

Parton fragmentation processes:
in the vacuum and in the medium

ECT* (Trento), 25-29 February 2008

Theory of (extended) Dihadron Fragmentation Functions (ext)DiFF

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Pavia

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Outline

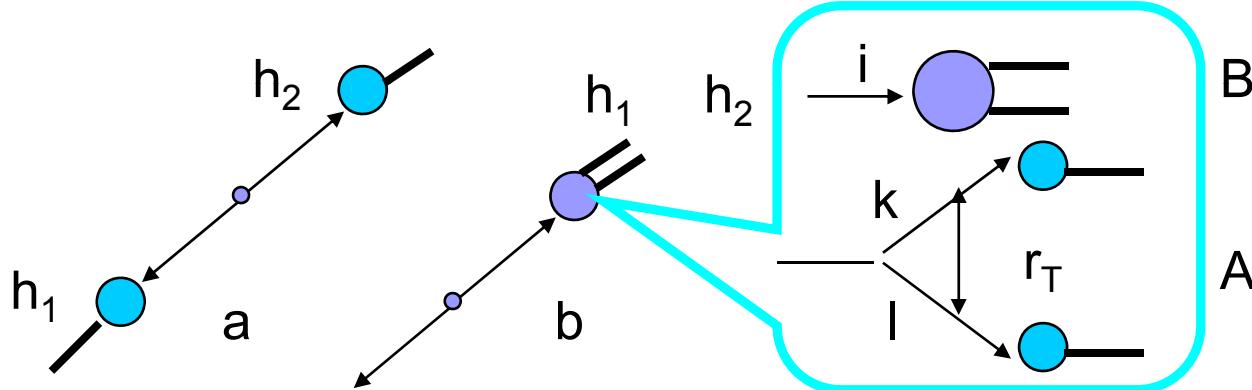
- History: why **DiFF** ?



Need **DiFF** in $e^+e^- \rightarrow h_1 h_2 X$

$$\frac{d\sigma^{h_1 h_2}}{dz_1 dz_2 dQ^2} = \sum_{kl} \hat{\sigma}_{kl}(Q^2) \otimes D^{k \rightarrow h_1}(z_1, Q^2) \otimes D^{l \rightarrow h_2}(z_2, Q^2) \quad A$$

$$+ \sum_i \hat{\sigma}_i(Q^2) \otimes D^{i \rightarrow h_1 h_2}(z_1, z_2, Q^2)$$



Konishi, Ukawa, Veneziano
P.L. **B78** (78) 243

	a	b
B	A	B
O(α_s^0)	A	B
O(α_s)	A	A+B

de Florian & Vanni,
P.L. **B578** (04) 139

1/r_T collinear singularities cancelled by $D(i \rightarrow h_1 h_2)$

time-like analogue of
Fracture Functions in SIDIS

Trentadue & Veneziano,
P.L. **B323** (94) 201

DiFF evolution equations

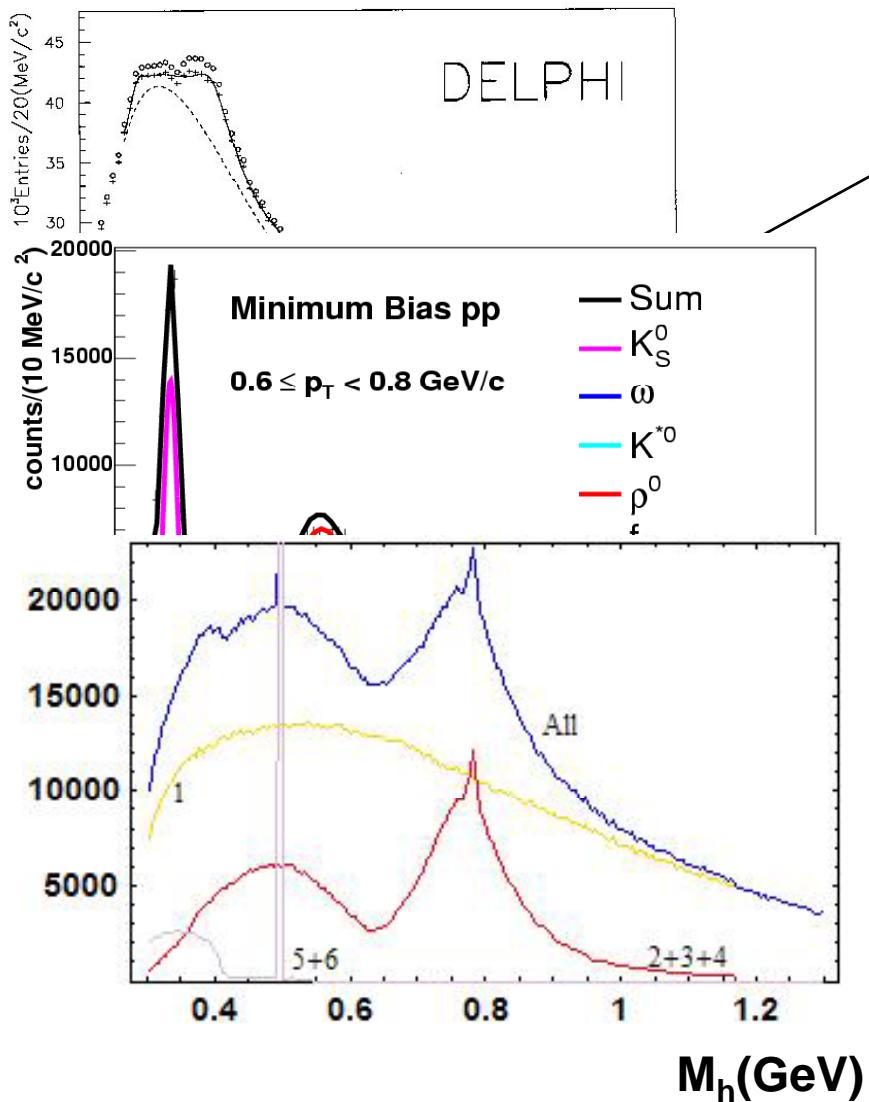
$$\frac{d}{d \log Q^2} D^{i \rightarrow h_1 h_2} = \frac{\alpha_s}{2\pi} [D^{k \rightarrow h_1 h_2} \otimes P_{ki} + D^{k \rightarrow h_1} \otimes D^{l \rightarrow h_2} \otimes \hat{P}_{kl}^i]$$

B

A

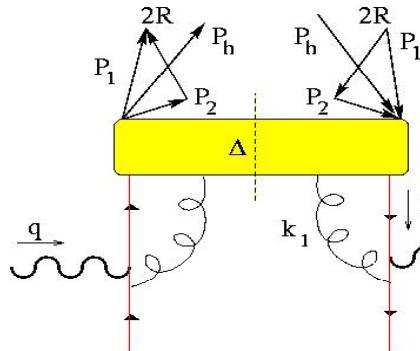
see also de Majumder & Wang,
P.R. **D70** (04) 014007; **D72** (05) 034007

Exp. info mostly on nontrivial spectrum in pair invariant mass M_h



Leading-twist analysis: full dependence of **DiFF**

$$\Delta(k, P_h, R) = \sum_X \int \frac{d^4\xi}{(2\pi)^4} e^{ik\cdot\xi} \langle 0 | U_{[\infty, \xi]}^T U_{[-\infty, \xi]}^+ \psi(\xi) | P_1, P_2, X \rangle \langle P_1, P_2, X | \bar{\psi}(0) U_{[0, -\infty]}^+ U_{[0, \infty]}^T | 0 \rangle \Big|_{\xi^- = 0}$$



