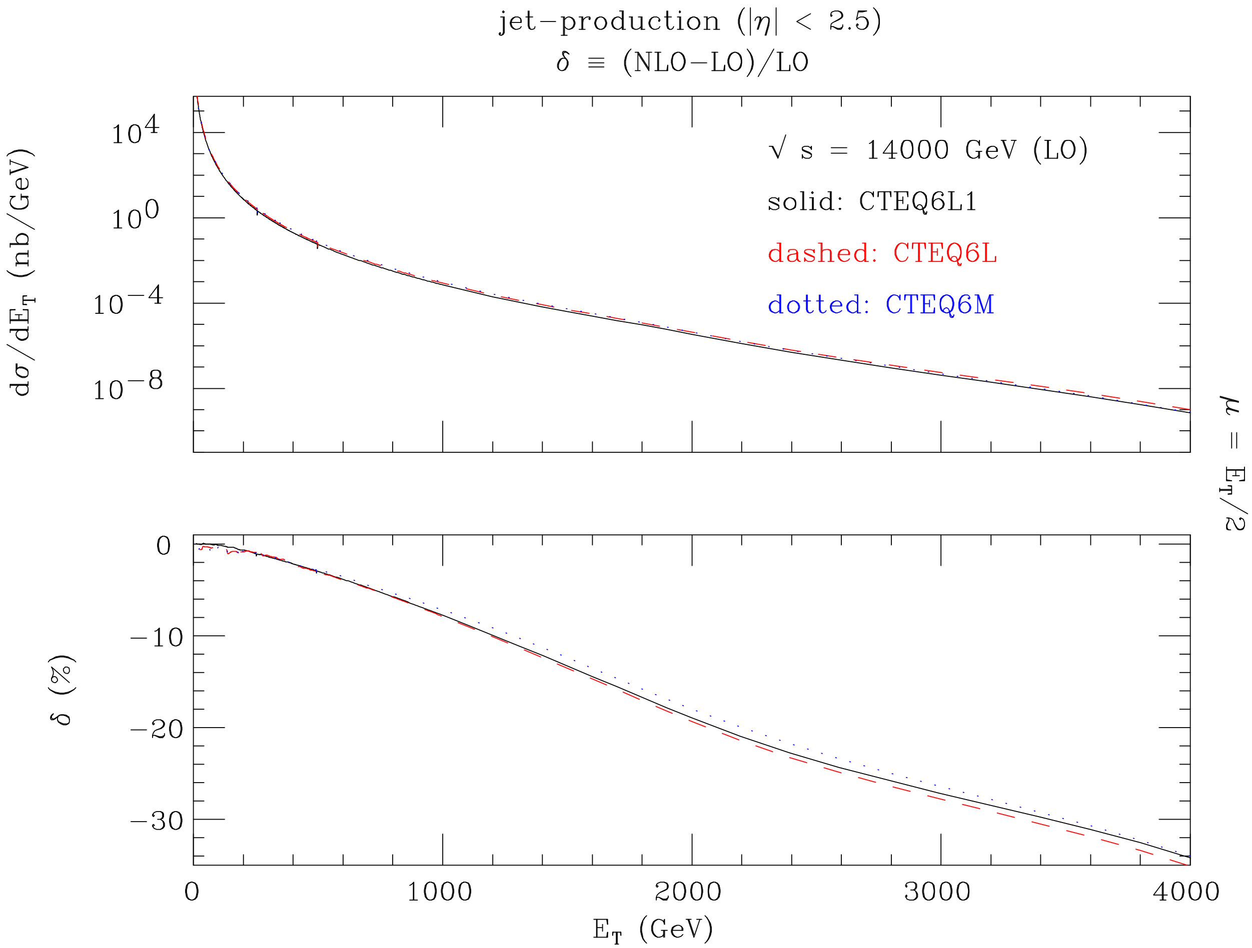
Horizontal line – one year projected LHC running.

Weak corrections



Jet cross-section a major example – calculation by Moretti, Nolten, Ross, goes like $(1 - \frac{1}{3}C_F\frac{\alpha_W}{\pi}\log^2(E_T^2/M_W^2))$.

Dominated by quark-(anti)quark processes $\rightarrow \approx 6\%$ correction at $E_T = 450\mathrm{GeV}$.

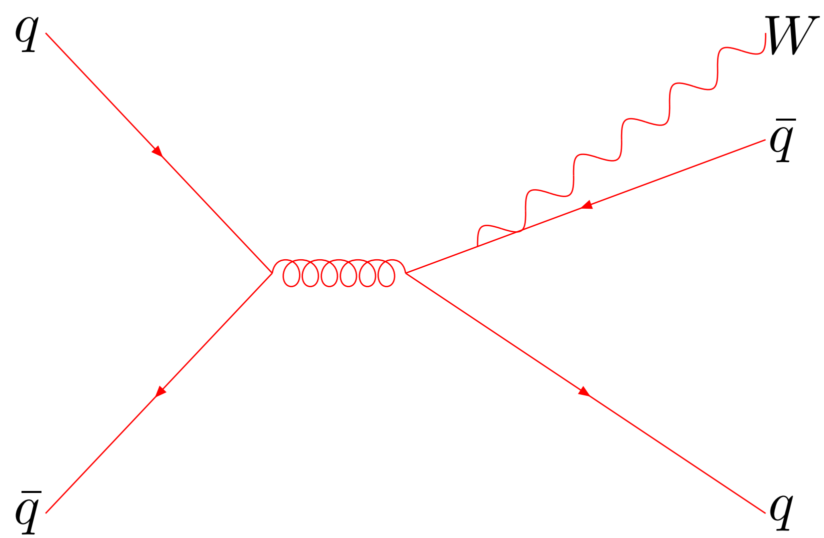


Much bigger at LHC energies. Up to 30%. Bigger than NLO QCD.

$\log^2(E_T^2/M_W^2)$ a very large number.

Similar results for corrections to other processes with a hard scale, e.g. Di-boson production (Accomando *et al*).

Only virtual corrections. Must have contributions of the form



Some electroweak bosons included with jets – some almost collinear with quark, and many decaying into hadrons.

Opposite sign, potentially large contribution. However, perfect cancellation will not happen. Total effect very possibly still large. Similar situation in variety of processes.

Needs calculation and decisions on experimental definitions. Recent calculation by Baur suggests positive 6% correction to inclusive jets at $4\mathrm{TeV}$. Very sensitive to jet veto in di-boson production.

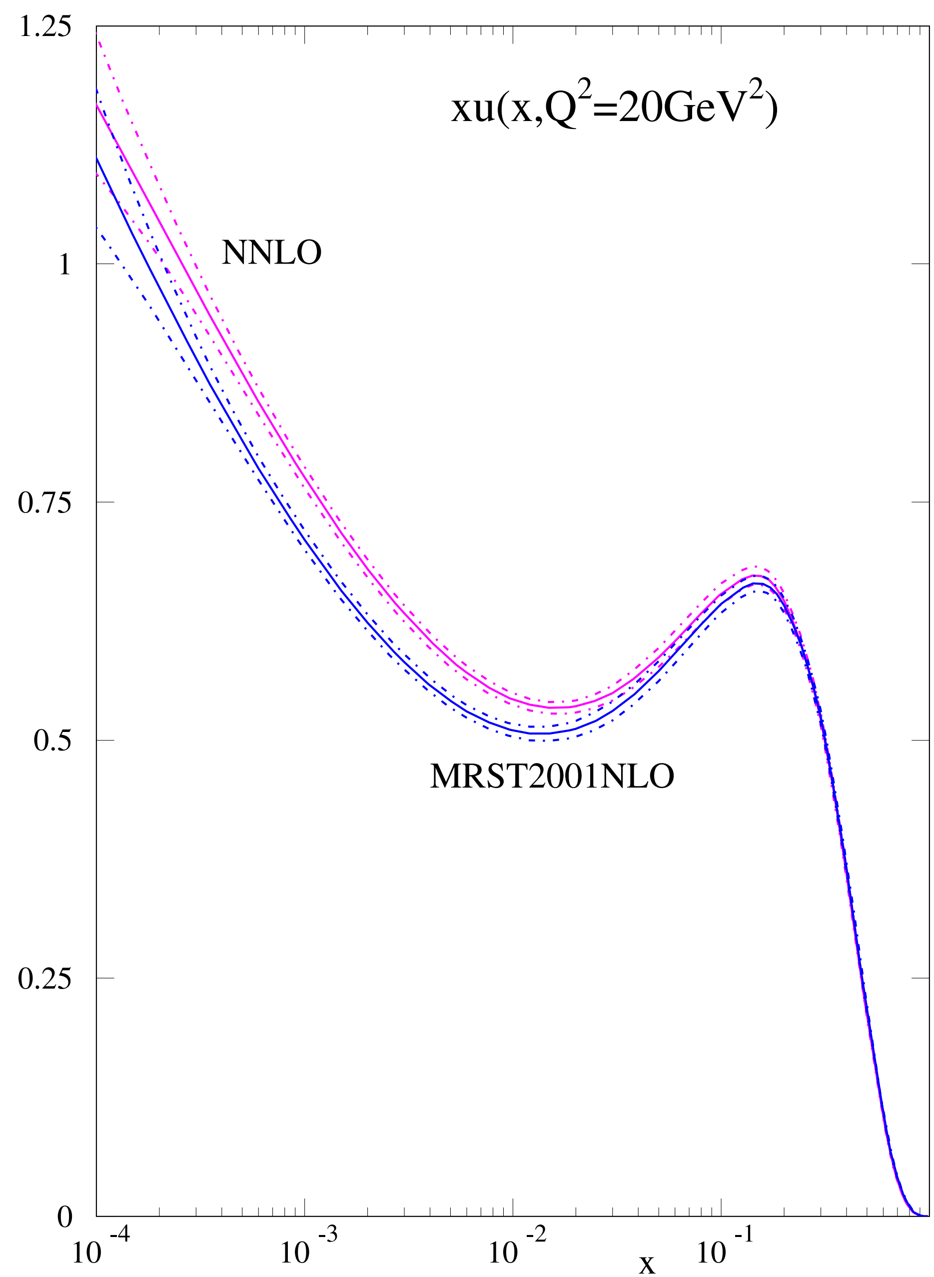
NNLO

Default has long been NLO. Essentially well understood. Now starting to go further.

