A Bayesian approach to the Constrained MSSM

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and

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hep-ph/0602028 $\longrightarrow$ JHEP06, hep-ph/0611173$\longrightarrow$ JHEP07 and arXiv:0705.2012

SuperBayes package, superbayes.org (to come 'very soon')

Outline

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- summary

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$\Rightarrow$ prepare proper tools for analyzing expected data (LHC,...)

Constrained MSSM ...aka mSUGRA

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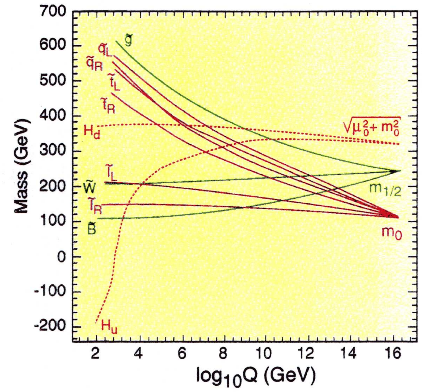
At $M_{\mathrm{GUT}} \simeq 2 \times 10^{16}$ GeV:

- gauginos $M_1 = M_2 = m_{\widetilde{g}} = m_{1/2}$ (c.f. MSSM)
- scalars $m^2_{\widetilde{q}_i} = m^2_{\widetilde{l}_i} = m^2_{H_b} = m^2_{H_t} = m^2_0$
- 3–linear soft terms $A_b = A_t = A_0$



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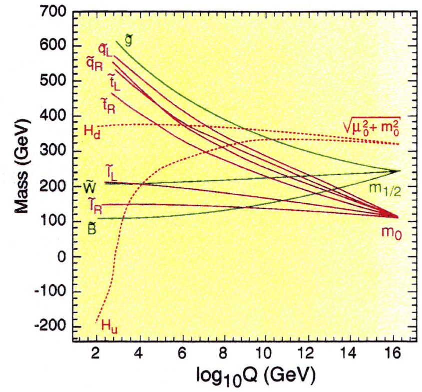

- radiative EWSB

$$\mu^2 = \frac{\left(m^2_{H_b} + \Sigma_b^{(1)}\right) - \left(m^2_{H_t} + \Sigma_t^{(1)}\right) \tan^2\beta}{\tan^2\beta - 1} - \frac{m_Z^2}{2}$$

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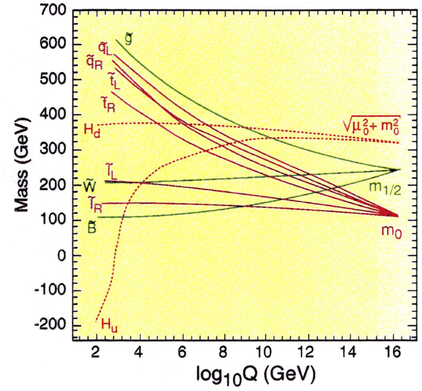

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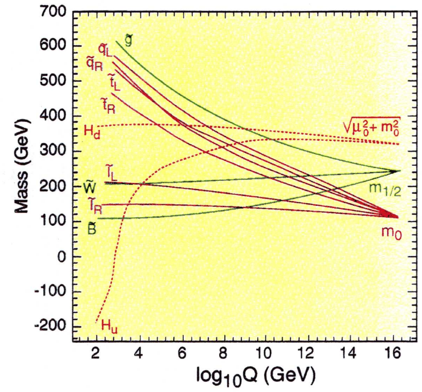

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- five independent parameters: $\tan\beta,\ m_{1/2},\ m_0,\ A_0,\ {\rm sgn}(\mu)$
- mass spectra at m_Z: run RGEs, 2–loop for g.c. and Y.c, 1-loop for masses
- some important quantities $(\mu, m_A, \ldots)$ very sensitive to procedure of computing EWSB & minimizing V_H

we use SoftSusy and FeynHiggs

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"usual" fixed-grid scans

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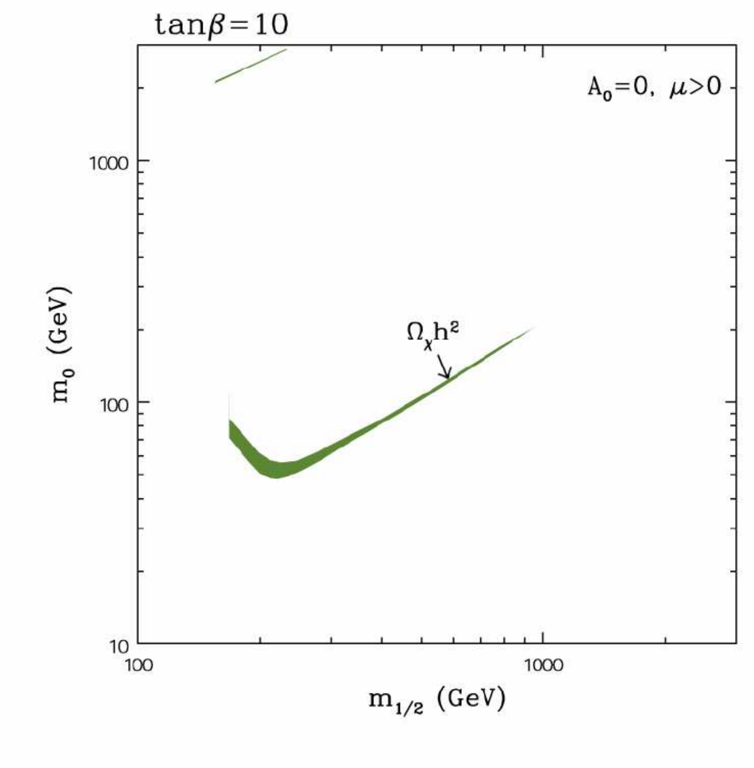
$\tan\beta \lesssim 45$



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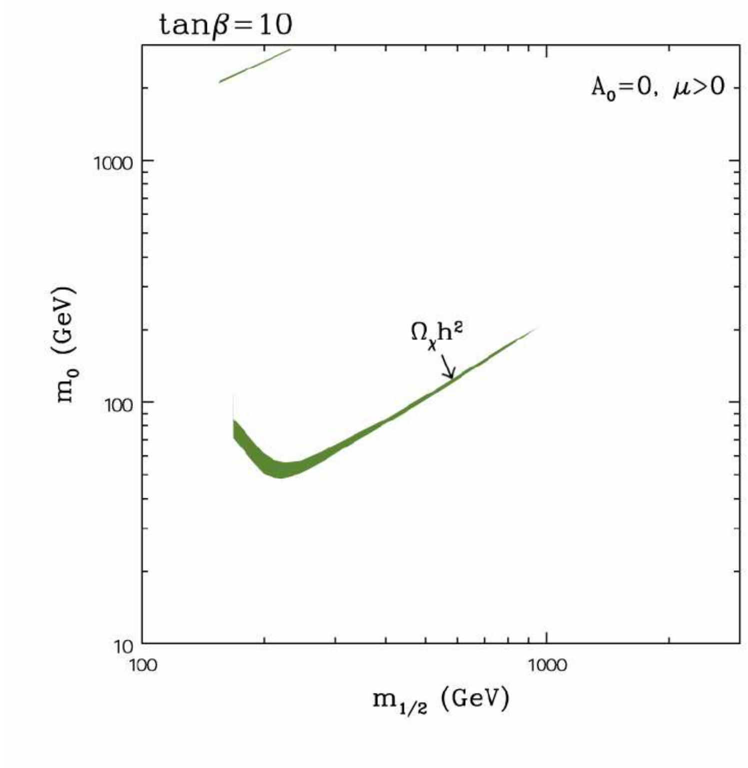

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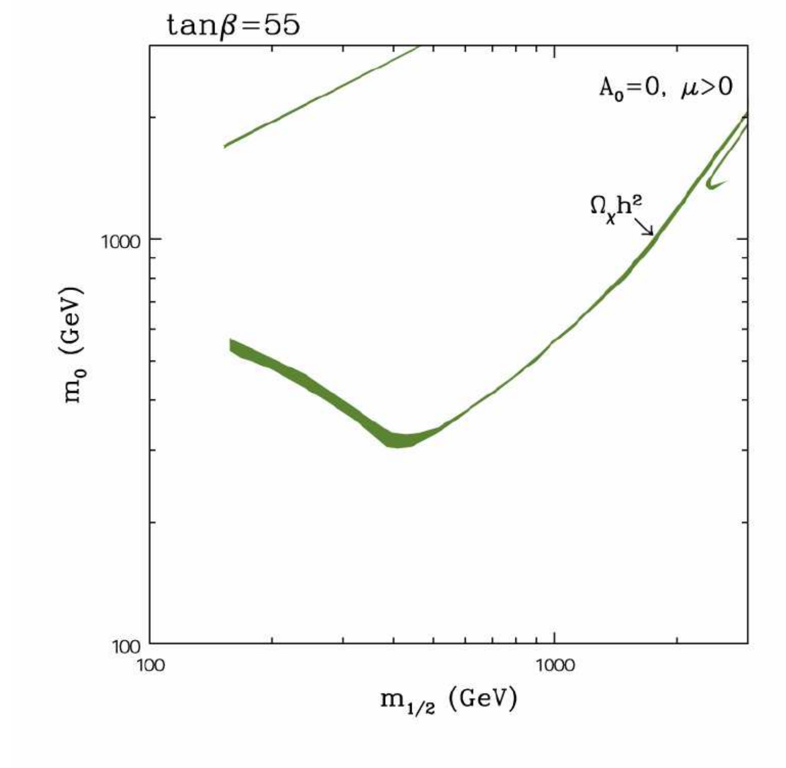

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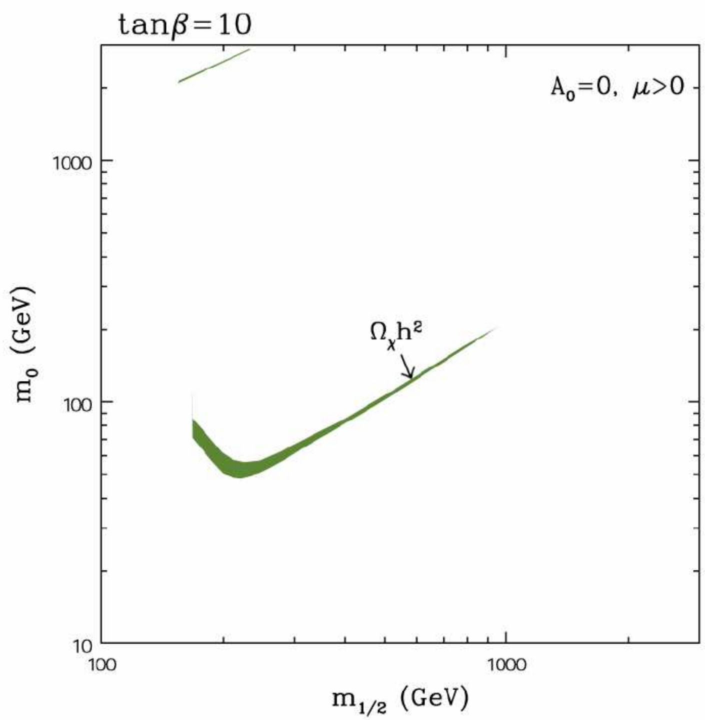

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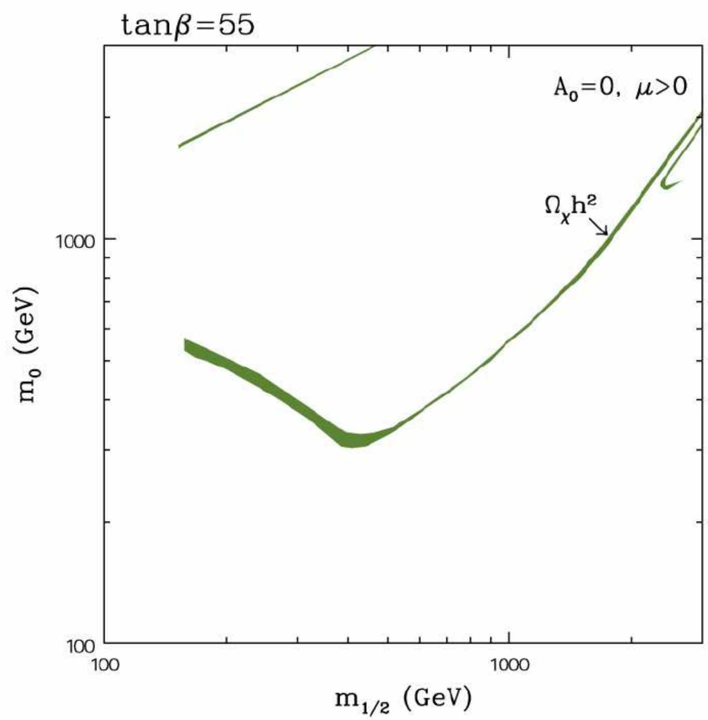

- fixed-grid scans, assuming rigid 1σ or 2σ experimental ranges
- green: consistent with WMAP-3yr (at 2σ)
- all the rest excluded by LEP, $\mathrm{BR}(\bar{B} \to X_s\gamma)$, $\Omega_\chi h^2$, EWSB, charged LSP,...

