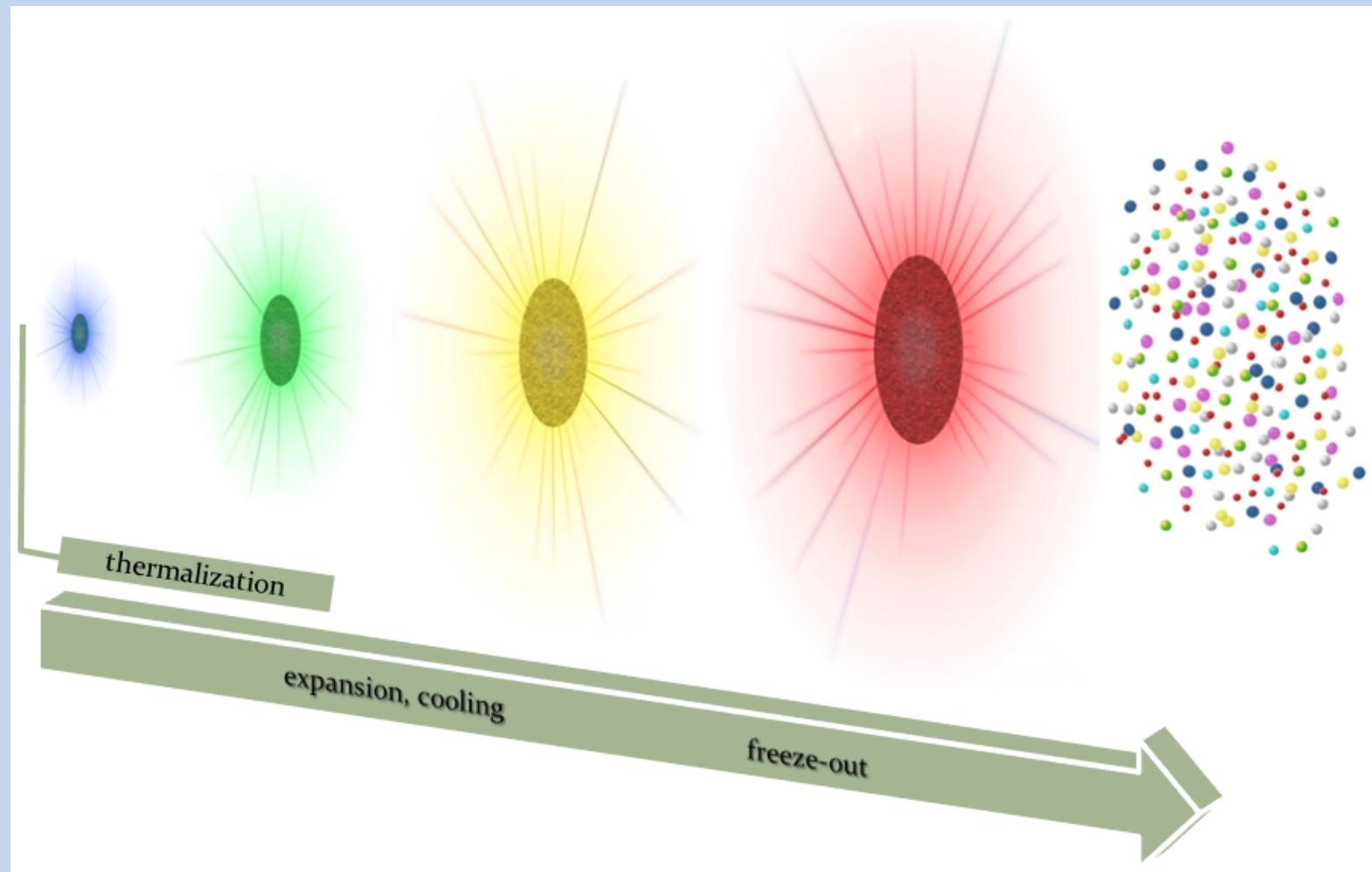


Máté Csanád, Márton Nagy, Sándor Lökös
Eötvös University, Budapest, Hungary

Hydrodynamics

- Thermal spectra, momentum anisotropy v_2 , scaling correlation radii: collective dynamics of sQGP is observed at RHIC
- Hydrodynamics applicable in high energy coll. PHENIX, Nucl. Phys. A **757**, 184 (2005)
- After thermalization: expansion & cooling
- Hadrons created at the freeze-out:



- Equations of hydro: highly non-linear, not straightforward to solve
- Exact, analytic solutions:** important to determine initial and final state
- Famous 1+1D solutions:
Landau, Izv. Acad. Nauk SSSR **17**, 51 (1953)
Hwa, Phys. Rev. D **10**, 2260 (1974)
Bjorken, Phys. Rev. D **27**, 40 (1983)
- Revival of interest, many new solutions, mostly 1+1D, few 1+3D
- Using constant Equation of State (EoS)!
- Time evolution of sQGP?
- Temperature dependent EoS?

Equations of hydrodynamics

With a conserved charge

- There may be a **conserved charge n**
- Energy-momentum tensor in perfect fluid:
 $T^{\mu\nu} = (\epsilon + p)u^\mu u^\nu - pg^{\mu\nu}$
- Basics: **continuity & en.-mom. conservation**
 $\partial_\mu(nu^\mu) = 0$ and $\partial_\nu T^{\mu\nu} = 0$
- EoS: $\epsilon = \kappa p$, and $c_s^2 = 1/\kappa$, if both const.
- This is a **full set of equations for u^μ , n and p**
- Temperature equation from $p = nT$
 $T\partial_\mu u^\mu + \kappa u^\mu \partial_\mu T = 0$
- Write up **solutions for $\{u^\mu, n, T\}$**

Without a conserved charge

- Let us introduce **entropy density σ**
- Fundamental thermodynamical relations:
 $\epsilon + p = T\sigma \Rightarrow d\epsilon = Td\sigma$ and $dp = \sigma dT$
- Same continuity equation for σ follows:**
 $\partial_\nu(\sigma u^\nu) = 0$,
- Same EoS, but **different $p - T$ relation!**
 $\epsilon = \kappa p$ and $p = T\sigma/(\kappa + 1)$
- If $\kappa = \text{const.}$: **same temperature-equation**
- Solutions valid for $\{u^\mu, n, T\}$ and $\{u^\mu, \sigma, T\}$

Solutions with arbitrary EoS?

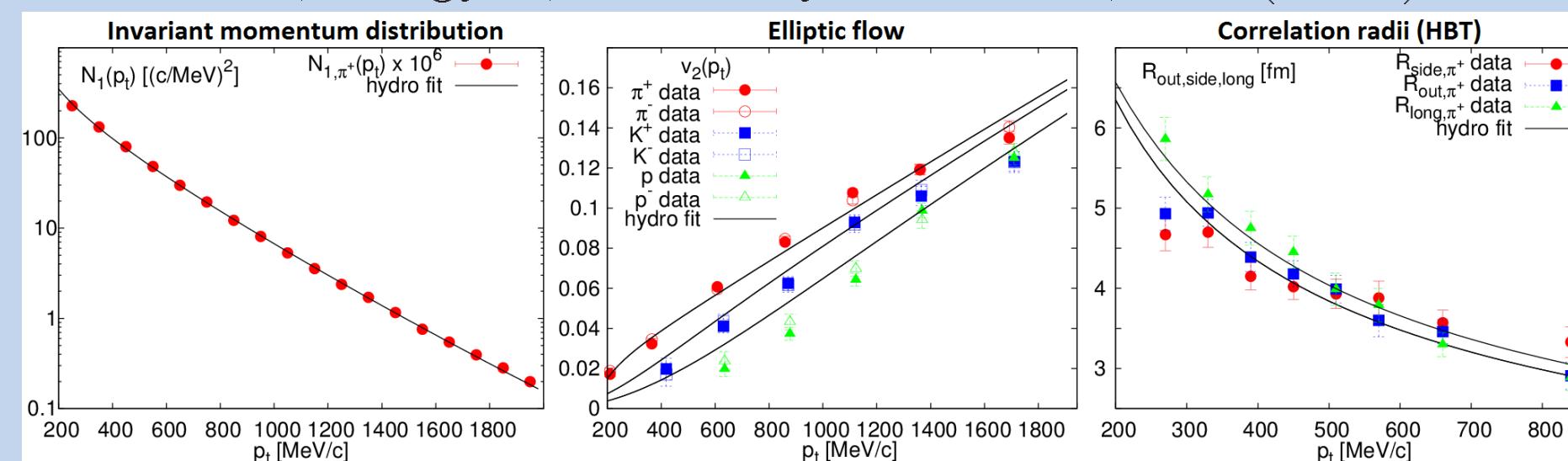
- Constant EoS **may not be realistic**
- If $\kappa(T)$, **new solutions** have to be found
- With n , the temperature equation is:
 $T\partial_\mu u^\mu + d(\kappa(T)T)/dT \cdot u^\mu \partial_\mu T = 0$.
- Works only if $d(\kappa(T)T)/dT > 0$!
- In case of no conserved charges:
 $T\partial_\mu u^\mu + \left[\kappa + \frac{T}{\kappa + 1} \frac{d\kappa(T)}{dT} \right] u^\mu \partial_\mu T = 0$,
- Remarkable: not the same, if $\kappa \neq \text{const.}$
- If u^μ known, **solve for arbitrary $\kappa(T)$!**

