Workshop on parton fragmentation processes in
the vacuum and in the medium
ECT* - Trento
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Evidence for power corrections in heavy quark fragmentation in e+e- collisions

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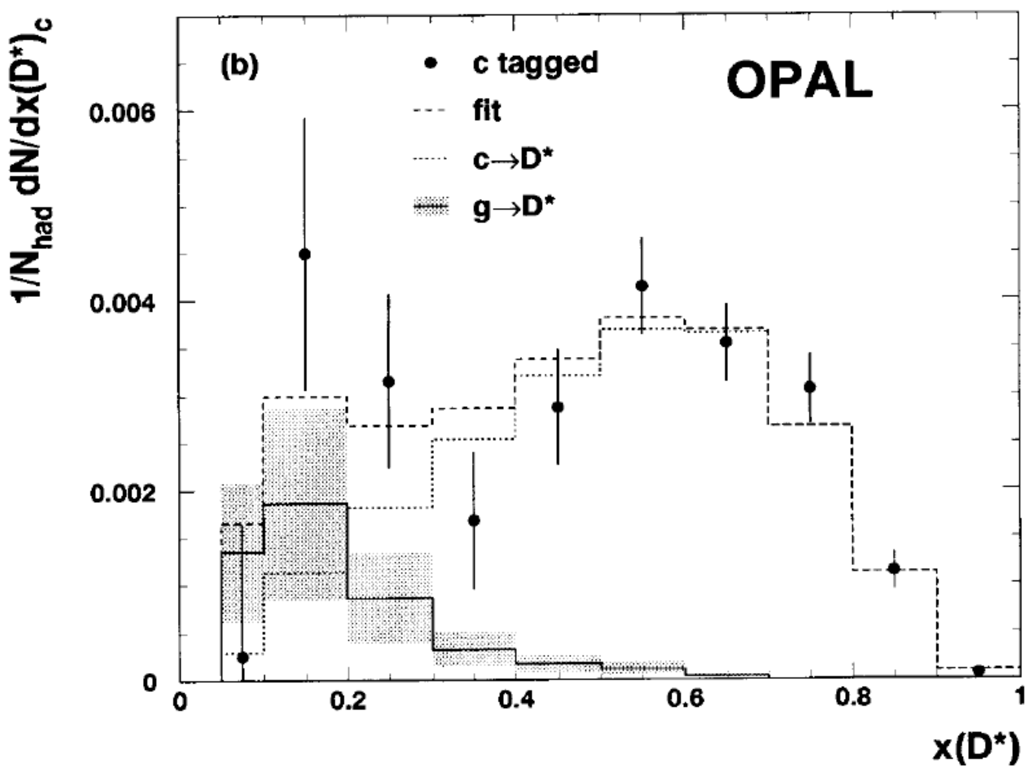
(hep-ph/0510032)

Outline

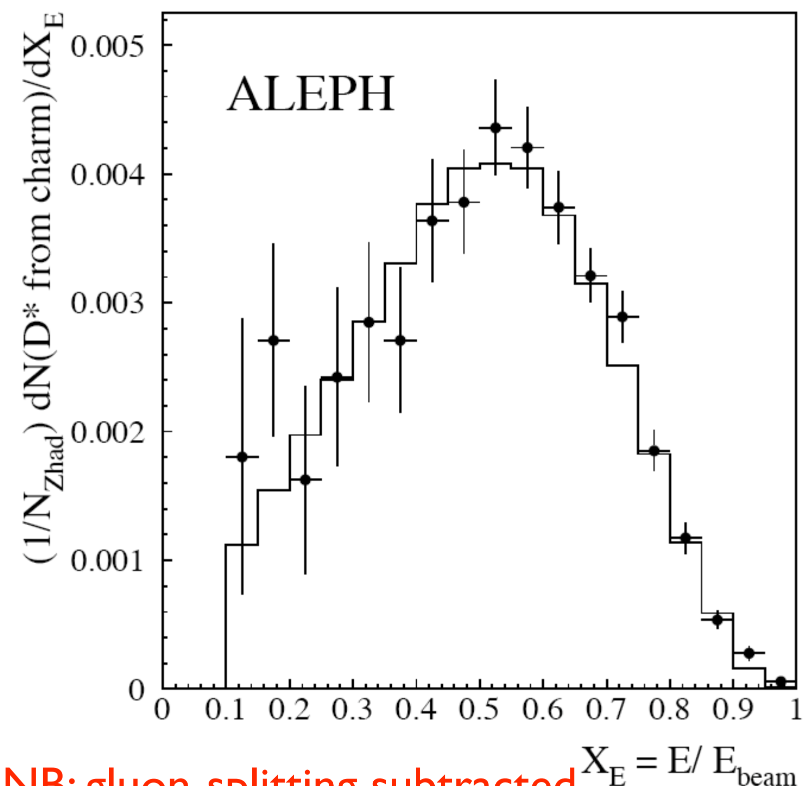
- Almost an experimental talk: global analysis of CLEO/BELLE and LEP data
- Comparisons with improved pQCD
- Evidence for power corrections?

The data: D* from LEP

OPAL 1994



ALEPH 1999

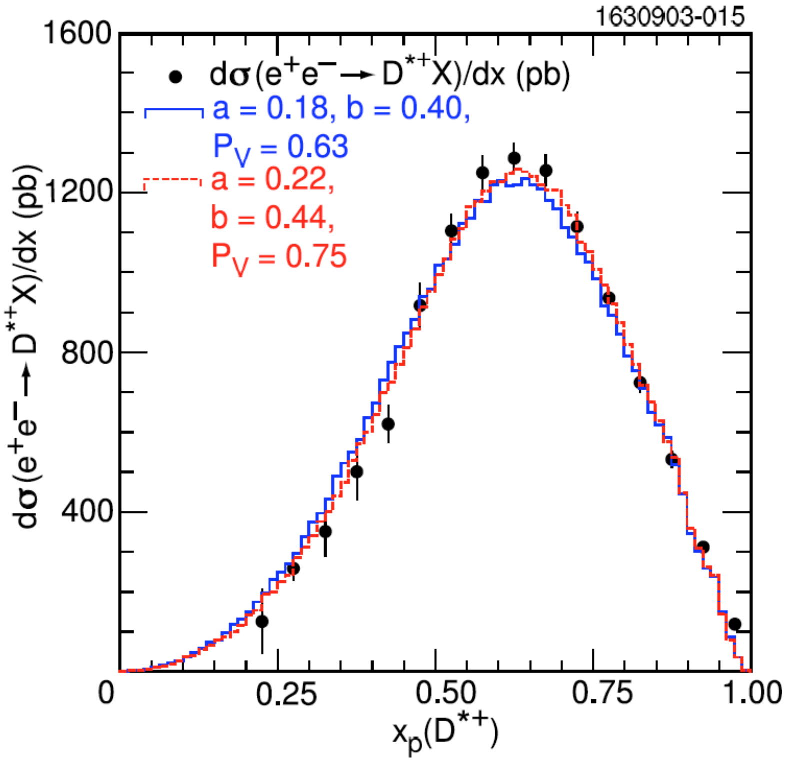


NB: gluon-splitting subtracted

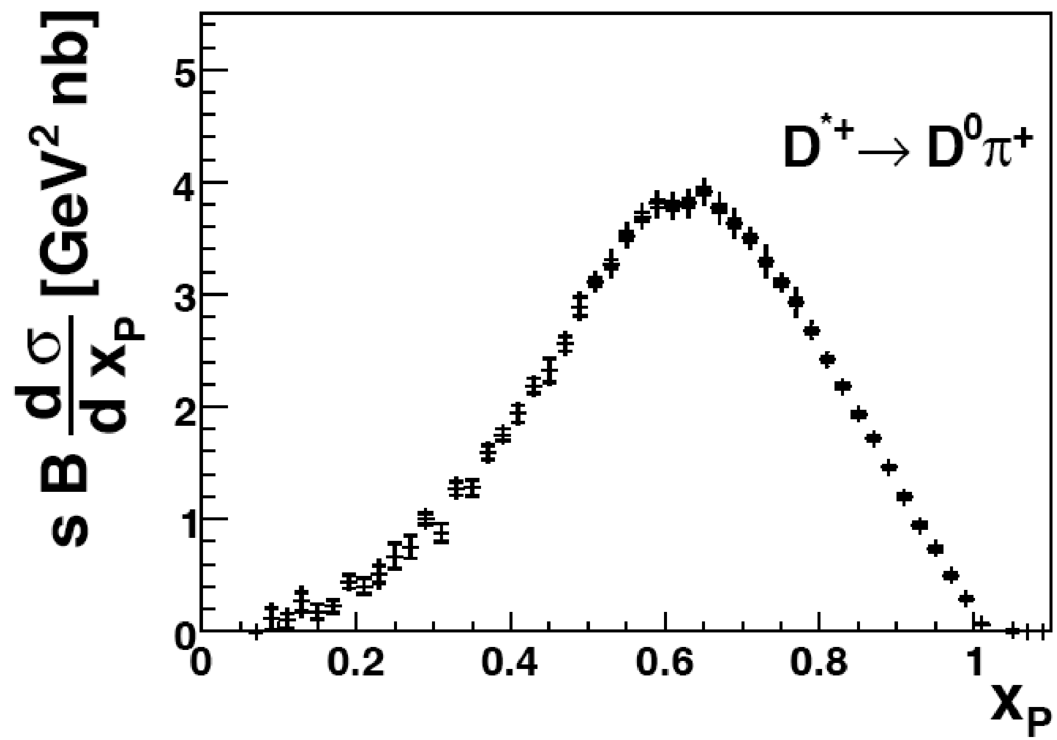
(All histograms are Monte Carlo fits)

