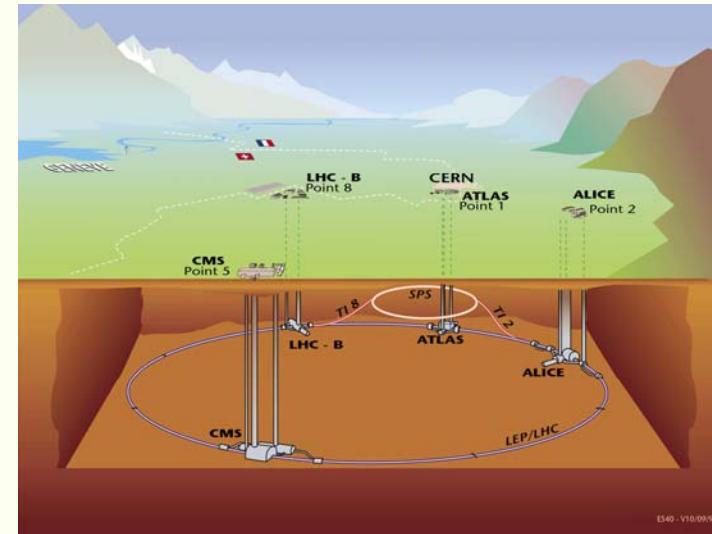


# Fragmentation function from the direct photon associated distributions.

Jan Rak

Jyväskylä University & Helsinki Institute of Physics, Finland



JYVÄSKYLÄN YLIOPISTO  
University of Jyväskylä



# Motivation

$$R_{AA}(p_T) = \frac{d^2N^{AA} / dp_T d\eta}{T_{AA} d^2\sigma^{NN} / dp_T d\eta}$$

## Great success of RHIC:

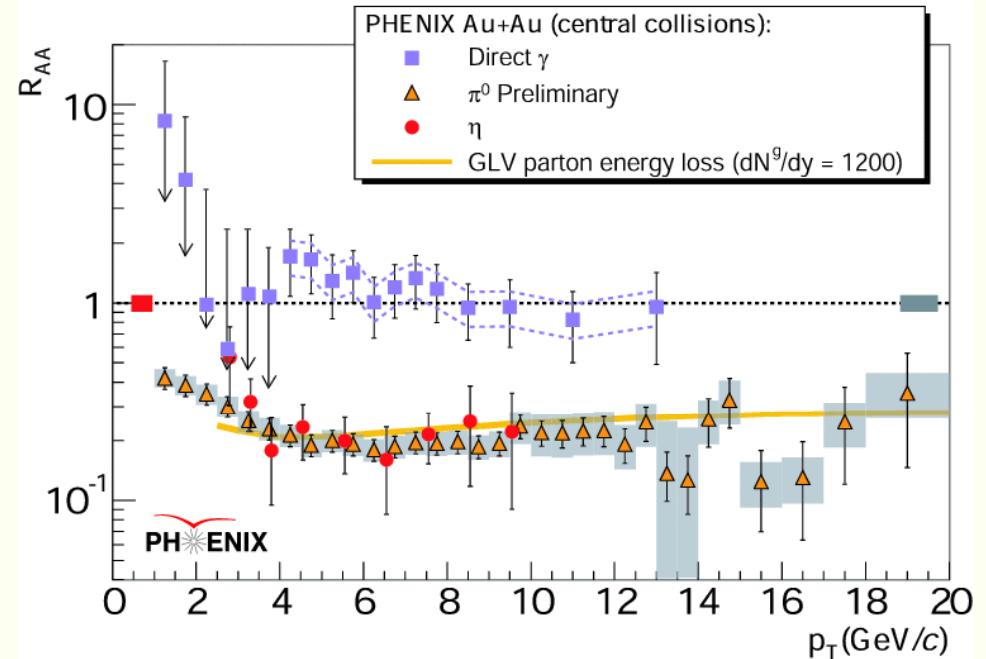
- perfect liquid, sQGP

Measured for variety of species

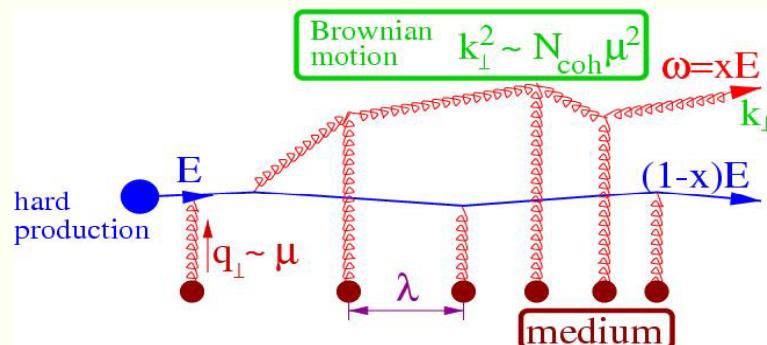
$\pi^0, \pi^\pm, \eta, \gamma_{\text{dir}}, p, K_S, \phi, \omega, J/\psi, \Omega\dots$

and CMS energies

$\sqrt{s}=17, 22.4, 62.4, 130, 200 \text{ GeV}/c$



Jet quenching - light mesons suppressed by factor of 5, direct- $\gamma$  unsuppressed => FS nature of observed suppression.



Transp. Coef.  
Scatt. power of  
QCD med:
Density of  
scattering  
centers
Range of  
color force

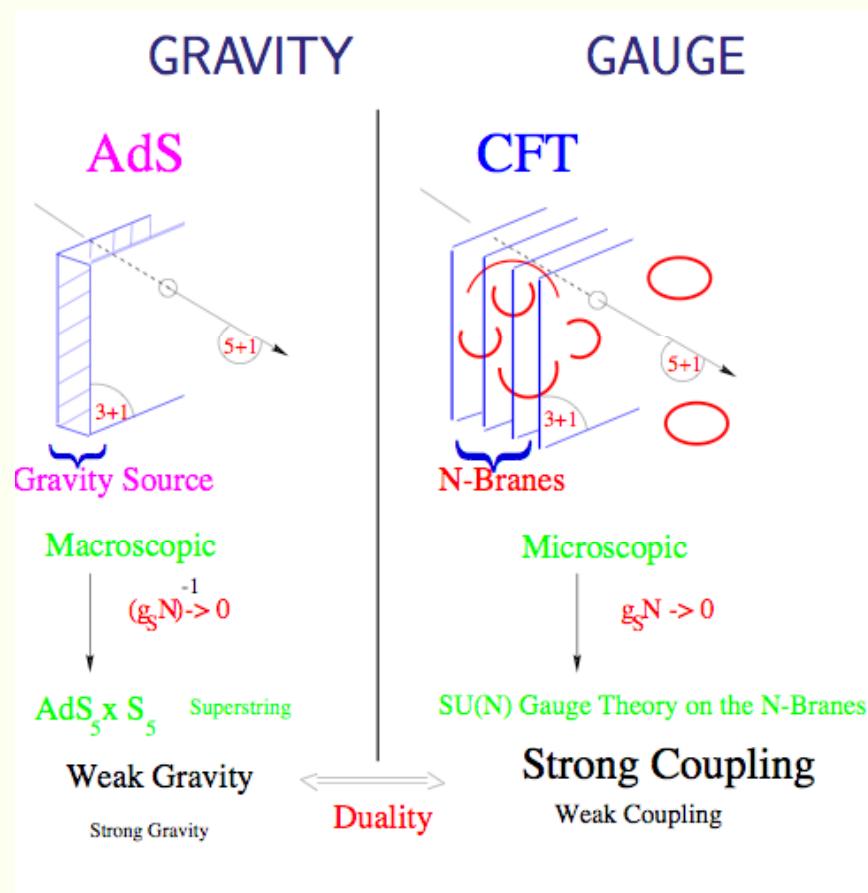
$$\dot{\varphi} = \rho \int q^2 dq^2 \frac{d\sigma}{dq^2} \equiv \rho \sigma \langle k_T^2 \rangle = \frac{\mu^2}{\lambda_f}$$

# Fun with the String Theory

## Great success of RHIC:

- test bench for string theory (AdS/CFT)

AdS/CFT Correspondence (Maldacena 1998)



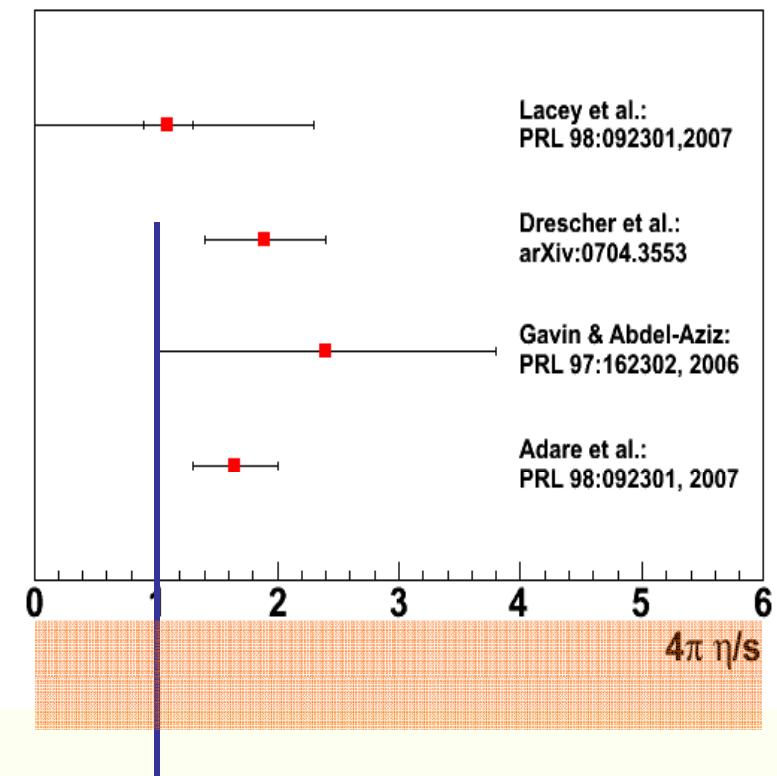
Strongly interacting matter

X

AdS/CFT duality

(Phys. Rev. Lett., 2005, 94, 111601)

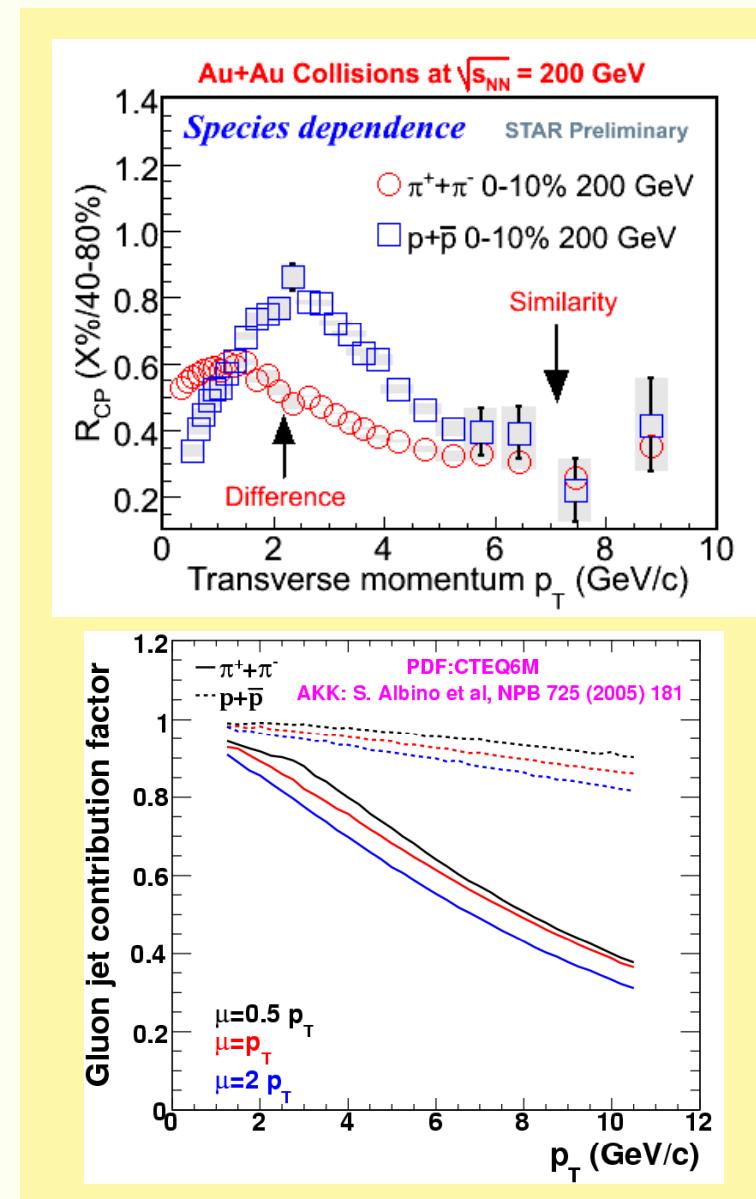
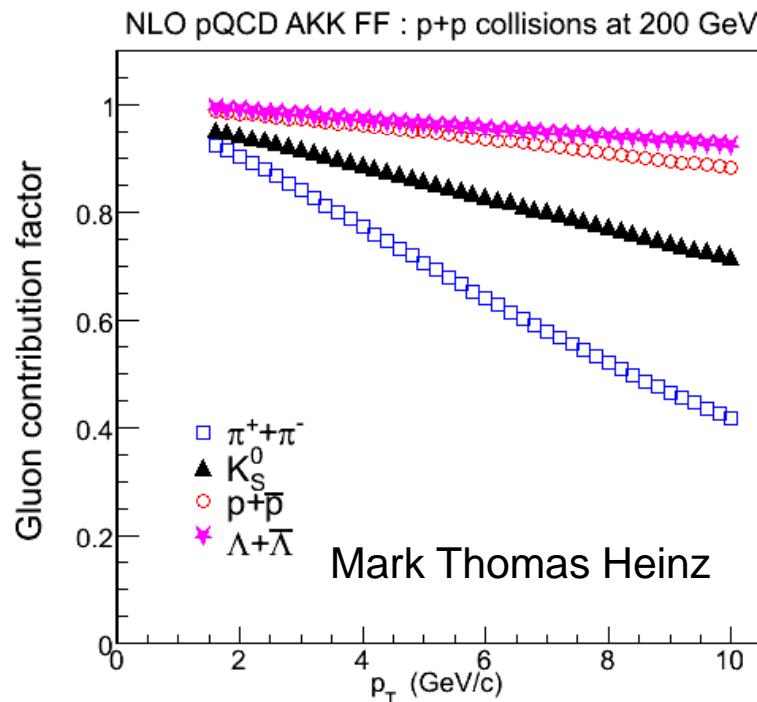
$$\frac{\eta}{s} \geq \frac{1}{4\pi}$$



