Workshop on parton fragmentation processes in the vacuum and in the medium
ECT* - Trento, February 25-29, 2008

The DSS Global QCD Analysis of Fragmentation Functions

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RIKEN

work done in collaboration with

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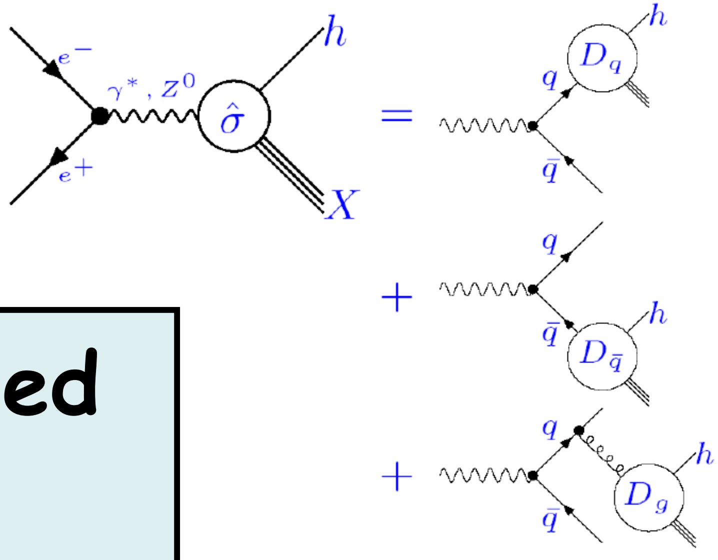
Rodolfo Sassot (Buenos Aires)

references

- *Global analysis of fragmentation functions for pions and kaons and their uncertainties,* **Phys. Rev. D75 (2007) 114010** (hep-ph/0703242)
- *Global analysis of fragmentation functions for protons and charged hadrons,* **Phys. Rev. D76 (2007) 074033** (arXiv:0707.1506 [hep-ph])

Fortran codes of the DSS fragmentation fcts are available upon request



- questions to be addressed in this talk:

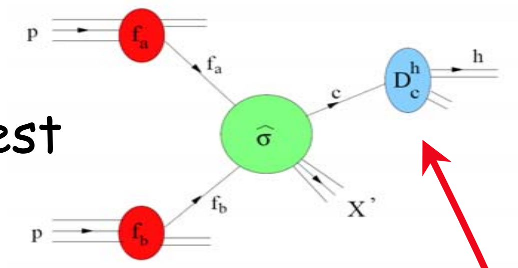
- Is it possible to arrive at a *unified* description of e^+e^-, ep, and pp inclusive hadron data in terms of a *universal set of fragmentation functions* ?
- If so, what are the typical *uncertainties* ?

precise knowledge of fragmentation functions crucial for interpretation & understanding of RHIC & LHC results and QCD in general

unpolarized pp cross sections are an important baseline for

- studies of saturation effects in dAu and AuAu collisions
- understanding of spin asymmetries & extraction of, e.g., Δg

incl. hadron data put fundamental ideas to the test

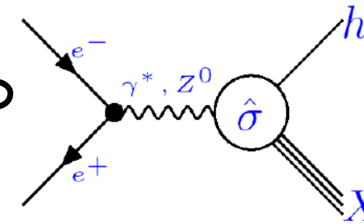


- fragmentation as fundamental as nucleon structure
- factorization and universality of fragmentation functions

most analyses so far

Bourhis et al., Kretzer; Kniehl et al.; Hirai et al.

- based on e^+e^- annihilation data mainly from LEP



- considerable progress but also shortcomings

  - singlet FF D_Σ at M_Z constrained but individual flavors differ wildly at scales relevant for ep, pp data

  - gluon fragmentation largely unknown but crucial for pp

  - cannot distinguish D_q^h and $D_{\bar{q}}^h$

$$D_{q+\bar{q}}^{h^+} = D_{q+\bar{q}}^{h^-}, \; D_q^{h^+ + h^-} = D_{\bar{q}}^{h^+ + h^-}$$

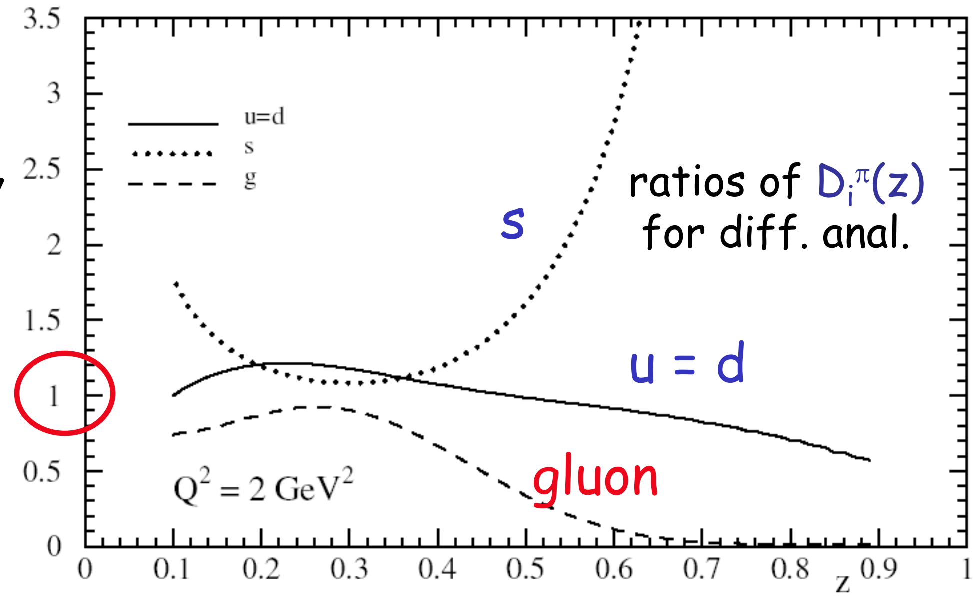


fig. by S. Kretzer

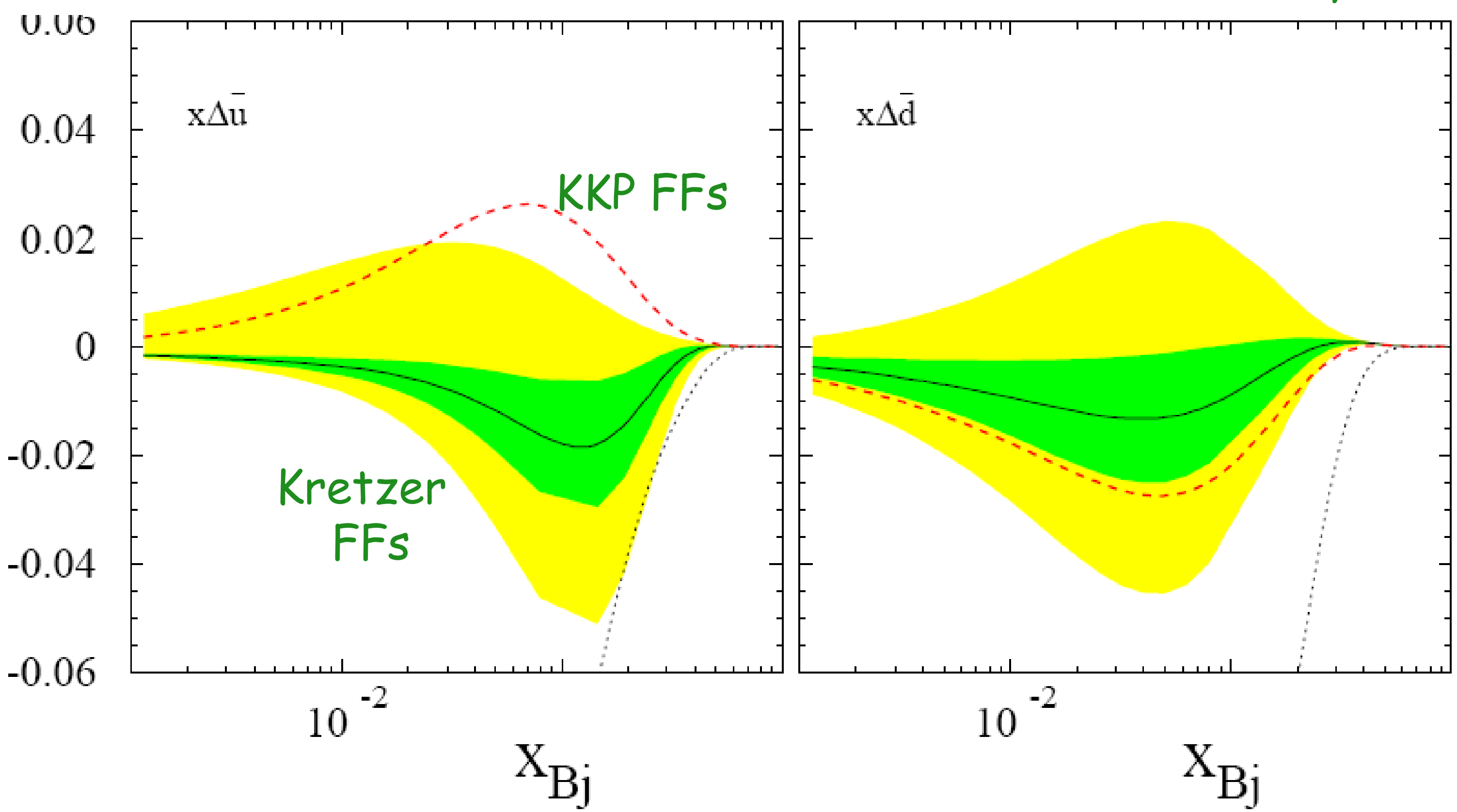
→ **extracted FFs cannot be used for many ep and pp processes without ad-hoc assumptions; limits predictive power mainly to e^+e^-**

actual example:

lack of flavor separation has profound impact on the extraction of pol. sea pdfs from SIDIS data

$$\frac{d\Delta\sigma^h}{dx\,dQ^2\,dz^h} \propto \sum_{f=q,\bar{q}} e_f^2\, \Delta f(x,\mu_f)\, D_f^h(z^h,\mu'_f) + \mathcal{O}(\alpha_s)$$

de Florian, Navarro, Sassot



upshot:

we can only hope for further progress on fragmentation functions by performing a global QCD analysis like CTEQ does for pdfs

it roughly goes like this …

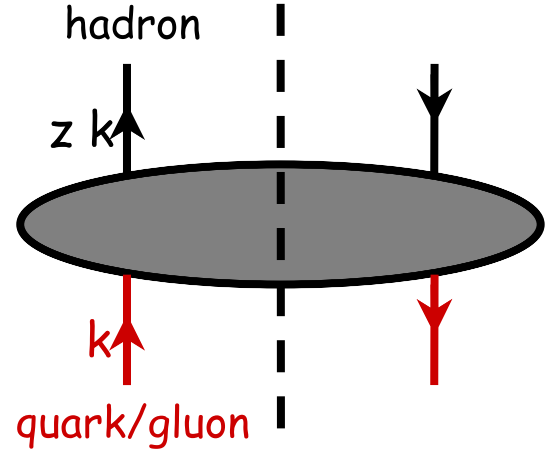
[redacted]

theory toolbox for DSS global analysis

content:

