

# Lattice chirality and the decoupling of mirror fermions

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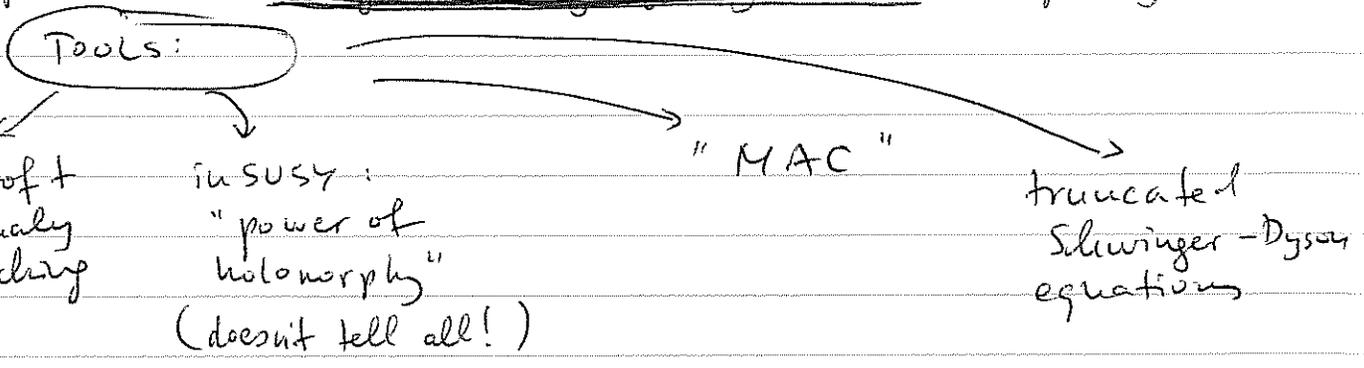
collaborators:

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Since this is an LHC-"BSM" Workshop -  
- what connection is there?

- # weak-coupling physics @ or  $\sim$  TeV is currently most popular scenario for LHC physics -  
- many arguments - EWPT, for one
- # one should keep in mind, however, that the kinds of strong-coupling gauge dynamics we understand - from experiment or theory or numerics - are limited; while QCD-like versions of TC are out by S(T--), other kinds of strong dynamics may be OK

In particular, strong chiral gauge dynamics is poorly understood.



tools you can trust

and now you don't know if you're right until confirmed by other

⇒ not very much there!

# Recent "AdS/CFT/QCD" approaches use large- $N$ ; this is not too useful for chiral theories:

$SU(5) \ 5^*, 10 \Rightarrow SU(N) \ w/ \ \square \ \& \ N-4 \ \square$

but different symmetry realizations than  $SU(5)$  case (see e.g. SUSY case)

Basically matter reps grow w/  $N$  & "quark" loops are not suppressed → "mesons" are not free at leading order, etc. ---

# Finally, SM is a chiral gauge theory. Do we have a nonperturbative formulation? (Does the SM "exist"?)

So, maybe we should not forget that lattice has been (still is) best approach to calculate many things in QCD (in addition to defining it). Especially spectra and various matrix elements.

Can we apply similar methods to non-QCD theories?

- \* chiral → ?
- \* supersymmetric

- ↓
- some progress of late mostly in lower dim. superrenormalizable cases (see Giedt's review)
- $N=4$ , 4d - where chiral symm. matters (global or gauge) - open.

This talk is about chiral lattice theories.

\* NOT about QCD physics via chiral lattice gauge theory  
(and hence, will not discuss a potential theory of nature)

\*\* Rather, I'd like to tell you where the lattice gauge theory problem is at and what attempts are being made at improvement & progress

\*\*\* Hope to convince you that's an interesting problem, theoretically appealing, and fun to think about — and may even be useful!

(many tools come together — theoretical & numerical "experiment")  
[non-technical, hopefully — for non-experts]

The particular approach I'd like to discuss is a combination of "old" and "new" (20 & 8 yrs, respectively).

(I'll put it in perspective later.)