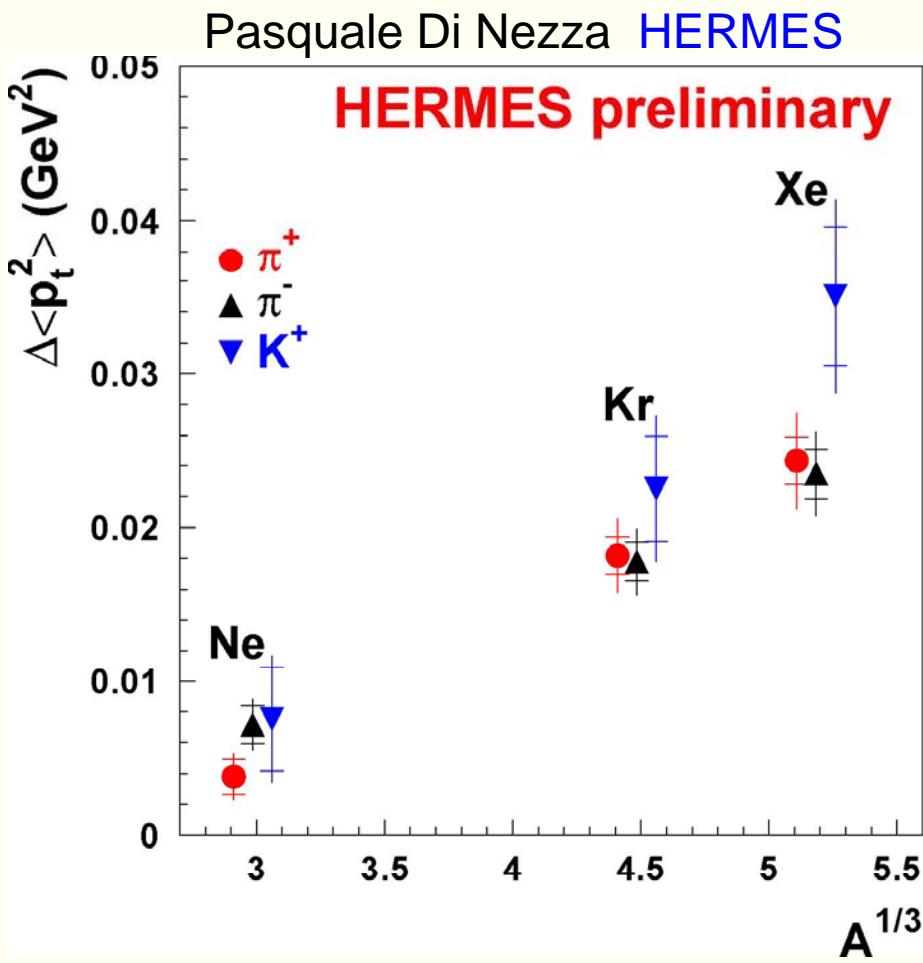
# Some open questions

however, even “simple” inclusive observable left us with with questions:

- *Light* and *heavy quarks* suppression
- *Quarks* and *gluons* suppression
- *Direct photon* suppression at *high  $p_T$*



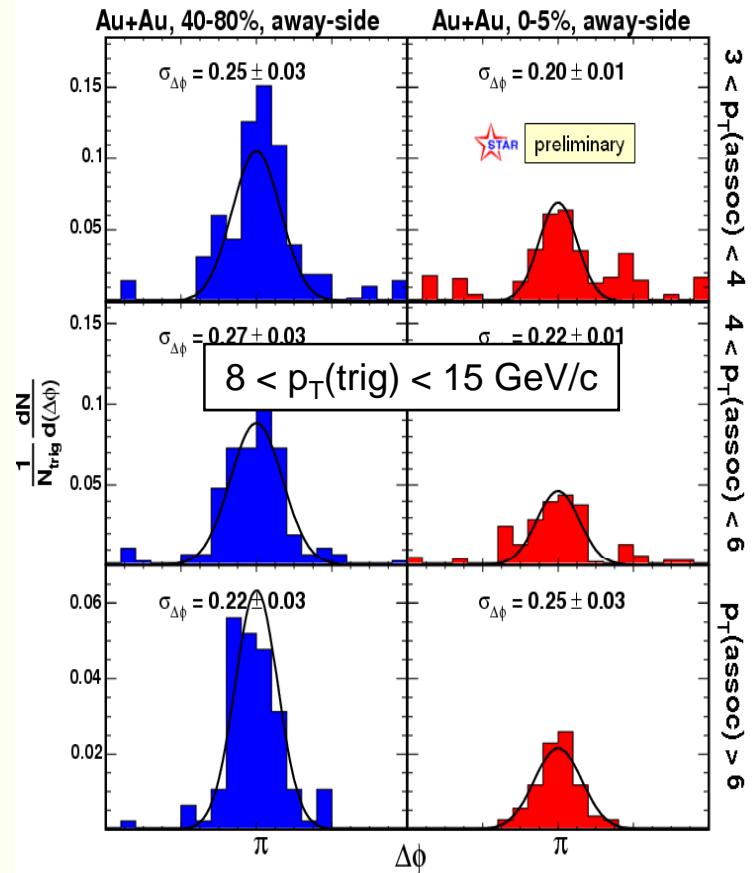
# $p_T$ broadening DIS x HI



away-side widths similar for central and peripheral

Collision Type	Width (rad)
d+Au	0.24+-0.07
Au+Au 20-40%	0.20+-0.02
Au+Au 0-5%	0.22+-0.02

**STAR Phys.Rev.Lett.97:2006**



# Direct photon correlations

## Two particle correlations

more detailed view into a nature of parton interactions with QCD medium.  
Access to parton intrinsic momentum

$k_T$  -> soft pQCD radiation, jet shape parameters

$j_T$  -> induced radiation, fragmentation function -> energy loss.

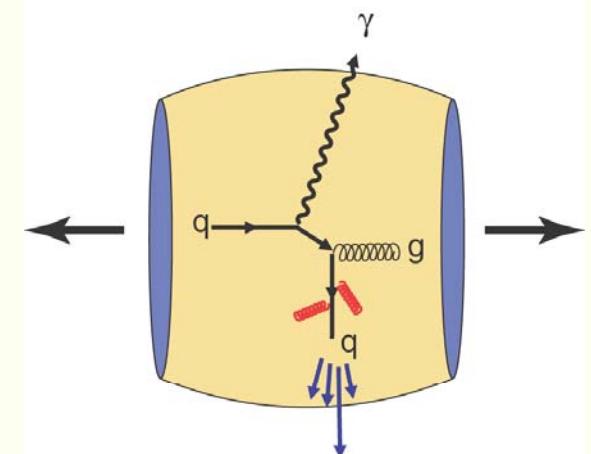
–Di-hadron correlations and conditional yields

–Direct photons-hadron correlations in  $p+p$  @  $\sqrt{s}=200$  GeV and 14 TeV

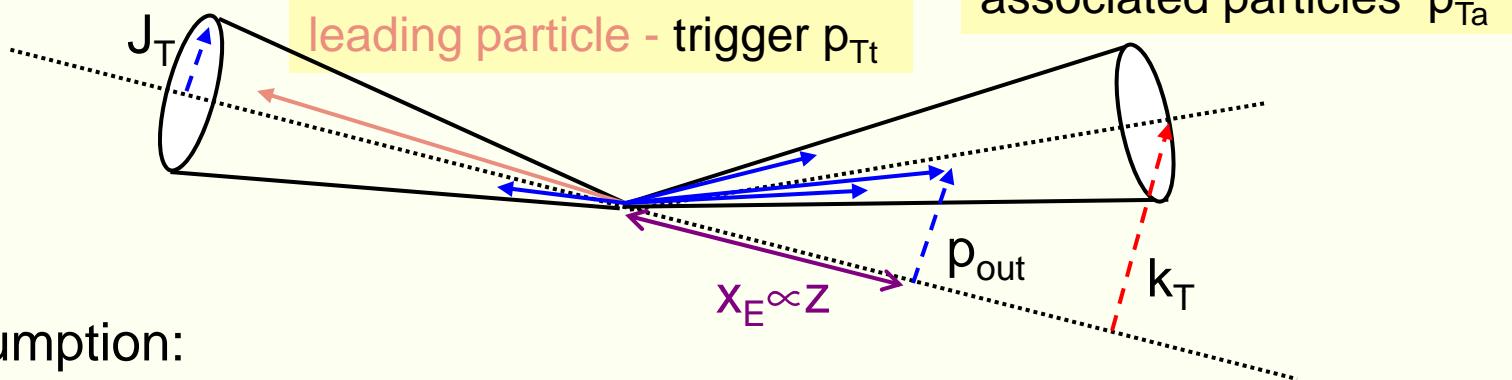
## Direct photon-hadron correlations

Jet reconstruction difficult/impossible small acceptance and in A+A collisions.

Photon energy balances the outgoing parton fixes the energy scale?