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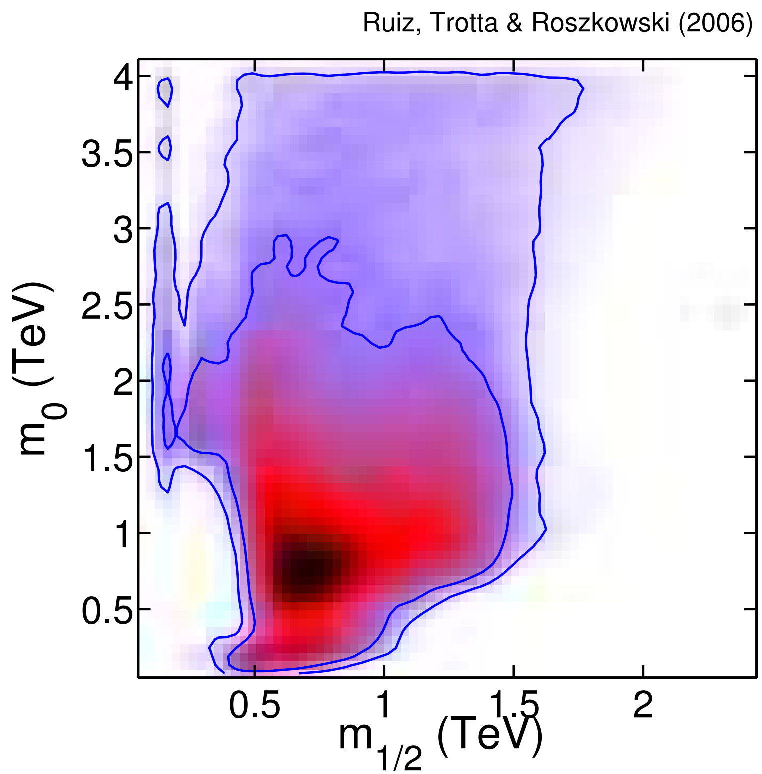
Bayesian pdf maps



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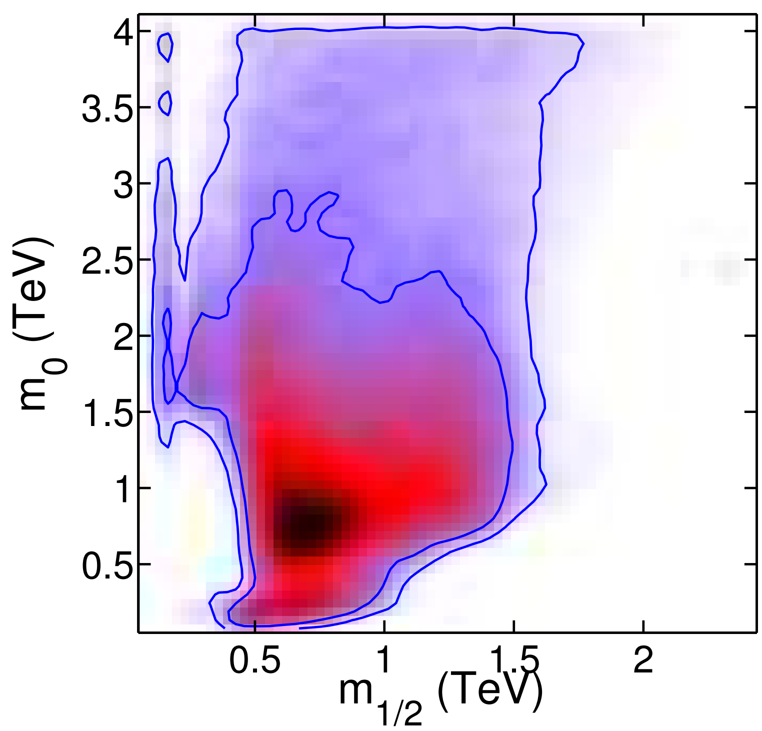

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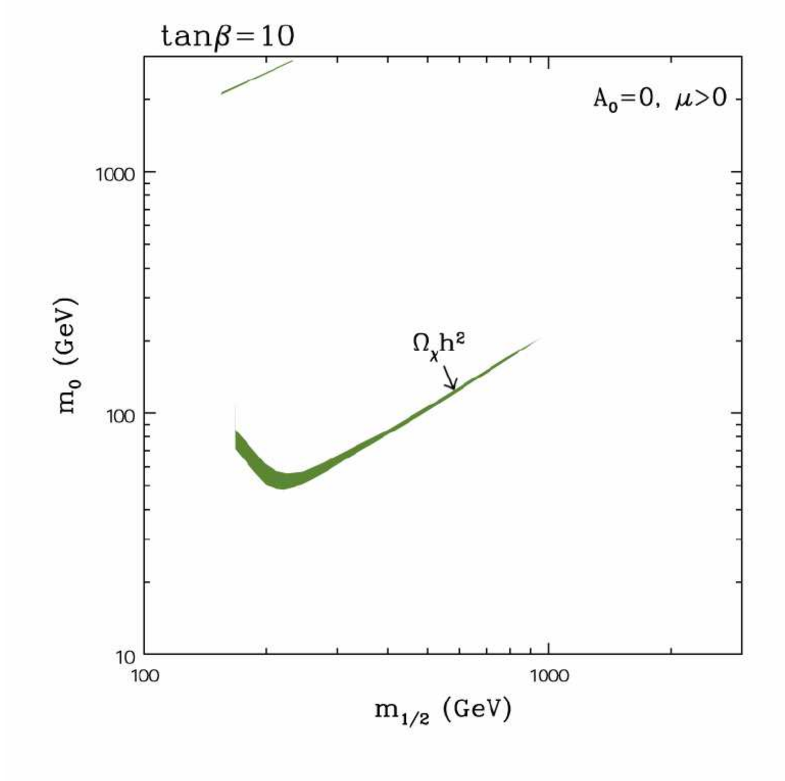

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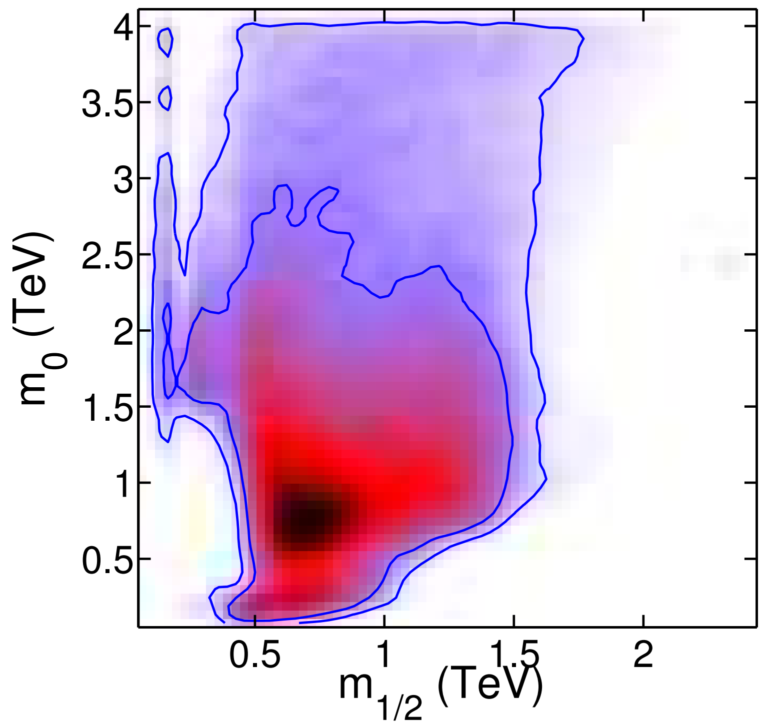

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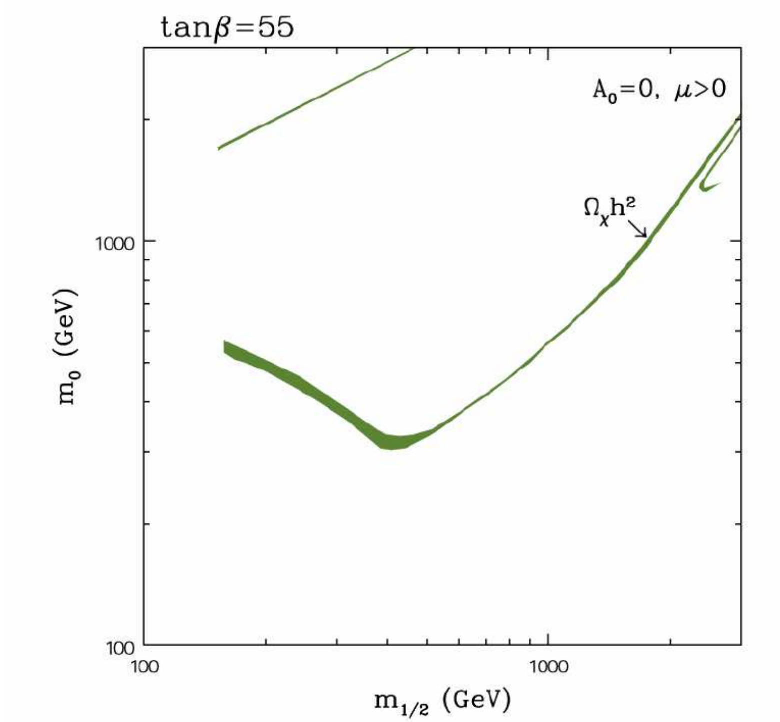

Note: In both an outdated SM value of $\mathbf{BR}(\bar{B} \to X_s\gamma)$ used. See below.

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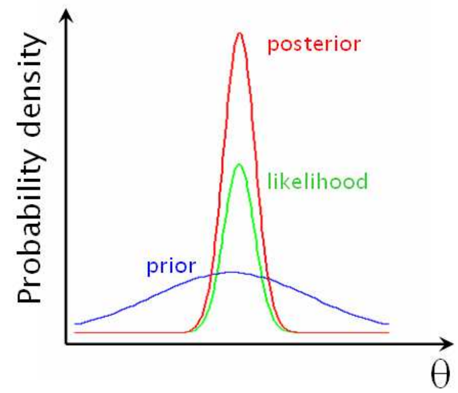

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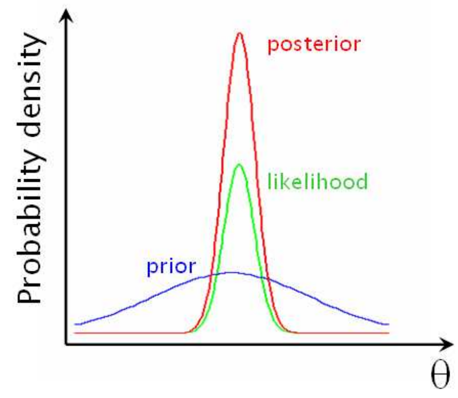

- Bayes' theorem: posterior pdf

$$p(\theta, \psi | d) = \frac{p(d|\xi)\pi(\theta,\psi)}{p(d)}$$

$$\text{posterior} = \frac{\text{likelihood} \times \text{prior}}{\text{normalization factor}}$$

- $p(d|\xi)$: likelihood
- $\pi(\theta, \psi)$: prior pdf
- $p(d)$: evidence (normalization factor)

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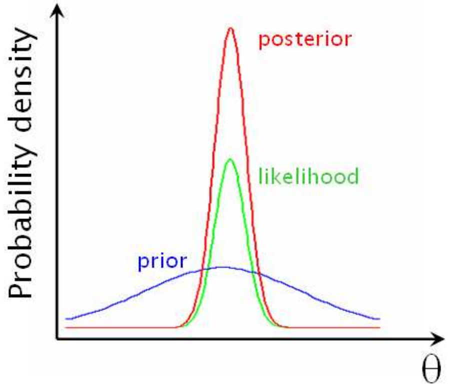

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- $\pi(\theta, \psi)$: prior pdf
- $p(d)$: evidence (normalization factor)
- usually marginalize over SM (nuisance) parameters $\psi \Rightarrow p(\theta|d)$

Bayesian Analysis of the CMSSM

- $\theta = (m_0, m_{1/2}, A_0, \tan\beta)$: CMSSM parameters
- $\psi = (M_t, m_b(m_b)^{\overline{MS}}, \alpha_{\rm em}(M_Z)^{\overline{MS}}, \alpha_s^{\overline{MS}})$: SM (nuisance) parameters
- priors – assume flat distributions and ranges as:

flat priors: CMSSM parameters
$50\,\text{GeV} < m_0 < 4\,\text{TeV}$
$50\,\text{GeV} < m_{1/2} < 4\,\text{TeV}$
$\vert A_0\vert < 7\,\text{TeV}$
$2 < \tan\beta < 62$

flat priors: SM (nuisance) parameters
$160\,\text{GeV} < M_t < 190\,\text{GeV}$
$4\,\text{GeV} < m_b(m_b)^{\overline{MS}} < 5\,\text{GeV}$
$0.10 < \alpha_s^{\overline{MS}} < 0.13$
$127.5 < 1/\alpha_{\rm em}(M_Z)^{\overline{MS}} < 128.5$

- vary all 8 (CMSSM+SM) parameters simultaneously, scan with MCMC
- include all relevant theoretical and experimental errors

Experimental Measurements

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SM (nuisance) parameter	Mean μ	Error σ (expt)
M_t	171.4 GeV	2.1 GeV
$m_b(m_b)^{\overline{MS}}$	4.20 GeV	0.07 GeV
$\alpha_s^{\overline{MS}}$	0.1176	0.002
$1/\alpha_{\rm em}(M_Z)^{\overline{MS}}$	127.918	0.018

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new $M_W = 80.413 \pm 0.048$ GeV (Jan 07, not yet included)
new $M_t = 170.9 \pm 1.8$ GeV (Mar 07, not yet included)
$\mathrm{BR}(\bar{\mathrm{B}} \to \mathrm{X_s}\gamma) \times 10^4$:
new SM: 3.15 ± 0.23 (Misiak & Steinhauser, Sept 06) used here

Derived observable	Mean	Errors	
	μ	σ (expt)	τ (th)
M_W	80.392 GeV	29 MeV	15 MeV
$\sin^2\theta_{\rm eff}$	0.23153	16×10^{-5}	15×10^{-5}
$\delta a_\mu^{\rm SUSY} \times 10^{10}$	28	8.1	1
$\mathrm{BR}(\bar{\mathrm{B}} \to \mathrm{X_s}\gamma) \times 10^4$	3.55	0.26	0.21
ΔM_{B_s}	17.33	0.12	4.8
$\Omega_\chi h^2$	0.119	0.009	$0.1\,\Omega_\chi h^2$

take as precisely known: $M_Z = 91.1876(21)$ GeV, $G_F = 1.16637(1) \times 10^{-5}$ GeV^{-2}

Experimental Limits

Derived observable	upper/lower limit	Constraints $\xi_{\rm lim}$	τ (theor.)
$\mathrm{BR}(B_s \to \mu^+\mu^-)$	UL	1.5×10^{-7}	14%
m_h	LL	114.4 GeV (91.0 GeV)	3 GeV
$\zeta_h^2 \equiv g_{ZZh}^2/g_{ZZH_{\rm SM}}^2$	UL	$f(m_h)$	3%
m_χ	LL	50 GeV	5%
$m_{\chi_1^\pm}$	LL	103.5 GeV (92.4 GeV)	5%
$m_{\tilde{e}_R}$	LL	100 GeV (73 GeV)	5%
$m_{\tilde{\mu}_R}$	LL	95 GeV (73 GeV)	5%
$m_{\tilde{\tau}_1}$	LL	87 GeV (73 GeV)	5%
$m_{\tilde{\nu}}$	LL	94 GeV (43 GeV)	5%
$m_{\tilde{t}_1}$	LL	95 GeV (65 GeV)	5%
$m_{\tilde{b}_1}$	LL	95 GeV (59 GeV)	5%
$m_{\tilde{q}}$	LL	318 GeV	5%
$m_{\tilde{g}}$	LL	233 GeV	5%
(σ_p^{SI}	UL	WIMP mass dependent	$\sim 100\%$)

Note: DM direct detection σ_p^{SI} not applied due to astroph'l uncertainties (eg, local DM density)