① = vectorlike gauge theories (like QCD) w/ no SUSY are no problem to deal with on the lattice

② = there are doublers, of course, and one has to deal with them, but there are ways (also- later)

③ = So, a natural question one might ask is:

Can one start from a v-like theory, e.g. 4d

$SU(5)$  w/  $5^*$   $5$  all L Weyl fermions

$\underbrace{10}_{\text{"light"}}$ 
 $\underbrace{10^*}_{\text{"mirror"}}$

simply names  $\rightarrow$

and then deform the theory in a way that only affects "mirrors", in such a way that they become heavy, without breaking  $SU(5)$ ?

— why we do that?

- current situation w/ chiral lattice gauge theories is that:

(exactly gauge invt.)

- existence of local action + measure has been proven for  $U(1)$  theories w/ anomaly-free content
- outside of perturbation theory, there is no explicit construction of measure - so while theoretically fascinating, not of practical use for calculations
- hence, attempts to decouple mirrors from v-like theories - which can be formulated explicitly - using new "stuff" are perhaps worthwhile
- e.g. Bhattacharya, Csaki, Martin, Shimmer, Terning '05 using warped domain walls -
- very cute, but never completely latticized...
- $\neq$  messy symmetry realization (limits only)

(5)

— so while working on realizing BCMST in 2d deconstructed setting, started thinking of other ways (in frustration)

Back to our question about decoupling mirrors w/out breaking SUS) ---

-- a "normal" field theorist would say: NO!

-- but lattice affords possibilities that continuum people rarely think of (and they should not!)

— for example, everybody knows that 4-fermi interactions, if taken strong enough, [e.g.  $\frac{g}{\Lambda^2} (\bar{\psi}\psi)(\bar{\psi}\psi)$ ,  $g > 8\pi^2/\mathcal{N}$  from gap eqn] break chiral symmetries

— few continuum people know, however, that if one takes  $g$  even stronger ( $g \rightarrow \infty$  <sup>in the</sup> limit) the symmetries can get restored and the theory enters "strong-coupling symmetric phase"

Why have most people not heard about it?

— because these phases are a "lattice artifact"

let me explain ---



Include mirror  $\chi$ -fermi interactions

$$10^* - 5 - 5 - 1 \sim g_1 \phi \cdot \rho^{ij} \chi_i \cdot \chi_j$$

$$10^* 10^* 10^* 5 \sim g_2 \rho^{ij} \cdot \rho^{kl} \cdot \rho^{mn} \cdot \epsilon_{ijklmn} \chi_n$$

Then, they said, taking  $g_1, g_2 \rightarrow \infty$  (in lattice units), in Euclidean path  $\int$  on lattice makes sense, one can neglect kinetic terms (i.e. do an  $1/g_{1,2}$  "hopping" expansion) & study spectrum ---

-- result can be told in words --

-- there are "bound states" of the mirror fermions that take part in strong  $\chi$ -fermi, which can get mass w/out  $SU(5)$  breaking --

for example  $10^* - 5 - 5$  fermion can ~~break~~ w/  $1$   $(\phi)$  <sup>get Dirac mass</sup>  
 $SU(5)$  singlet

or a  $(10^*)^5 \sim 5^*$  can ~~break~~ w/  $5$   $(\chi)$  <sup>get Dirac mass</sup>

w/out breaking  $SU(5)$  !

"strong-coupling symmetric phase"  
( $\leftarrow g_{1,2} \rightarrow \infty$ )

A flavor of how it works -

- pedagogical  $\leftarrow$  toy model w/  $SU(4)$  global, ignore  $\rightarrow$  spins

$$H_{4\text{-fermi}} = \sum_x (g \psi_a \psi_b \psi_c \psi_d \in^{abcd} + \text{h.c.})$$

$g \gg 1$ , 4-fermi  $H$  dominates hopping

$\{\psi_a(x), \psi_b(y)\} = \delta_{ab} \delta_{xy}$  - in Hamiltonian (spatial lattice, continuous time)

$$H = \sum_x H_{0x} + H_1$$

$\downarrow$  hopping  $\bar{\psi} \triangleright \psi$  ---

4-fermi @  $x$

rename  $\psi_{ax} \rightarrow a_a$   
 $\psi_{ax}^+ \rightarrow a_a^+$

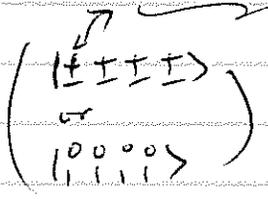
$$H_{0x} = g(\epsilon^{abcd} a_a a_b a_c a_d + \text{h.c.})$$

dup  $\uparrow$

$\forall$  site, a 4-fermion  $H$ , conserves  $F \pmod 4$

$2^4 = 16$  states

decompose  $\mathbb{1}_x + \mathbb{1}_x + 4_x + 6_x + 4_x^*$   
 $\downarrow \quad \downarrow$   
 $10000_x \quad 11111_x$   
etc. - under  $SU(4)$



