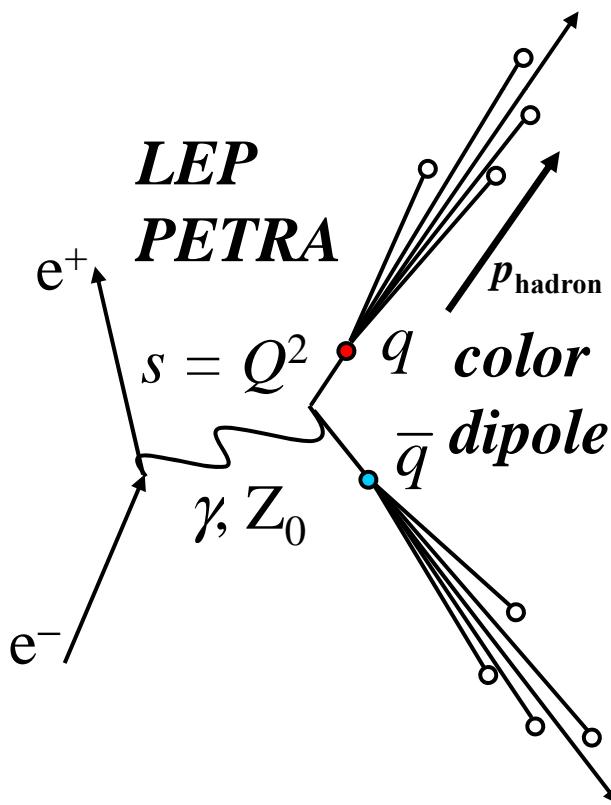


Fragmentation in e^+e^- Collisions at low Q^2



Dave Kettler

*Parton fragmentation processes: in the
vacuum and in the medium*

Trento

February 25-29, 2008

Agenda

- Fragment distributions from p-p spectra
- Parametrizing e^+e^- fragmentation functions
- Application to p-p correlations

QCD Issues for Nuclear Collisions

*we observe that a large fraction of RHIC correlations
are due to minijets = low- Q^2 parton fragments*

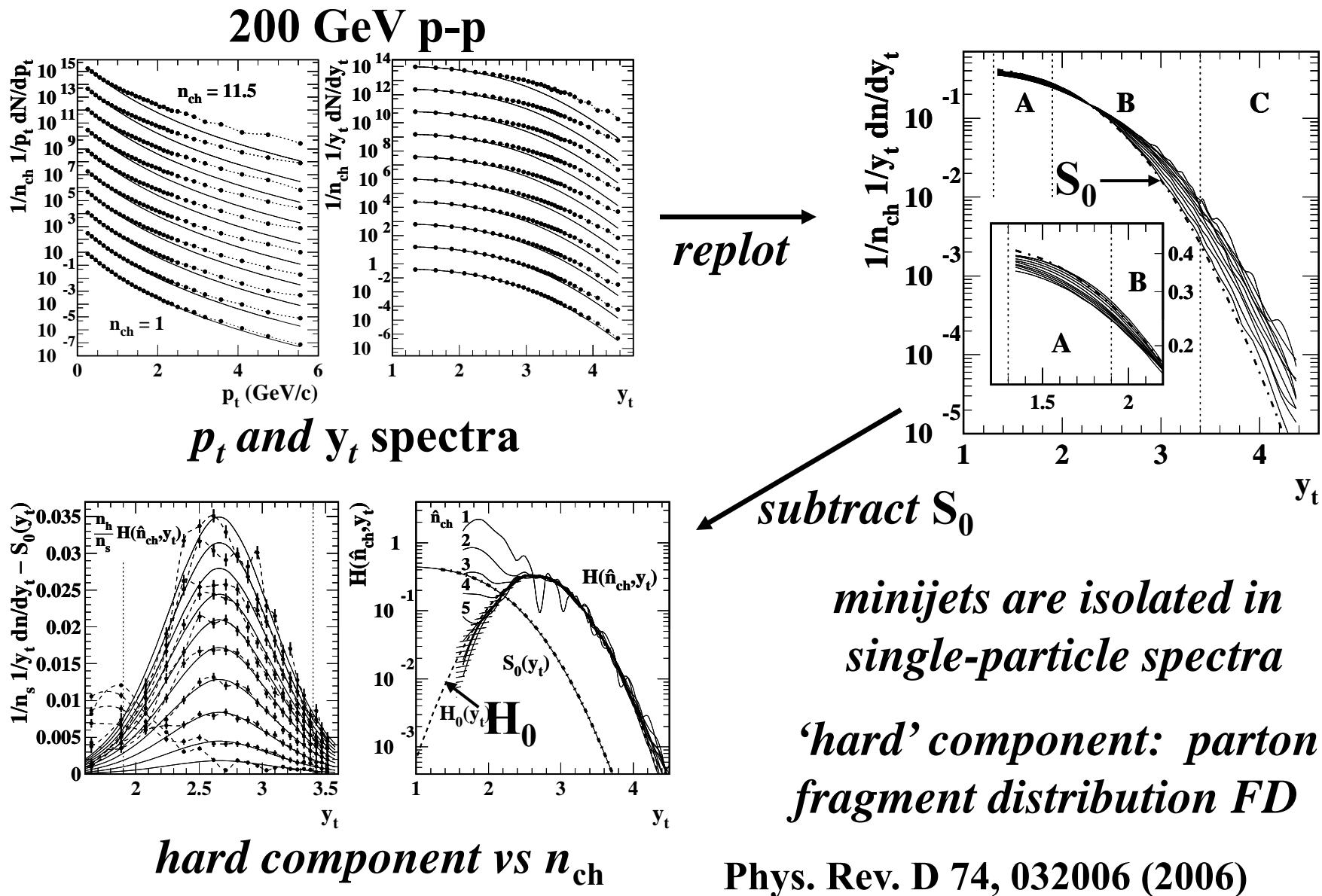
parton = QCD field quantum

- How do partons scatter at low Q^2 (invariant mass squared)
- How do low- Q^2 partons fragment to hadrons
- What happens to partons in heavy ion collisions

*to understand the second point we examine the systematics
of parton fragmentation in p-p and especially e⁺-e⁻ collisions*

Phys. Rev. D 74, 034012 (2006)

Two-component p-p Spectrum Model



Describing Fragment Distributions

how to access small Q^2

folding integral

$$E \frac{d^3\sigma_N}{dp_h^3} = \int \frac{dx dk_t^2 dk_t'^2}{\pi z} f_N^q(x, k_t) \frac{d\hat{\sigma}^q}{dt}(x, k_t, k_t') D_q^h(z, dk_t')$$

