On Kaon Fragmentation Functions

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based mainly on a paper written with

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Eur. J. Phys. **C51**, 825 (2007)

Plan of the talk

The FFs of Kaons are determined in:

1. $e^+e^- \to K + X \to$ sensitive to $D_q^{K^+ + K^-}$
2. SIDIS: $e + N \to e + K + X \to$ distinguishes $D_q^{K^+}$ & $D_q^{K^-}$

AKK, DSS, 2007

usually made assumptions, especially about unfav. FFs:

$$D_d^{K^+} = D_d^{K^-}, \quad etc.$$

We show:

1. how $D_d^{K^+ - K^-} = 0$ can be tested: LO, NLO, no knowledge of FFs required
2. how neutral Koans K_s^0 help to determine $D_{q,g}^{K^+ + K^-}$: LO, NLO...

Why K_s^0?

1. SU(2) inv. relates $K^+ + K^-$ and K_s^0
2. allows to work with NonSinglet and Singlet combinations of FFs

Why NS's?

We know
$$g_1^p - g_1^n = \frac{1}{6}\Delta q_3 \otimes \left(1 + \frac{\alpha_s}{2\pi}\delta C_q + \ldots\right)$$
$$\underbrace{\Delta q_3}_{\text{NS}} = (\Delta u + \Delta\bar{u}) - (\Delta d + \Delta\bar{d})$$

- in LO, NLO ... no new PD
- in Q^2 evolution – no new PD

– Δq_3 determined without any assumptions.

We ask:

- What are the meas. quantities that single out NS of FFs in e^+e^- and in SIDIS?
- What other info can we obtain without assumptions?

recall: NS are $D_q^{K^+-K^-}$, $D_{q-q'}^{K^++K^-}$ ...

The difference cross sections $\sigma_N^{h-\bar{h}}$

The general formula in SIDIS, $Q^2 \gg M^2$:

$$
\begin{aligned}
\sigma_N^h \;\; \propto \;\; \sum_q e_q^2 \Big\{ & q \otimes \hat{\sigma}_{qq}(\gamma q \to qX) \otimes D_q^h \\
& + \mathbf{q} \otimes \hat{\sigma}_{\mathbf{qg}}(\gamma \mathbf{q} \to \mathbf{gX}) \otimes \mathbf{D_g^h} \\
& + \mathbf{g} \otimes \hat{\sigma}_{\mathbf{gq}}(\gamma \mathbf{g} \to \mathbf{q\bar{q}X}) \otimes (\mathbf{D_q^h} + \mathbf{D_{\bar{q}}^h}) \Big\}
\end{aligned}
$$

$q(x,t)$ and $D_{q,g}^h(z,t) \Rightarrow$ from experiment

$\hat{\sigma}_{ff'} \Rightarrow$ theor. calculated in perturb. QCD:

C-inv. implies: $D_g^{h-\bar{h}} = 0$, $D_q^{h-\bar{h}} = -D_{\bar{q}}^{h-\bar{h}}$

$\Rightarrow$ In $\sigma_N^{h-\bar{h}}$, in all QCD orders, all gluons cancel: – no g, no D_g

$$\sigma_N^{h-\bar{h}} \propto \left[4u_V \otimes D_u^{h-\bar{h}} + d_V \otimes D_d^{h-\bar{h}} + (s-\bar{s}) \otimes D_s^{h-\bar{h}}\right] \otimes \hat{\sigma}(\gamma q \to qX)$$

$$\hat{\sigma}_{qq} = \hat{\sigma}_{qq}^{(0)} + \frac{\alpha_s}{2\pi}\hat{\sigma}_{qq}^{(1)} + \ldots$$

- only NS of PDs and FFs $\Rightarrow$ g and D_g^h do not reappear in Q^2-evol.

- each term is a NS

- we know from exp: $|s-\bar{s}| \leq 0.02$

[*C. Bourelly, J. Soffer, F. Buccella, 2007*]

Further: $\sigma_N^{h-\bar{h}}$ depends on the final hadron h.

Tests of $D_d^{K^+-K^-} = 0$

$s - \bar{s} = 0 \Rightarrow$ correct with an accuracy $\leq 2\%$

SIDIS: $e + N \to e + K^\pm + X \Rightarrow \sigma_N^{K^+-K^-}$

$$\textbf{LO}: \quad \sigma_p^{K^+-K^-} \simeq (4u_V D_u + d_V \mathbf{D_d})^{K^+-K^-}$$

$$\sigma_d^{K^+-K^-} \simeq (u_V + d_V)\,(4D_u + \mathbf{D_d})^{K^+-K^-}$$

usually $D_d^{K^+-K^-} = 0$ assumed $\to$ can we test it directly in $\sigma_N^{K^+-K^-}$?