The data: D/D* from CLEO and BELLE

CLEO 2004



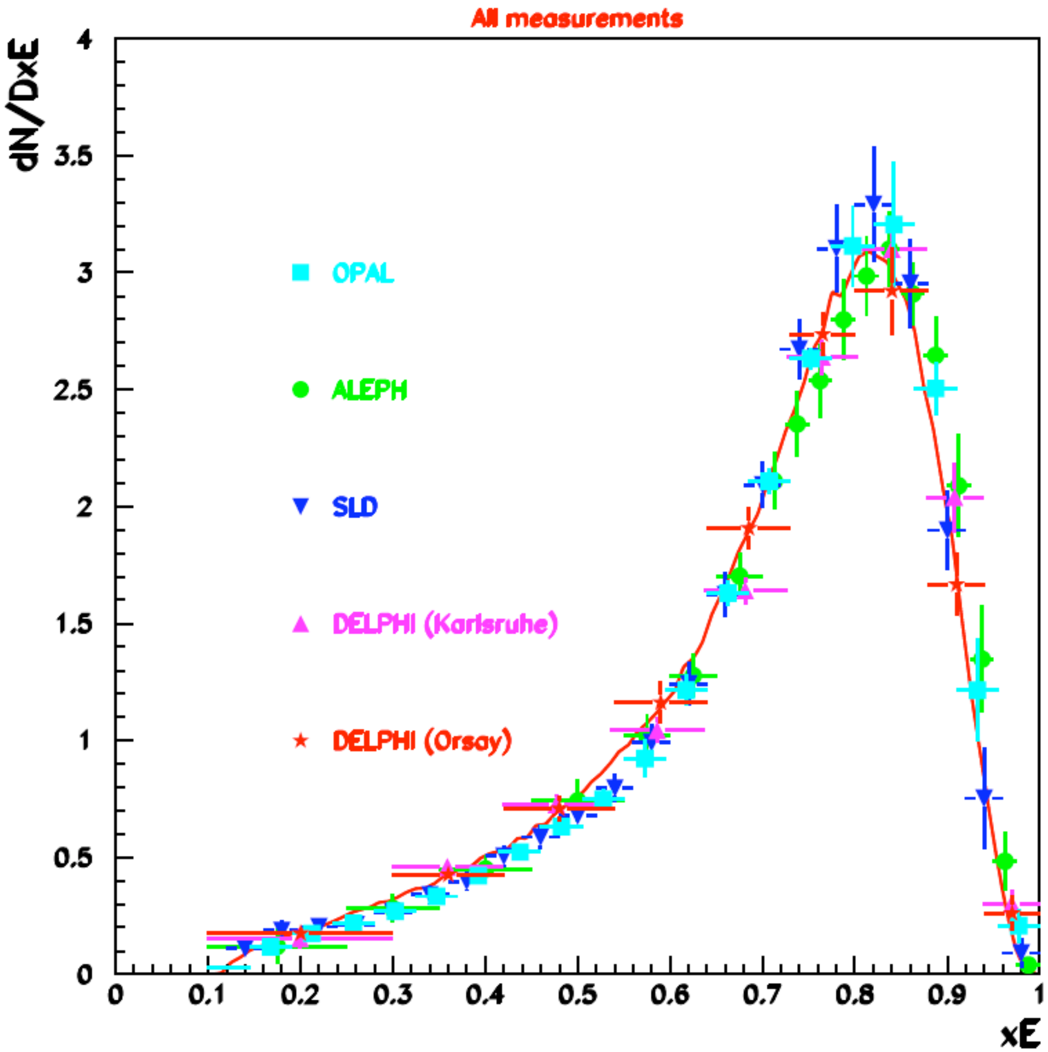
BELLE 2005



Extremely high-quality data at 10.6 GeV: possibility to test the 10.6 → 91.2 evolution and/or check the universality of the extracted non-perturbative fragmentation function

e.g. $\langle x\rangle_{10.6}$ = 0.642 ± 0.004 $\langle x\rangle_{91.2}$ = 0.492 ± 0.015

The data: B from LEP



A few per cent accuracy on low-N moments

Experiment	$< x_E >$	err. stat.	err. syst.
Cette thèse	0.704	0.001	0.008
ALEPH	0.716	0.006	0.006
DELPHI (Karl.)	0.715	0.001	0.005
OPAL	0.719	0.002	0.004
SLD	0.709	0.003	0.004

Table 1: *Différentes mesures de $< x_E >$ à l'énergie du Z^0.*

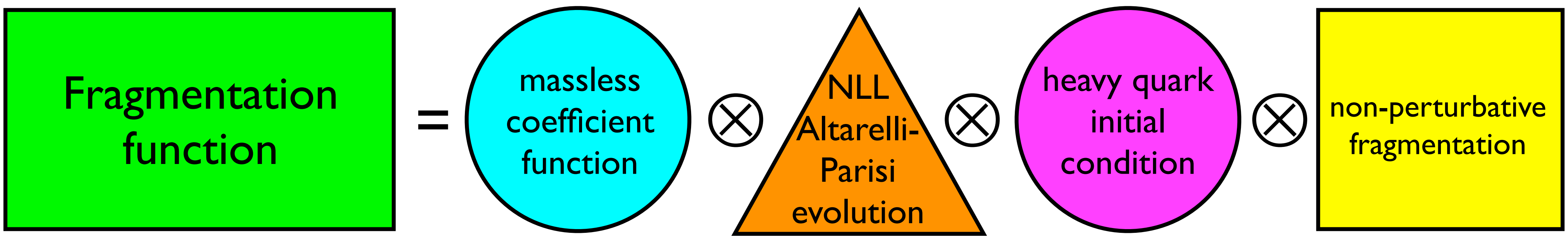
The theory

pQCD: matched O(alpha_s) +
NLL resummed (collinear) +
NLL resummed (soft)
in the $m/Q \to 0$ limit

[Mele, Nason '91]
[Dokshitzer, Khoze, Troyan '95] [MC, Catani '01]

Semi-numerical O(alpha_s^2) with mass terms exists [Nason, Oleari '97]
Phenomenological effect is fairly negligible

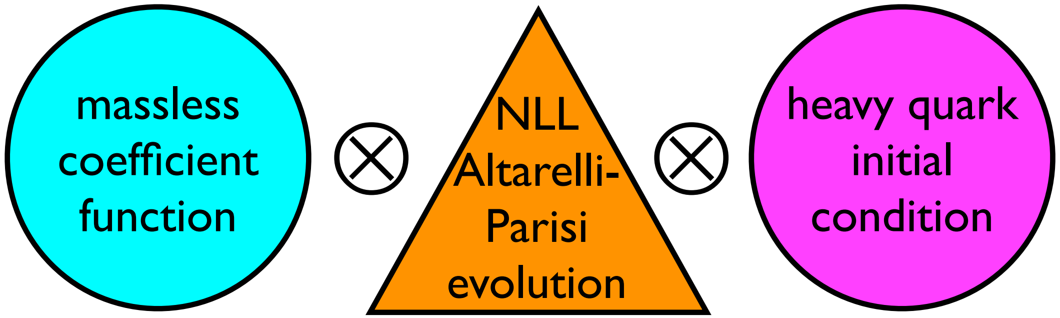
Schematically, we write the factorized expression (in MSbar):



Analytical massless O(alpha_s^2) initial conditions also exist [Melnikov, Mitov '04]
NNLO time-like evolution kernels recently calculated [Mitov, Moch, Vogt '06]

Power corrections -- the common wisdom

The perturbative factorisation is valid up to power suppressed terms.
An analysis of the pQCD structure gives:



$+O[(\Lambda/q)^2]$ $+O[(\Lambda/m)]$

Related to the coefficient function. Usually neglected

Related to the heavy quark part. Given by the non-perturbative fragmentation function

Four important issues (in order of increasing relevance):