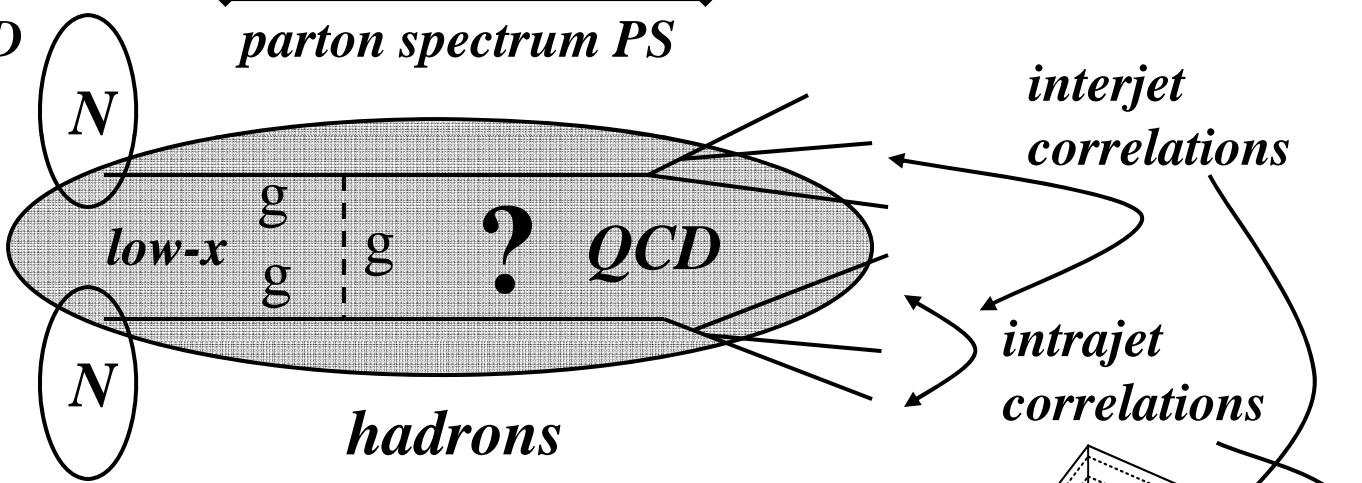
fragmentation function FF

fragment distribution FD

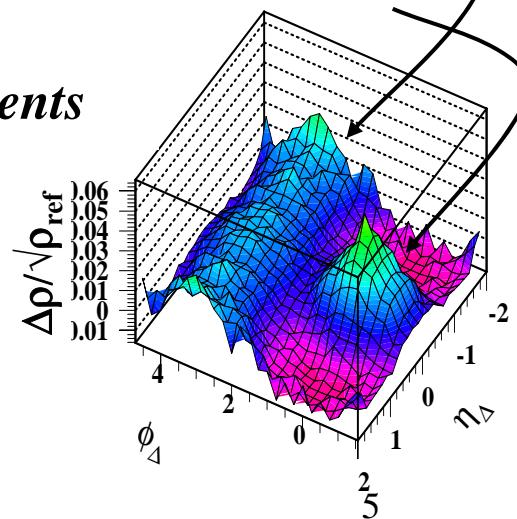
minimum bias FD

*factorization fails
at small energy scales*

assume: PS is simple

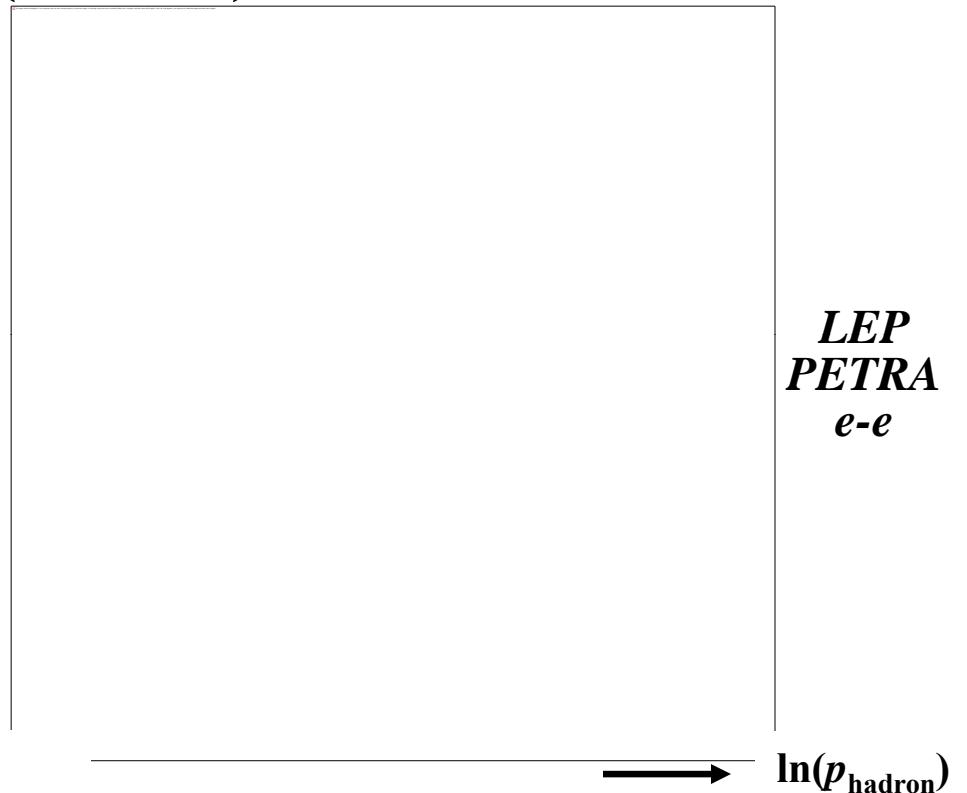
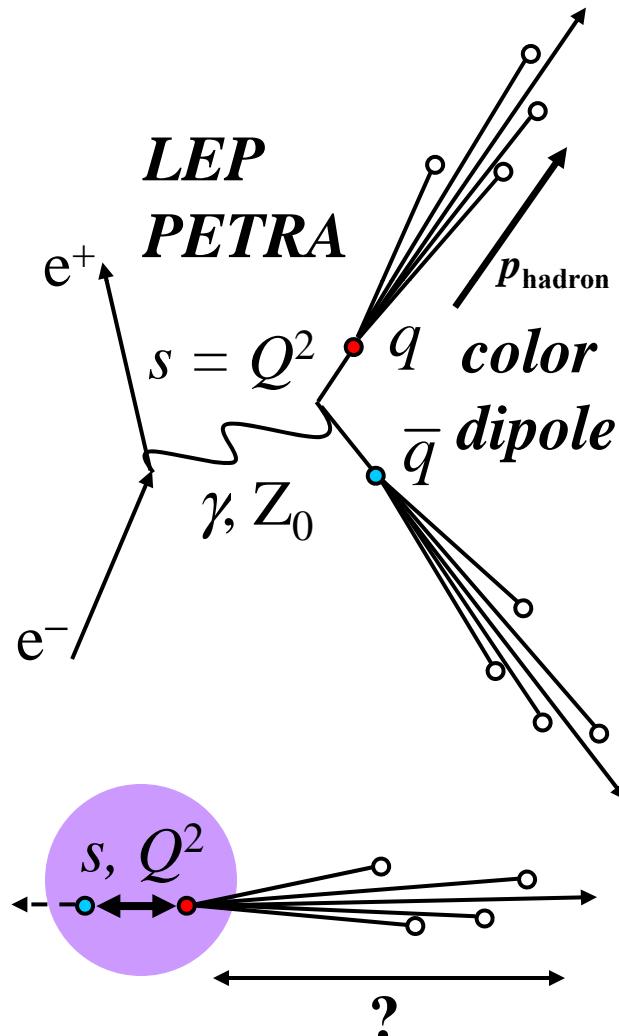


FF can be extrapolated



Parton Fragmentation in $e^+ - e^-$

how are parton fragments (hadrons) distributed on momentum



*color dipole radiation:
an equilibration process*

LEP, PETRA fragmentation data: 1985-2000

Conventional Fragmentation Functions

'leading-particle' strategy

trigger vs associated

*jet is not reconstructed, estimate
parton with high- p_t leading particle*

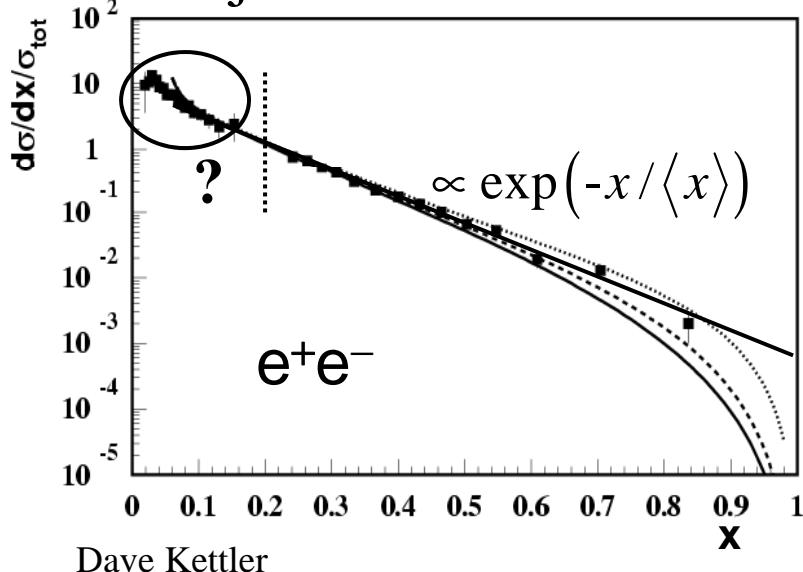
$$x = p_{t,\text{assoc}} / p_{t,\text{trig}}; \quad z_{\text{trig}} = p_{t,\text{trig}} / p_{t,\text{part}}$$

collinear approximation

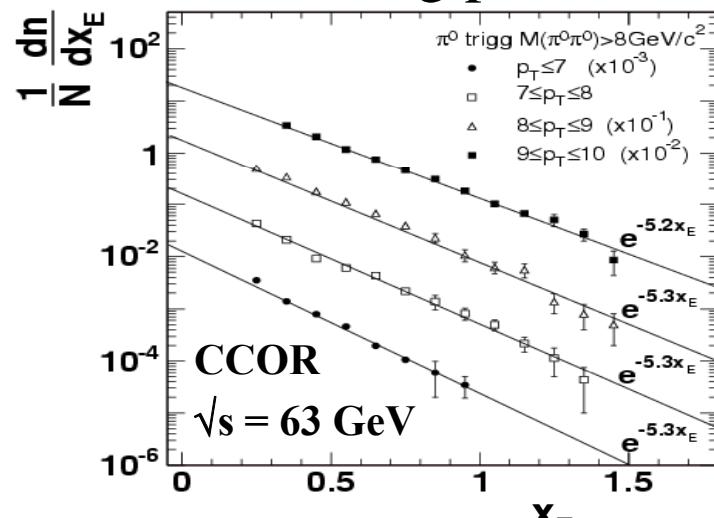
fragmentation function

$$D(x) = \frac{1}{N_{\text{trigger}}} \frac{dN_{\text{hadron}}}{dx} \propto \exp(-x/\langle x \rangle); \quad 1/\langle x \rangle \approx 6 - 7$$