NNLO coefficient functions for structure functions know for many years.

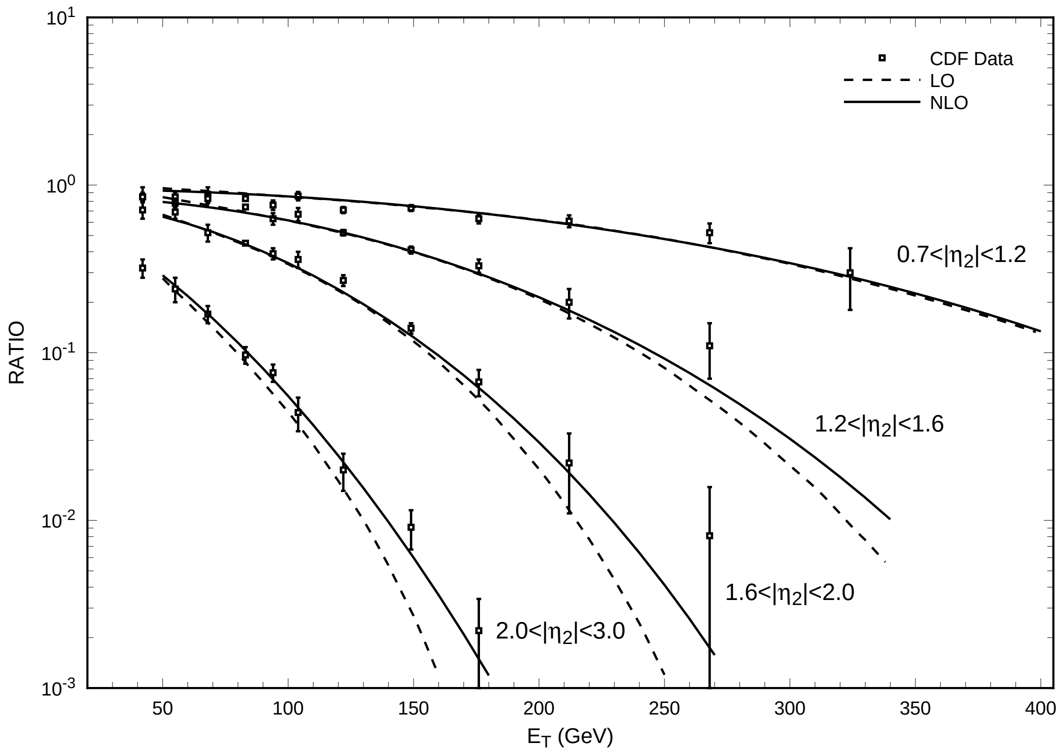
Splitting functions now complete. (Moch, Vermaseren and Vogt). Extremely similar to average of best estimates $\rightarrow$ no significant change in NNLO partons. Improve quality of fit very slightly (MRST), and reduces α_S.

Can be big change from NLO $\rightarrow$ NNLO



Do not know NNLO corrections to jet production in $pp(\bar{p})$ collisions. Stumbling block?

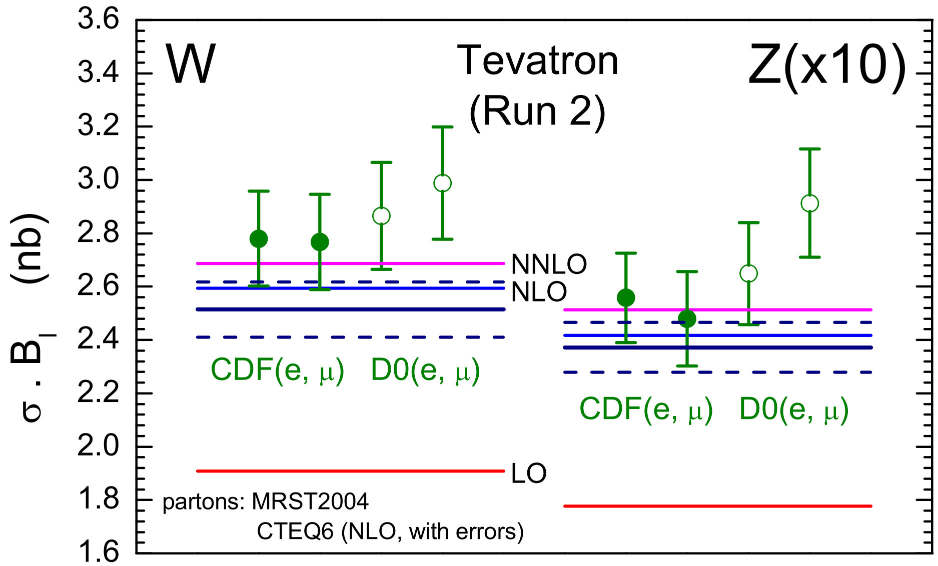
NLO corrections themselves not large, except at high rapidities. At central rapidities $\leq 10\%$. Similar to correlated errors.

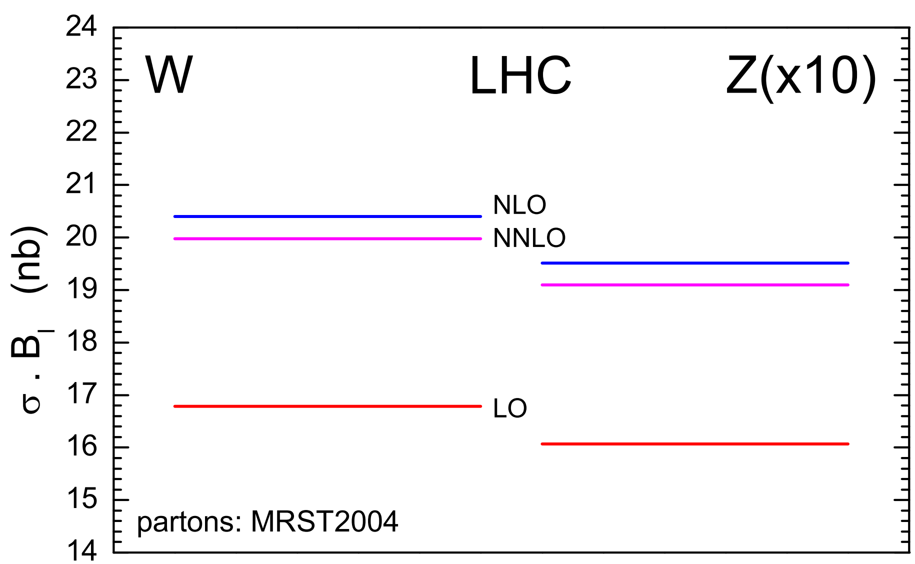


Also NNLO estimates Kidonakis, Owens. $\rightarrow 3-4\%$ correction. Consistent with what is known from NLO. Smaller than systematics on data. High-x gluon can change by more than 100% if jet data not included.

Reasonable stability order by order for (quark-dominated) W and Z cross-sections.