A known $\kappa = \text{const.}$ solution

- First exact, analytic 3D relativistic solution Csörgő *et al.*, Heavy Ion Phys. **A21**, 73 (2004)
- $u^\mu = \frac{x^\mu}{\tau}$, $\tau = \sqrt{x_\mu x^\mu}$, $n = n_0 \frac{V_0}{V} \nu(s)$, $V = \tau^3$
- $T = T_0 \left(\frac{V_0}{V} \right)^{1/\kappa} \frac{1}{\nu(s)}$, $s = \frac{r_x^2}{X^2} + \frac{r_y^2}{Y^2} + \frac{r_z^2}{Z^2}$
- $\nu(s)$ arbitrary function of scaling variable s
- X, Y, Z : principal axes of expanding ellipsoid
- $X(t) = \dot{X}_0 t$, $Y(t) = \dot{Y}_0 t$, $Z(t) = \dot{Z}_0 t$
- Non-accelerating, ie. $u^\nu \partial_\nu u^\mu = 0$.
- Can be written up for σ (i.e. no conserved n)
- Constant κ !**

Hadronic observables

- Hadronic source from the above solution:
 $S(x, p)d^4x = \mathcal{N} \frac{p_\mu d^3\Sigma^\mu(x) H(\tau) d\tau}{n(x) \exp(p_\mu u^\mu(x)/T(x)) - 1}$
- Observables calculable (spectra, flow, HBT)
- Compared to PHENIX data successfully
Csanád, Vargyas, Eur. Phys. J. A **44**, 473 (2010)