- factorization & properties of $D_i^h(z)$
- status of relevant pQCD calculations
- Mellin technique
- determination of uncertainties

some properties of $D_i^h(z,\mu)$



- non-perturbative ***universal*** objects
  scale μ-dep. predicted by pQCD

- needed to consistently absorb final-state collinear singularities like, e.g., in $pp \to \pi^0 X$ ("***factorization***")

- describe the *collinear* transition of a parton "i" into a massless hadron "h" carrying fractional momentum z

- bi-local operator: $D(z) \simeq \int dy^- e^{iP^+/zy^-} \mathrm{Tr}\gamma^+ \langle 0|\, \psi(y^-) |hX\rangle \langle hX| \overline{\psi}(0)\, |0\rangle$

  Collins, Soper '81, '83

no inclusive final-state
$\to$ no local OPE $\to$ no lattice formulation

also: power corrections are much less developed and entwined with mass effects unlike pdfs

- **"leading particle" picture incompatible with def. of $D_i^h(z)$**

can compute *incl. distributions* of hadrons with momentum fractions z but *not* a cross section for a "leading hadron"

(under certain kin. conditions it might be a good approximation though)

- **"energy-momentum conservation":** $\sum_h \int_0^1 z D_i^h(z,\mu) = 1$

(a parton fragments with 100% probability into *something* preserving its momentum)

of very limited practical use in fits because

- "mass effects" completely spoil framework for $D_i^h(z)$

  no systematic way to include entwined mass/higher twist effects

- timelike μ-evolution very singular as $z \to 0$, e.g. $P_{gg} \to \frac{2C_A}{z} - \frac{\alpha_s}{2\pi} \frac{4C_A^2}{z} \ln^2 z$

limits use of $D_i^h(z)$ to $z \gtrsim 0.05 \div 0.1$

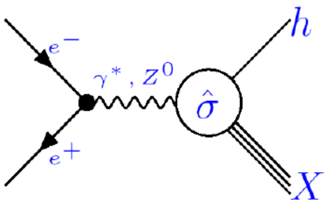
these properties are more or less at variance with hadronization models in PYTHIA et al.

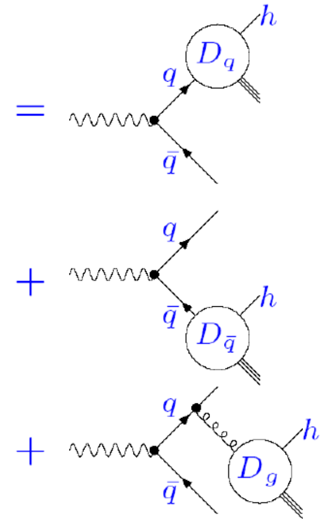
(color connected "strings", "soft physics", non-collinear)

pQCD framework based on factorization only applicable to certain class of processes characterized by a "hard scale"

single-inclusive e^+e^- annihilation (SIA)

relevant: "normalized distribution" $\frac{1}{\sigma_{tot}}\frac{d\sigma^h}{dz}$





$$= \frac{1}{\sigma_{tot}} \sum_{i=q,\bar{q},g} \left[\int_z^1 \frac{dy}{y} \, C_i\left(\frac{z}{y}, Q, \mu_r, \mu_f\right) \, D_i^h(y, \mu_f) \right]$$

$C_i(\frac{z}{y}, Q, \mu_r, \mu_f)$	$D_i^h(y,\mu_f)$
"coeff. fct." calculable in pQCD	$i \to h$ frag. fct.

total hadronic cross section

$$\sum_q \frac{4\pi\alpha^2}{s}\hat{e}_q^2 \left(1 + \frac{\alpha_s}{\pi} + \ldots\right)$$

$\equiv \sigma_0$

LO: $C_q = \delta(1-y)\,\sigma_0$; $C_g = 0$

$O(\alpha_s)$ NLO: Altarelli, Ellis, Martinelli, Pi '79; Furmanski, Petronzio '82

$O(\alpha_s^2)$ NNLO: Rijken, van Neerven '96,'97; Mitov, Moch '06

"scaling" variable $z \equiv \frac{2P^h \cdot q}{Q^2} = \frac{2E^h}{Q}$

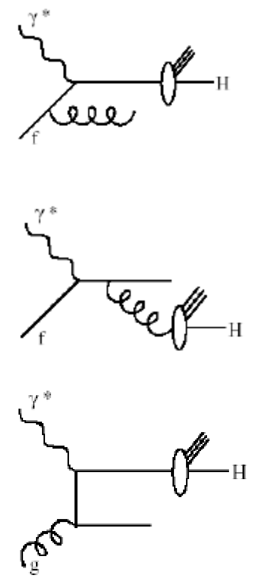
where

$$\begin{aligned} s &= q^2 = Q^2 \\ P_{e^\pm} &= (Q/2, 0, 0, \pm Q/2) \\ q &= P_{e^+} + P_{e^-} \end{aligned}$$

semi-inclusive DIS (SIDIS)

SIDIS = DIS plus one identified hadron with $x_F > 0$

$$\frac{d\sigma^h}{dx\,dQ^2\,dz^h} \approx \sum_{f=q,\bar{q}} e_f^2\, f(x,\mu_f)\, D_f^h(z^h,\mu_f') + \mathcal{O}(\alpha_s)$$



"scaling" variable $z^h \equiv \dfrac{P^h \cdot P^N}{P^N \cdot q}$

Altarelli et al. '79;
Furmanski, Petronzio '82;
de Florian, MS, Vogelsang '98

why important?

- charge separated data: π^+, π^-, K^+, K^- HERMES; h^+, h^- EMC

  → **valuable handle on flavor separation**

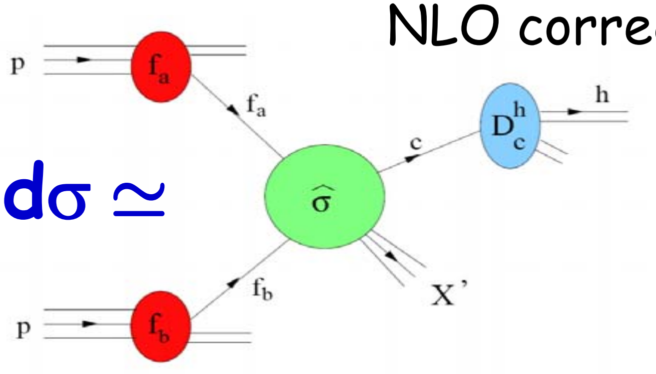
  LO analysis: $D_d^{\pi^+} \simeq (1-z)\, D_u^{\pi^+}$ Christova, Kretzer, Leader

pp → hX

Aversa et al.; Jäger, Schäfer, MS, Vogelsang; de Florian

well-know framework: factorization, NLO corrections

$d\sigma \simeq$



long-distance

from exp.; μ-dep.: $d\sigma/d\mu = 0$ (pQCD)

$$\frac{d\Delta\sigma^{\vec{p}\vec{p}\to\pi X}}{dp_T\,d\eta} = \sum_{abc}\int dx_a\,dx_b\,dz_c\,\Delta f_a(x_a,\mu_f)\,\Delta f_b(x_b,\mu_f)\,D_c^{\pi}(z_c,\mu_f')$$
$$\times\ \frac{d\Delta\hat{\sigma}^{ab\to cX'}}{dp_T\,d\eta}(x_aP_a, x_bP_b, P^{\pi}/z_c, \mu_f, \mu_f', \mu_r) + \mathcal{O}\left(\frac{\lambda}{p_T}\right)^n$$

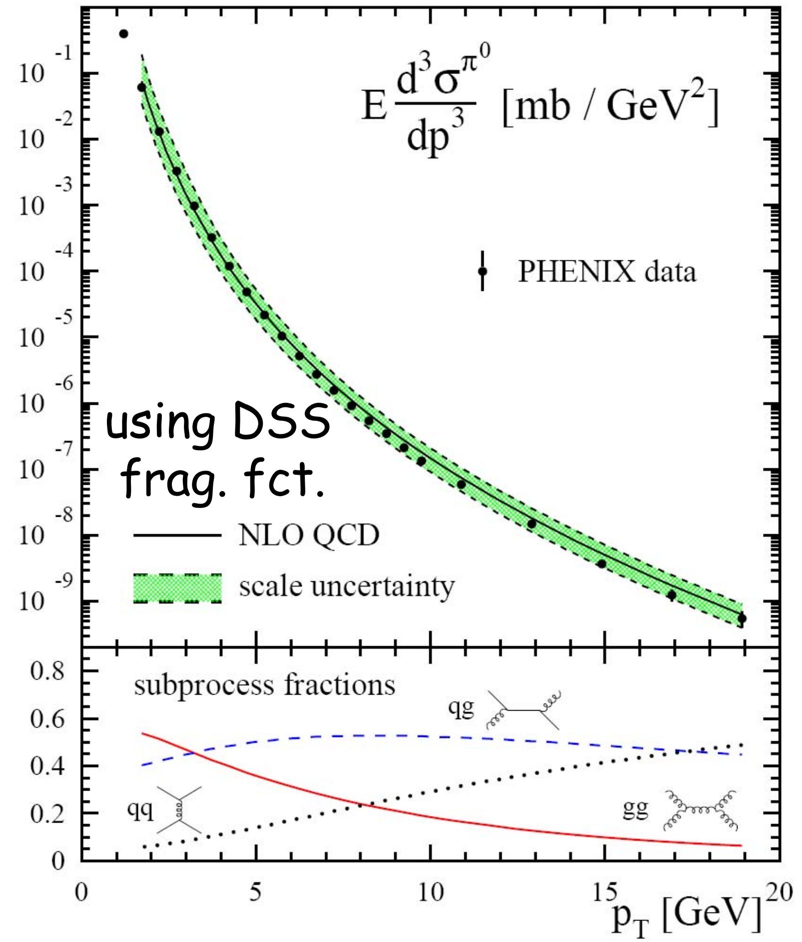
short-distance

calculable in pQCD; power series in α_s

power corrections

neglected

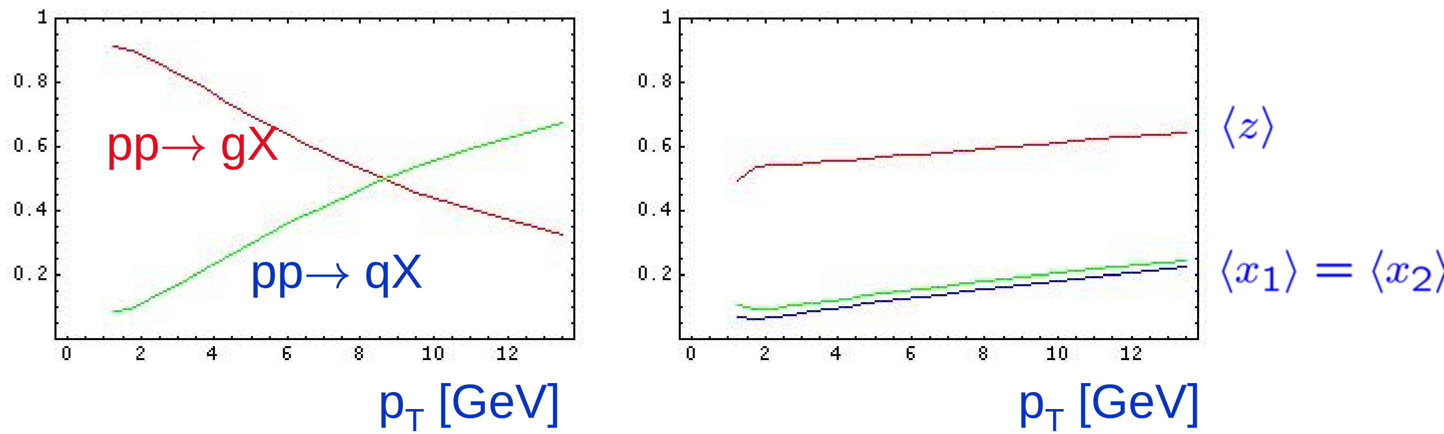
e.g., very nice data from RHIC:



plus STAR and BRAHMS

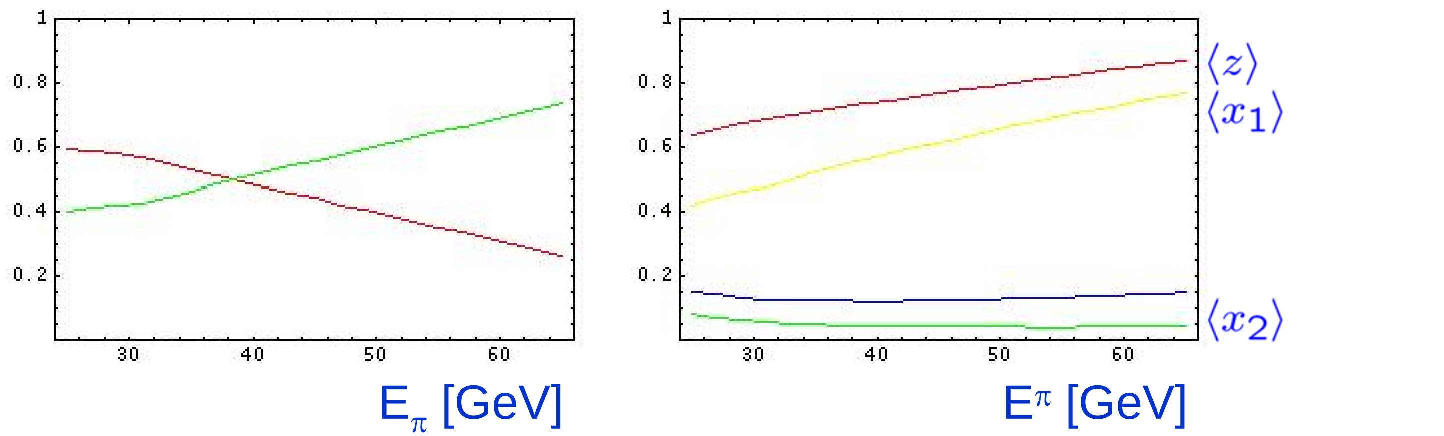
why important?

central rapidity
(PHENIX data)



→ low p_T data probe gluon fragmentation

forward rapidity
(STAR data)

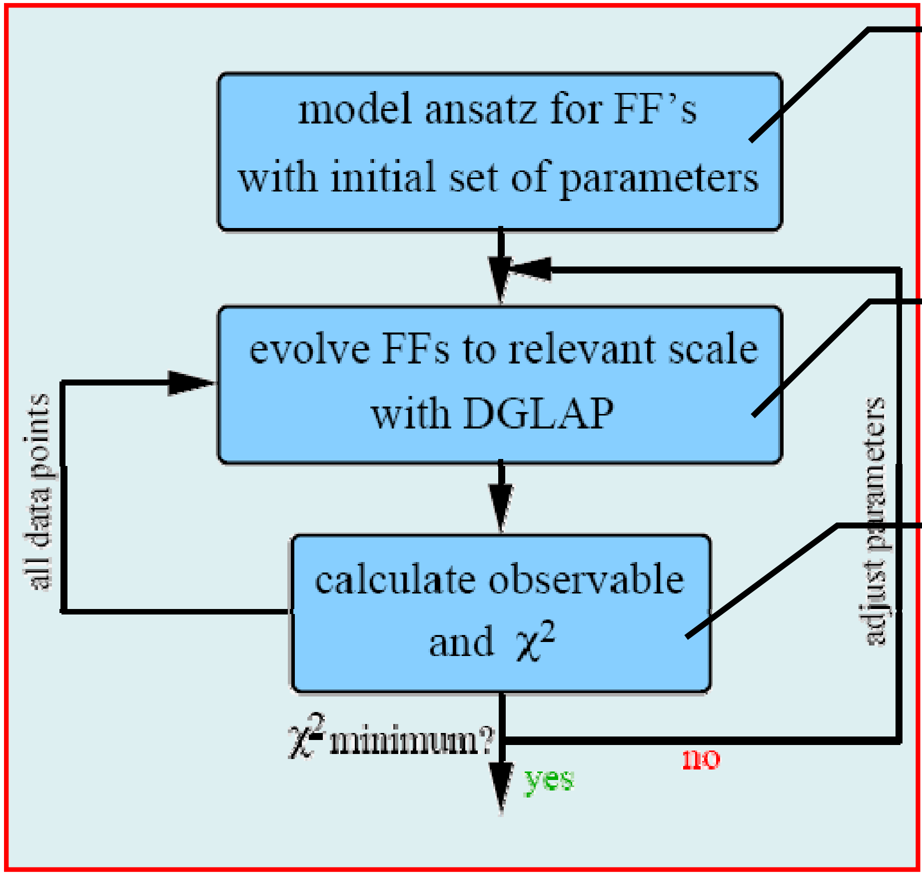


→ probe gluon and quark fragmentation at large z

BRAHMS $\pi^\pm$, $K^\pm$ data ($\eta \simeq 3$) → flavor separation from pp data

	SIA	SIDIS	pp
advantage	very precise data **clean process** sensitivity to heavy flavor contr.	**allows flavor/charge separation** scale $Q \ll M_Z$ $\rightarrow$ evolution effects	**very sensitive to D_g** **probes large z** charge sep. data different scales p_T
disadvantage	prec. data only at M_Z $\rightarrow$ **no handle on D_g** mainly determines D_Σ **no flavor/charge sep.** not precise at large z	**depends on pdfs** should be well constrained **rather low scales** $\rightarrow$ non-pert. corr. ? $D_{c,b}$ play no role	**depends on pdfs** should be well constrained fixed target data excluded resummations large **large scale uncert.** $D_{c,b}$ play no role

outline of a global analysis:



parametrize each parton flavor
must be flexible but
economical (# parameters)

μ_0(ansatz) $\rightarrow$ μ(data) evolution
can be done very fast!

1000's of evaluations of NLO
x-secs needed to minimize χ^2
(a proper error analysis adds to it)

bottleneck !

what's the problem:

- DIS and SIDIS data can be analyzed at no extra cost (as fast as evolution)
- other NLO expressions are numerically very time consuming

computing time for a global analysis at NLO becomes excessive

19^{th} century math comes to help …

idea: re-organize multi-convolutions by taking Mellin moments MS, Vogelsang

earlier ideas: Berger, Graudenz, Hampel, Vogt; Kosower

crucial property: convolutions factorize into simple products

example: pp$\rightarrow$ π X

$$d\sigma = \sum_{abc} \int f_a\, f_b\, d\hat{\sigma}_{ab\to cX}\, D_c\, dx_a dx_b dz_c$$

express frag. fct. by their Mellin inverse

$$\frac{1}{2\pi i}\int_{C_n} dn\, z_c^{-n}\, D_c^n$$

$$= \frac{1}{(2\pi i)}\sum_{abc}\ \underbrace{\int_{C_n} dn}_{\text{standard Mellin inv.}}\ \underbrace{D_c^n}_{\text{fit}}\ \underbrace{\iiint z_c^{-n}\, f_a\, f_b\, d\hat{\sigma}_{ab\to cX}\, dx_a dx_b dz_c \equiv d\tilde{\sigma}_{ab\to cX}(n)}_{\text{pre-calculated on look-up table}}$$

features:

- very fast and reliable; choice of contour C_n crucial
- not limited to single-incl. observables; 2D-grids needed for pdf fits
- fast grid prod. with VEGAS "events"; alternative to "fastNLO" method

de Florian, Sassot, Vogelsang, MS

estimating uncertainties:

pioneering work for pdfs by the CTEQ collaboration

many methods – in DSS we choose **"Lagrange multipliers"**

idea: see how fit deteriorates when forced to give a different O_i

$$\Phi(\{\lambda_i\},\{a_j\}) = \chi^2(\{a_j\}) + \sum_i \lambda_i O_i(\{a_j\})$$

fit parameters — some observable

- directly examines χ^2 profile; no assumptions like in Hessian method
- role of each data set can be assessed
- easy to implement

- z-dependent errors on $D_i^h(z)$ less straightforward

for the time being, we study

$$O_i(\{a_j\}) = \eta_i(\{a_j\}, z_{\rm min}) = \int_{z_{\rm min}}^{1} dz\, z\, D_i^h(z, Q^2)$$

"truncated energy fractions"

some details & results of the DSS global analysis

- setup
- comparison with data
- uncertainties

setup

- flexible input form

$$D_i^h(z, 1\,\mathrm{GeV}) = N_i z^{\alpha_i}(1-z)^{\beta_i}\left[1+\gamma_i(1-z)^{\delta_i}\right]$$

naïve ZM-VFNS for i = c, b with $Q_0 = m_{c,b}$

- take α_s from MRST [impossible to fit with precise SIA data only at M_Z]

- NLO (LO) sets for pions, kaons, protons, charged hadrons

[determined as "residuals" of π+K+p]

- try to avoid assumptions on $\{a_j\}$ unless data cannot discriminate

SU(2), SU(3) breaking: $D_{d+\bar{d}}^{\pi^+} = N D_{u+\bar{u}}^{\pi^+}$ $\quad D_s^{\pi^+} = D_{\bar{s}}^{\pi^+} = N' D_{\bar{u}}^{\pi^+}$

but we have to assume $D_{\bar{u}}^{\pi^+} = D_d^{\pi^+}$

$$D_{\bar{u}}^{K^+} = D_s^{K^+} = D_d^{K^+} = D_{\bar{d}}^{K^+}$$

data selection

included: ALEPH, DELPHI, OPAL, SLD, TASSO, TPC SIA w/o "flavor tag"
HERMES, EMC SIDIS
BRAHMS, PHENIX, STAR, CDF, UA1, UA2 "pp"

reluctantly included: SIA "flavor tagged" data
only constraint on c, b $\to$ light hadrons
many conceptual problems: no NLO interpretation;
leading hadron assumptions; …