# $D(z)$ measurement without jet reconstruction



Assumption:

Leading particle fixes the energy scale of the trigger & assoc. jet

associated yield

$$\frac{dN_{assoc}}{dx_E} \propto \sum_i \int_x^1 \frac{dz}{z} C_i(s; z, \alpha_s) D_i^h(x/z, s) \equiv D^h(z)$$

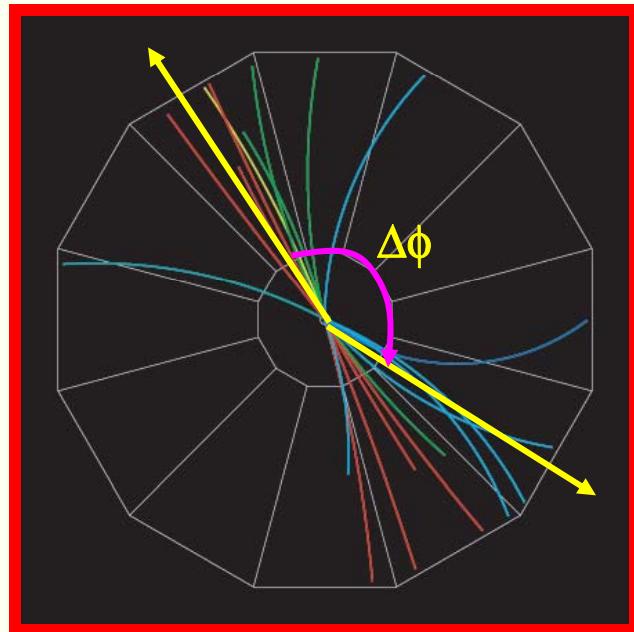
=>

acoplanarity (CF width)

$$\frac{dN_{assoc}}{dp_{out}} \propto \frac{dN_{assoc}}{d\Delta\phi} \propto \frac{d\sigma}{dk_T}$$

$$x_E = \left| \frac{\mathbf{p}_{Ta} \cdot \mathbf{p}_{Tt}}{p_{Tt}^2} \right| = -\frac{p_{Ta}}{p_{Tt}} \cos \Delta\phi \approx -\frac{p_{Ta}}{p_{Tt}}$$

# Azimuthal correlation function in $p+p$ @ $\sqrt{s}=200$ GeV

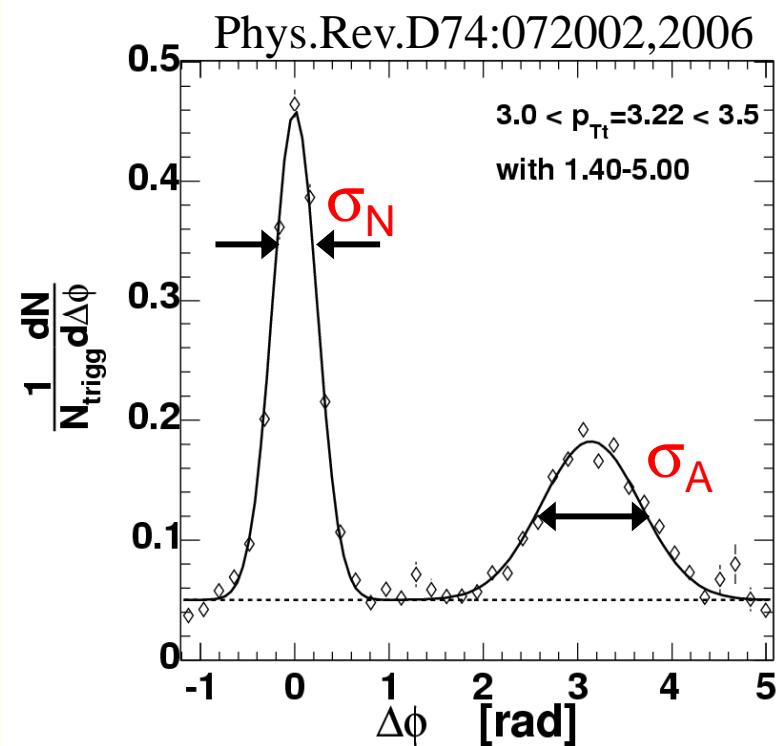


$p + p \rightarrow \text{jet} + \text{jet}$

$\sigma_N \propto \langle j_T \rangle$  jet fragmentation transverse momentum

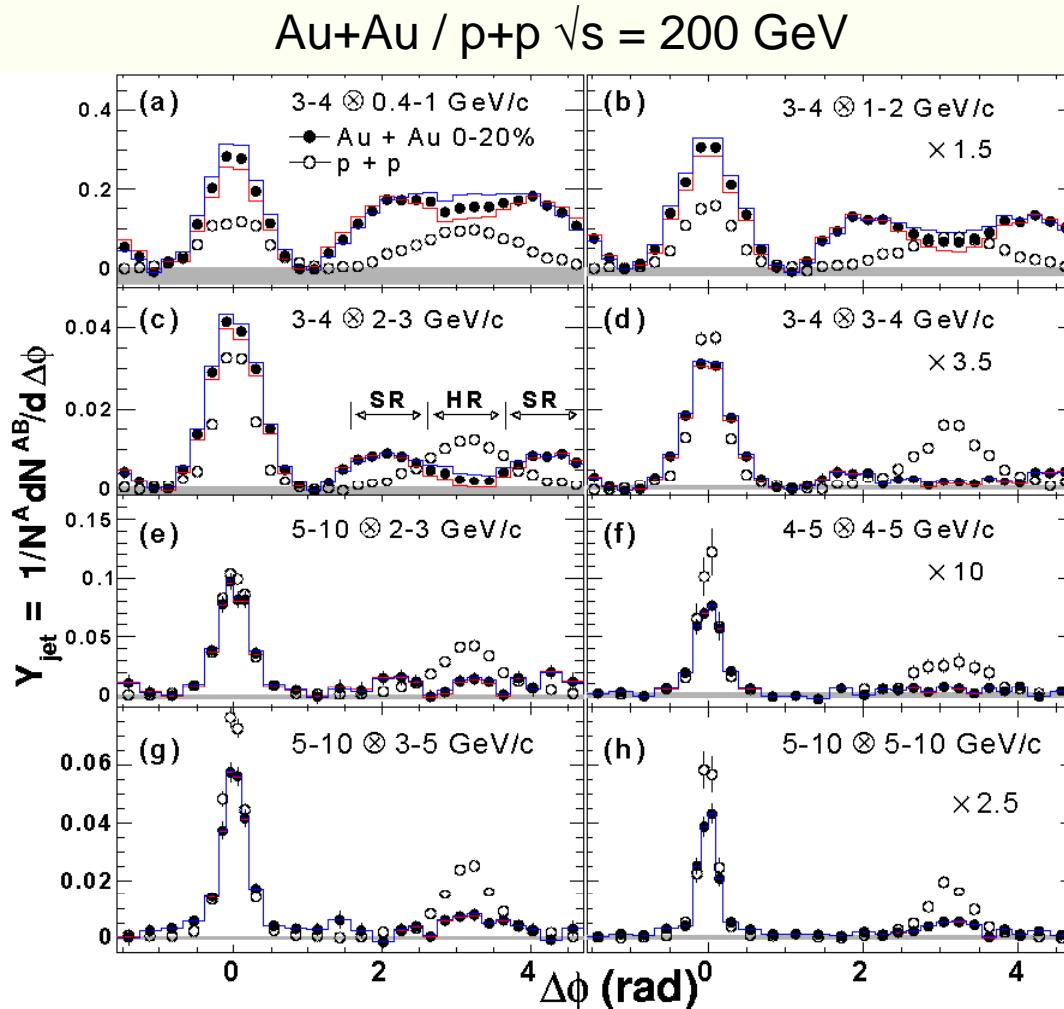
$\sigma_F \propto \langle k_T \rangle$  parton transverse momentum

$Y_A \propto$  folding of  $D(z)$  and final state parton dist.



$$\frac{1}{N_{\text{trigg}}} \frac{dN}{d\Delta\phi} = \frac{R_{\Delta\eta}}{N_{\text{trigg}} \epsilon(p_T)} \frac{dN_{\text{uncorr}}(\Delta\phi)/d\Delta\phi}{dN_{\text{mix}}(\Delta\phi)/d\Delta\phi}$$

# Jet shape evolution with trigger and assoc. $p_T$



Per-trigger yield vs.  $\Delta\phi$  for various trigger and partner  $p_T$  ( $p_A^A \otimes p_B^B$ ), arranged by increasing pair momentum ( $p_A^A + p_B^B$ )

- **Flat region:**  
celebrated b2b disappearance
- **Punch through region (HR):**  
reappearance at high- $p_T$
- **Shoulder region (SR):**  
Medium induced “Mach cone”
- **Low  $p_T$ :**  
Enhancement in SR & suppression in HS
- **High  $p_T$ :**  
Reappearance of away-side jet **not due to merging** of side peaks

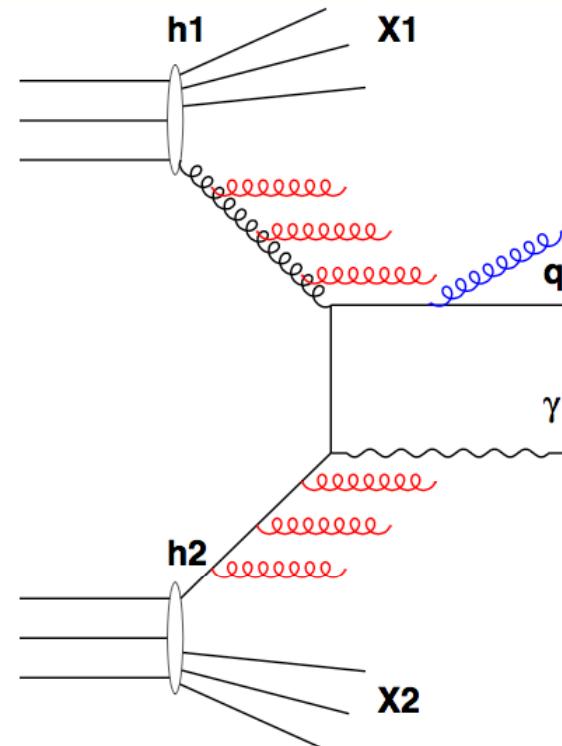
# Two-particle correlations in $p+p$

Fragmentation function  $D(z)$  and  
Intrinsic momentum  $k_T$

# Soft + hard QCD radiation $k_T$ phenomenology

Compton photo-production

$$q + g \rightarrow \text{quark}(\vec{q}_T) + \text{photon}(p_{T\gamma})$$

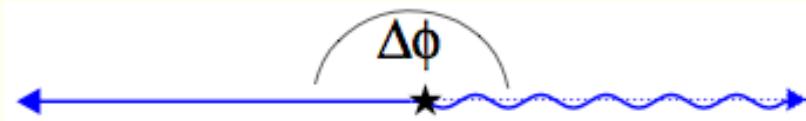


Back-to-back

$$\frac{d\sigma}{d\Delta\phi} = \delta(\varphi - \pi)$$

balanced

$$\left. \frac{d\sigma}{dq_T} \right|_{p_{T\gamma}} = \delta(q_T - p_{T\gamma})$$



Soft QCD radiation

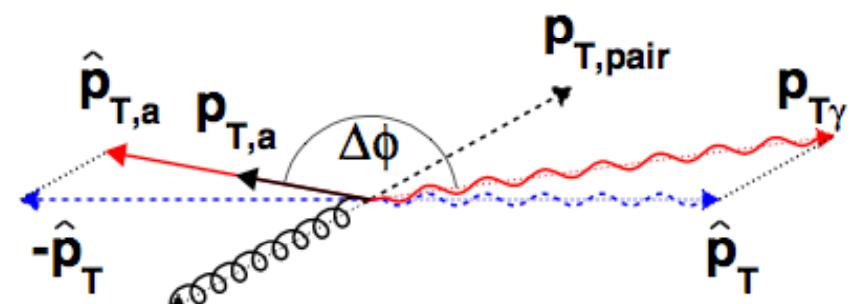
$$\frac{d\sigma}{d\Delta\phi} \propto \text{Gauss}(\Delta\phi)$$

$$\left. \frac{d\sigma}{dq_T} \right|_{p_{T\gamma}} \propto \text{Gauss}(p_{T\gamma})$$

Hard NLO radiation

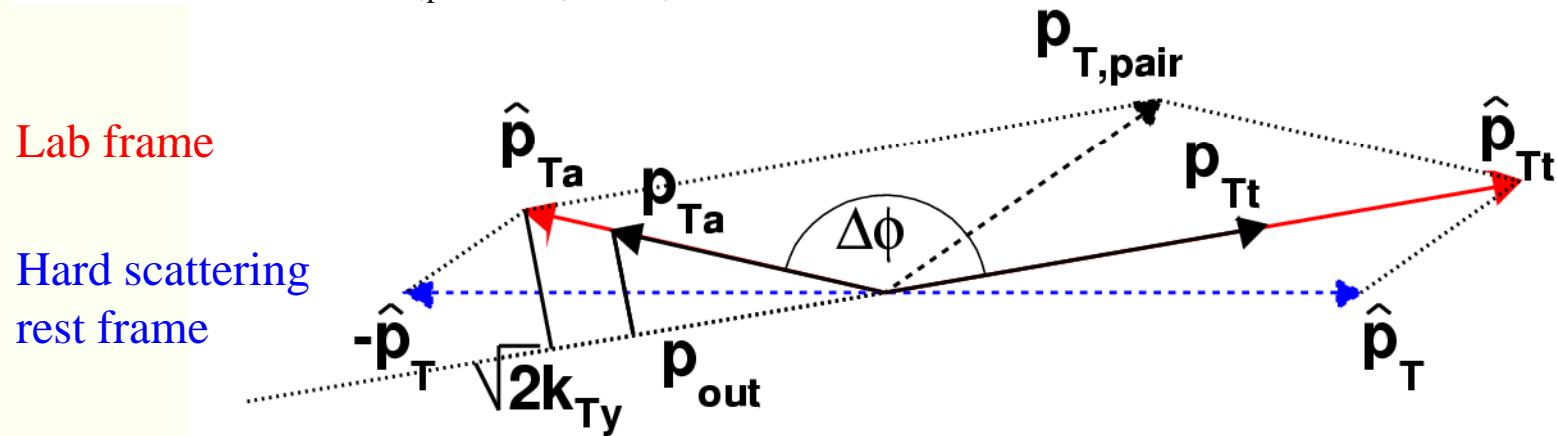
$$\frac{d\sigma}{d\Delta\phi} \propto \frac{1}{\Delta\phi^{-n}}$$

$$\left. \frac{d\sigma}{dq_T} \right|_{p_{T\gamma}} \propto \frac{1}{p_{T\gamma}^{-n}}$$