- crossing of bottom threshold when evolving charm FF

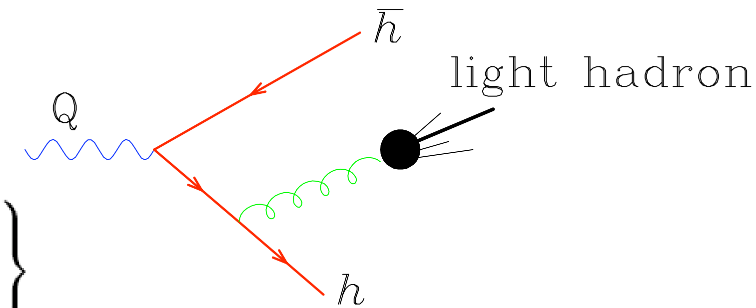
- inclusion of gluon splitting effects. Or, more generally, mixings

- deconvolution of initial state electromagnetic radiation

- treatment of Landau pole in soft-gluon resummation expressions

Bottom threshold [MC, Oleari, Nason, '05]

Production via fragmentation of a given hadron h can be considered either in the n_L or in the $n = n_L + 1$ schemes



n_L

$$\frac{d\sigma}{dx} = \int_x^1 \frac{dy}{y} \left\{ \sum_{i \in \mathbb{I}_{n_L}} D_i^{(n_L)}(x/y,\mu) \frac{d\hat{\sigma}_i^{(n_L)}(y,\mu)}{dy} + D_g^{(n_L)}(x/y,\mu) \frac{d\sigma_{h\bar{h}g}(y)}{dy} \right\}$$

$$\frac{d\sigma_{h\bar{h}g}(y)}{dy} = \sigma_{h\bar{h}} \frac{\alpha_S}{2\pi} C_F \, 2 \frac{1+(1-y)^2}{y} \left\{ \log \frac{Q^2}{m^2} + \log(1-y) - 1 \right\}$$

MASSIVE QUARK

$n = n_L + 1$

$$\frac{d\sigma}{dx} = \int_x^1 \frac{dy}{y} \sum_{i \in \mathbb{I}_n} D_i^{(n)}(x/y,\mu) \frac{d\hat{\sigma}_i^{(n)}(y,\mu)}{dy}$$

Difference:

$$\frac{d\hat{\sigma}_g^{(n)}(y,\mu)}{dy} - \frac{d\hat{\sigma}_g^{(n_L)}(y,\mu)}{dy} = \frac{d\hat{\sigma}_{h\bar{h}g}(y,\mu)}{dy}$$

MASSLESS MSBAR

$$\frac{d\hat{\sigma}_{h\bar{h}g}(y,\mu)}{dy} = \sigma_{h\bar{h}} \frac{\alpha_S}{2\pi} C_F 2 \frac{1+(1-y)^2}{y} \left\{ 2\log y + \log(1-y) + \log \frac{Q^2}{\mu^2} \right\}$$

$$D_h^{(n)}(x,\mu) = D_{\bar{h}}^{(n)}(x,\mu) = \int_x^1 \frac{dy}{y} D_g(x/y,\mu) \frac{\alpha_S}{2\pi} C_F \frac{1+(1-y)^2}{y} \left[\log \frac{\mu^2}{m^2} - 1 - 2\log y \right]$$

$$D_g^{(n)}(x,\mu) = D_g^{(n_L)}(x,\mu) \left(1 - \frac{T_F \alpha_S}{3\pi} \log \frac{\mu^2}{m^2} \right)$$

$$D_{i/\bar{i}}^{(n)}(x,\mu) = D_{i/\bar{i}}^{(n_L)}(x,\mu) \qquad \text{for } i = q_1, \ldots, q_{n_L} \, .$$

Time-like equivalent of Collins-Tung relations for parton distribution functions

ISR effects

The data use the observed heavy hadron energy normalized to the beam energy. However, before the hard interaction the beams lose energy due to electromagnetic radiation

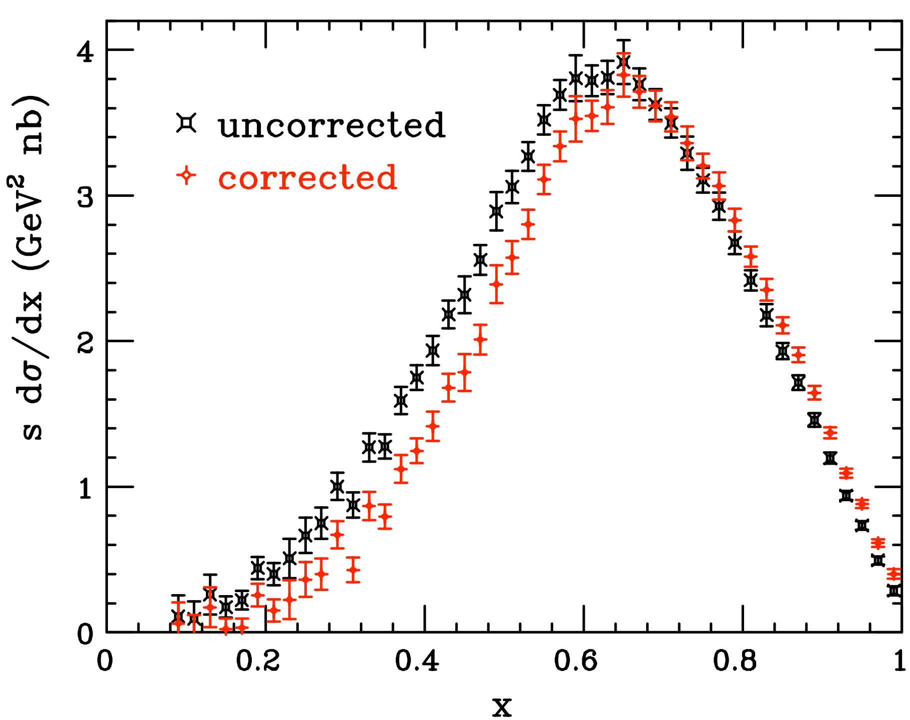
Either include this effect in the calculation of the fragmentation function, or deconvolute the data

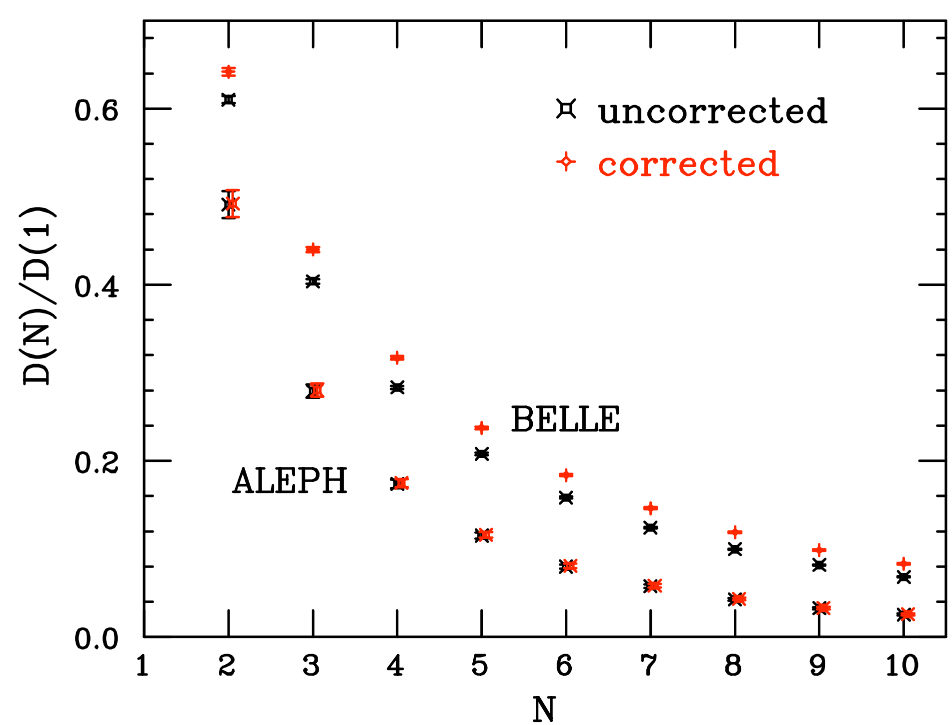
The latter is more convenient (do it once, get a 'clean' set of data to be used for multiple fits)

measured **corrected**

$$D(x_i) = \int_{\frac{4m_h^2}{s}}^{1} dz \int dy\, d\cos\theta\, \frac{1}{\sigma_0(s)} \frac{d\sigma_0(zs,\cos\theta)}{d\cos\theta} \frac{dP}{dz} D_c(y)\, \delta(x_i - x(z,y,\theta))$$

$$\frac{dP}{dz} = \delta\beta(1-z)^{\beta-1} - \frac{\beta}{2}(1+z) \qquad \beta = 2\frac{\alpha_{\rm em}}{\pi}\left[\log\frac{s}{m_e^2} - 1\right], \quad \delta = 1 + \frac{3}{4}\beta + \frac{\alpha_{\rm em}}{\pi}\left(\frac{\pi^2}{3} - \frac{1}{2}\right), \quad z = \frac{s_{\rm had}}{s}$$





Large-x region

Branch point in Sudakov resummation factor prevents going beyond

$$N_{\rm ini}^{L} = \exp\left(\frac{1}{2\, b_0 \alpha_{\rm s}(\mu_0^2)}\right) \simeq \frac{\mu_0}{\Lambda_{\rm QCD}}$$

i.e. N_L ~ 5-10 for charm and N_L ~ 30 for bottom. This corresponds to x ~ 0.8 for charm.