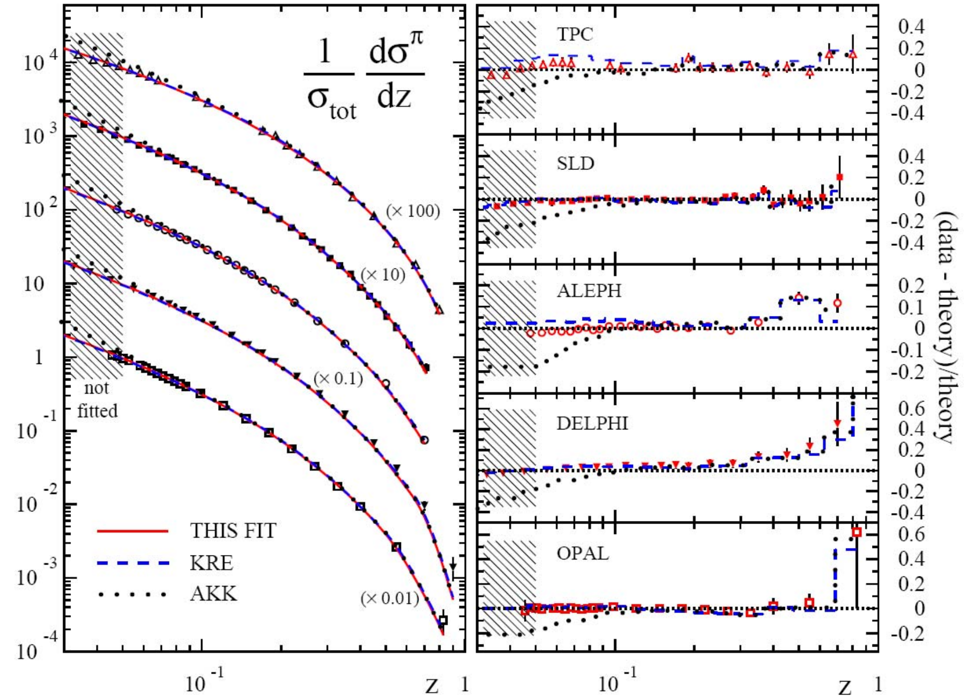
excluded: ep photoproduction data [uncert. from photon structure]
UA1 data for kaons [inconsistent with STAR]

like in pdf fits we allow for

- cuts: $z > 0.05$ pions, $z > 0.1$ otherwise
- relative normalizations/shifts of data sets
- extra "TH errors": scale uncertainty (pp); flavor tag; bin size, …

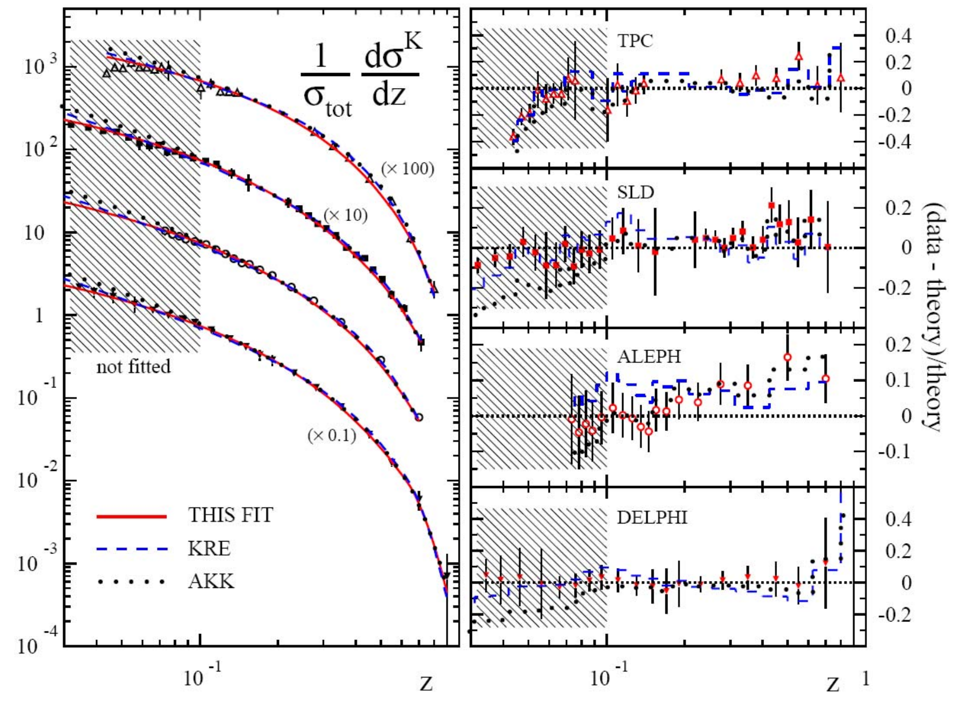
some results

SIA data still work very well within a global fit



pions

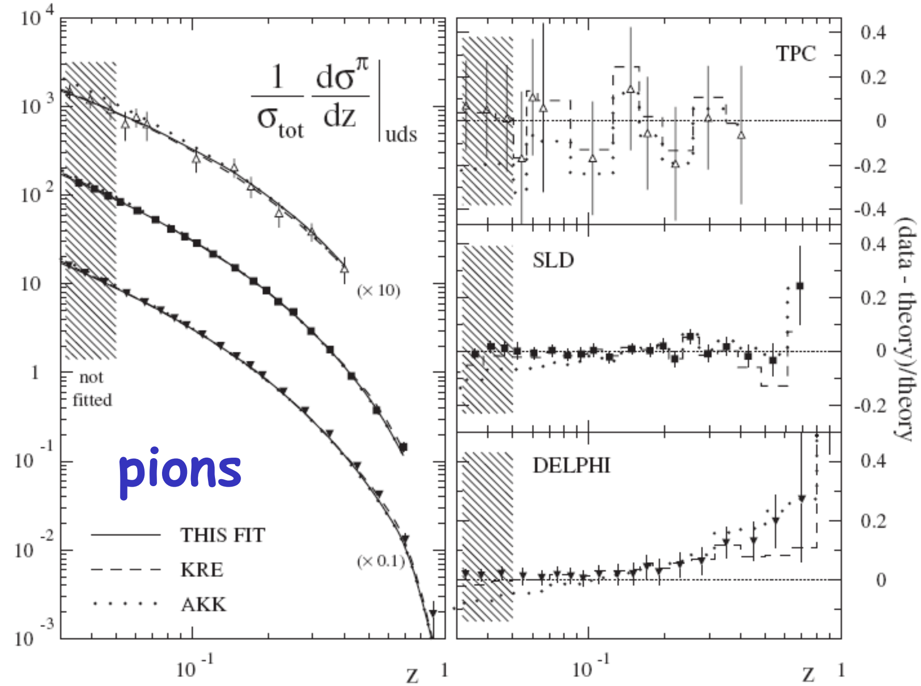
kaons



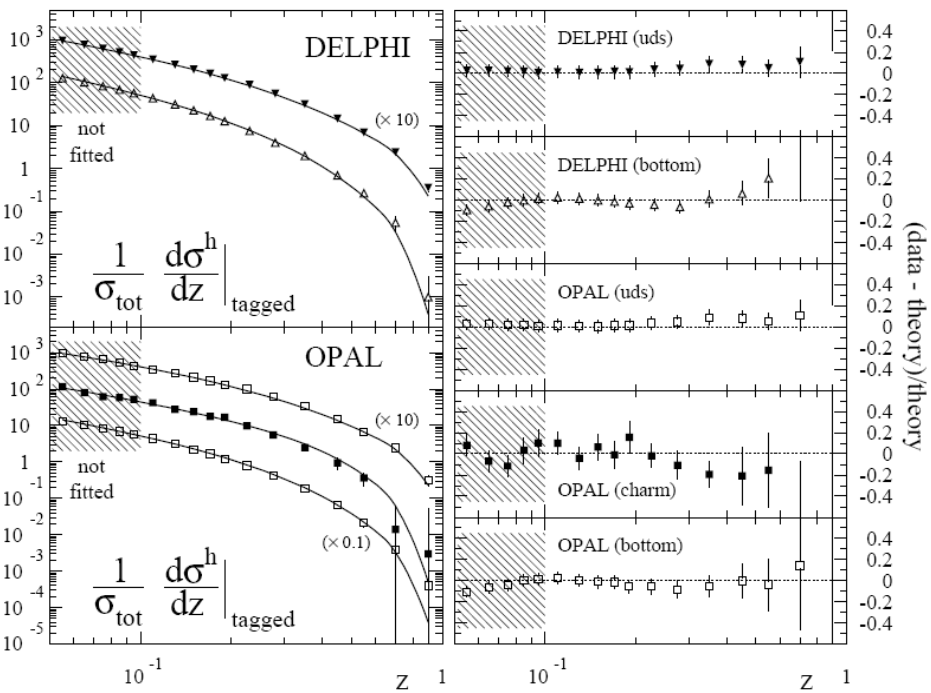
similar for protons and and charged hadrons

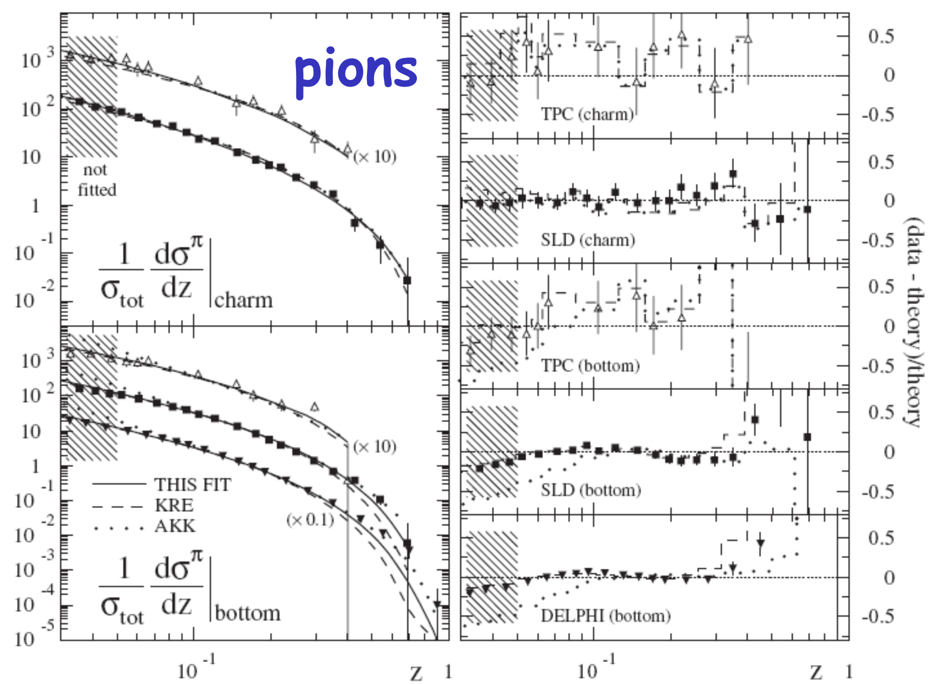
KRE: S. Kretzer
AKK: S. Albino et al.
AKK uses z > 0.1