Bianconi *et al.*,
P.R. D62 (00) 034008

Subleading-twist
Bacchetta & Radici,
P.R. D69 (04) 074026

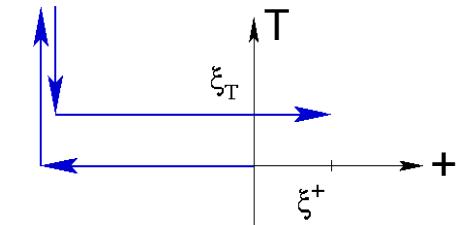
Radici *et al.*,
P.R. D65 (02) 074031
Bacchetta & Radici
P.R. D74 (06) 114007

$$P_h = P_1 + P_2 \\ R = (P_1 - P_2)/2$$

$$z_1 + z_2 = \frac{P_1^- + P_2^-}{k^-} = \frac{P_h^-}{k^-} \equiv z$$

$$dk_T$$

$$\Delta^{(\Gamma)}(z_1, z_2, k_T, R_T) = \frac{1}{4z} \int dk^+ \text{Tr} [\Gamma \Delta(k, P_h, R)] \Big|_{k^- = P_h^- / z}$$



$$\Delta^{[\gamma^-]} \rightarrow D_1 = \bullet \rightarrow \circlearrowleft$$

jet handedness

Efremov *et al.*

P.L. B284 (92) 394

$$D = \frac{N_R - N_L}{N_R + N_L} \propto P_{\vec{q}}$$

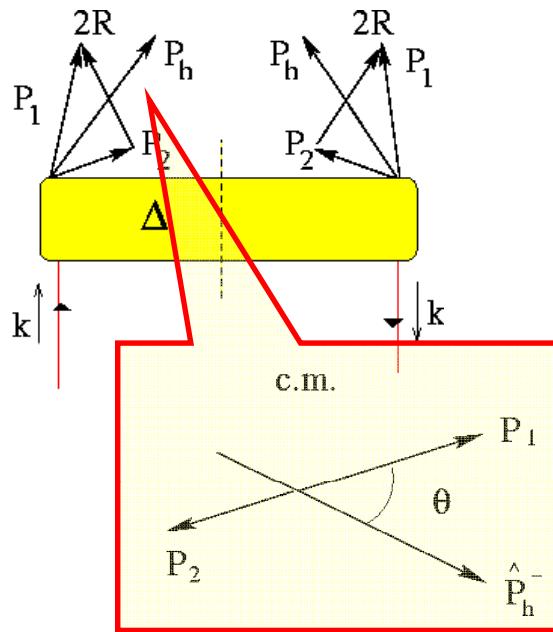
$$>/< 0$$

$$\Delta^{(\gamma^- \gamma_5)} \rightarrow (\mathbf{k}_T \times \mathbf{R}_T) \mathbf{G}_1^\perp = \left(\bullet \rightarrow \circlearrowleft \right) - \left(\bullet \rightarrow \circlearrowright \right)$$

$$\Delta^{(i\sigma^i - \gamma_5)} \rightarrow (\mathbf{S}_T^q \times \mathbf{k}_T) H_1^\perp + (\mathbf{S}_T^q \times \mathbf{R}_T) H_1^\not\perp \left(\bullet \rightarrow \circlearrowleft \right) - \left(\bullet \rightarrow \circlearrowright \right)$$

$$\frac{1}{4\pi} D_1(z_1, z_2, R_T^2), \quad \frac{\epsilon_T^{ij} R_{Tj}}{4\pi M_h} H_1^\not\perp(z_1, z_2, R_T^2) \quad R_T^2 = \frac{z_1 z_2}{z_1 + z_2} \left[\frac{M_h^2}{z_1 + z_2} - \frac{M_1^2}{z_1} - \frac{M_2^2}{z_2} \right]$$

Partial wave expansion



$$\frac{1}{4\pi} D_1(z_1, z_2, R_T^2), \quad \frac{\epsilon_T^{ij} R_{Tj}}{4\pi M_h} H_1^\Delta(z_1, z_2, R_T^2)$$

$$[z_1, z_2] \rightarrow [z] = z_1 + z_2, [\zeta] = \frac{z_1 - z_2}{z} = a(M_h) + b(M_h) \cos\theta$$

DiFF($z, \zeta(\cos\theta), M_h^2$) = \sum_n **DiFF**_n(z, M_h^2) **P**_n($\cos\theta$)

(1) \longleftrightarrow (2)

partial wave (LM)

Bacchetta & Radici,
P.R. D67 (03) 094002

$$\Delta(k, P_h, R) = \int_X \int \frac{d^4\xi}{(2\pi)^4} e^{ik\cdot\xi} \langle 0 | \psi(\xi) | P_1, P_2, X \rangle \langle P_1, P_2, X | \bar{\psi}(0) | 0 \rangle$$

$$|(\pi\pi)_{L=0}\rangle \quad |(\pi\pi)_{L=0}\rangle$$

Collins *et al.*,
N.P. B420 (94) 565

$$|(\pi\pi)_{L=1}\rangle \quad |(\pi\pi)_{L=1}\rangle$$

diagonal part →

$$D_1^{ss+pp}(z, M_h^2)$$

Im [interference] → naïve T-odd

$$H_1^{\Delta sp}(z, M_h^2)$$



Naive T - reversal transformation

$$|a\rangle = \begin{array}{c} \text{yellow circle} \\ \uparrow \\ \longrightarrow \end{array}$$

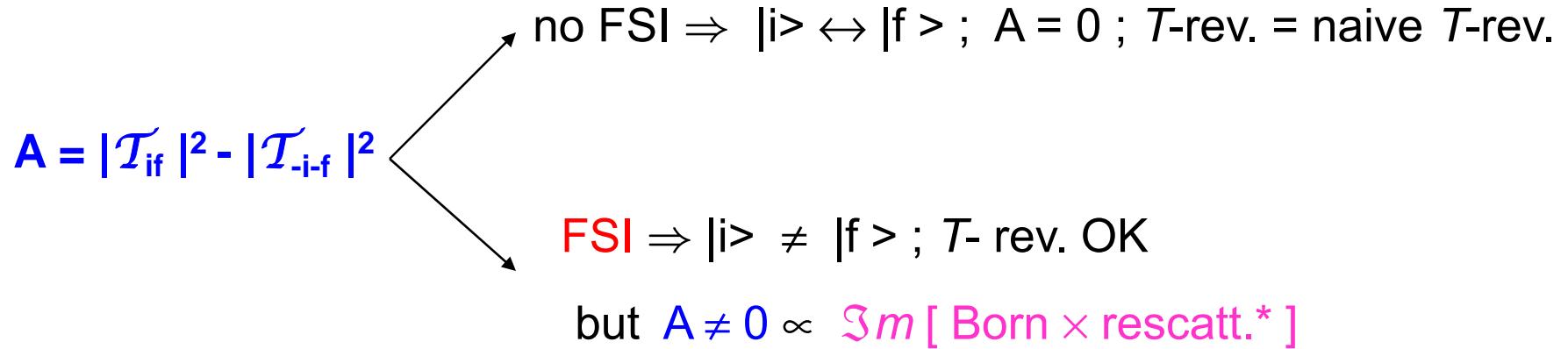
system with some spin and momentum

$$|-a\rangle = \begin{array}{c} \text{yellow circle} \\ \downarrow \\ \longleftarrow \end{array}$$

flipping spin and momentum

$|i\rangle, |f\rangle$ initial, final states of the system; \mathcal{T}_{if} trans. matrix; T -rev. $\rightarrow |\mathcal{T}_{if}|^2 = |\mathcal{T}_{-f-i}|^2$

naive T - reversal transformation : \mathcal{T}_{-i-f}



Outline

- History: why **DiFF** ?