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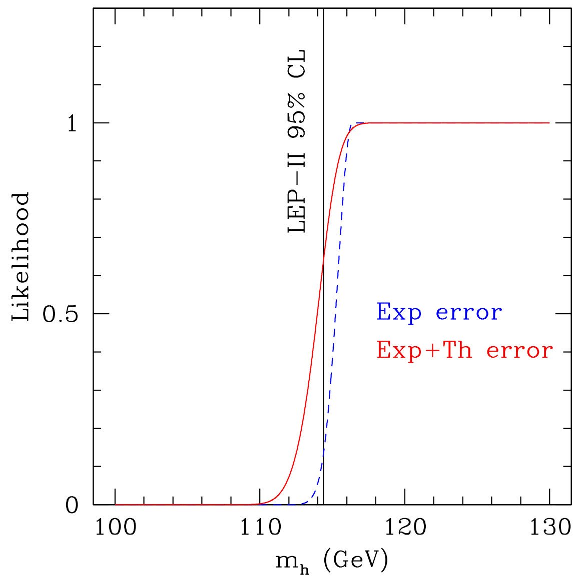
- for several uncorrelated observables (assumed Gaussian):

$$\mathcal{L} = \exp\left[-\sum_i \frac{\chi_i^2}{2}\right]$$

Light Higgs in the CMSSM

LEP: $m_h > 114.4$ GeV (95% CL) - if SM-like

- include both experimental and theoretical error: $\sigma \longrightarrow s = \sqrt{\sigma^2 + \tau^2}$



- we find $\zeta_h^2 \equiv \frac{g^2(m_h ZZ)_{\rm MSSM}}{g^2(m_h ZZ)_{\rm SM}} \simeq 1$

$\Rightarrow$ the light Higgs boson of the CMSSM is very SM-like

LEP-II limit applies

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Bayesian analysis, relative probability density fn (pdf), flat priors, $\mu > 0$

computed with SoftSusy v2.08

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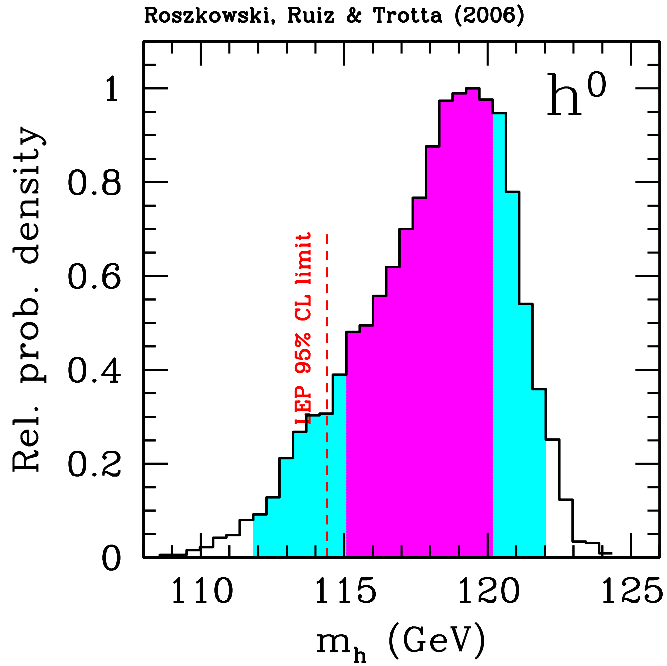
relative pdf $p(m_h|d)$



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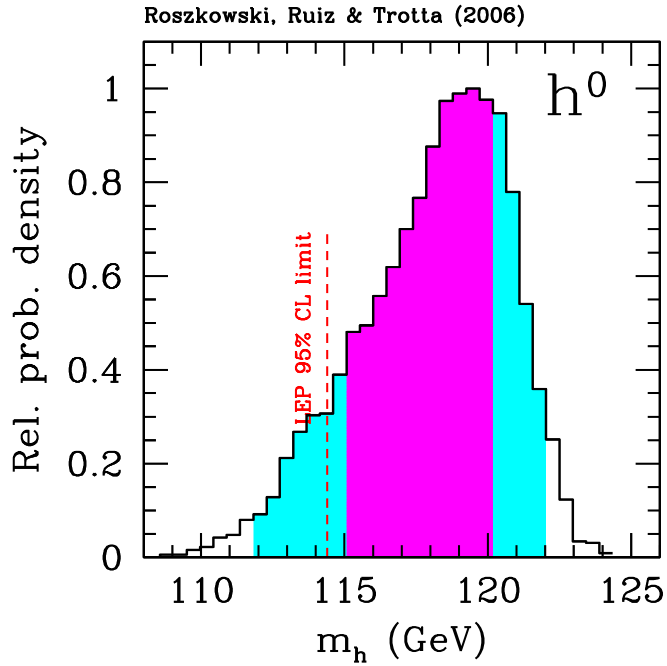

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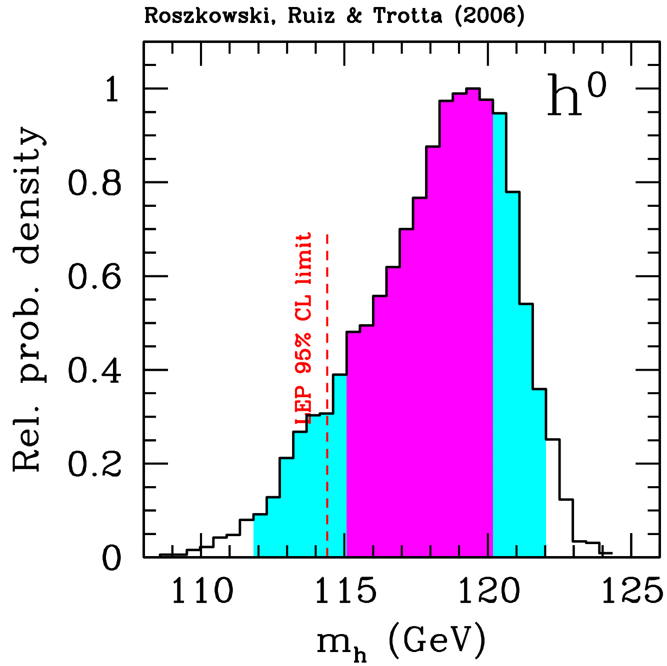

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sharp drop-off on rhs from no solutions at large $m_{1/2}$ and/or cutoff at $m_0 < 4$ TeV

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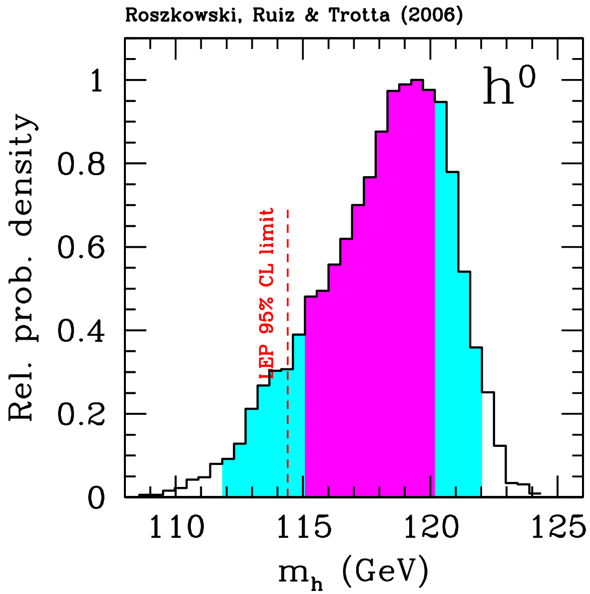

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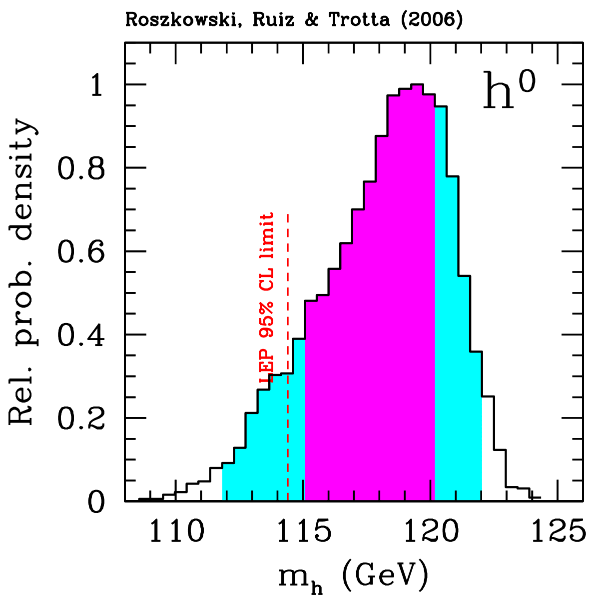
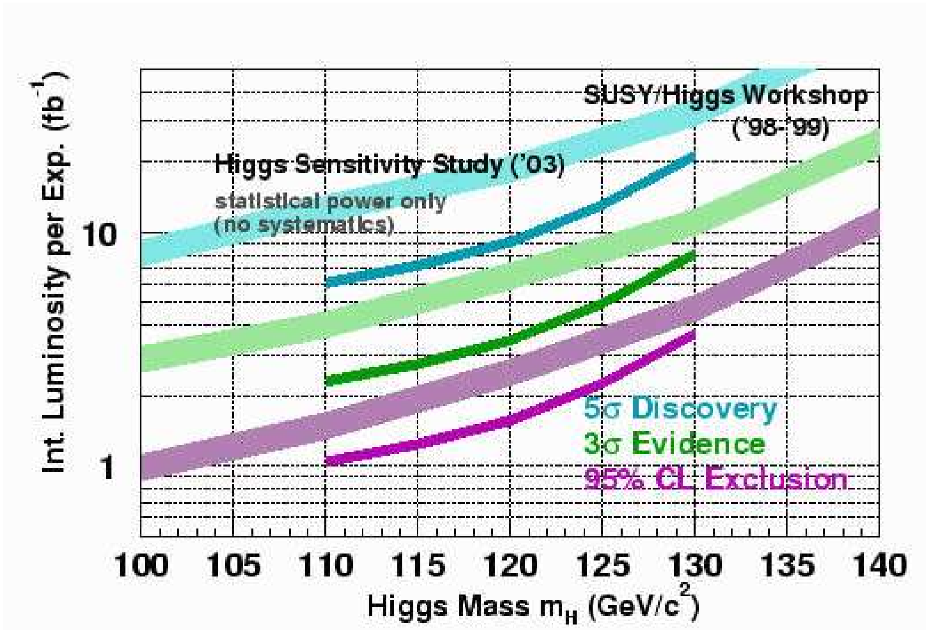


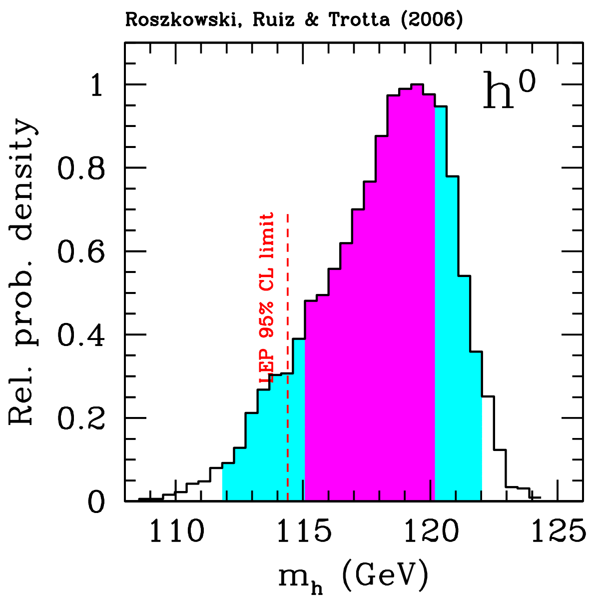
Tevatron reach (CDF and D0 WG (Oct 03))



CMSSM Higgs Boson & the Tevatron

CMSSM: light Higgs boson h^0 is SM-like (SM-like couplings)

MCMC scan, Bayesian analysis



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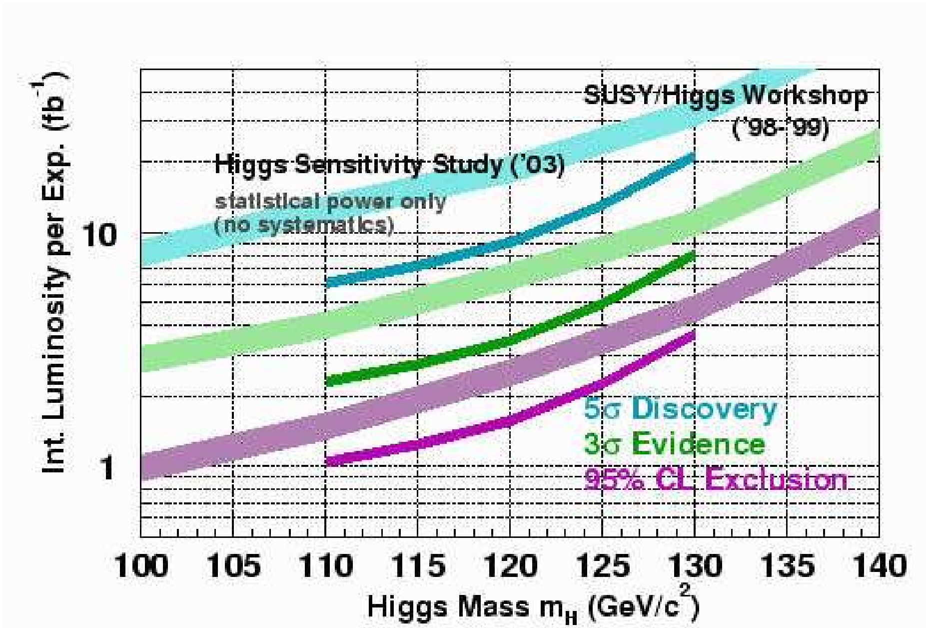

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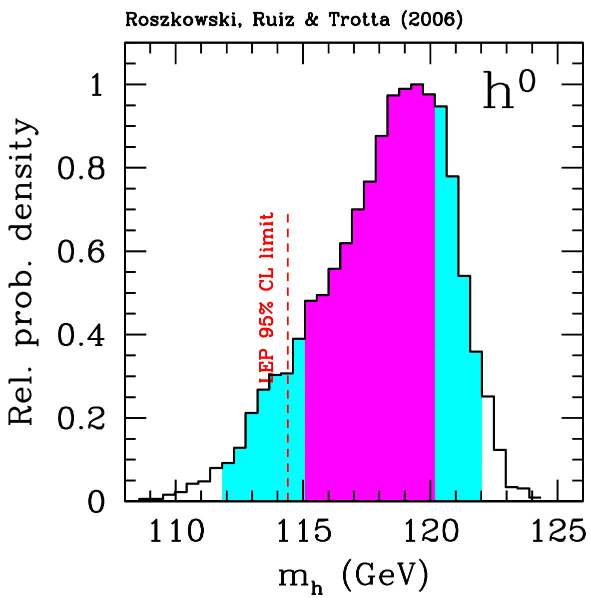