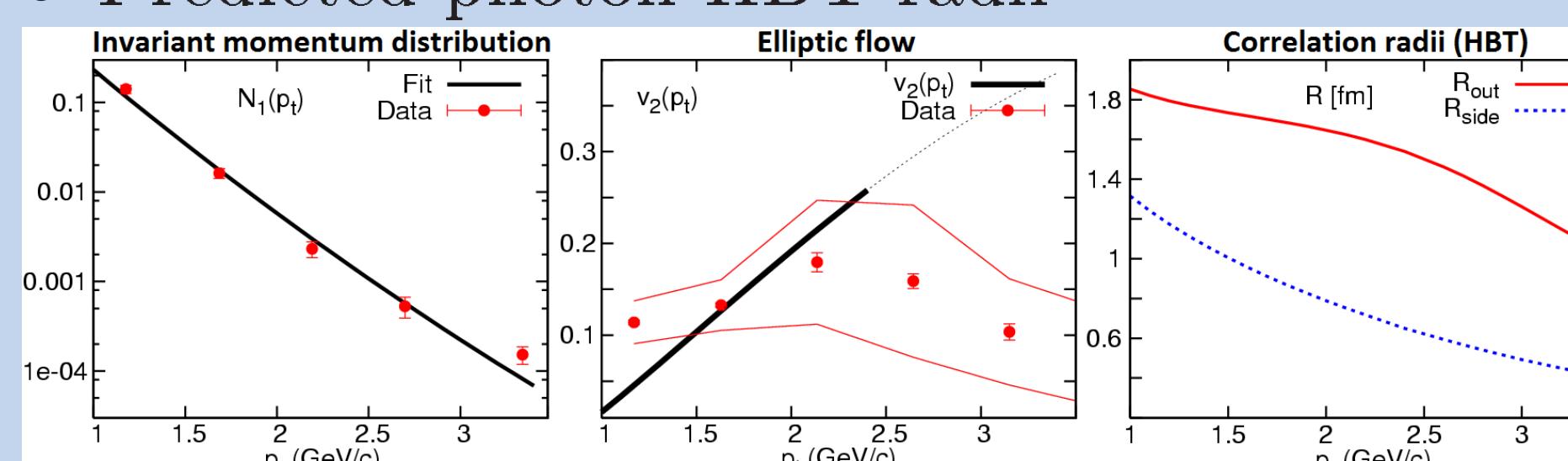


- Hadronic data describe the final state
- Freeze-out state fixed from these fits
- Different EoS lead to different initial states!

Csanád, Nagy, Csörgő, Eur. Phys. J. ST, 19 (2008)

Direct photon observables

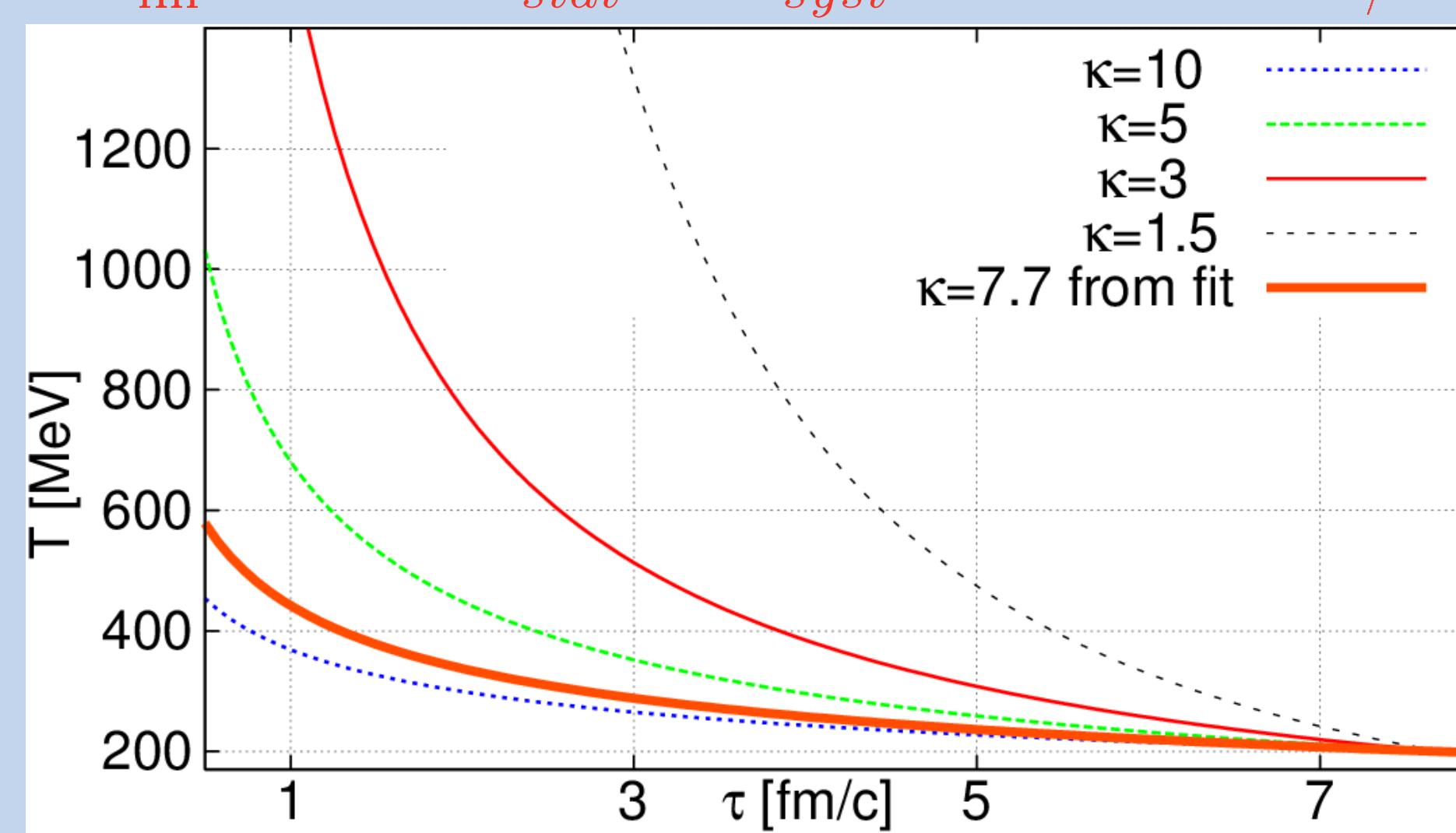
- Photons are created throughout the evolution
- Their distribution reveals info about EoS!
- Photon source from the above solution:
 $S(x, p)d^4x = \mathcal{N} \frac{p_\mu u^\mu}{\exp(p_\mu u^\mu(x)/T(x)) - 1} d^4x$
- Integrated over momentum: emission $\propto T^4$
- Analyzed T power dependence, based on rate($A + B \rightarrow X$) = $n_A n_B \langle v \sigma_{A+B \rightarrow X} \rangle \propto T^6$
- Varying T power gives systematic uncertainty
- Observables calculable (spectra, flow, HBT)
Csanád, Májer, Central Eur. J. Phys. **10** (2012)
- Compared to PHENIX data successfully
- Predicted photon HBT radii



