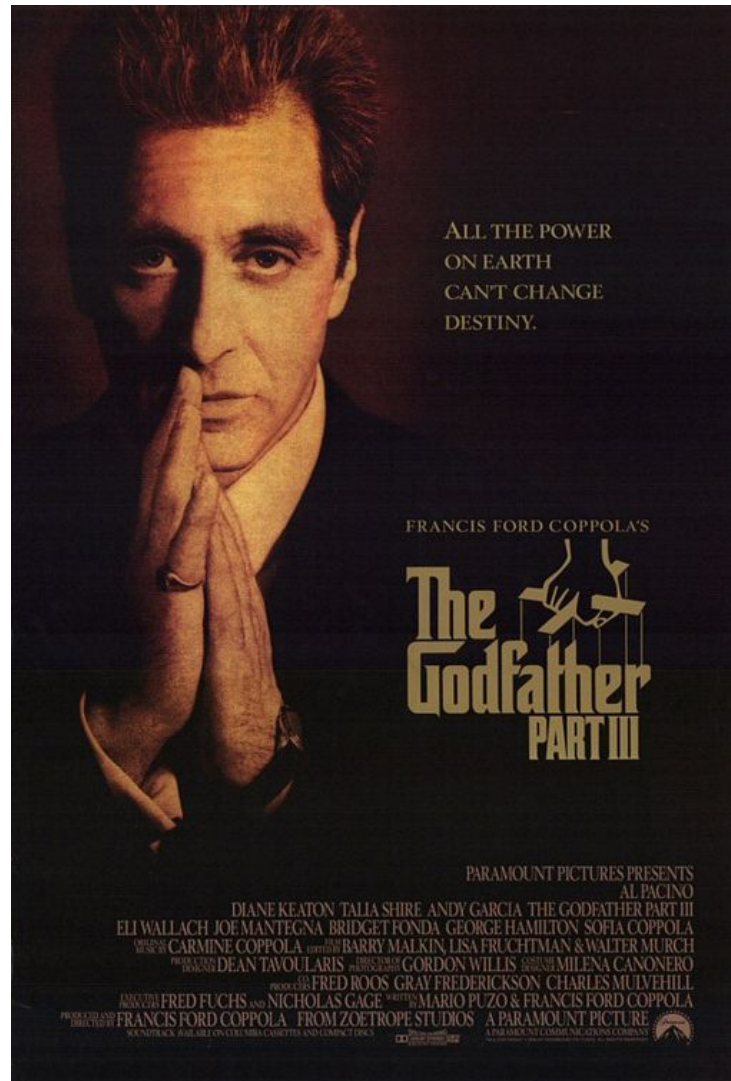


(Extra)Ordinary Gauge Mediation

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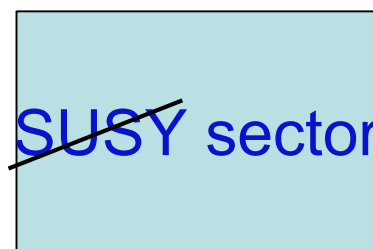
Based on: [hep-th/0703196](https://arxiv.org/abs/hep-th/0703196)
and [work in progress](#) with
Clifford Cheung and Liam Fitzpatrick



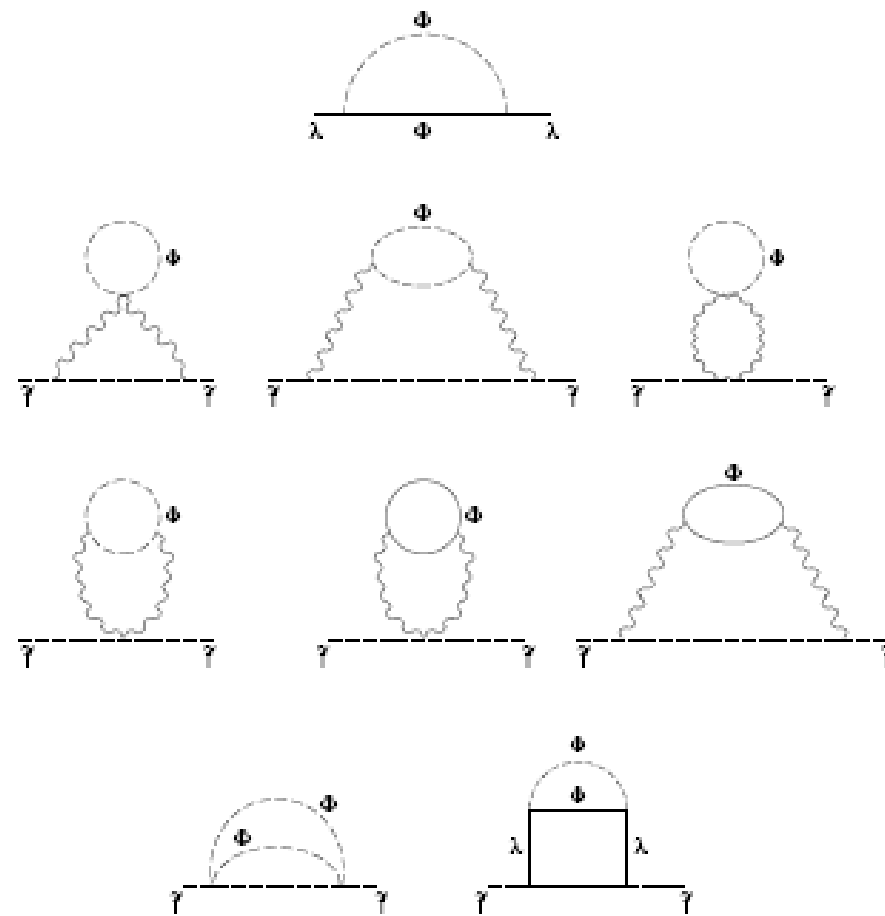


Gauge Mediation

Dine, Fischler; Alvarez-Gaume, Claudson, Wise; Nappi, Ovrut; Dimopoulos, Raby;
Dine, Nelson, et al.
(cf. review by Giudice & Rattazzi)



