Summary of the Intra Beam Scattering Mini Workshop held at the Cockcroft Institute
August 28 - 29, 2007

Michel Martini, CERN

CLIC Workshop

CERN, October 16-18, 2007

Injectors and Damping rings Working Group

Programme

- Introduction
  - Aim: get a new look at the intra-beam interaction effects relevant for
    - high brightness beams in synchrotron radiation facilities
    - damping rings for linear colliders.
  - Contributions to the most recent developments in
    - Status of IBS theory and available measurement data
    - New approaches to compute phase space distribution and beam tails for IBS dominated beams
    - IBS with polarized beams
    - IBS at very low beam energies and high beam brightness
    - Diagnostic systems for IBS measurement
    - Lattice design for IBS dominated beams

The Cockcroft Institute

Programme

- Presentations
  - ➢ Review of IBS measurements at ATF (K. Kubo, KEK)
  - ➢ IBS for ILC damping ring (A. Wolski, Cockroft Inst., Univ. of Liverpool)
  - ➢ IBS for CLIC damping ring (M. Korostelev, F. Zimmerman, CERN)
  - ➢ Quantum effects in IBS (S. Nikitin, BINP)
  - – Simulation of CTF-II emittance growth measurements using the string space charge formalism (R. Talman, Cornell)
  - – SC damping Wiggler developments (R. Rossmanith, KIT)
  - ➢ Beyond Piwinski & Bjorken-Mtingwa: theories, codes and benchmarking (J. Wei, BNL, IHEP)
  - ➢ Lattice design for IBS dominated beams (Y. Papaphilippou, CERN)
  - – IBS at very low beam energies (A. Adelmann, PSI)
  - – IBS effects in a wiggler-dominated light source (B. Podobedov, BNL)
  - ➢ Beyond Maxwell-Vlasov: space time correlations (G. Bassi, Cockcroft Inst.)
  - – The CESR test accelerator and intra-beam scattering (D. Sagan, Cornell)
  - ➢ Polarization measurement at VEPP-4M with the help of IBS (S. Nikitin)

Review of IBS measurements at ATF

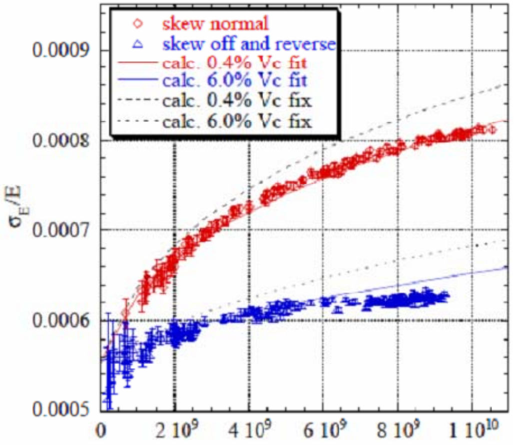
Kiyoshi Kubo

- Outline of experiments
  - Parameter measurement of IBS dominated beam
    - Momentum spread (extracted beam) : Screen monitor at large dispersion in extraction line
    - Bunch length (in damping ring) : streak camera
    - Horizontal and vertical emittance (in DR and extracted beam) : laser wire in damping ring, wire scanners in Extraction line

    as function of
    - Bunch intensity
    - Horizontal/vertical coupling :
      - Normal skew quad correctors (small ε_y ✸ strong IBS)
      - All skew correctors off
      - Half off and half reversed (large ε_y ✸ very weak IBS)
- Experimental results compared with SAD calculations
  - Issue calculations:
    - How to include the impedance effect for comparison with computation
    - Coulomb log factor in SAD

Review of IBS measurements at ATF **Kiyoshi Kubo**

- Measured beam parameters, comparison with IBS calculations (SAD)