$H_{0x}$  only connects these

→ the 4, 6, 4 have zero energy

→  $10000 > -11111 >$  has  $-g$

$10000 > +11111 >$  has  $+g$

So, in  $g \rightarrow \infty$  limit ground state at  $\forall x$

- is
- (a) unique
  - (b)  $SU(5)$  singlet

Turn on hopping?

$$H = \begin{pmatrix} -g & & & & \\ & -g & & & \\ & & -g & & \\ & & & \ddots & \\ & & & & -g \end{pmatrix} + \underbrace{\begin{pmatrix} 0 & 1 & & & \\ 1 & 0 & 1 & & \\ & 1 & 0 & 1 & \\ & & 1 & 0 & 1 \\ 1 & & & 1 & 0 \end{pmatrix}}$$

↑  
(in lowest state space  $\forall x$ )

hopping, small  
causes  $SU(4)$  singlet  
states to propagate,  
"mass"  $\sim \left(g \times \frac{1}{a}\right)$

moreover, hopping expansion  
is convergent (= strong coupling  
expansion); ~~not~~ so as long as  $g$  is sufficiently large,  
<sup>very</sup> (naively  $g > 2d$ ) this should represent the true  
spectrum of the theory.

Gives an idea.  $SU(5)$  of E-P has more  
complicated group theory -- singlet was needed to

have "static limit", b/c.

- So "dream" is that mirrors get heavy in this manner. Gauge field is in hopping terms and should be only weakly affected

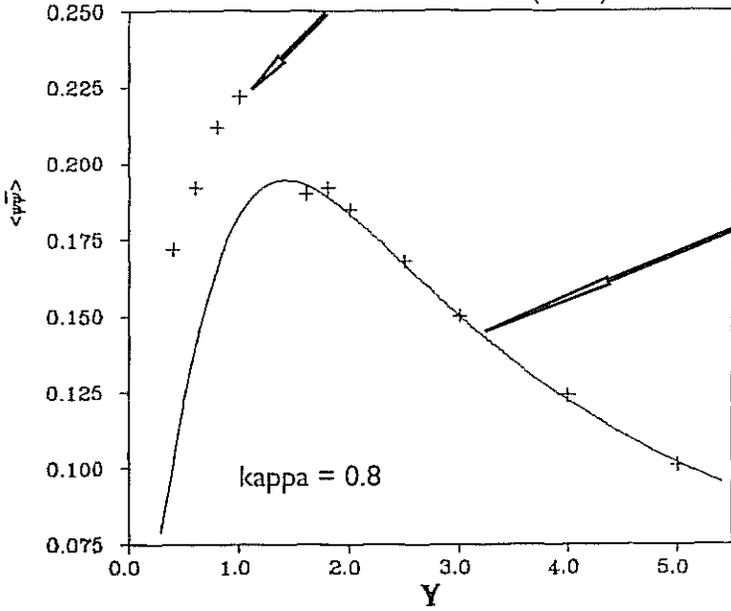
$$\left( \frac{1}{g_0^2} + \frac{1}{g^2} + \dots \right) \text{tr } U_{\square} + \frac{1}{g^2} (\text{tr } U_{\square})^2 + \dots$$

$\uparrow$  ~~the~~ gauge couplings  
 $\uparrow$  contribution from hopping, small at  $g \rightarrow \infty$

Before continuing, for the still disbelieving, some older results in Yukawa models:

↳ p. 11

A. Hasenfratz, T. Neuhaus, MC simulation  
8x8x8x16 lattice (1988)



M. Stephanov, M. Tsypin  
mean field(1/d) calculation (1989)

a 4d Z2-chiral invariant Yukawa-Higgs  
lattice theory, naive fermions

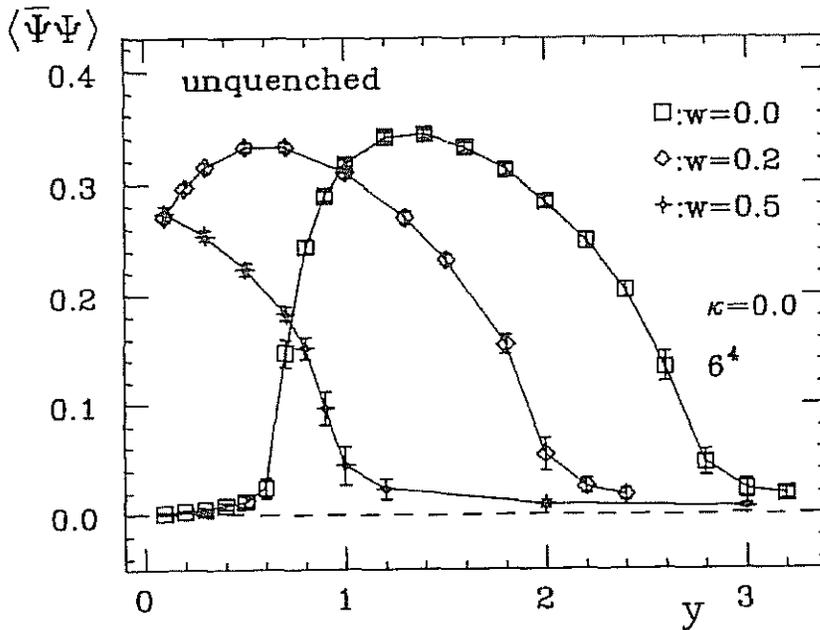
$$S_B = -2k \sum_{x,\mu} \phi_x \phi_{x+\mu},$$

$$S_F = \frac{1}{2} \sum_{x,\mu} \bar{\psi}_x \gamma_\mu (\psi_{x+\mu} - \psi_{x-\mu}) \equiv \sum_{x,\mu} \bar{\psi}_x K_{xy} \psi_y$$

$$S_Y = \sum_x Y \phi_x \bar{\psi}_x \psi_x, \quad \mu=1, 2, \dots, d, \quad \phi_x = \pm 1.$$

W. Bock, A. De, K. Jansen, J. Jersak, T. Neuhaus, J. Smit (1990)

6x6x6x6 lattice



a 4d SU(2)\_L x SU(2)\_R model with naive fermions  
(or Wilson, w>0, - explicitly broken chiral symmetry)

-- DID "E-P dream" come true?

↳ NOT back then. Why?

- because, on the lattice, pre-1998, there was no way to separate light from mirror components of fermions  $\mathbf{5}$  in other words, the 4-fermi interactions of the mirrors  $\sim 10^* 55 1$ ,  $(10^*)^3 5$  are supposed to break all global symmetries of mirror sector & preserve those of light fermions.

But there was no notion of exact chiral symmetry on the lattice  
(w/out doublers, that is)

- So light  $\mathbf{5}^*$ ,  $\mathbf{10}$  also "felt" strong interactions; all fermions (L & M) acquired mass in the end.

What's changed?



(A) Most importantly,  $\forall a$  - lattice spacing  
an exact <sup>(chiral)</sup> symmetry can be defined.

• Not the usual  $\gamma_5$  for all modes, but reduces to it for low-lying ones.

• Exact symmetry of <sup>lattice</sup> action

• Anomaly - through non-invariance of measure, à la Fujikawa (also @  $\forall a$ !)

}	Ginsparg-Wilson '82
	Kaplan '92
	Narayanan, Neuberger
	P. Hasenfratz, Niedermeyer
	Neuberger
Lüscher '98	

(B)  $\Psi_x$ -Dirac  $\Rightarrow \Psi_{L_x}, \Psi_{R_x}$  : Weyl components

$$\bar{\Psi} D \Psi = \bar{\Psi}_L D \Psi_L + \bar{\Psi}_R D \Psi_R$$

as in continuum!

‡

without doublers!

So, then why can't we try to formulate a theory w/ the goal of decoupling mirrors?