However, there are plenty of data beyond that point, not to mention that the singularity distorts the spectrum even for lower x.

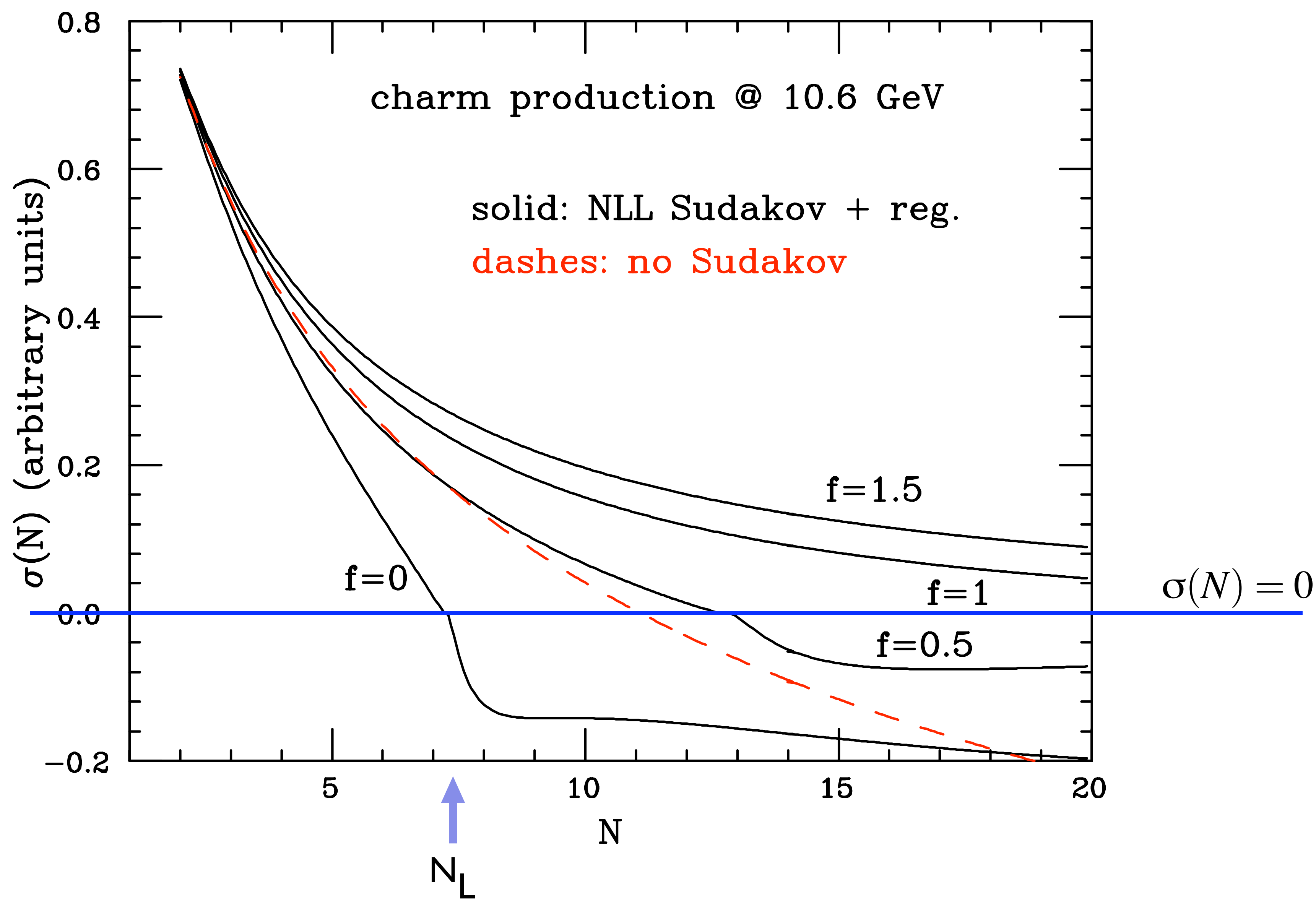
Two options:

1. resum all subleading logs (for instance via DGE) and regularize the ensuing Borel antitransform
2. minimally modify the Sudakov factor, such that the resummation prescription
   - a - is consistent with all known perturbative results
   - b - yields physically acceptable results
   - c - does not introduce power corrections larger than generally expected, i.e. NΛ/m for the initial condition and N(Λ/Q)^2 for the coefficient function

This is achieved by replacing:

$$-\ln\frac{1}{N} \to -\ln\left(\frac{\frac{1}{N}+\frac{f}{N_L}}{1+\frac{f}{N_L}}\right) \to \begin{cases} \text{large } N \to \ln\frac{N_L}{f} \\ \text{small } N \to \ln\left[N\left(1-\frac{f(N-1)}{N_L}+\cdots\right)\right] \end{cases}$$

Large-x region



The regularization prescription with f > 1 prevents the distribution from becoming unphysical beyond the Landau pole

Non-perturbative fragmentation function

A **non-perturbative** component is of course needed to describe the data. It is assumed **universal** (for a given quark and a given heavy hadron) and convoluted to the perturbative component:

$$\sigma_H(N, q^2) = \sigma_Q(N, q^2, m^2) D_{\rm NP}(N)$$

One possible choice for the non-perturbative FF, flexible enough to lead to particularly good fits, is

$$D_{\rm NP}(x) = {\rm Norm.} \times \frac{1}{1+c}\left[\delta(1-x) + cN_{a,b}^{-1}(1-x)^a x^b\right]$$

Simpler choices can of course also be made. For instance, the Karvelishvili et al. one,

$$D_{\rm NP}(x) = (\alpha+1)(\alpha+2)x^\alpha(1-x)$$

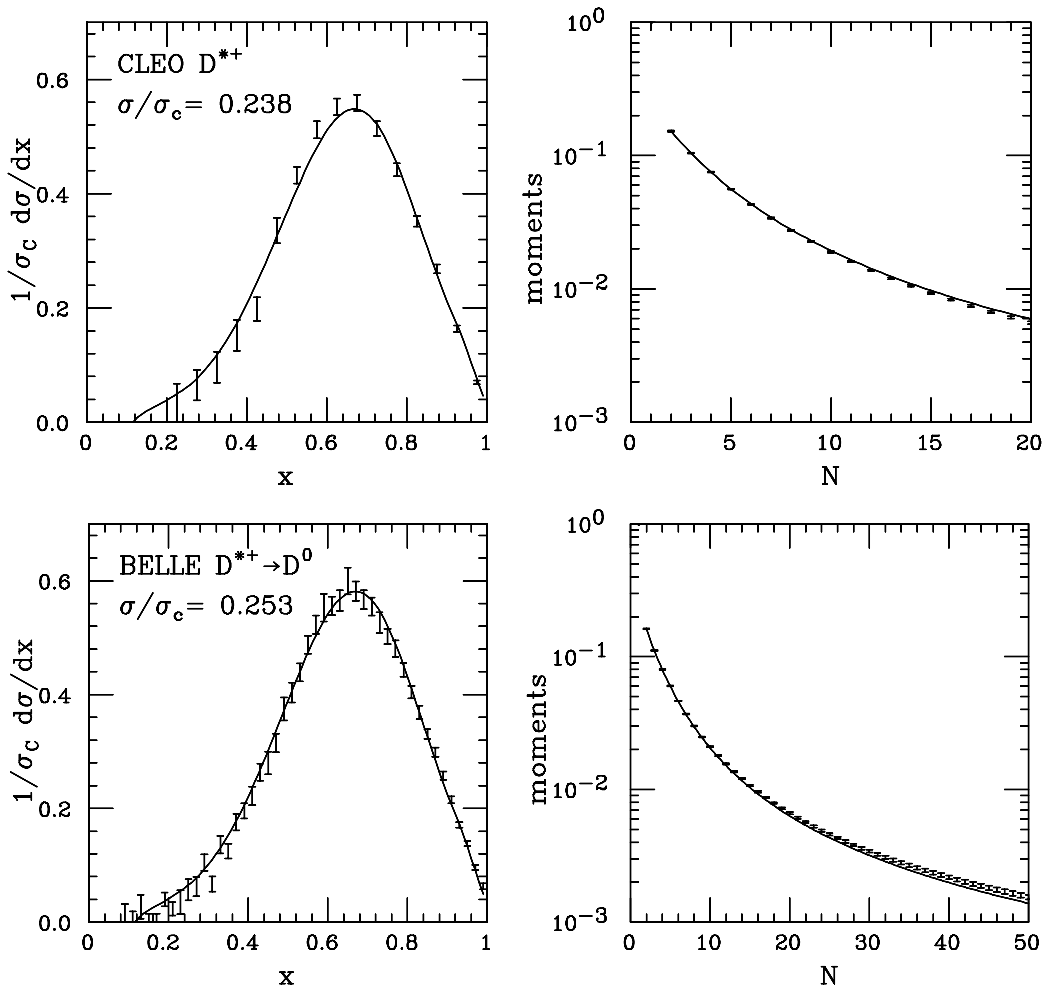
whose Mellin transform,

$$D_{\rm NP}(N) = \frac{(\alpha+1)(\alpha+2)}{(\alpha+N)(\alpha+N+1)}$$

can easily be written as a power correction series, by interpreting $\alpha \to 2m/\Lambda$

$$D_{\rm NP}(N) = 1 - (N-1)\frac{2}{\alpha} + \mathcal{O}\left(\frac{1}{\alpha^2}\right)$$