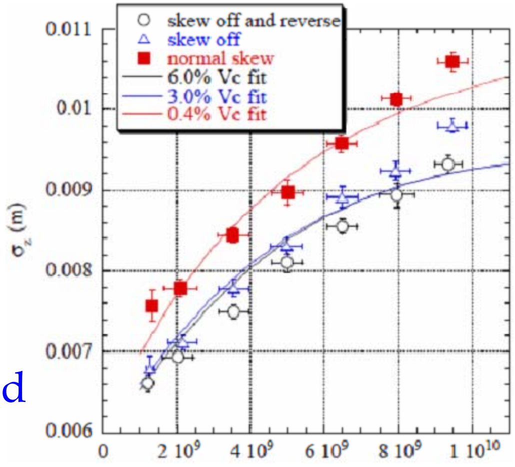
small ε_y

Good agreement

large ε_y

Momentum spread vs. intensity

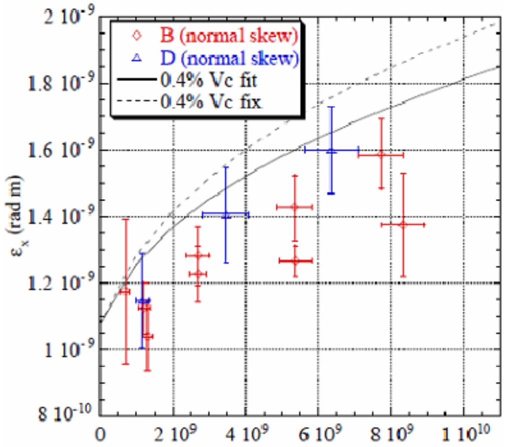
Not too bad



small ε_υ

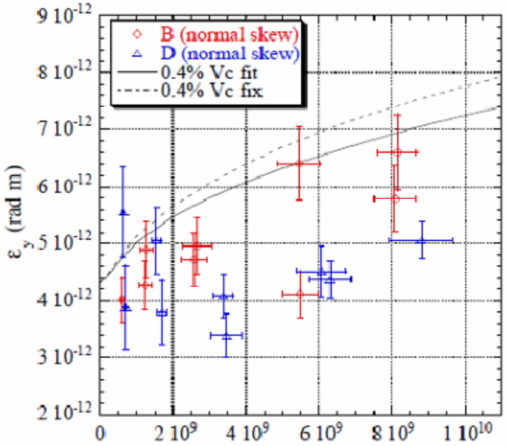
large ε_y

Bunch length vs. intensity



Horizontal emittance vs. intensity

Bad agreement



Vertical emittance vs. intensity

Review of IBS measurements at ATF **Kiyoshi Kubo**

- Summary
  - Strong IBS effect observed in ATF damping ring
  - Calculation with SAD is mostly consistent with experimental data.
    - Momentum spread : agreed well
    - Choice of Coulomb log factor seems reasonable
    - Bunch length : hard to use as a model test because it was affected by impedance effect (though included in the calculations)
  - Transverse emittances: do not agree very well.
    - Possibly due to measurement error
    - Discrepancy much smaller than a factor 2

IBS for ILC damping rings **Andy Wolski**

- Outline
  - IBS was not expected to be a dominant effect (≈10%) (though included in the lattice evaluation)
  - Low-emittance beam extraction from damping rings is critical for luminosity production. Any effect that can increase the emittance must be taken into account
  - Most electron storage rings operate with larger (physical) emittances than specified for the ILC damping rings: IBS is not a significant effect
    - » Synchrotron light sources have a few nm natural emittance and operate with ≈1% coupling (for reasonable beam lifetime)
  - ATF has produced the world's lowest physical emittance beam (≈4.5 pm vertical) including IBS effects measurements
  - ILC damping rings will produce a vertical emittance less than half that of the ATF (stronger IBS)…but the beam energy will be ≈4 higher (5 GeV in ILC, 1.28 GeV in ATF) (weeker IBS) which will counteract the effects of smaller emittance

IBS for ILC damping rings **Andy Wolski**

- IBS calculations for e^- and e^+ beams
  - IBS growth time calculation (high-energy beam approximation)
    - Bane's approximation
    - Completely-integrated modified Piwinski (CIMP)
  - Equilibrium vertical emittance depends on how the emittance is produced
    - The equilibrium vertical emittance is found by iteratively using the formula

$$\varepsilon_y = (1-\kappa)\frac{\varepsilon_{y0}}{1-\tau_y / T_y(\varepsilon_x,\varepsilon_y,\varepsilon_p)} + \kappa\frac{\varepsilon_{y0}}{1-\tau_x / T_x(\varepsilon_x,\varepsilon_y,\varepsilon_p)}$$

    - ε_y is the vertical emittance (with IBS), ε_{y0} the zero-current equilibrium emittance (without IBS), $\tau_{x,y}$ and $T_{x,y}$ are the betatron radiation damping and transverse IBS growth times
    - κ varies from 0 (ε_y generated from dispersion) to 1 (ε_y generated from betatron coupling)
  - Cross-checking of the calculations against :
    - Calculations of the IBS growth times using the Bjorken-Mtingwa formulae
    - Experimental data from the ATF

IBS for ILC damping rings **Andy Wolski**

- Summary
  - Bane & CIMP approximations to the IBS growth rates
    - Good agreement with each other
    - Overestimate the IBS growth when benchmarked against ATF data
  - ILC damping rings
    - The strongest IBS effect is observed in horizontal when half the vertical emittance is generated by dispersion and half by betatron coupling
    - OCS lattice : horizontal emittance growth of ≈20% (2×10^{10} particles/bunch, and 6 mm rms bunch length), vertical emittance growth of ≈10%.
    - IBS should not prevent to reach the specified 8 μm normalized emittance
    - Damping ring beam energy should probably not be reduced below 5 GeV
  - IBS is not a high priority for the ILC damping rings … however
    - What is the reason for the difference between IBS calculations and ATF data?
    - What will be the best value of κ to use?
    - Could IBS affect the beam distribution, perhaps generating tails?

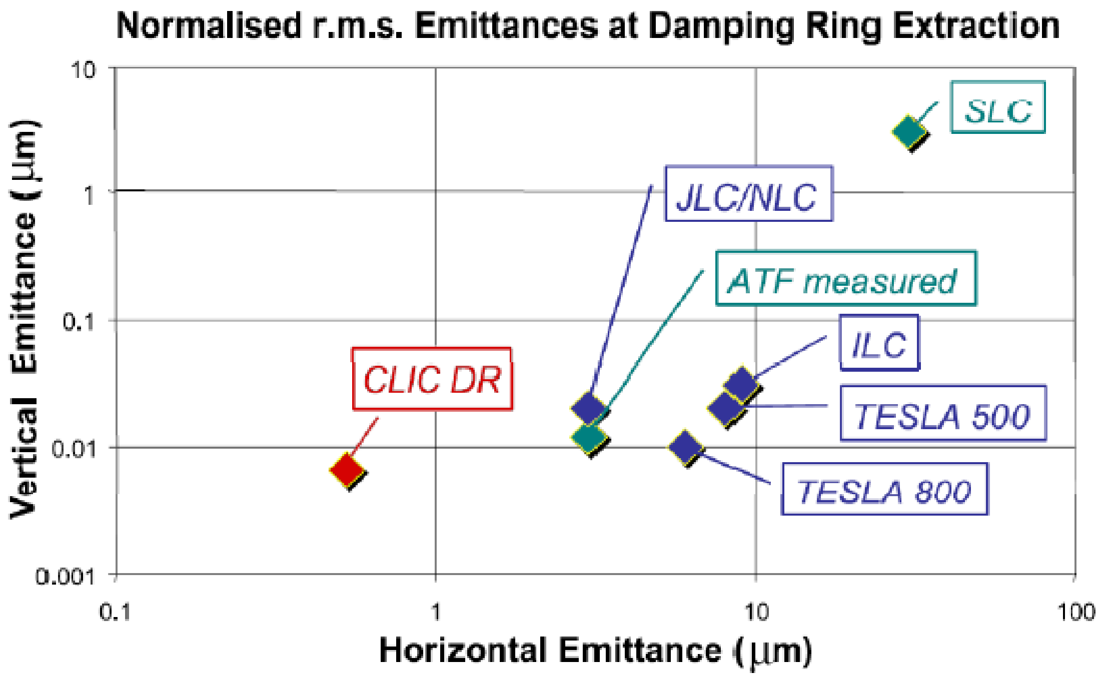
IBS for CLIC damping ring/Lattice design for IBS dominated beams