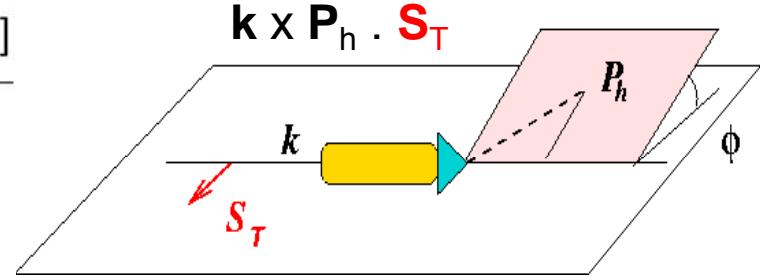
Spin analyzers
- SIDIS: extract **transversity**
with leading-twist
Single-Spin Asymmetry (SSA)
via **DiFF**
(HERMES, COMPASS)

Single-Spin Asymm. (SSA) in $e+p^\uparrow \rightarrow e'+\pi+X$: from Collins effect...

$$A_{UT}^{\sin \phi} = \frac{\int d\mathbf{P}_{hT} |\mathbf{P}_{hT}| \sin \phi [d\sigma(p^\uparrow) - d\sigma(p^\downarrow)]}{\int d\mathbf{P}_{hT} [d\sigma(p^\uparrow) + d\sigma(p^\downarrow)]}$$

$$\propto \frac{\sum_q e_q^2 h_1^q(x) H_1^{\perp q(1)}(z)}{\sum_q e_q^2 f_1^q(x) D_1^q(z)}$$

$$H_1^{\perp(1)}(z) = \int d\mathbf{k}_T \frac{\mathbf{k}_T^2}{2M^2} H_1^{\perp}(z, \mathbf{k}_T)$$

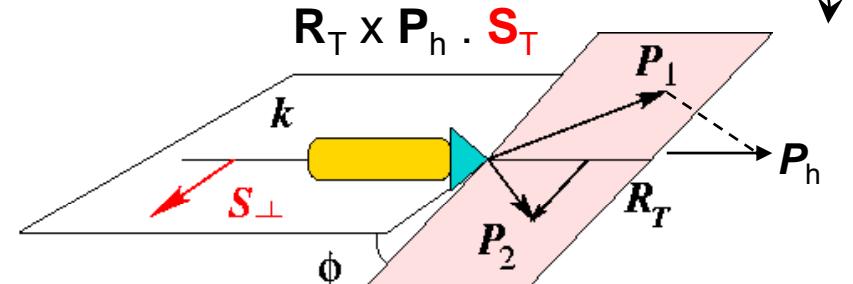


$$\int d\mathbf{k}_T$$

... to the **DiFF** effect in $e+p^\uparrow \rightarrow e'+(\pi\pi) + X$

$$A_{UT}^{\sin \phi} = \frac{1}{\sin \phi} \frac{\int d \cos \theta [d\sigma(p^\uparrow) - d\sigma(p^\downarrow)]}{\int d \cos \theta [d\sigma(p^\uparrow) + d\sigma(p^\downarrow)]}$$

$$\propto \frac{\sum_q e_q^2 h_1^q(x) H_{1,q}^{\leftarrow sp}(z, M_h^2)}{\sum_q e_q^2 f_1^q(x) D_{1,q}^{ss+pp}(z, M_h^2)}$$

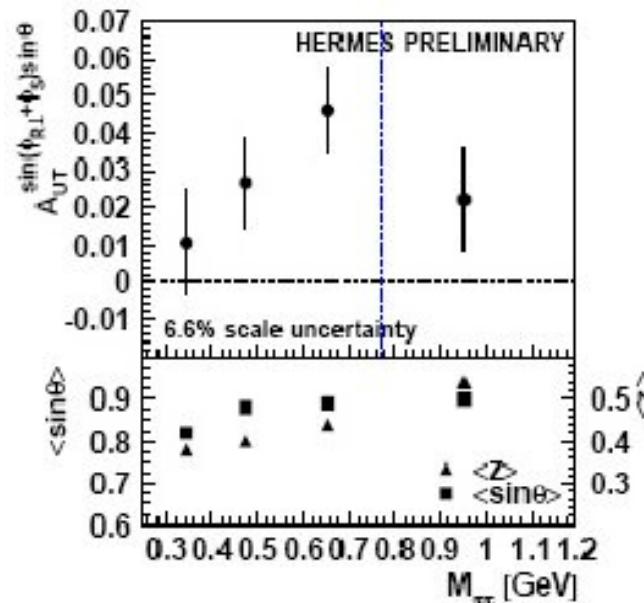


SSA measurements



$Q^2 > 1 \text{ GeV}^2$ $s = 56.2 \text{ GeV}^2$

Van der Nat
hep-ex/0512019



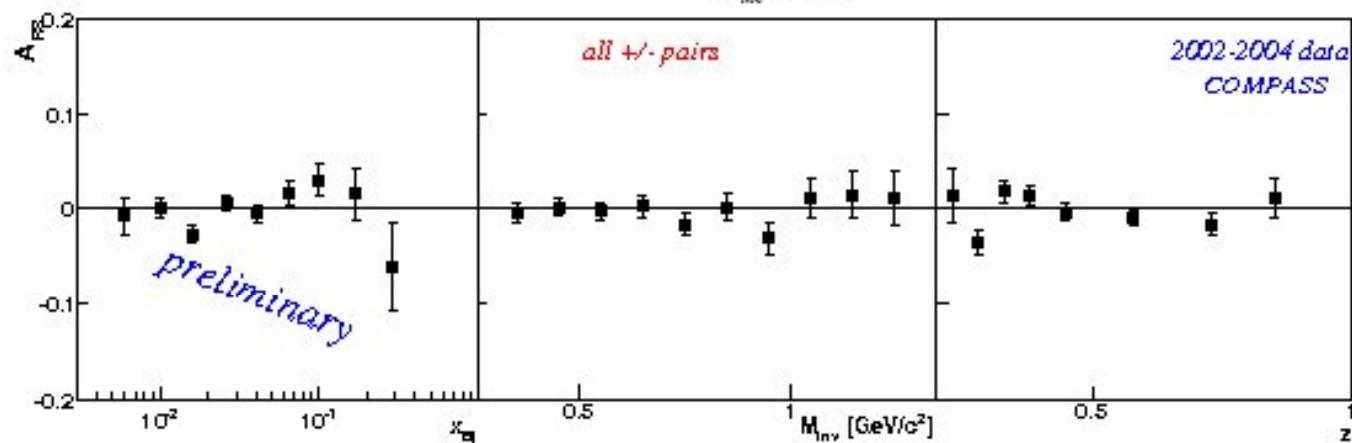
N.B. in Hermes analysis
no $\int d\cos\theta$ but
projection of $h_1^q H_{1,q}^{< s p}$
term
 $\Rightarrow A_{UT}^{\sin \phi \sin \theta}$

6.6% scale uncertainty



$Q^2 > 1 \text{ GeV}^2$
 $s = 604 \text{ GeV}^2$

Martin
hep-ex/0702002



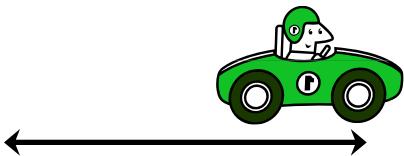
N.B. deuteron target: cancellation by isospin symmetry ?

Outline

- History: why **DiFF** ?

Spin analyzers

- SIDIS: extract **transversity** with leading-twist Single-Spin Asymmetry (SSA) via **DiFF** (HERMES, COMPASS)



- e^+e^- : determine unknown
DiFF
(BELLE)

