Evidence for critical isoscalar fluctuations at the SPS

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Observational critical QCD

- The critical system in transverse space consists of σ-clusters with self-similar geometry characterized by the fractal dimension:

$$d_F = \frac{2(\delta - 1)}{\delta + 1}$$

leading to a power law:

$$< \sigma^2 > \sim |\vec{x}|^{-\frac{4}{\delta+1}}$$

for the σ-field fluctuations, within each cluster (δ : isotherm critical exponent)

- In transverse momentum space the σ-fluctuations obey a power law:

$$< \sigma^2 > \sim |\vec{p}_\perp|^{-\frac{2(\delta-1)}{\delta+1}}$$

leading to observable intermittent behaviour of factorial moments: $F_2(M) \sim (M^2)^{\frac{\delta-1}{\delta+1}}$ (M^2 = number of $2D$ cells)

- The density fluctuations of pion pairs ($\pi^+\pi^-$) with invariant mass close to two-pion threshold ($2m_\pi$) simulate to a good approximation the sigma-field fluctuations, $(\delta\sigma)^2 \approx< \sigma^2 >$, at the critical point, under the assumption that the sigma mass reaches the two-pion threshold ($m_\sigma \gtrsim 2m_\pi$) at a freeze-out temperature close to the critical value

- The QCD critical point belongs to the universality class of the $3D$ Ising system in which $\delta \approx 5$. In this universality class the above critical clusters interact weakly.

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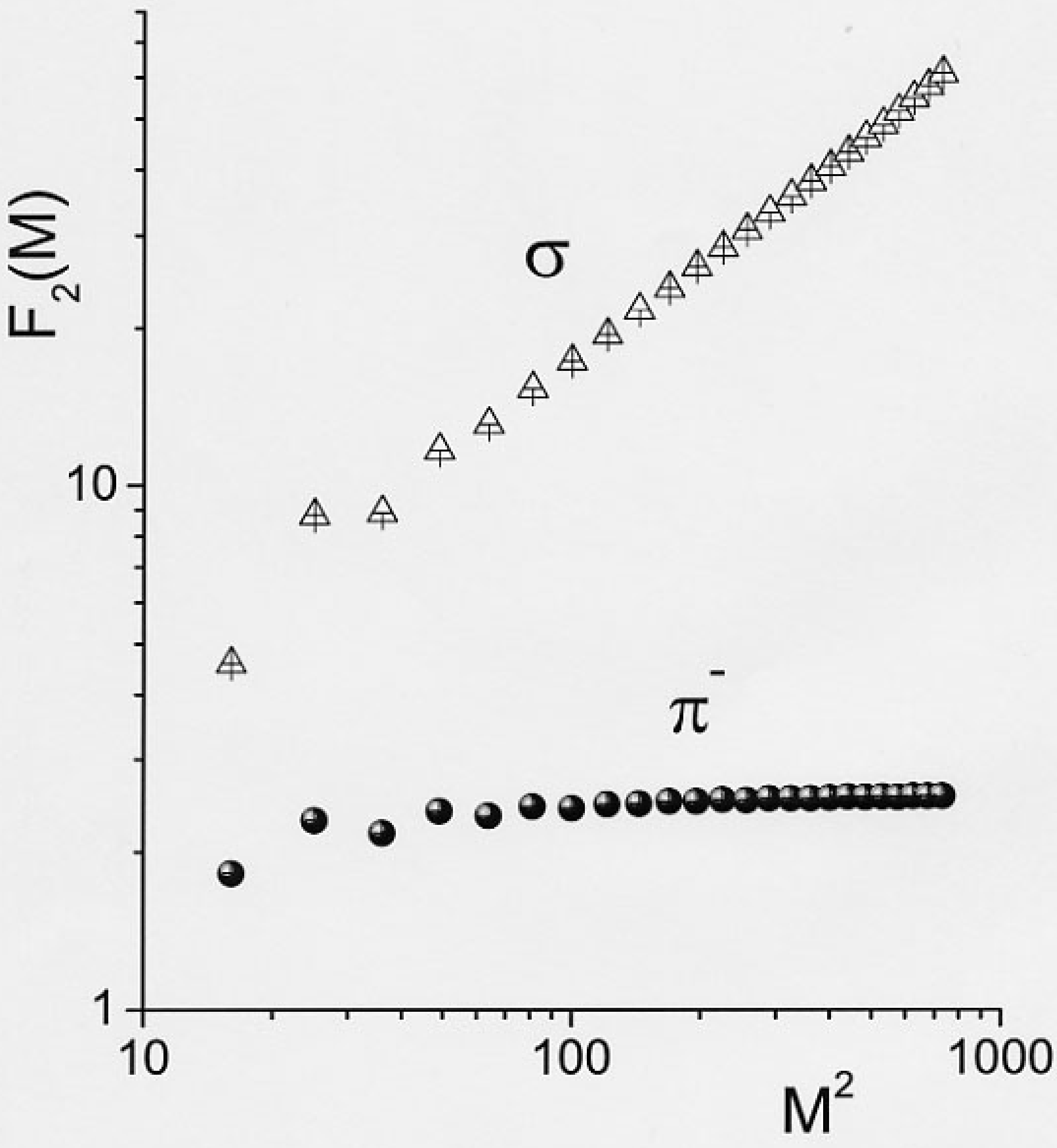
Generating and analyzing critical events

- The picture of a weakly interacting gas of self-similar clusters in momentum space is simulated in a **critical event generator** (Critical Monte-Carlo: CMC) containing only isoscalar particles (sigmas)
- Check if the critical fluctuations in the sigma sector are properly described by the CMC events (use tools like factorial moments in transverse momentum space)
- Produce critical events incorporating pions by letting the sigmas in each critical event decay into pions. Use pure kinematics to describe this process. Due to kinematic distortion the critical fluctuations are potentially suppressed in the pionic sector

- Use the produced charged pions in these events to reconstruct the sigma sector
- As a self-consistency test of the sigma reconstruction algorithm check if the critical fluctuations are properly restored in the reconstructed isoscalar sector
- Apply the reconstruction algorithm to systems not involving critical fluctuations for a comparative study
- Use the reconstruction algorithm to explore critical fluctuations in SPS data



F_2(M)
σ
π^-
10
1
10
100
1000
M^2

The sigma reconstruction algorithm

In each event involving charged pions:

- Look for (π^+, π^-) pairs fulfiling the criterion:

$$4m_\pi^2 \leq (p_{\pi^+} + p_{\pi^-})^2 \leq (2m_\pi + \epsilon)^2$$

  with $\frac{\epsilon}{2m_\pi} \ll 1$

- For each such pair form the momentum of the corresponding sigma particle:

$$\vec{p}_\sigma = \vec{p}_{\pi^+} + \vec{p}_{\pi^-}$$

  Take into account **all** possible combinations!

- Record only those events which have sigma multiplicity $n_o > 2$

Calculating the correlator $G(M, \epsilon)$

Use the events consisting from reconstructed sigmas (r.s.) for further analysis

$$\Downarrow$$

reveal the correlated fluctuating part of the isoscalar sector (if any)

The relevant steps are:

- For a given data set form a corresponding data set consisting of mixed events (same number of events)
- Apply the sigma reconstruction algorithm to both the original data set as well as the set of mixed events using a fixed value for ϵ. Obtain this way two data sets consisting of events incorporating isoscalar particles. Observe that in general:

$$\underset{\text{(original data)}}{\sharp \text{ of events with r.s.}} \neq \underset{\text{(mixed events)}}{\sharp \text{ events with r.s.}}$$

- Use these two sets to calculate the correlator:

$$G(M,\epsilon) = F_2(M,\epsilon) - W(\epsilon)\left(\frac{< n_\sigma^{(m)} >}{< n_\sigma >}\right)^2 F_2^{(m)}(M,\epsilon)$$

index (m): mixed events

F_2: second scaled factorial moment in transverse momentum space

$< n_\sigma >$: mean multiplicity of sigmas/event

- The function $W(\epsilon) = \frac{N_{ev}^{(m)}}{N_{ev}}$ is a measure for the relative weight of the "fake" sigmas!