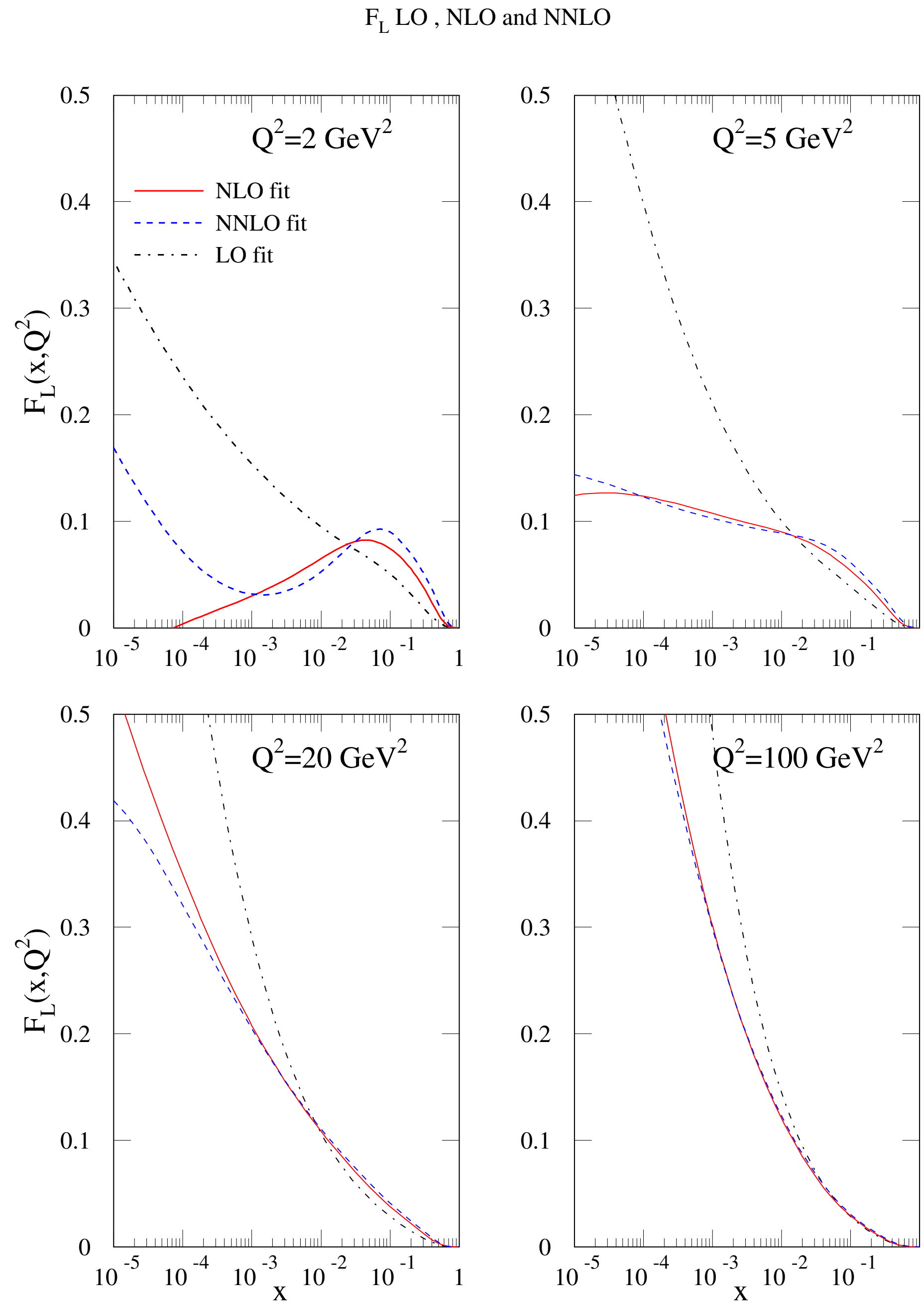
This fairly good convergence is largely guaranteed because the quarks are fit directly to data.





Instability in physical, gluon dominated, quantity $F_L(x,Q^2)$ going from LO $\rightarrow$ NLO $\rightarrow$ NNLO.

Similar problems possible for charm and/or bottom production, and low-mass Drell-Yan (γ) production at the LHC.



Small-x Theory

Reason for this problem.

It is known that at each order in α_S each splitting function and coefficient function obtains an extra power of $\ln(1/x)$ (some accidental zeros in P_{gg}), i.e.

$$P_{ij}(x, \alpha_s(Q^2)), \quad C_i^P(x, \alpha_s(Q^2)) \sim \alpha_s^m(Q^2) \ln^{m-1}(1/x).$$

$\rightarrow$ no guarantee of convergence at small x!

$$x < 0.01, \qquad \ln(1/x) > 5, \qquad \rightarrow \alpha_S \ln(1/x) > 1.$$

The global fits usually assume that this turns out to be unimportant in practice, and proceed regardless.

Fits work fairly well at small x, but could be better.

Good recent progress in incorporating $\ln(1/x)$ resummation into global fits – now at next-to-leading $\ln(1/x)$ level (White, RT). More work needed for reliable predictions.

Approach to Look for Safe Theoretical Regions.

In order to investigate real quality of fit and regions with problems vary kinematic cuts on data.

Procedure – change W^2_{cut}, Q^2_{cut} and x_{cut}, re-fit and see if quality of fit to remaining data improves and/or input parameters change dramatically. Continue until quality of fit and partons both stabilize.

Raising Q^2_{cut} from $2\mathrm{GeV}^2$ in steps there is a slow continuous and significant improvement for higher Q^2 up to $> 10\mathrm{GeV}^2$.

Raising x_{cut} from 0 to 0.005 continuous improvement. At each step moderate x gluon becomes more positive.

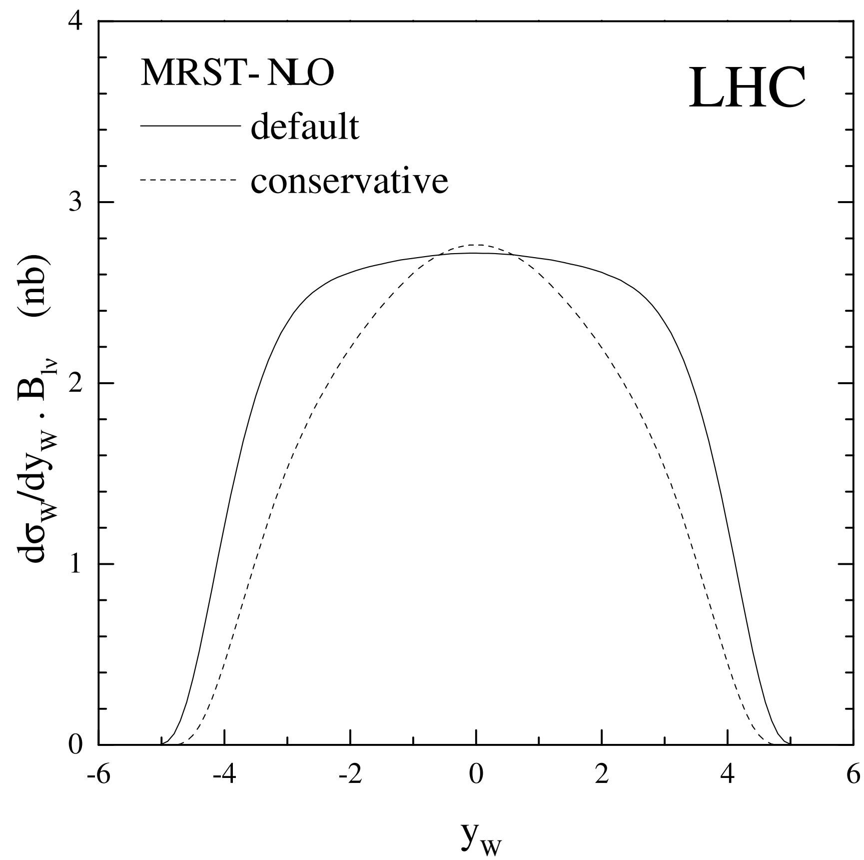
$\rightarrow$ conservative partons. Should be most reliable method of parton determination ($\Delta\chi^2 = -70$ for remaining data), but only applicable for restricted range of x, Q^2.
$\rightarrow \alpha_S(M_Z^2) = 0.1165 \pm 0.004$. Errors sensible and consistency with $\Delta\chi^2 \sim 5$.

Also NNLO conservative partons. Similar cuts and improvement in fit quality (bit smaller), but change in partons considerably less. Already includes important theoretical corrections.

Some doubt in predictions at high rapidity.

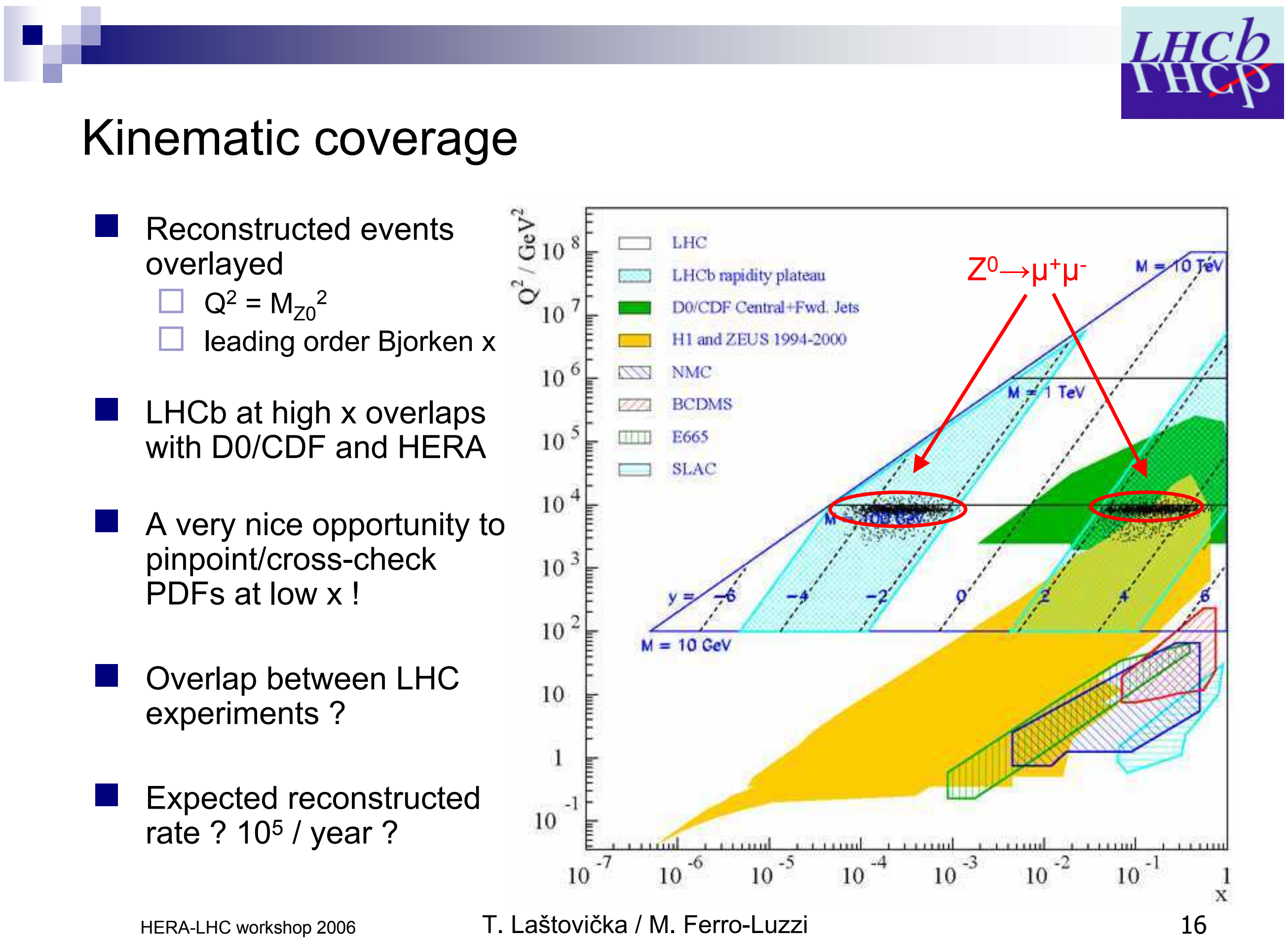
Comparison of prediction for $(d\sigma_W/dy_W)$ for the standard MRST partons and a set which represents the possible type of theoretical uncertainty in this region when working at NLO.