even for "uds", "c", "b" flavor tagged data



chg. hadrons



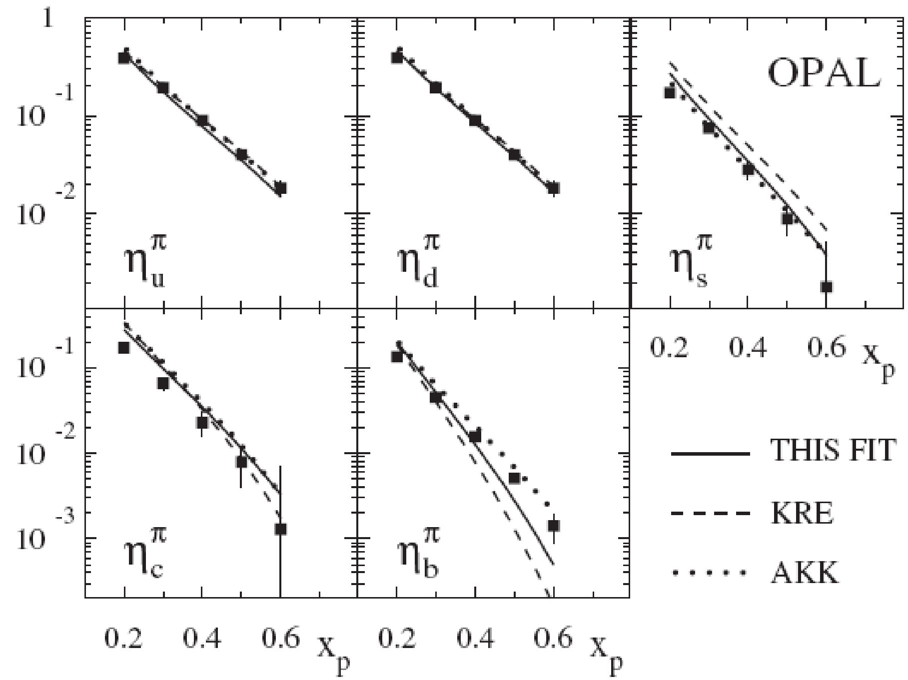


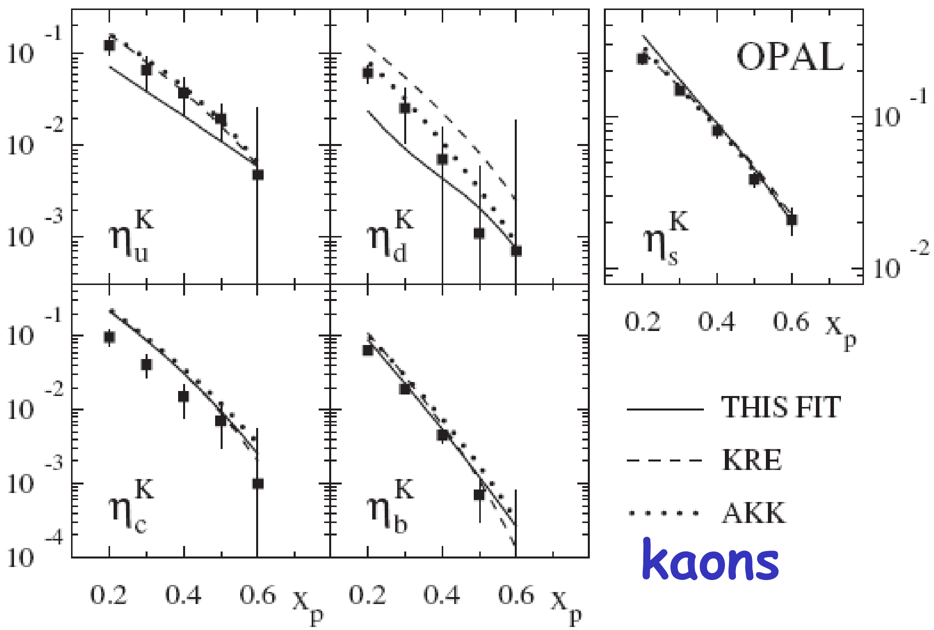
some tension with OPAL “tagging probabilities”

[recall: not really defined in NLO]

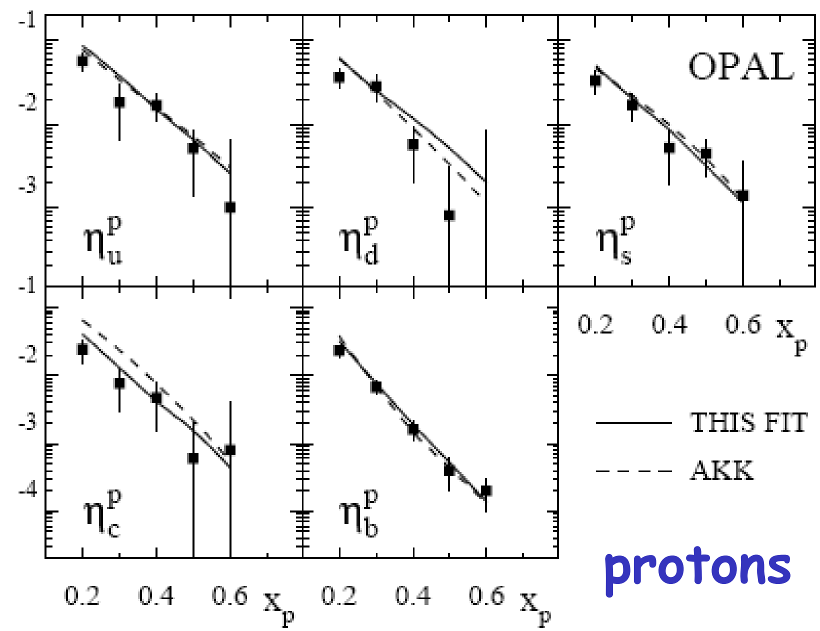
also tension with other data sets in particular for c,b

pions





kaons



protons

good description of SIDIS multiplicities

HERMES data (not final)
A. Hillenbrand (thesis)