N_{ev}=$\sharp$ of events with r.s. using the original data set

$N_{ev}^{(m)}$=$\sharp$ of events with r.s. using the data set with mixed events

Revealing the correlated critical fluctuations

- Calculate $W(\epsilon)$ for several values of ϵ approaching $\epsilon \to 0$.

(1) If $W(\epsilon) \approx 1$ "fake" sigmas dominate

(2) If $W(\epsilon) \to 0$ real critical isoscalars are present

- Calculate the correlator $G(M, \epsilon)$ for these values of ϵ.

(3) Large ($\gg 1$) values of the correlator

increasing as the number of bins M^2 increases

$$\Downarrow$$

Strong correlations are present in the isoscalar sector

- Check for a power-law behaviour: $G(M, \epsilon) \sim (M^2)^{\phi_2(\epsilon)}$

in a range of M-values $\Longrightarrow$ (4) self-similar structures in $p_\perp$

If (2),(3),(4) are simultaneously fulfiled the corresponding system possesses critical isoscalar sector. Calculate $\phi_2(0)$ using linear extrapolation to check universality class

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Critical (CMC) vs. conventional events (HIJING)

Apply the sigma reconstruction algorithm and the corresponding analysis to the following data sets:

- 100000 CMC events using the following values of the input parameters:

  $\Delta = 6$ (size in rapidity)

  $R_{\perp} = 15\ fm$ (size in transverse space)

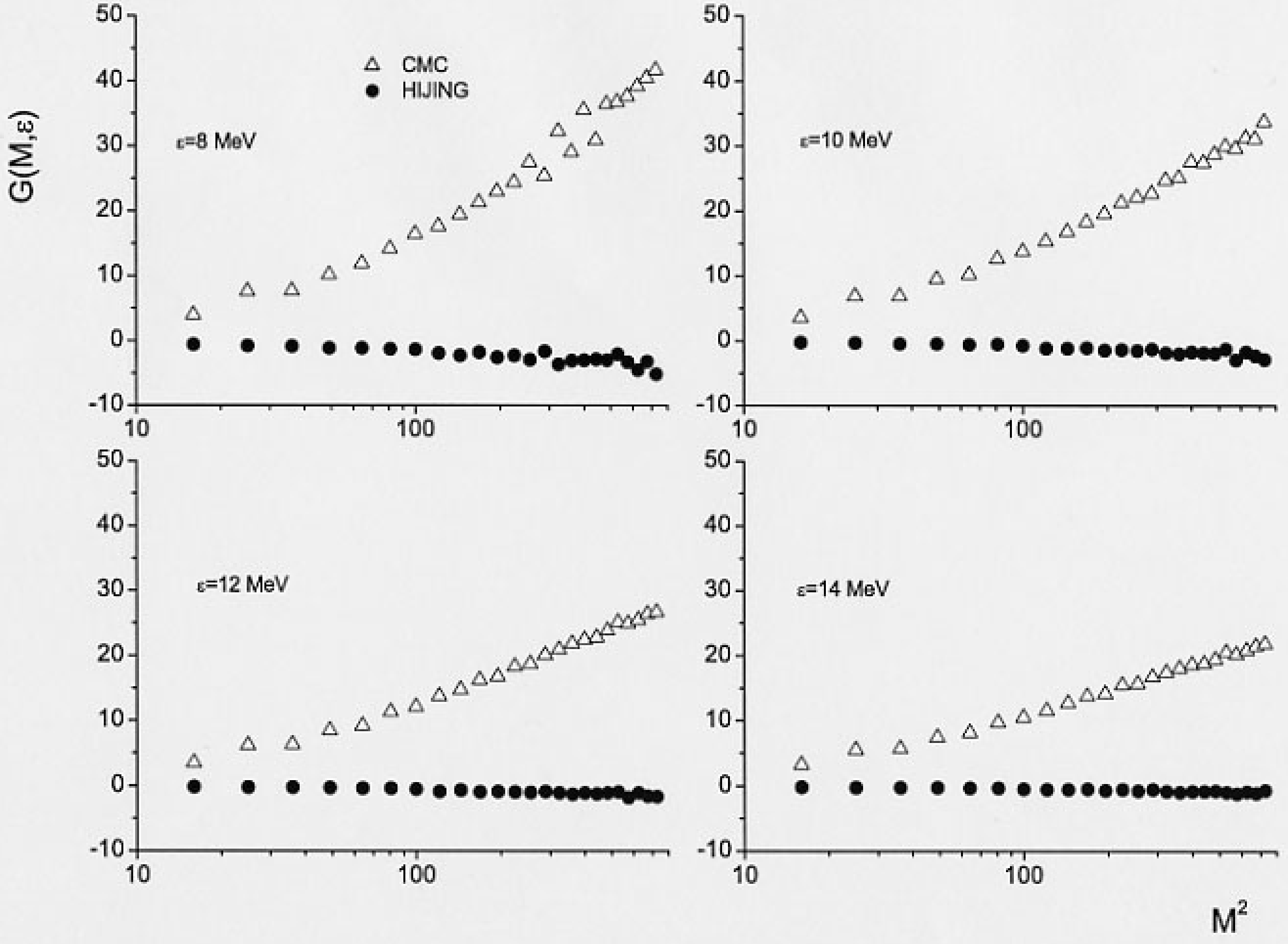
  $T_c = 140 - 170\ MeV$ (critical temperature)