Gauge interactions



“Ordinary” Gauge Mediation

- Gauge mediation is a successful theory of the soft masses. It has many attractive features:
 - flavor blindness
 - calculability
 - predictivity
 - distinctive phenomenology
- In most phenomenological studies, what is usually assumed is some form of “ordinary” or “minimal” gauge mediation:

$$\begin{aligned}
 W &= \sum_{i=1}^N \lambda_i X \phi_i \tilde{\phi}_i & \longrightarrow & & M_r &= \frac{\alpha_r}{4\pi} \Lambda_G, & \Lambda_G &= \frac{NF}{M} \\
 \langle X \rangle &= M + \theta^2 F & & & m_{\tilde{f}}^2 &= 2 \sum_{r=1}^3 C_{\tilde{f}}^{rr} \left(\frac{\alpha_r}{4\pi} \right)^2 \Lambda_S^2, & \Lambda_S^2 &= \frac{NF^2}{M^2}
 \end{aligned}$$

$$\phi_i \in \mathbf{5}, \tilde{\phi}_i \in \bar{\mathbf{5}} \text{ of } SU(5)$$

(other representations are also possible, of course)

$$m_{\tilde{f}}^2 \sim M_r^2 \sim \left(\frac{\alpha}{4\pi} \frac{F}{M} \right)^2$$

(Giudice & Rattazzi; valid to leading order in F/M)

Features of OGM

$$M_r = \frac{\alpha_r}{4\pi} \Lambda_G, \quad \Lambda_G = \frac{NF}{M}; \quad m_{\tilde{f}}^2 = 2 \sum_{r=1}^3 C_{\tilde{f}}^r \left(\frac{\alpha_r}{4\pi} \right)^2 \Lambda_S^2, \quad \Lambda_S^2 = \frac{NF^2}{M^2}$$

(Applies at the messenger scale! Still need to run down to weak scale with MSSM RGEs!)

- Colored particles (squarks, gluinos) are **heavier** than uncolored particles (sleptons, wino, bino).
- The scale of the sfermion masses relative to the gaugino masses is controlled by the **messenger number**

$$N = \Lambda_G^2 / \Lambda_S^2$$

The sfermions become lighter than the gauginos as N increases.

- Perturbativity up to the GUT scale bounds the messenger number

$$\frac{150}{\log(M_{GUT}/M)} \sim 5 \gtrsim N \geq 1$$

Features of OGM

$$M_r = \frac{\alpha_r}{4\pi} \Lambda_G, \quad \Lambda_G = \frac{NF}{M}; \quad m_{\tilde{f}}^2 = 2 \sum_{r=1}^3 C_{\tilde{f}}^r \left(\frac{\alpha_r}{4\pi} \right)^2 \Lambda_S^2, \quad \Lambda_S^2 = \frac{NF^2}{M^2}$$

(Applies at the messenger scale! Still need to run down to weak scale with MSSM RGEs!)

- Gravitino LSP always!

$$m_{3/2} \sim \left(\frac{\Lambda_G}{100 \text{ TeV}} \right) \left(\frac{M}{100 \text{ TeV}} \right) \times 2.4 \text{ eV}$$

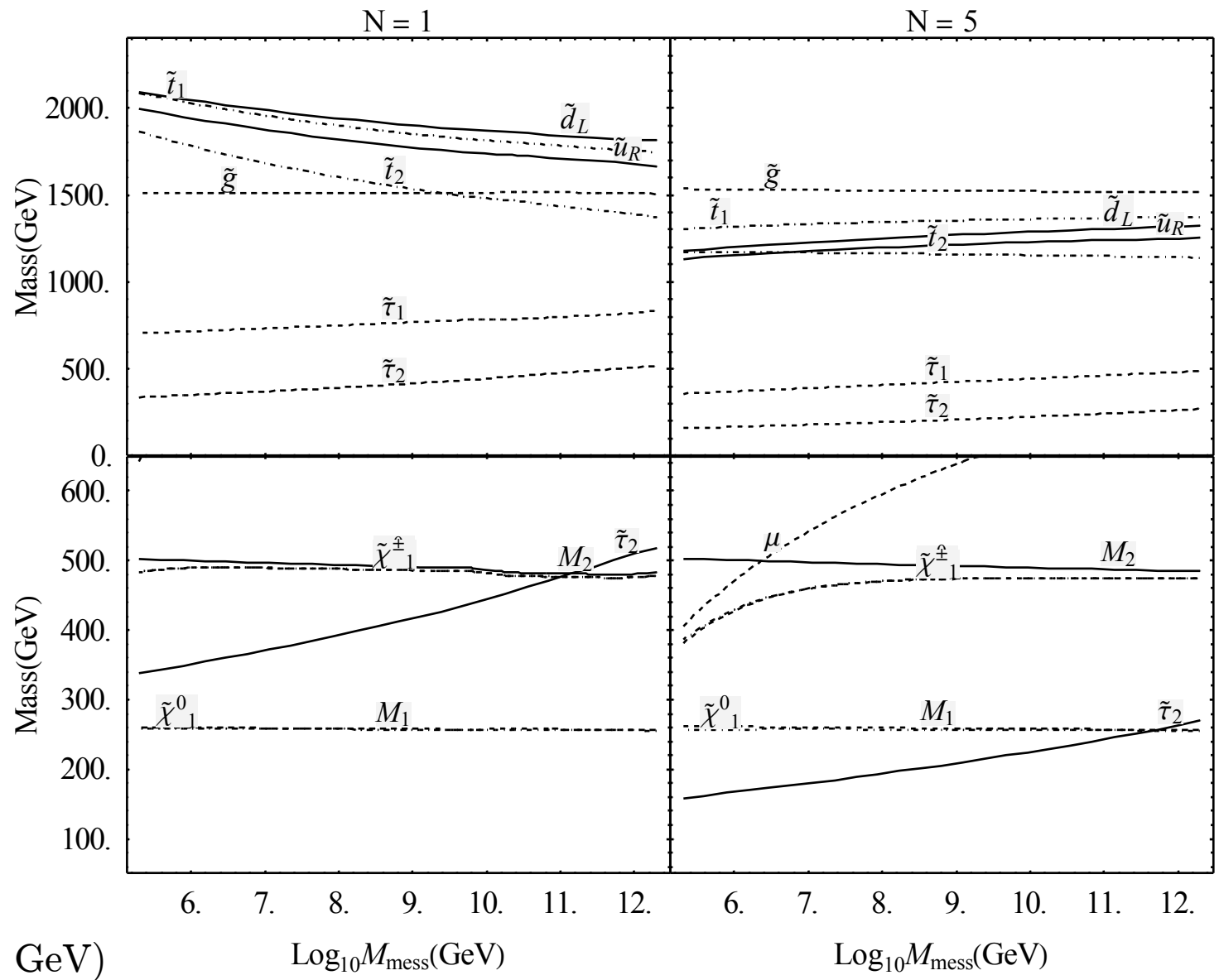
- NLSP is either **bino-like neutralino** (small N) or **stau** (large N)
- LEP bound on Higgs mass, $m_h \geq 114.4 \text{ GeV}$ means squark masses and μ are always large,

$$\mu \gtrsim 0.5 - 1 \text{ TeV}, \quad m_{\tilde{q}} \gtrsim 1 - 1.5 \text{ TeV}$$