# Correl. fcn width - $k_T$ and acoplanarity

Lorentz boost  $\Rightarrow p_{T,\text{pair}} \parallel k_{T,t} \parallel k_{T,a}$  colinearity



$$\langle |p_{out}| \rangle = \sqrt{2} \langle |k_{Ty}| \rangle \frac{p_{Ta}}{\langle \hat{p}_{Ta} \rangle} \quad \Rightarrow \quad \sqrt{\langle p_{out}^2 \rangle} = \langle z_t \rangle \sqrt{\langle k_T^2 \rangle} \frac{x_h}{\hat{x}_h}$$

$$k_T\text{-induced jet imbalance } \hat{x}_h(x_h) = \frac{\hat{p}_{Ta}}{\hat{p}_{Tt}} \quad \text{particle pair imbalance } x_h = \frac{p_{Ta}}{p_{Tt}}$$

partonic	$\frac{\langle z_t \rangle}{\langle \hat{x}_h \rangle} \sqrt{\langle k_T^2 \rangle} = \frac{1}{x_h} \sqrt{\langle p_{out}^2 \rangle - \langle j_{Ty}^2 \rangle (1 + x_h^2)}$	hadronic
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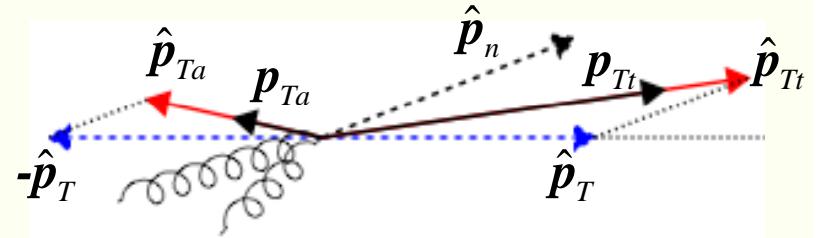
# Associated yield

$$\frac{\langle z_t \rangle \left( \sqrt{\langle k_T^2 \rangle}, p_{Tt}, p_{Ta} \right)}{\langle \hat{x}_h \rangle \left( \sqrt{\langle k_T^2 \rangle}, p_{Tt}, p_{Ta} \right)} \sqrt{\langle k_T^2 \rangle} = \frac{1}{x_h} \sqrt{\langle p_{out}^2 \rangle - \langle j_{Ty}^2 \rangle (1 + x_h^2)}$$

$$x_h = \frac{p_{Ta}}{p_{Tt}}; \quad p_{out} \approx \sigma_{away}; \quad j_T \approx \sigma_{near}$$

**Assumption** (*Phys.Rev.D74:072002,2006 for details*):

Invariant mass of mass-less partons in hard scattering CMS and in LAB is the same -> non-Gaussian  $k_T$ -smearing.

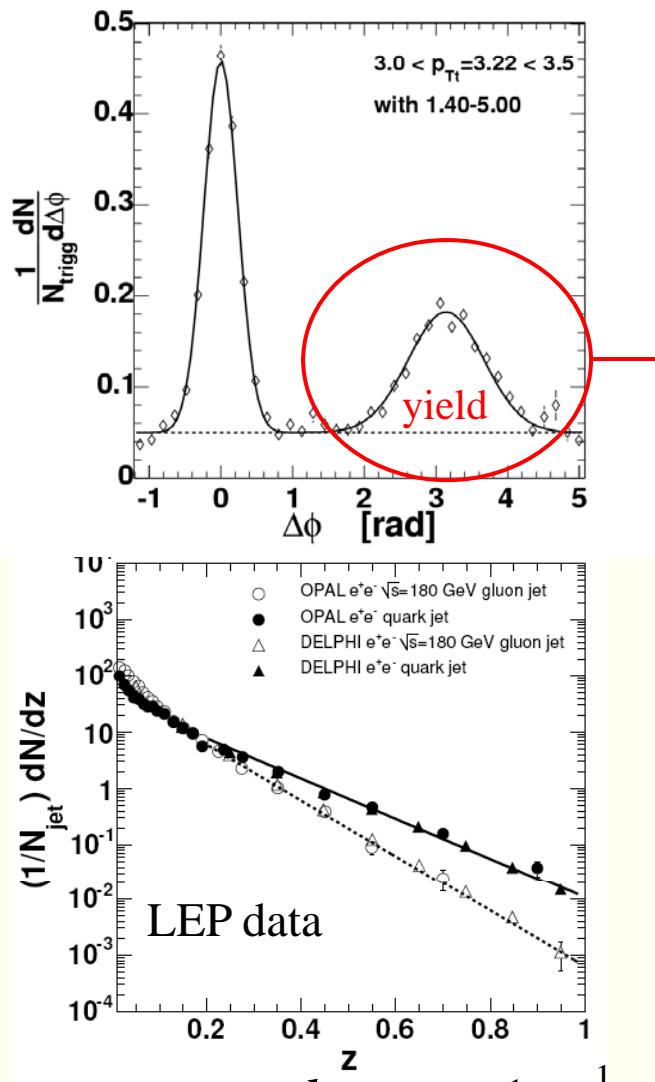


$$\langle z_t \rangle \left( \sqrt{\langle k_T^2 \rangle}, p_{Tt}, p_{Ta} \right) = \int_{xTt}^1 dz_t \ z^{n-1} \cdot D^\pi(z_t) \cdot \Sigma'_Q(z_t)$$

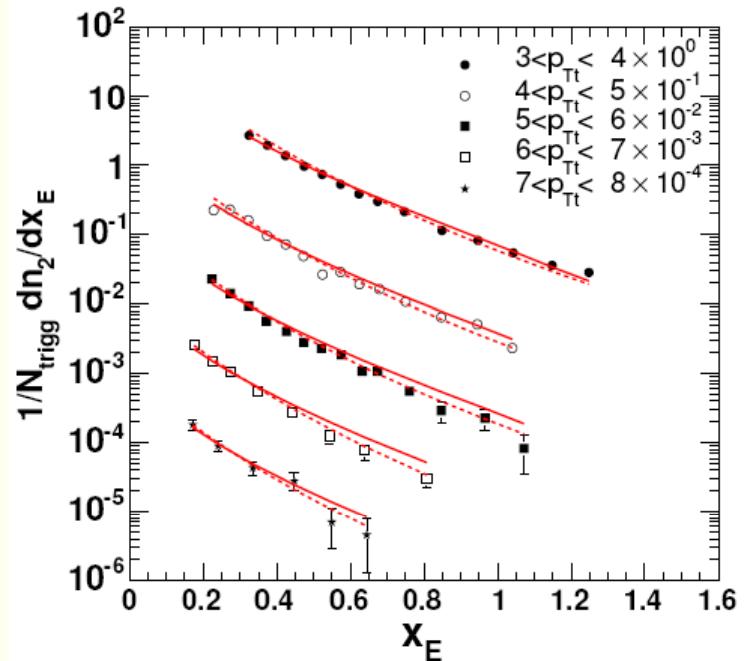
$$\langle \ddot{x}_h \rangle \left( \sqrt{\langle k_T^2 \rangle}, p_{Tt}, p_{Ta} \right) = \int_{pTt}^{\sqrt{s}/2} d\ddot{\mathbf{p}}_{Tt} \ \ddot{\mathbf{p}}_{Tt}^{n-1} \cdot D^\pi\left(\frac{p_{Tt}}{\ddot{\mathbf{p}}_{Tt}}\right) \cdot \Sigma'_Q\left(\frac{p_{Tt}}{\ddot{\mathbf{p}}_{Tt}}\right)$$

where  $k_T$ -smeared parton dist.  $\Sigma'_Q(z_t) = \int_0^{\sqrt{s}/2} d\ddot{\mathbf{p}}_T \ \Sigma_Q(\ddot{\mathbf{p}}_T) \int_0^\pi d\phi \ \ddot{\mathbf{p}}_n \cdot G\left(\ddot{\mathbf{p}}_n, \sqrt{\langle k_T^2 \rangle}\right) D^\pi\left(\frac{p_{Ta}}{\ddot{\mathbf{p}}_{Ta}}\right) \cdot \frac{1}{\ddot{\mathbf{p}}_{Ta}}$

# Trigger associated spectra are insensitive to D(z)



$$x_E = \left| \frac{\vec{p}_{Ta} \cdot \vec{p}_{Tt}}{p_{Tt}^2} \right| = -\frac{p_{Ta}}{p_{Tt}} \cos \Delta\phi \approx -\frac{p_{Ta}}{p_{Tt}}$$

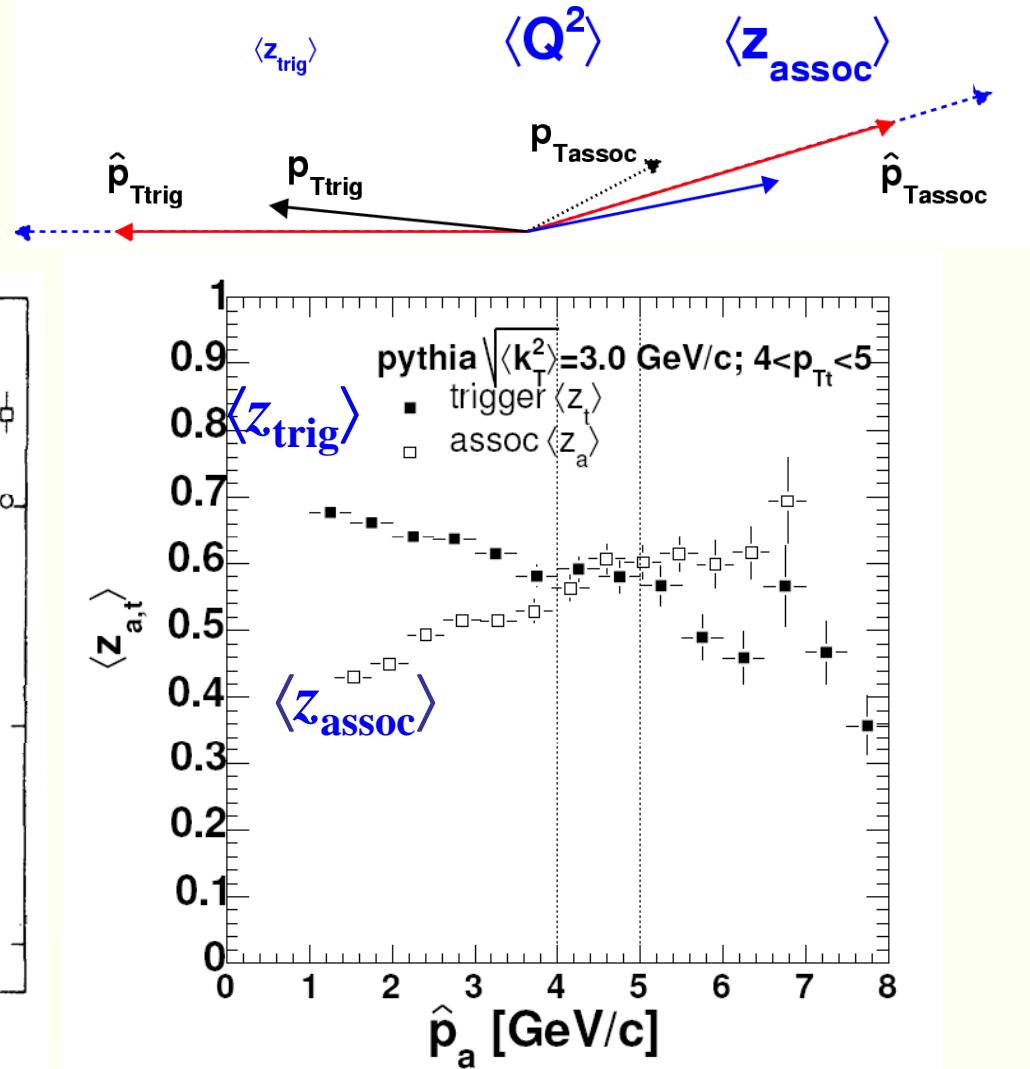
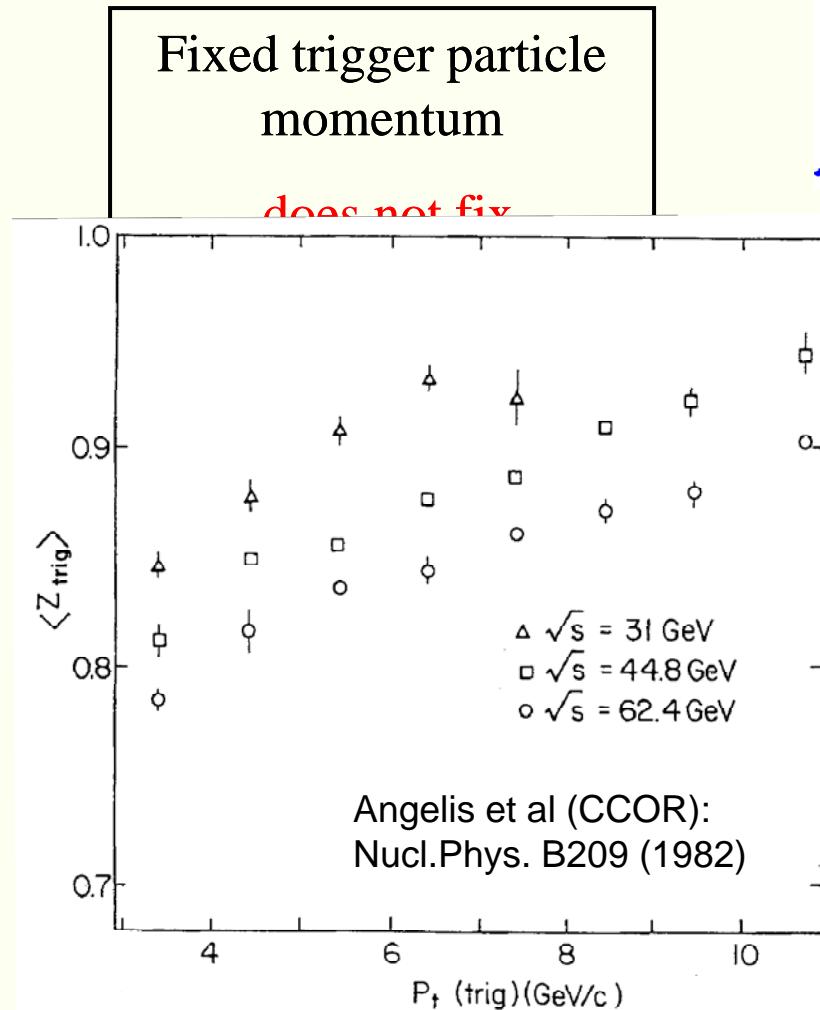