- models of **DiFF**

DiFF from $e^+e^- \rightarrow (\pi^+\pi^-)_{jet\,1}(\pi^+\pi^-)_{jet\,2}X$

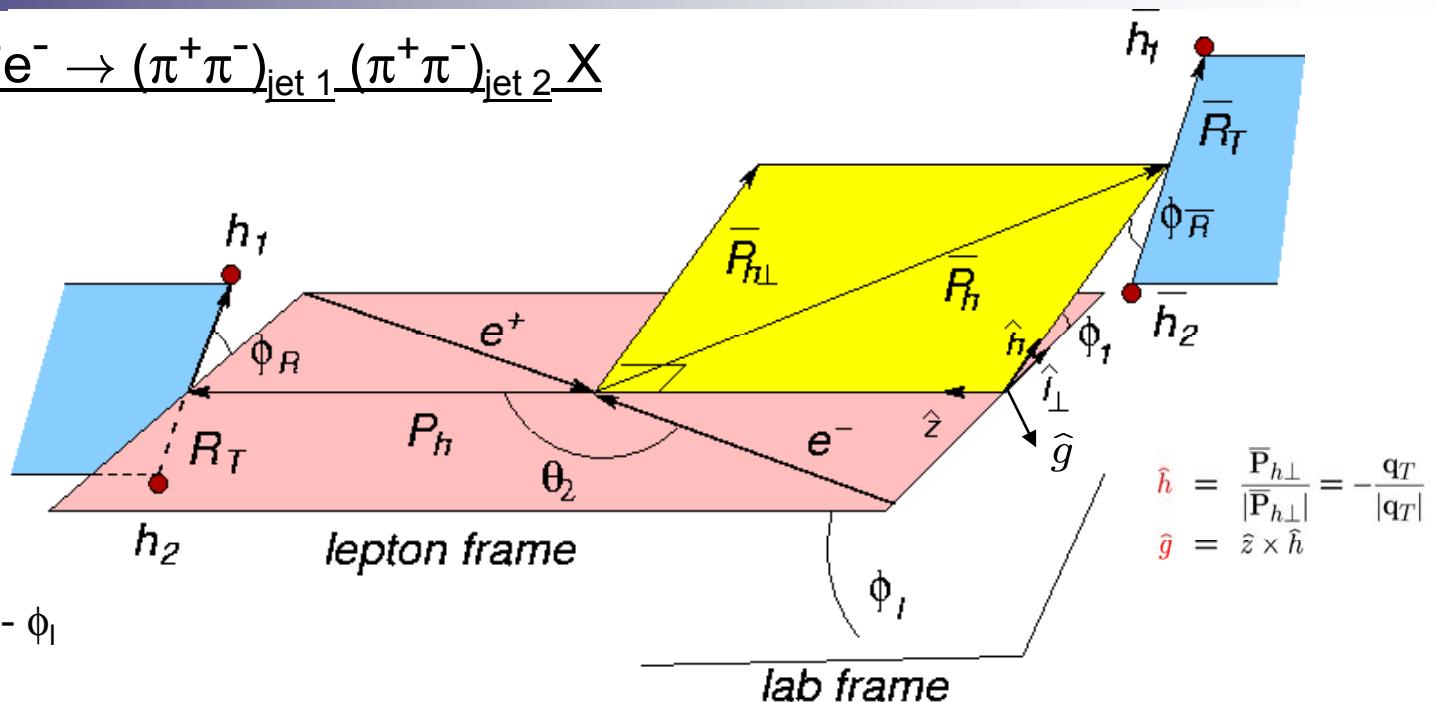
$$q^2 = (k + \bar{k})^2 = Q^2$$

$$\mathbf{P}_h \parallel \hat{z}, \quad \mathbf{P}_{h\perp} = 0$$

$$\overline{\mathbf{P}}_{h\perp} = -\bar{z}\mathbf{q}_T$$

$\swarrow \searrow$

$$[(P_h, R) \quad (P_h, k)] = \phi_R - \phi_l$$



leading-twist cross section $d\sigma$:

$$\int_0^{2\pi} \frac{d\phi_l}{2\pi} d\mathbf{q}_T \cos(\phi_R + \bar{\phi}_R - 2\phi_l) d\sigma \equiv d\sigma^{\langle \cos(\phi_R + \bar{\phi}_R - 2\phi_l) \rangle}$$

“Artru-Collins” azimuthal asymmetry :

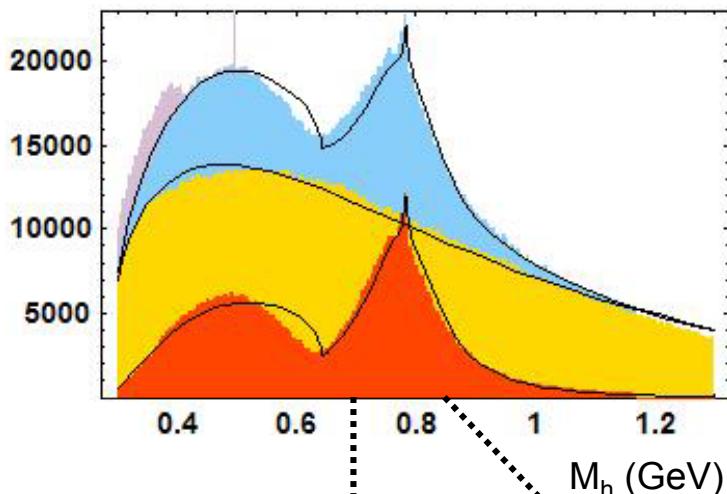
$$A_H = \frac{d\sigma^{\langle \cos(\phi_R + \bar{\phi}_R - 2\phi_l) \rangle}}{d\sigma^{(1)}} \propto \frac{\sum_q e_q^2 H_{1,q}^\Delta(z, M_h^2) H_{1,\bar{q}}^\Delta(\bar{z}, \bar{M}_h^2)}{\sum_q e_q^2 D_1^q(z, M_h^2) D_1^q(\bar{z}, \bar{M}_h^2)}$$

Boer, Jakob, Radici,
P.R. D67 (03) 094003

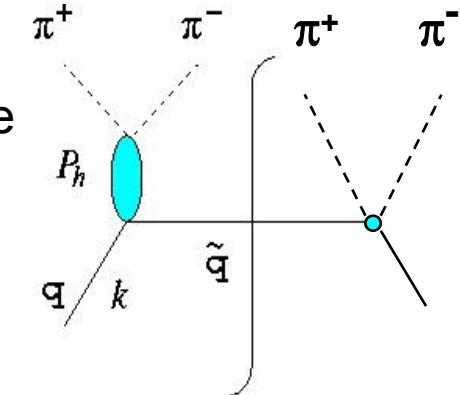
same as in SIDIS: check of universality
at leading twist

Survey of models: the spectator approx.

$$|P_1, P_2, X\rangle \sim |(\pi^+ \pi^-)_L, \tilde{q}\rangle$$



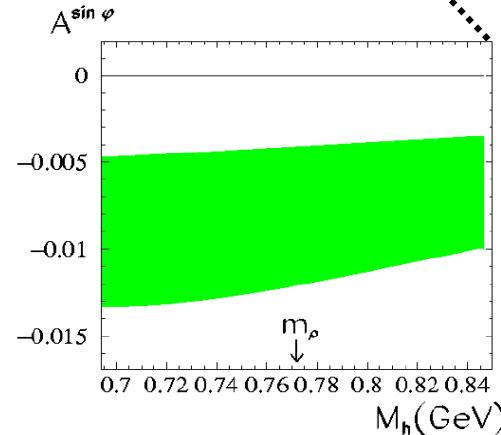
1. All-(2+3+4)= background
non resonant s-wave
2. $q \rightarrow \rho X_2 \rightarrow \pi^+ \pi^- X_2$ p-wave
3. $q \rightarrow \omega X_3 \rightarrow \pi^+ \pi^- X_3$ “
4. $q \rightarrow \omega X'_4 \rightarrow \pi^+ \pi^- (\pi^0 X_4)$ “
5. $q \rightarrow \eta X'_5 \rightarrow \pi^+ \pi^- (\chi X_5)$
6. $q \rightarrow K^0 X_6 \rightarrow \pi^+ \pi^- X_6$ “



Bacchetta and Radici,
P.R. D74 (06) 114007

fit $D_1(M_h)$ & $D_1(z)$ → predict SSA

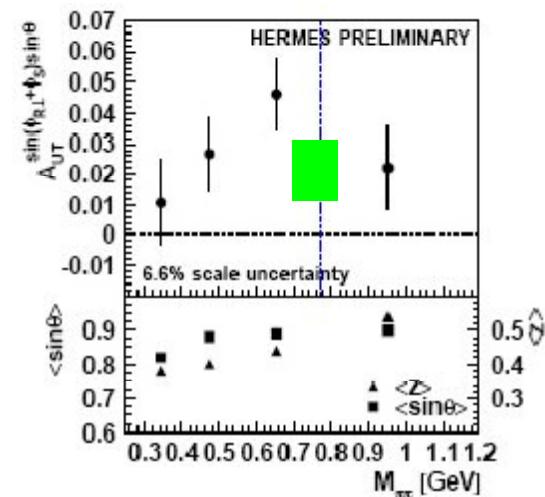
Radici, Jakob,
Bianconi,
P.R. D65 (02)
074031