π^+

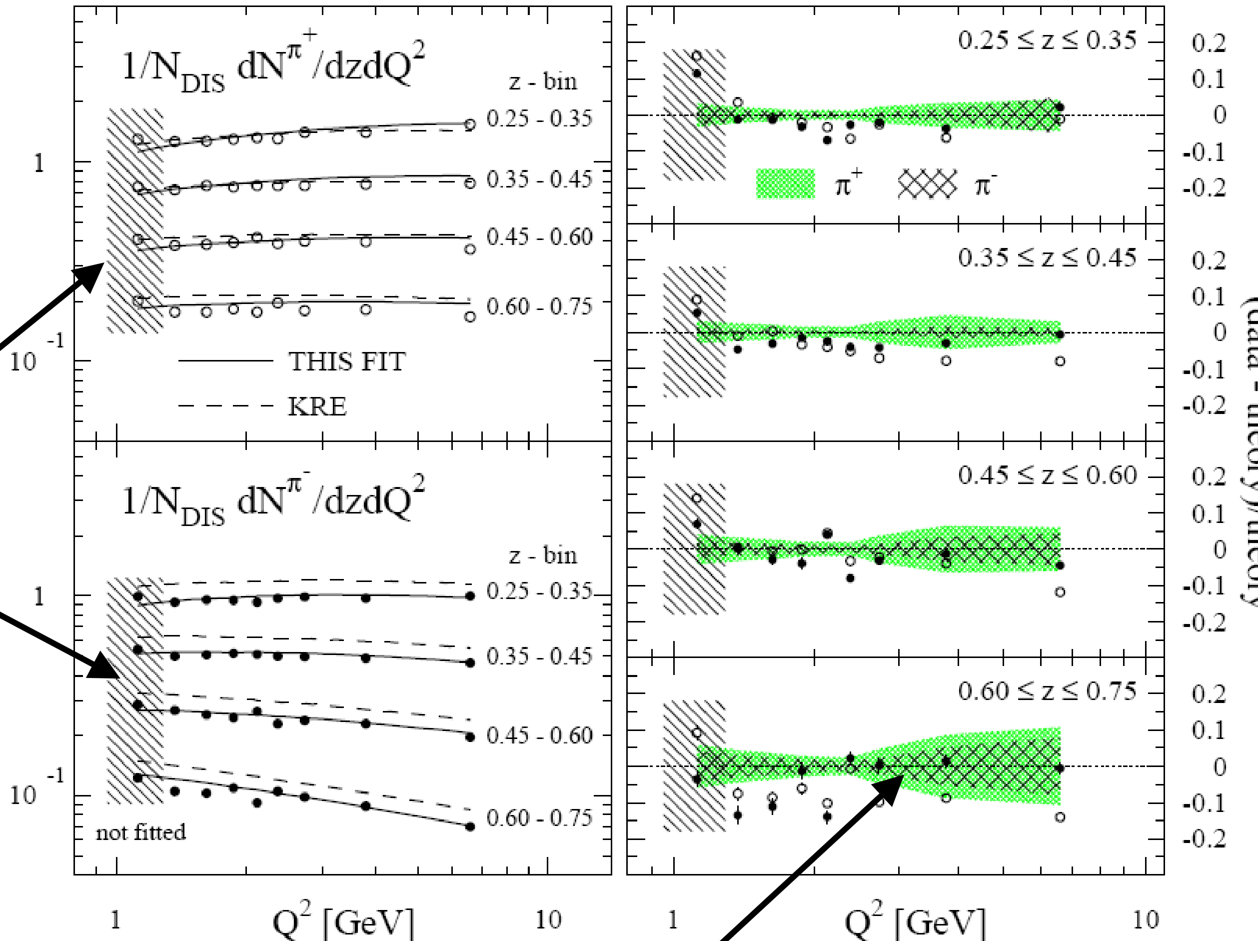
Kretzer's assumption

$$D_d^{\pi^+} \simeq (1-z) D_u^{\pi^+}$$

works for π^+

but not for π^-

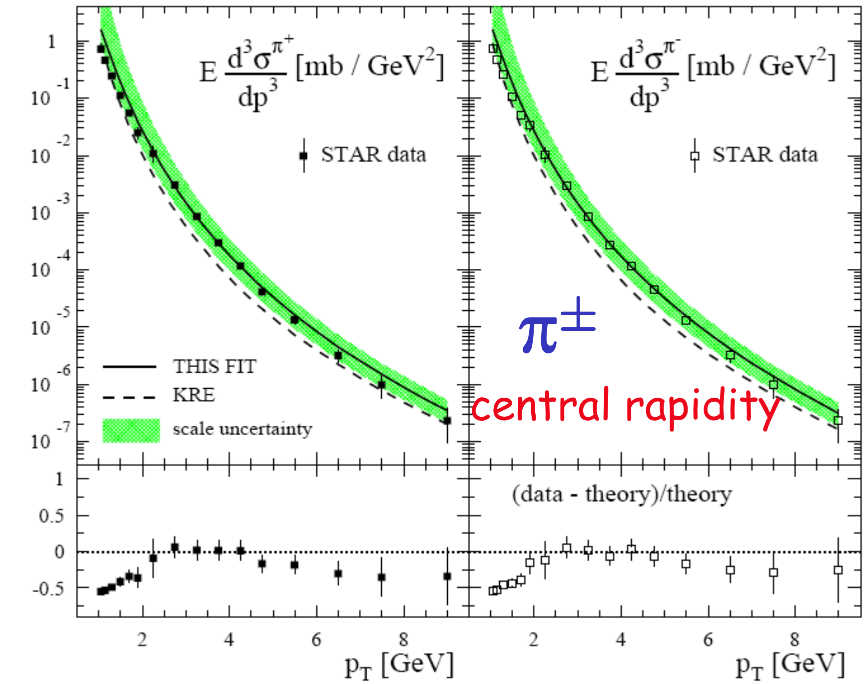
π^-

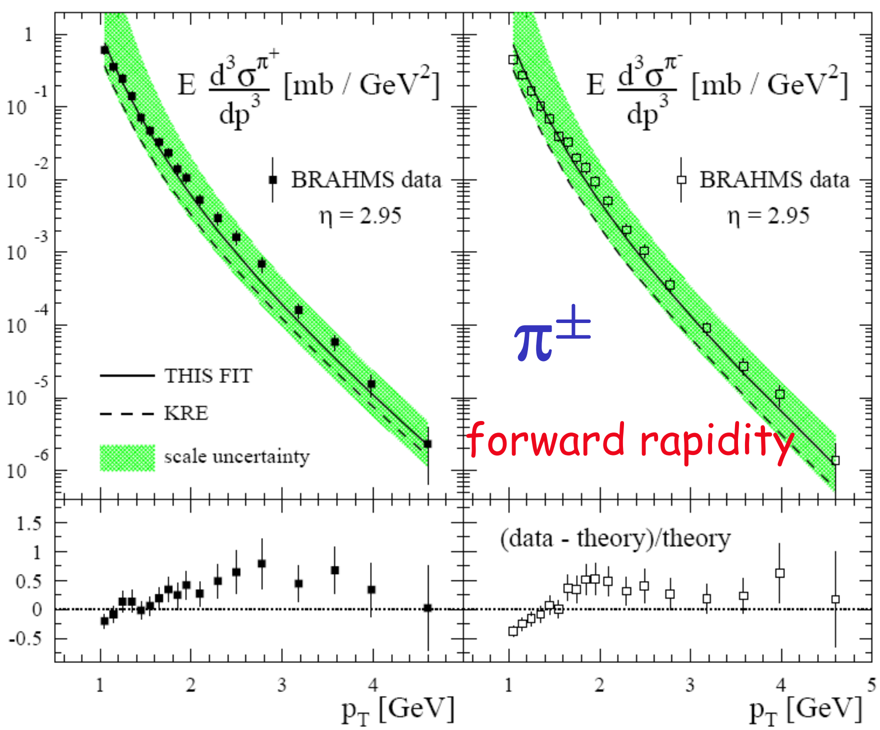


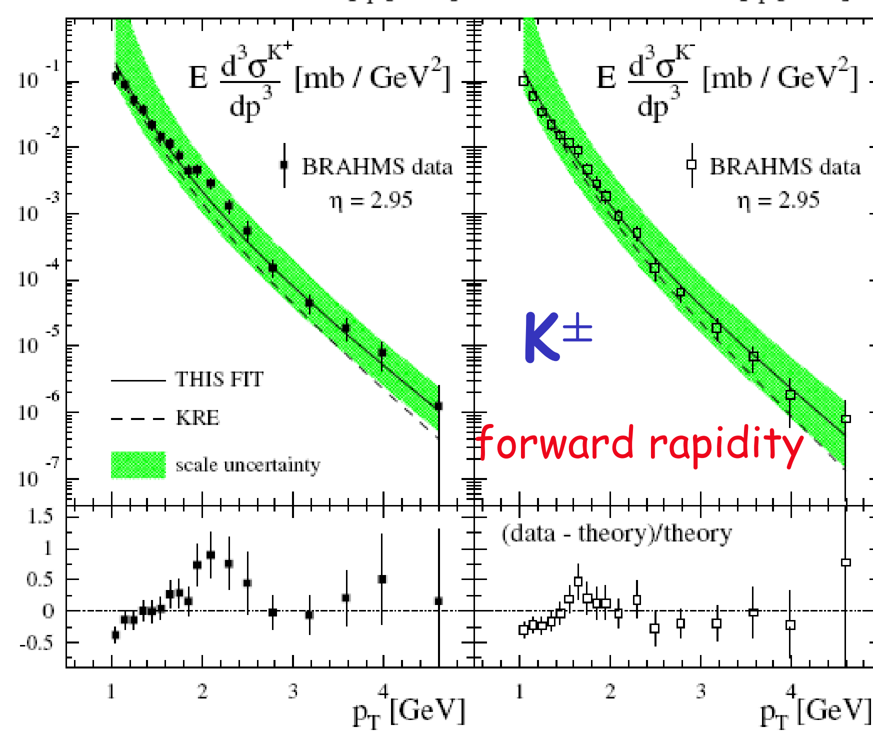
shaded bands:
our estimate of
"Q^2-binning effects"

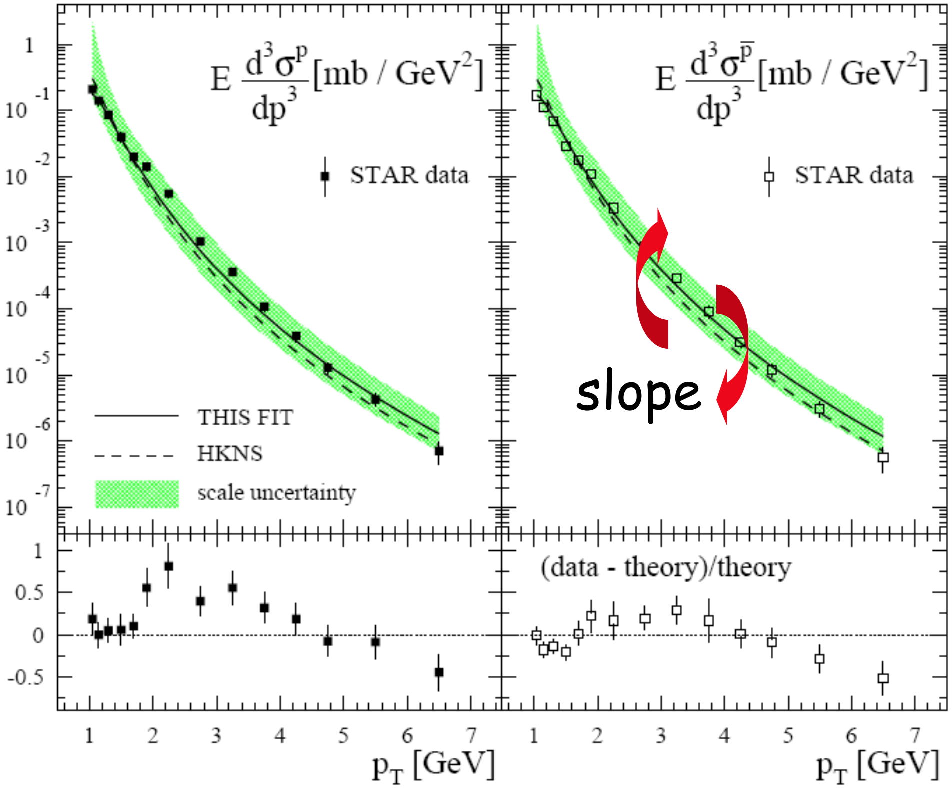
“pp” data also well reproduced

- large scale uncertainties
- probe z values well below 0.1 but x-sec mainly samples z > 0.5
- $p_{T,min}$ cut has no impact on fit









(anti-)protons:

fit works too well (?) at $p_T \simeq m_p$

need large p_T data to check slope

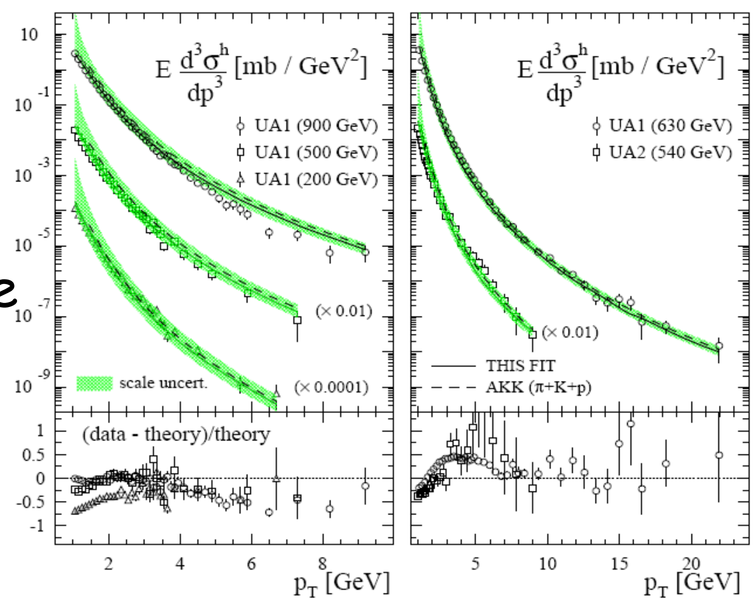
issue with BRAHMS data at $y \simeq 3$

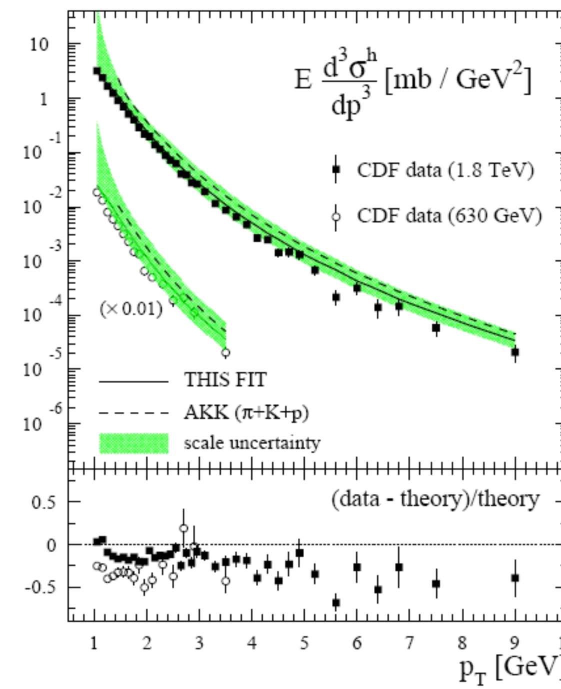
find proton = 10 $\times$ antiproton yield

charged hadrons:

mainly a prediction:

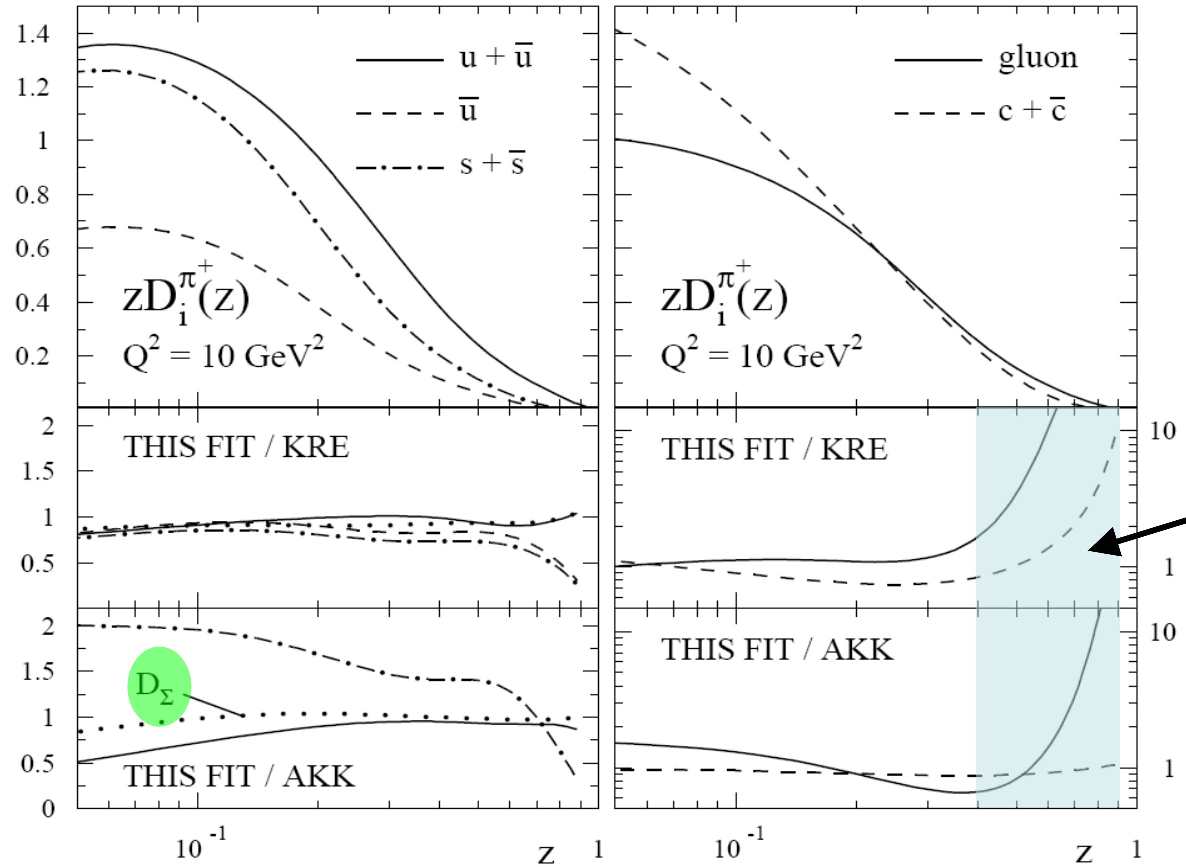
π+K+p almost saturate $D^h(z)$ with a small and positive $D^{res}(z)$