Good stability at central rapidity– $x = 0.005$.



Increased uncertainty if worrying about theory for very small x.

Useful early measurement at LHCb – even probe very small x with high rapidity Z (Lastovicka, Ferro-Luzzi), (Dublin, Liverpool). W production good for flavour separation.



Good test of parton distributions. Also at lowest rapidity $y = 1.8$ could be luminosity monitor if cross-checked with ATLAS and QCD calculations.

Heavy Quarks – Essential to treat these correctly. Two distinct regimes:

Near threshold $Q^2 \sim m_H^2$ massive quarks not partons. Created in final state. Described using **Fixed Flavour Number Scheme** (FFNS).

$$F(x,Q^2) = C_k^{FF}(Q^2/m_H^2) \otimes f_k^{n_f}(Q^2)$$

High scales $Q^2 \gg m_H^2$ massless partons. Behave like up, down, strange. Sum $\ln(Q^2/m_H^2)$ terms via evolution. **Zero Mass Variable Flavour Number Scheme** (ZM-VFNS). Ignores $\mathcal{O}(m_H^2/Q^2)$ corrections.

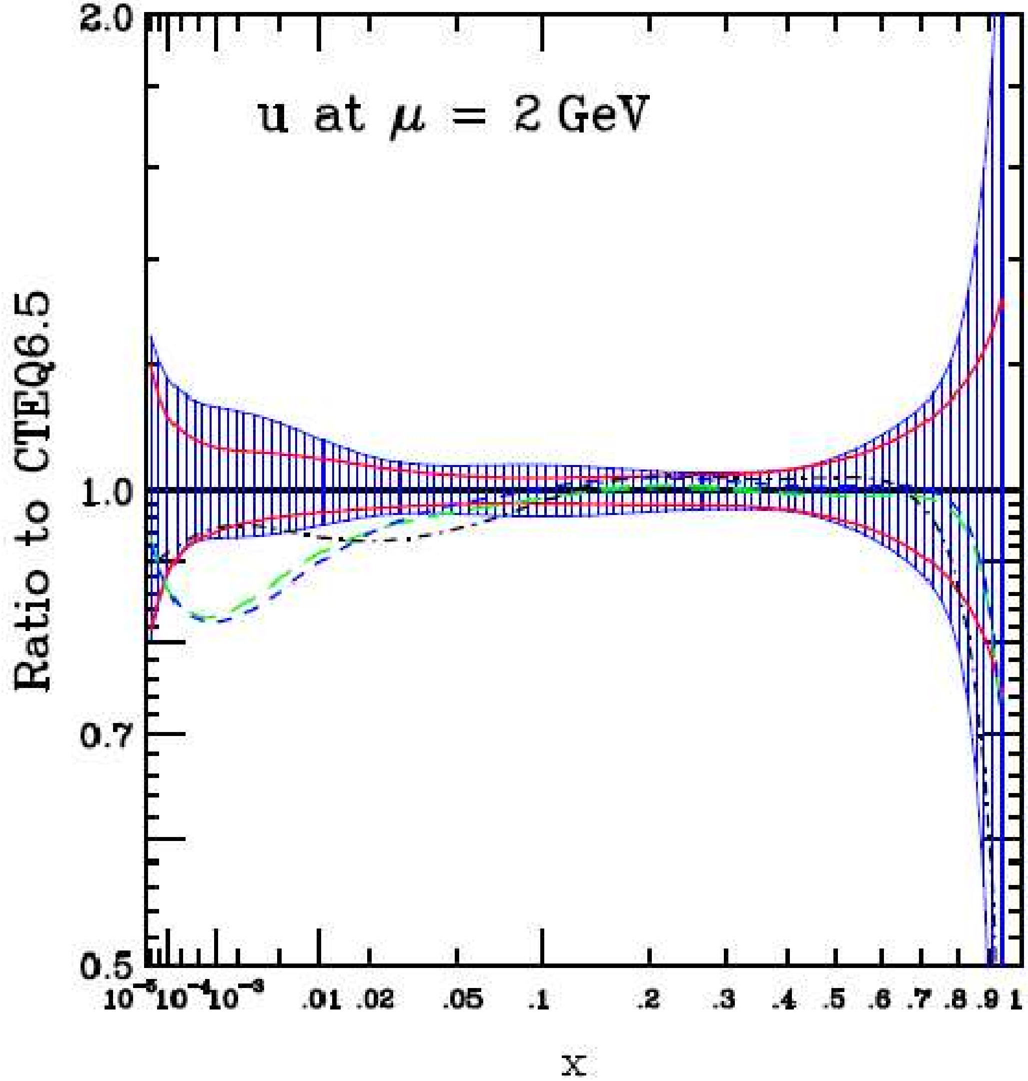
$$F(x,Q^2) = C_j^{ZMVF} \otimes f_j^{n_f+1}(Q^2).$$

Need a general **Variable Flavour Number Scheme** (VFNS) interpolating between the two well-defined limits of $Q^2 \leq m_H^2$ and $Q^2 \gg m_H^2$.

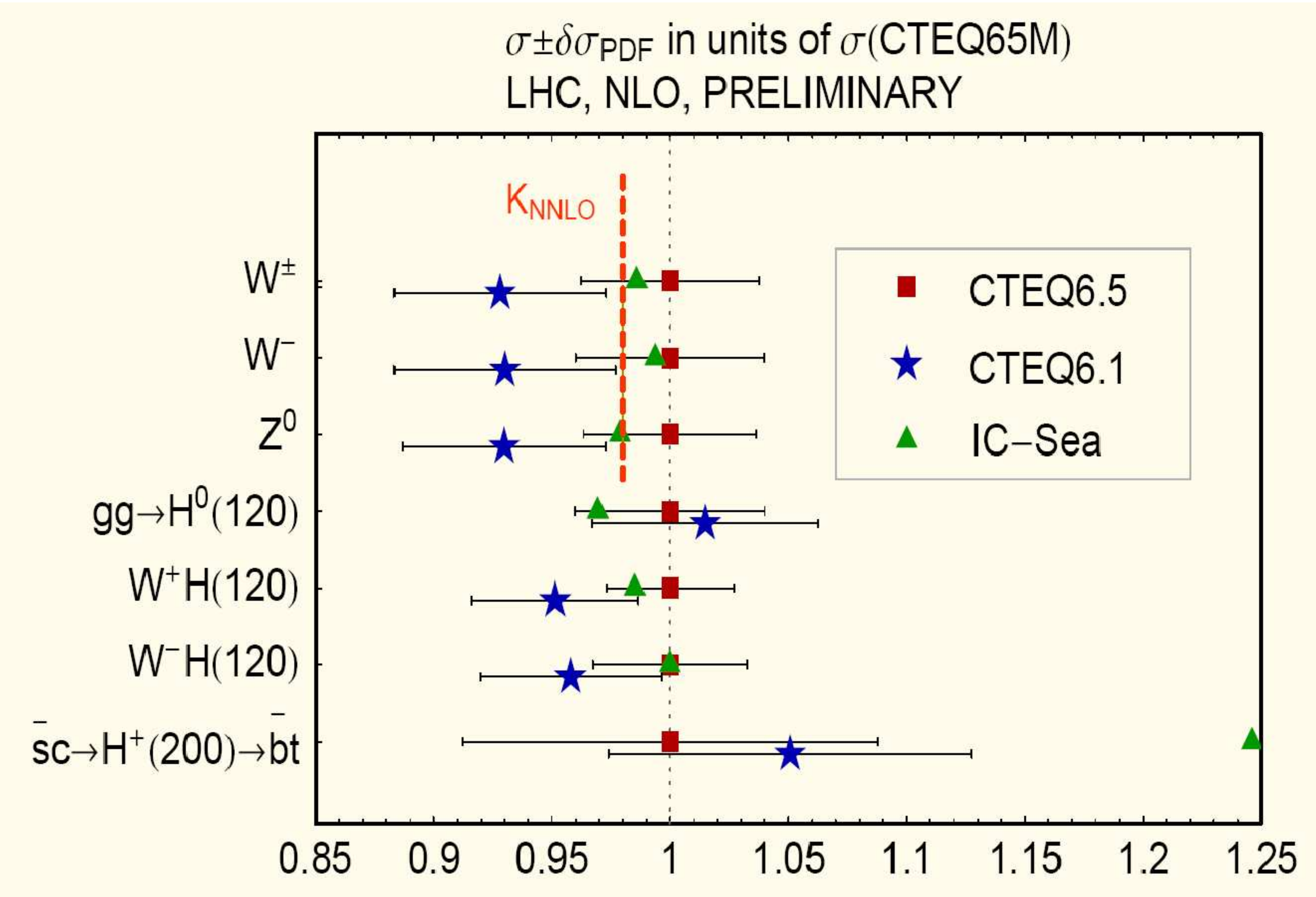
Inclusive processes (mainly $ep \to X$) prescriptions ACOT – CTEQ and TR – MRST, latter now to NNLO). More work needed for LHC.