Maxim Korostelev, Frank Zimmermann/Yannis Papaphilippou

- Outline
  - Damping rings need a small output emittance and a good transmission of the very intense e+/e- beam
  - IBS effect can lead to 6-D phase space emittance growth
  - Lattice design : actions should be taken to reduce the IBS effect
    - Energy, lattice choice (TME, NBA …)
    - Optics function parameters (phase advance …)
    - Damping wiggler parameters
    - Alignment tolerances, orbit and coupling correction
  - Like to iterate the lattice design until a robust optimum is found with regard to emittance with IBS (including other optics considerations)
  - Optimisation of CLIC damping rings performed by M. Korostlelev for various ring parameters

IBS for CLIC damping ring/Lattice design for IBS dominated beams
Maxim Korostelev, Frank Zimmermann/Yannis Papaphilippou

- Damping ring target normalized emittance figures
  - CLIC DR parameters of the extracted beam at 2.4 GeV (recent data)
    - Bunch population 4×10^9 charges/bunch
    - Normalized horizontal/vertical emittances (with IBS) : 380 nm/4 nm
    - Physical horizontal/vertical emittances (with IBS) : 80.2 pm/0.8 pm



IBS for CLIC damping ring/Lattice design for IBS dominated beams

Maxim Korostelev, Frank Zimmermann/Yannis Papaphilippou

- Intra-beam scattering and synchrotron radiation
  - Evolution of beam emittances for e^- and e^+ beams
    - $\varepsilon_{x,y,p}$ are the rms transverse and longitudinal emittances (radiation damping, quantum excitation and IBS), $\varepsilon_{x0,y0,p0}$ are the zero-current equilibrium rms emittances (radiation damping, quantum excitation, no IBS)
    - $\tau_{x,y,p}$ are the betatron and synchrotron radiation damping times, $T_{x,y,p}$ the transverse and longitudinal IBS growth times (nonlinear functions)

$$\frac{d\varepsilon_{x,y,p}}{dt} = -\frac{2}{\tau_{x,y,p}}(\varepsilon_{x,y,p} - \varepsilon_{x0,y0,p0}) + \frac{2\varepsilon_{x,y,p}}{T_{x,y,p}(\varepsilon_x, \varepsilon_y, \varepsilon_p)}$$

  - Equilibrium emittances
    - Follows from the solution of the 3 equations (fixed point of above equations)
    - Since $T_{x,y,p}$ depend on ε_x, ε_y, ε_p the 3 equations for the equilibrium emittances ε_x, ε_y, ε_p must be solved iteratively until a proper equilibrium is found

$$\frac{d\varepsilon_{x,y,p}}{dt} = 0 \qquad \varepsilon_{x,y,p} = \frac{\varepsilon_{x0,y0,p0}}{1 - \tau_{x,y,p} / T_{x,y}(\varepsilon_x, \varepsilon_y, \varepsilon_p)}$$

IBS for CLIC damping ring/Lattice design for IBS dominated beams

Maxim Korostelev, Frank Zimmermann/Yannis Papaphilippou

- Special case of e^- / e^+ beam vertical emittances
  - For week betatron coupling and in the presence of fallacious vertical dispersion the latter equation for the equilibrium vertical emittance is to be replaced by

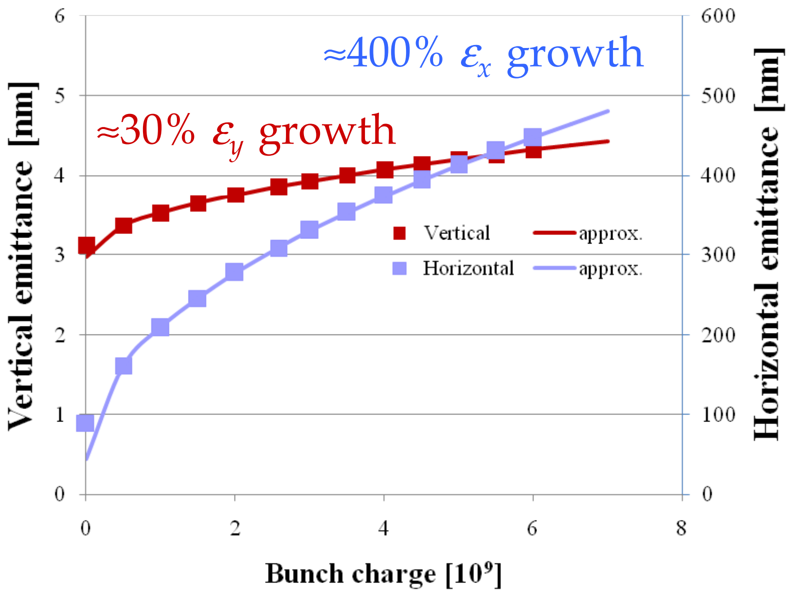
$$\varepsilon_y = (1-\kappa)\frac{\varepsilon_{y0}}{1-\tau_y / T_y(\varepsilon_x,\varepsilon_y,\varepsilon_p)} + \kappa\frac{\varepsilon_{y0}}{1-\tau_x / T_x(\varepsilon_x,\varepsilon_y,\varepsilon_p)}$$