M.J.Tannenbaum Approximation - Incomplete Gamma function when assumed power law for final state PDF and exp for  $D(z)$

$$\frac{d\sigma_\pi^z}{dp_{Tt}} = \frac{1}{p_{Tt}^{n-1}} \int_{xT_t}^1 dz_t z_t^{n-2} e^{-b.z_t} \approx \langle m \rangle (n-1) \frac{1}{x_h} \left( 1 + \frac{x_E}{x_h} \right)^{-n}$$

# Unavoidable z-bias in di-hadron correlations

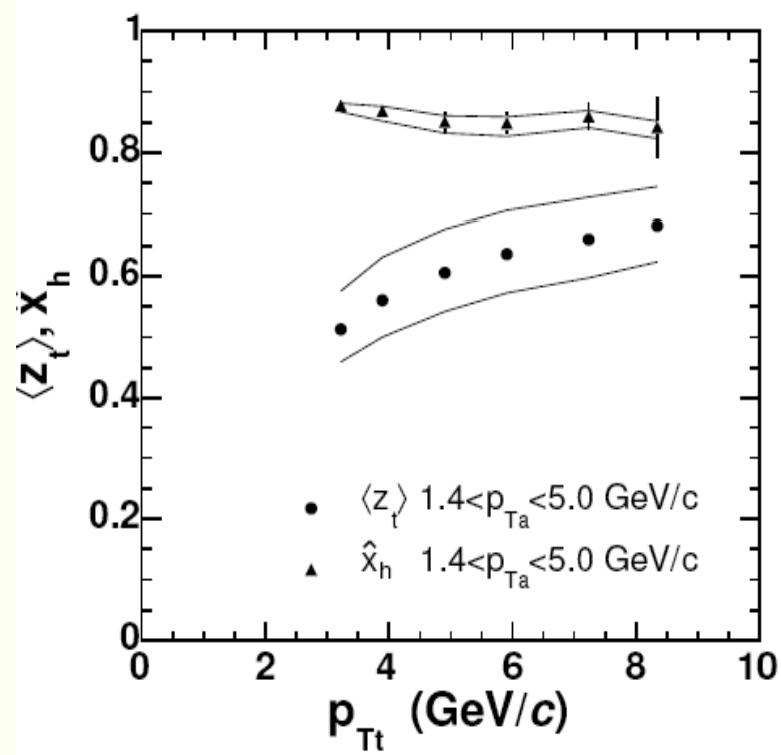
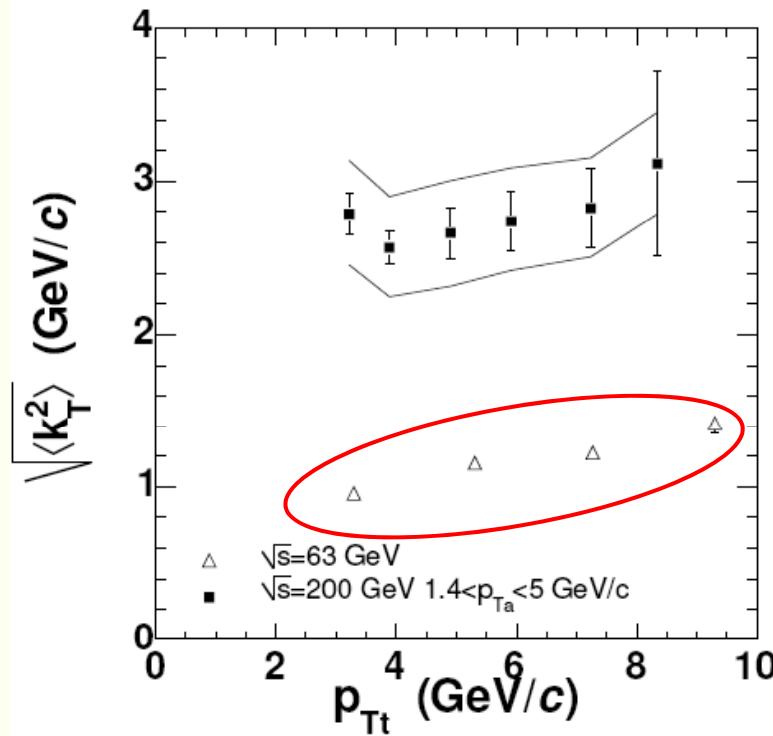
**z-bias;** steeply falling/rising  $D(z)$  &  $\text{PDF}(1/z)$



# $\sqrt{\langle k_T^2 \rangle}$ and $\langle z_t \rangle$ in p+p @ 200 GeV from $\pi^0$ -h CF

Phys.Rev.D74:072002,2006

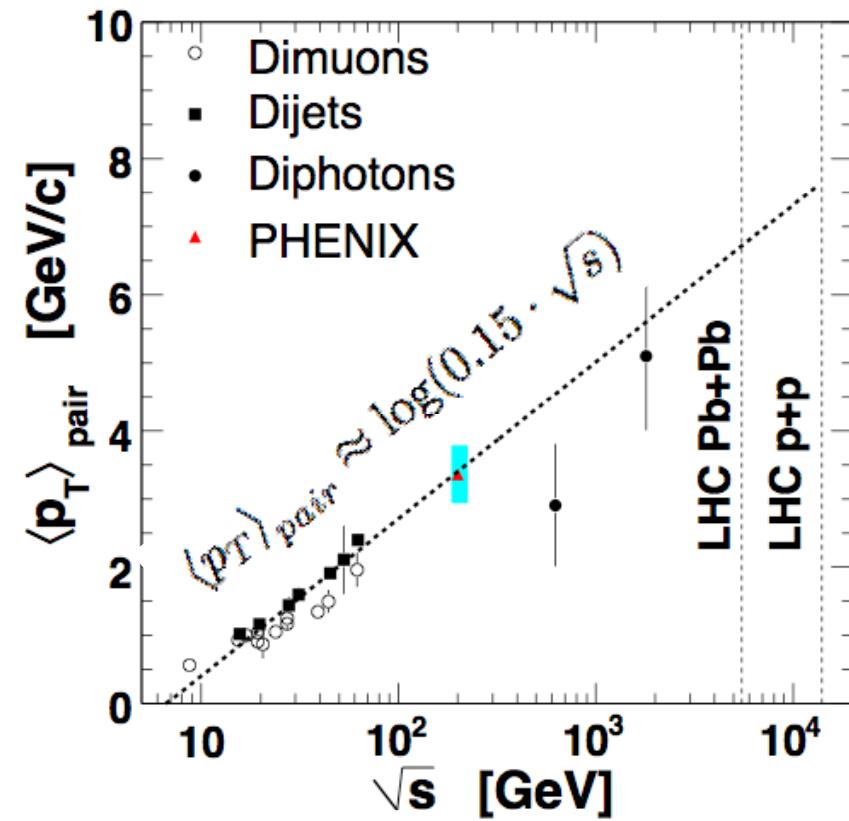
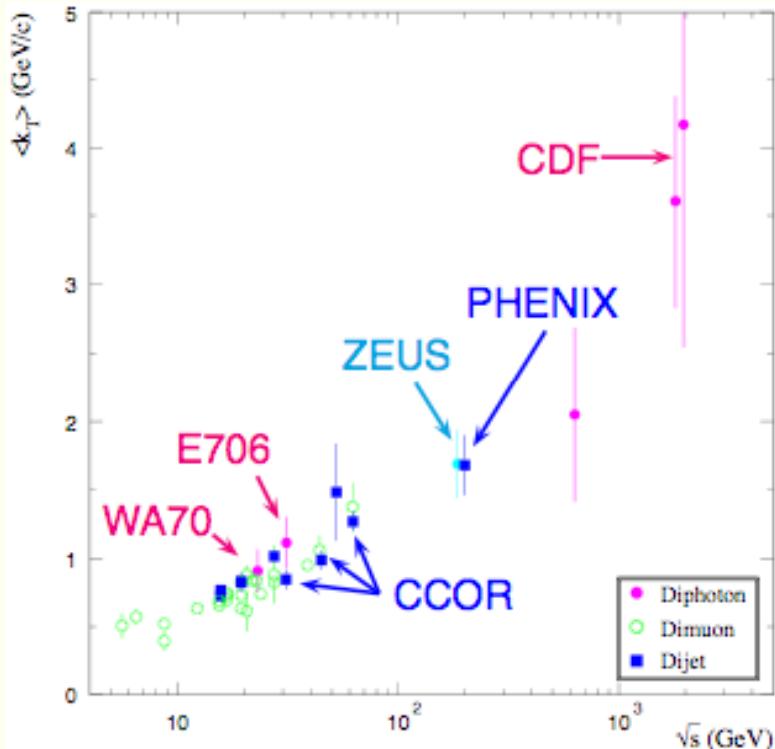
For D(z) the LEP date were used. Main contribution to the systematic errors comes from unknown ratio gluon/quark jet  $\Rightarrow$  D(z) slope.



Base line measurement for the  $k_T$  broadening - collisional energy loss. Direct width comparison is biased.

Still, we would like to extract FF from our own data  $\rightarrow$  direct photon-h correl.