“Little hierarchy problem”

($\Lambda_G = 200 \text{ TeV}$
 $\tan \beta = 20$)

A Snapshot of OGM



($m_h \gtrsim 115 \text{ GeV}$)

Some undesirable features of OGM

Despite its many successes, OGM also has some undesirable features:

- It is not a complete model. Origin of SUSY and R-symmetry breaking not explained.
- It is “indirect”: messenger and ~~SUSY~~ sectors completely distinct
- Fine-tuned (“little hierarchy problem”)
- μ , $B\mu$ problem
- ...
- **It is non-generic:** $W = \lambda_i X \phi_i \tilde{\phi}_i$ is not the most general superpotential allowed by the symmetries!

Indeed, the most general superpotential allowed by gauge symmetry is

$$W = \lambda_{ij} X \phi_i \tilde{\phi}_j + m_{ij} \phi_i \tilde{\phi}_j$$

Undesirables, cont'd

$$W = \lambda_{ij} X \phi_i \tilde{\phi}_j + m_{ij} \phi_i \tilde{\phi}_j$$

These models have not been much explored. Why not??

- “They have explicit mass parameters. This seems unnatural.”
Actually, there are many examples (e.g. most recently SQCD with massive flavors and its generalizations) of SUSY gauge theories which dynamically generate renormalizable messenger superpotentials. (These models generally also come with an accidental, approximate R-symmetry, a point which we will return to momentarily...)
- “They can’t possibly give rise to any phenomenology that is qualitatively different than OGM.”
Actually...

Undesirables, cont'd

$$W = \lambda_{ij} X \phi_i \tilde{\phi}_j + m_{ij} \phi_i \tilde{\phi}_j$$

- We will see that these models can exhibit features not ordinarily associated with gauge mediation.

Indeed, many of the classic predictions of OGM can be modified in this more general (but just as “ordinary”!) class of models.

Thus, there is a wider space of phenomenological possibilities that should be attributed to gauge mediation!


- By studying these models, we will naturally be led to a completion of OGM which is minimal and direct.

Part I: Phenomenology of (Extra)Ordinary Gauge Mediation

R-symmetry

$$W = \lambda_{ij} X \phi_i \tilde{\phi}_j + m_{ij} \phi_i \tilde{\phi}_j \quad (i, j = 1, \dots, N)$$

- Starting from the most general renormalizable superpotential for the messengers (actually, not quite – still haven't included d/t splitting), we will impose a continuous $U(1)_R$ symmetry:

Comes from $W_{hidden} \sim FX$ 

$$R(X) = 2, \quad \begin{cases} \lambda_{ij} \neq 0 & \text{only if } R_i + \tilde{R}_j = 0 \\ m_{ij} \neq 0 & \text{only if } R_i + \tilde{R}_j = 2 \end{cases}$$

- Motivation: R-symmetry is necessary for SUSY breaking ([Nelson & Seiberg](#)), so it must be present if this theory is to be part of a direct mediation model
- Apology: if a model of this form is generated dynamically (as in SQCD with massive flavors), then the R-symmetry could be accidental and approximate

Determinant Identity

Because of the R-symmetry, the messenger mass matrix $\mathcal{M} \equiv \lambda X + m$ satisfies an identity:

$$\det \mathcal{M} = X^n G(m, \lambda), \quad n = N - \frac{1}{2} \sum_{i=1}^N (R_i + \tilde{R}_i) \in \mathbb{Z}$$

Proof:

$$\det \mathcal{M} = \sum_{\sigma} \text{sign}(\sigma) \prod_{i=1}^N \mathcal{M}_{i\sigma(i)}, \quad \mathcal{M}_{ij} = \begin{cases} \lambda_{ij} X & \text{if } R_i + \tilde{R}_j = 0 \\ m_{ij} & \text{if } R_i + \tilde{R}_j = 2 \\ 0 & \text{otherwise} \end{cases}$$

So the power of X in each (nonvanishing) term is:

$$n = \sum_{i=1}^N \left(1 - \frac{1}{2} (R_i + \tilde{R}_{\sigma(i)}) \right) = N - \frac{1}{2} \sum_i (R_i + \tilde{R}_i)$$

which proves the claim.

This determinant identity, which follows directly from the R-symmetry, has a number of important consequences.

Soft masses

- Soft masses to leading order in F/M^2 , extending technique of Giudice and Rattazzi:

$$M_r = \frac{\alpha_r}{4\pi} \Lambda_G, \quad \Lambda_G = F \partial_X \log \det \mathcal{M}$$
$$m_{\tilde{f}}^2 = 2 \sum_{r=1}^3 C_{\tilde{f}}^r \left(\frac{\alpha_r}{4\pi} \right)^2 \Lambda_S^2, \quad \Lambda_S^2 = \frac{1}{2} |F|^2 \partial_{XX^*}^2 \sum_{i=1}^N (\log |M_i|^2)^2$$

- Using determinant identity, $\det \mathcal{M} \propto X^n$, so

$$\Lambda_G = \frac{nF}{M}$$

Independent of the couplings!

Gauginos always obey the GUT relations in these models.

This is a direct consequence of R-symmetry!

Soft masses, cont'd

- On the other hand, the sfermion masses in general depend on the messenger superpotential couplings.
- So, by analogy with OGM we will define the “effective messenger number”

$$N_{eff}(M, m, \lambda) = \frac{\Lambda_G^2}{\Lambda_S^2} = \left[\frac{1}{2n^2} |X|^2 \partial_{X X^*}^2 \sum_{i=1}^N (\log |M_i|^2)^2 \right]^{-1}$$

- By varying the parameters, we can change the effective messenger number of the model. This, combined with doublet/triplet splitting, can lead to some **interesting deviations** from the predictions of OGM.