jet reconstruction



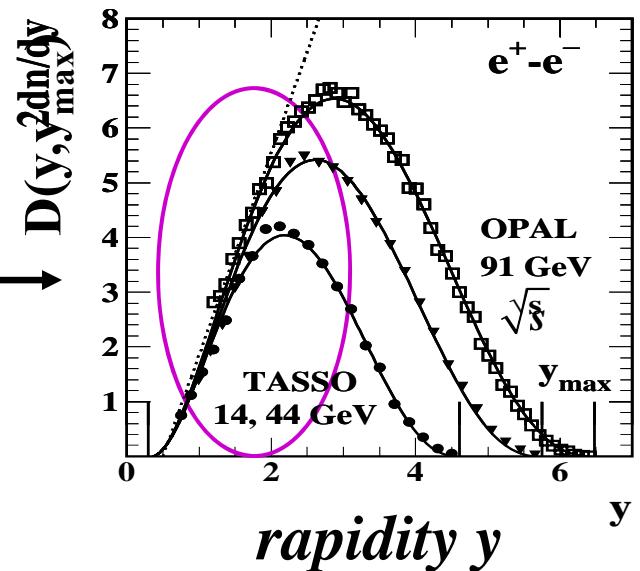
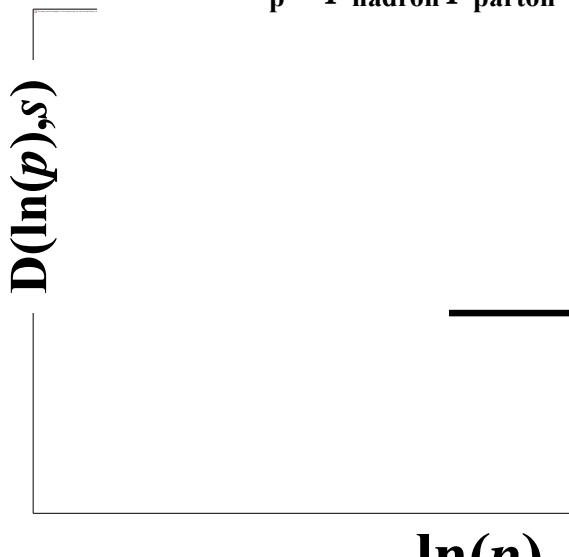
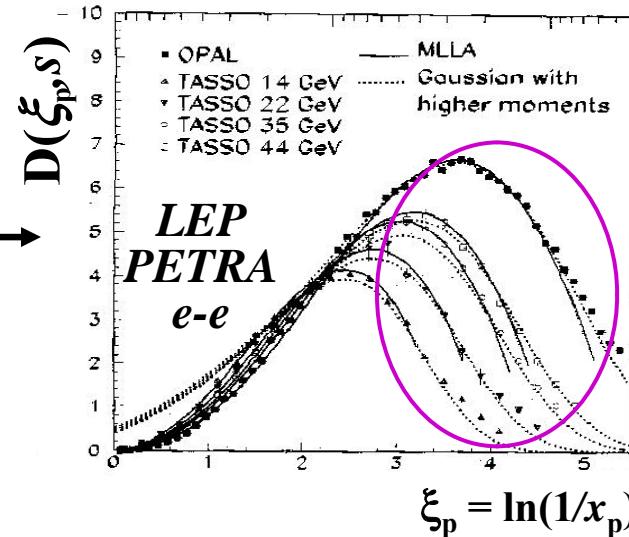
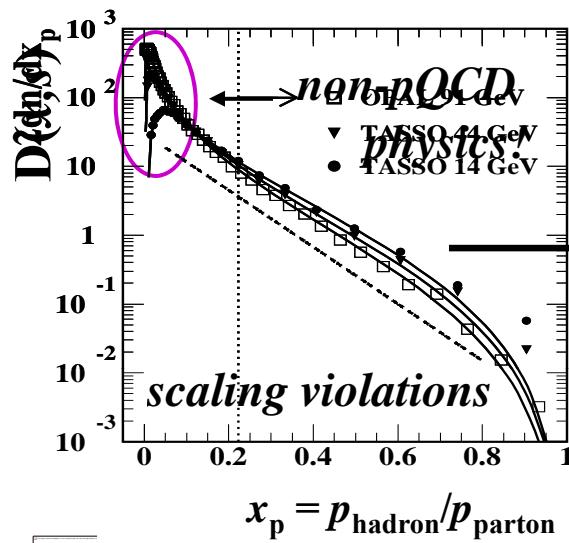
leading particle



A.L.S. Angelis *et al.*, NPB 209 (1982)

Fragment Distributions on Momentum

fragmentation functions on logarithmic variables



unidentified hadrons

conventional:
fragment momentum
relative to
parton momentum

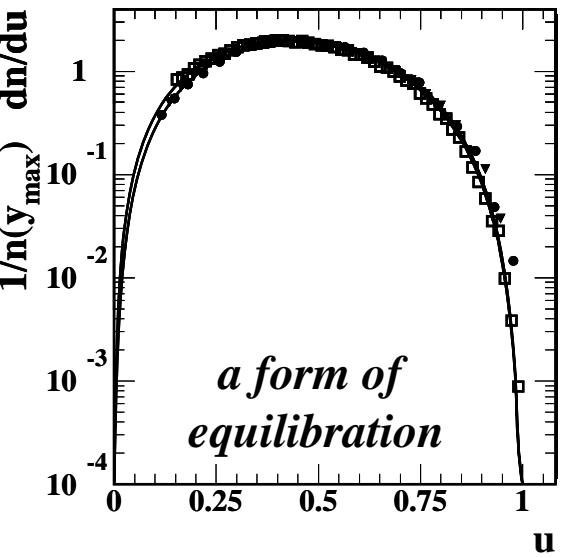
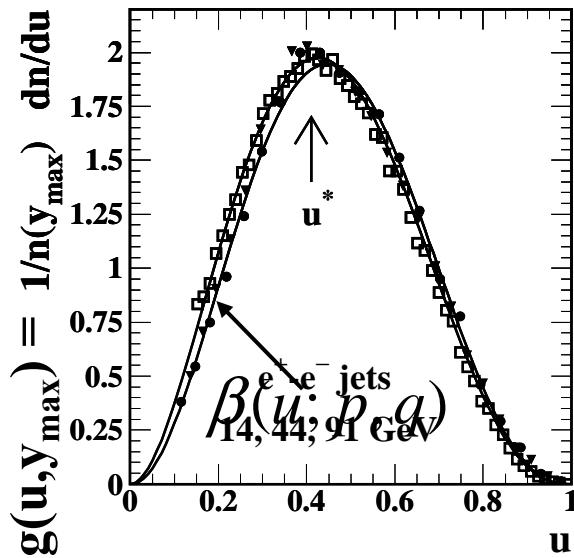
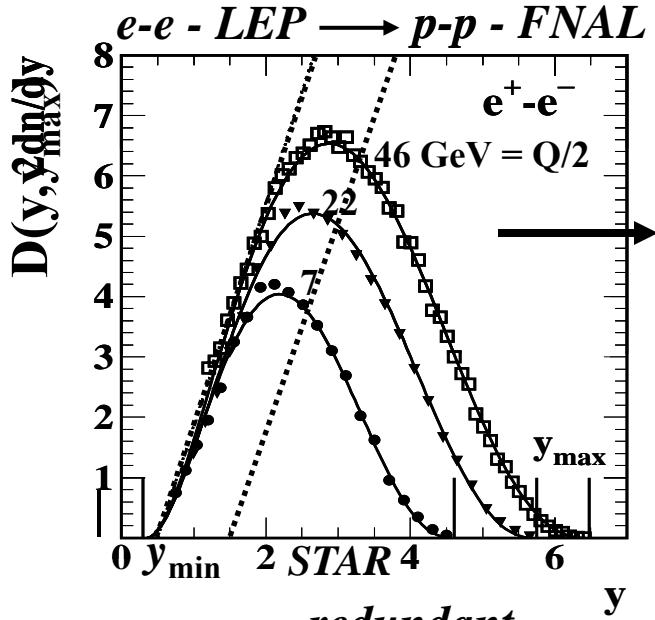
fragmentation function
 $D(x, s) \rightarrow D(y, y_{\max})$

alternative:
fragmentation functions
on rapidity y

$$y \equiv \ln \left\{ (E + p)/m_\pi \right\}$$

Precision Analysis of Fragmentation

fragmentation functions well described by simple model function



$$D(y, y_{\max}) \xrightarrow{\text{redundant}} 2n(y_{\max}) g(u, y_{\max})$$

dijet multiplicity

$$u \equiv \frac{(y - y_{\min})}{(y_{\max} - y_{\min})}$$

normalized rapidity

$$B(p, q) = \frac{\Gamma(p)\Gamma(q)}{\Gamma(p+q)}$$

$$g(u, y_{\max}) \approx \beta(u; p, q) \equiv u^{p-1}(1-u)^{q-1} / B(p, q) \quad (\text{normalized})$$

*beta distribution on normalized rapidity u
precisely models fragmentation functions*

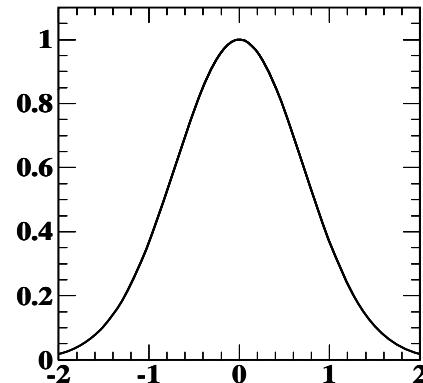
The Beta Distribution

Maximum Entropy Distributions

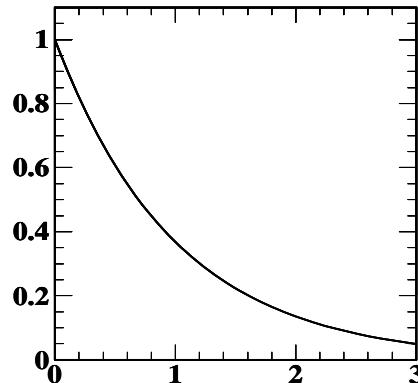
Maximize Shannon Entropy $S = -\int dx p(x) \ln[p(x)]$
 x is arbitrary variable