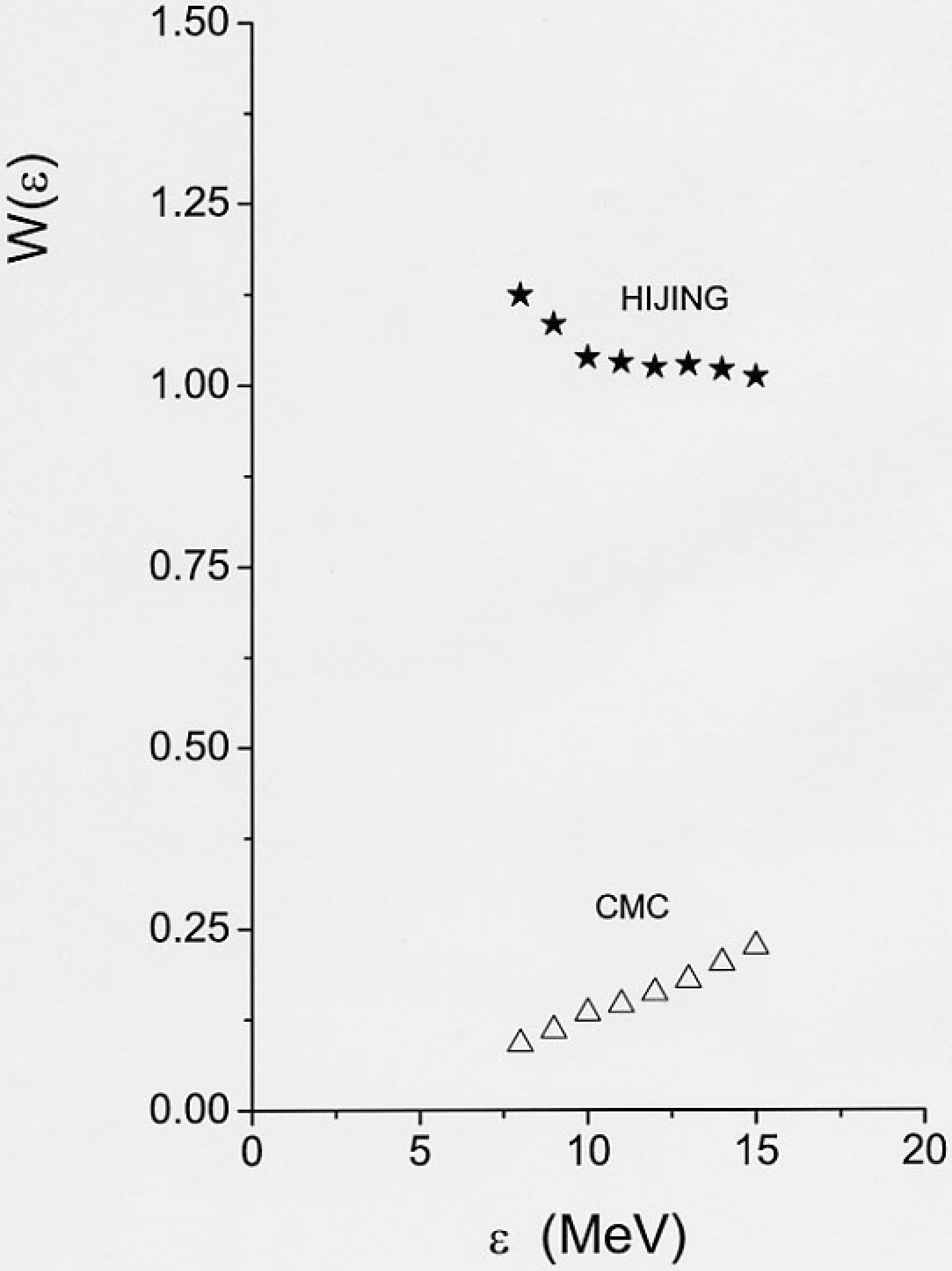
  $\tau_c = 10\ fm$ (characteristic time scale for the formation of the condensate)

  This choice corresponds to a rather small system (average total charged pion multiplicity ≈ 40)

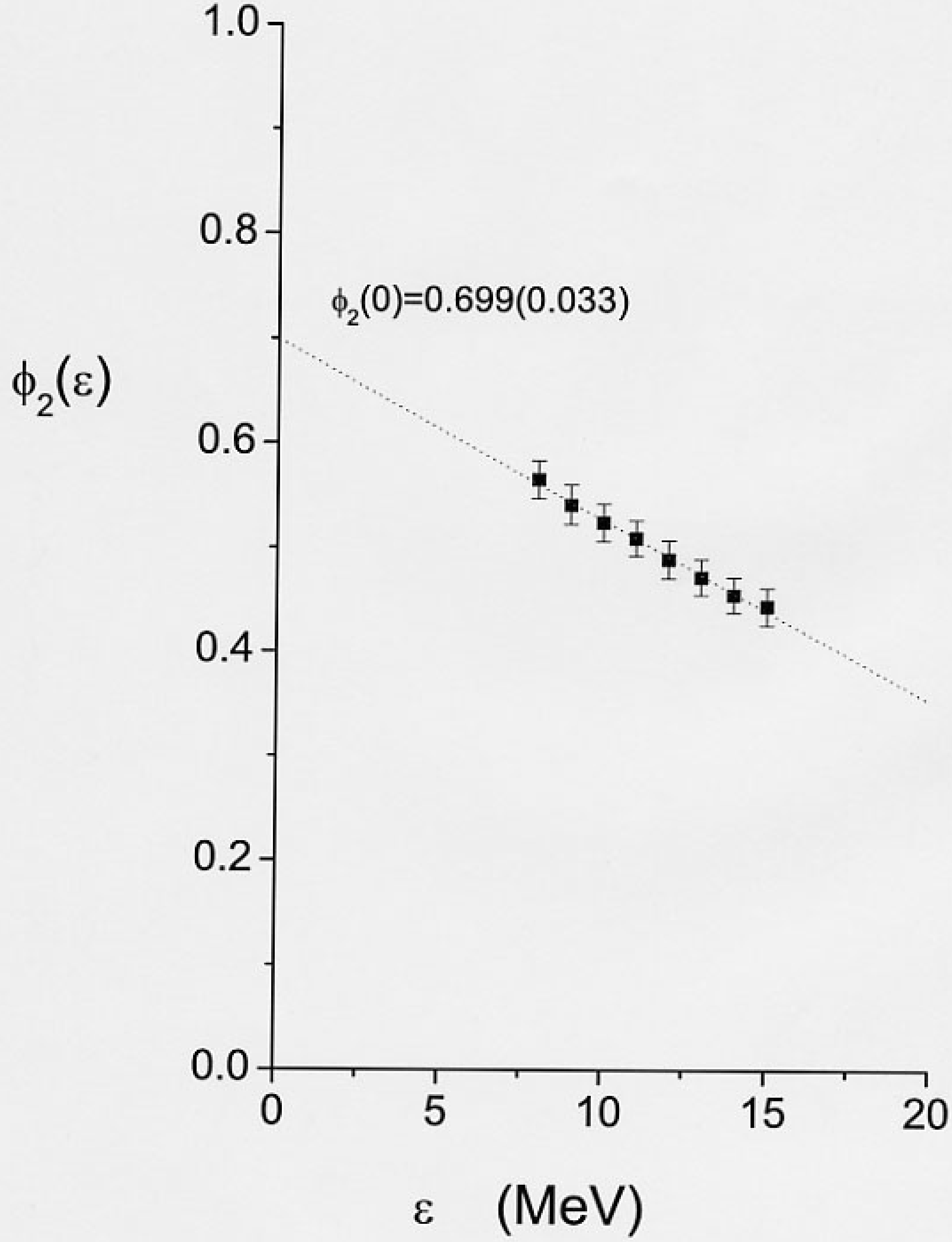
- 33000 HIJING events simulating the $C{+}C$ system at 158 GeV/n.

  This system is characterized by a similar value for the average total charged pion multiplicity

