Multi-turn Extraction: Splitting the PS Beam in Transverse Phase Space

An Update on my Progress

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Importance of my Project

- Big Picture: PS $\rightarrow$ SPS beam transfer for fixed target experiments.

  Nonlinear magnetic fields create stable islands in the beam's horizontal phase space, separating the beam into parts for clean extraction.

- Goal: Understand/predict population ratios of separated beams.

Current Status

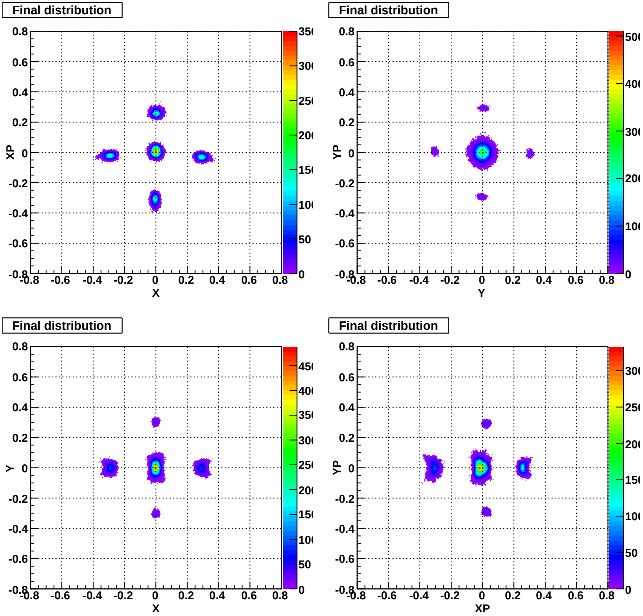
- My Project:

  - ✓ Get accustomed to the simulations and the code.
  - ✓ Learn *Mathematica* in order to analyze their output.
  - ✓ Become acquainted with the theory of normal forms.
  - ✓ Study some fundamentals of beam dynamics.

  - ✓ Optimize beam-splitting in 2 and 4 dimensions using simulations.
  - ✓ Investigate simple Hamiltonian to understand trapping.

  - … Predict trapping fractions for realistic Hamiltonian.

Beam Splitting



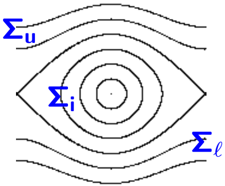
Particle Trapping

- We make use of the **adiabatic invariant**, since we cannot solve the system

$$\mathcal{H} = \frac{1}{2}\left[p - \delta(t)\right]^2 - \left[1 + \beta(t)\right] \cos q \, .$$

- Adiabatic trapping is a quasi-**random** process:

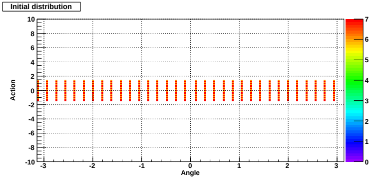
$$P(u \to i) = \frac{\dot{\Sigma}_i}{\dot{\Sigma}_i + \dot{\Sigma}_\ell}$$

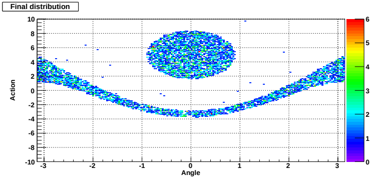


- When (**say at $\mathbf{t}_*$**) the area under the separatrix matches the area (**say $\boldsymbol{\Sigma}$**) under a particle's trajectory, the particle meets the separatrix.
- With $\rho(\Sigma)\, d\Sigma$ particles having orbit-areas between Σ and $\Sigma + d\Sigma$,

$$\#\text{ particles captured at } t_* \;=\; \frac{\dot{\Sigma}_i}{\dot{\Sigma}_i + \dot{\Sigma}_\ell}\, \rho(\Sigma)\, d\Sigma \, .$$

Predicting Trapping Fractions



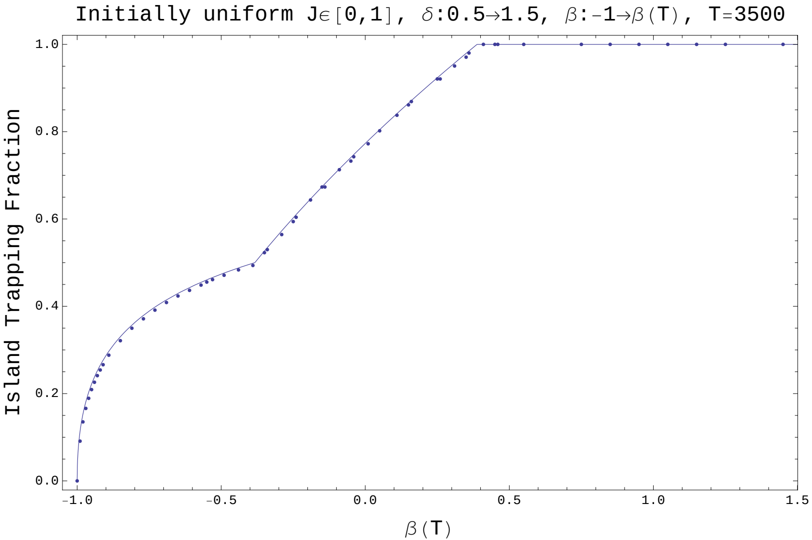


- If δ varies linearly $(\,1/2 \to 3/2\,)$ and β quadratically $(\,-1 \to \beta(T)\,)$ in

$$\mathcal{H} = \frac{1}{2}\,[p - \delta(t)]^2 - [1 + \beta(t)]\,\cos q \qquad \text{then}\ldots$$

$$\text{Trapping Fraction} = \begin{cases} \dfrac{4\sqrt{1+\beta(T)}}{\pi + 4\sqrt{1+\beta(T)}} & \text{for} \quad -1 \le \beta(T) \le \frac{\pi^2}{16} - 1\,, \\ \frac{4}{\pi}\,\sqrt{1+\beta(T)} - \frac{1}{2} & \text{for} \quad \frac{\pi^2}{16} - 1 \le \beta(T) \le \frac{9\pi^2}{64} - 1\,, \\ 1 & \text{for} \quad \beta(T) \ge \frac{9\pi^2}{64} - 1\,. \end{cases}$$

Agreement with Simulations



What's Next?

- Carry out the same technique with the realistic Hamiltonian:

$$\mathcal{H} = \alpha\, p + \beta\, p^2 + \gamma\, p^2 \cos(4q)\,.$$