Tests of $D_d^{K^+-K^-} = 0$, LO

- $\mathcal{R}_p^{K^+-K^-}(x,z) = \frac{\sigma_p^{K^+-K^-}}{u_V} \simeq D_u(z)\left[1 + \frac{d_V}{u_V}(x)\frac{\mathbf{D_d}}{\mathbf{4D_u}}(z)\right]^{K^+-K^-}$
- $\mathcal{R}_d^{K^+-K^-}(x,z) = \frac{\sigma_d^{K^+-K^-}}{u_V+d_V} \simeq D_u(z)\left[1 + \frac{\mathbf{D_d}}{\mathbf{4D_u}}(z)\right]^{K^+-K^-}$

the x dependence in $\mathcal{R}_p^{K^+-K^-}$ is induced solely by $\frac{\mathbf{D_d}}{\mathbf{4D_u}}(z) \neq 0$

$\Rightarrow$ **tests of** $\mathbf{(D_d/D_u)^{K^+-K^-} = 0}$:

$1.$ $\mathcal{R}_p^{K^+-K^-} = \mathcal{R}_d^{K^+-K^-}$, $2.$ $\mathcal{R}_p^{K^+-K^-}(x,z) = f(z)(= \mathbf{D_u^{K^+-K^-}})$

$\Rightarrow$ **test of LO**: $\mathcal{R}_d^{K^+-K^-}(x,z) = f(z)$

Tests of $D_d^{K^+-K^-} = 0$, NLO

$\sigma_N^{K^+-K^-}$, NLO: the same PDs & FFs, but simple products $\to$ convolutions

NLO :

$$\sigma_p^{K^+-K^-} \simeq (4u_V \otimes D_u + d_V \otimes \mathbf{D_d})^{K^+-K^-} \otimes (1 + \alpha_s C_{qq})$$
$$\sigma_d^{K^+-K^-} \simeq (u_V + d_V) \otimes (1 + \alpha_s C_{qq}) \otimes (4D_u + \mathbf{D_d})^{K^+-K^-}$$

If $D_d^{K^+-K^-} = 0$: only *one* FF enters *both* σ_p and σ_d:

$$\sigma_p^{K^+-K^-} \simeq u_V \otimes (1 + \alpha_s C_{qq}) \otimes D_u^{K^+-K^-}$$
$$\sigma_d^{K^+-K^-} \simeq (u_V + d_V) \otimes (1 + \alpha_s C_{qq}) \otimes D_u^{K^+-K^-}$$

If we fit data on **both** $\sigma_p^{K^+-K^-}$ **and** $\sigma_d^{K^+-K^-}$ with **the same** FF ($=D$) and obtain an acceptable fit $\Rightarrow D_d^{K^+-K^-} = 0$ and $D = D_u^{K^+-K^-}$.

• Independently of our knowledge of the FF's and without any assumptions we obtain info about $D_d^{K^+-K^-} = 0$ & $D_u^{K^+-K^-}$ – LO and NLO!

measurability:

1) difference cross sections: $\Rightarrow$ high precisions needed

2) data in bins in both x and z required

very precise data of HERMES $\Rightarrow \sigma_d^{K^\pm}$, $\sigma_p^{K^\pm}$, in bins $[x_i, z_j]$

A. Hillenbrand, DESY, 2005

If $K^{\pm}$ and K_s^0 measured

no new FFs appear: **SU(2) relates $K^{\pm}$ and K_s^0**:

$$SU(2): \qquad D_u^{K^+ + K^-} = D_d^{K^0 + \bar{K}^0}, \qquad D_d^{K^+ + K^-} = D_u^{K^0 + \bar{K}^0}$$
$$D_s^{K^+ + K^-} = D_s^{K^0 + \bar{K}^0}, \qquad D_g^{K^+ + K^-} = D_g^{K^0 + \bar{K}^0}$$

the processes are: $[K_s^0 = (K^0 + \bar{K}^0)/\sqrt{2}]$

$$e^+e^- \to K^{\pm} + X, \qquad e^+e^- \to \mathbf{K_s^0} + X$$
$$e + N \to e + K^{\pm} + X, \qquad e + N \to e + \mathbf{K_s^0} + X$$