with
constraints

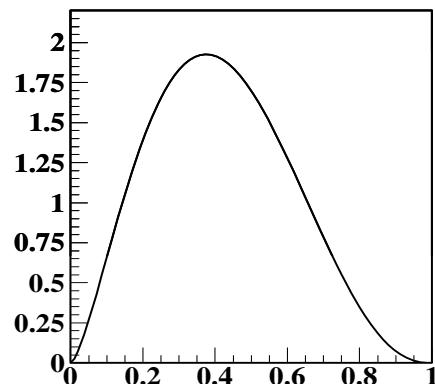
Gaussian
 \bar{x}, \bar{x}^2



Exponential
 $-x$



Beta
Distribution
 $\ln x, \ln(1-x)$

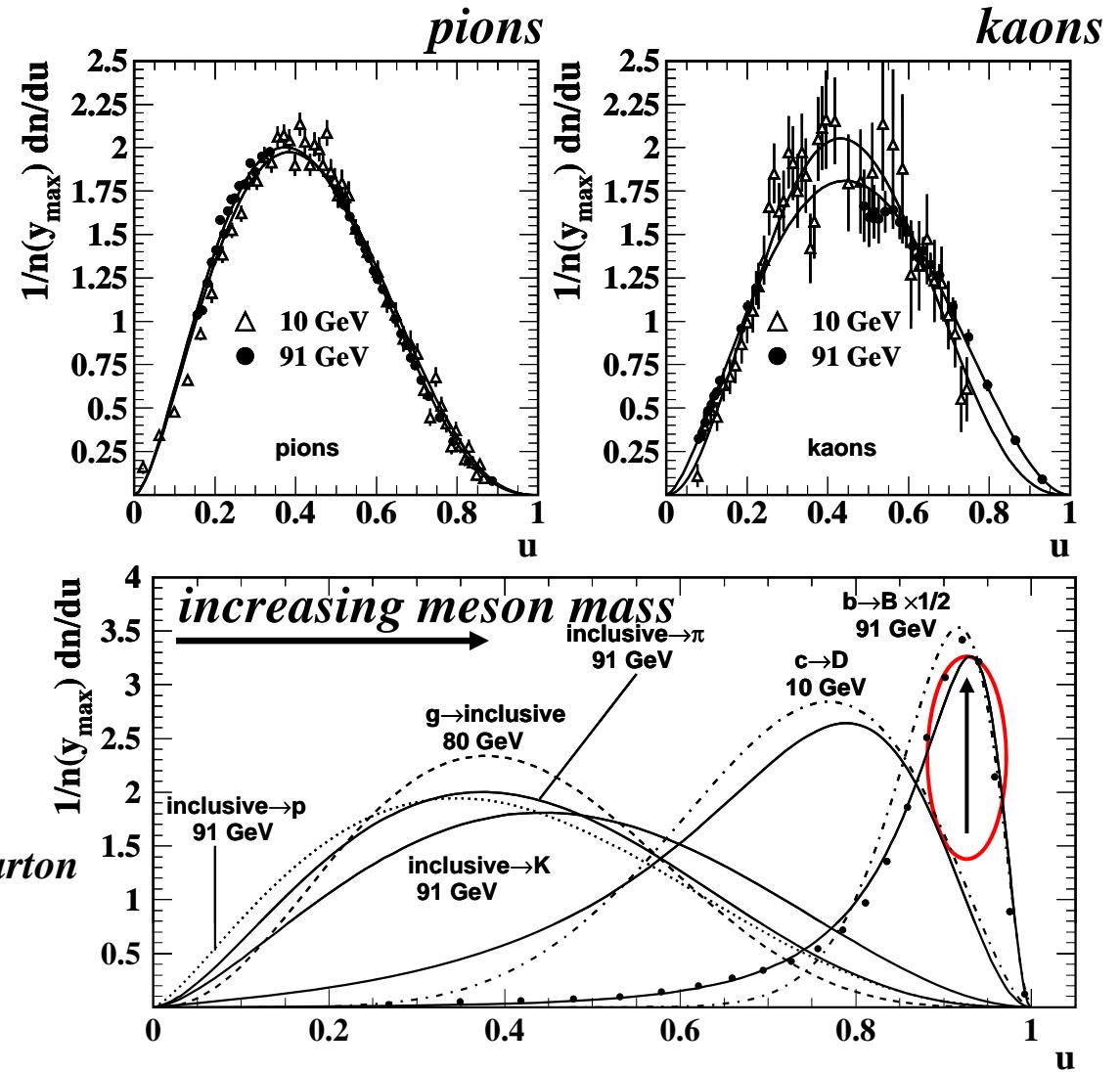


*Fragmentation as a
form of equilibration?*

Identified Hadrons

the flavor/color chemistry of fragmentation

identified hadron fragments



*heavier fragments stay close to parent parton
Not well described by a beta dist.*

Identified Partons

the flavor/color chemistry of fragmentation

identified partons

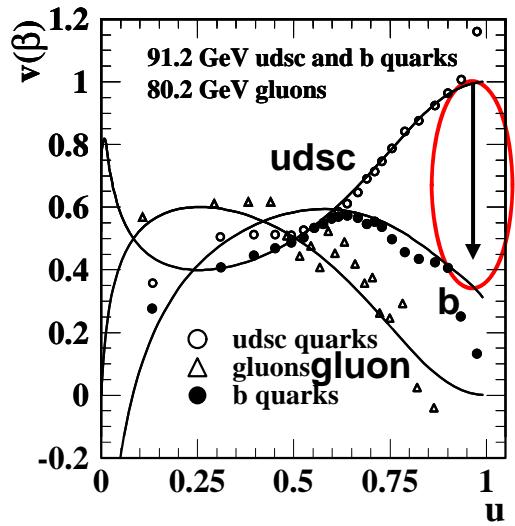
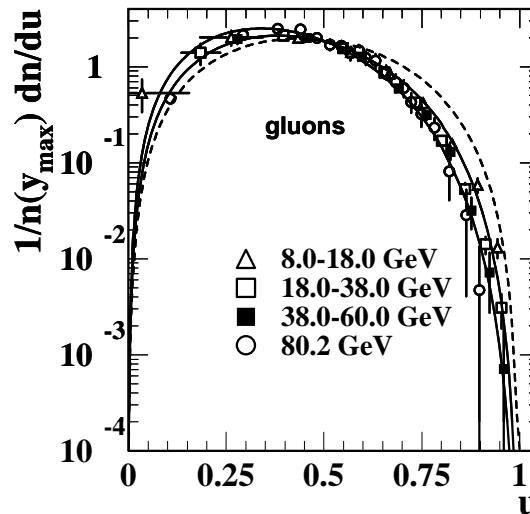
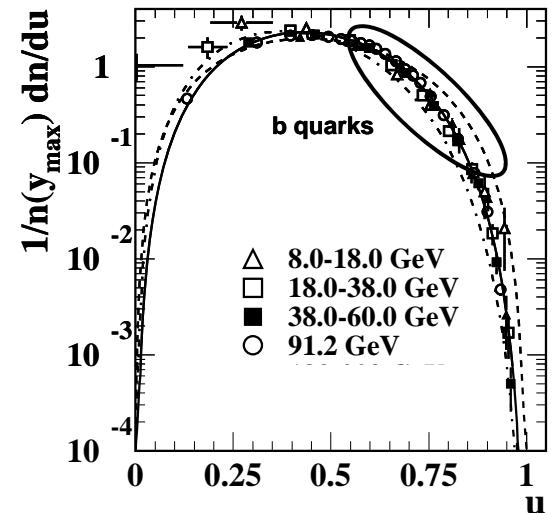
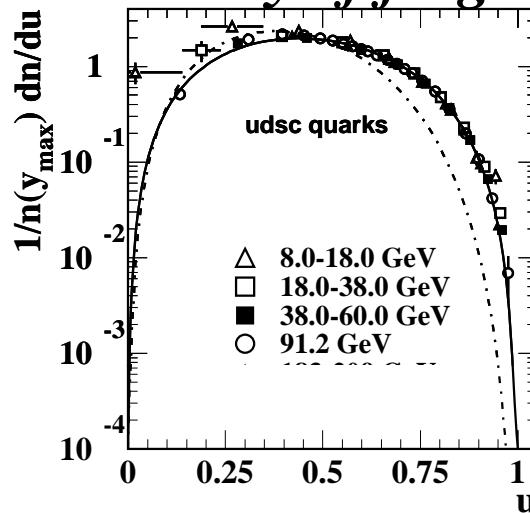
*quark and gluon shapes
are different*

$$\beta = \frac{1}{n(y_{\max})} \frac{dn}{du}$$

$$v_{\max} = \ln(\beta_q + \beta_g) \quad \text{Limiting}$$

$$v_{\min} = -\ln(1/\beta_q + 1/\beta_g) \quad \text{cases}$$

$$v(\beta) = \frac{\ln \beta - v_{\min}}{v_{\max} - v_{\min}}$$



bottom quark is anomalous

Sum Rule

$$\int_{2m_0/\sqrt{s}}^1 dx_E \quad D(x_E, s) = 2n_{\text{tot}}(s) \quad \text{Total dijet multiplicity}$$

$$\int_{2m_0/\sqrt{s}}^1 dx_E \quad x_E \quad D(x_E, s) = 2f \quad \begin{array}{l} \text{Energy sum rule} \\ f \text{ is fraction of particles detected} \end{array}$$

We obtain $2n_{\text{det}} \langle x_E \rangle = 1.18 \pm 0.05$ ($f = 0.59$)