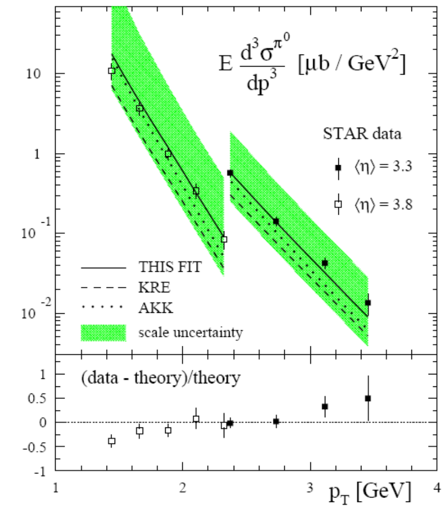




meet the $D_i(z)$'s: pions

$\langle z \rangle \gtrsim 0.6$

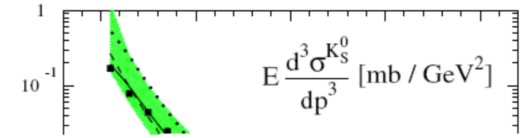


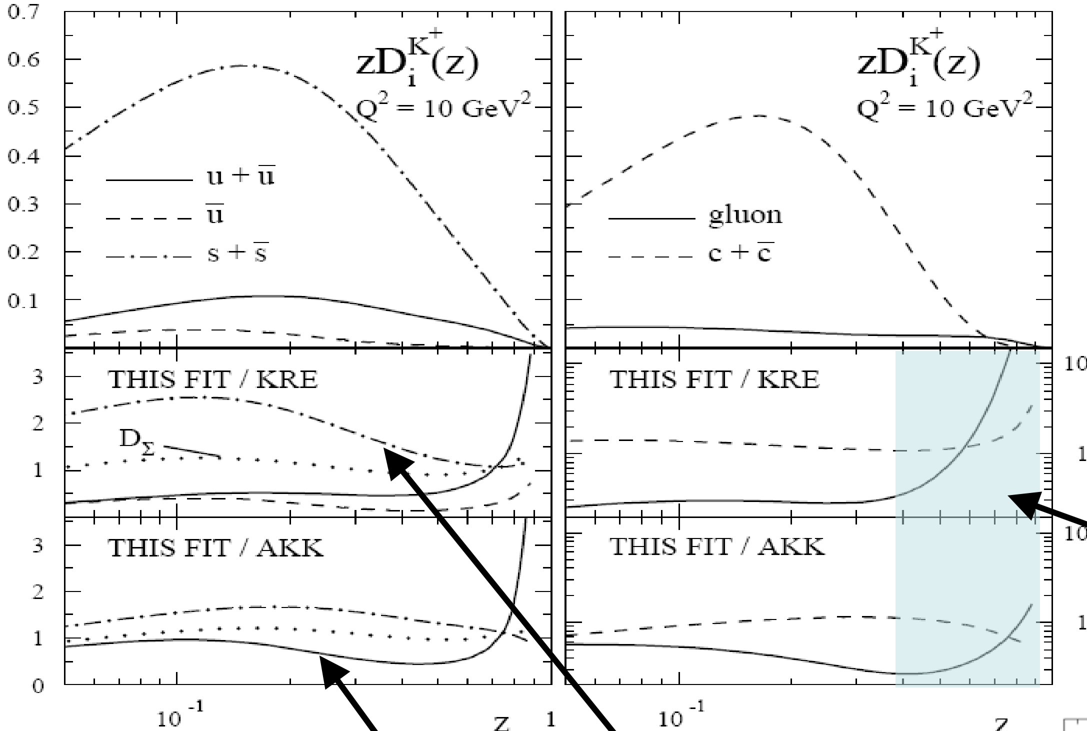


z-range of RHIC pp data explains difference in D_g

- singlet fragmentation D_Σ very similar (fixed by SIA at M_Z)
- u-frag. smaller than in AKK (due to SIDIS) ; compensated by larger D_s in SIA
- find: SU(2) violation < 10%; SU(3) violation $\simeq$ 20%

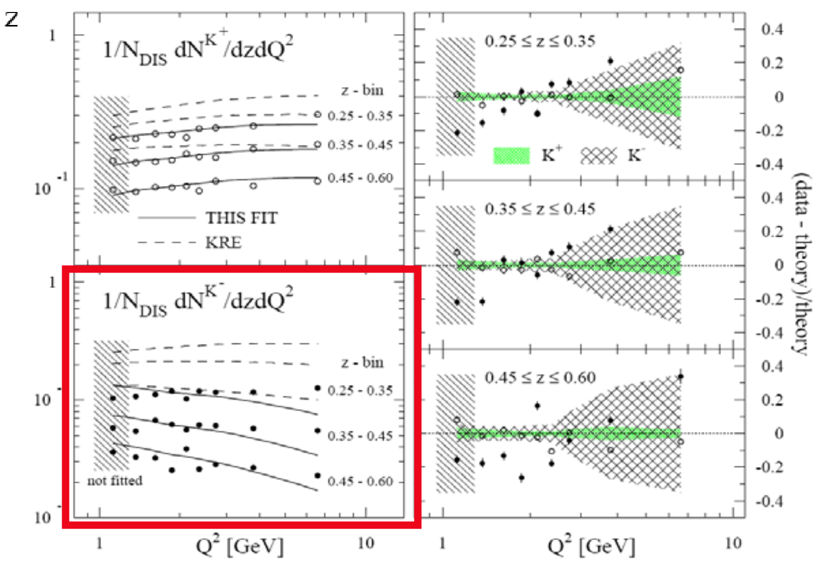
meet the $D_i(z)$'s: kaons





again, RHIC pp data explain different D_g

smaller u & larger s-frag. required by SIDIS



note: some issues with K^- data (slope!)

await eagerly final HERMES data

uncertainties from Lagrange multipliers: pions

recall:

$$O_i(\{a_j\}) = \eta_i(\{a_j\}, z_{\rm min}) = \int_{z_{\rm min}}^{1} dz\, z\, D_i^h(z, Q^2)$$

here:

z_{min}= 0.2, Q = 5 GeV

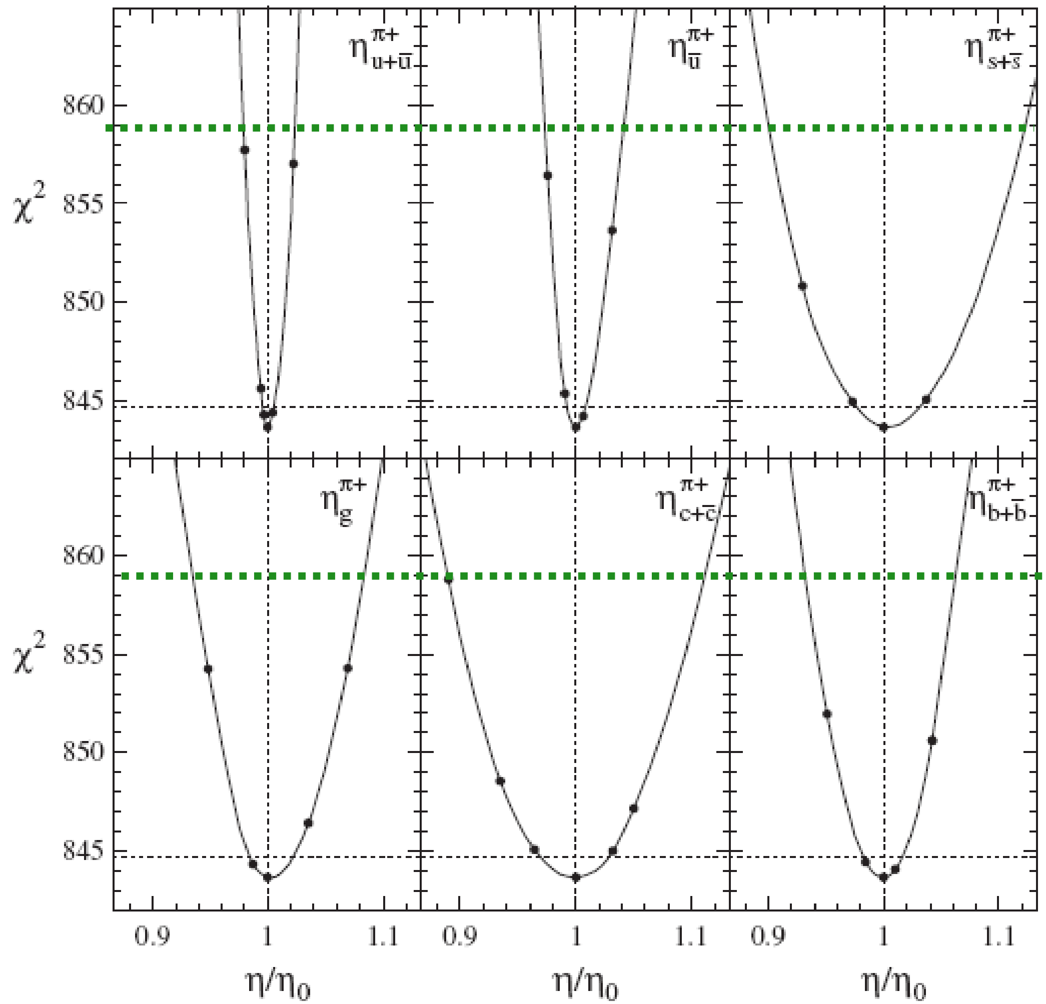
η_0: best fit value

next:

generate the χ^2 profiles

choose the $\Delta\chi^2$ ***you*** want to tolerate, e.g., $\Delta\chi^2$=15 and read off uncertainties

$\delta\eta^{\pi^+}_{u+\bar{u}} \leq 3\%$ $\delta\eta^{\pi^+}_{\bar{u}} \leq 5\%$ $\delta\eta^{\pi^+}_{s+\bar{s}} \simeq 10\%$