Fits to D* data



Simultaneous fit to CLEO and BELLE D* data

Very good description up to x=1

Eq. (61): $a = 1.8 \pm 0.2$, $b = 11.3 \pm 0.6$, $c = 2.46 \pm 0.07$, total $\chi^2 = 139$					
Set	(C) D^{*+}	(B) $D^{*+} \to D^0$	(B) $D^{*+} \to D^+$	(C) D^{*0}	(B) D^{*0}
Norm.	0.238	0.253	0.227	0.225	0.211
χ^2/pts	33/16	63/46	13/46	13/16	17/46

Table 2: Results of the fit to D^* CLEO (C) and BELLE (B) data. The last line reports the χ^2 over the number of fitted points for each data set.

Fits to D data

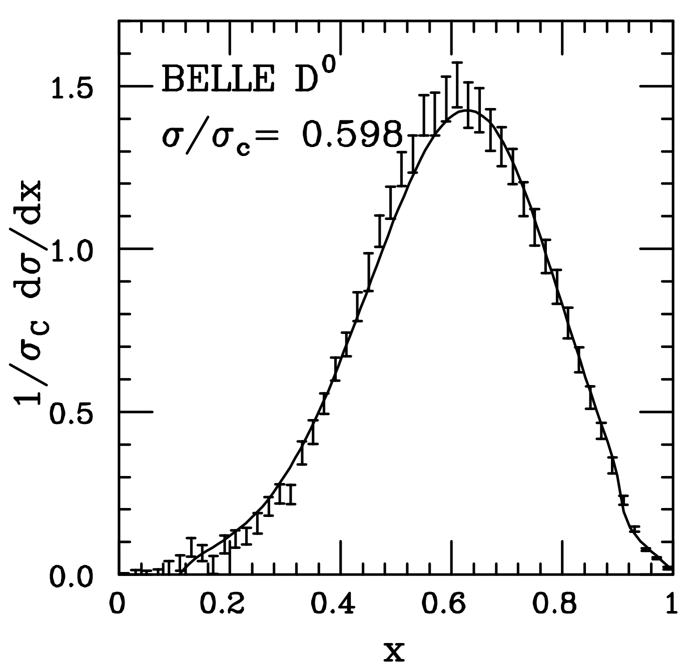
The D*→ DX decays can be modeled kinematically and lead to the following fragmentation functions:

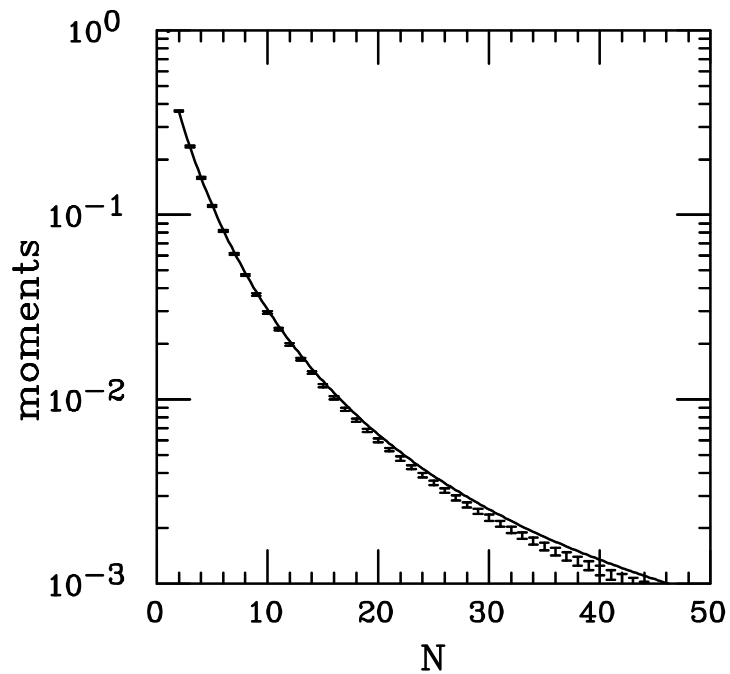
$$\tilde{D}^D_\pi(N) = D^{D^*}_{\rm NP}(N)\left[\frac{m_D}{m_{D^*}}\right]^{N-1}$$

$$\begin{aligned}\tilde{D}^D_\gamma(N) &= D^{D^*}_{\rm NP}(N)\int\frac{d\cos\theta}{2}\left[\frac{p_D\cos\theta+m_D}{m_{D^*}}\right]^{N-1}\\ &= D^{D^*}_{\rm NP}(N)\,\frac{m_{D^*}}{2p_D}\,\frac{(m_D+p_D)^N-(m_D-p_D)^N}{Nm^N_{D^*}}\end{aligned}$$

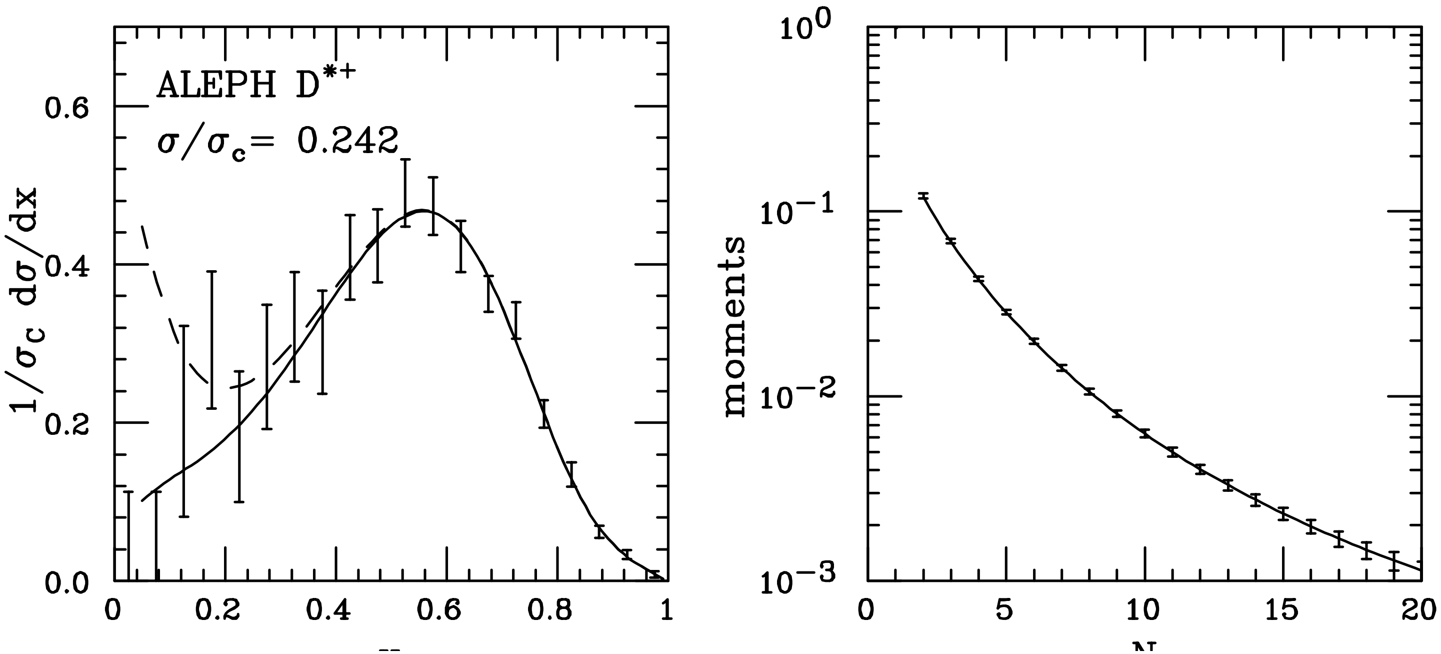
$$\begin{aligned}D^{D^+}_{\rm NP}(x) &= D^{D^+,p}_{\rm NP}(x)+B(D^{*+}\to D^+\pi^0)\tilde{D}^{D^+}_\pi(x)\\ &\quad +B(D^{*+}\to D^+\gamma)\tilde{D}^{D^+}_\gamma D(x)\,, \qquad (73)\\ D^{D^0}_{\rm NP}(x) &= D^{D^0,p}_{\rm NP}(x)+[B(D^{*+}\to D^0\pi^+)+B(D^{*0}\to D^0\pi^0)]\tilde{D}^{D^0}_\pi(x)\\ &\quad +B(D^{*0}\to D^0\gamma)\tilde{D}^{D^0}_\gamma(x)\,. \qquad (74)\end{aligned}$$