Importance of doing it correctly illustrated by CTEQ6.5 up quark with uncertainties compared with previous versions, e.g. CTEQ6 in green.

MRST in dash-dot line. Reasonable agreement. Already used heavy flavour treatment in default sets.



Leads to large change in predictions using CTEQ partons at LHC



Some disagreements with MRST partons due to different gluon distributions, as seen.

Similar effect at NNLO with MRST partons.

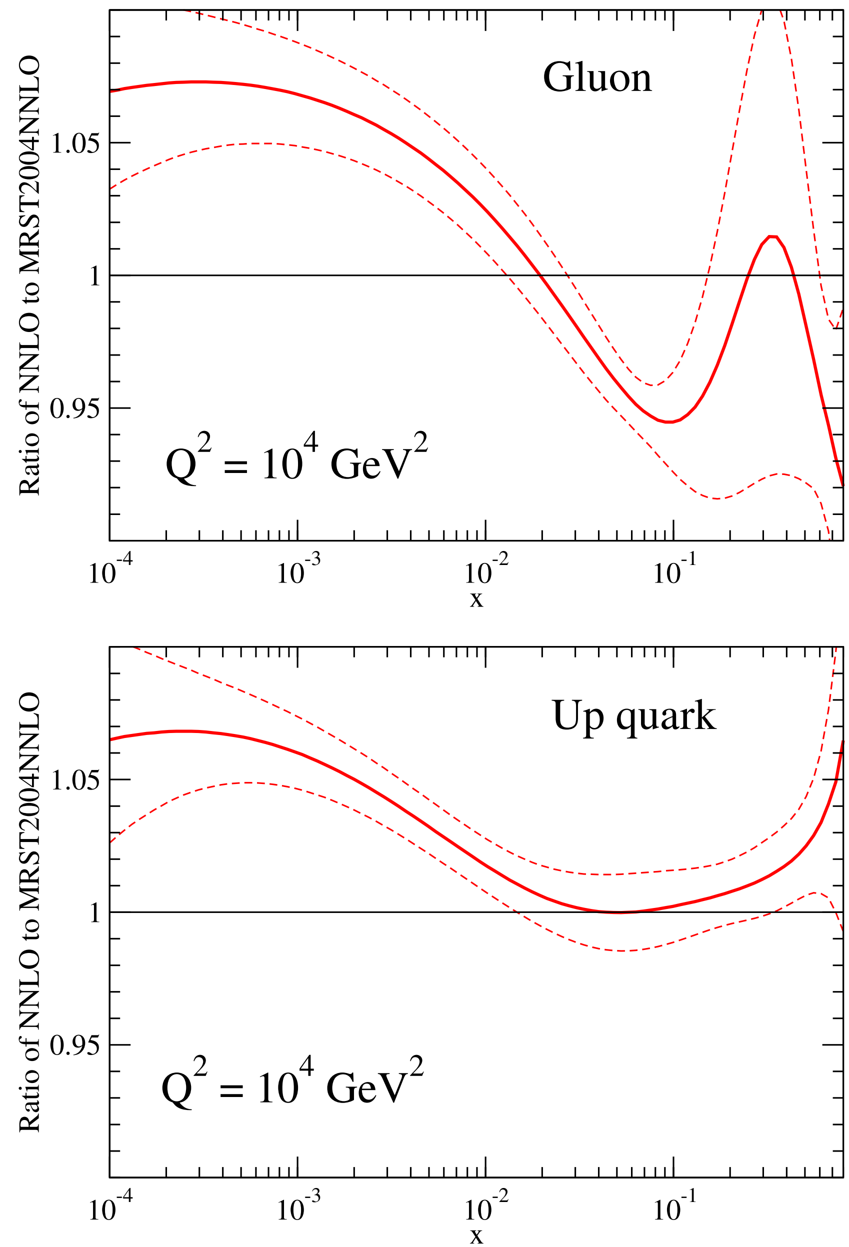
Previous approximate NNLO sets used (declared) approximate VFNS at flavour thresholds.

Full VFNS $\longrightarrow$ flatter evolution of charm

$\longrightarrow$ bigger gluon and more evolution of light sea.

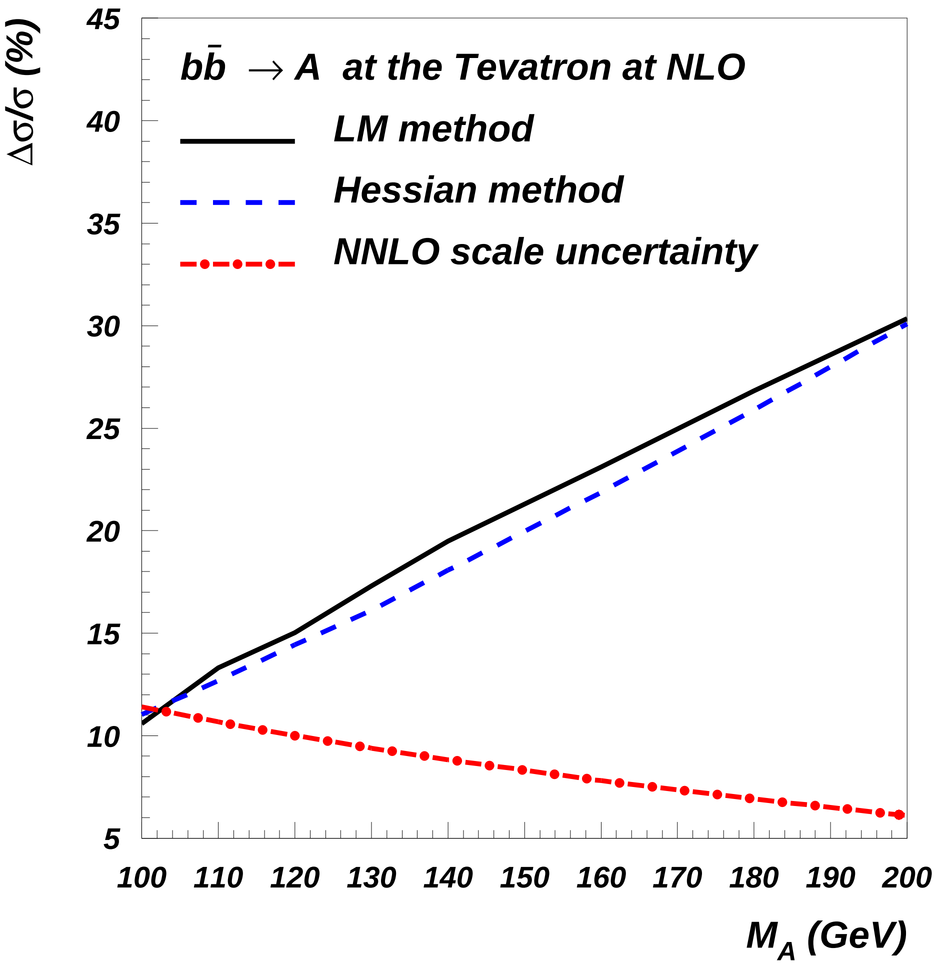
$\longrightarrow$ 6% in crease in σ_W and σ_Z at the LHC.

