

Tev-scale gravity in Hořava-Witten theory  
on a compact complex hyperbolic threefold.  
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The observed physical universe is a very stiff structure, approximately flat up to distances larger, by a factor of  $10^{61}$ , than the radius of curvature expected on the basis of the Standard Model, plus General Relativity, in 3+1 dimensions.

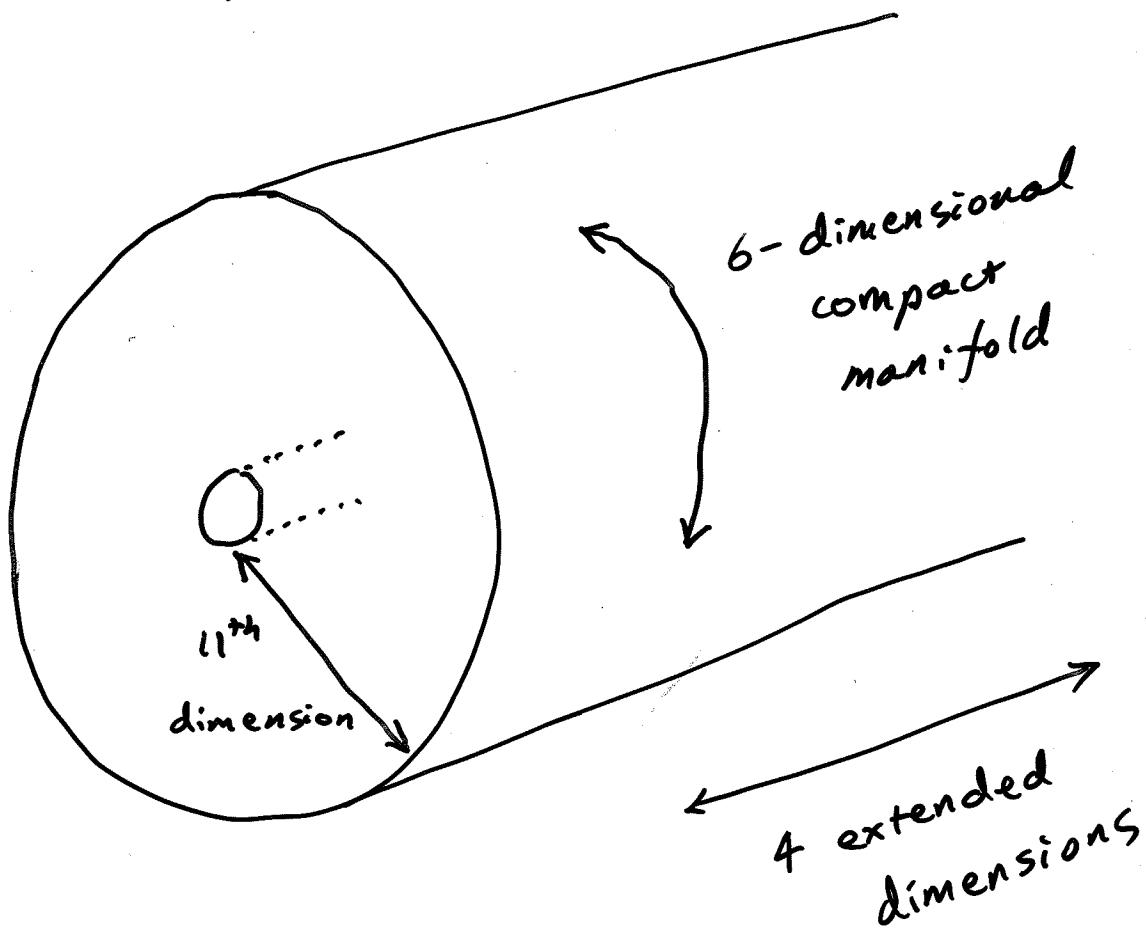
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Perhaps compact additional spatial dimensions, not yet observed, play an active role in stiffening the universe.

The larger such unobserved compact extra dimensions are, the stronger we would expect their stiffening effects to be.

To study the possibility of such a mechanism, consider the compactification of Horava-Witten theory on a smooth compact quotient of  $\mathbb{C}H^3$  or  $H^6$ .

Look for solutions that realize the ADD mechanism by a form of thick pipe geometry:



We live on the inner surface of the thick pipe.