$$p_D = \frac{m^2_{D^*}-m^2_D}{2m_{D^*}}$$





Fits to LEP D* data



Fit to ALEPH data

$$a = 2.4 \pm 1.2\,, \quad b = 13.9 \pm 5.7 \quad c = 5.9 \pm 1.7$$

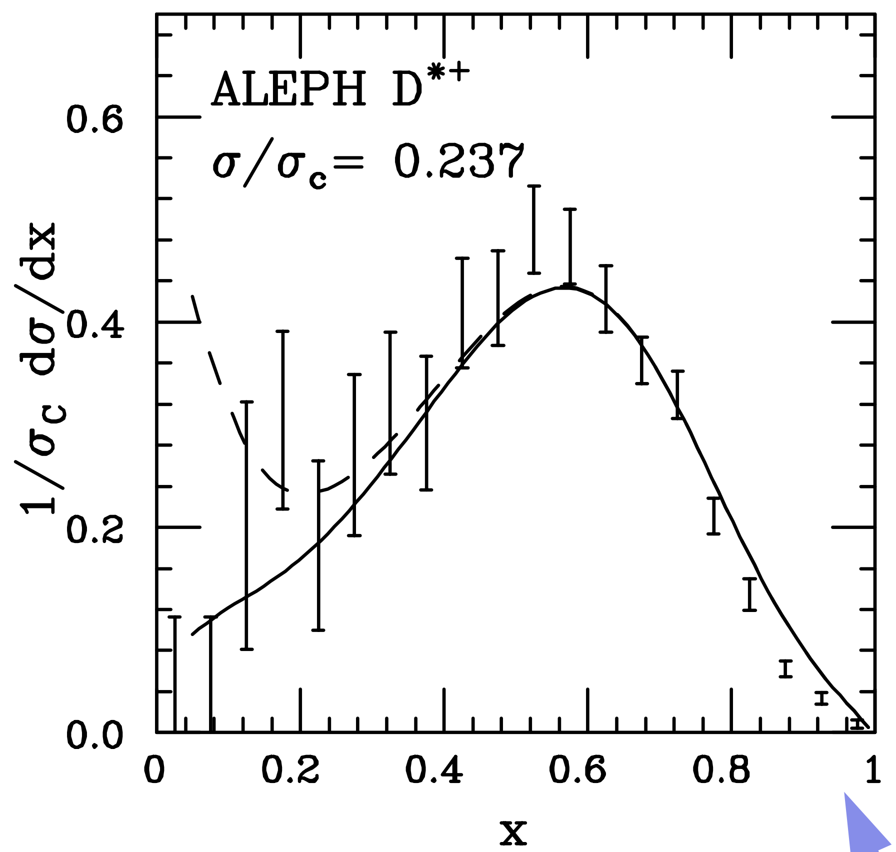
Compare to CLEO/BELLE parameters

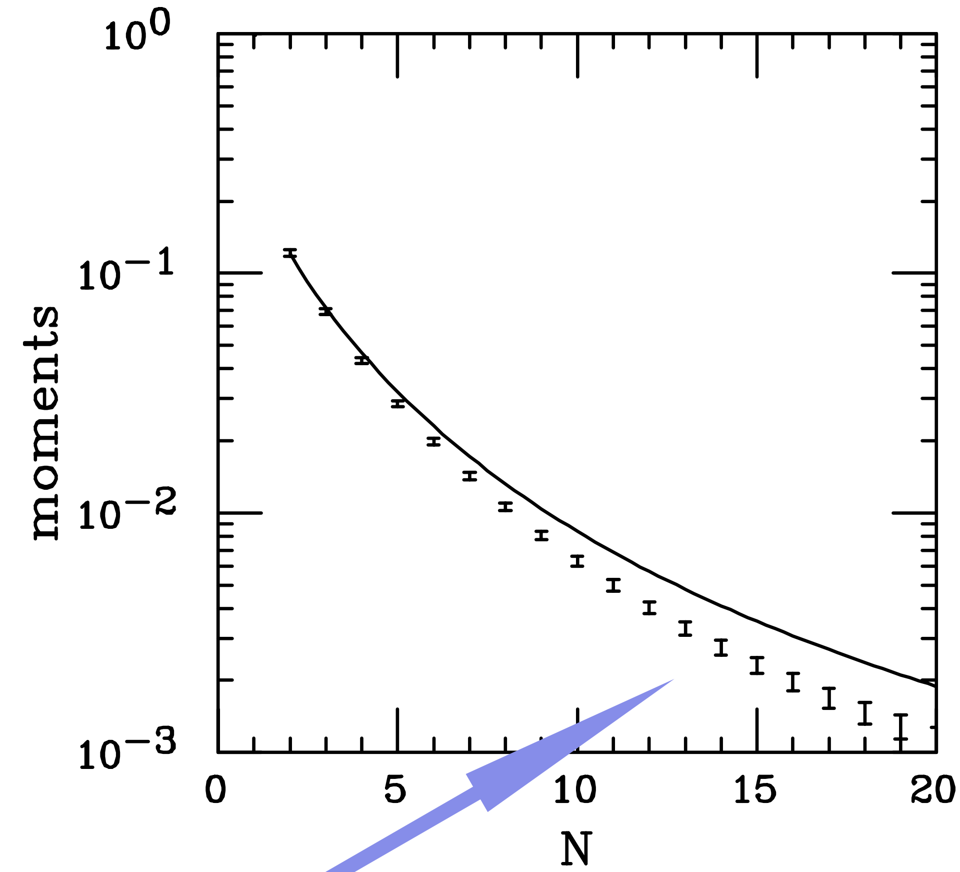
Eq. (61):	$a = 1.8 \pm 0.2$, $b = 11.3 \pm 0.6$, $c = 2.46 \pm 0.07$, total $\chi^2 = 139$				
Set	(C) D^{*+}	(B) $D^{*+} \to D^0$	(B) $D^{*+} \to D^+$	(C) D^{*0}	(B) D^{*0}
Norm.	0.238	0.253	0.227	0.225	0.211
χ^2/pts	33/16	63/46	13/46	13/16	17/46

Table 2: Results of the fit to D^* CLEO (C) and BELLE (B) data. The last line reports the χ^2 over the number of fitted points for each data set.

ALEPH vs CLEO/BELLE

Compare the ALEPH data to the PREDICTION given by the fit to CLEO/BELLE + pQCD evolution





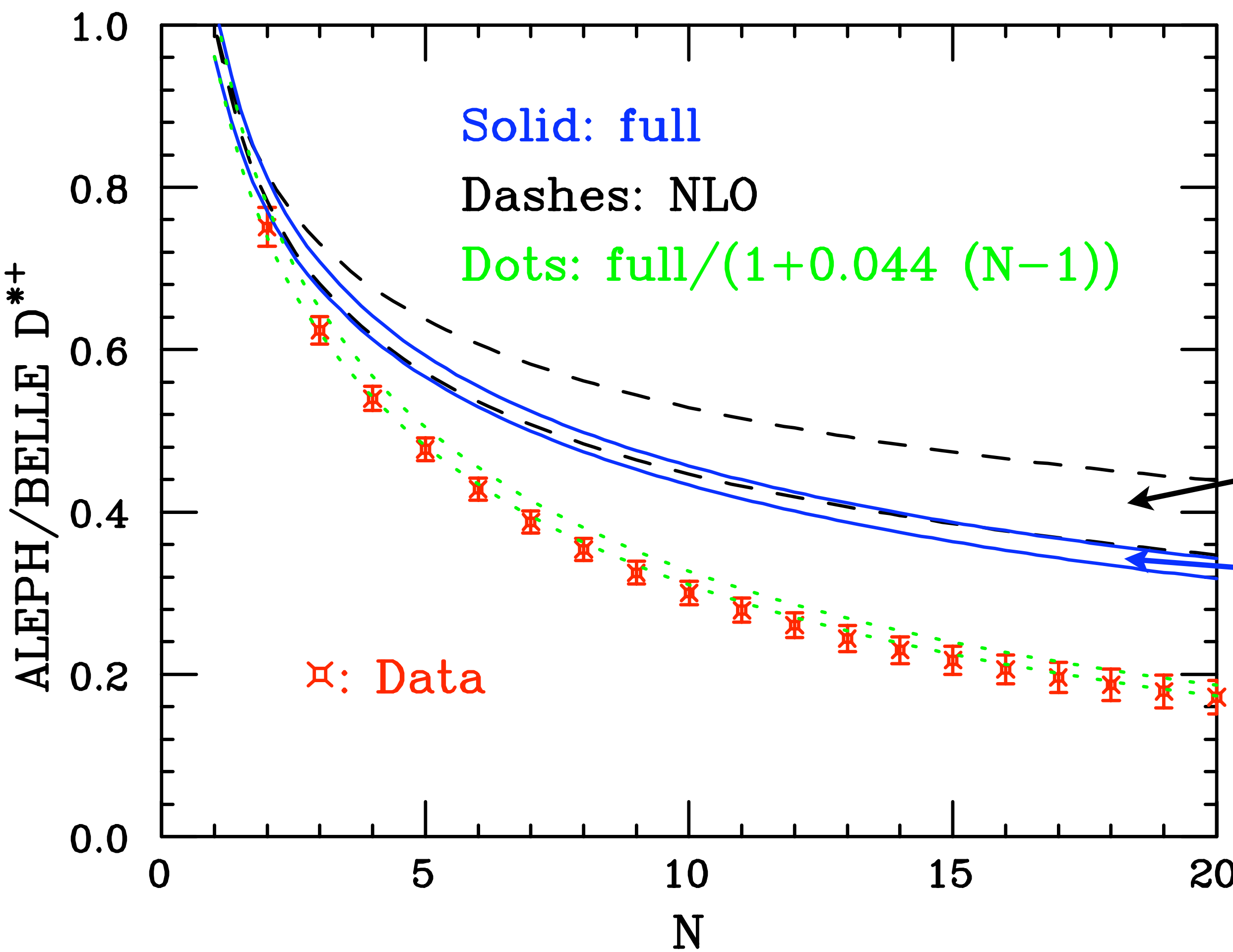
Discrepancy in the large-x/large-N region