Trento conventions

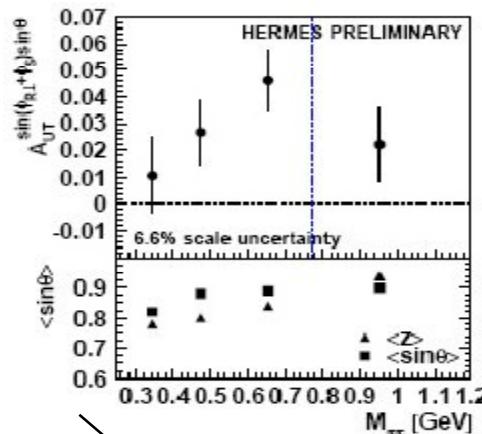


Bacchetta et al.,
P.R. D70 (04) 117504



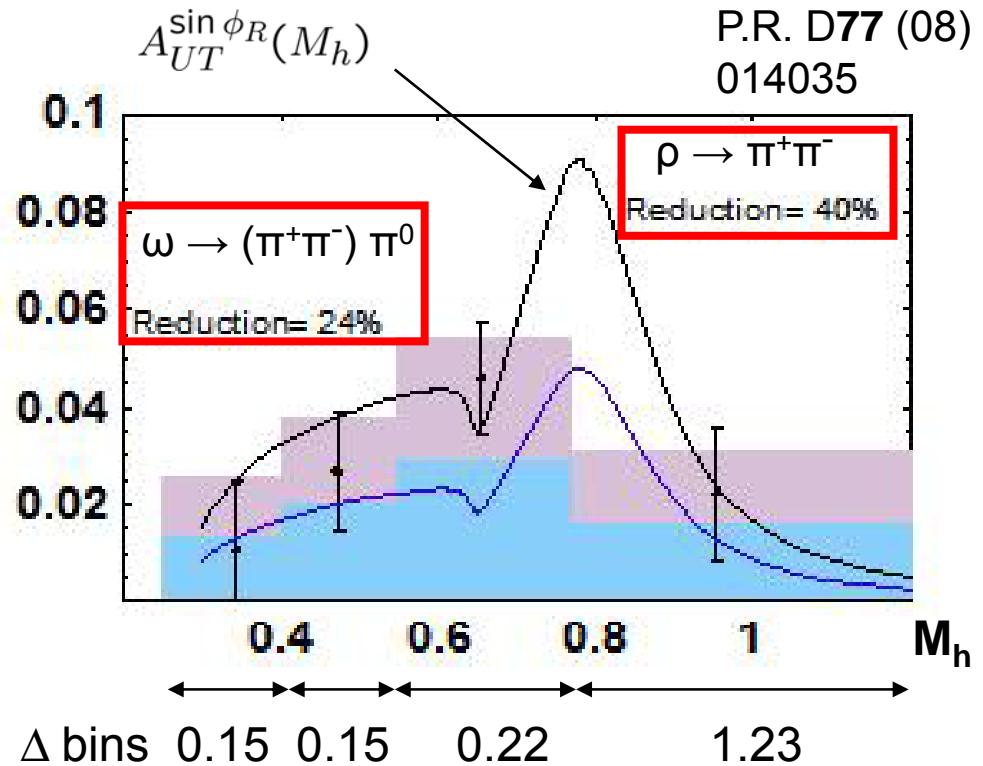
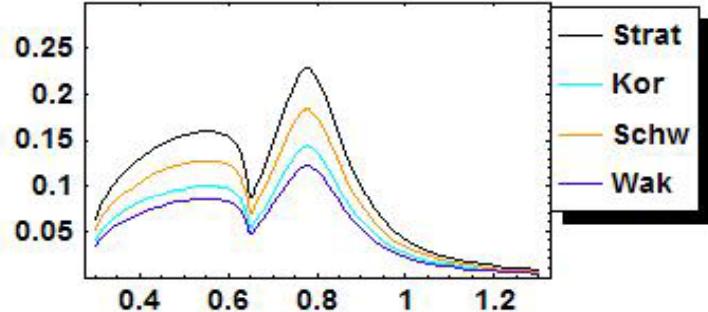
Status of the art of spectator model

see also
 Jun She *et al.*
 P.R. D77 (08)
 014035



“bin” the model and fit the exp. data

model prediction

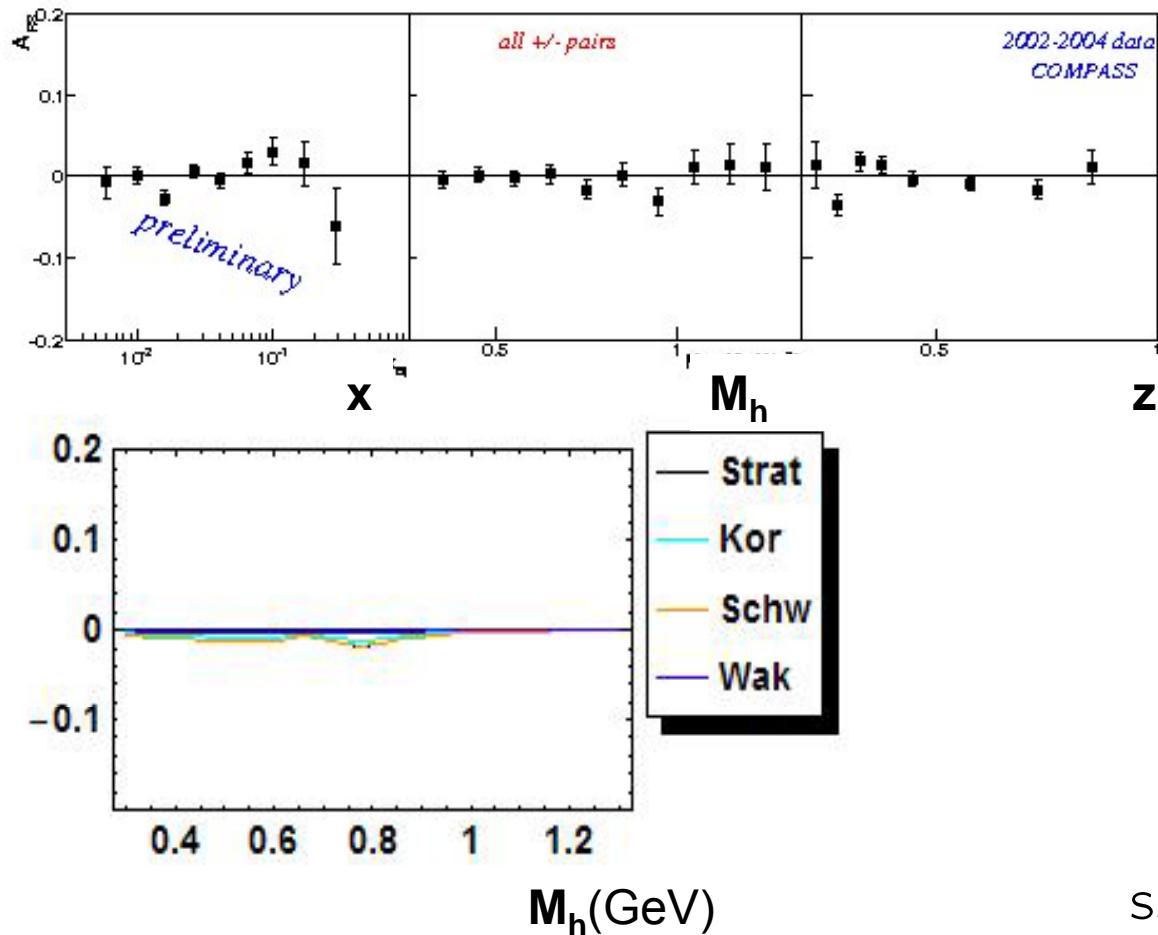


- $X_1=X_2=X_3=X_4=X \rightarrow p\text{-wave channel} = |2+3+4| \rightarrow \text{max. } (\pi^+ \pi^-) \text{ interference}$
- warning: $\omega \rightarrow [(\pi \pi)_{L=1} \pi]_{J=1}$
- narrow-width approx. $M_{3\pi} = m_\omega \rightarrow \text{cure dip at } M_h \sim 0.65 \text{ GeV}$



Prediction of SSA @ : deuteron target

$$A_{UT}^{\sin \phi_R}$$



$Q^2 > 1 \text{ GeV}^2$ $s = 604 \text{ GeV}^2$
 $0.004 < x < 0.25$

Martin
hep-ex/0702002

explain in spect. model as flavor symmetry of **DiFF**

$$\begin{aligned} D_1^u &= D_1^{\bar{d}} = D_1^d = D_1^{\bar{u}} \\ H_1^{\not\leftrightarrow u} &= H_1^{\not\leftrightarrow \bar{d}} = -H_1^{\not\leftrightarrow d} = -H_1^{\not\leftrightarrow \bar{u}} \\ u &\leftrightarrow d \\ + & R \leftrightarrow -R \quad z_1 \leftrightarrow z_2 \\ d = \{p, n\} \text{ isospin symmetry} \end{aligned}$$

$$\begin{aligned} f_1^d &\approx \frac{1}{2} f_1^u \\ h_1^d &\approx -\frac{1}{4} h_1^u \end{aligned}$$

$$\frac{\text{SSA}(d)}{\text{SSA}(p)} \approx \frac{\frac{1}{4} h_1^u H_1^{\not\leftrightarrow u}}{\frac{5}{6} f_1^u D_1^u} \left[\frac{\frac{17}{36} h_1^u H_1^{\not\leftrightarrow u}}{\frac{1}{2} f_1^u D_1^u} \right]^{-1} \approx \frac{1}{3}$$

Outline

- History: why **DiFF** ?