we suggest 2 possible combinations [use SU(2)-inv.]:

$$K^+ + K^- - 2K_s^0 \qquad \& \qquad K^+ + K^- + 2K_s^0$$

$\underline{\underline{e^+e^- \to K^\pm, K_s^0 + X}}$

The general expressions – **all** FFs enter:

$$\frac{d\sigma^{K^+ + K^-}}{dz} = 6\sigma_0 \Big\{ [\hat{e}_u^2 \, D_u + \hat{e}_d^2 (D_d + \underline{D_s})](1 + \alpha_s \otimes C_q) + \underline{\underline{\alpha_s \, (\hat{e}_u^2 + 2\,\hat{e}_d^2) C_g \otimes D_g}} \Big\}^{K^+ + K^-}$$

$$\frac{d\sigma^{K^0 + \bar{K}^0}}{dz} = 6\sigma_0 \Big\{ [\hat{e}_u^2 \, D_d + \hat{e}_d^2 (D_u + \underline{D_s})](1 + \alpha_s \otimes C_q) + \underline{\underline{\alpha_s \, (\hat{e}_u^2 + 2\,\hat{e}_d^2) C_g \otimes D_g}} \Big\}^{K^+ + K^-}$$

In $d\sigma^{K^+ + K^- - 2K_s^0}$ – only **one** FF:

$$d\sigma^{K^+ + K^- - 2K_s^0} = 6\sigma_0 (\hat{e}_u^2 - \hat{e}_d^2)(1 + \alpha_s \, C_q \otimes) D_{u-d}^{K^+ + K^-}$$

$$d\sigma^{K^+ + K^- - 2K_s^0} = d\sigma^{K^+} + d\sigma^{K^-} - 2\, d\sigma^{K_s^0}$$

$\underline{\underline{K^+ + K^- - 2K_s^0}}$

In the general expressions for $\sigma^{K^+ + K^-}$ & $\sigma^{K^0 + \bar{K}^0}$, **both** in e^+e^- & SIDIS, **all** FFs enter;

In $d\sigma^{K^+ + K^- - 2K_s^0}$ **both** in e^+e^- & SIDIS – always only **one** FF enters:

$$d\sigma^{K^+ + K^- - 2K_s^0} = 6\sigma_0(\hat{e}_u^2 - \hat{e}_d^2)(1 + \alpha_s\, C_q\otimes)\, \mathbf{D_{u-d}^{K^+ + K^-}}$$

$$d\sigma_p^{K^+ + K^- - 2K_s^0} = [(4u + d) \otimes (1 + \alpha_s\, C_{qq}\otimes) + \alpha_s\, g \otimes C_{gq}\otimes]\, \mathbf{D_{u-d}^{K^+ + K^-}}$$

$$d\sigma_d^{K^+ + K^- - 2K_s^0} = [(u + d) \otimes (1 + \alpha_s\, C_{qq}\otimes) + \alpha_s\, g \otimes C_{gq}\otimes]\, \mathbf{D_{u-d}^{K^+ + K^-}}$$

$$d\sigma^{K^+ + K^- - 2K_s^0} = d\sigma^{K^+} + d\sigma^{K^-} - 2\, d\sigma^{K_s^0}$$

$K^+ + K^- - 2K_s^0$

1. all 3 processes measure the same NS: $(D_u - D_d)^{K^+ + K^-}$

$\Rightarrow$ the combination $(K^+ + K^- - 2K_s^0)$ is a NS of the FFs

$\Rightarrow$ it can be easily evolved in Q^2

2. no s-quarks due to SU(2), but $g(x)$ enters

$\Rightarrow$ the combination $(K^+ + K^- - 2K_s^0)$ is **not** a NS of the PDs

3. holds in any QCD order!

4. no assumptions

How can we use this?

Test of factorization:

Factorization:

SIDIS: $\sigma_N^h \simeq PD_N \times \hat{\sigma}_\alpha \times \mathbf{FF^h}$

$\mathbf{e^+e^-}$: $\sigma^h \simeq \hat{\sigma}_\alpha \times \mathbf{FF^h}$

two kind of processes for the FFs at very different Q^2:

$$e^+e^- \to K + X \qquad high \quad Q^2, \quad \sim \quad Z^0 - exchange$$
$$eN \to e + K + X \qquad low \quad Q^2, \quad \sim \quad \gamma - exchange$$

if $\mathbf{K^+ + K^- - 2K_s^0}$ – both measure only $D_{u-d}^{K^+ + K^-}$. As it is NS, its Q^2-evolution does not involve any other FFs. Comparing these measurements one tests factorization directly, without any assumptions.