- Average EoS: $c_s = 0.36 \pm 0.02_{\text{stat}} \pm 0.04_{\text{syst}}$
- $\tau_{\text{ini}} \approx 0.7 \text{ fm}/c$, $\tau_{\text{final}} = 7.7 \text{ fm}/c$

Time evolution for $\kappa = \text{const.}$

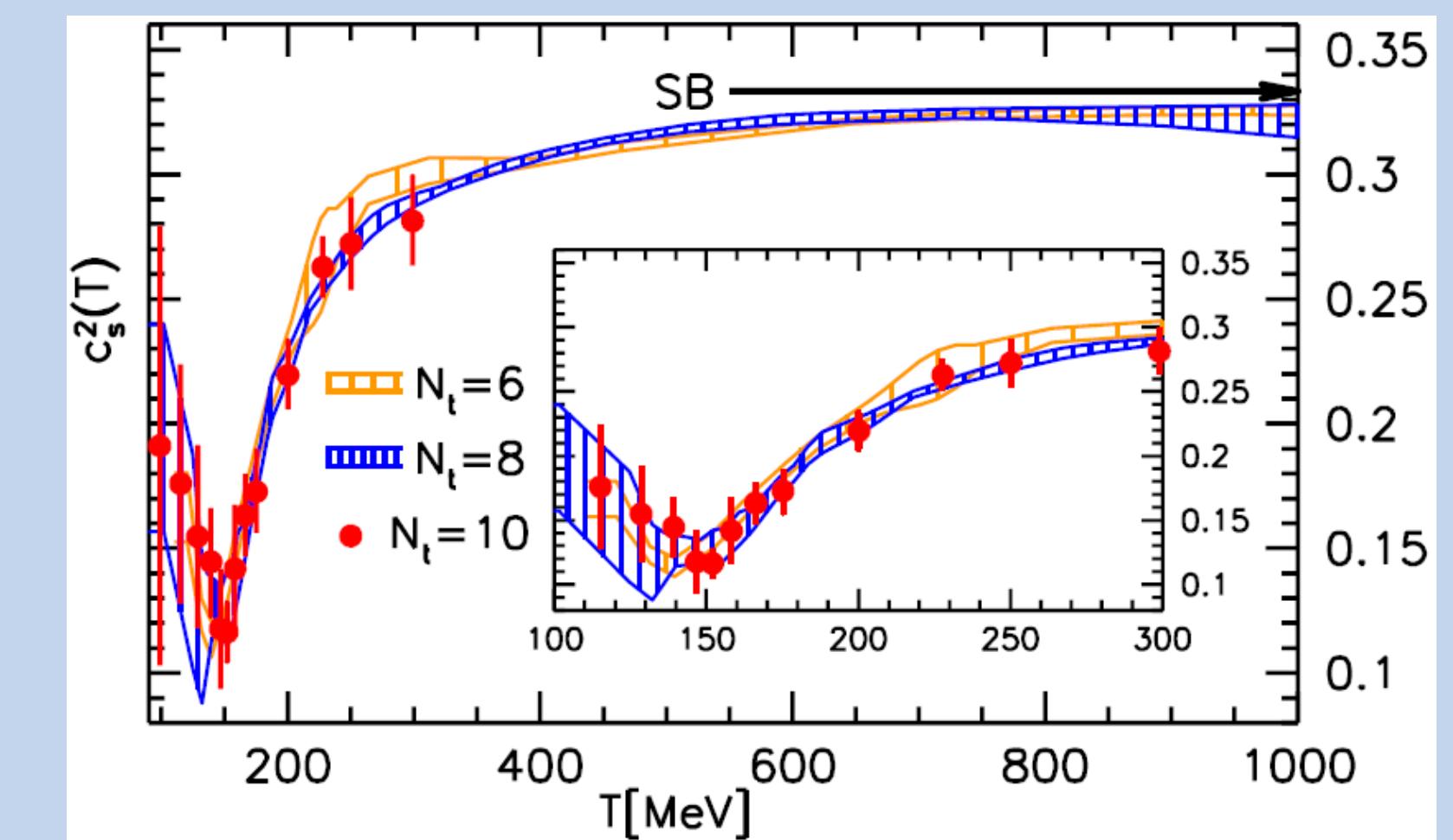
- The $c_s = 0.36$ result means $\kappa = 7.7$
- $T_{\text{ini}}: 507 \pm 12_{\text{stat}} \pm 90_{\text{syst}} \text{ MeV}$ at $0.7 \text{ fm}/c$