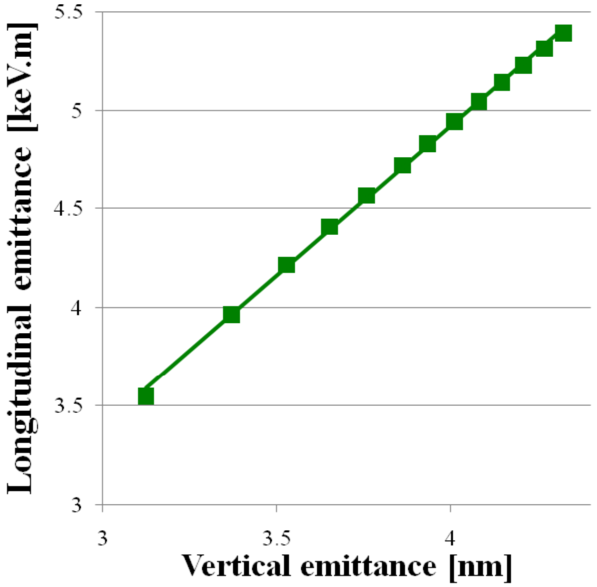
  - where κ varies between 0 and 1, κ=0 in the presence of vertical dispersion only and κ=1 in case of betatron coupling only
  - in fact κ tends to be closer to 1 than to 0 (ε_y being rather generated from betatron coupling due to magnet imperfections and misalignments, which may in turn induce a vertical dispersion)

IBS for CLIC damping ring/Lattice design for IBS dominated beams

Maxim Korostelev, Frank Zimmermann/Yannis Papaphilippou

- Equilibrium normalized emittances with IBS vs. bunch charge
  - Rough scaling laws can be derived for damping ring design
  - CLIC damping rings : horizontal normalized emittance scales roughly as $\gamma\varepsilon_x \propto (N_b/\sigma_p)^{1/2}$ (the scaling is more exact when the longitudinal emittance is kept constant (≈5000 eV.m for the CLIC damping rings)
  - Vertical & longitudinal emittances are weakly dependent on bunch charge and almost linear with each other
  - IBS growth times calculation is based on the modified Piwinski formalism





IBS for CLIC damping ring/Lattice design for IBS dominated beams
Maxim Korostelev, Frank Zimmermann/Yannis Papaphilippou

- Summary
  - Complete design of damping ring reaching CLIC target parameters
  - Equilibrium emittance dominated by IBS
    - Damping ring lattice parameters can be optimised to reach the target emittance with IBS
    - IBS effect is evaluated ex-post (after setting up the basic lattice features)
    - Correction scheme for errors (e.g. magnet misaligment) recovers emittance
  - IBS computation scheme developed
  - Iterative process can be used to scan the full parameter space and reach the optimum using numerical tools
  - Lack of a unique tool to do all the optimisation steps
    - MATLAB based package using accelerator toolbox should be a good choice
  - Proposal: derive analytically the optics parameters to reach minimum IBS dominated emittance in selected lattices (FODO, TME,…)

Quantum effects in IBS **Sergei Nikitin**

- Quantum lower limit on scattering angle in the calculation of multiple Touschek-effect
  - IBS impact parameter is bounded above (e.g. by the limited beam size)
  - Lower quantum particle scattering angle exists (uncertainty principle).
  - Rutherford cross section used in IBS classical approach is valid in a quantum approach (Coulomb nature of scattering potential)
  - IBS quantum consideration concerns the limits of scattering parameters
  - In what conditions a quantum scattering angle lower limit is important?
  - Can a large quantum limit lead to a major increase of IBS diffusion compared to classical consideration?
- IBS parameters
  - Coulomb logarithm
    - » $\theta_{\max}$, $\theta_{\min}$, $b_{\max}$, $b_{\min}$ are upper & lower bounds for the scattering angles and impact parameters, V_b is the beam volume (lab frame), N_b the bunch charge

$$(\log) = \ln\left(\frac{\theta_{\max}}{\theta_{\min}}\right) = \ln\left(\frac{b_{\max}}{b_{\min}}\right) \qquad b_{\max} = \min\left\{\sigma_y, \left(\frac{\gamma V_b}{N_b}\right)^{1/3}\right\}$$

Quantum effects in IBS **Sergei Nikitin**

- Classical definition

$$(\log) = \ln\left(\frac{\chi}{\chi_m^c}\right) \qquad \chi_m^c = \frac{r_0\, p_0^2}{b_{\max}\sigma_p^2} \qquad \chi = \frac{p^2}{\sigma_p^2}$$