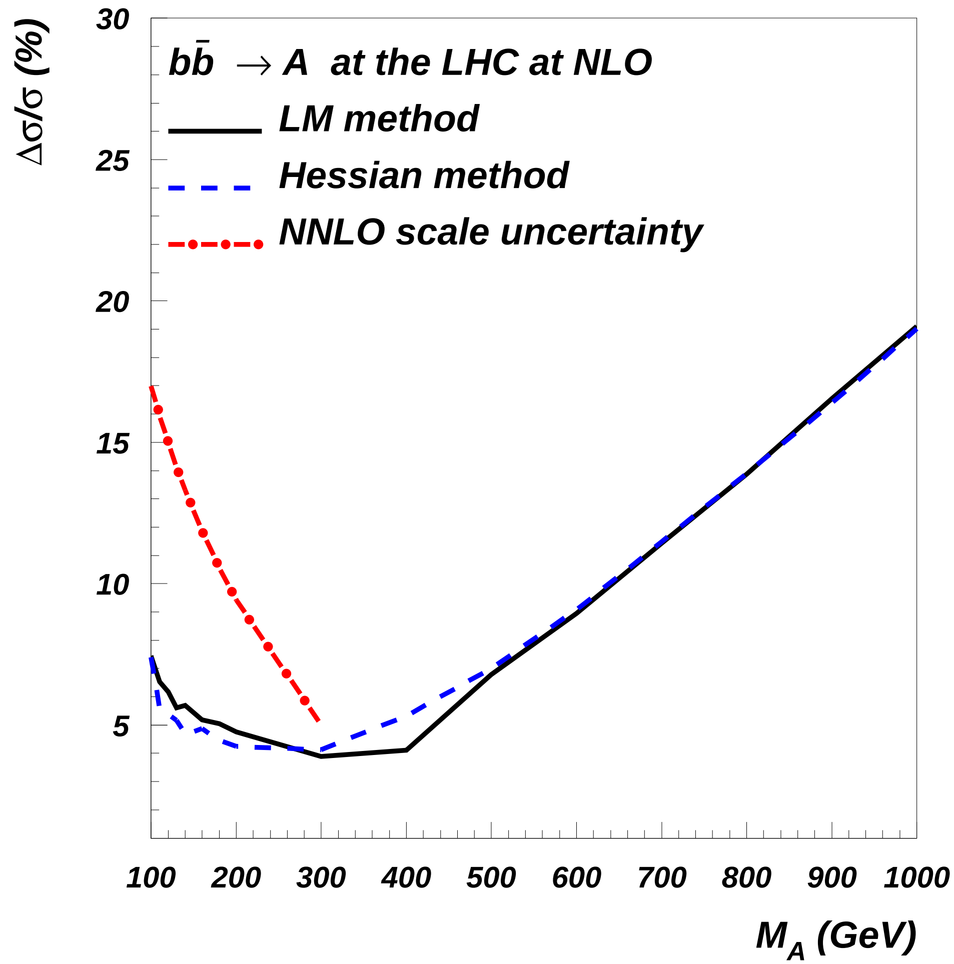
With hindsight this and CTEQ result are corrections not uncertainty.



Example of need to understand both heavy flavours and small-x physics for LHC.

Production of supersymmetric Higgs depends on parton uncertainties (Belyaev, Pumplin, Tung and Yuan), heavy flavour procedure and high-energy (small-x) treatment.





Conclusions

One can determine the parton distributions and predict cross-sections at the LHC, and the fit quality using NLO or NNLO QCD is fairly good.

Various ways of looking at uncertainties due to errors on data. For **Global Fits** using $\Delta\chi^2 = 1$ is not a sensible option. Uncertainties rather small – $\sim 1 - 5\%$for most LHC quantities, and fairly similar between approaches.

Central values differ. Uncertainty from input assumptions e.g. cuts on data, data used, *etc.*, comparable and potentially larger than statistical uncertainties. Flavour decomposition still uncertain. Electroweak corrections potentially large at very high energies – $\ln^2(E^2/M_W^2)$. Requires careful definitions of theory and measurement.

Errors from higher orders/resummation potentially large. Direct measurement of $F_L(x, Q^2)$ at HERA an important means of testing this. At LHC measurement at high rapidities, e.g. W, Z would be useful in testing understanding of QCD, and particularly quantities sensitive to low x at low scales, e.g. low mass Drell-Yan.

Theory often the dominant source of uncertainty. Much progress – more processes at NLO, some NNLO, heavy flavours treatments, resummations .... Pretty much full NNLO parton determinations now possible. Theoretical improvements necessary for full understanding of best predictions and uncertainties.

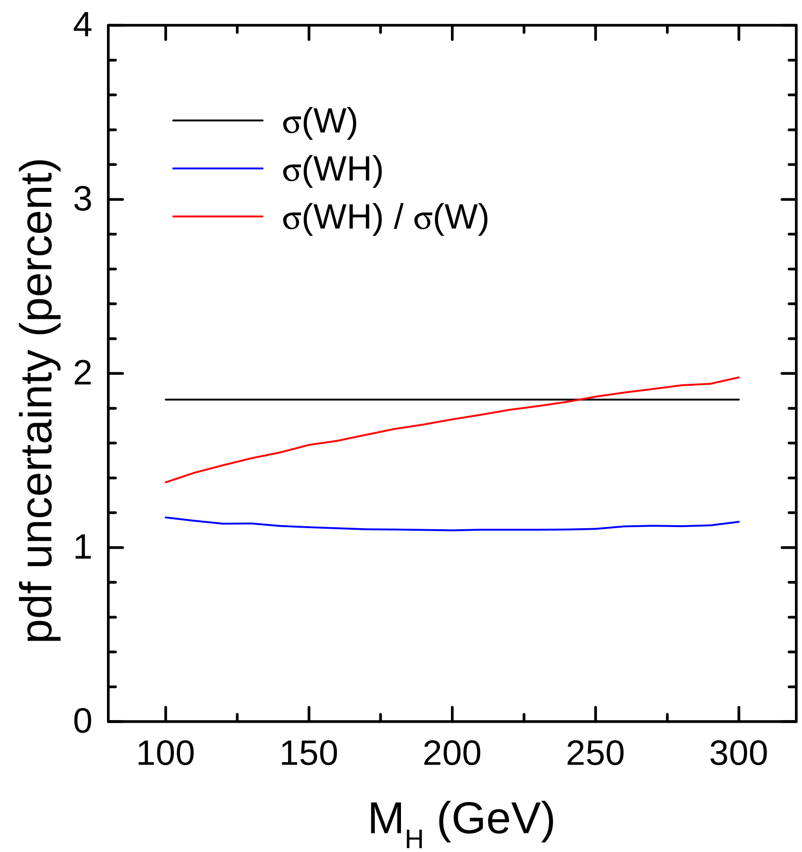
Could $\sigma(W)$ or $\sigma(Z)$ be used to calibrate other cross-sections, e.g. $\sigma(WH)$, $\sigma(Z')$?

$\sigma(WH)$ more precisely predicted because it samples quark pdfs at higher x, and scale, than $\sigma(W)$.

However, ratio shows no improvement in uncertainty, and can be worse.

Partons in different regions of x are often anti-correlated rather than correlated, partially due to sum rules.

pdf uncertainties on W, WH
cross sections at LHC (MRST2001E)



No obvious advantage in using $\sigma(t\bar{t})$ as a calibration SM cross-section, except maybe for very particular, and rather large, M_H.

However, a light (SM or MSSM) Higgs dominantly produced via $gg \to H$ and the cross-section has small pdf uncertainty because $g(x)$ at small x is well constrained by HERA DIS data.

Current best (MRST) estimate, for $M_H = 120$ GeV: $\delta\sigma_H^{\rm NLO}(\text{expt pdf}) = \pm 2 - 3\%$ with less sensitivity to small x than $\sigma(W)$.

Much smaller than the uncertainty from higher-order corrections, for example, Catani et al,

$\delta\sigma_H^{\rm NNLL}(\text{scale variation}) = \pm 8\%$

pdf uncertainties on top, (gg→) H cross sections at LHC (MRST2001E)

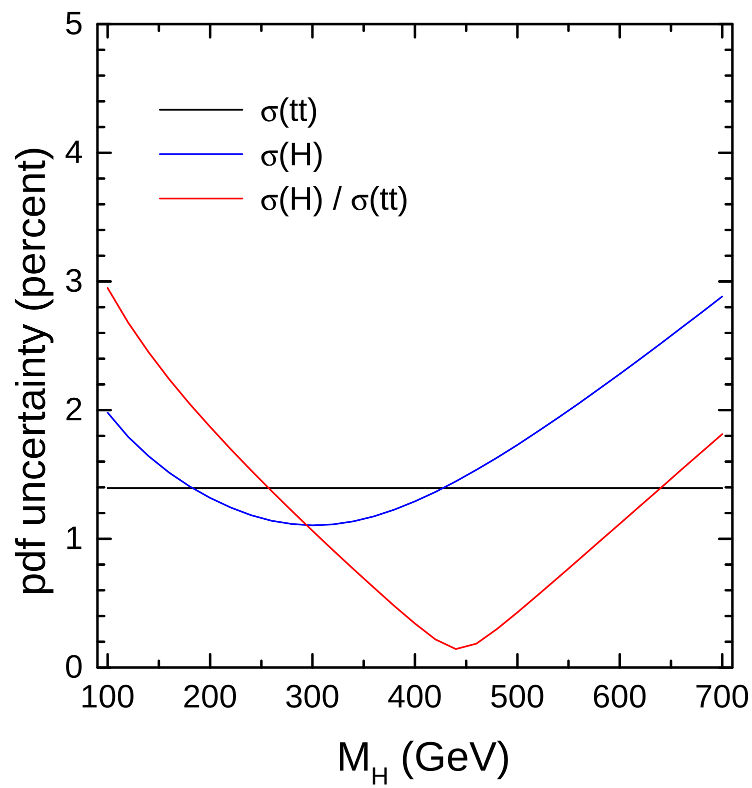


Table 1: Cross sections for Drell-Yan pairs (e^+e^-) with `PYTHIA` 6.206. The errors shown are the statistical errors of the Monte-Carlo generation.

PDF set	Comment	xsec
$81 < M < 101$ GeV		
CTEQ5L	PYTHIA internal	1516 ± 5 pb
CTEQ5L	PDFLIB	1536 ± 5 pb
CTEQ6	LHAPDF	1564 ± 5 pb
MRST2001	LHAPDF	1591 ± 5 pb
Fermi2002	LHAPDF	1299 ± 4 pb
$M > 1000$ GeV		
CTEQ5L	PYTHIA internal	6.58 ± 0.02 fb
CTEQ5L	PDFLIB	6.68 ± 0.02 fb
CTEQ6	LHAPDF	6.76 ± 0.02 fb
MRST2001	LHAPDF	7.09 ± 0.02 fb
Fermi2002	LHAPDF	7.94 ± 0.03 fb

Note anti-correlation between deviations at high and low mass, i.e. high and low x. Typical result from sum rules and evolution.