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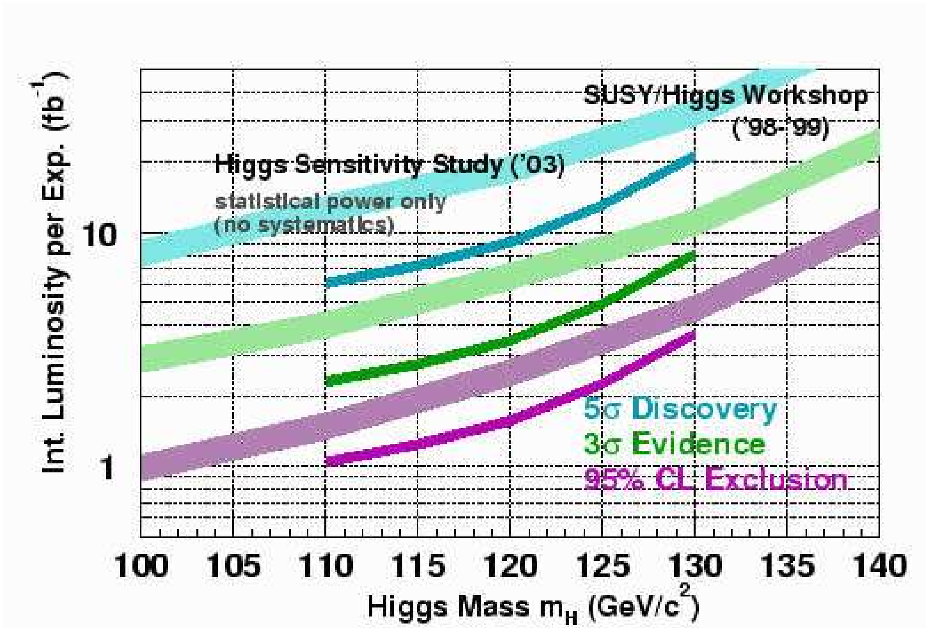

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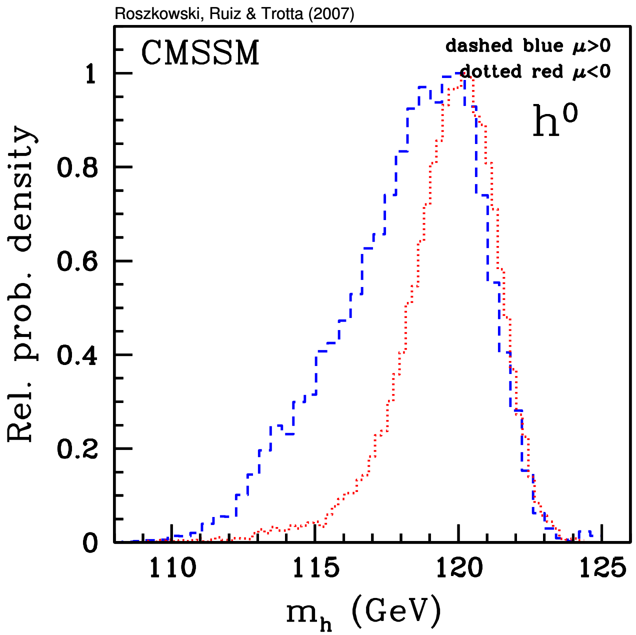
...or else...

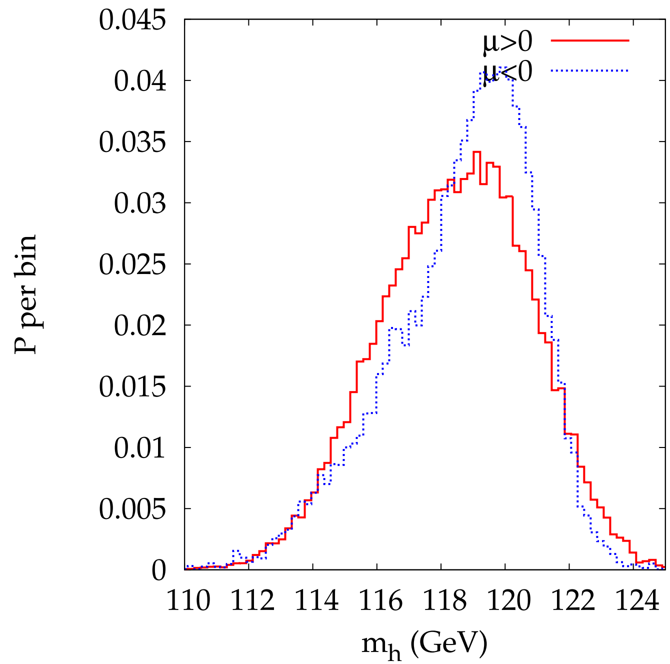
$\Rightarrow$ with $\sim 4\,\text{fb}^{-1}$/expt: 3σ evidence over entire 95% range of m_h

$\Rightarrow$ with $\sim 10-12\,\text{fb}^{-1}$/expt: 5σ discovery over entire 95% range of m_h

Tevatron: hope for up to $\sim 8\,\text{fb}^{-1}$/expt

Light Higgs Boson Mass





Allanach, Lester, Weber, hep-ph/0609295
(used previous value of $\mathbf{BR}(\bar{B} \to X_s\gamma)$)

$\Rightarrow$ Tevatron reach similar for both signs of μ

Light Higgs in the CMSSM

Bayesian analysis, relative probability density fn (pdf), flat priors, $\mu > 0$

computed with SoftSusy v2.08

$$\mathbf{115.2}\,\text{GeV} < m_h < \mathbf{120.4}\,\text{GeV} \quad \mathbf{(68\%)}$$

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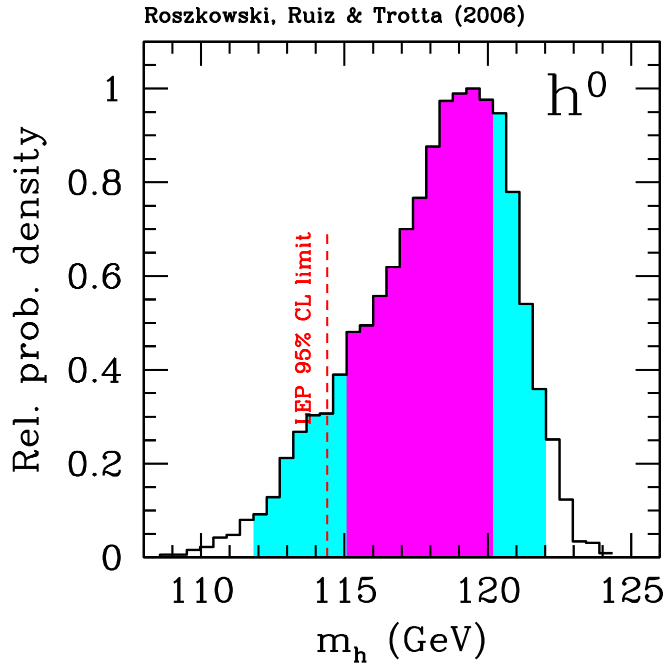

sharp drop-off on rhs from no solutions at large $m_{1/2}$ and/or cutoff at $m_0 < 4$ TeV

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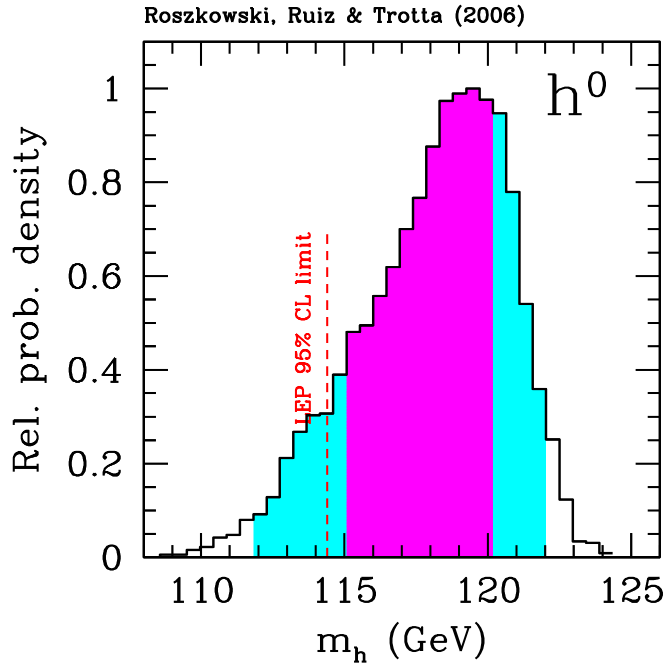

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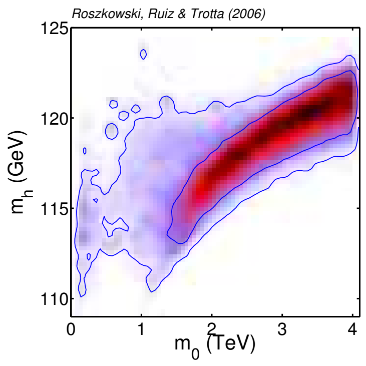

if $m_0 < 8$ TeV then $m_h \lesssim 125.6$ GeV (95% CL)

Probability maps of the CMSSM

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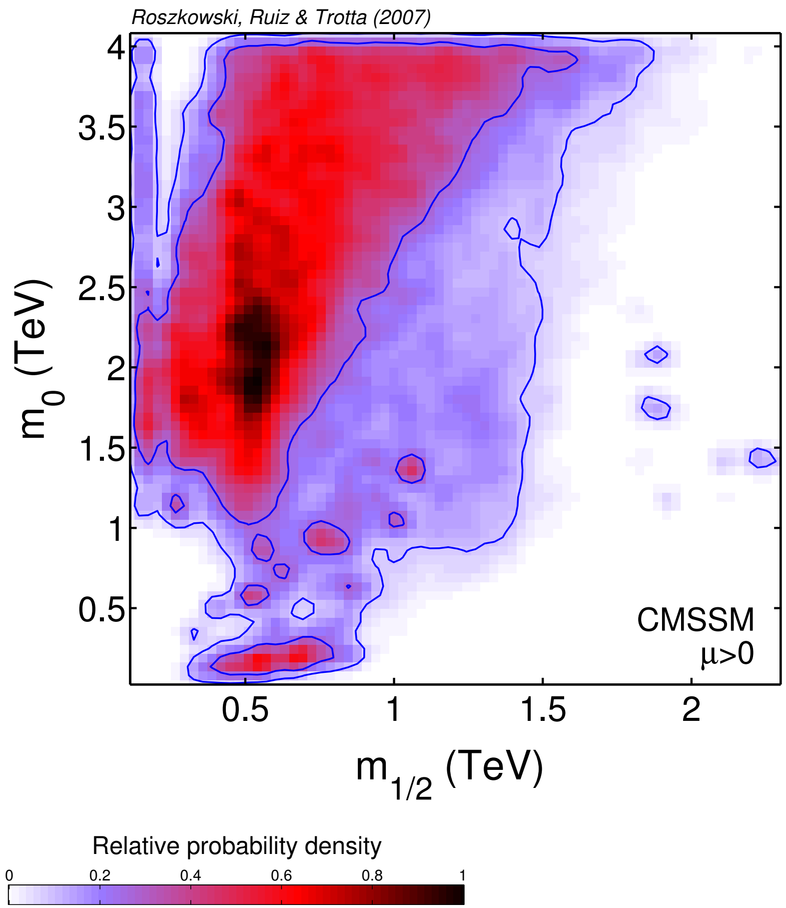
arXiv:0705.2012



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- 95% total prob. – outer contours
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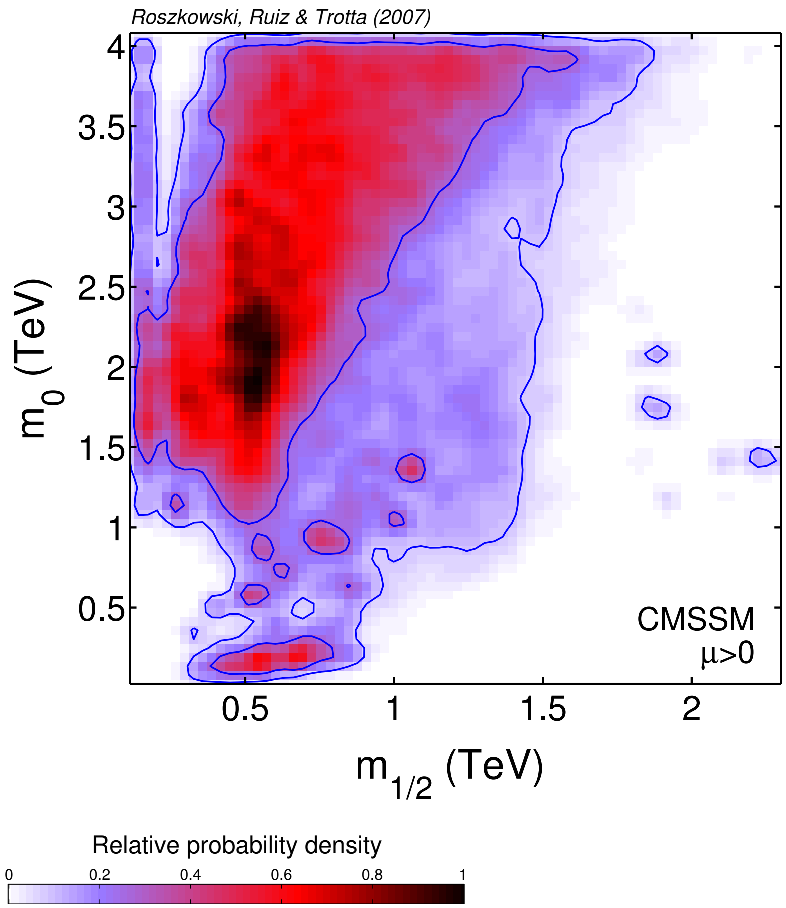

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similar study by Allanach+Lester(+Weber) (but no mean qof),
see also, Ellis et al (EHOW, χ^2 approach, no MCMC, they fix SM parameters!)