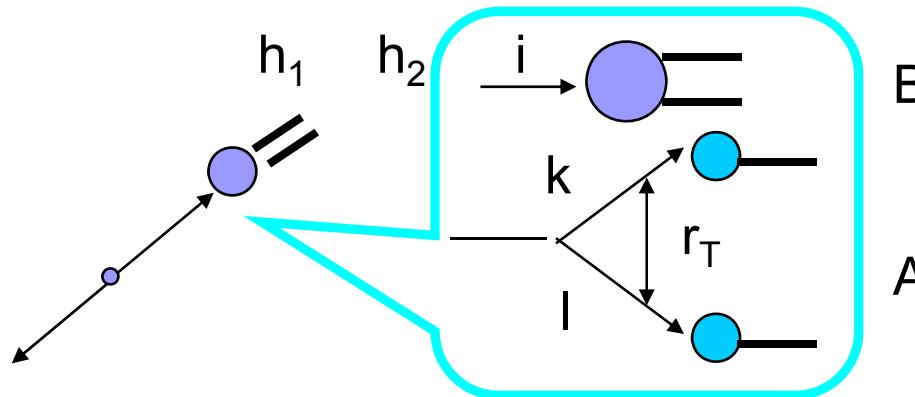
Spin analyzers

- SIDIS: extract **transversity**
with leading-twist
Single-Spin Asymmetry (SSA)
via **DiFF**
(HERMES, COMPASS)
 $\langle Q^2 \rangle \approx 2.5 \text{ GeV}^2$



- e^+e^- : determine unknown
DiFF
(BELLE)
 $Q^2 \approx 100 \text{ GeV}^2$

DiFF $D(z_1, z_2, Q^2)$ evolution equations



de Florian & Vanni,
P.L. **B578** (04) 139

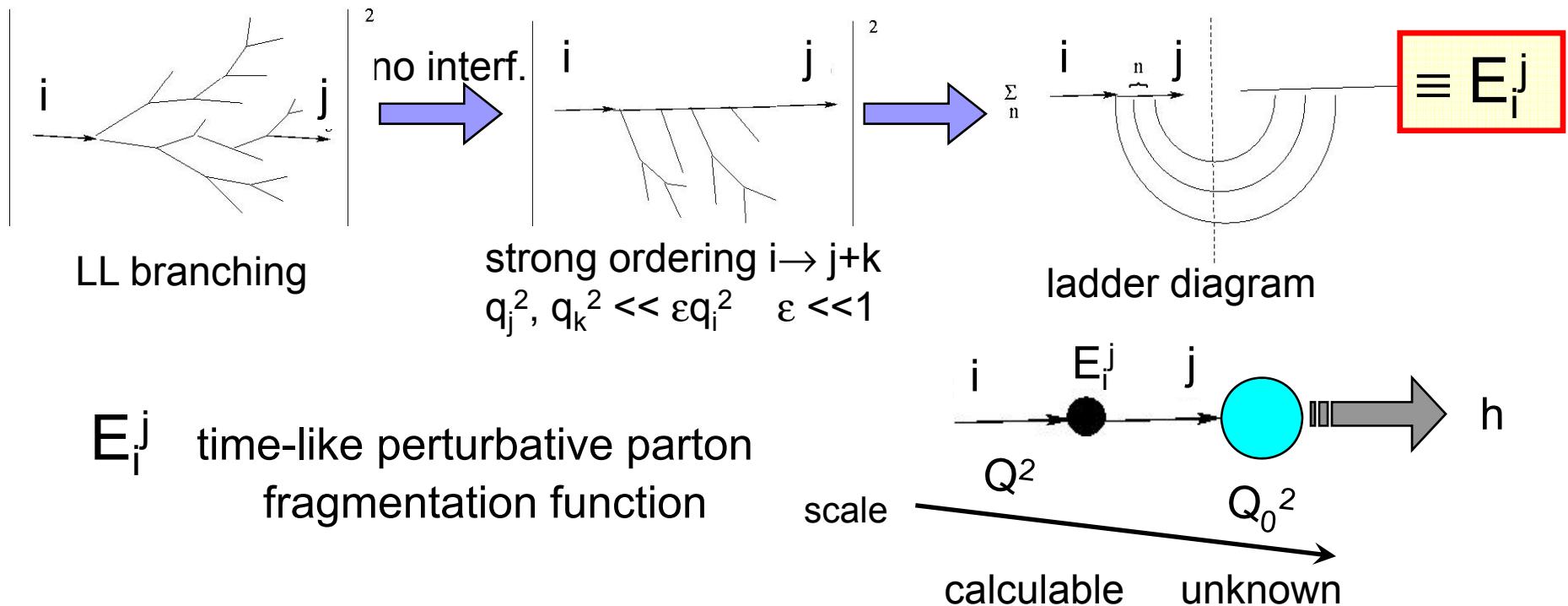
$$\frac{d}{d \log Q^2} D^{i \rightarrow h_1 h_2}(z_1, z_2, Q^2) = \frac{\alpha_s}{2\pi} \left[\underset{B}{D^{k \rightarrow h_1 h_2}} \otimes P_{ki} + \underset{A}{D^{k \rightarrow h_1}} \otimes \underset{A}{D^{l \rightarrow h_2}} \otimes \hat{P}_{kl}^i \right]$$

which evolution equations for
extended DiFF (extDiFF) $D(z_1, z_2, M_h^2, Q^2)$?

1. recover **DiFF** evolution with Jet Calculus technique
2. deduce **extDiFF** evolution “ “ “ “ → LL

single-hadron fragmentation in Jet Calculus

Konishi, Ukawa, Veneziano
N.P. B157 (79) 45

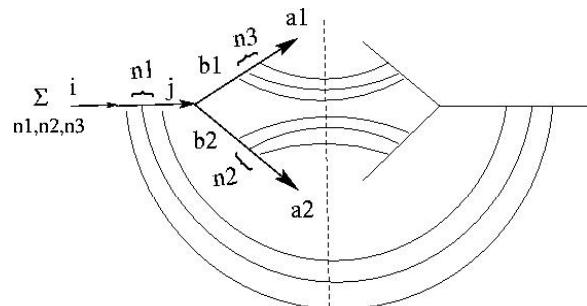


$$D^{i \rightarrow h}(z, Q^2) = E_i^j(Q^2, Q_0^2) \otimes D^{j \rightarrow h}(z, Q_0^2)$$

$$\frac{dE_i^j}{d\log Q^2} = \sum_k E_k^j \otimes P_{ki}^{(0)}$$

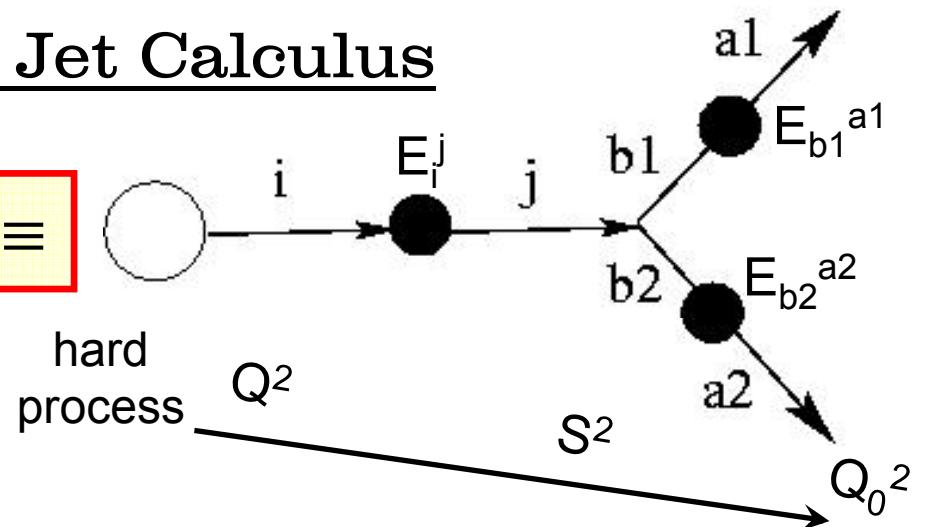
[at LL, resums all collinear $\alpha_s^n \log^n(Q^2/Q_0^2)$]