- Quantum definition

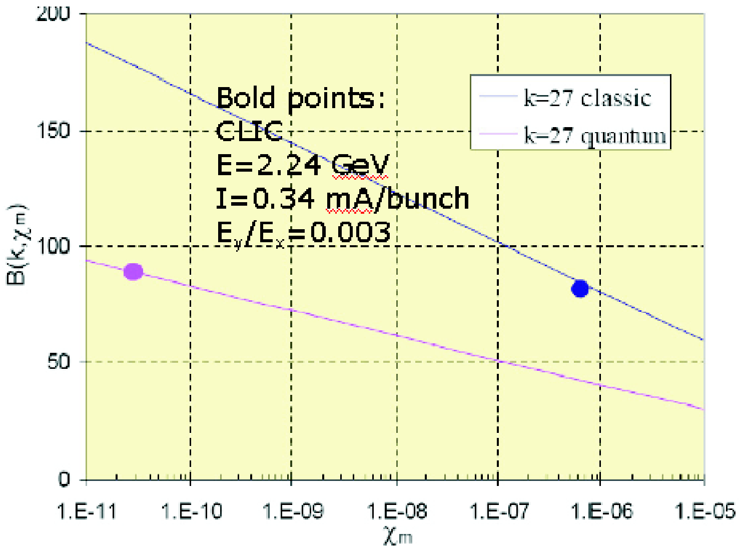
$$(\log) = \frac{1}{2}\ln\left(\frac{\chi}{\chi_m^q}\right) \qquad \chi_m^q = \left(\frac{\hbar}{b_{\max}\sigma_p}\right)^2 \qquad \chi_m^c >> \chi_m^q$$

  - » Both classical & quantum Coulomb logarithm would be similar if $\chi^c_m \approx \chi^q_m$ ($b_{max} \approx 10^{-10}$ m) corresponding to unrealistic superdense/superthin beams (p, $\sigma_p = mc\gamma(\sigma_{x'}^2 + \sigma_{y'}^2)^{1/2}$ are the momentum and its transverse spread, $p_0 = mc$)

- Application to CLIC and VEPP-4M
  - Touschek quantum calculations for CLIC and VEPP-4M are almost similar to those obtained in the classical approximation
    - the diffusion factor $B(k, \chi_m)$ of classical and quantum approximations (that account for the scattering) are close to each other in spite of a huge difference between χ^c_m and χ^q_m (cf, below), $k = \sigma_{x'}/\sigma_{y'}$ is the velocity coupling parameter ($k=1$ for round and k✹∞ for flat beams)

Quantum effects in IBS **Sergei Nikitin**

- CLIC diffusion factor in classical and quantum approximations



- Summary
  - Quantum lower scattering angle limit must be included in IBS process
  - However, CLIC and VEPP-4M Touschek calculations show that considering the quantum limit of minimal scattering angle instead of the classical one does not change notably the results
    - This seems to be true for all existing and designed storage rings since an apparent difference between classical and quantum approximations occur only in non-realistic cases of super-dense/super-thin beams

Polarization measurement at VEPP-4M with the help of IBS

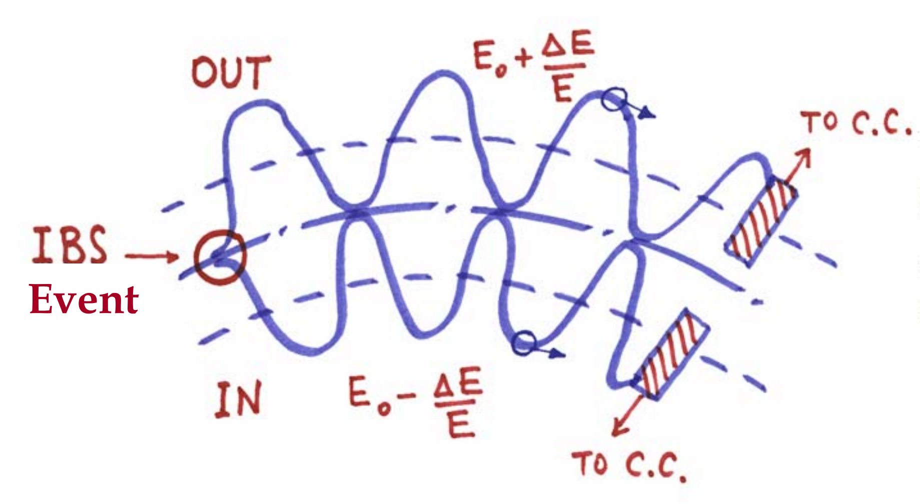
Sergei Nikitin

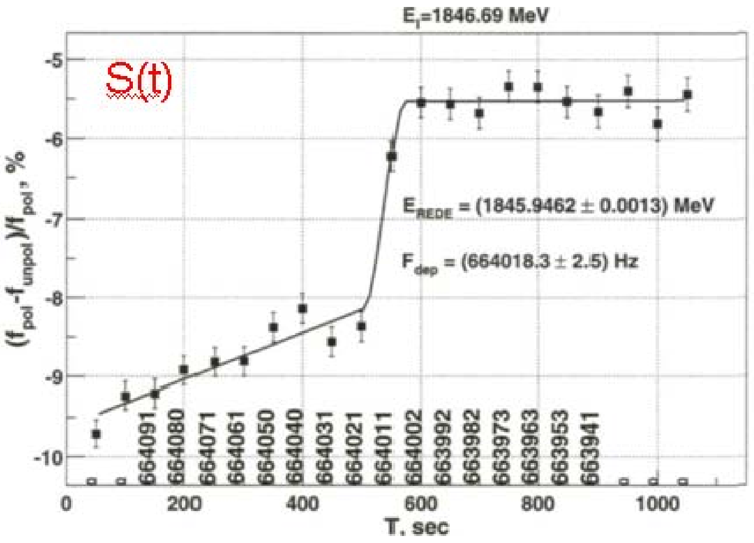
- Use of IBS in the precision experiment with polarized beams
  - Measurement of IBS rate is applied (Touschek particles detection), using the resonant depolarization technique, to observe the beam polarization and to measure the beam energy in e^-/e^+ storage rings
    - » Resonant Depolarization technique : beam polarization (and spin precession frequency) measurement by mean of a fast change in the counting rate during scanning an external field depolarizer frequency (depolarization jump in the counting rate of scattered particles occurs at time when the beam becomes un-polarized)
    - » Polarization contribution to the beam emittance and energy spread is negligible.
    - » Depolarization influence of IBS is usually small because of its insignificant contribution to energy diffusion as compared with SR.
    - » Practically, a few percent change in beam lifetime related to polarization is too small to be measured because of large systematic errors.
- Experiments at VEPP-4M
  - J/✡, ✡′ meson mass measurement and ♦ lepton mass measurement at its production threshold
  - Study of beam energy long-term stability
  - High resolution comparison of the bunch depolarization frequencies