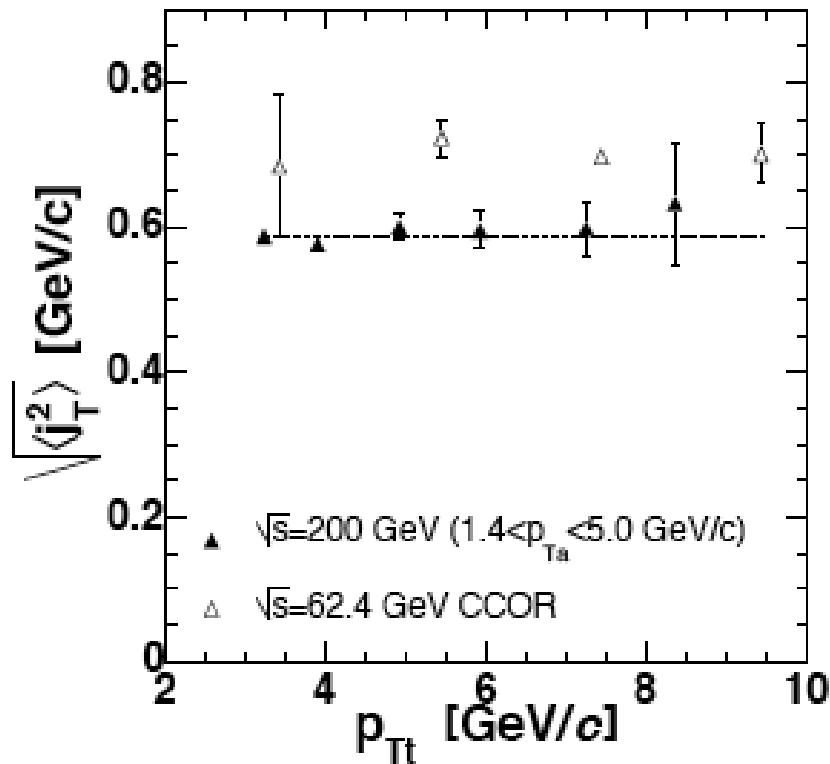
# High $p_T$ : ref. for HI, resummation, detailed NLO tests



PHENIX measured  $\langle p_T \rangle_{\text{pair}} = 3.36 \pm 0.09 \pm 0.43 \text{ GeV}/c$

extrapolation to LHC  $\langle k_T^2 \rangle \approx 36 \text{ GeV}^2/c^2$

# Parton evolution - $j_T$



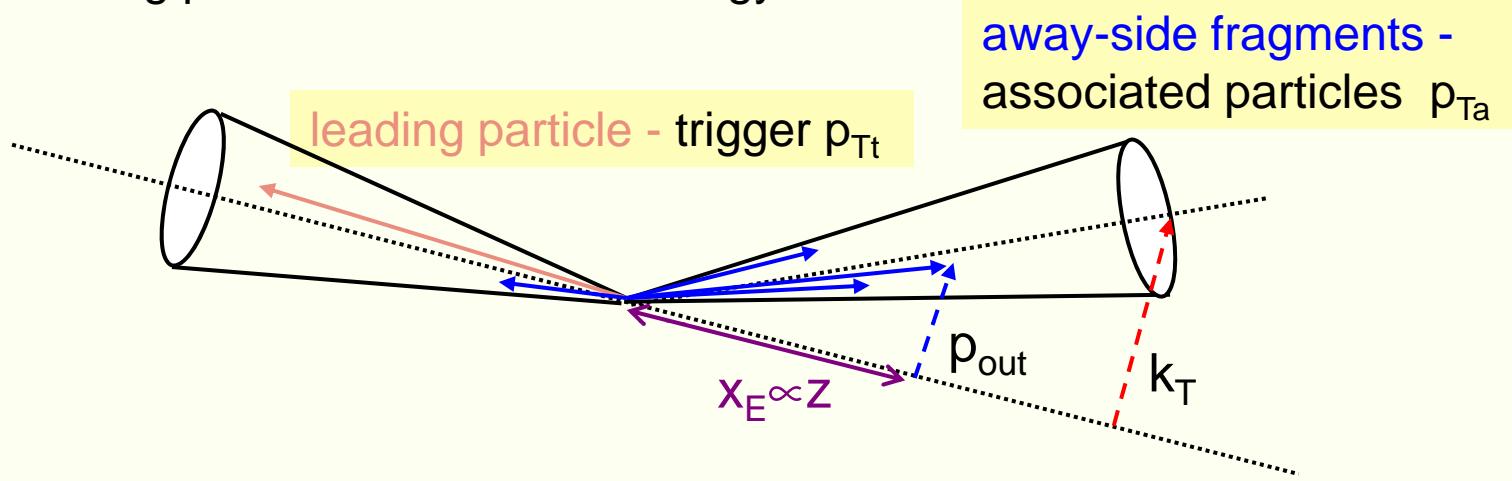
H.J. Pirner talk from yesterday

A. Basseto, M. Ciafaloni, G. Marchesini  
And Nucl Phys B **163** (1980) 477

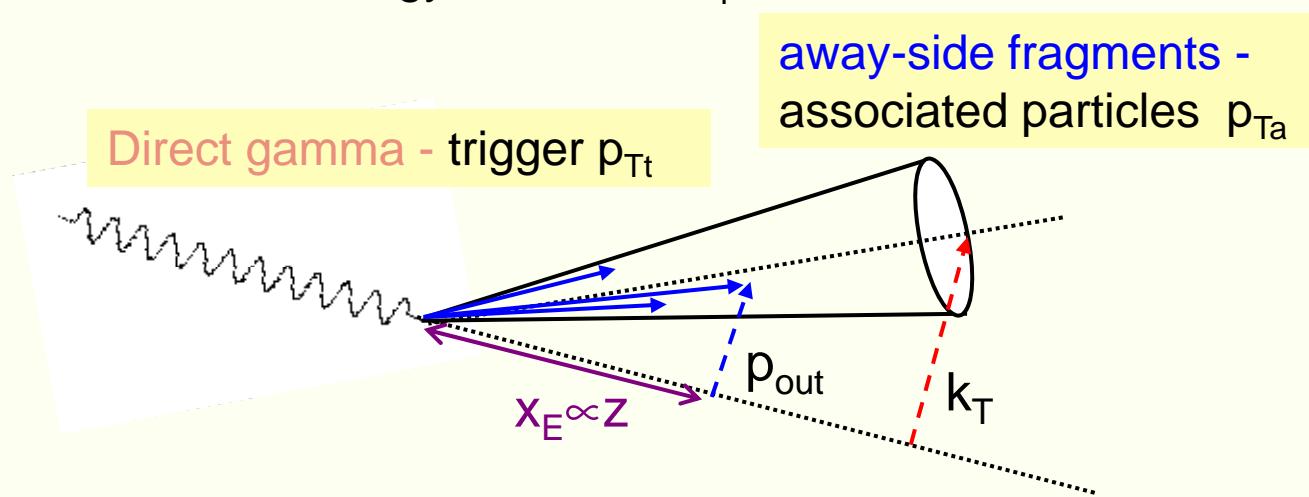
$$\begin{aligned}
 & Q^2 \frac{D_i^j(z, Q^2, \vec{p}_t)}{\partial Q^2} = \\
 &= \frac{\alpha_s(Q^2)}{2\pi} \int_z^1 \frac{du}{u} P_i^r(u, \alpha_s(Q^2)) \frac{d^2 \vec{q}_t}{\pi} \delta(u(1-u)Q^2 - Q_0^2/4 - q_t^2) D_r^j(z/u, Q^2, \vec{p}_t - z/u \vec{q}_t)
 \end{aligned} \tag{19}$$

# $D(z)$ from gamma tagged correlation

**h-h:** Leading particle **does not** fix Energy scale.



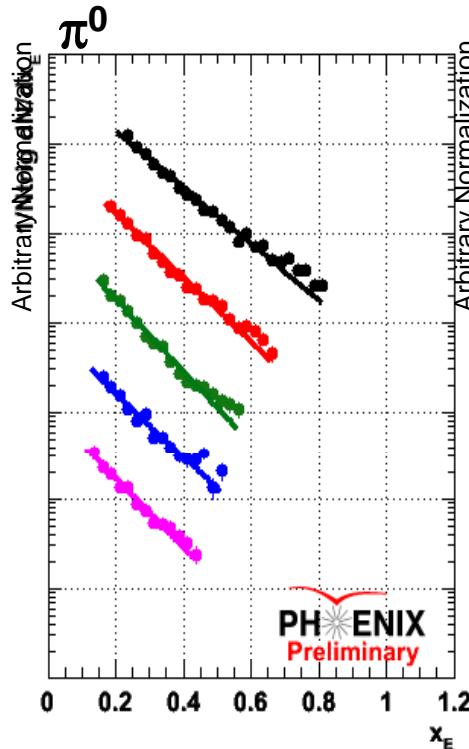
**$\gamma$ -h:** direct gamma **does** fix Energy scale if no  $k_T$



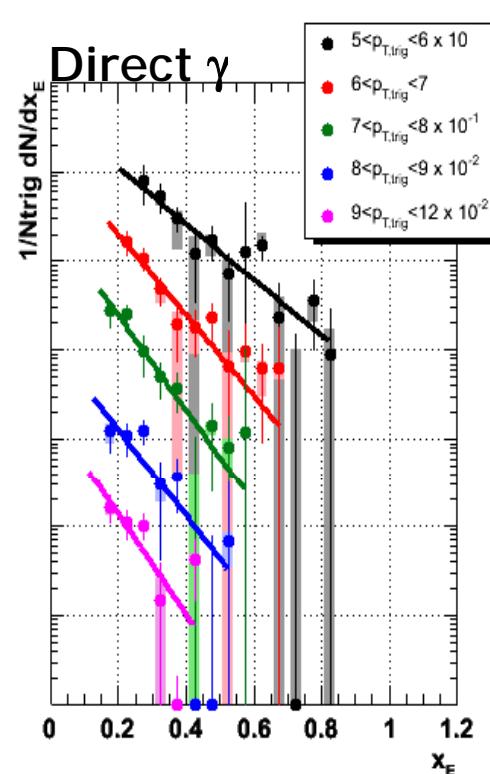
# PHENIX $\sqrt{s}=200$ GeV $\pi^0$ and dir- $\gamma$ assoc. distributions

Matthew Nguyen

APS Spring Meeting, April 15, 2007



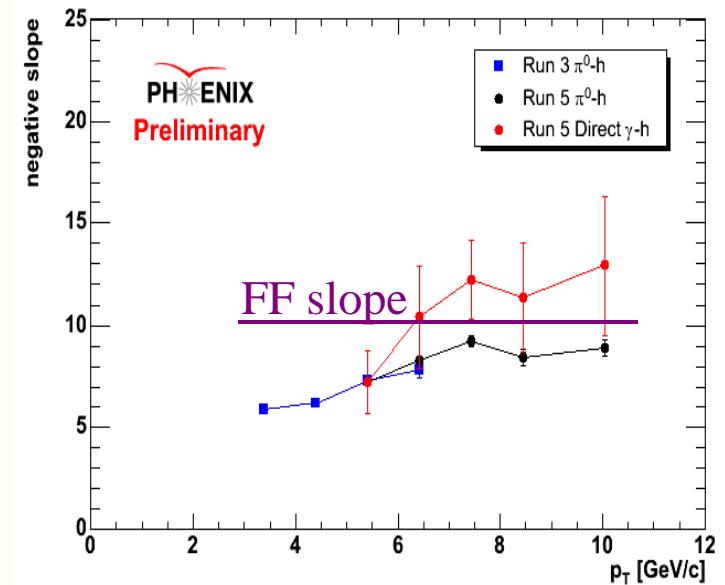
Run 5 p+p @ 200 GeV  
Statistical Subtraction Method



$$x_E = \left| \frac{\frac{r}{p_{Ta}} \cdot \frac{r}{p_{Tt}}}{p_{Tt}^2} \right| = -\frac{p_{Ta}}{p_{Tt}} \cos \Delta\phi \approx -\frac{p_{Ta}}{p_{Tt}}$$

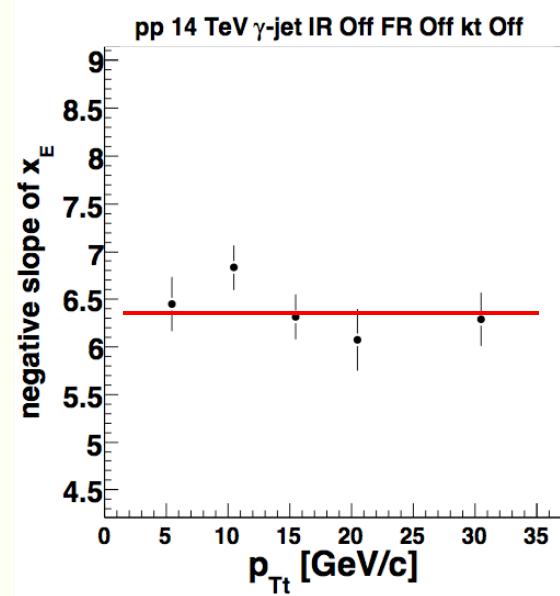
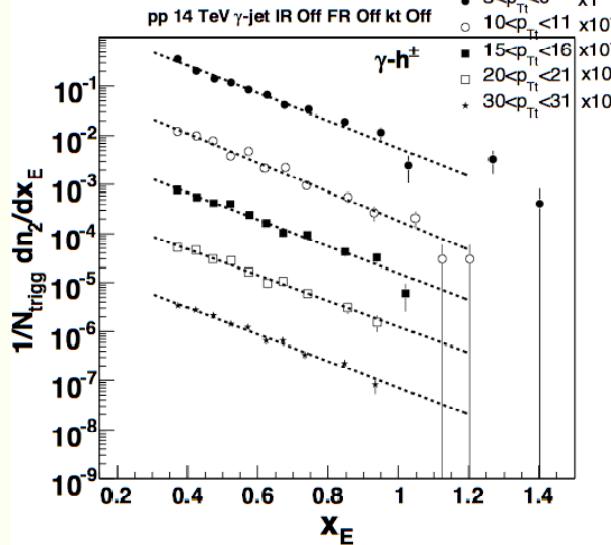
Exponential slopes still vary with trigger  $\gamma p_{T\gamma}$ .