$$D(u, y_{\max}) = 2n_{\text{tot}}(y_{\max})g(u, y_{\max})$$

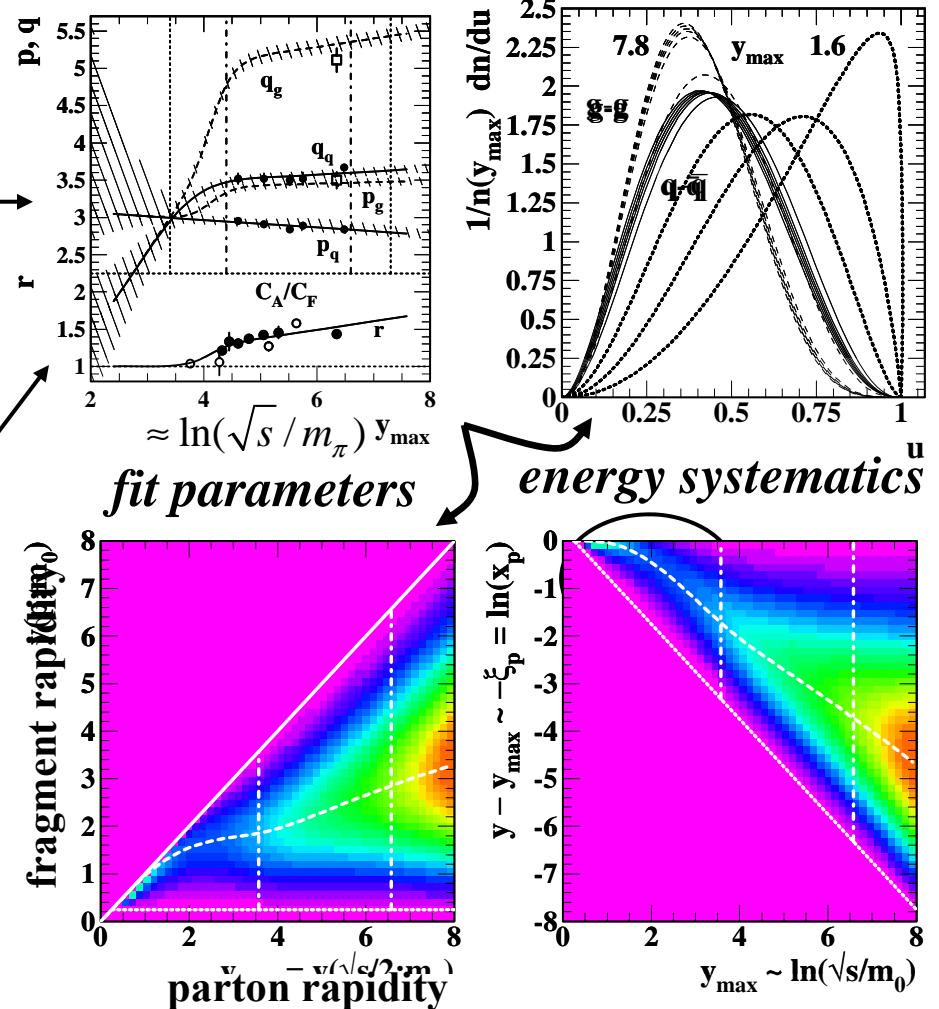
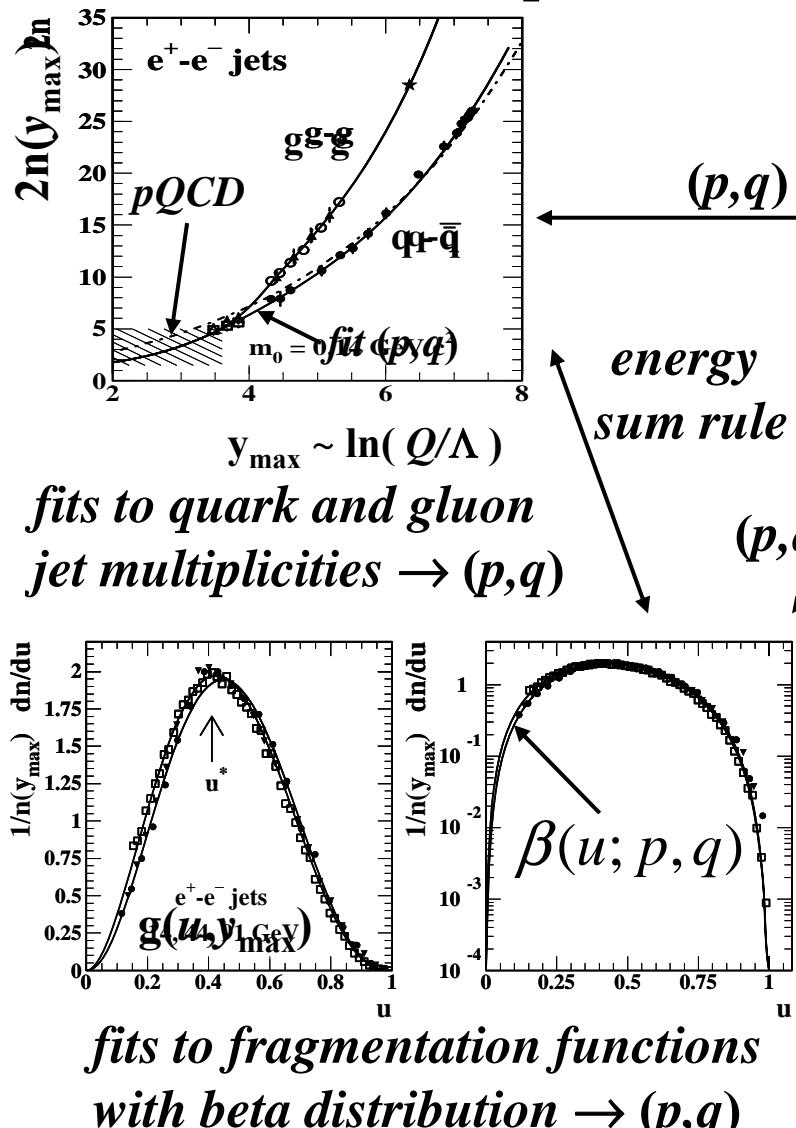
$$g(u, y_{\max}) \approx \beta(u; p, q)$$

$$\Rightarrow 2n_{\text{det}}(y_{\max}) = 1.18 / \int_0^1 du \quad x_E(u, y_{\max}) \quad \beta(u; p, q)$$

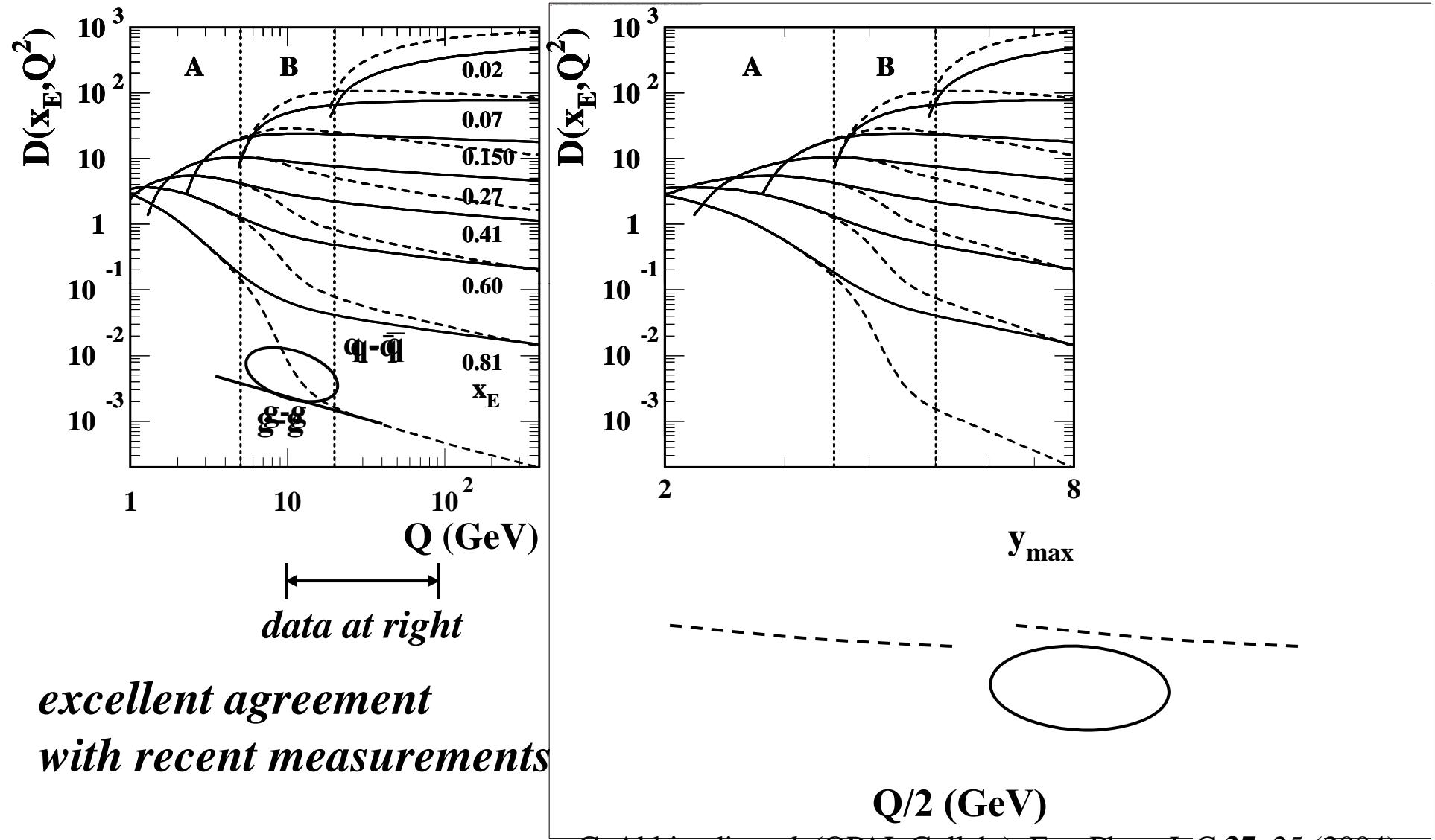
Dijet multiplicity directly relates to p and q parameters