Doublet/Triplet Splitting

$$\mathbf{5} \rightarrow (\mathbf{3}, 1, -\frac{1}{3}) \oplus (1, \mathbf{2}, \frac{1}{2})$$

$$W = X(\lambda_{3ij}q_i\tilde{q}_j + \lambda_{2ij}l_i\tilde{l}_j) + m_{3ij}q_i\tilde{q}_j + m_{2ij}l_i\tilde{l}_j \quad (i, j = 1, \dots, N)$$

- In OGM, doublet/triplet splitting has no effect on the soft masses to leading order,

$$\Lambda_G = \frac{NF}{M}, \quad \Lambda_S^2 = \frac{NF}{M} \text{ independent of } \lambda_{2,3}$$

- However, in these more general models it can lead to different effective messenger numbers for the doublets and triplets

$$N_{eff,2} = N_{eff}(M, m_2, \lambda_2), \quad N_{eff,3} = N_{eff}(M, m_3, \lambda_3)$$

$$\Lambda_{S,2}^2 = \frac{\Lambda_G^2}{N_{eff,2}}, \quad \Lambda_{S,3}^2 = \frac{\Lambda_G^2}{N_{eff,3}}, \quad \Lambda_{S,1}^2 = \frac{3}{5}\Lambda_{S,2}^2 + \frac{2}{5}\Lambda_{S,3}^2$$

D/T splitting and Focusing

$$m_{H_u}^2 \sim m_{H_u}^2(M_{mess}) - \frac{3}{4\pi^2} y_t^2 m_{\tilde{t}}^2 \log \frac{M_{mess}}{m_{\tilde{t}}}$$

$$\sim \Lambda_G^2 / N_{eff,2} \qquad \qquad \qquad \sim \Lambda_G^2 / N_{eff,3}$$

- As noticed before ([Agashe & Graesser](#); [Agashe](#)), having different numbers of doublets and triplets can lead to “focusing” in the running of the Higgs mass parameter:

$$m_{H_u}^2 \ll m_{\tilde{t}}^2$$

- And through the relation:

$$\mu^2 \approx -\frac{m_Z^2}{2} - m_{H_u}^2$$

this can lead to very small μ . (In practice, need $N_{eff,3}/N_{eff,2} \gtrsim 3.5$)

Consequences: higgsino-like charginos and (NLSP) neutralinos(!)

Unification?

Doublet/triplet splitting can potentially spoil gauge coupling unification. However the R-symmetry improves the situation:

$$\alpha_a^{-1}(M_{GUT}) = \alpha_{GUT, MSSM}^{-1} - \frac{1}{2\pi} (N \log M_{GUT} - \log \det \mathcal{M}_a)$$

By the determinant identity, $\det \mathcal{M}_a = \det(\lambda_a X + m_a) = X^n G(m_a, \lambda_a)$

In general, $G(m, \lambda)$ is independent of some of the couplings.

(Example: $n = N \Rightarrow \det \mathcal{M}_a = X^N \det \lambda_a$, independent of m .)

So these couplings can be split arbitrarily without spoiling gauge coupling unification.

Classification and Examples

Classification of Models: Type I

$$W = \lambda_{ij} X \phi_i \tilde{\phi}_j + m_{ij} \phi_i \tilde{\phi}_j$$

- Type I models: $\det m \neq 0$ ($\Rightarrow \det \mathcal{M} = \det m, n = 0$)

These models include those based on original O’Raifeartaigh model (e.g. [Nomura, Izawa, Tobe, Yanagida](#)), as well as those based on SQCD with massive flavors (e.g. [Csaki et al](#); [Kitano et al](#); [Murayama & Nomura](#); and more)

Because $n = 0$, gaugino masses vanish to leading order in the SUSY breaking.

Consequently, there is generally a large hierarchy between gauginos and squarks, and these models tend to be even more fine-tuned than OGM.

Classification of Models: Type II

- Type II models: $\det \lambda \neq 0$ ($\Rightarrow \det \mathcal{M} = X^N \det \lambda$, $n = N$)
 - These models include OGM, and continuous deformations thereof.
 - Here the gaugino masses are nonzero at leading order in the SUSY breaking.
 - These models are also nice, because $\det \mathcal{M}$ is independent of m , so we can get arbitrary doublet/triplet splitting without sacrificing unification.

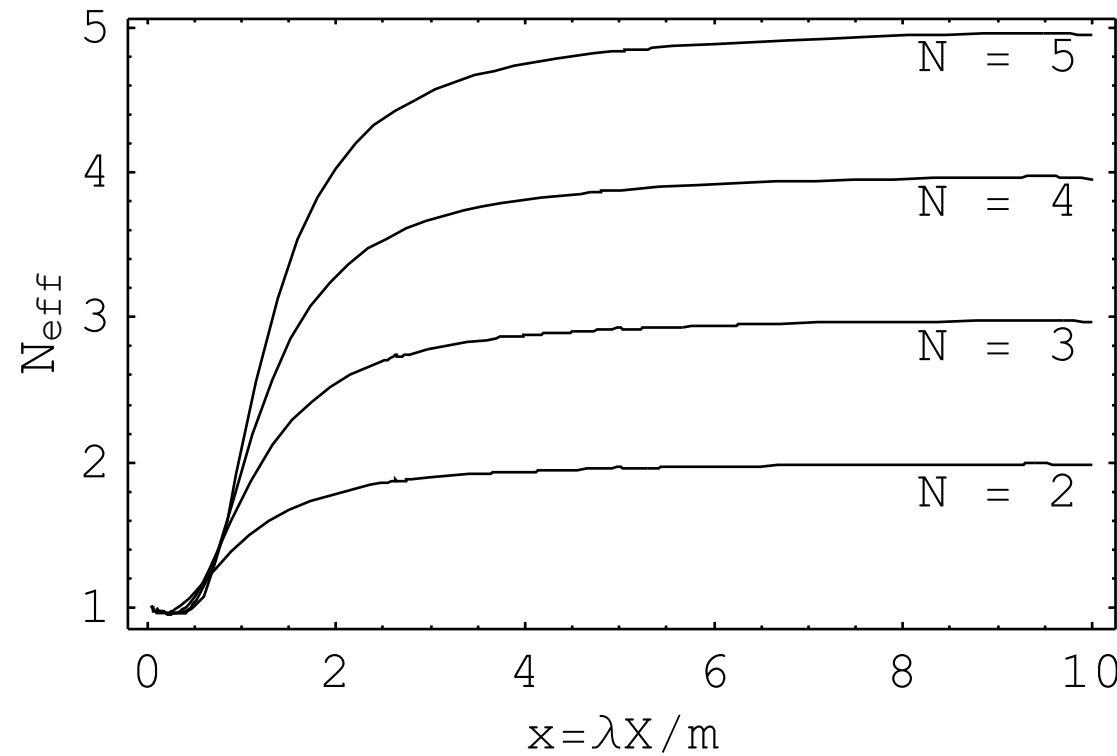
- Let us illustrate these facts with a simple example:

$$W = \lambda X \phi_i \tilde{\phi}_i + m \phi_i \tilde{\phi}_{i+1} \quad \text{e.g. } \mathcal{M} = \begin{pmatrix} \lambda X & m & 0 \\ 0 & \lambda X & m \\ 0 & 0 & \lambda X \end{pmatrix}$$