2-h fragmentation in Jet Calculus

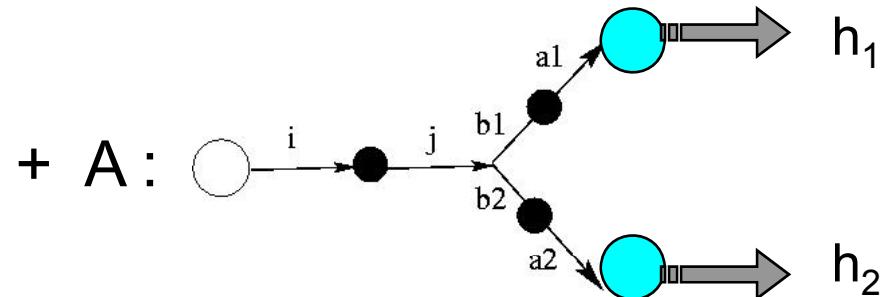
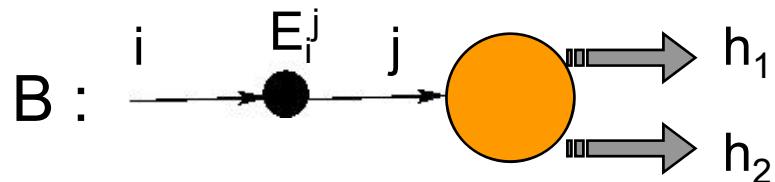


multi-ladder diagram

$$\equiv E_i^{a_1, a_2} \equiv$$



$$E_i^{a_1, a_2}(Q^2, Q_0^2) = \int_{Q_0^2}^{Q^2} dS^2 E_i^j(Q^2, S^2) \otimes \hat{P}_{b_1 b_2}^j \otimes E_{b_1}^{a_1}(S^2, Q_0^2) \otimes E_{b_2}^{a_2}(S^2, Q_0^2)$$



$$D^{i \rightarrow h_1 h_2}(z_1, z_2) = E_i^j(Q^2, Q_0^2) \otimes D^{j \rightarrow h_1 h_2} + E_i^{a_1, a_2}(Q^2, Q_0^2) \otimes D^{a_1 \rightarrow h_1} \otimes D^{a_2 \rightarrow h_2}$$

$$\frac{d}{d \log Q^2} D^{i \rightarrow h_1 h_2} = \frac{\alpha_s}{2\pi} \left[D^{j \rightarrow h_1 h_2} \otimes P_{ji} + D^{b_1 \rightarrow h_1} \otimes D^{b_2 \rightarrow h_2} \otimes \hat{P}_{b_1 b_2}^i \right]$$



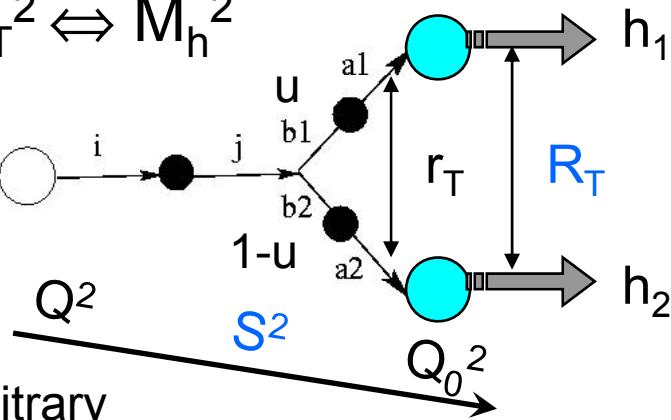
introducing the scale $R_T^2 \Leftrightarrow M_h^2$

1. take $Q_0^2 < R_T^2 <$

$$k_j^2 = \frac{k_{b1}^2}{u} + \frac{k_{b2}^2}{1-u} + \frac{r_T^2}{4u(1-u)} \stackrel{\text{LL}}{\approx} r_T^2 \approx R_T^2$$

fixing $R_T^2 \rightarrow$ scale $k_j^2 \Leftrightarrow S^2$ no longer arbitrary

A:

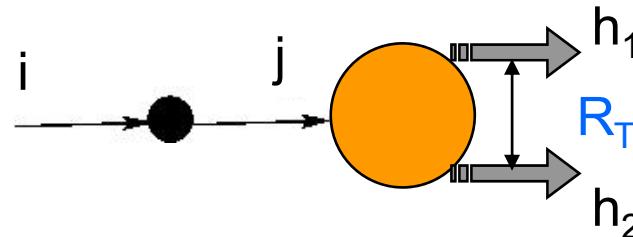


$$\begin{aligned} E_i^{a1,a2}(Q^2, Q_0^2) &= \int_{Q_0^2}^{Q^2} dS^2 E_i^j(Q^2, S^2) \otimes \hat{P}_{b1b2}^j \otimes E_{b1}^{a1}(S^2, Q_0^2) \otimes E_{b2}^{a2}(S^2, Q_0^2) \\ D_A^{i \rightarrow h_1 h_2}(z_1, z_2, R_T^2, Q^2) &= \frac{d}{dR_T^2} D_A^{i \rightarrow h_1 h_2}(z_1, z_2, Q^2) = \frac{d}{dR_T^2} E_i^{a1,a2} \otimes D^{a1 \rightarrow h_1} \otimes D^{a2 \rightarrow h_2} \\ &= E_i^j(Q^2, R_T^2) \otimes \hat{P}_{b1b2}^j \otimes D^{b1 \rightarrow h_1}(R_T^2) \otimes D^{b2 \rightarrow h_2}(R_T^2) \end{aligned}$$

2. take $R_T^2 \leq Q_0^2$

scale k_j^2 arbitrary \rightarrow set to Q_0^2

B:



$$D_B^{i \rightarrow h_1 h_2}(z_1, z_2, R_T^2, Q^2) = E_i^j(Q^2, Q_0^2) \otimes D_B^{j \rightarrow h_1 h_2}(z_1, z_2, R_T^2, Q_0^2)$$

evolution equations for **extDiFF**

Ceccopieri, Radici, Bacchetta,
P.L. **B650** (07) 81

$$D^{i \rightarrow h_1 h_2}(z_1, z_2, R_T^2, Q^2) = \text{B} + \text{A}$$
$$= E_i^j(Q^2, Q_0^2) \otimes D^{j \rightarrow h_1 h_2} + E_i^j(Q^2, R_T^2) \otimes \hat{P}_{b_1 b_2}^j \otimes D^{b_1 \rightarrow h_1} \otimes D^{b_2 \rightarrow h_2}$$

$$\frac{d}{d \log Q^2} D^{i \rightarrow h_1 h_2}(z_1, z_2, R_T^2, Q^2) = D^{k \rightarrow h_1 h_2}(z_1, z_2, R_T^2, Q^2) \otimes P_{ki}$$

same for **polarized extDiFF**
with δP_{ij}

scale R_T^2 breaks degeneracy of terms A-B \rightarrow homogeneous evolution
conjecture: at LL, factorization with same kernel as 1h-fragmentation