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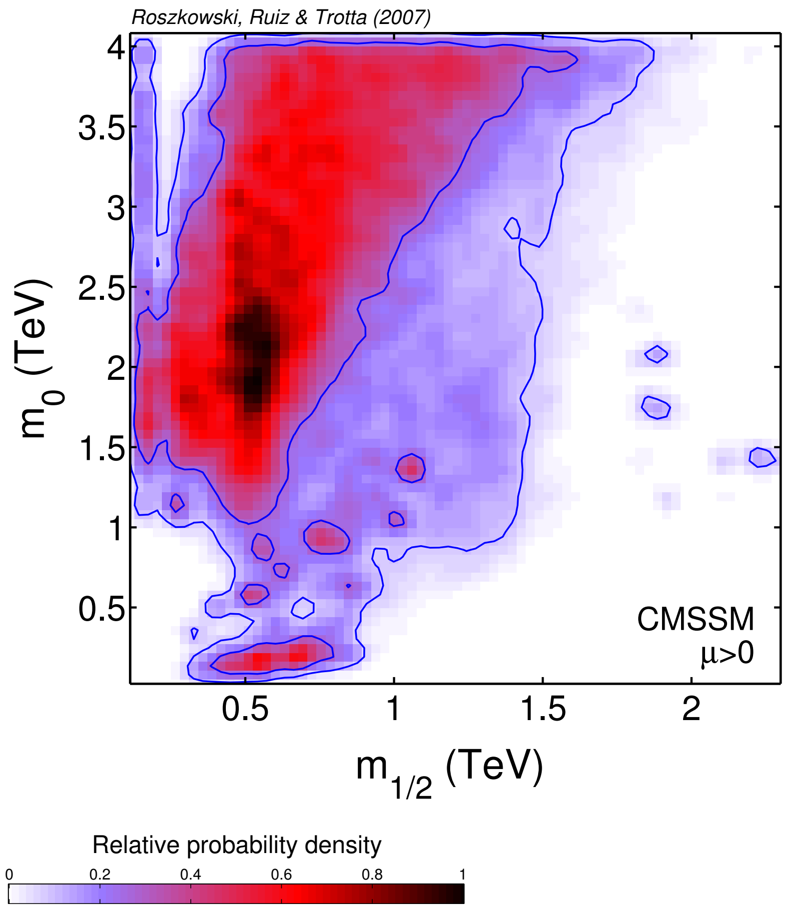

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unlike others (except for A+L), we vary also SM parameters

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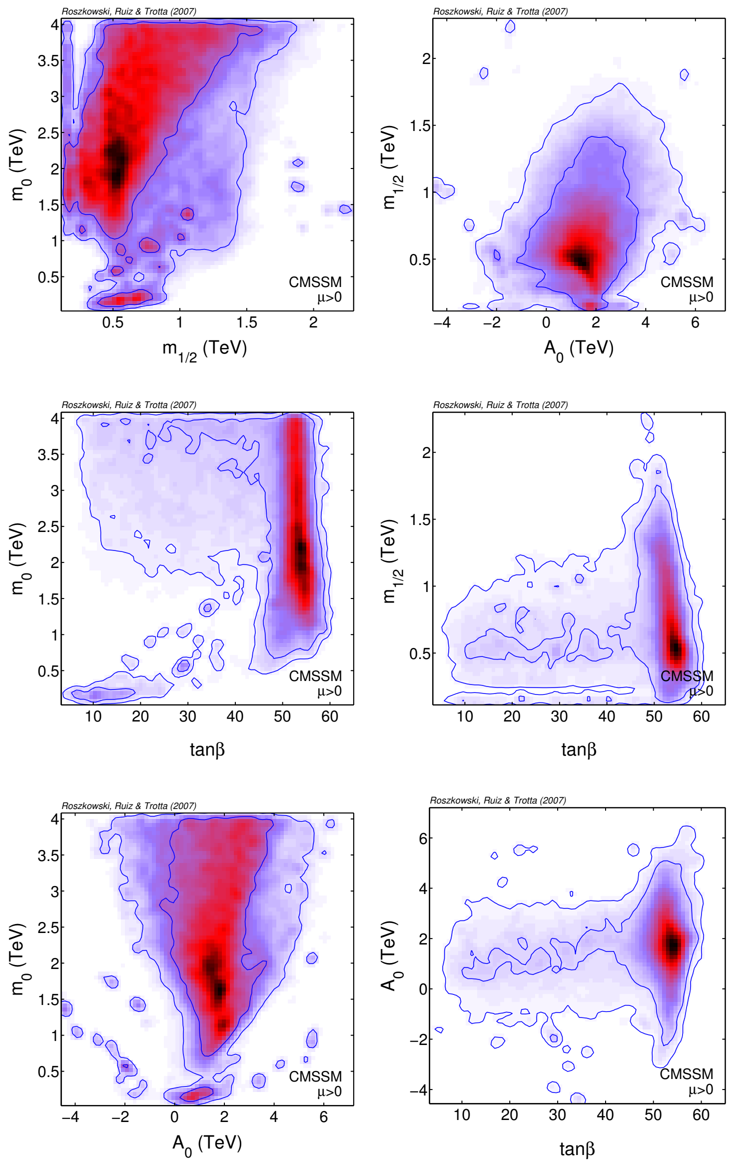

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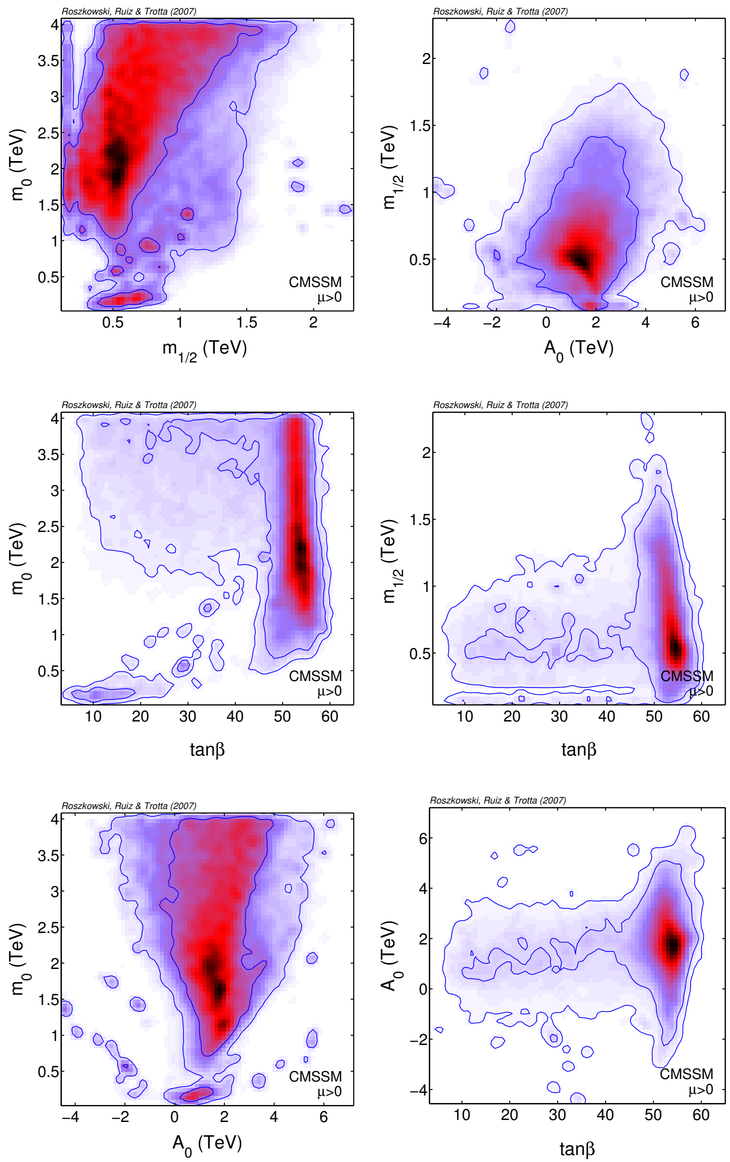

$\mu > 0$:

- large $m_0 \sim$ TeV favored, dep. on prior
- $m_{1/2}, A_0, \tan\beta$ within priors
- large $50 \lesssim \tan\beta \lesssim 60$ favored
- peak at $A_0 \simeq 1.5$ TeV (usually assumed: $A_0 = 0$)

Impact of $b \to s\gamma$

- recall

$$BR(B \to X_s\gamma) = B(W^-/t) + B(H^-/t) - \mathrm{sgn}(\mu)\, B(\chi^-/\widetilde{t})$$

compute SM: full NLO + NNLO of m_c (from M. Misiak); SUSY: dominant NLO $\propto \tan\beta, \log(M_S/m_W)$

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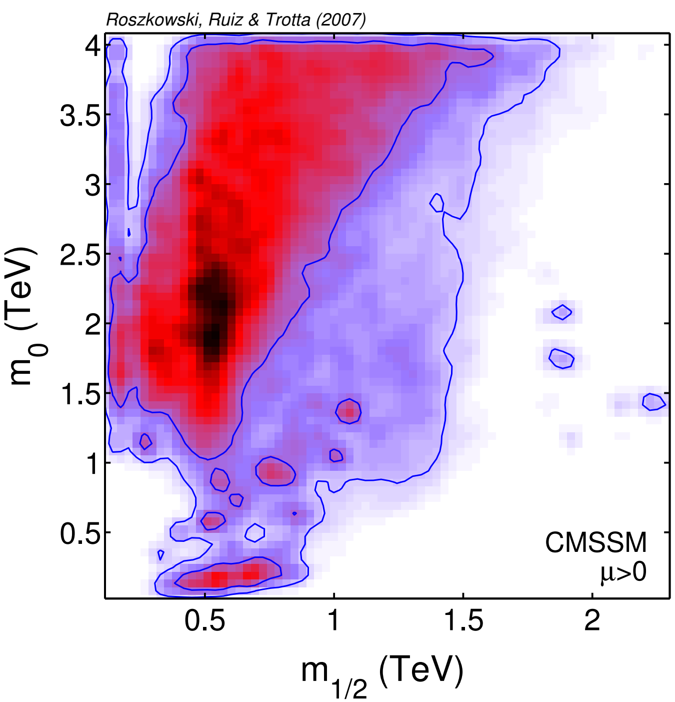
NEW: $BR(B \to X_s\gamma) \times 10^4$

EXPT: 3.55 ± 0.26, TH: 3.11 ± 0.21

(with our inputs), (May 07)



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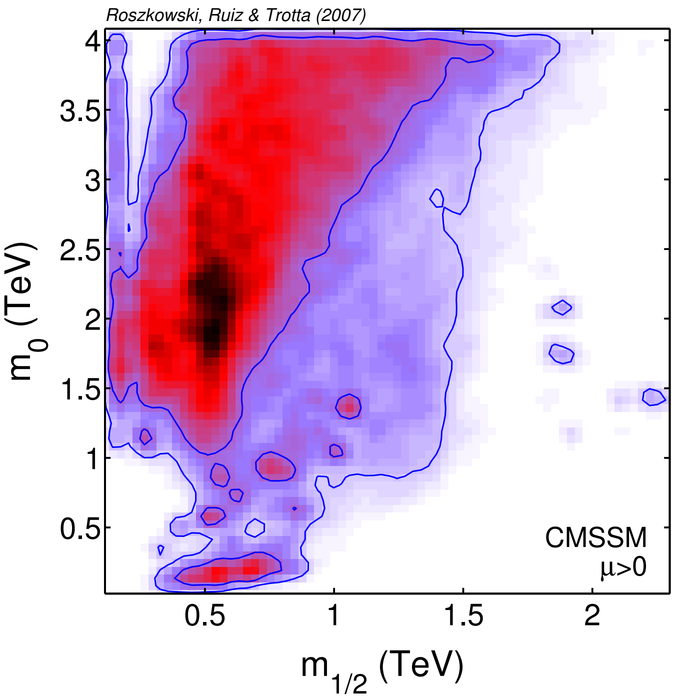
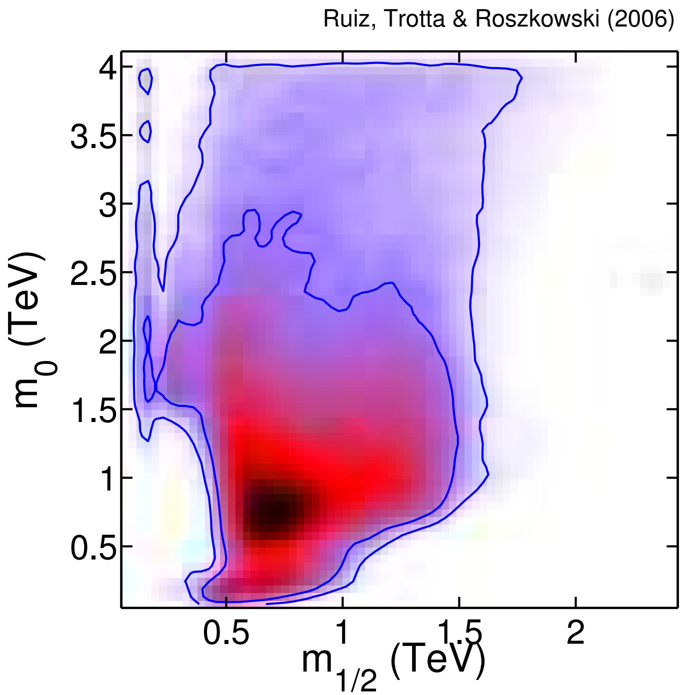
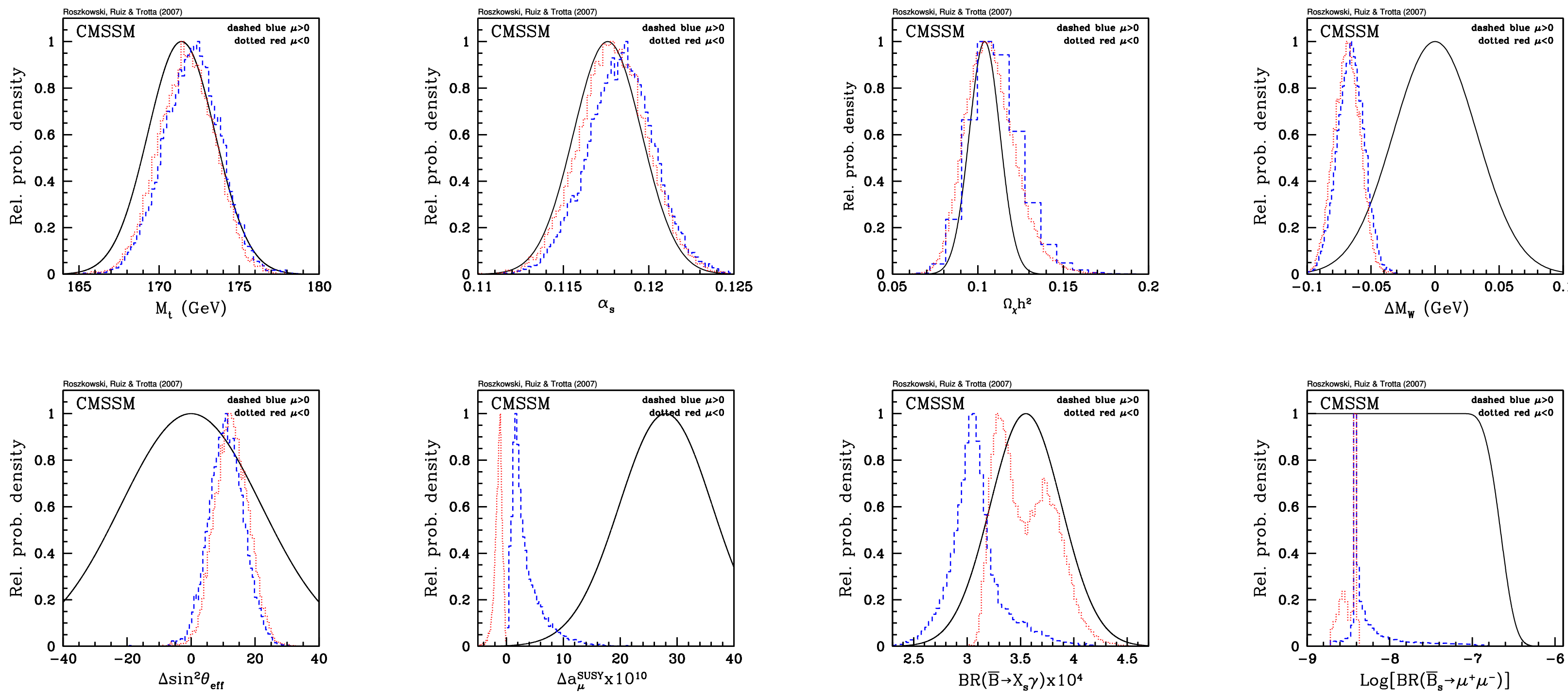
OLD: $BR(B \to X_s\gamma) \times 10^4$
EXPT: 3.39 ± 0.68, TH: 3.70 ± 0.30
(Feb 2006)