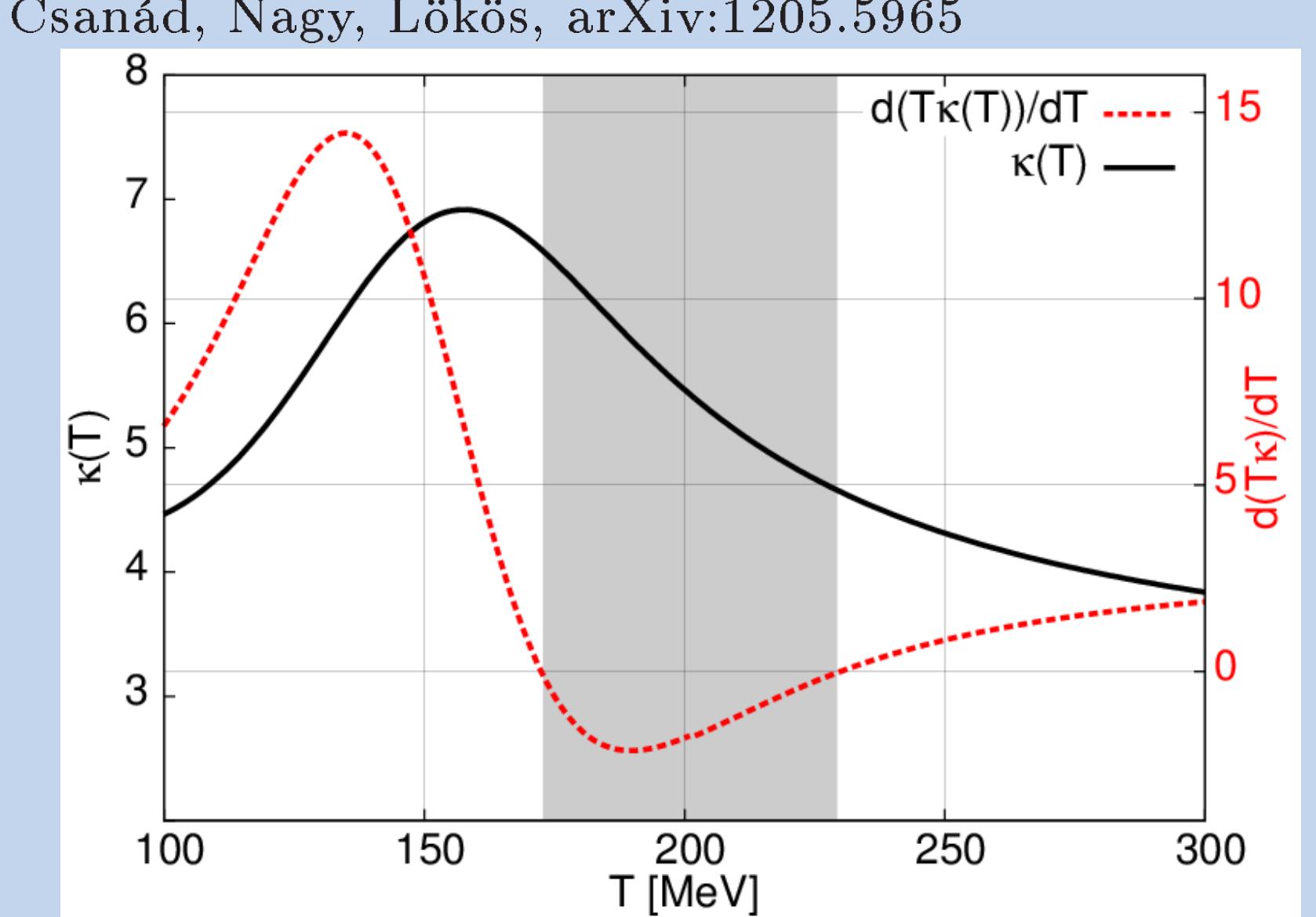
Csanád, Vargyas, Eur. Phys. J. A **44**, 473 (2010)