$$\frac{d\sigma}{dz_1 dz_2 dR_T^2} = \sum_i \hat{\sigma}_i^{NLO}(Q^2) \otimes D^{i \rightarrow h_1 h_2}(z_1, z_2, R_T^2, Q^2)$$

effect of evolution on extDiFF

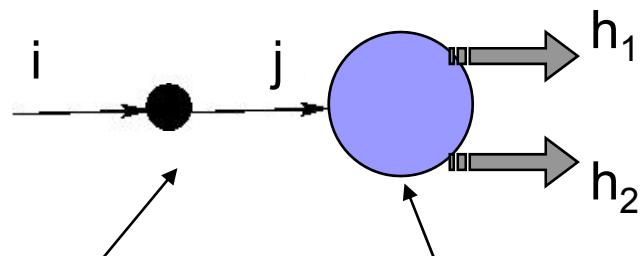
2. partial-wave expansion “commutes” with evolution

$$\frac{d}{d\log Q^2} D^{i \rightarrow h_1 h_2}(z_1, z_2, R_T^2, Q^2) = \frac{\alpha_s(Q^2)}{2\pi} \int_{z_1+z_2}^1 \frac{du}{u^2} D^{k \rightarrow h_1 h_2}\left(\frac{z_1}{u}, \frac{z_2}{u}, R_T^2, Q^2\right) P_{ki}(u)$$

$$\frac{d}{d\log Q^2} D^{i \rightarrow h_1 h_2}(z, \zeta, M_h^2, Q^2) = \frac{\alpha_s(Q^2)}{2\pi} \int_z^1 \frac{du}{u^2} D^{k \rightarrow h_1 h_2}\left(\frac{z}{u}, \zeta, M_h^2, Q^2\right) P_{ki}(u)$$

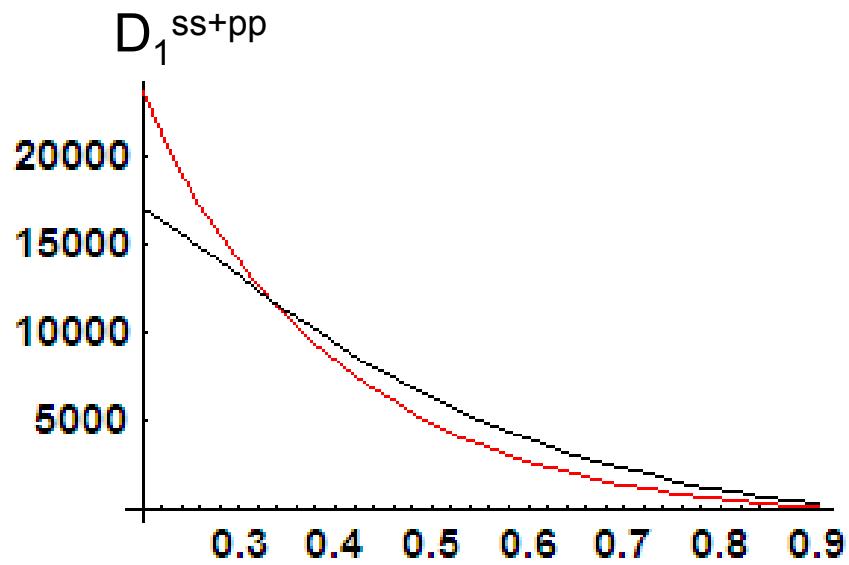

$$\begin{aligned} \frac{d}{d\log Q^2} [D_i^{ss+pp}(z, M_h^2, Q^2) &= \frac{\alpha_s(Q^2)}{2\pi} \left\{ \int_z^1 \frac{du}{u^2} D_k^{ss+pp}\left(\frac{z}{u}, M_h^2, Q^2\right) P_{ki}(u) \right. \\ &\quad \left. + D_i^{sp}(z, M_h^2, Q^2) \cos \theta + \dots \right] \\ &\quad + \cos \theta \int_z^1 \frac{du}{u^2} D_k^{sp}\left(\frac{z}{u}, M_h^2, Q^2\right) P_{ki}(u) + \dots \} \end{aligned}$$

$$z = z_1 + z_2 \quad \zeta = \frac{z_1 - z_2}{z_1 + z_2}$$



1. evolution affects “perturbative sector” z , but not the “soft” ζ

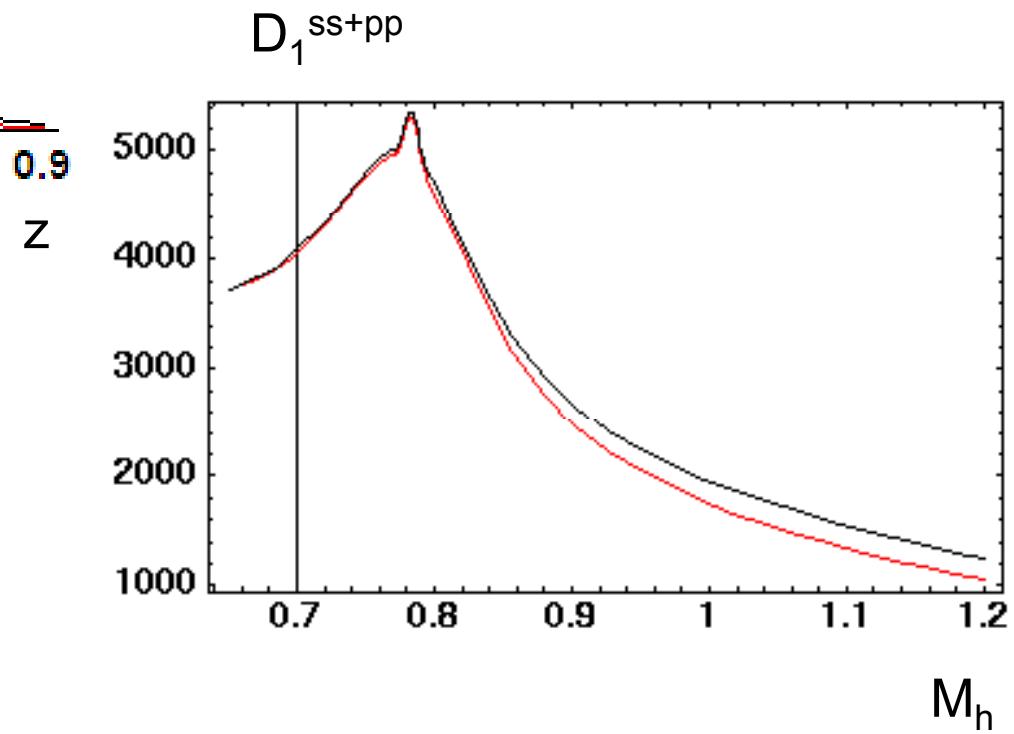
effect of evolution on [extDiFF](#)



$Q_0^2 = 2.5 \text{ GeV}^2$ HERMES

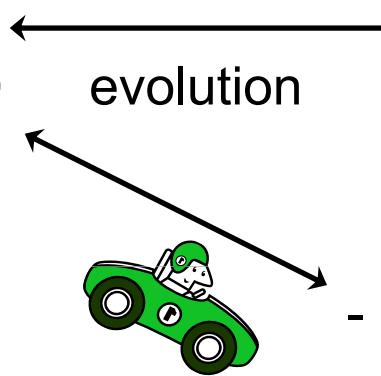
$Q^2 = 100 \text{ GeV}^2$ BELLE

PRELIMINARY !



M_h

Outline

- History: why **DiFF** ?
Spin analyzers
 - SIDIS: extract **transversity** with leading-twist
Single-Spin Asymmetry (SSA)
via **DiFF**
(HERMES, COMPASS)
 $\langle Q^2 \rangle \approx 2.5 \text{ GeV}^2$
 - e^+e^- : determine unknown
DiFF
(BELLE)
 $Q^2 \approx 100 \text{ GeV}^2$
 - pp collision: determine all the unknowns with two measurements
(self extraction of **transversity**)
- 
- The diagram illustrates the evolution of the DiFF method. It shows three main experimental contexts: SIDIS, e^+e^- annihilation, and pp collision. Horizontal arrows indicate the progression from SIDIS to e^+e^- and then to pp collision, labeled "evolution". A green toy car icon is positioned below the arrows, pointing towards the pp collision direction.