$\Rightarrow$ big shift towards large m_0, FP region!

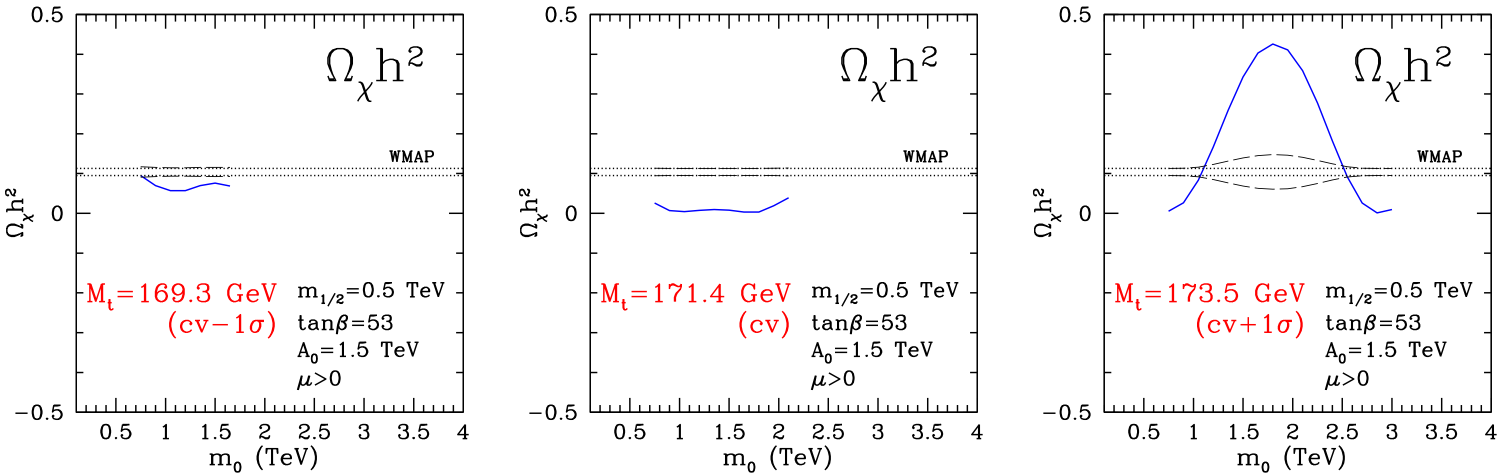
Fits of Observables



- good fits: M_t, α_s, $\Omega_\chi h^2$, $\mathrm{BR}(\bar{B} \to X_s\gamma)$ (for $\mu < 0$!)
- not so good: M_W, $\sin^2\theta_{\mathrm{eff}}$, $\mathrm{BR}(\bar{B} \to X_s\gamma)$ (for $\mu > 0$!)
- bad: $\Delta a_\mu^{\mathrm{SUSY}}$ (for both signs of μ!)

FP: Dependence on SM parameters

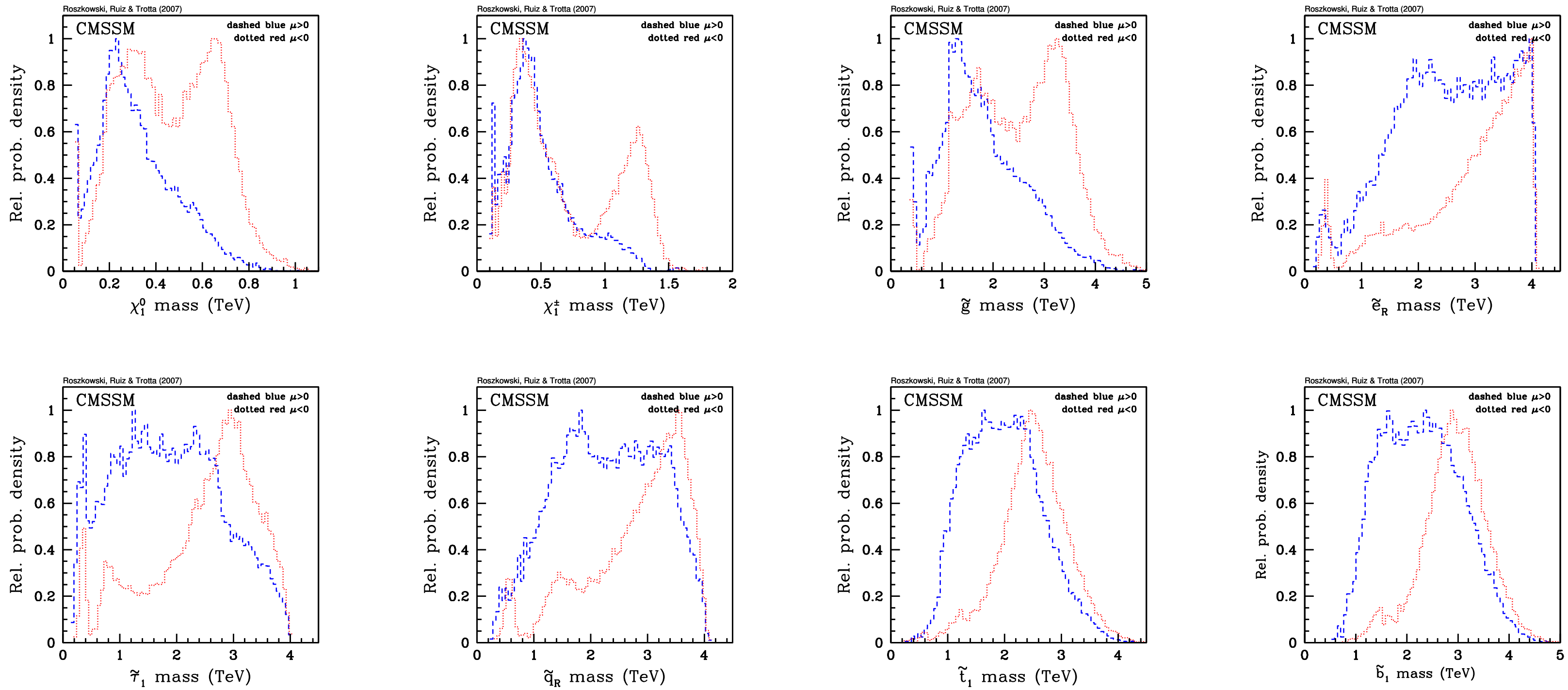
Illustration:



vary M_t within 1σ around its central value:

- range of m_0 increases by a factor of two!
- m_0 consistent with $\Omega_{\rm CDM}h^2$ at one/zero/two values
- in a probabilistic sense the whole range of m_0 is allowed with varying probability

Sparticle Mass Ranges



- large ranges beyond LHC reach (esp. for spin-0 particles)
- best prospects for the gluino? (LHC: $m_{\tilde{g}} \lesssim \mathbf{2.7}$ TeV?)

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another statistical measure

Regions of high posterior probability do not always give the best fits

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$$m = (\theta, \psi)$$

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need to do both to see if that is the case

Posterior pdf vs. mean qof: $\mu > 0$

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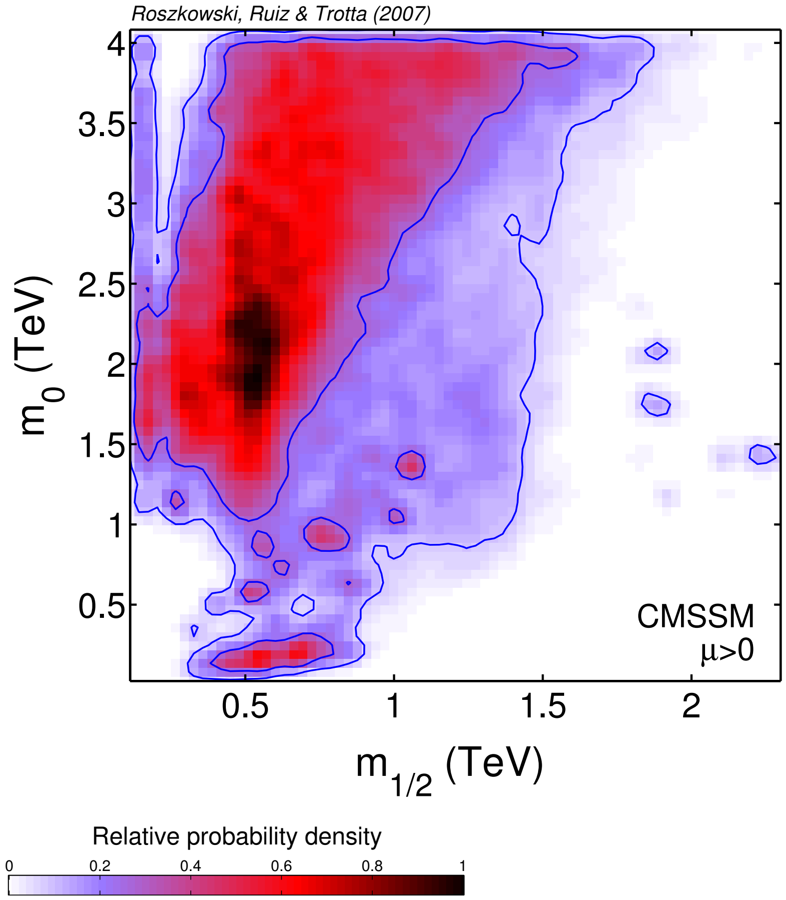
posterior pdf



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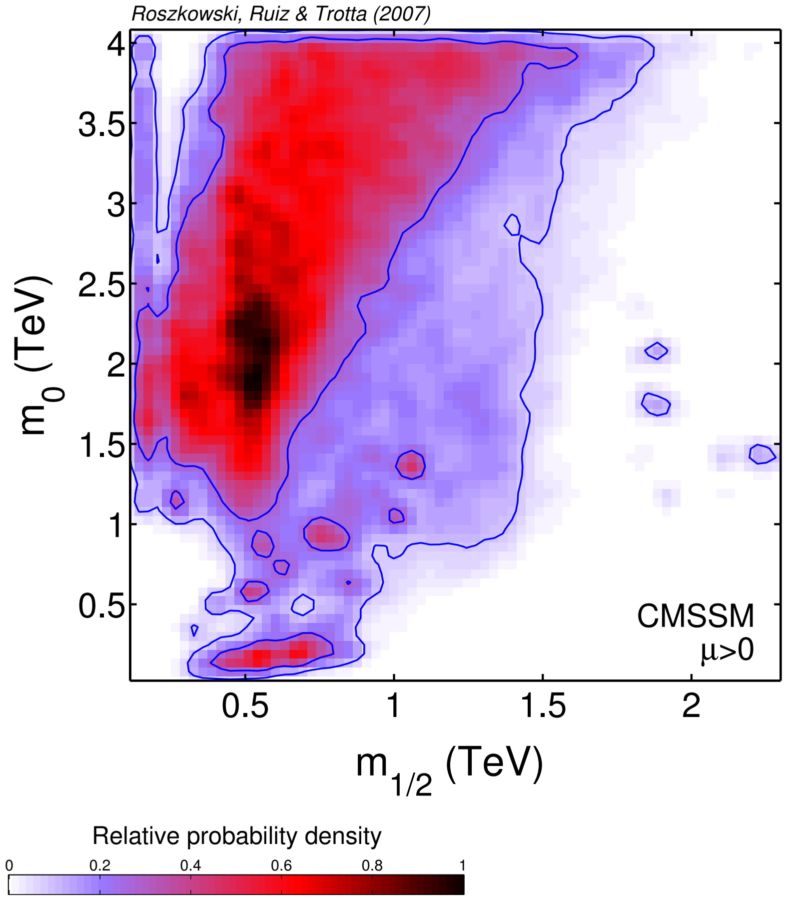