- This is the most general superpotential allowed by the R-charge assignments

$$R_i = -2i, \quad \tilde{R}_i = 2i$$

Type II example: N_{eff}

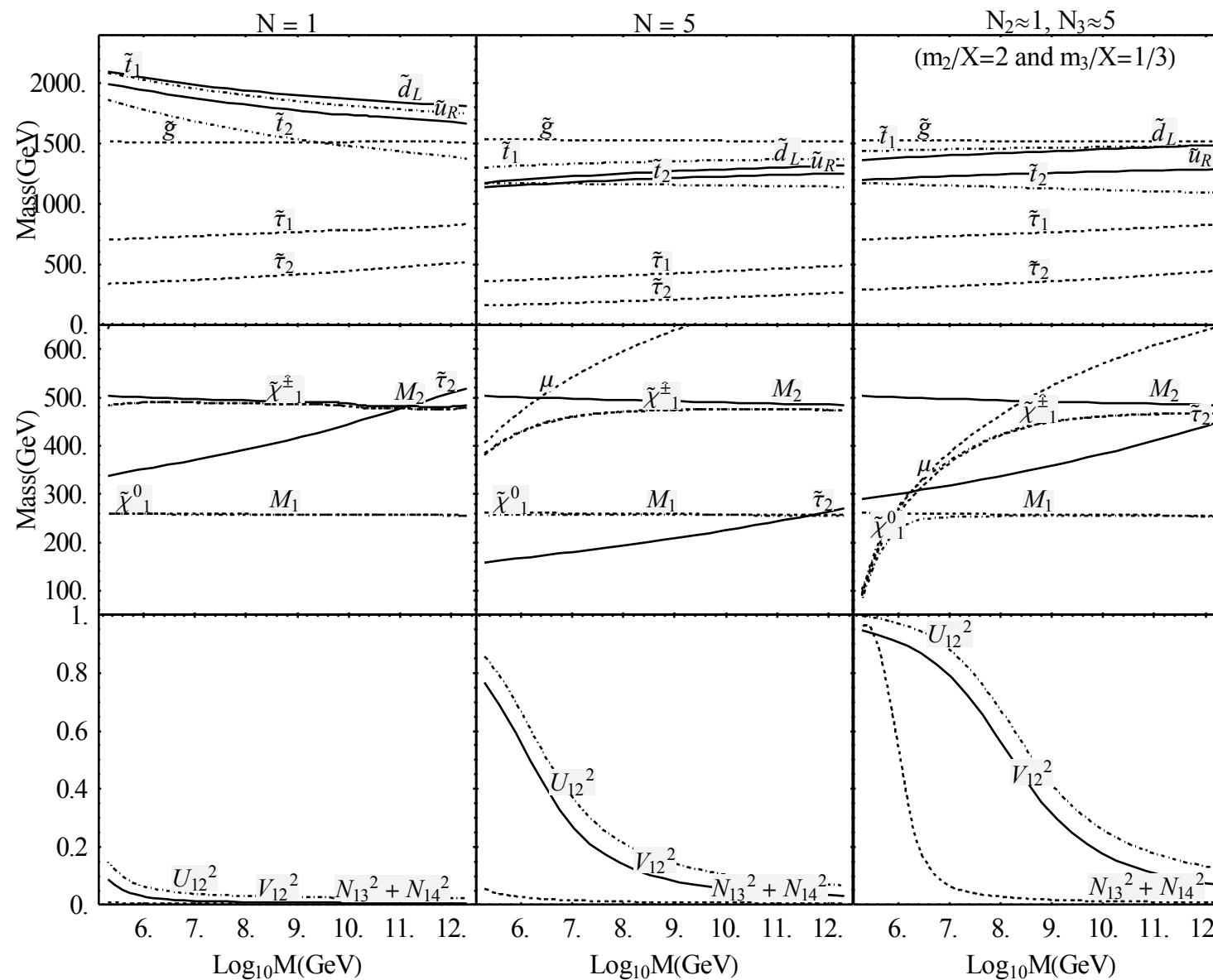


For small μ focusing, we would like $N_{eff,3} \gg N_{eff,2}$

So we would like to be in the regime $m_3 \ll \lambda X \ll m_2$

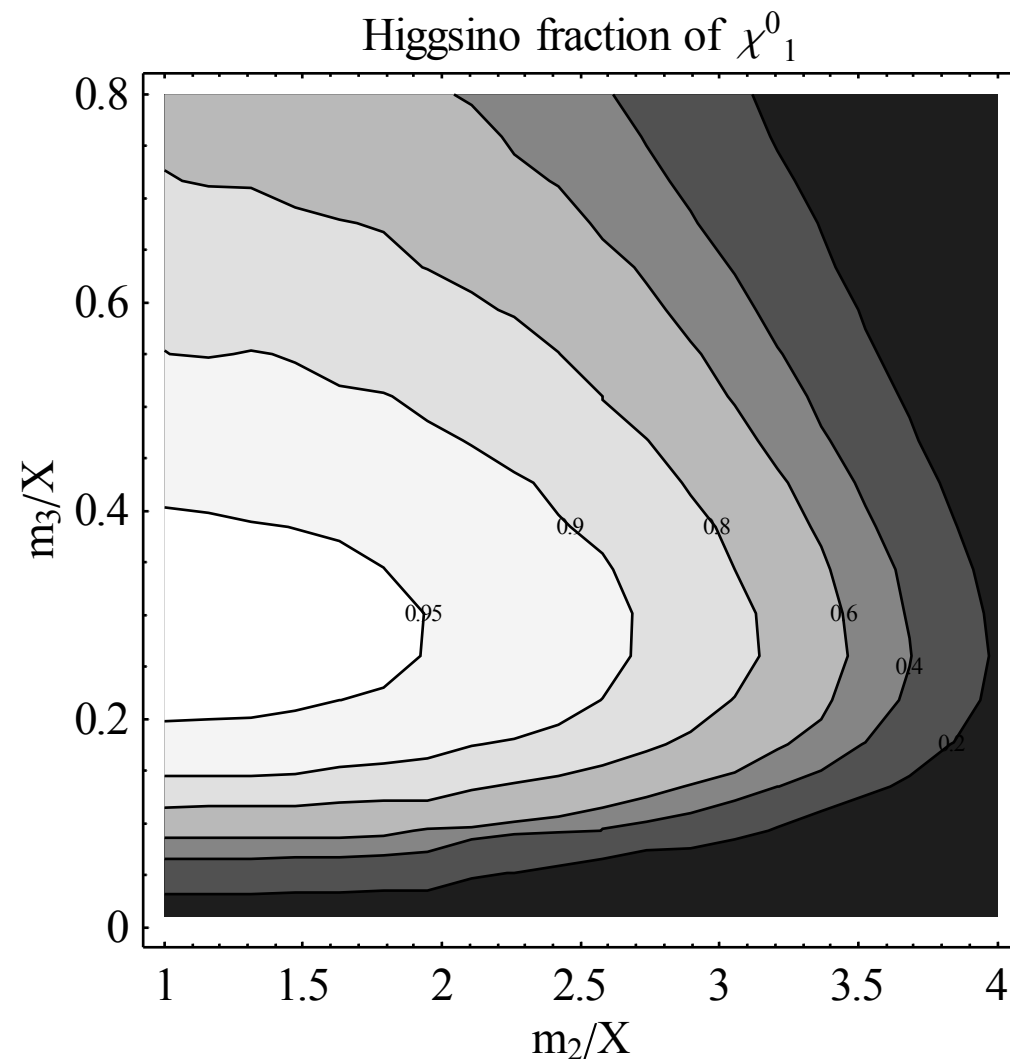
Type II example: spectra

($\Lambda_G = 200$ TeV
 $\tan \beta = 20$)



($m_h \gtrsim 115$ GeV)