If  $dN/dx_E \propto dN/dz$  then the local slope should be  $p_{T\gamma}$  independent.

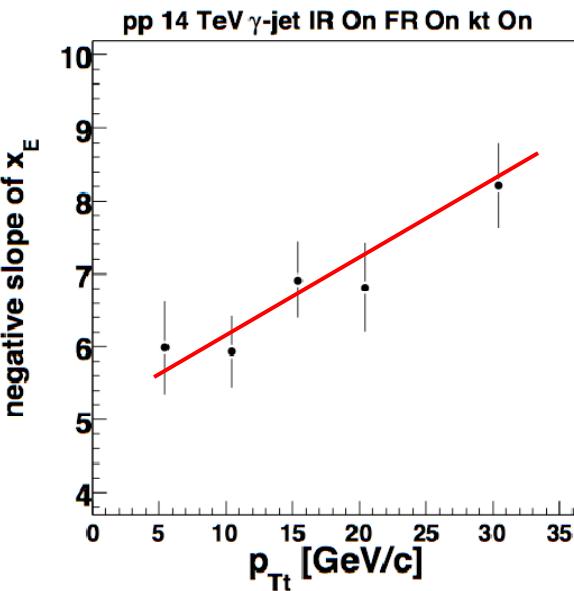
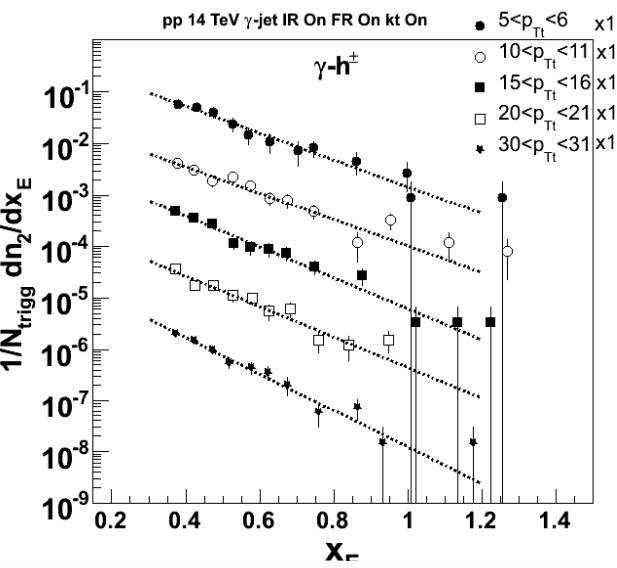


# Pythia Initial/Final st. radiation & $k_T$

Initial/Final state radiation OFF,  $\langle k_T^2 \rangle = 0$  GeV/c

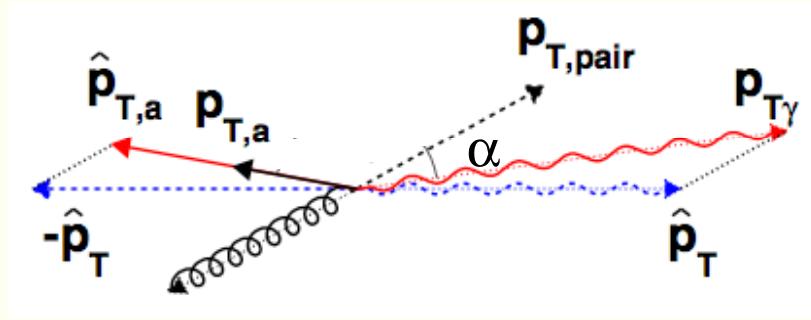


Initial/Final state radiation ON,  $\langle k_T^2 \rangle = 5$  GeV/c



# $\gamma$ -h correlation and $k_T$ bias

Can one extract the Fragmentation Function from  $\gamma$ -h associated distribution despite the  $k_T$  bias?



$$\hat{p}_{Tg} + \hat{p}_{Ta} = 2\hat{p}_T \sqrt{(g^2 - 1)}(\cos a, \sin a) = \hat{p}_{T,pair}$$

$$\hat{p}_{Tg} - \hat{p}_{Ta} = 2\hat{p}_T (1 + (g - 1)\cos^2 a, (g - 1)\sin a \cos a)$$

Solve for net-pair momentum

Lorentz invariance:

$$(\hat{p}_{Tg} + \hat{p}_{Ta})^2 = 2\hat{p}_{Tg}\hat{p}_{Ta} - 2\cos f \hat{p}_{Tg}\hat{p}_{Ta} = 4\hat{p}_T^2$$

the boost:

$$b = \frac{\hat{p}_{T,pair}}{E_{pair}} = \frac{\hat{p}_{T,pair}}{\sqrt{\hat{p}_{T,pair}^2 + 4\hat{p}_T^2}} \quad h = gb \quad \text{and}$$

$$\hat{p}_{Tg} = \pm \hat{p}_T + \frac{\hat{E}_+ \hat{h} \hat{p}_T}{\hat{A} + g} + \hat{p}_T \hat{\tilde{z}}$$

$$\hat{p}_{T,pair}^2 = (\hat{p}_{Tg} + \hat{p}_{Ta})^2 - 4\hat{p}_T^2$$

$$\hat{p}_{T,pair} = \left( \pm (\hat{p}_{Tg} - \hat{p}_{Ta}), \pm 2\sqrt{\hat{p}_{Tg}\hat{p}_{Ta} - \hat{p}_T^2} \right)$$

## $\gamma$ -h correlation and $k_T$ bias

when changing variable  $(\hat{p}_{T,pair,x}, \hat{p}_{T,pair,y})$  to  $(\hat{p}_{Tg,x}, \hat{p}_{Ta,y})$

$$|J| = \frac{\hat{p}_{Tg} + \hat{p}_{Ta}}{\sqrt{\hat{p}_{Tg}\hat{p}_{Ta} - \hat{p}_T^2}}$$

And assuming 2D Gaussian distribution for  $p_{T,pair} \Rightarrow$

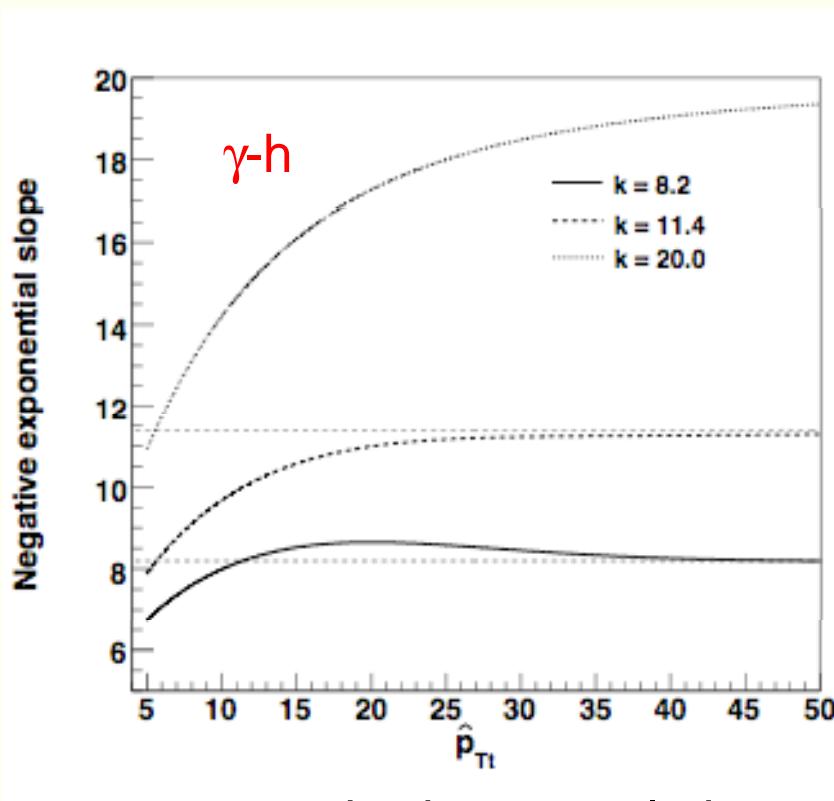
Conditional probability for detecting given photon-associated particle pair:

$$P(\hat{p}_{T\gamma} \& \hat{p}_{Ta}) \Big|_{p_T} = \frac{\hat{p}_{T\gamma} + \hat{p}_{Ta}}{\sqrt{\hat{p}_{T\gamma}\hat{p}_{Ta} - \hat{p}_T^2}} \exp\left(\frac{(\hat{p}_{T\gamma} + \hat{p}_{Ta})^2 - 4\hat{p}_T^2}{\pi \langle k_T^2 \rangle}\right)$$

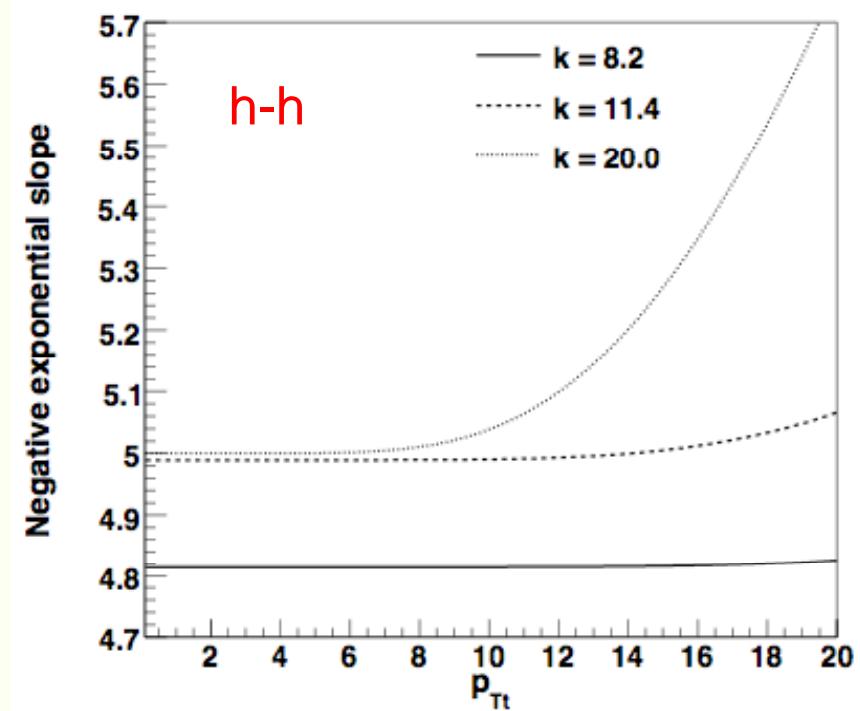
$$\text{provided } \langle k_T^2 \rangle = \frac{2}{\pi} \langle p_{T,pair}^2 \rangle$$

# Folding with a Fragmentation function

$$\left. \frac{dN}{dp_{Ta}} \right|_{pT\gamma} = D(z_t) \otimes \Sigma_Q \left( \frac{p_{Tt}}{z_t} \right) \otimes \int d\hat{p}_{T\gamma} P(\hat{p}_{T\gamma} \& \hat{p}_{Ta}) \Big|_{\hat{p}_T}$$