Fragmentation Energy Systematics

extrapolation to non-perturbative regime

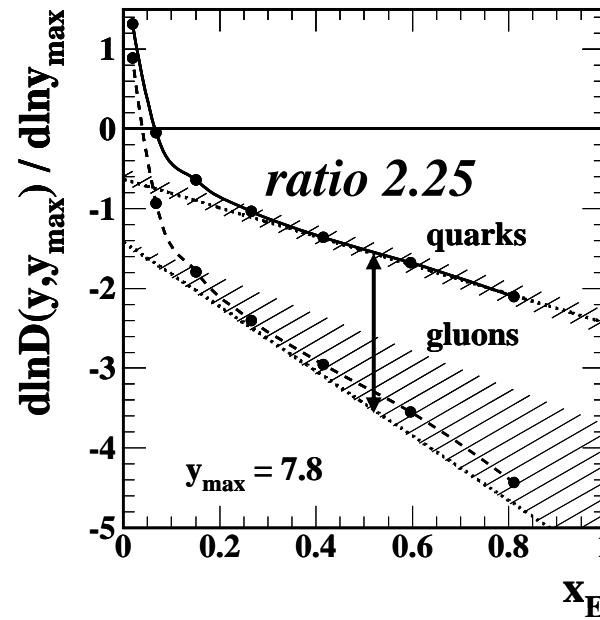
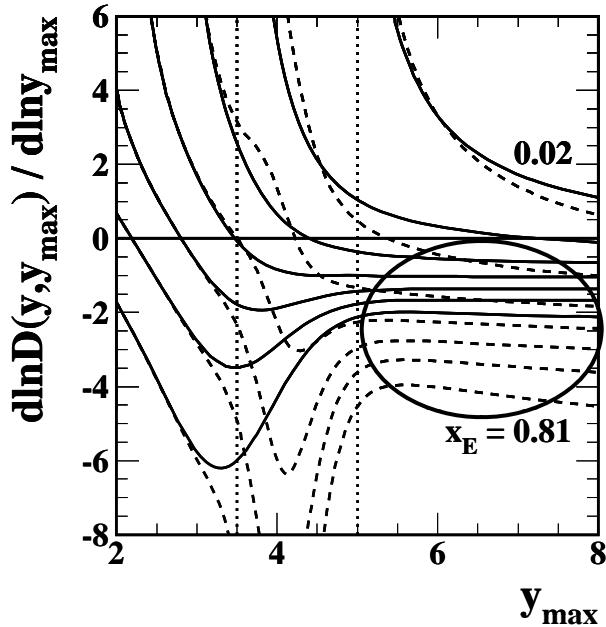


Scaling Violations – Standard



G. Abbiendi *et al.* (OPAL Collab.), Eur. Phys. J. C 37, 25 (2004)

Scaling Violations – Logarithmic



related to anomalous dimensions of QCD

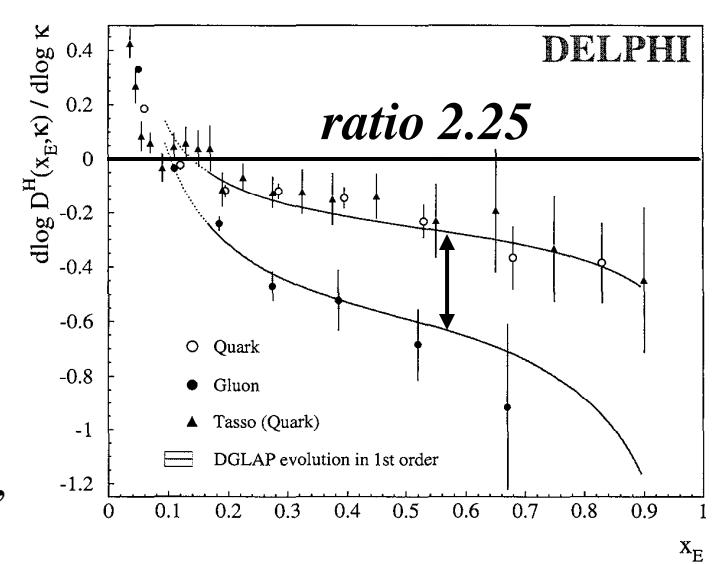
$$C_A/C_F = 2.25$$

near uniformity to right of dotted line

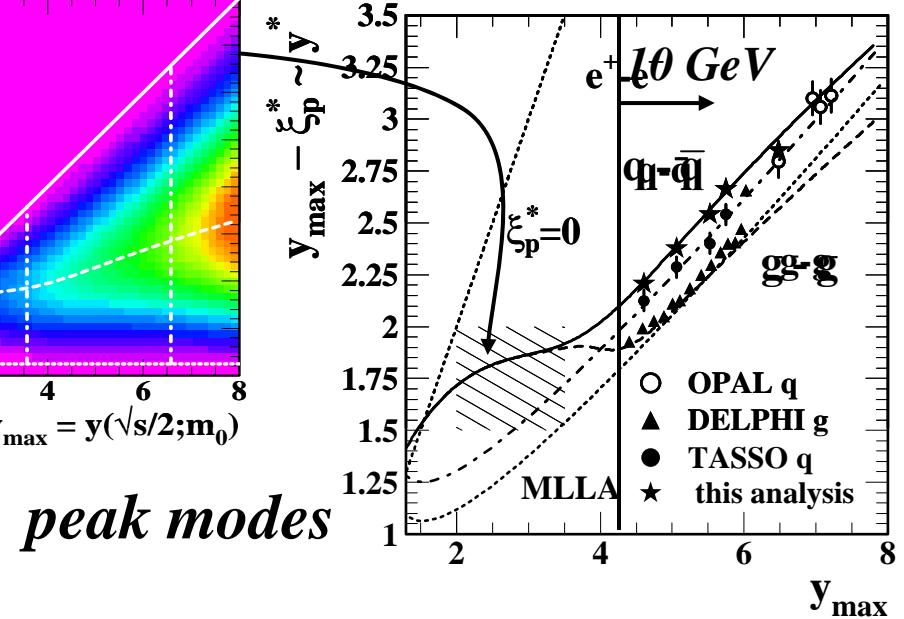
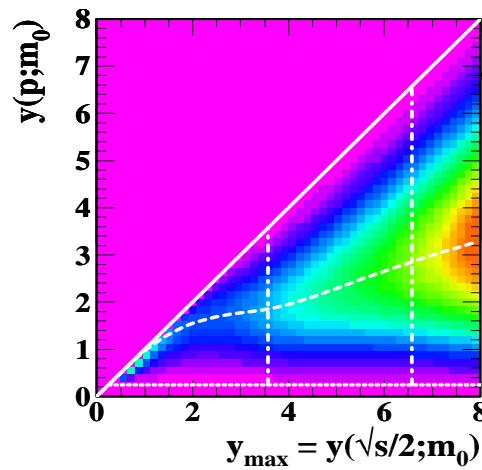
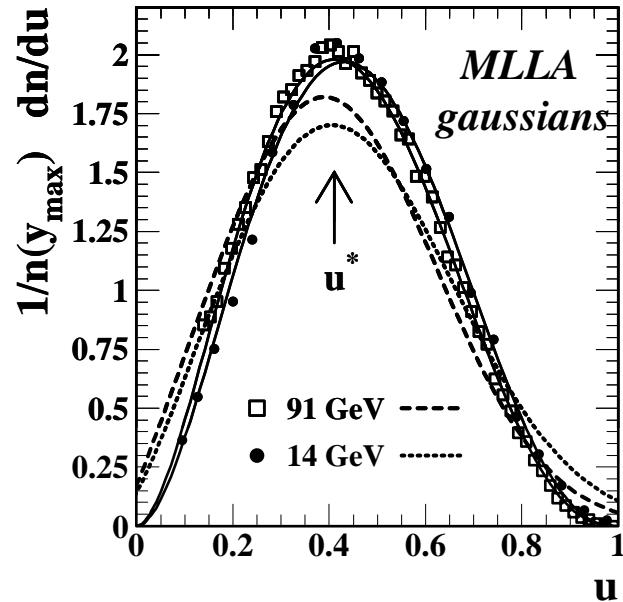
$$\frac{d \ln D_g(x_E, s) / d \ln s}{d \ln D_q(x_E, s) / d \ln s} \rightarrow \frac{C_A}{C_F} = \frac{3}{4/3} = 2.25$$

for $x_E \rightarrow 1$ and s large

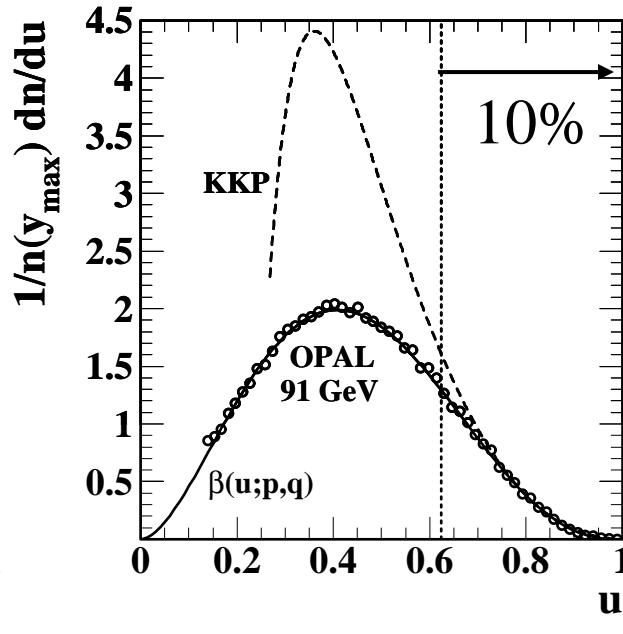
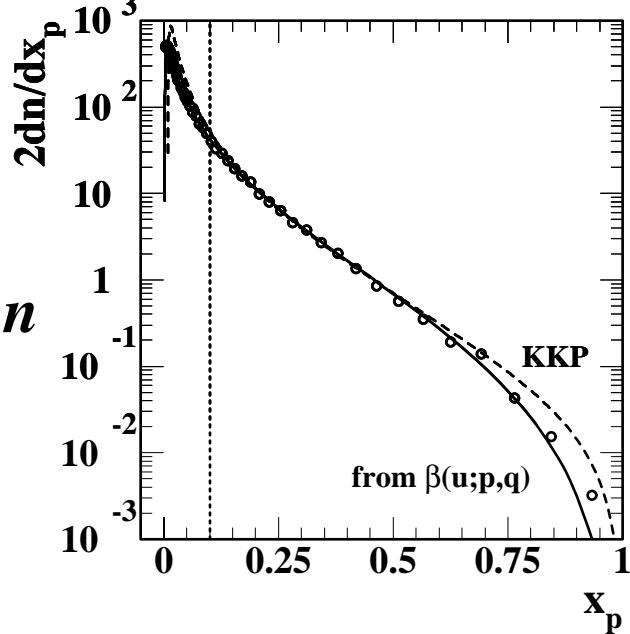
P. Abreu *et al.* (DELPHI Collab.),
Eur. Phys. J. C **13**, 573 (2000)