(Bhattacharya, Martin, E.P. '2006)

$\hookrightarrow$  we certainly can.

\* Light - Major separation is exact

\* all light chiral <sup>(global)</sup> symmetries, and their anomalies, are as in desired

target chiral gauge theories; all is gauge int.

\* measure is explicitly defined (usual  $\nu$ -like one)  
(& so is action)

- one can stop and say "but we're done!"

- based on symmetry "what else can it be?"

why do we still dwell on it?

- because I didn't tell all --

$\psi_x$ : Dirac is a local field  $\rightarrow \psi_{L_x}$  is not exactly local, but smeared  $\rightarrow$

$$\psi_{L_x} = C_{xx'} \psi_{x'}$$

$$\left( \text{w/ } C_{xx'} \sim e^{-\frac{|x-x'|}{a}} \right)$$

(is also gauge-dependent)

①# So, with there slightly nonlocal Yukawa / 4-fermi interactions of the mirrors, is it still true that a strong-coupling symmetric phase happens? Are the mirrors heavy?

②# In typical models there's more than one strong Yukawa / 4-fermi mirror coupling - (like in SU(5) there were 2) - needed to break all mirror global symmetries [else there'd be x-tra <sup>(mirror)</sup> instanton 0-modes, for one!]

& there can be a nontrivial phase structure as their ratios change

- these questions can be addressed w/out gauge fields; need numerics, really! ~~stress!~~ adding these brings more interesting questions:

③# What if one tries to decouple an anomalous mirror rep?

④# Unitarity, w/ gauge fields - can be addressed at small gauge, large Yukawa-4 fermi, ? (-perhaps also need numerics-)

Where are we?

① ( & ② ) do strong-coupling symmetric phases exist?  
(w/ these slightly nonlocal interactions)

→ phase structure? (p. 17)

- YES! Geedt, E.P. '0701 - 2d, 2 couplings  
Gerhold, Jansen '0706 - 4d, different structure (1 coupling) & motivation (p. 18)  
→ (non trivial phase structure - yes, but not too complex)

are mirrors heavy?

- in 2d model - YES (p. 18)  
- but mirror rep is anomalous -- how come?

③ -- on anomalies --

Y. Shao, E.P. '0706 -- studied how splitting of  $\mathbb{Z}$  <sup>show Yukawa /  $\psi$ -fermi here</sup>

$$\mathbb{Z}(\text{light}; \text{mirror}; A_\mu) = \mathbb{Z}(\text{light } A_\mu) \times \mathbb{Z}(\text{mirror}, A_\mu)$$

is affected by anomaly -- turns out that the splitting is a singular function of  $\{A_\mu\}$  if <sup>mirror/light</sup> rep. is anomalous

toy 2d Yukawa-unitary Higgs model with Ginsparg-Wilson fermions

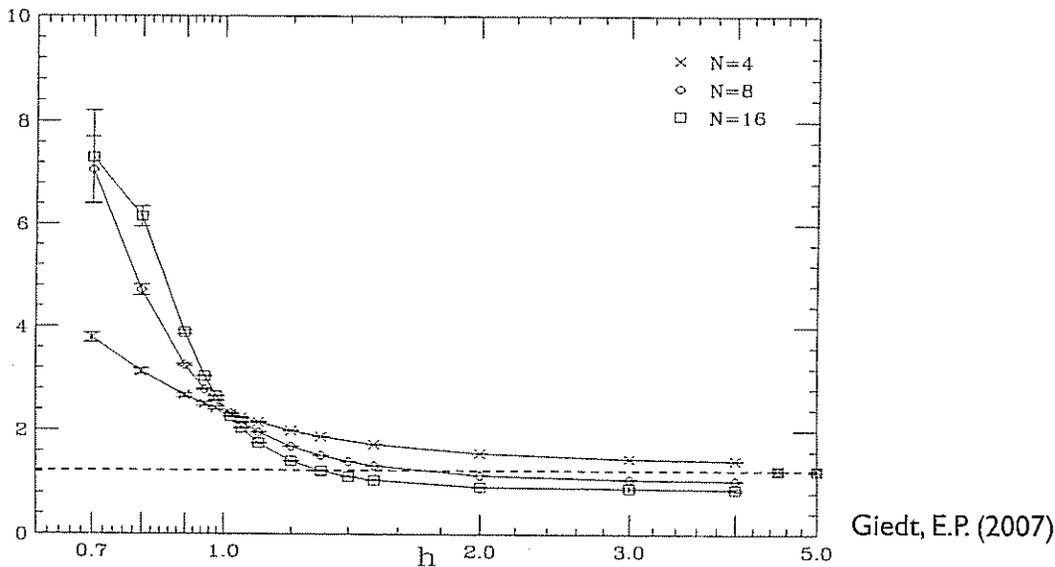
exact chiral symmetry, zero gauge fields in simulation:

$$\begin{aligned}
 S &= S_{light} + S_{mirror} \\
 S_{light} &= (\bar{\psi}_+, D_1 \psi_+) + (\bar{\chi}_-, D_0 \chi_-) \\
 S_{mirror} &= (\bar{\psi}_-, D_1 \psi_-) + (\bar{\chi}_+, D_0 \chi_+) \\
 &+ y \{ (\bar{\psi}_-, \phi^* \chi_+) + (\bar{\chi}_+, \phi \psi_-) + h [ (\psi_-^T, \phi \gamma_2 \chi_+) - (\bar{\chi}_+, \gamma_2 \phi^* \bar{\psi}_-^T) ] \} \\
 S_\kappa &= \frac{\kappa}{2} \sum_x \sum_{\hat{\mu}} [ 2 - ( \phi_x^* U_{x, x+\hat{\mu}} \phi_{x+\hat{\mu}} + \text{h.c.} ) ]
 \end{aligned}$$

all simulations done at infinite  $y$   
 (for economical reasons!) i.e. dropping mirror kinetic terms

2d Yukawa-unitary Higgs model with Ginsparg-Wilson fermions,

- scalar susceptibility at infinite  $y$ , as function of  $h$ ,  $\kappa=0.1$   
 (~ inverse "mass squared" of scalar in lattice units)



also measured other order parameters:

3-inder cumulant, fermion composite -"Dirac" and "Majorana"- susceptibilities, and vortex density -  
 all show similar behavior as a function of  $h$ , no indication of long-range correlations for  $h > 1$

--- strong coupling symmetric phase exists

--- strong coupling symmetric phase exists also in at least one 4d model:  
 SU(2)<sub>L</sub> × SU(2)<sub>R</sub> chirally invariant Yukawa-Higgs model with Ginsparg-Wilson fermions