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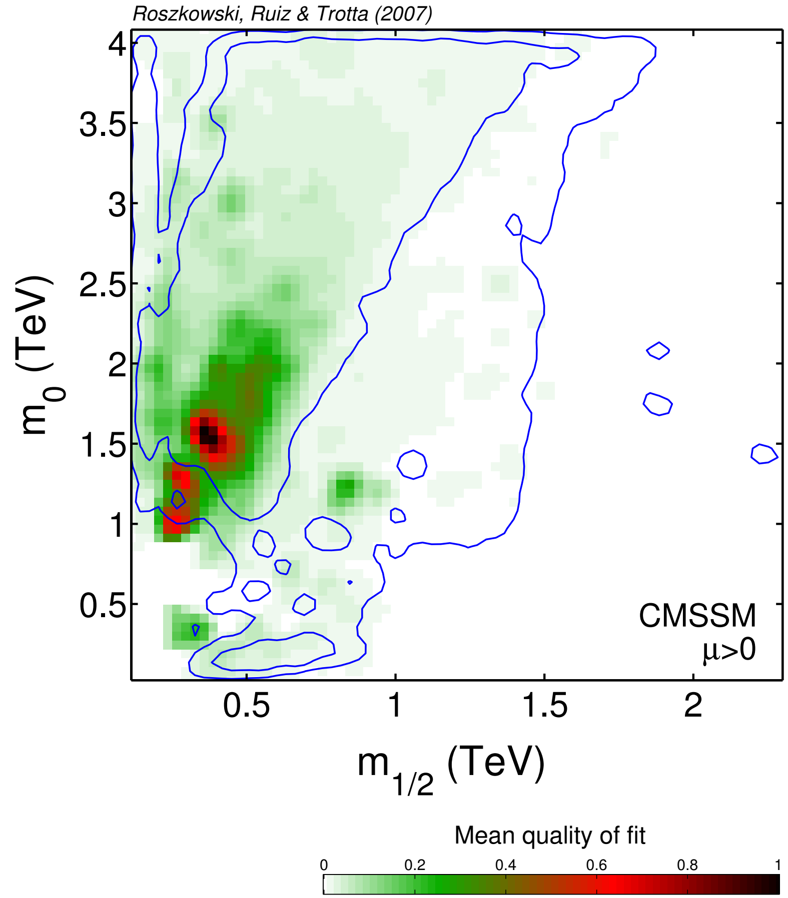

$\Rightarrow$ data not yet constraining enough

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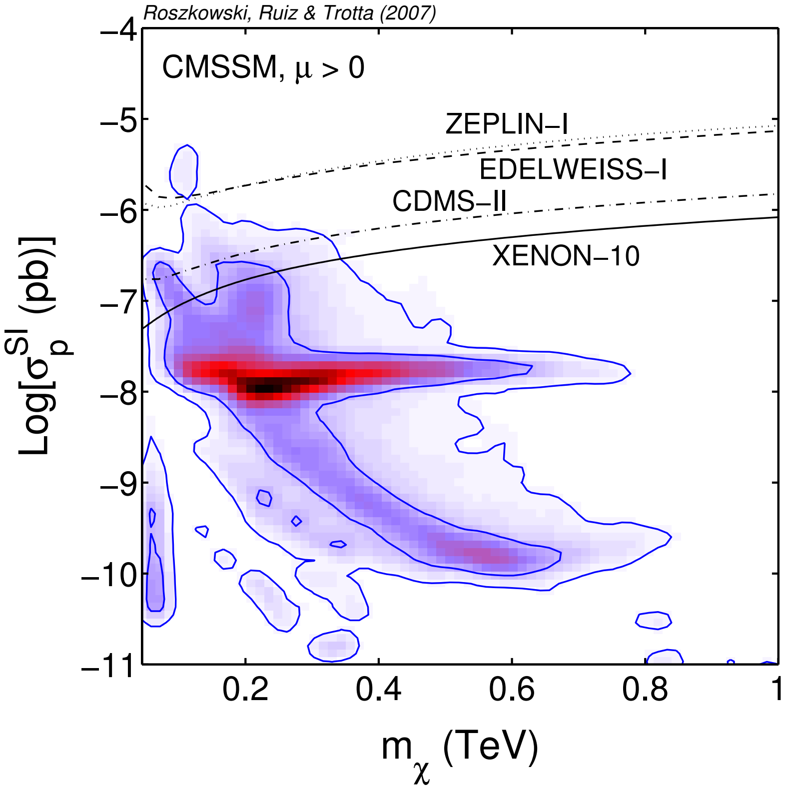

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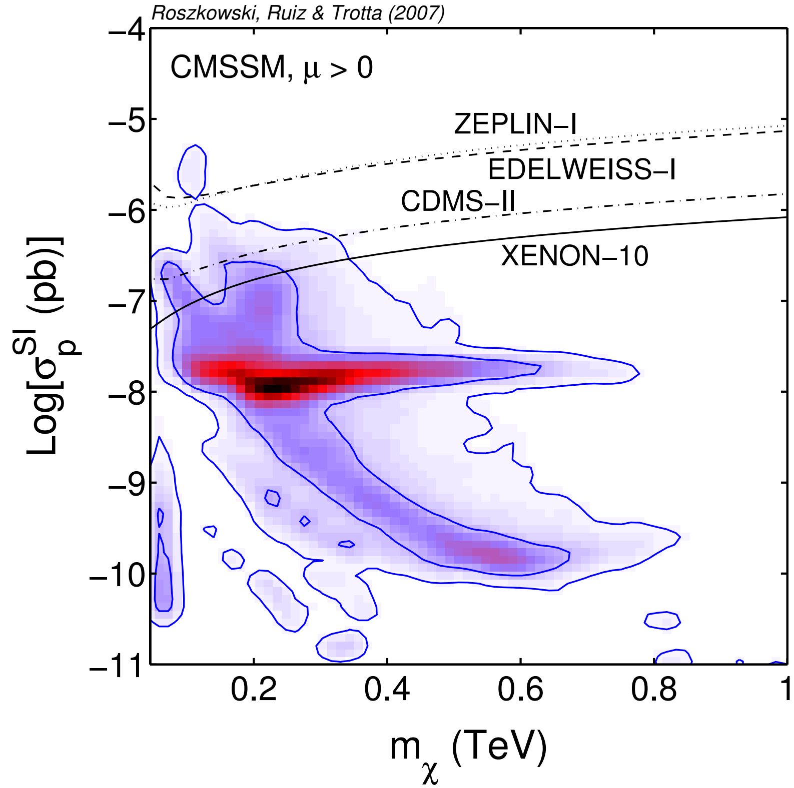
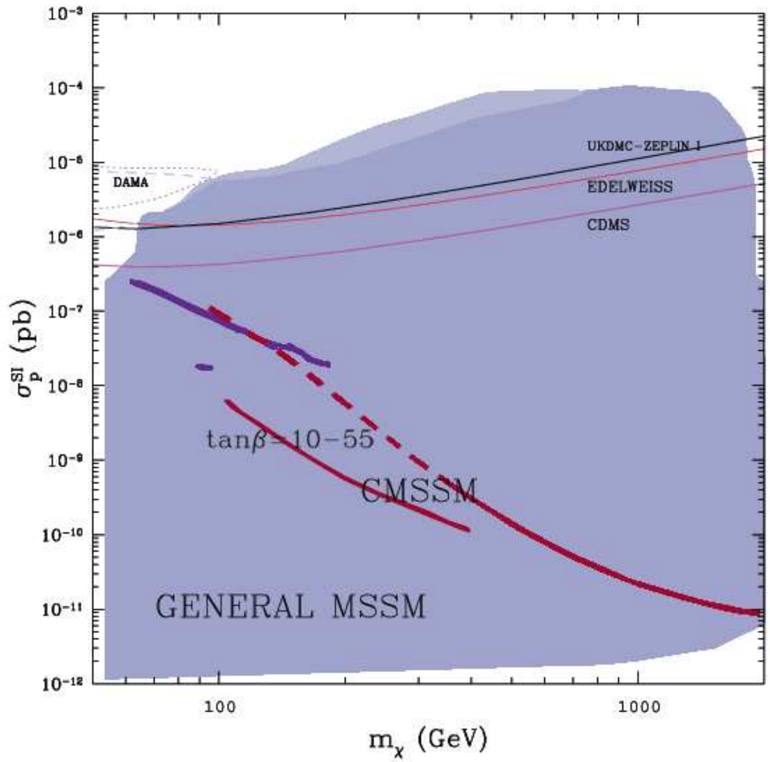


compare: fixed grid scan



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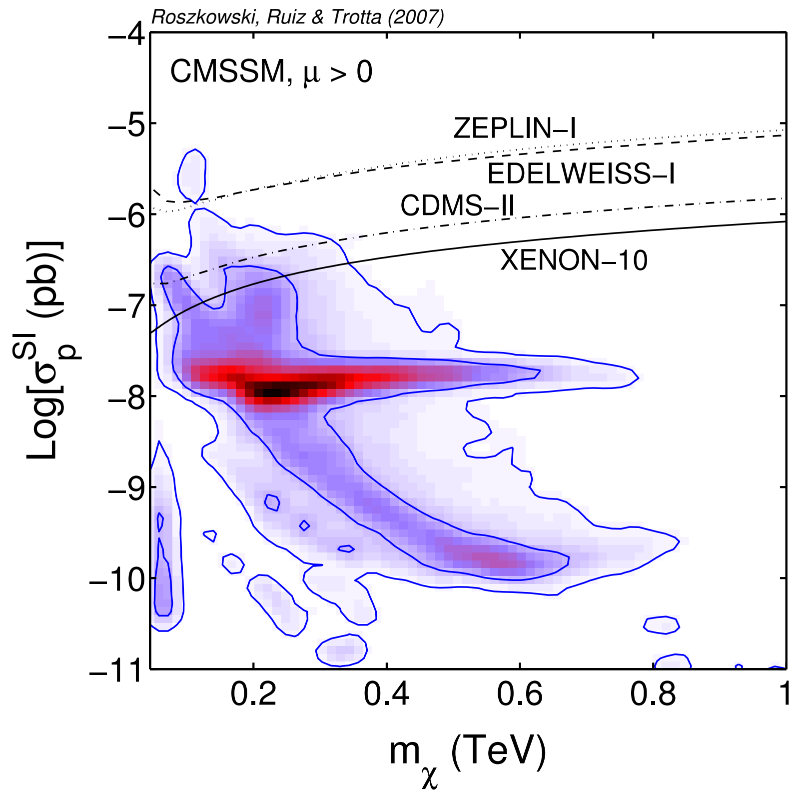

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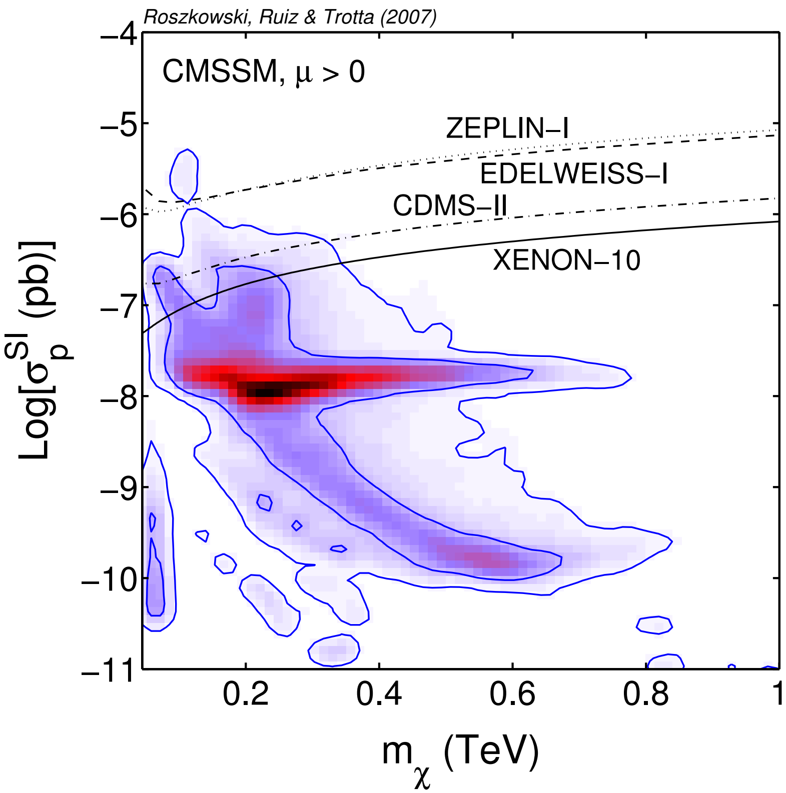

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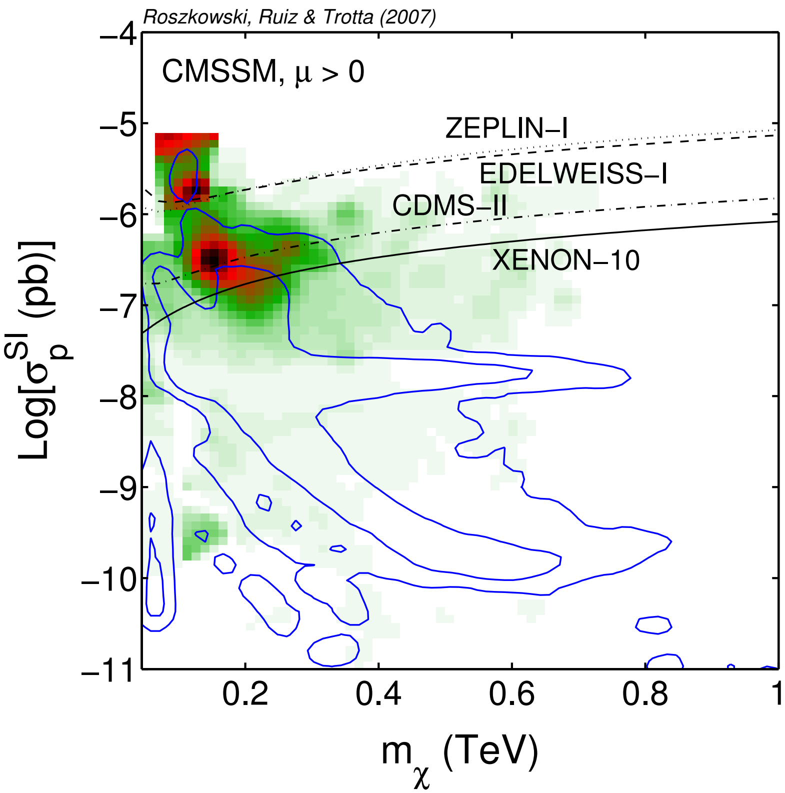

$\Rightarrow$ best-fit regions excluded by DM searches

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- DM direct detection: expt now probing favored 68% CL region

  largest $\sigma_p^{SI} \simeq 10^{-8}$ pb for large m_0, beyond LHC reach!
- LHC: CMSSM spartners often beyond reach? (except for $\widetilde{g}$)

Backup

The Likelihood

incorporates information about the observational data

- the mapping $\xi(m)$ comes with uncertainties
- experimental uncertainty σ_i
- theoretical uncertainty τ_i
- introduce "exact" mapping $\hat{\xi}(\theta, \chi)$
- the likelihood:

$$p(d|\xi) = \int p(d|\hat{\xi}) p(\hat{\xi}|\xi) d^m \hat{\xi}$$

where

$$p(\hat{\xi}|\xi) = \frac{1}{(2\pi)^{m/2}|C|^{1/2}} \exp\left(-\frac{1}{2}(\xi - \hat{\xi}) C^{-1} (\xi - \hat{\xi})^T\right)$$