Conventional Parametrizations



*conventional
FF description*

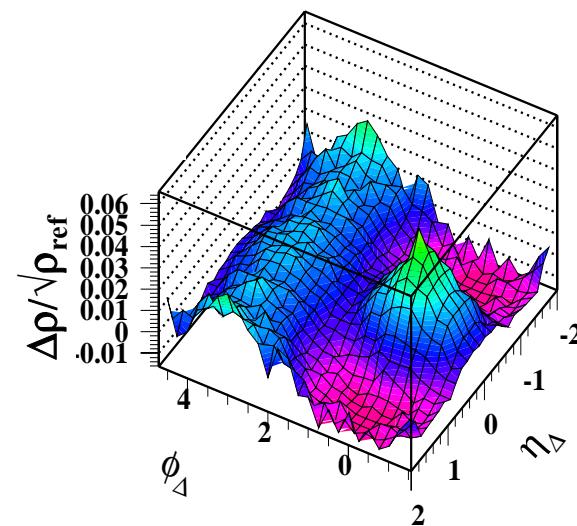
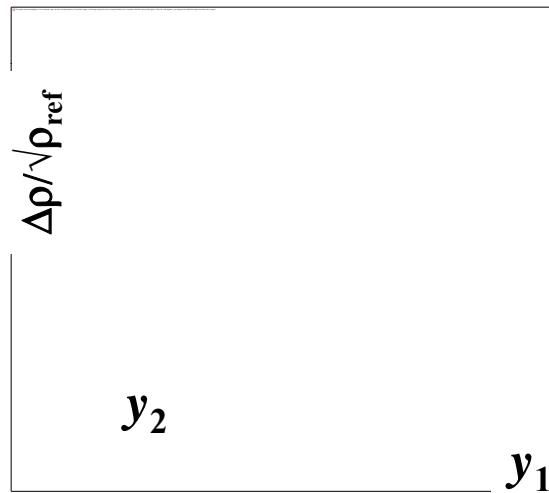


form
factor
on u

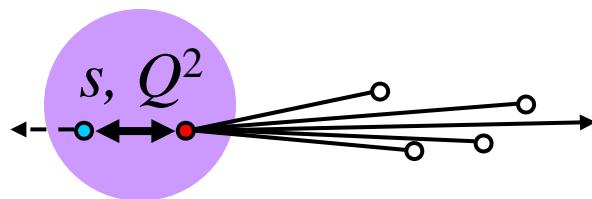
p-p Parton Fragment Correlations

two-particle fragment distributions

6D space: $(y_{t1}, \eta_1, \phi_1, y_{t2}, \eta_2, \phi_2)$



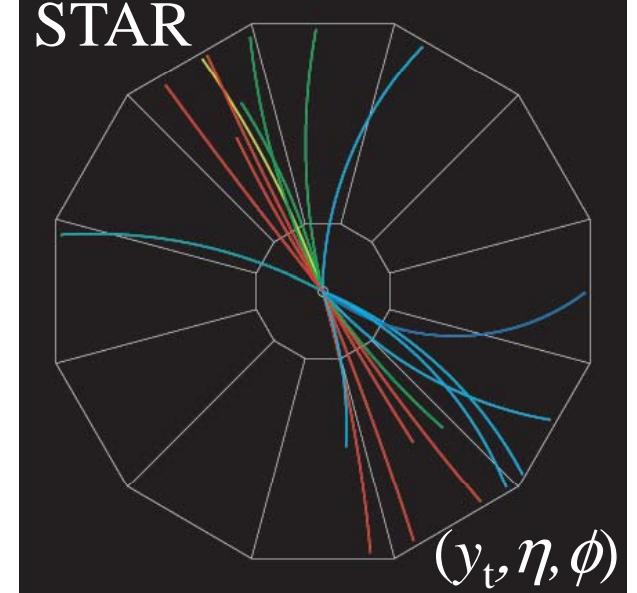
autocorrelation



minimum-bias: no trigger condition

RHIC p-p collision

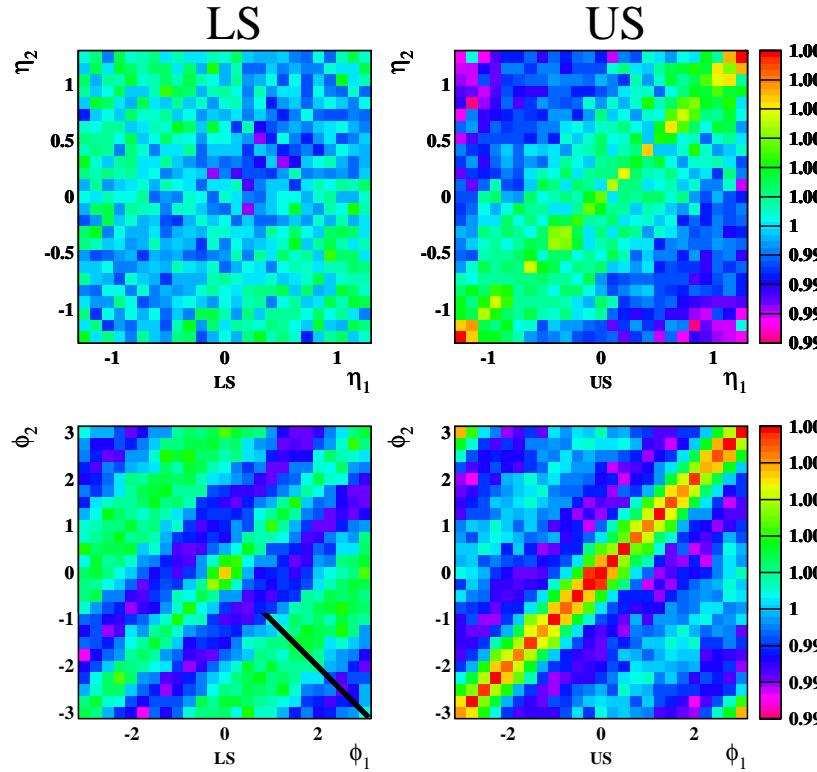
STAR



TPC end-on view

**autocorrelations are
unbiased – access
low- Q^2 partons**

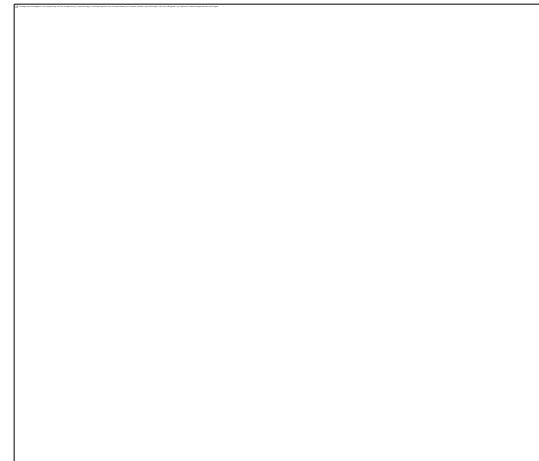
Angular Autocorrelations



6D Space:

$$(y_{t1}, \eta_1, \phi_1, y_{t2}, \eta_2, \phi_2)$$

How to preserve as much information as possible

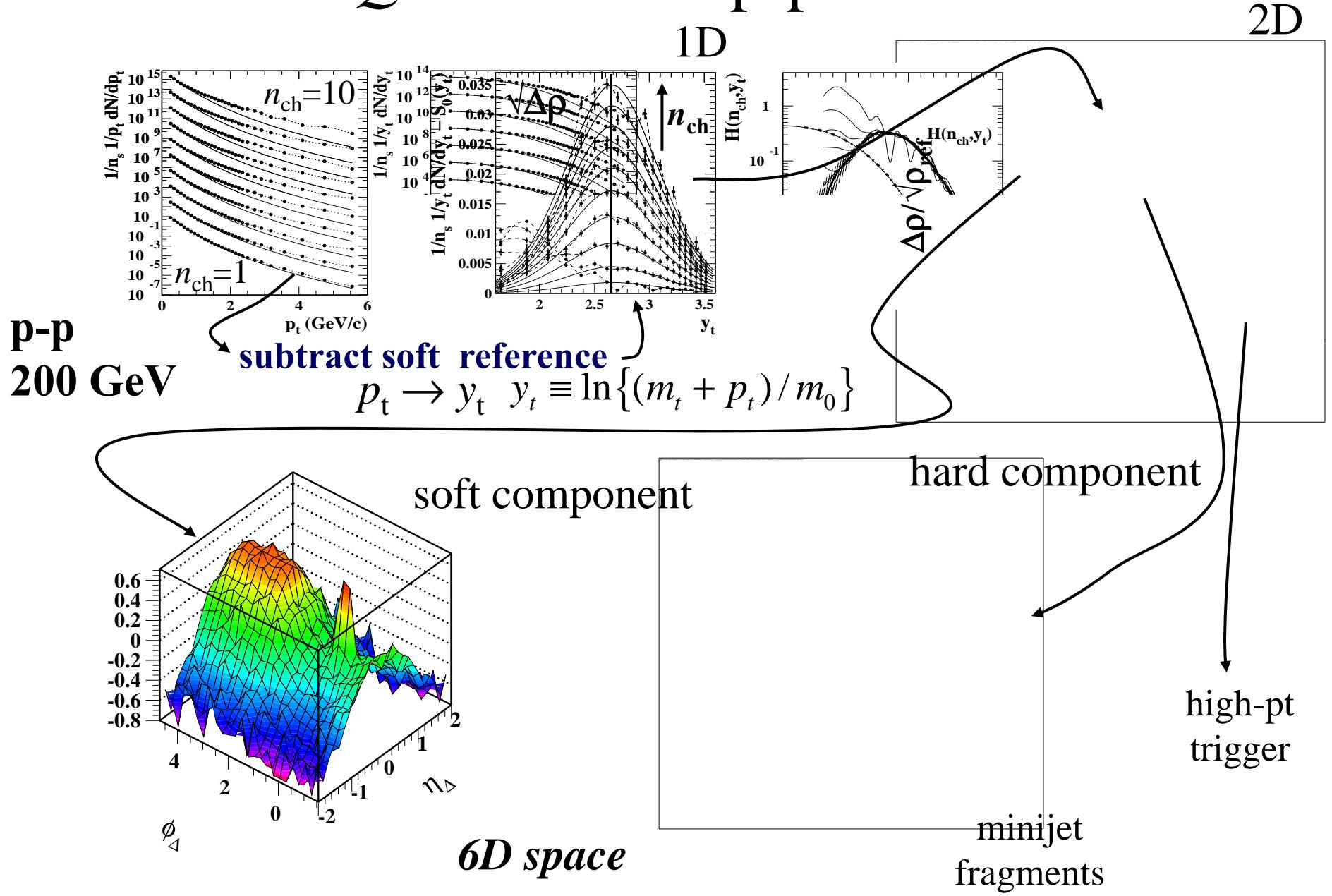


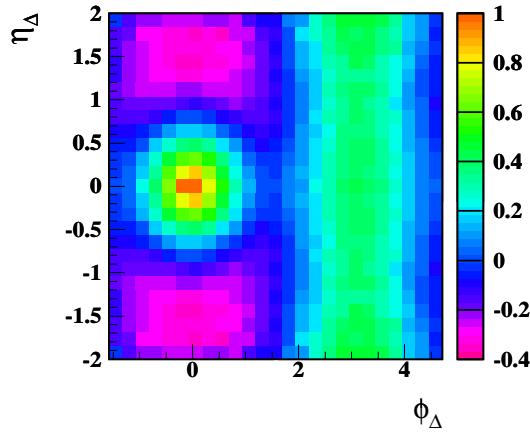
Project without loss of information

$$(\eta_1, \phi_1, \eta_2, \phi_2) \rightarrow (\eta_\Delta, \phi_\Delta)$$

Joint Angular Autocorrelation

Low- Q^2 Partons in p-p Collisions

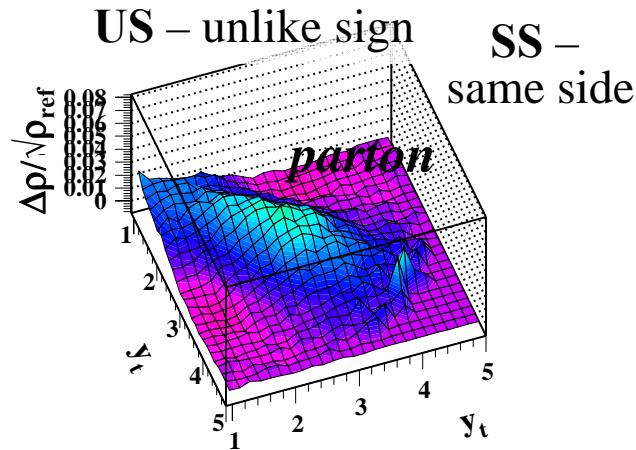
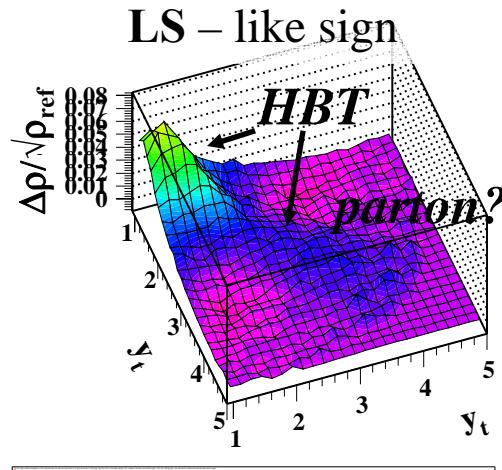




p-p Correlations on (y_{t1}, y_{t2})

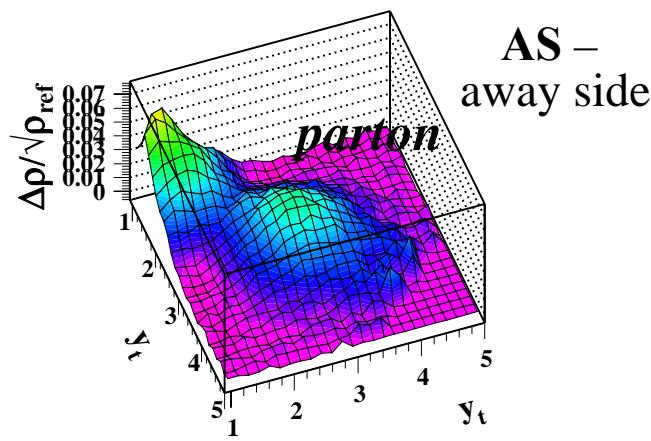
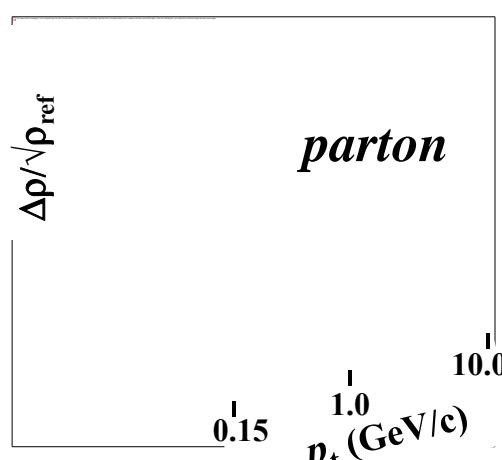
first two-particle *fragment distributions*

(except OPAL on ξ)



SS – same side

same-side parton fragmentation is restricted to US pairs



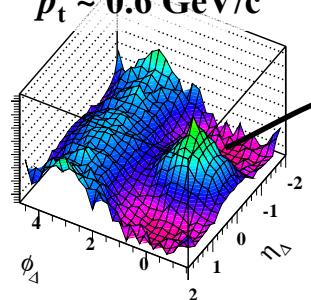
away-side parton fragmentation is independent of charge combination

Low- Q^2 Parton Fragment Distributions

p-p 200 GeV

STAR preliminary

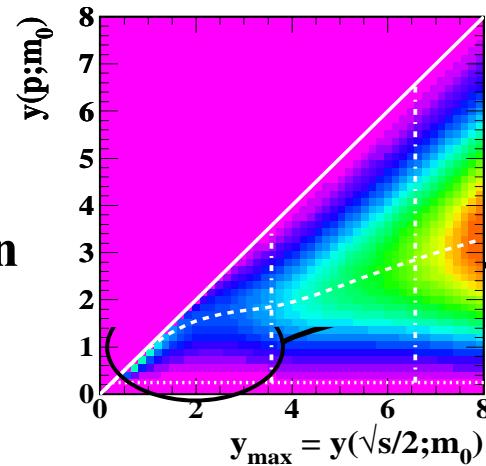
$p_t \sim 0.6 \text{ GeV}/c$



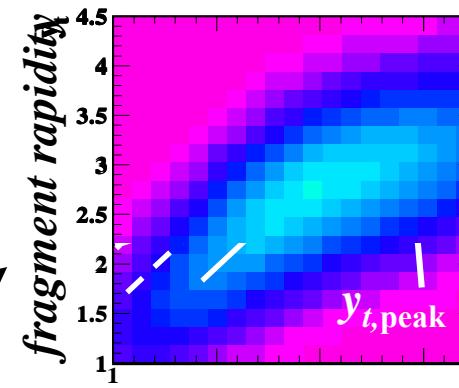
non-PID hadrons

$$y_t \equiv \ln \left\{ (m_t + p_t) / m_\pi \right\}$$

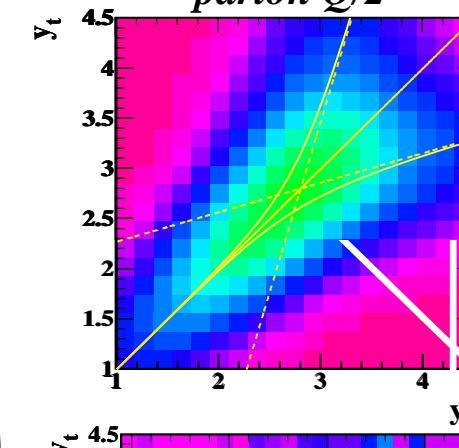
p-p *intrajet two-particle
fragment distribution*



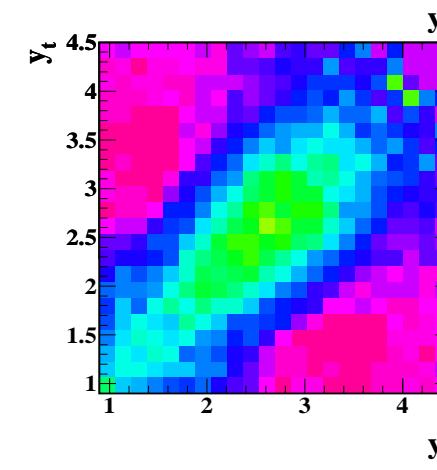
e-e fragmentation
functions on y_t
universal form



parton $Q/2$



fragment-parton
joint distribution
on $(y_t, y_{t,max}) \sim (x_p, Q^2)$



symmetrize to
fragment-fragment
distribution on (y_t, y_t)

compare with data

hacking QCD

Summary

- We have described all measured e^+e^- fragmentation functions with a precise (few %) model function
- The model function (beta distribution) allows us to extrapolate fragmentation trends to low Q^2
- Fragmentation parametrization provides excellent description of aspects of p-p correlations
- Ready to probe Au-Au fragmentation

‘theoretical’ basis for minijet correlations in nuclear collisions