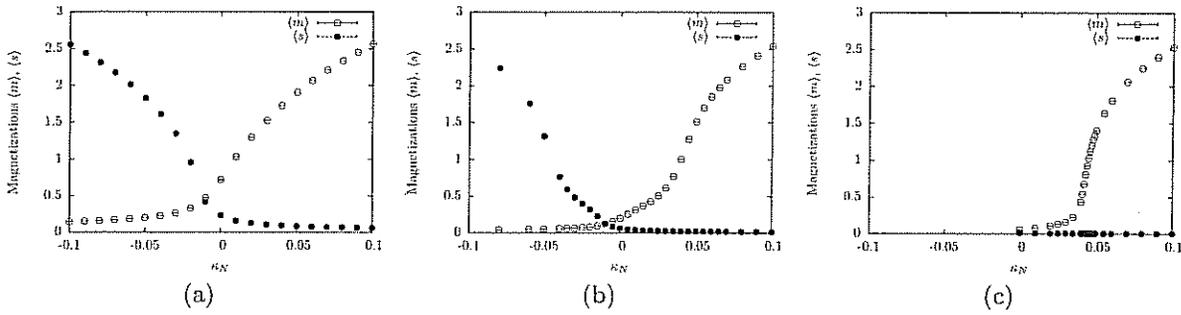
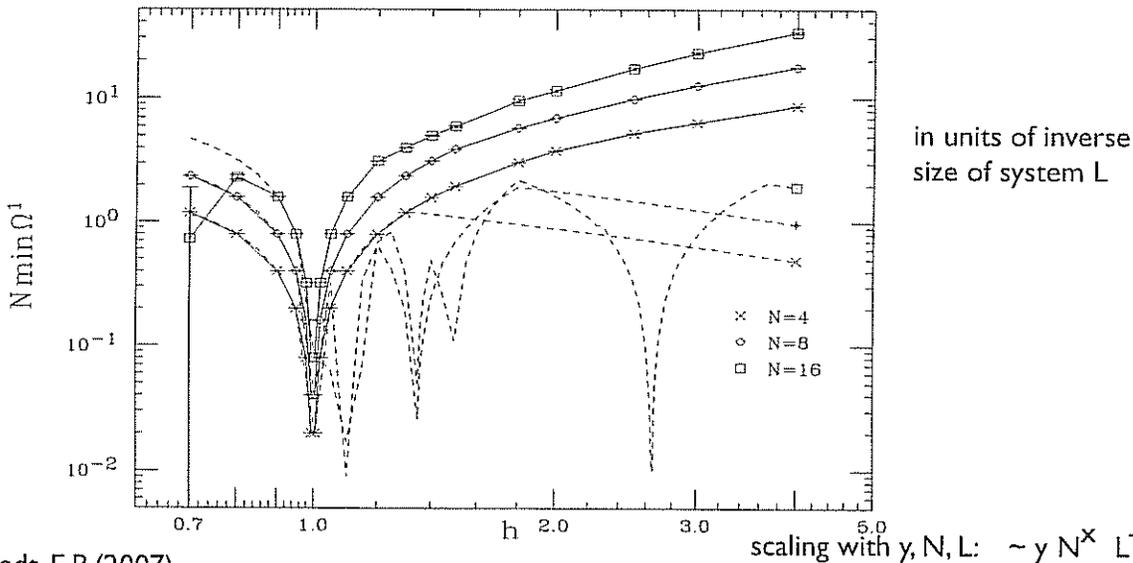


FIG. 8: The behaviour of the average magnetization  $\langle m \rangle$  and staggered magnetization  $\langle s \rangle$  as a function of  $\kappa_N$  on a 4<sup>4</sup>- (a), 8<sup>4</sup>- (b) and 16<sup>4</sup>-lattice (c). In the plots we have chosen  $\bar{y}_N = 30$ ,  $\bar{\lambda}_N = 0.1$  and  $N_f = 2$ .

P. Gerhold, K. Jansen (2007)  
 (different motivation, single Yukawa coupling- only Dirac, no Majorana)

2d Yukawa-unitary Higgs model with Ginsparg-Wilson fermions  
 · lowest, wrt momentum, inverse eigenvalue of the L-R components of mirror fermion Green's function  
 [zero-massless pole; dotted lines: "broken" phase values, where perturbation theory good, agrees with MC]



Giedt, E.P. (2007)

scaling with  $y, N, L$ :  $\sim y N^x L^{-1}$   
 (values of exponent  $x$  depend weakly on  $\kappa, h$ , but  $x$  is usually about 1)

--- mirrors heavy ... but... more to come - simulation with dynamical gauge fields

v.v. if rep is anomaly-free, the splitting is smooth. Used results of Neuberger, Lüscher (~00)

+ our own "splitting theorem" to show that (it is!)

$Z(\text{minor}, A_\mu)$  is a smooth func if { anomaly free.

Simulation used to find minor spectrum <sup>(in 2d toy model)</sup> used

this singular minor-light split (it only simulated minor @  $A_\mu=0$ ; however  $\frac{\partial Z_{\text{minor}}}{\partial A_\mu} \Big|_{A_\mu=0} = \infty$ ) ---

--- need: (a) simulation w/ gauge fields in, + light  $\Rightarrow$  TBA & study of (b) ? unitarity? w/  $A_\mu$  dynamical - dumm... (vs. anomaly freedom?)  $\hookrightarrow$  recall Jadhav/Rajaraman!

Will all this be useful?

given that heavy minors, unbroken symmetry, unitarity work

most practical issue: how badly does phase of  $Z$  fluctuate? (can it be included in observable man MC simulation - one of not too fluctuating, bad if so...)

only one data point:

in 2d model,  $Z_{\text{minor}}$  @  $y \rightarrow \infty$  and  $A_\mu=0$  is real --- [hope?] -- at  $y < \infty$  and  $A_\mu=0$  - small phase fluctuations