C: $m \times m$ covariance matrix

if uncorrelated: $C = \mathrm{diag}\left(\tau_1^2, \ldots, \tau_\mathrm{m}^2\right)$

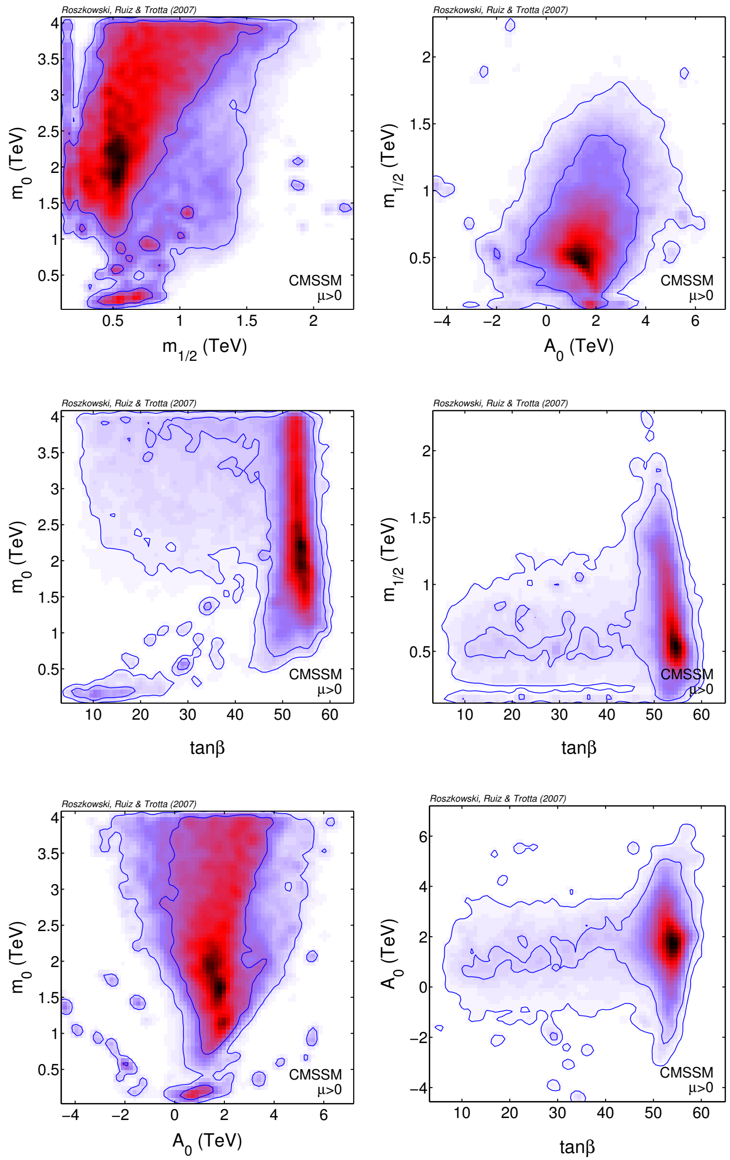
$$p(d|\hat{\xi}) = \frac{1}{(2\pi)^{m/2}|D|^{1/2}} \exp\left(-\frac{1}{2}(d - \hat{\xi}) D^{-1} (d - \hat{\xi})^T\right)$$

if uncorrelated: $D = \mathrm{diag}\left(\sigma_1^2, \ldots, \sigma_\mathrm{m}^2\right)$

- total error for each observable: $s_i = \sqrt{\sigma_i^2 + \tau_i^2}$

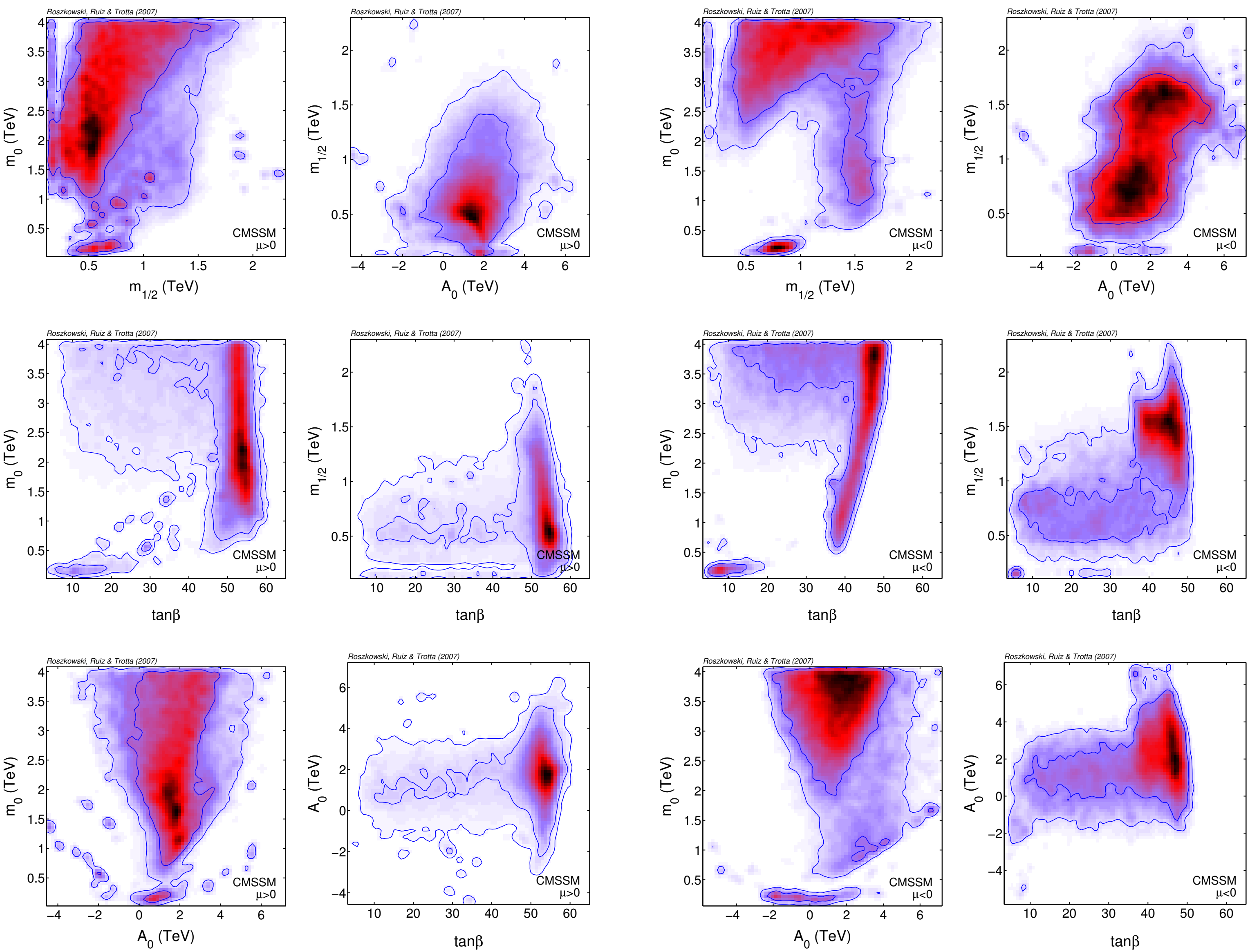
Probability maps of the CMSSM

$\mu > 0$:



Probability maps of the CMSSM

$\mu > 0$: $\mu < 0$:



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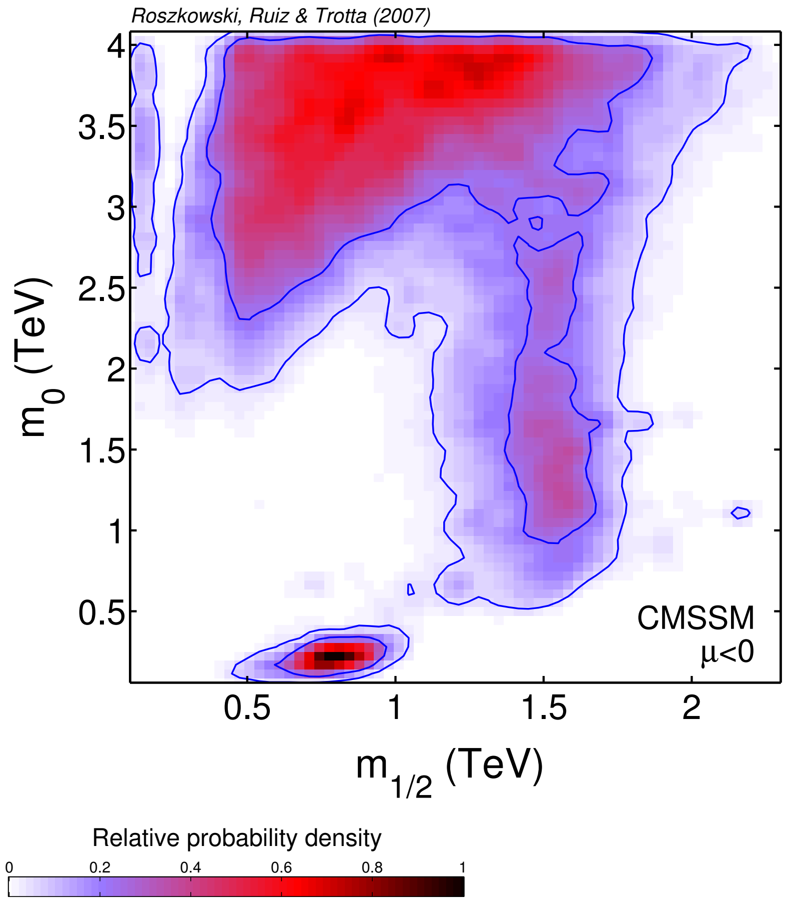
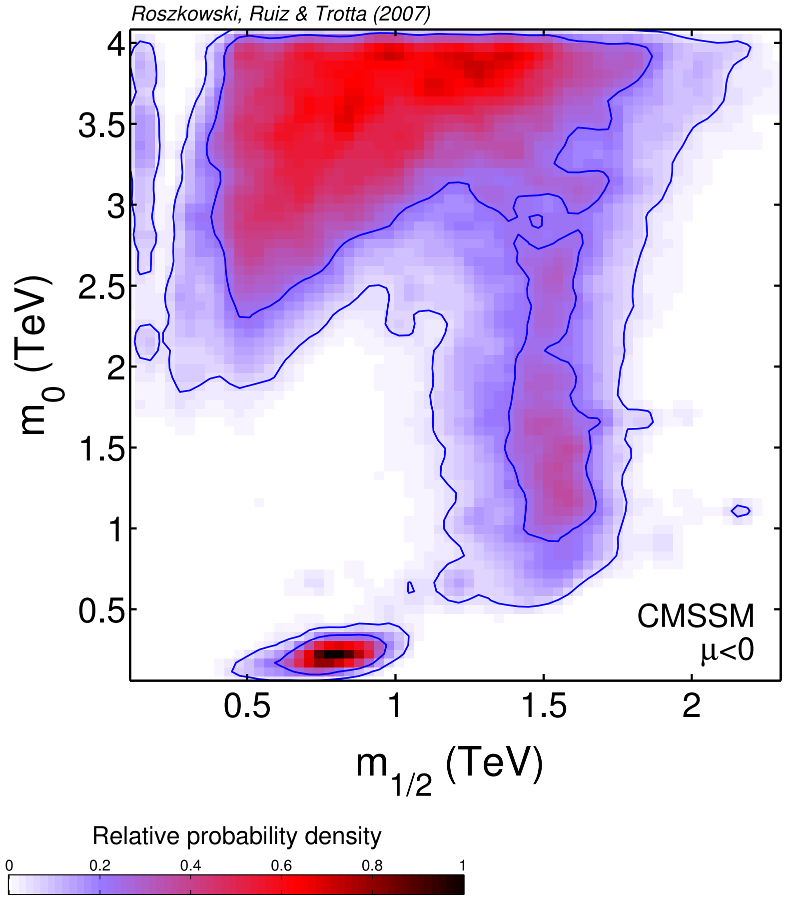
Posterior pdf vs. mean qof: $\mu < 0$

posterior pdf

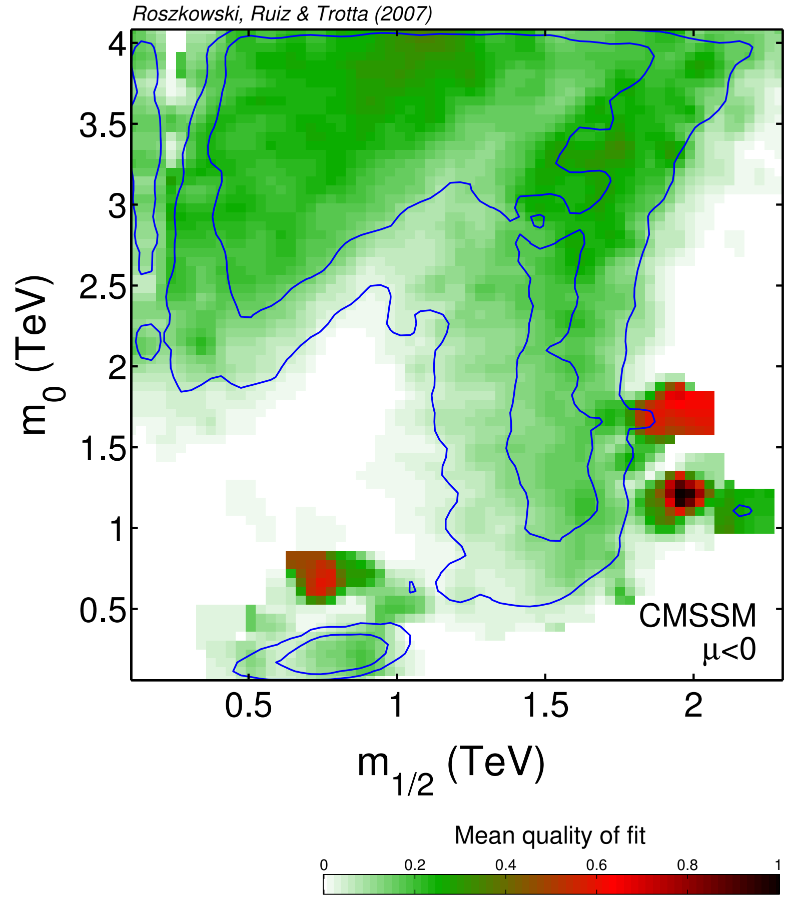


Posterior pdf vs. mean qof: $\mu < 0$

posterior pdf



mean qof



$\mu < 0$ generally poorer fit to the data