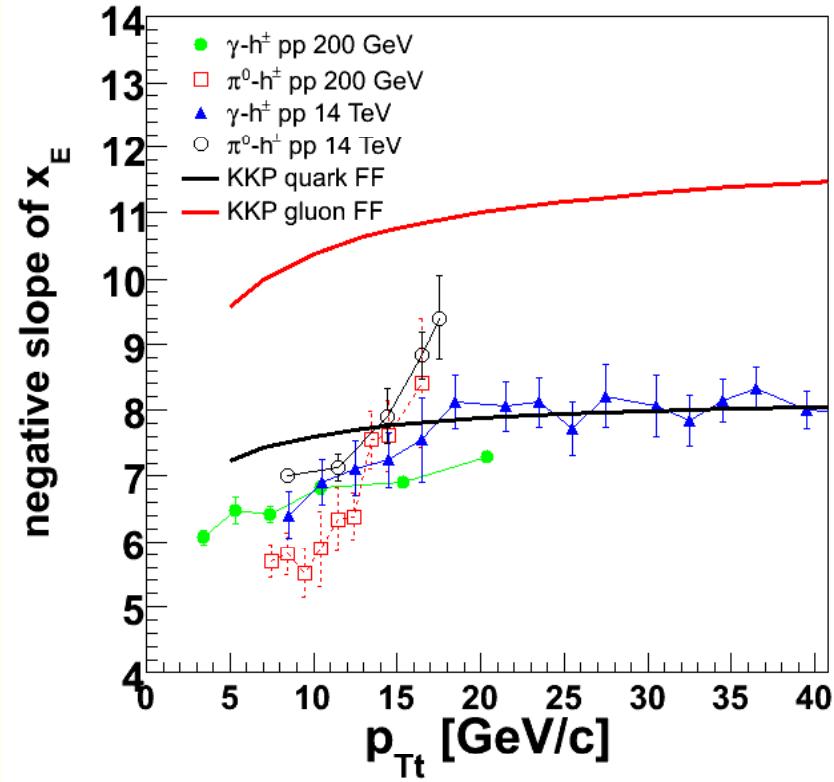
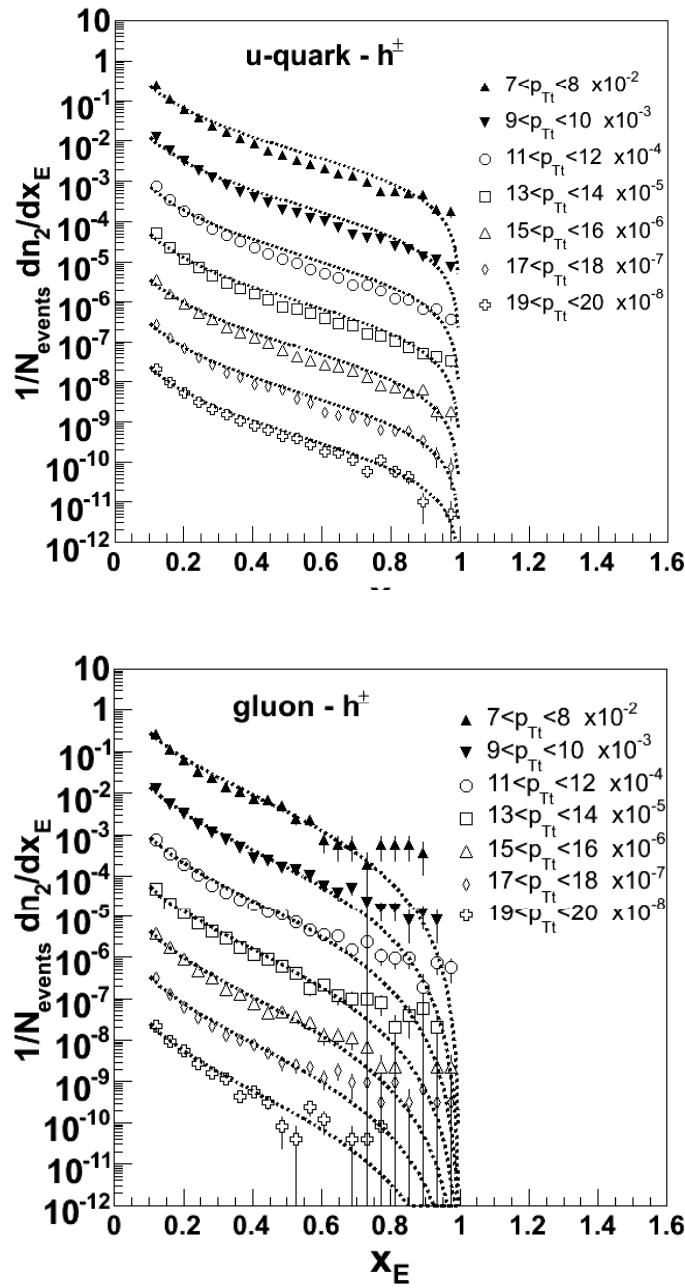
gamma-hadron correlation slopes are converging at high photon-p<sub>T</sub>



Di-hadron correlation slopes is increasing with p<sub>T</sub> as in the data

$\gamma$ -gluon jet (Annihilation) 17 %

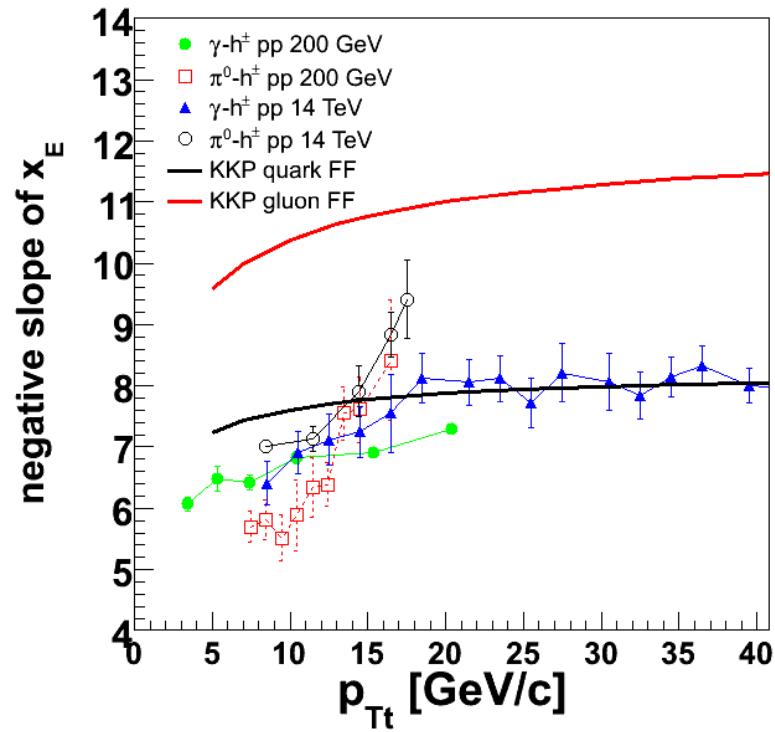
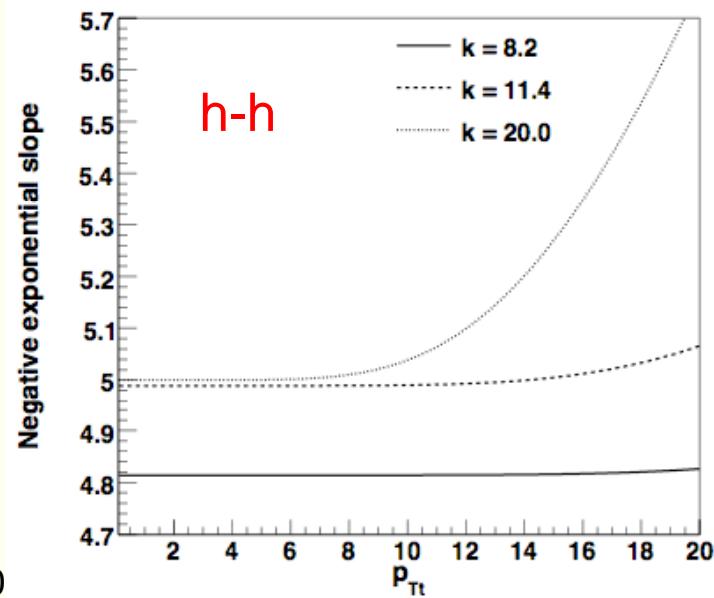
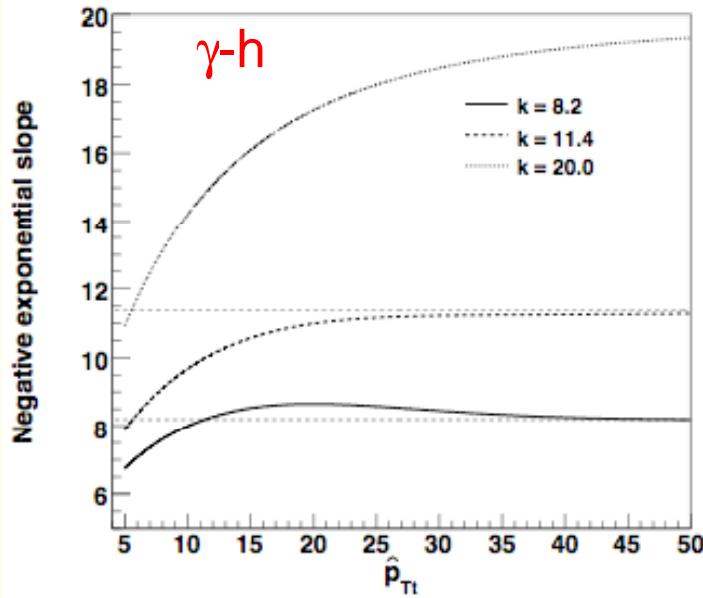
$\gamma$ -u quark jet (Compton) 66 %



points are p+p 14 TeV PYTHIA xE distribution,  
dashed line KKP FF parameterization

Nucl. Phys., 2001, B597, 337-369

# Comparison to PYTHIA



When we used PYTHIA FF slopes for quarks and gluons we got quite satisfactory agreement.

# Summary

I discussed: di-hadron and direct photon-h correlations - base line measurement for nuclear modification study:

- $k_T$  and initial/final state QCD radiation, resummation vs NLO
- $j_T$  near-side jet shape modifications
- fragmentation function - can be measured using jets - not from the first data.  
Despite our expectation FF is not accessible in di-hadron correlations. FF can be extracted from direct photons correlation only at relatively small trigger-photon momenta.
- $k_T$ -bias still present - pushes the minimum photon-trigger  $p_T$  above 10 GeV/c at RHIC and 20 GeV/c at LHC.

What we need is just data...

# Final state parton dist and FF from data of pQCD?

Effective FS parton distribution:

$$\text{in LO: } \Sigma_Q(\ddot{\vec{p}}_T) = \frac{d^3\sigma}{d\ddot{\vec{p}}_T^2 dy_1 dy_2} = \frac{1}{s^2} \sum_{ab} f_p(x_1) f_p(x_2) \frac{\pi \alpha_s^2(Q^2)}{x_1 x_2} \Sigma^{ab}(\cos \theta^*)$$

or

$$\Sigma_Q(\ddot{\vec{p}}_T) \propto \ddot{\vec{p}}_T^{-n}$$

$\pi^0$  invariant cross section provides a constrain:

$$\text{if } \frac{1}{\hat{p}_T} \frac{d\hat{\sigma}}{d\hat{p}_T} \propto \hat{p}_T^{-n} \text{ than } \frac{1}{p_T^\pi} \frac{d\sigma_\pi}{dp_T^\pi}; \frac{1}{p_T^\pi} \int_{xT}^1 A \cdot D^\pi(z) \cdot \left( \frac{p_T^\pi}{z} \right)^{-n+1} \frac{dz}{z^2}; \frac{A}{(p_T^\pi)^n} \int_{xT}^1 D^\pi(z) \cdot z^{n-3} dz \Rightarrow$$

$$\Rightarrow \text{partons } \frac{1}{\hat{p}_T} \frac{d\hat{\sigma}}{d\hat{p}_T} \propto \frac{1}{p_T^\pi} \frac{d\sigma_\pi}{dp_T^\pi} \propto \frac{1}{(p_T^\pi)^8} \Rightarrow \Sigma_Q(\hat{p}_T) \propto \frac{1}{\hat{p}_T^8} \text{ same power law}$$

Fragmentation function:

$$D^\pi(z) \equiv \sum_i \int_x^1 \frac{dz}{z} C_i(s; z, \alpha_s) D_i^\pi(x/z, s) \text{ or}$$

$$D^\pi(z) = z^\alpha \cdot (1-z)^\beta \cdot (1+z)^\gamma \text{ where } \alpha, \beta, \gamma \text{ from LEP or fit}$$