Type II example: Higgsino NLSP



$$N = 5$$

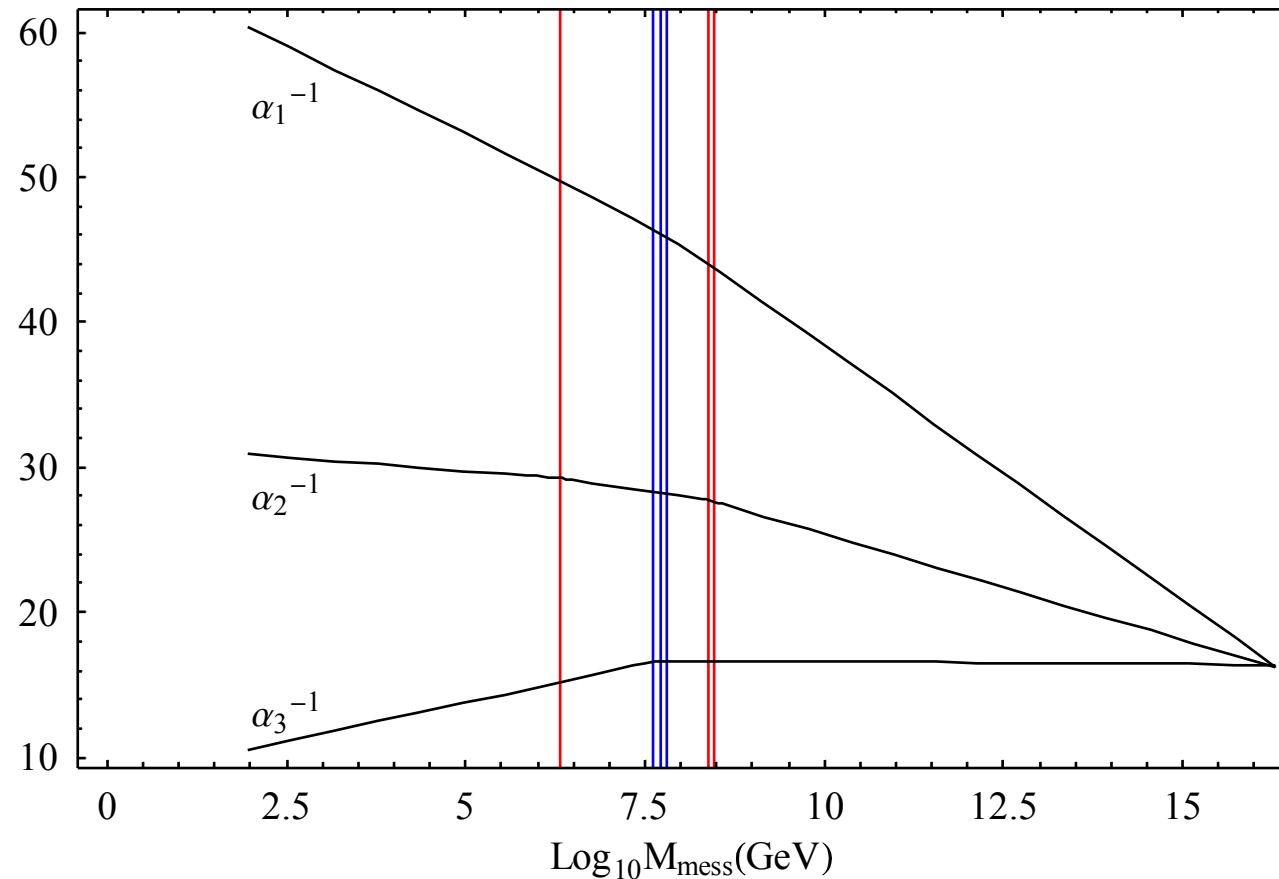
$$M_{mess} = 220 \text{ TeV}$$

$$\Lambda_G = 200 \text{ TeV}$$

NLSP neutralino has a significant Higgsino fraction in a sizeable region of parameter space!

Type II example: unification

$N_2 \approx 1, N_3 \approx 3; \quad N=3, m_2/X=5, m_3/X=0.3$



R-symmetry guarantees that the heavy doublet messengers come in just right to fix up the running!