Polarization measurement at VEPP-4M with the help of IBS

Sergei Nikitin

- Touschek particle pair detection using IBS-based polarimeter
  - <u>Left figure</u> : dual coincidence circuits identify the IBS contribution from a total counting rate of 2 counters (trajectories of scattered Touschek e^- or e^+ symmetrically lie on each side of the reference orbit)
  - <u>Right figure</u> : typical jump in the counting rate of Touschek coincidences. Quantity under observation is S(t)=1-å$_2$/å$_1$, where å$_2$, å$_1$ are counting rates for Touschek e^- or e^+ from a polarized and unpolarized bunches

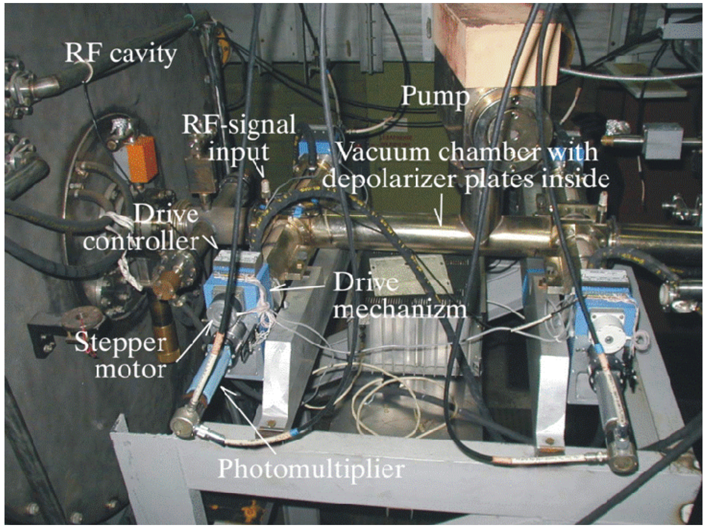




Polarization measurement at VEPP-4M with the help of IBS

Sergei Nikitin

- IBS-based polarimeter at VEPP-4M



- Summary
  - High efficient IBS-based polarimeter is developed for various precision experiments with polarized beams
  - J/✡, ✡′ meson and ♦ lepton masses are defined more accurately
  - Record resolution in the depolarization frequency of 3×10^{-9} achieved gives an incentive to next studies of possibility to realize the CPT test experiment at a storage ring
  - Developed methods and skills may be useful to study other IBS aspects

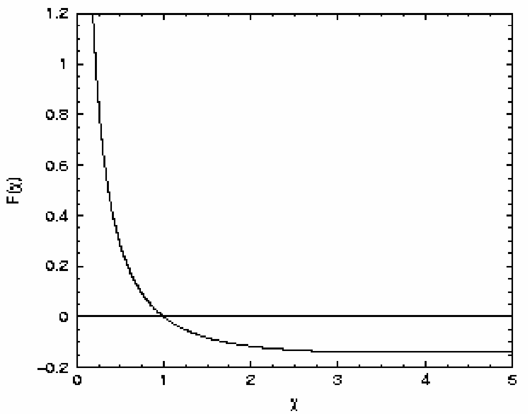
Beyond Piwinski & Bjorken-Mtingwa: IBS theories, codes, and benchmarking **Jie Wei**

- Intra-beam scattering models
  - Conventional Gaussian-beam models
    - Predicts rms beam dimension growth rates (Bjorken-Mtingwa, Piwinski), scaling laws and asymptotic rules (G. Parzen, J. Wei)
  - Fokker-Planck approach for arbitrary distributions
    - Beam loss and beam shape study
  - Molecular dynamics method for particle-particle interaction
    - Predicts ultra-cold beam behavior
    - Crystalline beam formation and heating due to Coulomb interactions (J. Wei, X.P. Li, A.M. Sessler)
- Benchmarking experiments in RHIC
  - Beam emittance growths in 3 directions
    - Agreement on rms beam size growth: longitudinal ≈20%, transverse ≈40%
  - Beam loss at tail & de-bunching
    - Agreement on de-bunching beam loss for both Gaussian & hollow beams
  - Beam distribution evolution: Gaussian-like vs. hollow beams
    - Agreement on longitudinal profile for both beams

Beyond Piwinski & Bjorken-Mtingwa: IBS theories, codes, and benchmarking **Jie Wei**

- Conventional Gaussian-beam models
  - IBS rates computed in the moving beam rest frame using the Rutherford formula, next transformed to the lab frame and averaged over phase-space and time
  - Emittance and momentum growths obtained from multiple small-angle scattering among particles with 6-D phase-space Gaussian distribution

$$\begin{bmatrix} \frac{1}{\sigma_p}\frac{d\sigma_p}{dt} \\ \frac{1}{\sigma_x}\frac{d\sigma_x}{dt} \\ \frac{1}{\sigma_y}\frac{d\sigma_y}{dt} \end{bmatrix} = \frac{Z^4 N}{A^2}\,\frac{r_0^2 m_0 c^2 (\log)}{8\gamma\varepsilon_x\varepsilon_y S}\,F(\chi)\begin{bmatrix} n_b(1-d^2) \\ -\frac{a^2}{2}+d^2 \\ -\frac{b^2}{2} \end{bmatrix}$$