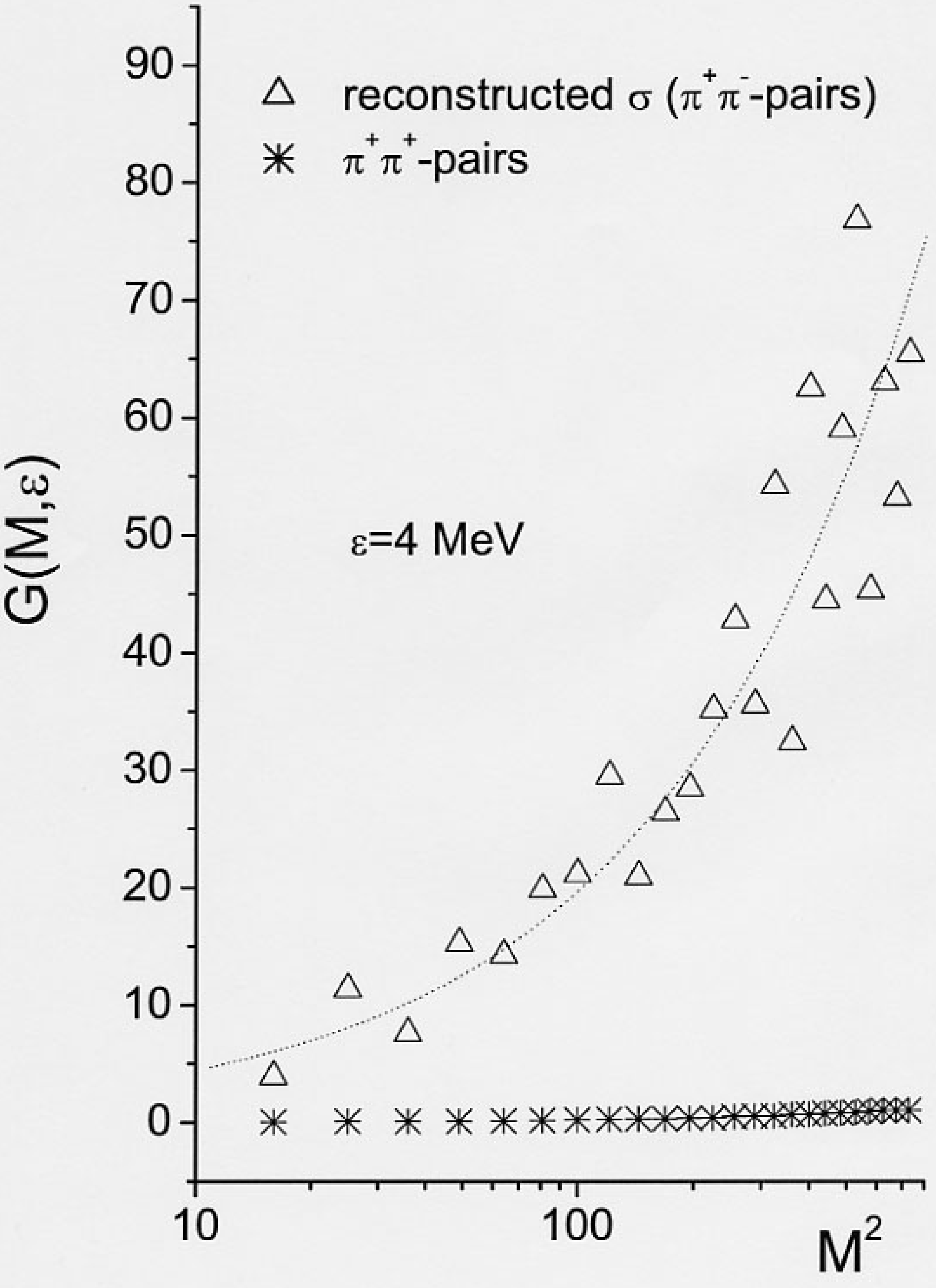

G(M,ε)
CMC
HIJING
ε=8 MeV
ε=10 MeV
ε=12 MeV
ε=14 MeV
M²



1.50
1.25
1.00
0.75
0.50
0.25
0.00
W(ε)
HIJING
CMC
0
5
10
15
20
ε (MeV)



1.0
0.8
0.6
0.4
0.2
0.0
$\phi_2(0)$=0.699(0.033)
$\phi_2(\varepsilon)$
0
5
10
15
20
ε (MeV)



90
80
70
60
50
40
30
20
10
0
10
100
M²
G(M,ε)
reconstructed σ (π⁺π⁻-pairs)
π⁺π⁺-pairs
ε=4 MeV

Analysis of the NA49 data

Analyse data of $A + A$ collisions for different size (A) nucleii at

158 GeV/n

$\Downarrow$

systematic study of the QCD phase diagram in a wide region of

the (ρ_B, T) plane

$\Downarrow$

increasing baryonic density ($\rho_B \sim A^{1/3}$) as we move from the

smallest ($C + C$) to the largest ($Pb + Pb$) system

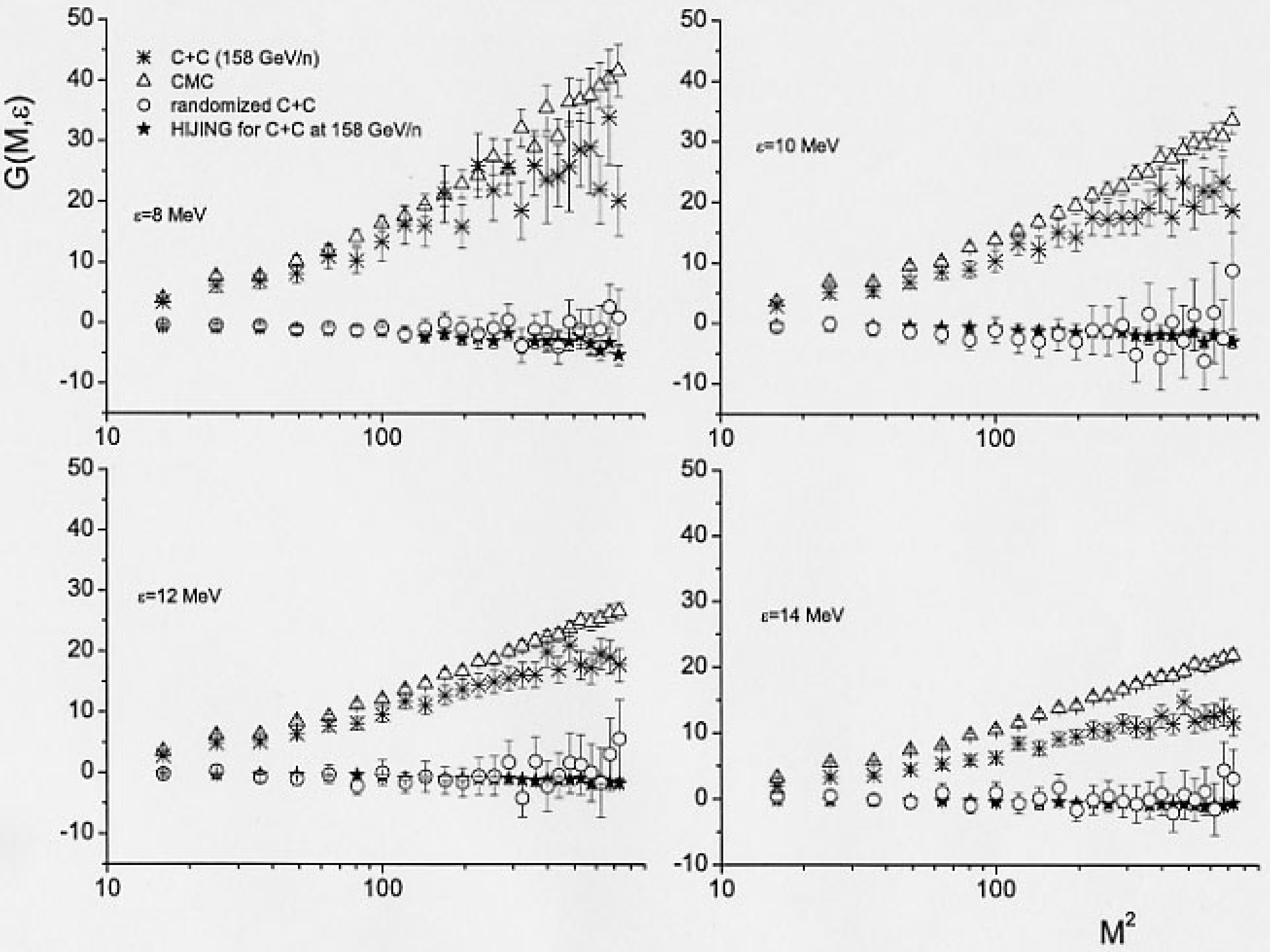
Here we use:

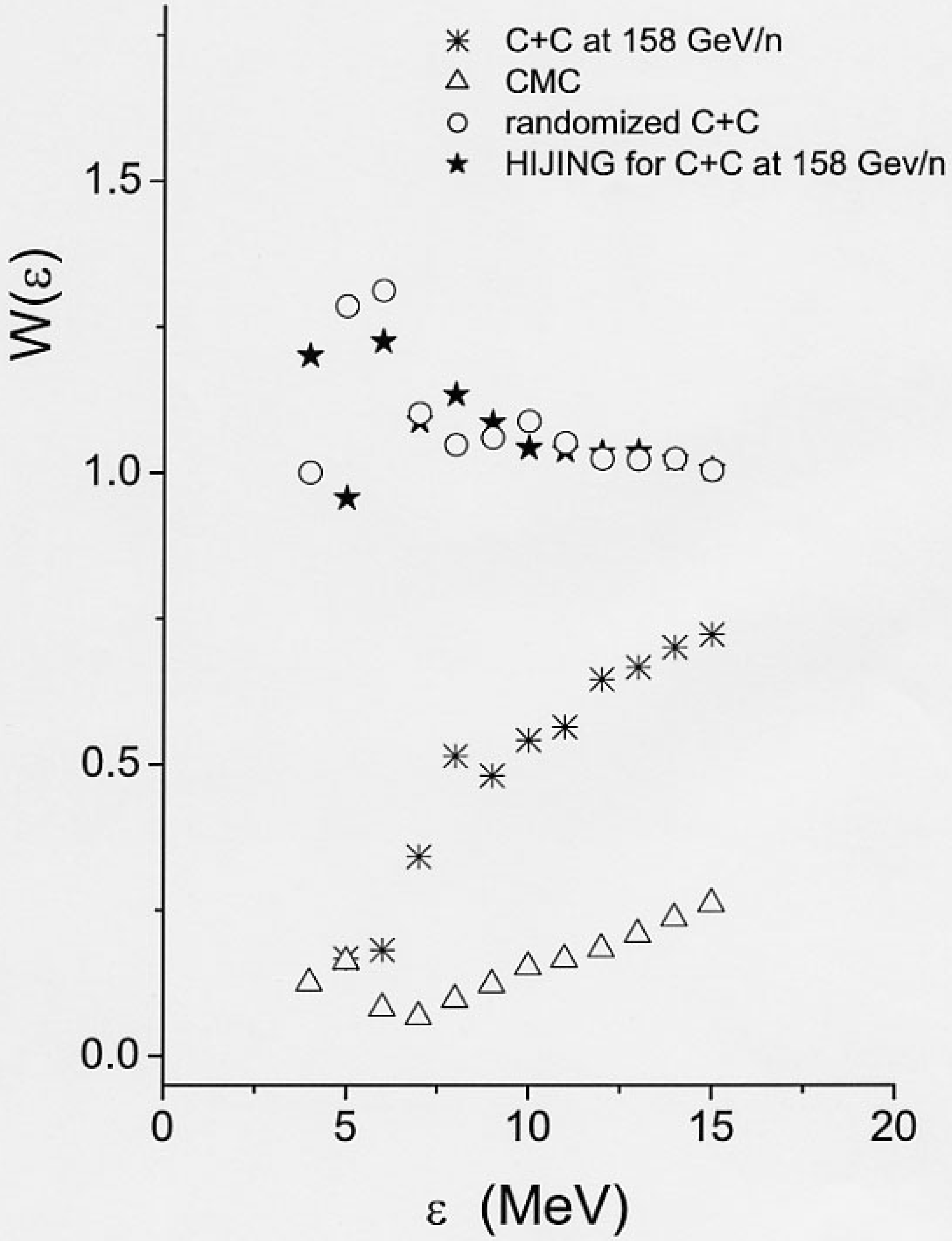
- 33689 $C + C$ events (1998 run period)
- 16953 $Si + Si$ events (1998 run period)
- 1487 $Pb + Pb$ events (1996 run period)

This choice leads to almost equal total number of charged pion tracks for all systems