Classification of Models: Type III

- Type III models:

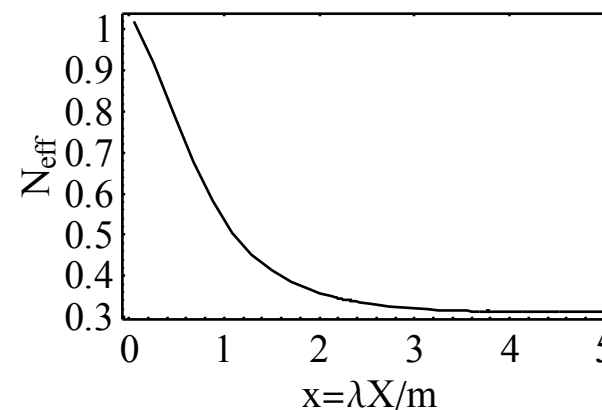
$$\det \lambda = \det m = 0 \quad (\Rightarrow \det \mathcal{M} = X^n G(m, \lambda), \quad 0 < n < N)$$

These models include neither OGM, nor previously studied models.

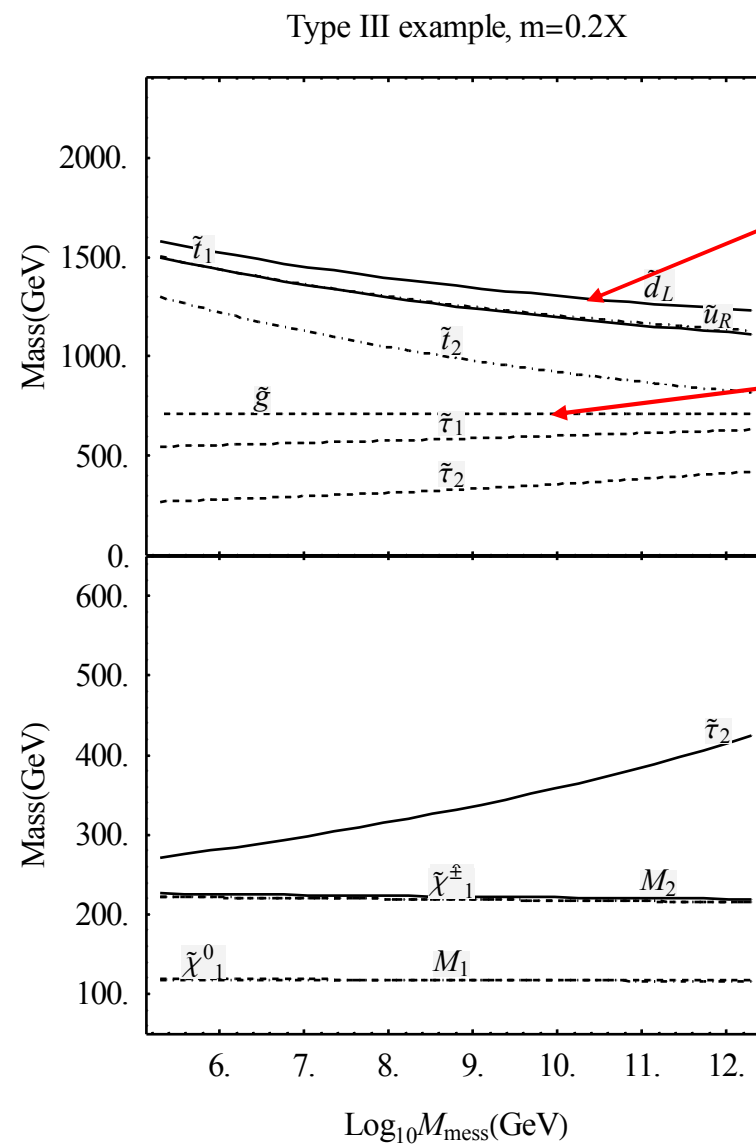
These models are quite exotic – in particular, here one can get

$$N_{eff} \ll 1 (!)$$

- For example: $W = \lambda X(\phi_1 \tilde{\phi}_1 + \phi_2 \tilde{\phi}_2) + m(\phi_1 \tilde{\phi}_2 + \phi_1 \tilde{\phi}_3 + \phi_3 \tilde{\phi}_1)$



Type III example: spectra



Given that the squarks must be heavy for the LEP higgs mass bound,

$N_{eff} < 1$ allows for the gluino to remain light(er)

($\Lambda_G = 300$ TeV
 $\tan \beta = 20$)

($m_h \gtrsim 115$ GeV)

Part II: Minimal Completions of Gauge Mediation

Minimal Completions = O'R models

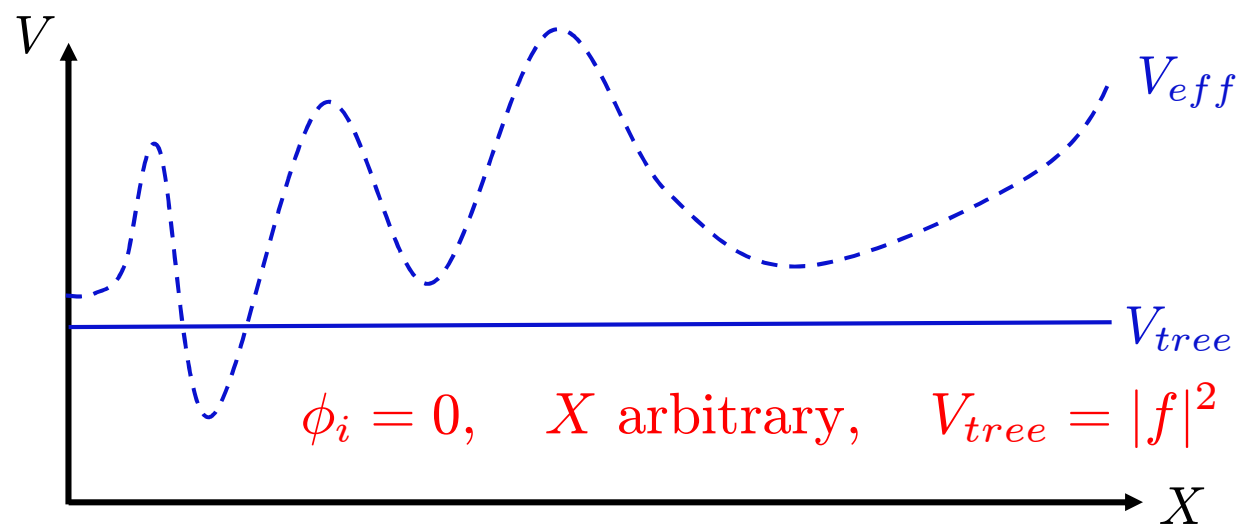
- So far we have studied these minimal extensions of OGM, treating the SUSY-breaking field X as a spurion. We have seen that some interesting and exotic phenomenology can emerge from these models.
- Now let us address the issue of completeness: what is the minimal thing we can do to achieve SUSY breaking **and** R-symmetry breaking?
- Because of the R-symmetry, these models are one step away from being generalized O'Raifeartaigh models. Indeed, the only renormalizable operator we can add consistent with symmetries is

$$\delta W = fX$$

Pseudo-moduli space

$$W = \lambda_{ij} X \phi_i \tilde{\phi}_j + m_{ij} \phi_i \tilde{\phi}_j + f X$$