Test of factorization, LO

In LO it's particularly simple:

For example one could test the relation

$$\frac{9\, d\sigma_p^{K^+ + K^- - 2K_s^0}(x,z,Q^2)}{d\sigma^{K^+ + K^- - 2K_s^0}(z,m_Z^2)_{\downarrow Q^2}} = \frac{[4(u+\bar{u})-(d+\bar{d})](x,Q^2)}{6\,\sigma_0\,(\hat{e}_u^2-\hat{e}_d^2)_{m_Z^2}}$$

Here $d\sigma^{K^+ + K^- - 2K_s^0}(z,m_Z^2)_{\downarrow Q^2}$ denotes that data is measured at $\sim m_Z^2$ and then evolved to Q^2 according to the DGLAP equations.

• **Independently of our knowledge of the FF's and without any assumptions one tests factorization in SIDIS: PDs $\times$ FFs.**

$\mathcal{K} = K^+ + K^- + 2K_s^0$, LO

$$LO : d\sigma^{\mathcal{K}} = 6\sigma_0 \left[(\hat{e}_u^2 + \hat{e}_d^2)\, (\mathbf{D_u + D_d}) + \hat{e}_d^2 \mathbf{D_s} \right]^{K^+ + K^-}$$

$$d\sigma_p^{\mathcal{K}} = \left[(4\tilde{u} + \tilde{d})(\mathbf{D_u + D_d}) + 2\tilde{s}\,\mathbf{D_s} \right]^{K^+ + K^-}$$

$$d\sigma_d^{\mathcal{K}} = \left[(\tilde{u} + \tilde{d})(\mathbf{D_u + D_d}) + 4\tilde{s}\,\mathbf{D_s} \right]^{K^+ + K^-}$$

Due to SU(2) all three processes measure only 2 FFs:

$$(\mathbf{D_u + D_d})^{\mathbf{K^+ + K^-}}, \qquad \& \quad \mathbf{D_s}^{\mathbf{K^+ + K^-}}$$

This holds in all QCD orders!

- SIDIS with $K^\pm$ **and** K_s^0 are enough to determine all FF in LO, $(D_u \pm D_d)^{K^+ + K^-}$ and $D_s^{K^+ + K^-}$. No need of data from e^+e^- at very different Q^2, whose evolution requires D_g.

$\mathcal{K} = K^+ + K^- + 2K_s^0$, NLO

$\sigma_N^{\mathcal{K}}$ are NS neither in PDs and nor in FFs $\longrightarrow$ in NLO D_g enters:

$$\mathbf{(D_u + D_d)^{K^+ + K^-}}, \quad \mathbf{D_s^{K^+ + K^-}} \quad \& \quad \mathbf{D_g^{K^+ + K^-}}$$

measurements of e^+e^- are needed $\longrightarrow$ we have 3 measurements for the 3 unknown FFs and **no assumptions** needed.

Summary

two complementary approaches to data:

1. a simultaneous analysis of **all** data to determine **all** FFs

The problem : data come with errors and biases are possible, assumptions

2. split data into singlets and non singlets and proceed step by step, analyzing them separately – precisions of data is important!

3. NS are especially attractive: we work out some NS for 1) $K^\pm$ and 2) $K^\pm$ & K_s^0 in both e^+e^- and eN semi inclusive processes. We show:

$K^\pm$: SIDIS: $\sigma_p^{K^+-K^-}$ and $\sigma_d^{K^+-K^-} \to D_d^{K^+-K^-} = 0?$, $D_u^{K^+-K^-}$
in LO, NLO, no assumptions, no knowledge of FFs

$K^\pm$ & K_s^0: $\to$ 1) $K^+ + K^- - 2K_s^0$ in e^+e^- & SIDIS $\to D_{u-d}^{K^++K^-} =$ NS
test of factorization: SIDIS = PDs $\times$ FFs, e^+e^- = FFs
in LO, NLO, no assumptions

2) SIDIS $\sigma_N^{K^++K^--2K_s^0}$ & $\sigma_N^{K^++K^-+2K_s^0}$ are enough to determine $D_{u,d,s}^{K^++K^-}$

in LO, no assumptions, no need of e^+e^- data at very different Q^2.