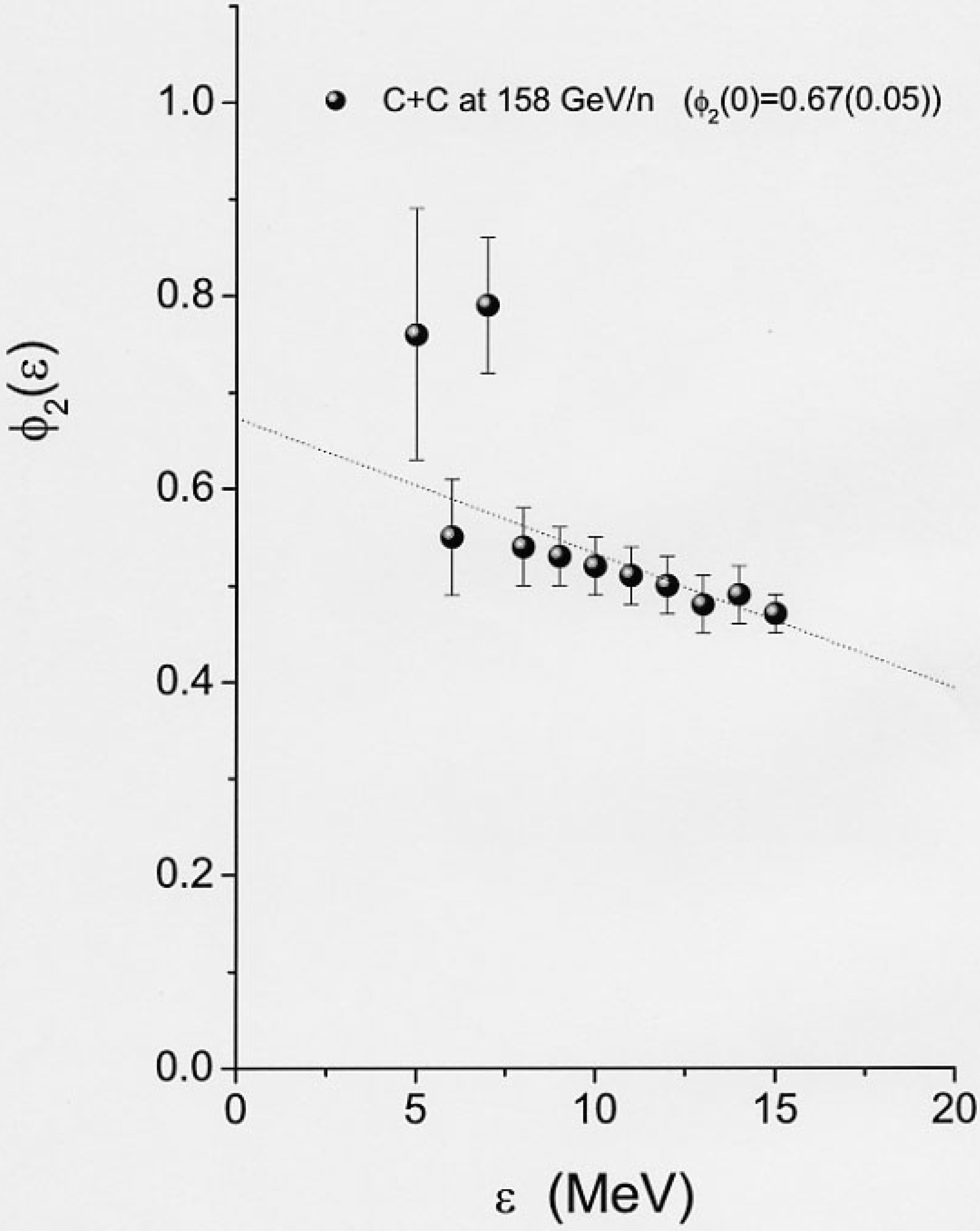
Selection criteria for the tracks

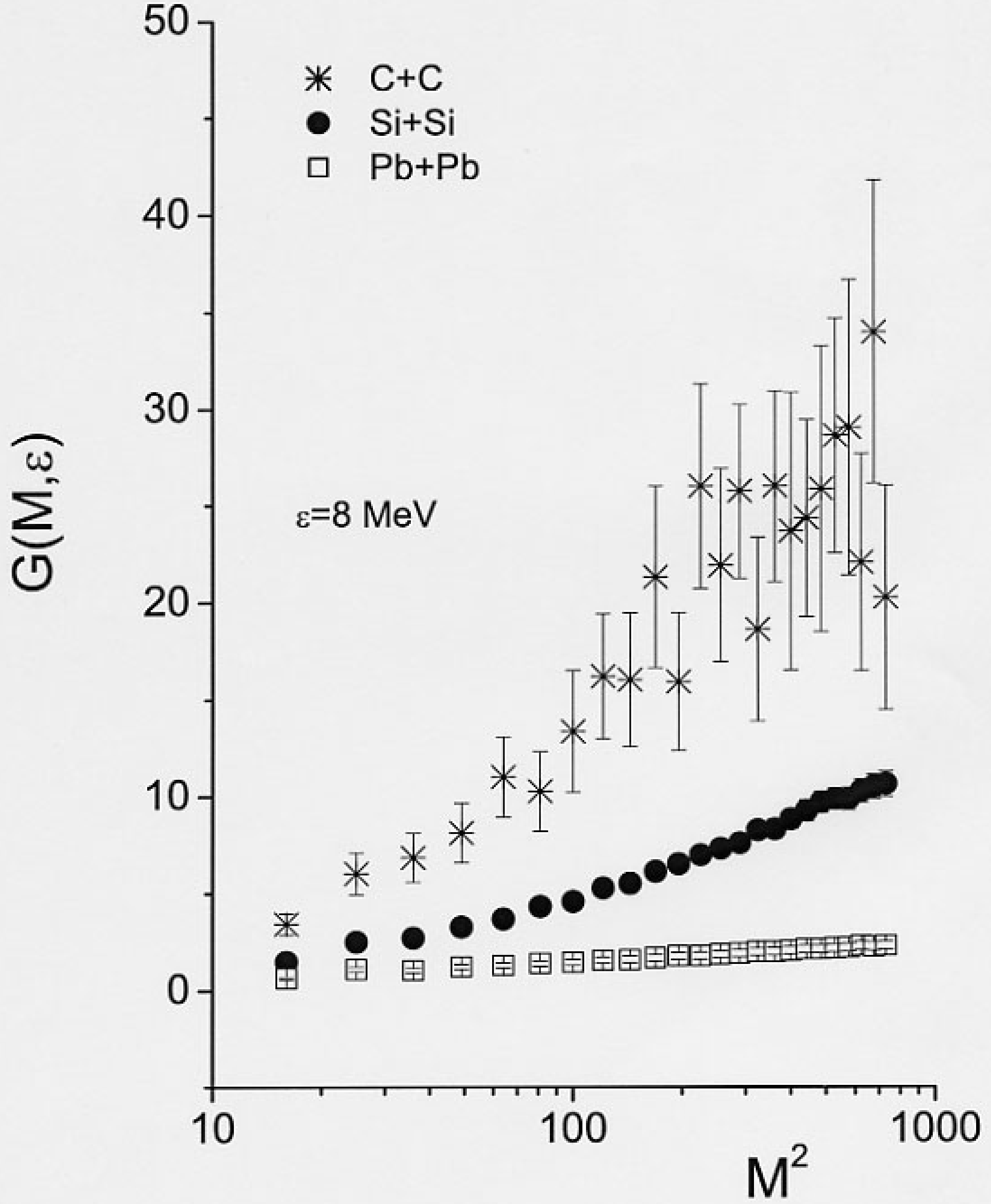
- $\sharp$ of reconstructed points in the TPC's between 20 and 235
- ratio of reconstructed to potential points > 0.5
- Charged pion identification up to 50 GeV using $\frac{dE}{dx}$
- 1σ acceptance around pion peak for each analysis bin


G(M,ε)
C+C (158 GeV/n)
CMC
randomized C+C
HIJING for C+C at 158 GeV/n
ε=8 MeV
ε=10 MeV
ε=12 MeV
ε=14 MeV
M²