  - Analytic expression in case of nearly constant D_x/ e rms longitudinal bunch area, (log)≈20, a, b, d are functions of D_x, $\beta_{x,y}$, σ_x, γ; F(χ) is an analytical function of $\chi=(a^2+b^2)/2$, n_b=1 (bunched) or 2 (coasting)
  - Inversely proportional to the 6-D phase-space area, proportional to the bunch intensity N and to Z^4/A^2
  - Conventional models deficient for lossy and non-Gaussian beams

Beyond Piwinski & Bjorken-Mtingwa: IBS theories, codes, and benchmarking **Jie Wei**

- Fokker-Planck approach for arbitrary distributions
  - Evolution of particle distribution in phase space
    - » Start from general 6-D Fokker-Planck equation for the longitudinal distribution $\psi_{T,L}$(x,x′,y,y′,Q,J,t) time change due to IBS in the lab frame
      - using angle-action variables Q, J instead of RF phase and energy deviations (ϕ, $W=\Delta E/h\omega_s$)
    - » Simplify to 1-D Fokker-Planck for the longitudinal distribution ψ(J) (if growth time much longer than the synchrotron period)
    - » Use Rutherford scattering in the beam rest frame
  - Fokker-Planck for a bunch in a single-harmonic bucket
    - » IBS growth typically much slower than synchrotron/betatron oscillation period - averaging over phase angles
    - » For RHIC, averaging over transverse directions: using time-evolving Gaussian transverse distributions and arbitrary longitudinal distribution yields, with the boundary condition at J=0 for the flux $\partial\psi/\partial t$ and at the separatrix J_{max} for ψ

$$\frac{\partial \psi}{\partial t} = -\frac{\partial}{\partial J}(F\psi) + \frac{1}{2}\frac{\partial}{\partial J}\left(D\frac{\partial \psi}{\partial J}\right) \qquad \begin{cases} J = 0 & : \quad -F\psi + \dfrac{D}{2}\dfrac{\partial \psi}{\partial J} = 0 \\ J = J_{\max} & : \quad \psi = 0 \end{cases}$$

Beyond Piwinski & Bjorken-Mtingwa: IBS theories, codes, and benchmarking **Jie Wei**

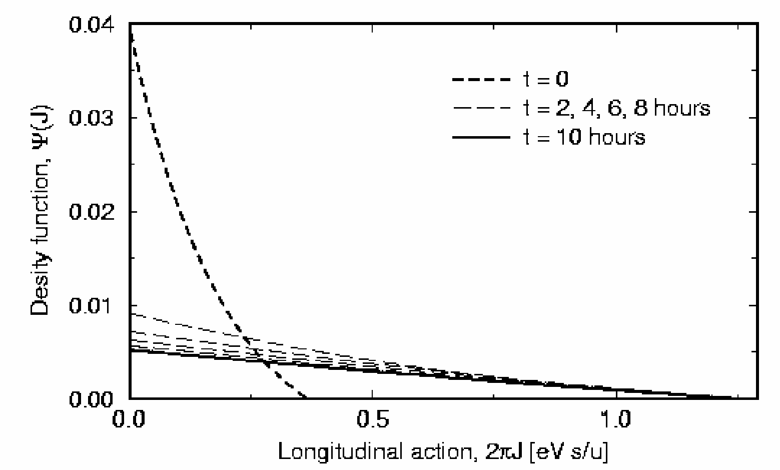
$$F(J) = \oint \frac{2}{\pi R} ds \int_0^{\frac{1}{4}} dQ \frac{\partial W}{\partial J}\bigg|_{\phi}^{-1} (Q,J) \int_{J_{\min}}^{J_{\max}} \frac{\partial W}{\partial J}\bigg|_{\phi} (Q',J')\left[A_F(\lambda_1) + A_F(\lambda_2)\right]\psi(J')dJ'$$

$$D(J) = \oint \frac{2}{\pi R} ds \int_0^{\frac{1}{4}} dQ \left(\frac{\partial W}{\partial J}\bigg|_{\phi}^{-1} (Q,J) \right)^2 \int_{J_{\min}}^{J_{\max}} \frac{\partial W}{\partial J}\bigg|_{\phi} (Q',J')\left[A_D(\lambda_1) + A_D(\lambda_2)\right]\psi(J')dJ'$$

where *F(J)*, *D(J)* are the drift and diffusion coefficients

- » Below transition *F(J)* dominates : energy transfer between the 3 directions, there is an equilibrium state
- » Above transition *D(J)* dominates : diffusive growth of all beam dimensions

- – Time evolution of the longitudinal density distribution under IBS
  - » 10-hour store of gold beam at RHIC

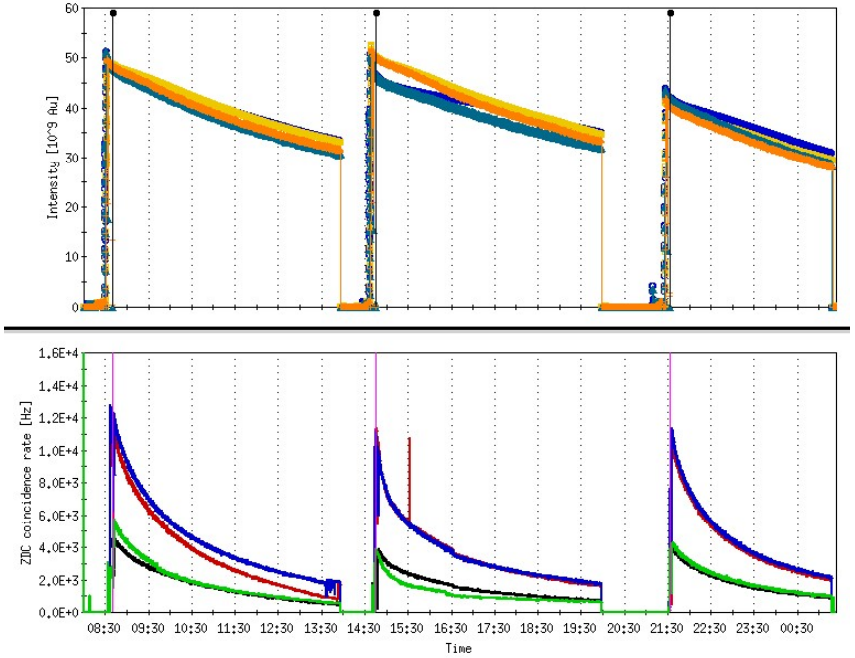


Beam loss through the RF bucket separatrix given by the reduction of the integrated area ⇩✡(J)dJ