CLEO/BELLE data are too hard (or, conversely, ALEPH is too soft...)

ALEPH vs CLEO/BELLE

$$\frac{\sigma_Q(N, M_Z^2, m^2)}{\sigma_Q(N, M_\Upsilon^2, m^2)} = \frac{\bar{a}_q(N, M_Z^2, \mu_Z^2)}{1 + \alpha_S(\mu_Z^2)/\pi} E(N, \mu_Z^2, \mu_\Upsilon^2) \frac{1 + \alpha_S(\mu_\Upsilon^2)/\pi}{\bar{a}_q(N, M_\Upsilon^2, \mu_\Upsilon^2)}$$

NB. heavy quark mass scale effects cancel in this ratio

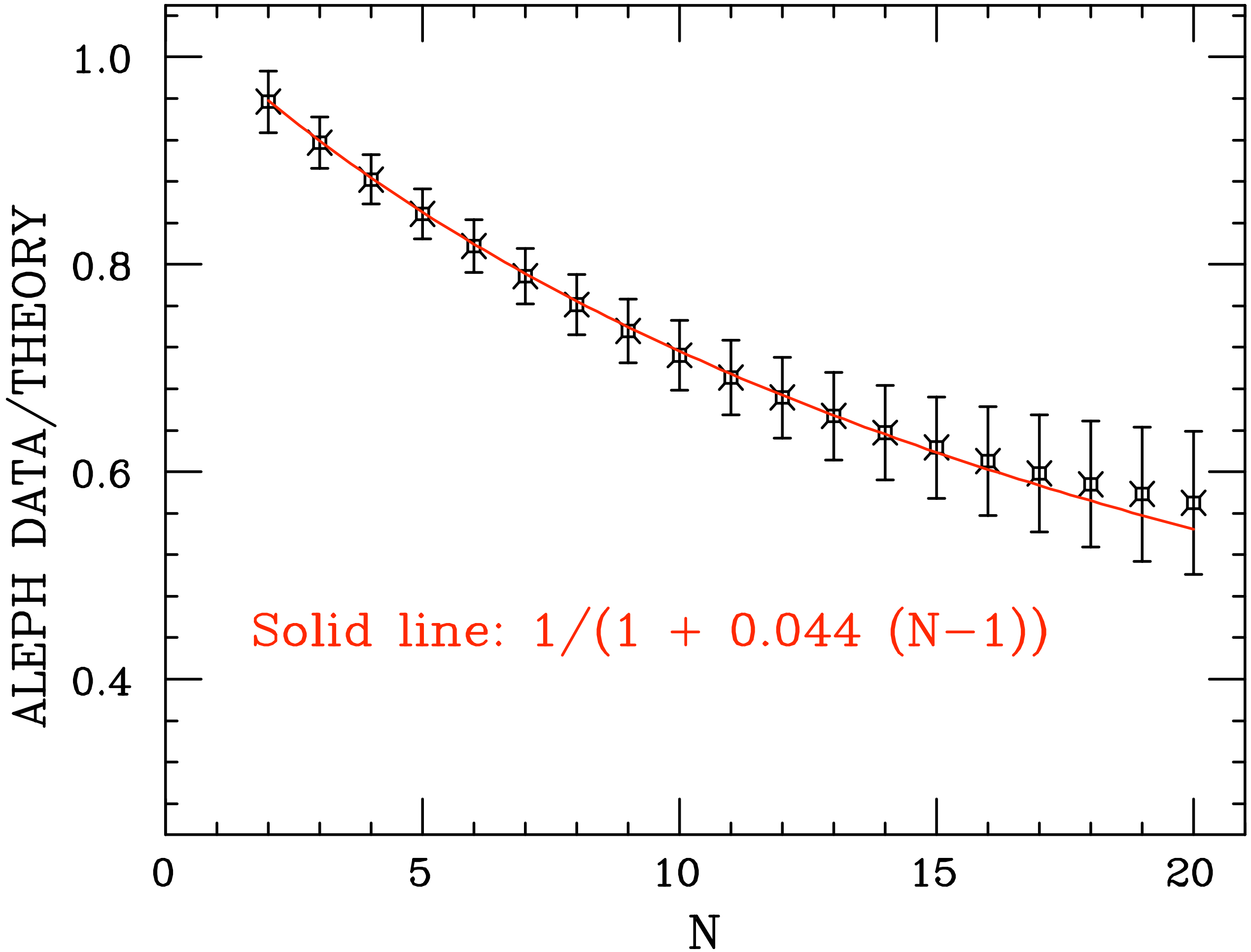


<u>Not a perturbative uncertainty issue</u>: difference is larger than uncertainty band for perturbative evolution

NLO uncertainty

NLO+NLL uncertainty

ALEPH vs Theoretical prediction



Gap with theoretical prediction increases with N. Fitted by 1/(1 + 0.044(N-1))

0.044 corresponds to

$$\frac{5\ \mathrm{GeV}^2}{M_\Upsilon^2}$$ quadratic power correction [expected, but coefficient fairly large]

$$\frac{0.52\ \mathrm{GeV}}{M_\Upsilon}$$ linear power correction [fair coefficient, but unexpected]

Checks

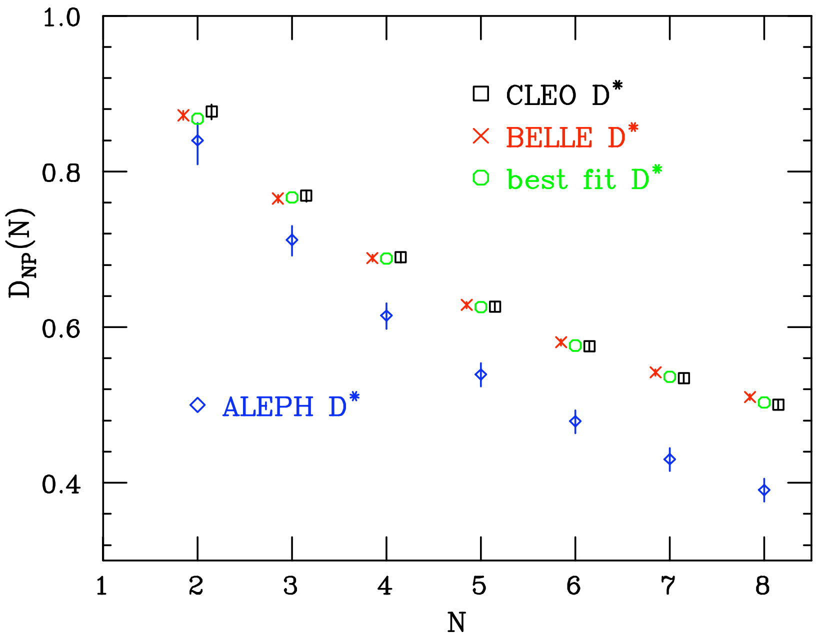
the inclusion of the correction factor (80) (dotted lines). We have also checked that our full result is essentially unchanged if, instead of formula (87), we use the fully exponentiated formula (42). Furthermore, the change of variable given in Eq. (44) to deal with the Landau pole has very little impact on our curves. Using the very large value $\Lambda^{(5)}_{\rm QCD} = 0.3\,\text{GeV}$ would lower the theoretical predictions by no more than 11% for $N \leq 20$, very far from explaining the observed effect.