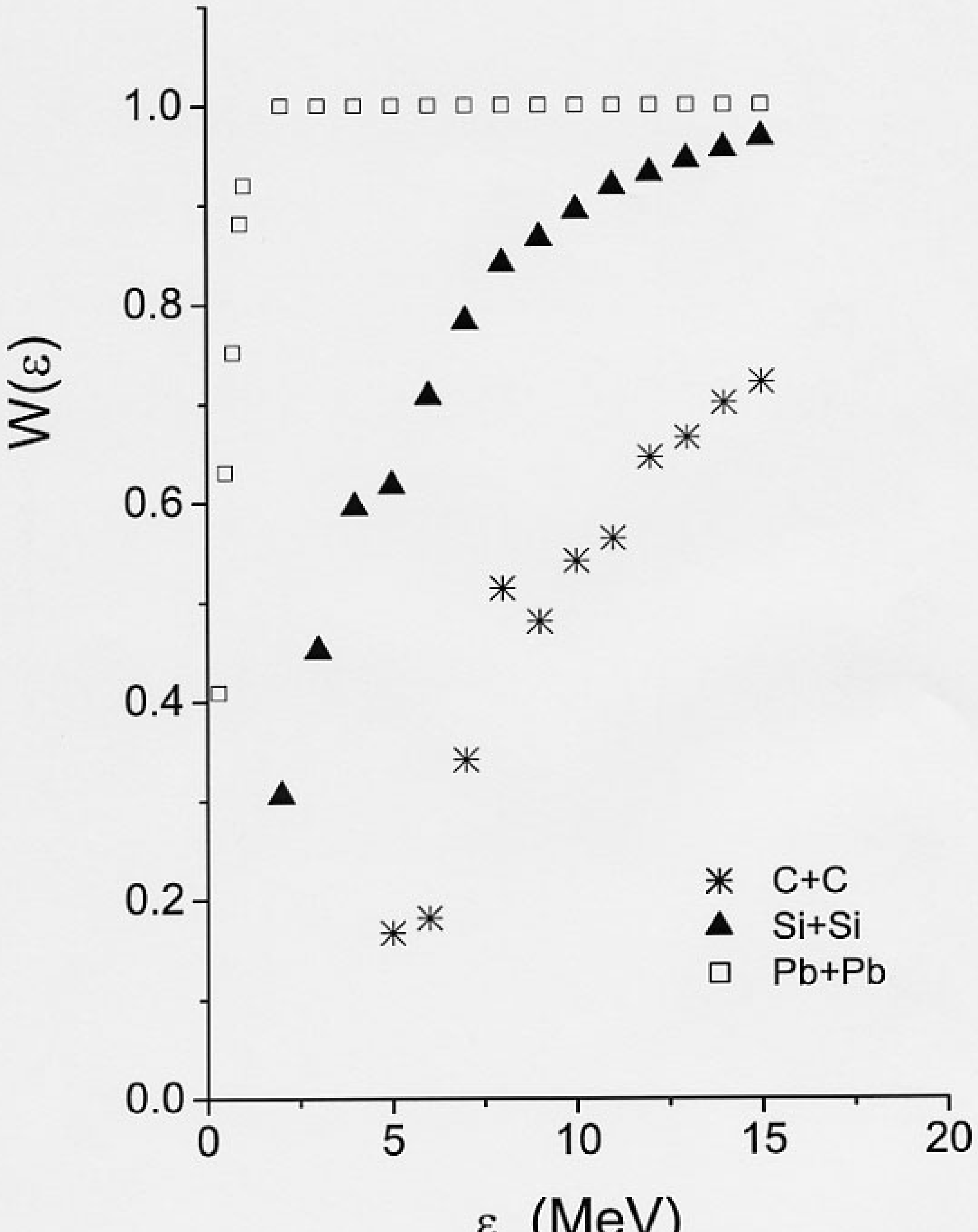


C+C at 158 GeV/n
CMC
randomized C+C
HIJING for C+C at 158 Gev/n
W(ε)
ε (MeV)
0.0
0.5
1.0
1.5
0
5
10
15
20

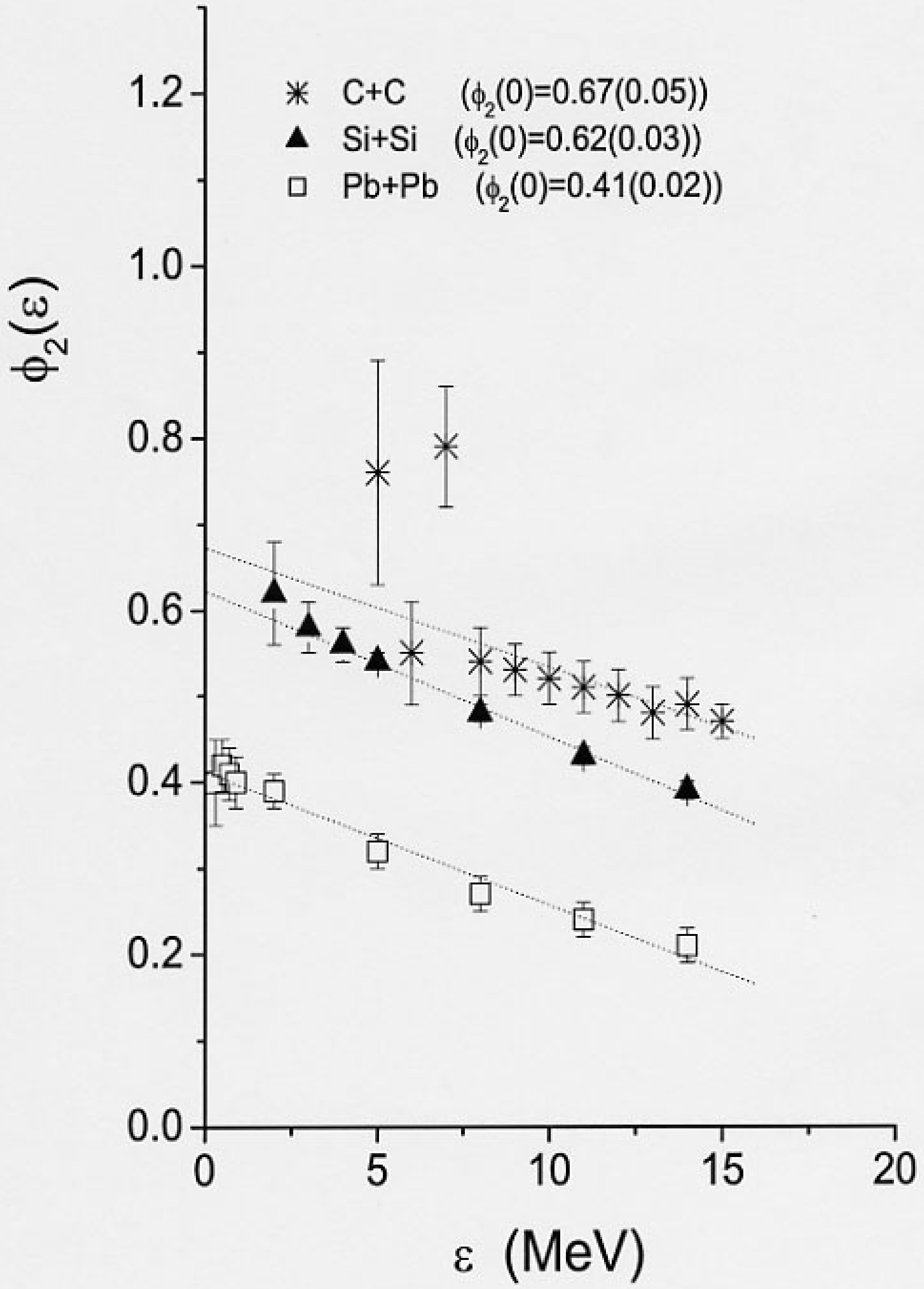

C+C at 158 GeV/n ($\phi_2(0)$=0.67(0.05))
1.0
0.8
0.6
0.4
0.2
0.0
$\phi_2(\varepsilon)$
0
5
10
15
20
ε (MeV)



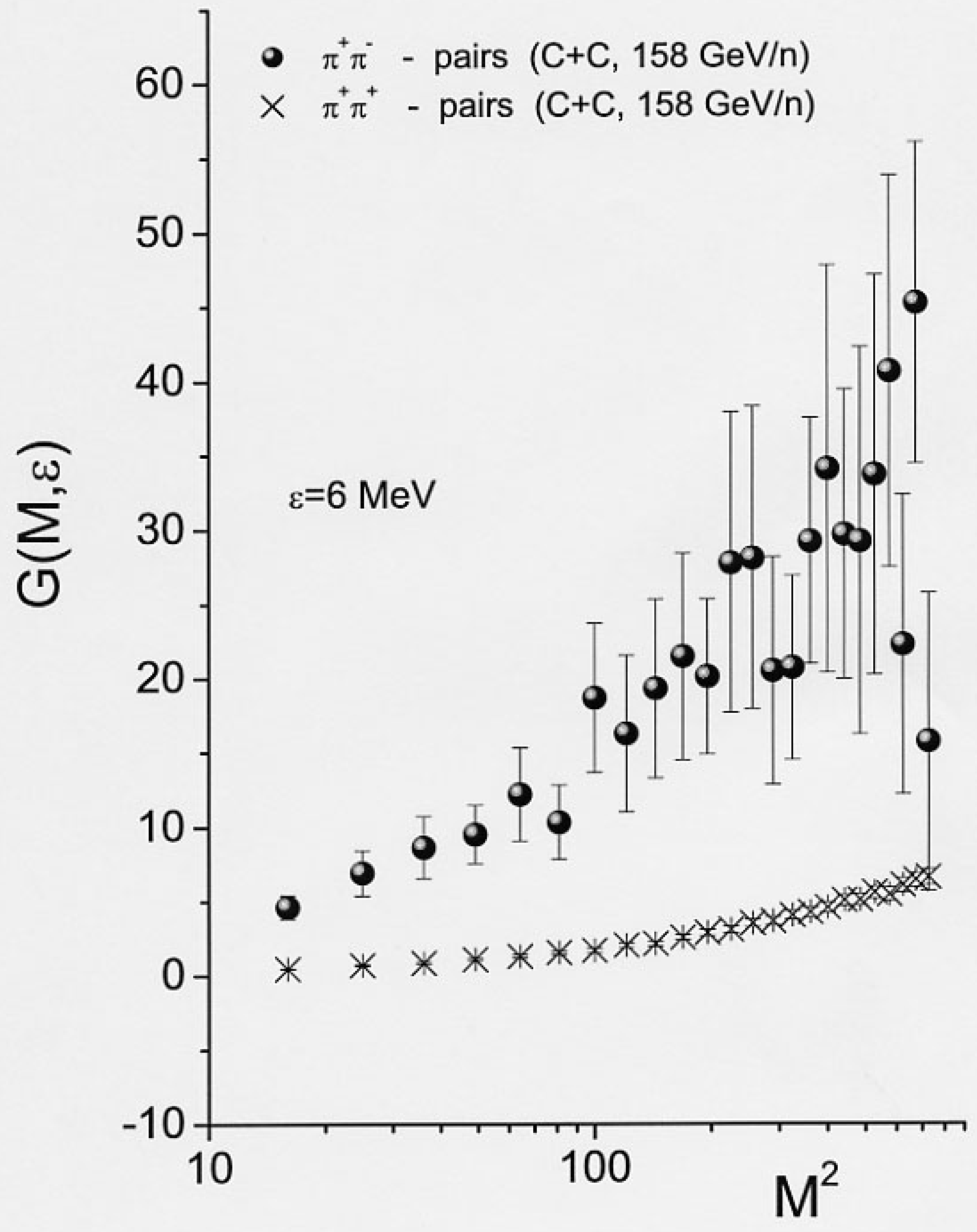
C+C
Si+Si
Pb+Pb
ε=8 MeV
G(M,ε)
M²
50
40
30
20
10
0
10
100
1000