- These models spontaneously break SUSY (at least locally) because their F-terms are incompatible. What about R-symmetry?
- SUSY-breaking leads to a pseudo-moduli space:



If $m_X^2 < 0$,
R-symmetry
definitely
broken!

But the lore is that
it's always
positive...

$$V_{eff}^{(1)} = \frac{1}{64\pi^2} \text{Tr} (-1)^F \mathcal{M}^4 \log \frac{\mathcal{M}^2}{Q^2} = V_0 + m_X^2 |X|^2 + (|X|^4)$$

Examples

- Original O'R model:

$$W = fX + m\phi_1\phi_2 + \frac{1}{2}\lambda X\phi_1^2$$

- Witten's inverted hierarchy

$$W = fX + \frac{1}{2}h\text{Tr} A^2 B + \frac{1}{2}\lambda X\text{Tr} A^2$$

- ITIY model

$$W = fX + m\vec{S} \cdot \vec{V} + \frac{1}{2}X\vec{V} \cdot \vec{V}$$

- massive SQCD in the free-magnetic phase

$$W = \mu^2\text{Tr} \Phi + h\text{Tr} \Phi q\tilde{q}$$

In all of these models, all the fields have R=0 or R=2!

R-charge Condition

Indeed, one can show that if all the fields have $R = 0, 2$, then there is always an R-symmetry preserving vacuum at the origin. (DS)

So spontaneous R-breaking requires a field with $R \neq 0, 2$

The simplest models with this property exhibit spontaneous R-breaking, so it seems to be a generic phenomenon...

Interestingly, all type II models have this property:

$$\det \lambda \neq 0 \quad \Rightarrow \quad (R_i, \tilde{R}_i) = (R_i, -R_i)$$

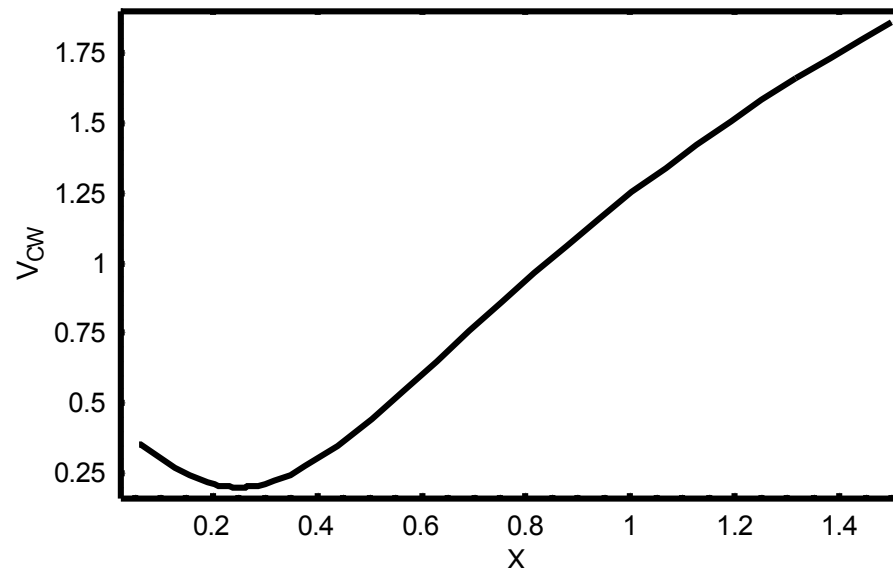
$$m_{ij} \neq 0 \quad \Rightarrow \quad 2 = R_i + \tilde{R}_j = R_i - R_j = \tilde{R}_j - \tilde{R}_i$$

So any renormalizable perturbation of OGM typically leads to a complete model of direct mediation!?

Simplest example with spontaneous ~~R~~

- Perturb N=2 OGM with the only renormalizable operators allowed by R-symmetry:

$$W = \lambda X (\phi_1 \tilde{\phi}_1 + \phi_2 \tilde{\phi}_2) + m \phi_1 \tilde{\phi}_2 + f X$$



Straightforward to find messenger masses, compute Coleman-Weinberg potential:

$$V_{CW} = \text{Tr} M_B^4 \log \frac{M_B^2}{\mu^2} - \text{Tr} M_F^4 \log \frac{M_F^2}{\mu^2}$$

An R-symmetry breaking minimum is generated at one-loop!

Pheno of the complete models

- These models are more constrained than those of Part I. Turns out it is not as straightforward to get exotic phenomenology as before, when X was a spurion.
- Nevertheless, in various corners of parameter space, one can still find the features discussed in Part I.
- We are still working on exploring the parameter space of these models. There are likely to be more surprises...

Conclusions, future directions

- We have argued that OGM is part of a much wider model space which is not forbidden by any symmetries.
- By exploring this model space, we have seen that many of the classic features of OGM can qualitatively change.
 - higgsino-like neutralino NLSP
 - small μ
 - $N < 1$
 - squashed slepton/squark spectrum
- Thus, gauge mediation, even in its simplest form, allows for richer phenomenological possibilities than previously thought.

Conclusions, cont'd

- We have also seen how the more general space of OGM models can lead naturally to minimal, complete models of direct mediation.
- Some future directions/open questions are:
 - Collider phenomenology of these models, esp. higgsino NLSP
(work in progress; cf. [Matchev & Thomas](#))
 - Cosmological implications – R-axion, (nearly) stable messengers?
 - Can these types of models be generated dynamically?
 - What happens if we give up R-symmetry altogether?
 - What do known solutions to the mu problem look like in this framework?