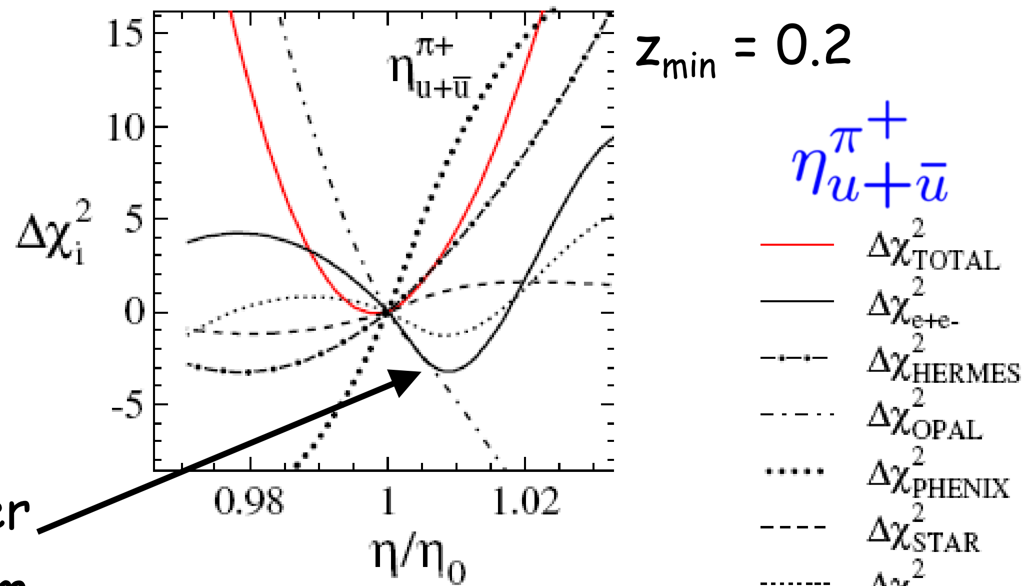
$\delta\eta^{\pi^+}_{g} \leq 10\%$ $\delta\eta^{\pi^+}_{c+\bar{c}} \geq 10\%$ $\delta\eta^{\pi^+}_{b+\bar{b}} \leq 10\%$

assessing the role of each data set:

define:

$$\Delta\chi_i^2 = \chi_i^2 - \chi_i^2\big|_0$$

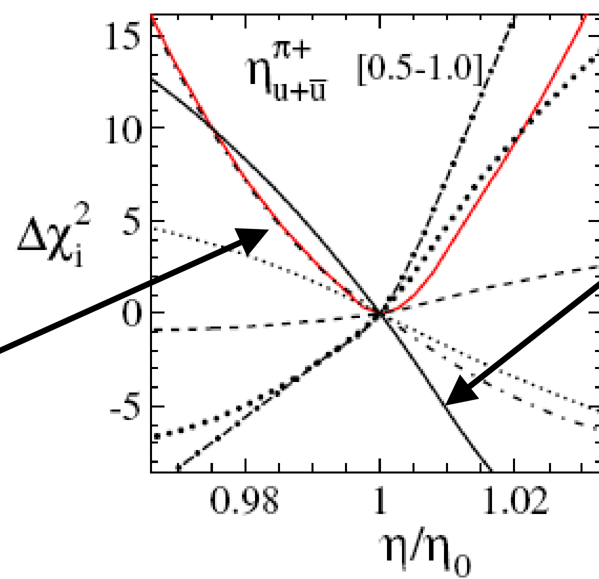
partial contribution of data subset i to $\Delta\chi^2$



$z_{min} = 0.2$

$\eta^{\pi+}_{u+\bar{u}}$

"**tension**": SIA data prefer slightly different minimum



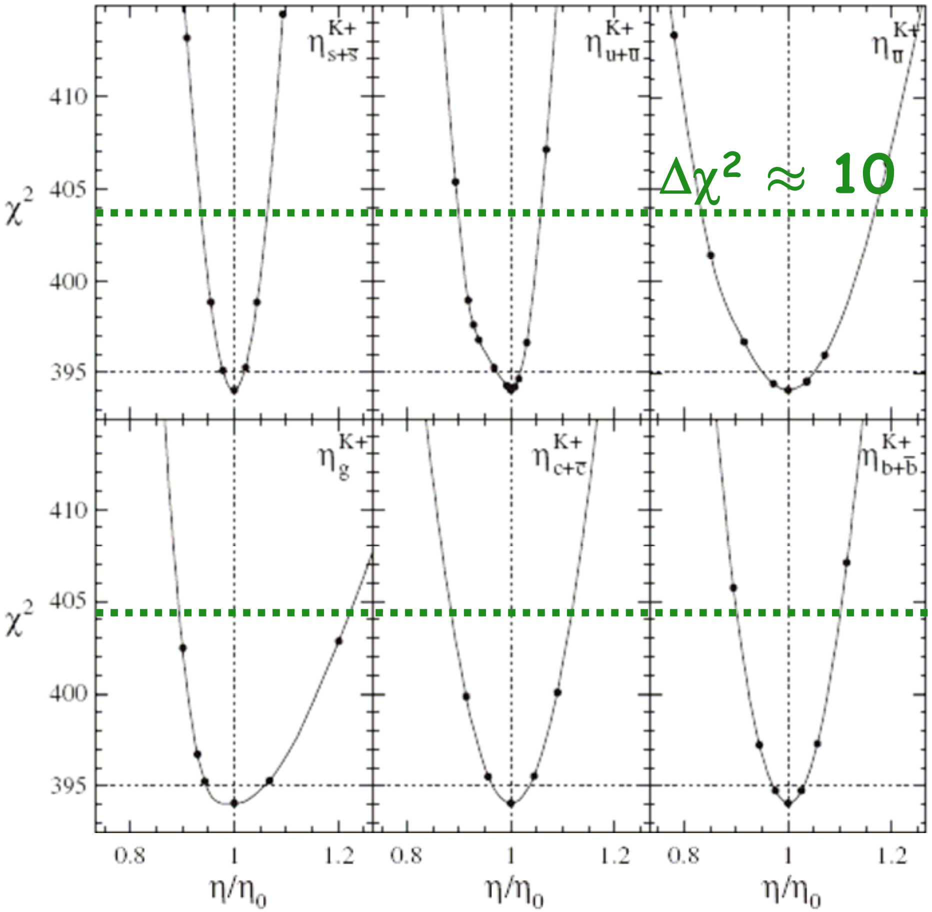
$z_{min} = 0.5$

"**complementarity**": data sets conspire to a constraining χ^2 - profile

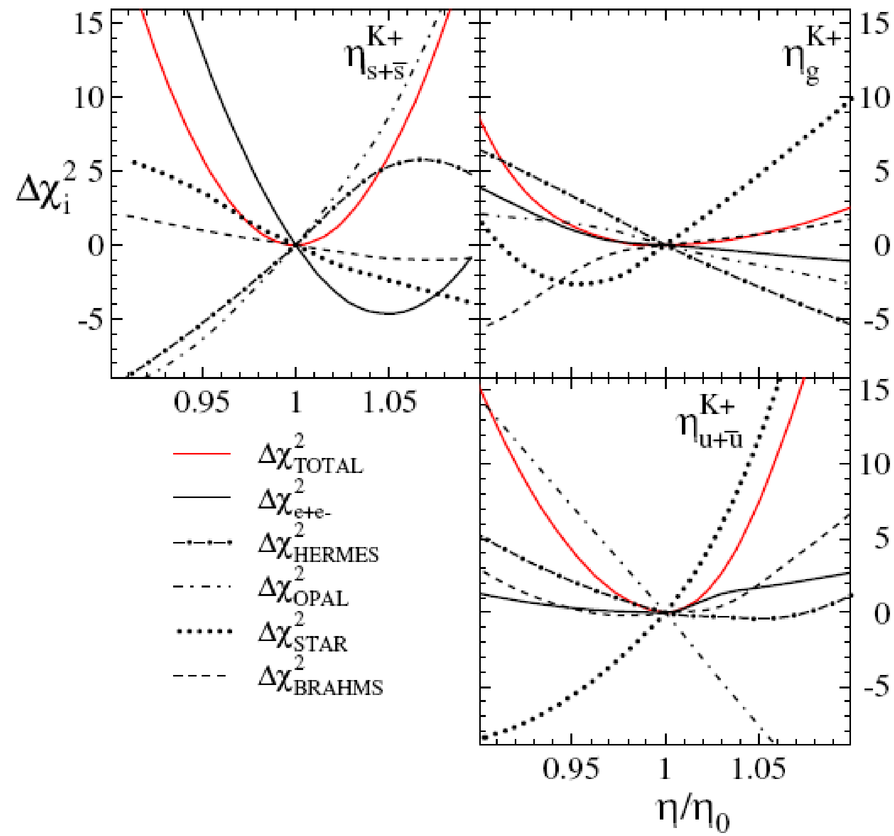
SIA alone gives no constraint for z > 0.5 (no data!)

uncertainties: kaons

at least twice as large than for pions



χ^2 - profiles less parabolic



partial $\Delta\chi^2_i$ from subsets show again "tension" and "complementarity"

remarks on the overall quality of the fits

- typically χ^2/d.o.f. $\simeq 2$

  mainly from a few isolated points, e.g., SIDIS π^- and K^-

  some tension among data sets with flavor tagging

- χ^2 grows $\approx$ 25% for LO fits

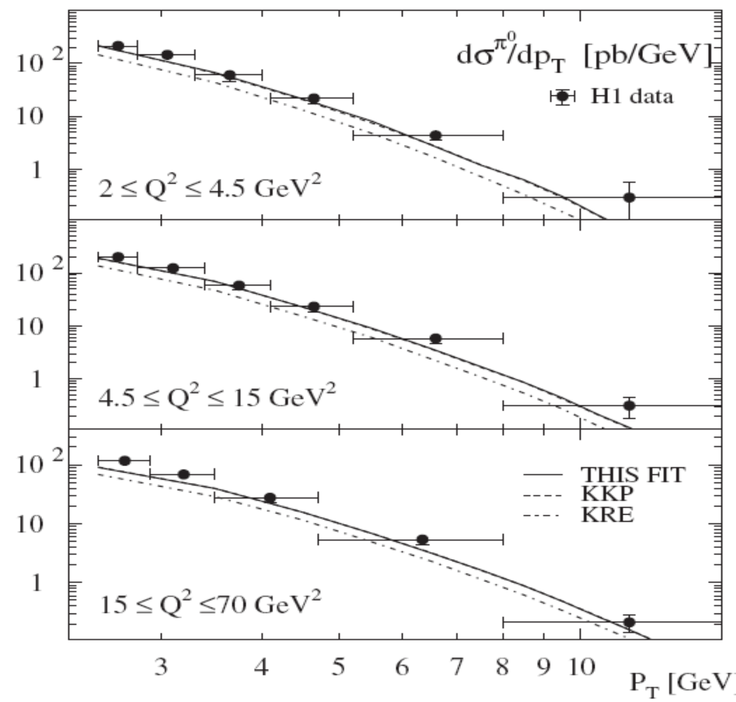
  mainly from pp data (fits try to make up for large NLO corrections)

- predictive power

NLO p_T distribution of forward π^0

crucially dependent on D_g

Daleo, de Florian, Sassot



not included in fit

summary & outlook

first global analysis of fragmentation functions

it works!

can be only the beginning:

must be an ongoing effort like CTEQ/MRST for pdfs

more/new data usually call for refinements

more groups (AKK, …) essential for progress on

“the known knowns, the known unknowns, and the unknown unknowns”*)

studies of uncertainties must be further refined

treatment of charm and bottom contributions needs improvement

need more data: HERMES (final set), BELLE/BaBar, RHIC,…

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*) courtesy of a Rumsfeld poem