Essentially full NNLO determination of partons possible. Surely this is best, i.e. most accurate.

Yes, but ... only know some hard cross-sections at NNLO.

Processes with two strongly interacting particles largely completed

DIS coefficient functions and sum rules

$pp(\bar{p}) \rightarrow \gamma^\star, W, Z$ (including rapidity dist.), H, A^0, WH, ZH.

But for many other final states NNLO not known. NLO still more appropriate.

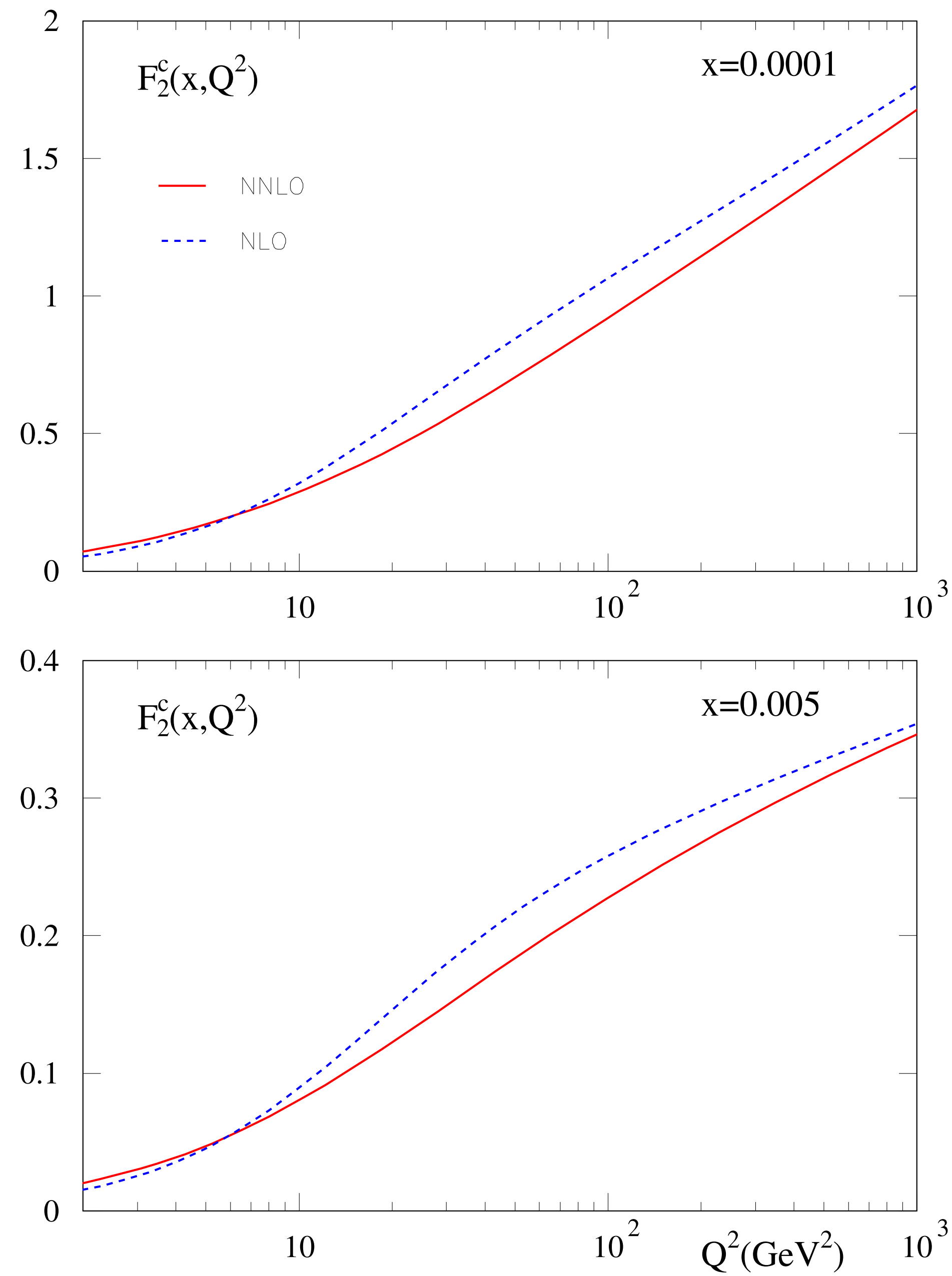
(If NLO known. NLO calculations largely complete for $2 \rightarrow 2$ and $2 \rightarrow 3$ processes. Beyond this only LO. Absolute cross-section not under control. e.g. $pp \rightarrow t\bar{t} + b\bar{b}$ or $\rightarrow t\bar{t} + \mathrm{jets}$, background to $t\bar{t} + H$.)

Resummations may be important even beyond NNLO in some regions, as may higher twist.

NNLO $F_2^c(x,Q^2)$ starts from higher value at low Q^2.

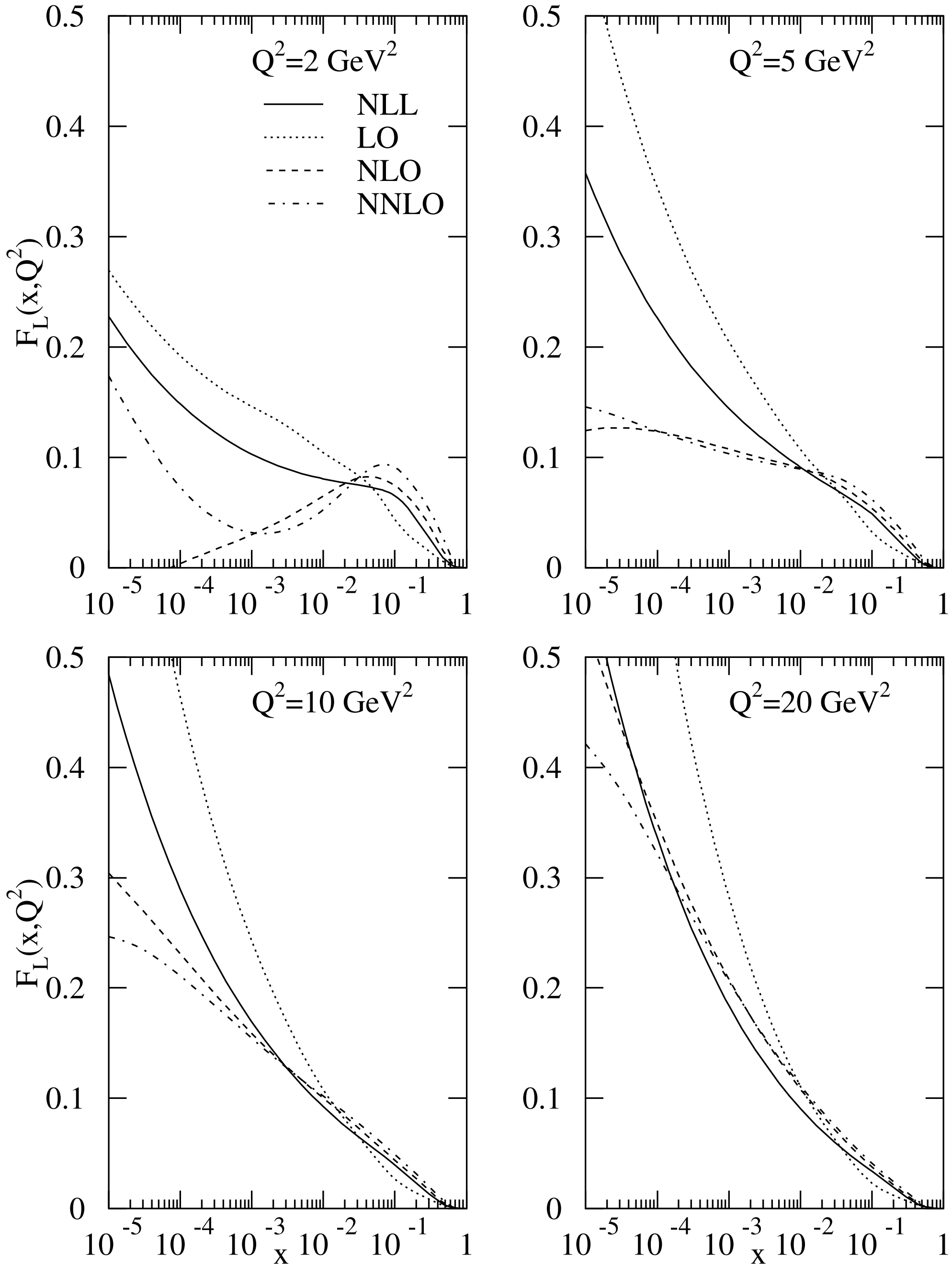
At high Q^2 dominated by $(c+\bar{c})(x,Q^2)$. This has started evolving from negative value at $Q^2 = m_c^2$. Remains lower than at NLO for similar evolution.

General trend – $F_2^c(x,Q^2)$ flatter in Q^2 at NNLO than at NLO. Important effect on gluon distribution going from one to other.