A lattice QCD EoS

- Physical quark masses, continuum limit EoS Borsányi, Fodor, Katz *et al.* JHEP **1011**, 077 (2010)



- Sound speed $c_s^2 = \frac{dp}{de}$ given, $\kappa = \frac{e}{p}$ needed
- Trace anomaly $I(T)/T^4$ is parametrized
- Pressure is given by $\frac{p(T)}{T^4} = \int \frac{dT}{T} \frac{I(T)}{T^4}$
- From $I = \epsilon - 3p \Rightarrow \kappa = I/p + 3$
- EoS in form of $\kappa(T)$ analytically given**
Csanád, Nagy, Lökös, arXiv:1205.5965



- Problem: $\frac{d(\kappa(T)T)}{dT} \leq 0$ for $T = 173 - 225 \text{ MeV}$
- Conserved n not compatible with this EoS!**

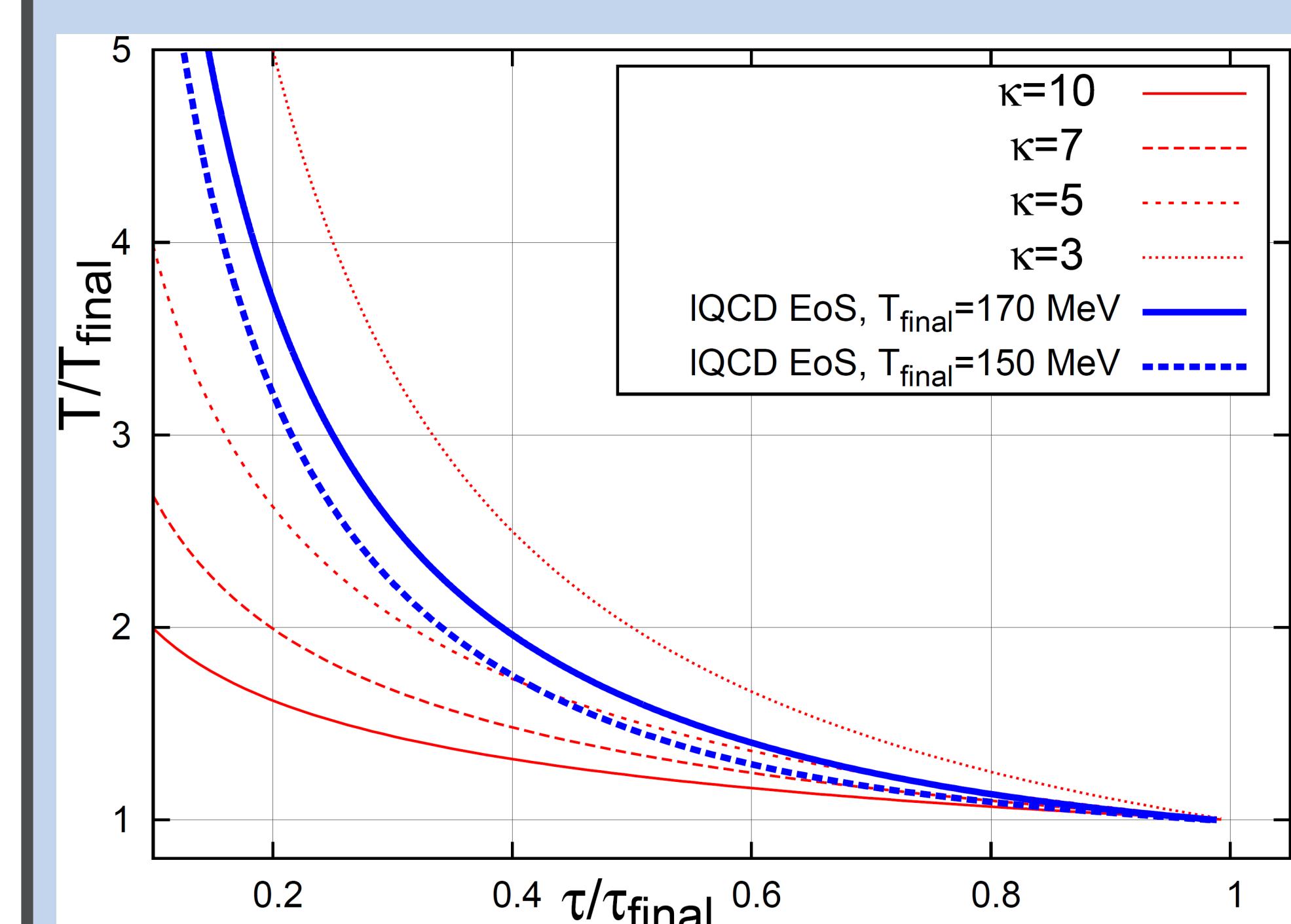
New solutions

- A new solution with conserved charge n :
Csanád, Nagy, Lökös, EPJ A, arXiv:1205.5965

$$n = n_0 \frac{\tau_0^3}{\tau^3} \quad u^\mu = \frac{x^\mu}{\tau},$$

$$\frac{\tau_0^3}{\tau^3} = \exp \int_{T_0}^T \left(\frac{1}{\beta} \frac{d\kappa(\beta)\beta}{d\beta} \right) d\beta$$

- $\kappa(T)$ arbitrary
- If $\frac{d(\kappa(T)T)}{dT} \leq 0$: ill-defined
- A new solution without conserved n :
 $\sigma = \sigma_0 \frac{\tau_0^3}{\tau^3}$, $u^\mu = \frac{x^\mu}{\tau}$,
- $\frac{\tau_0^3}{\tau^3} = \exp \int_{T_0}^T \left(\frac{\kappa(\beta)}{\beta} + \frac{1}{\kappa(\beta) + 1} \frac{d\kappa(\beta)}{d\beta} \right) d\beta$
- Let's assume the above 1QCD EoS
- $T(t)$ can be calculated:



- Result applicable widely
- Plug in $\tau_{\text{final}}/\tau_{\text{ini}}$, get $T_{\text{final}}/T_{\text{ini}}$
- Other solutions with $\kappa(p)$ possible:

$$\sigma = \sigma_0 \frac{\tau_0^3}{\tau^3}, \quad u^\mu = \frac{x^\mu}{\tau},$$

$$\frac{\tau_0^3}{\tau^3} = \int_{p_0}^p \left(\frac{\kappa(\beta)}{\beta} + \frac{1}{\kappa(\beta) + 1} \frac{d\kappa(\beta)}{d\beta} \right) \frac{d\beta}{\kappa(\beta) + 1}$$