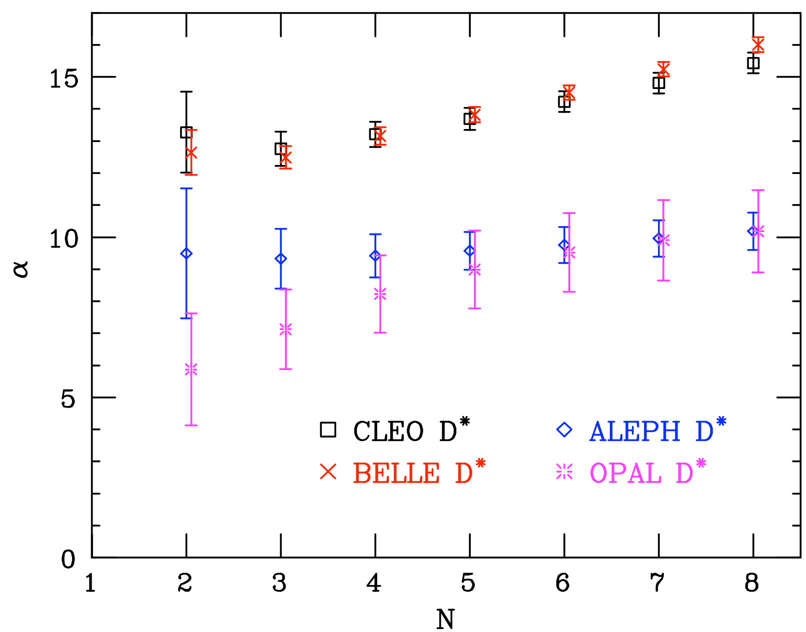
The deconvolution of ISR effects, that hardens the $\Upsilon(4S)$ data, but is insignificant on the Z^0, widens the discrepancy. However, if we did not apply the deconvolution, the effect would still be partially visible.

Because of the relatively low energy of the data on the $\Upsilon(4S)$, it is legitimate to wonder whether charm-mass effects could be responsible for the discrepancy between LEP and $\Upsilon(4S)$ data. We have not included mass effects in the present calculation. However, in Ref. [33], mass effects in charm production on the $\Upsilon(4S)$ where computed at order $\alpha_{\rm S}^2$, and found to be small. We thus believe that it is unlikely that mass effects could play an important role in explaining this discrepancy.

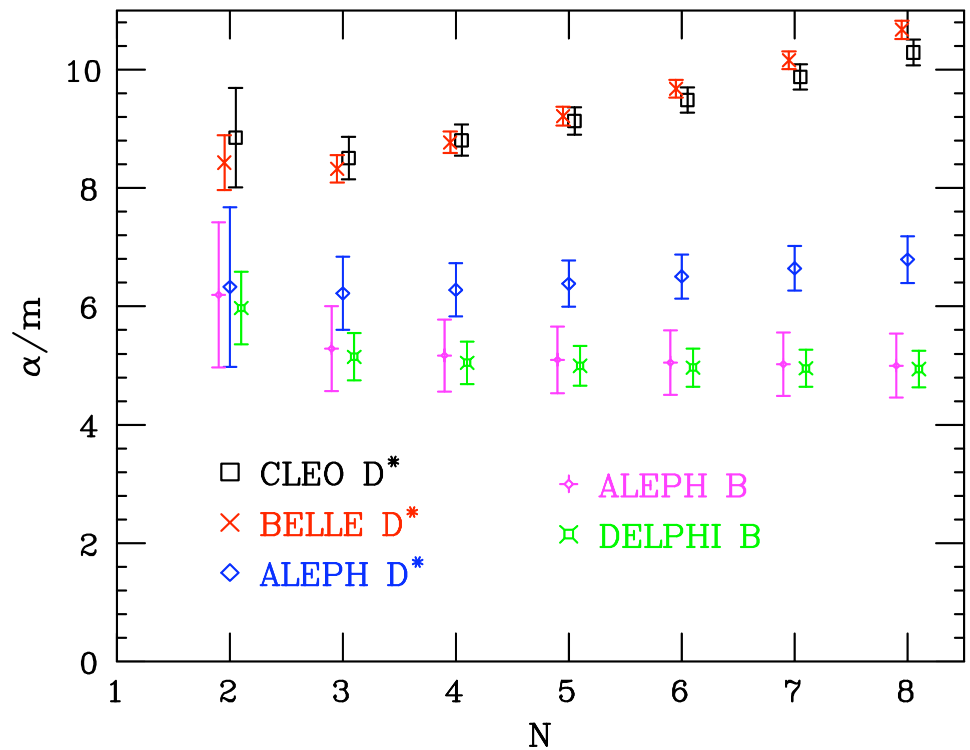
Single-parameter fits

Extract non-perturbative contribution from single moments, extract Kartvelishvili's α



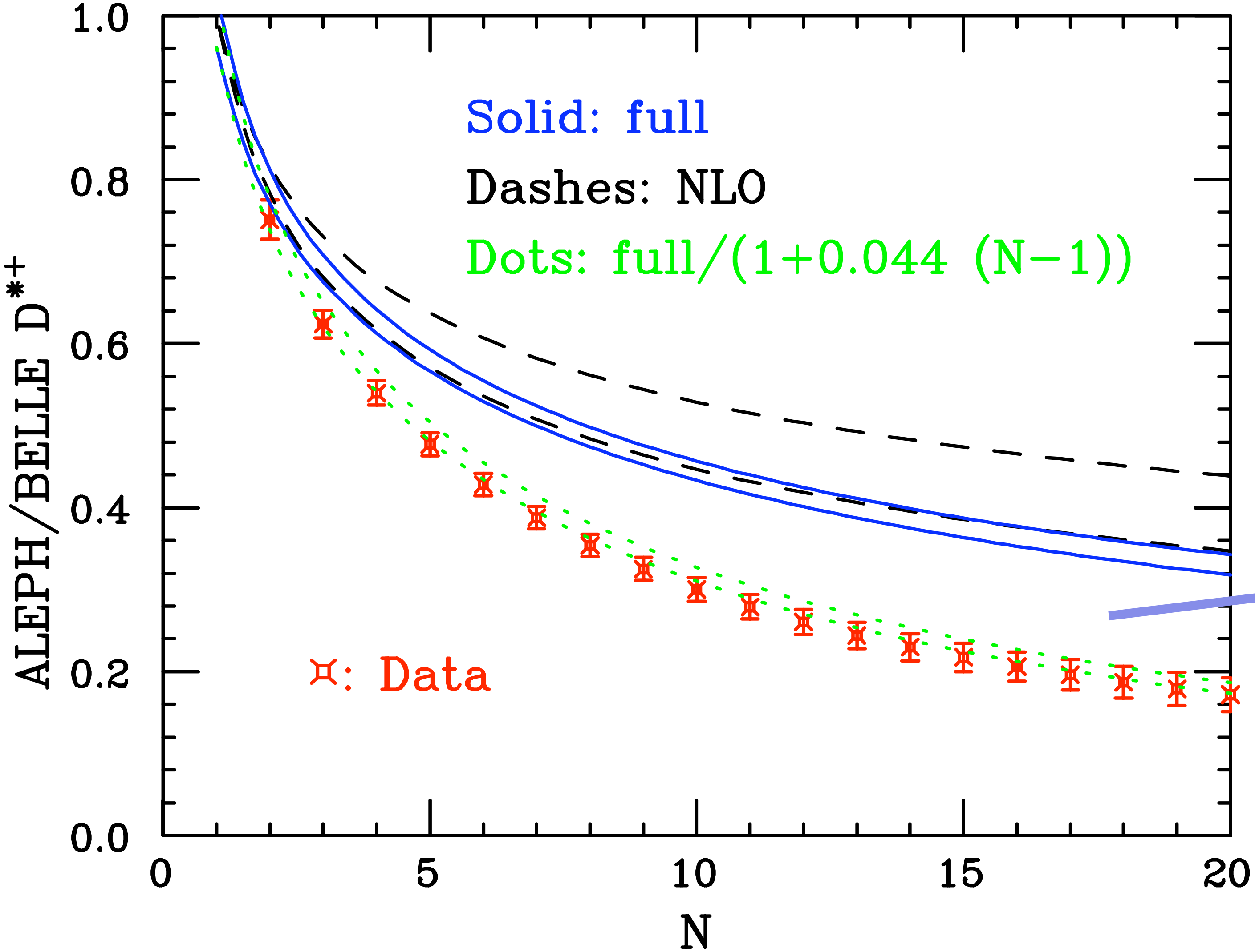


Compare with B mesons. Check scaling of α with the heavy quark mass m



Disagreement of CLEO/BELLE data **mildly** support view that these data might be affected by large power corrections

Conclusions



What's this gap due to?

Is it a $1/q^2$ correction with a large coefficient, or a novel $1/q$ correction?