Longitudinal variable J is linked to the energy deviation W via a canonical transformation

Beyond Piwinski & Bjorken-Mtingwa: IBS theories, codes, and benchmarking **Jie Wei**

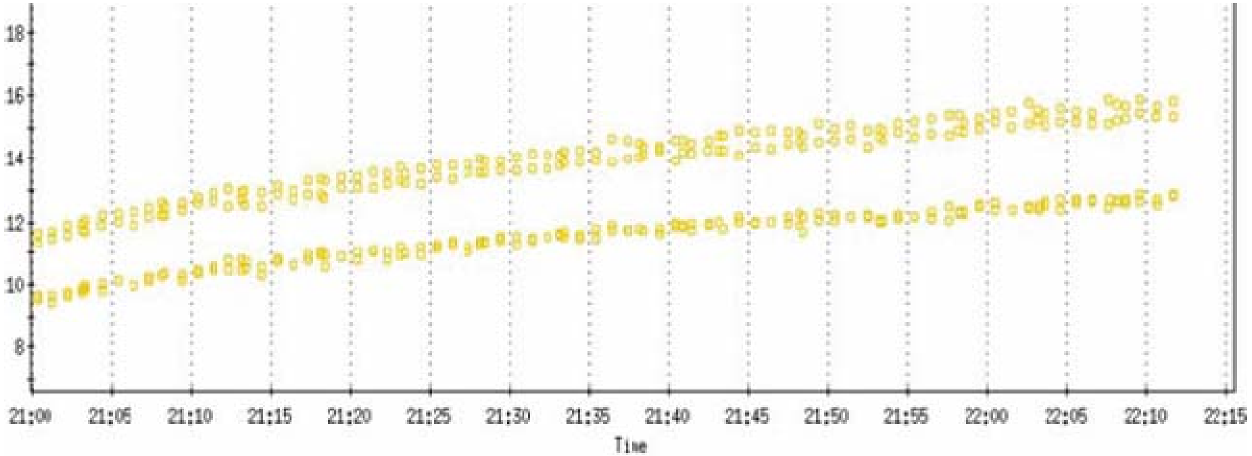
- Intra Beam Scattering experiments at RHIC
  - 5-hour store of gold beam
    - Emittance grows by more than a factor 4, intensity loss of ≈40% escaping RF bucket (de-bunching), luminosity decreases by a factor 10 from start to end (transverse emittance growth, longitudinal growth & beam loss due to RF voltage limitation)



Beam loss in the blue & yellow rings (top) luminosity decrease at different interaction points (bottom) of a $^{197}Au^{79+}$ beam (≈5 hours per fill)

Beyond Piwinski & Bjorken-Mtingwa: IBS theories, codes, and benchmarking **Jie Wei**

- Intra Beam Scattering experiments at RHIC
  - IBS beam experiment diagnostics
    - Ionization profile monitor, simultaneous measurement of emittance on different bunches



Normalized, 95% vertical emittance growths (≈30%) for 2 different intensities of 6×10^8 and 3×10^8 per bunch in the yellow ring (≈70 minutes)

    - Conventional Gaussian-beam models foresees rms beam size growth rates
    - Fokker-Planck approach foresees beam de-bunching loss

Beyond Piwinski & Bjorken-Mtingwa: IBS theories, codes, and benchmarking **Jie Wei**

- Summary
  - The mechanism of intra-beam scattering is well understood.
  - The theory of Bjorken-Mtingwa and Piwinski is usually good within a factor of 2 in growth rates under proper conditions (Gaussian distribution, coupling …)
  - Several efforts were made as an extension or beyond these theories
    - Approximate/analytical formulae and scaling laws
    - Fokker-Planck solver for the longitudinal phase space (tail, loss, hollow bunch …)
    - Molecular dynamics method for ultra-low emittance beams
  - Benchmarking is satisfactory given measurement and machine uncertainties

Beyond Vlasov Maxwell: Space-Time Correlations Gabriele Bassi

- Outline
  - IBS is an important collective effect in beam dynamics
  - IBS may degrade the beam quality and cause emittance growth
  - A limitation of existing IBS models: assumes Gaussian beams, not self-consistent
  - Proposed method and future work: study IBS within the framework of non-equilibrium statistical mechanics
    - » **Klimontovich approach** (originally established to study the nonlinear brownian motion, characterized by nonlinear dissipative forces)
    - » **BBGKY hierarchy** (Bogoliubov, Born, Green, Kirkwood, Yvon) : space correlations, corrections to the Vlasov-Poisson equation
      - Time evolution of the N-particle distribution function in terms of the (N+1)-particle distribution function
    - » **Generalized BBGKY hierarchy** : space-time correlations, corrections to the Vlasov-Maxwell equation (approximate account of retardation effects)
      - Calculations of Coherent Synchrotron Radiation (CSR) effect from curved orbits on bunches moving through bunch compressors done using a 4-D Vlasov-Maxwell approach