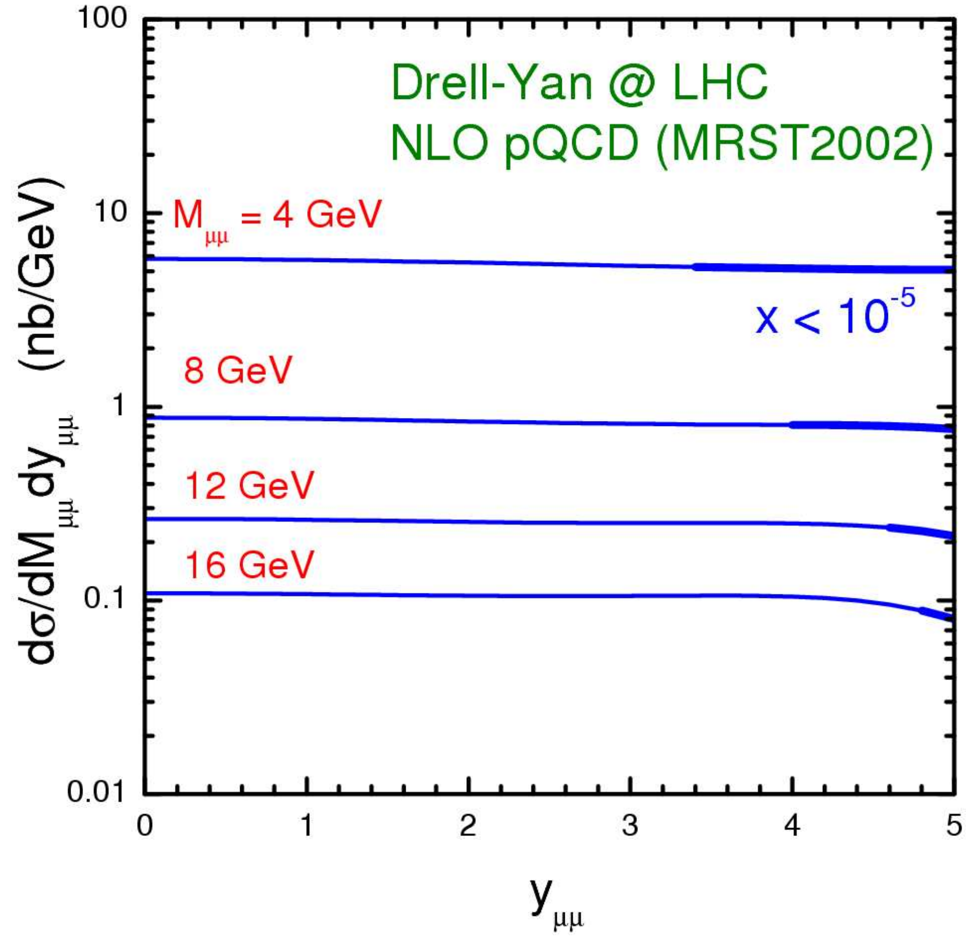
Instability in physical, gluon dominated, quantity $F_L(x,Q^2)$ going from LO $\rightarrow$ NLO $\rightarrow$ NNLO.

Improved by next-to-leading $\ln(1/x)$ resummation in the global fit and prediction (White, RT).



Possible to get to very low values of x at the LHC, particularly LHCb.

Can probe below $x = 10^{-5}$ - beyond range tested at HERA.



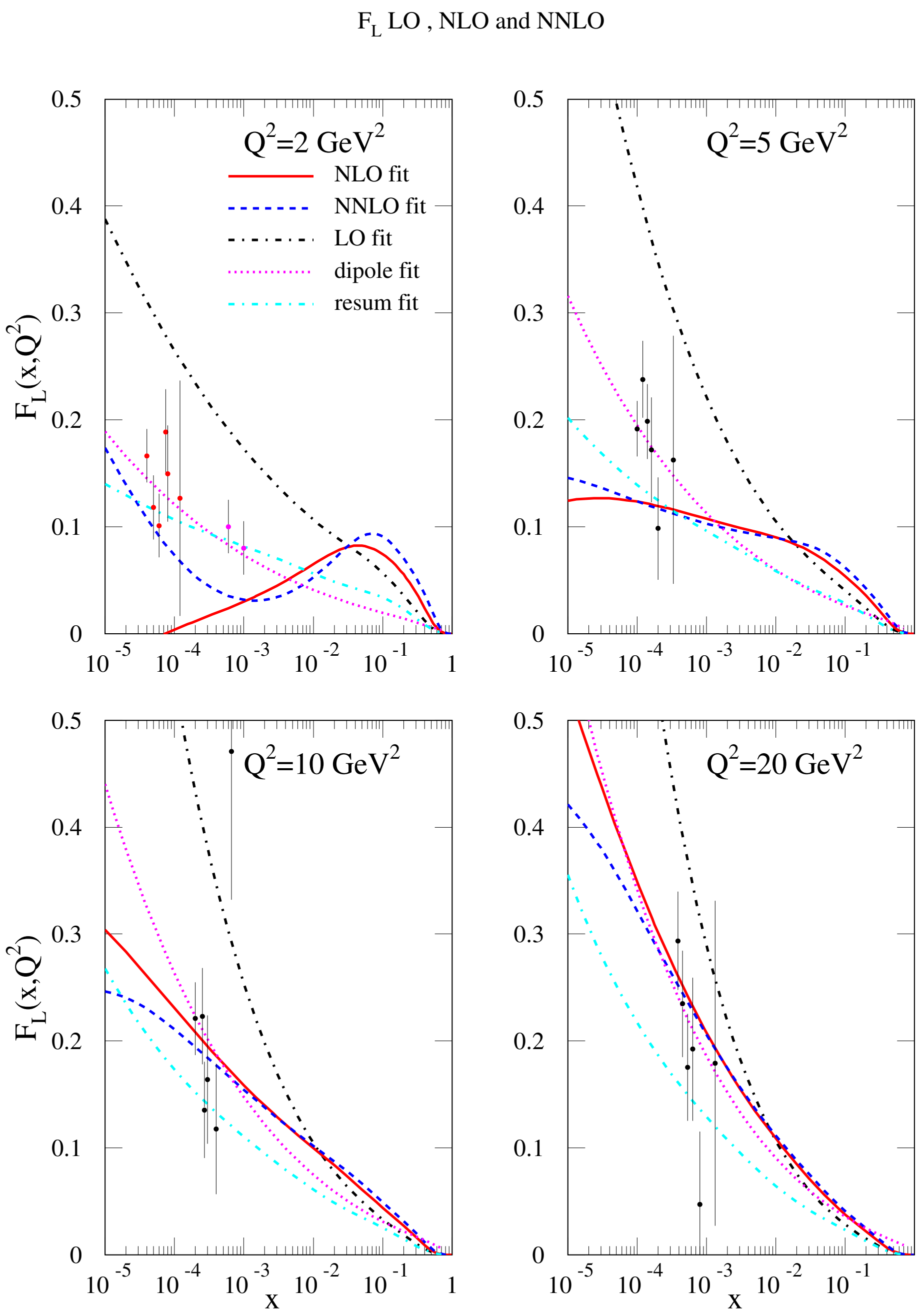
Theories with extensions at small x, both resummations and higher twist, produce rather different shape and size prediction for $F_L(x, Q^2)$ from that at NLO and NNLO.

Similar variation expected for other gluon-sensitive quantities.

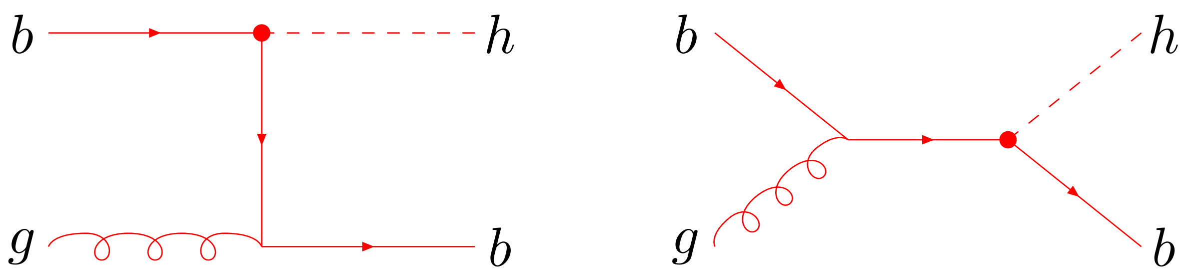
Currently working with HERA to determine if this can be measured if beam energy is lowered to measure $F_L(x, Q^2)$. Now very likely.

Clearly some reasonable power to differentiate.

Important for understanding LHC physics.



Consider bottom production along with a Higgs boson.



In Standard Model tiny since Higgs-bottom coupling $g_{b\bar{b}h} = m_b/v$, (v Higgs vacuum expectation value.) $m_b = 4.5\mathrm{GeV}$, $v = 246\mathrm{GeV}$.

In Minimal Supersymmetric Standard Model two Higgs doublets coupling separately to d-type and u-type quarks. Expectation values v_d and v_u.

Ratio $\tan\beta = v_u/v_d$.

Enhancement of Higgs-bottom coupling

$$g_{b\bar{b}h} \propto \frac{g^{SM}_{b\bar{b}h}}{\cos\beta}.$$

Bounds from LEP, $\tan\beta$ large $\rightarrow$ $\cos\beta$ small. Enhancement of Higgs-bottom coupling.

Search at Tevatron for enhancement in jets with b quarks.

Produces upper limit on parameter $\tan\beta$.