W(ε)
1.0
0.8
0.6
0.4
0.2
0.0
0
5
10
15
20
ε (MeV)
C+C
Si+Si
Pb+Pb



C+C (φ2(0)=0.67(0.05))
Si+Si (φ2(0)=0.62(0.03))
Pb+Pb (φ2(0)=0.41(0.02))
φ2(ε)
ε (MeV)
0.0
0.2
0.4
0.6
0.8
1.0
1.2
0
5
10
15
20



π⁺π⁻ - pairs (C+C, 158 GeV/n)
π⁺π⁺ - pairs (C+C, 158 GeV/n)
ε=6 MeV
G(M,ε)
M²
60
50
40
30
20
10
0
-10
10
100

Conclusions

- Guided by the properties of the CMC critical events we develop a self-consistent algorithm for the reconstruction of the critical strongly correlated isoscalar sector (if any) in $A+A$ collisions. It is shown that events produced through conventional MC generators do not possess such a critical sector

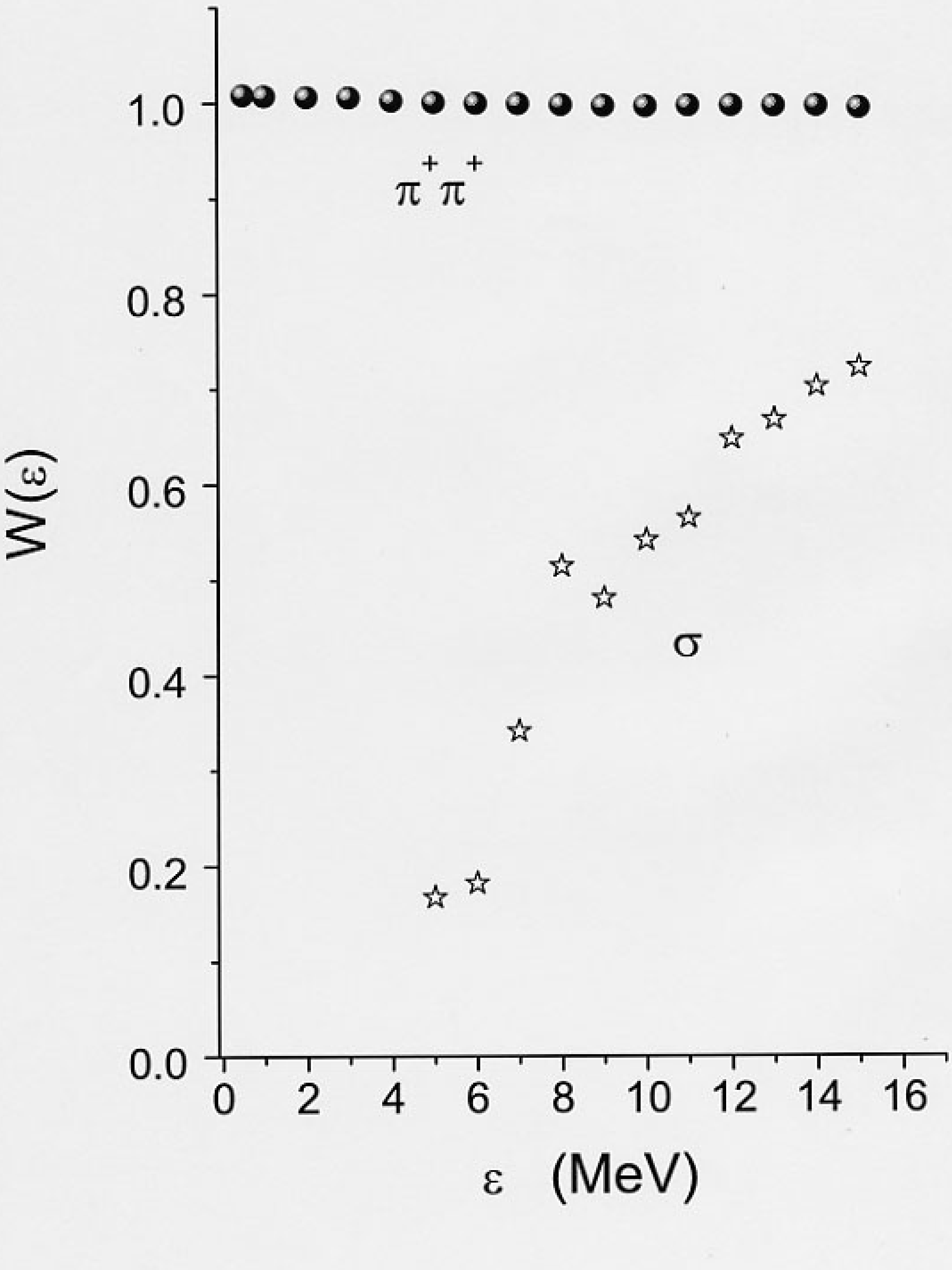
- The sigma-reconstruction algorithm is applied to 3 processes of NA49 at 158 GeV using nucleii of increasing size: $C+C$, $Si+Si$ and $Pb+Pb$ exploring a wide region of the QCD phase diagram in the (ρ_B, T) plane

- It turns out that all three SPS processes lie in the neighbourhood of the critical point. The $C+C$ system possesses the strongest critical isoscalar sector indicating its freezing out very close to the critical endpoint (CEP).

As the size of the system is increased we move away from CEP (consistently with the increase of the baryonic density) and the correlations as well as the fluctuations in the isoscalar sector are correspondingly reduced

- The fluctuations in the isoscalar sector of the $C+C$ system are consistent with the QCD predictions for the universality class of the CEP

- Further investigations in other systems using the above mentioned sigma reconstruction algorithm would allow a more detailed exploration of the QCD phase diagram near the CEP



W(ε)
1.0
0.8
0.6
0.4
0.2
0.0
0
2
4
6
8
10
12
14
16
ε (MeV)
π⁺π⁺
σ