The Hořava-Witten action in the bulk is the standard Cremmer-Julia-Scherk (CJS) action of  $d=11$  supergravity. In the "upstairs" picture, on the orbifold  $M^{10} \times S^1 / \mathbb{Z}_2$ , this is:

$$S_{\text{CJS}} = \frac{1}{\kappa^2} \int_{M^{10} \times S^1 / \mathbb{Z}_2} d^{10}x \sqrt{-g} \left( -\frac{1}{2} R - \frac{1}{2} \bar{\Psi}_I \Gamma^{IJK} D_J \Psi_K - \frac{1}{48} G_{IJKL} G^{IJKL} \right.$$

$$- \frac{\sqrt{2}}{192} (\bar{\Psi}_I \Gamma^{IJKLMN} \Psi_N + 12 \bar{\Psi}^J \Gamma^{KL} \Psi^M) G_{JKLM}$$

$$\left. - \frac{\sqrt{2}}{3456} g^{I_1 I_2 \dots I_{10}} C_{I_1 I_2 I_3} G_{I_4 \dots I_7} G_{I_8 \dots I_{10}} \right)$$

where terms quartic in the gravitino have been omitted,  $g^{I_1 I_2 \dots I_{10}}$  is the tensor  $g^{I_1 I_2 \dots I_{10}} = \frac{1}{\sqrt{-g}} \epsilon^{I_1 I_2 \dots I_{10}}$ ,  $\epsilon^{012\dots 910} = 1$ , and  $G_{IJKL} = 24 \partial_{[I} C_{JKL]}$ .

The supersymmetric Yang-Mills action, on the orbifold fixed point set at  $y_i$ ,  $i=1, 2$ , is:

$$S_{YM}^{[i]} = -\frac{1}{\lambda^2} \int_{M^{10}[i]} d^{10}x \sqrt{-g} \text{tr} \left( \frac{1}{4} F_{uv}^{[i]} F^{uv} + \frac{1}{2} \bar{X} \Gamma^u D_u X \right)$$

where indices  $u, v, w, \dots$  run over all directions on  $M^{10}$ , and  $\text{tr}$  denotes  $\frac{1}{30}$  of the trace in the adjoint representation of E8.

Seeking to extend the SUSY Yang-Mills actions, on the orbifold fixed point hyperplanes, to locally supersymmetric actions, coupled in a locally supersymmetric manner to the bulk supergravity multiplet, Horava and Witten found that the Chern-Simons term,  $C_{GG}$ , in the CJS action, acquires a non-vanishing variation, under Yang-Mills gauge transformations, which is localized on the orbifold fixed point hyperplanes, and precisely cancels the one-loop quantum gauge anomaly of the Majorana-Weyl fermions in the SUSY Yang-Mills multiplets, provided that

$$\lambda^2 = 2\pi (4\pi k^2)^{2/3}$$

(Conrad found an extra factor of  $2^{1/3}$  in the right-hand side.)

I shall now denote the full eleven-dimensional metric by  $G_{IJ}$ . (Distinguished from  $G_{IJKL}$  by context, and the number of indices.)  $y$  is the coordinate in the  $S'$  direction, which is the radial direction of the thick pipe, and is also the coordinate index in the  $S'$  direction.

We use the gauge freedom of general coordinate invariance, in order to choose Gaussian normal coordinates, such that  $G_{yy} = 1$ , and  $G_{uy} = 0$ , and seek a solution of the quantum corrected field equations and boundary conditions of Horava-Witten theory, such that the metric has the form:

$$ds_{11}^2 = G_{IJ} dx^I dx^J = \\ = a(y)^2 g_{\mu\nu} dx^\mu dx^\nu + b(y)^2 h_{AB} dx^A dx^B + dy^2$$

where  $g_{\mu\nu}$  is the metric on a four-dimensional locally de Sitter space, whose de Sitter radius  $g$  shall set equal to 1, so that  $R_{\mu\nu}(g) = -3g_{\mu\nu}$ , and  $h_{AB}$  is a standard metric on a smooth compact quotient of either  $CH^3$  or  $H^6$ .

In the case of  $\mathbb{CH}^3$ , the standard metric  $h_{AB}$  is locally given by

$$h_{AB} = \begin{pmatrix} 0 & h_{ab} \\ h_{\bar{a}\bar{b}} & 0 \end{pmatrix}$$

where

$$h_{ab} = h_{\bar{b}\bar{a}} = \frac{\delta_{ab}}{(1-z^c z^c)} + \frac{z_a z_b}{(1-z^c z^c)^2} = \frac{1}{(1-z^c z^c)} \left( \delta_{ab} + \frac{z_a z_b}{(1-z^c z^c)} \right)$$

The indices of the complex coordinates,  $z^a$ , are lowered and raised by the flat space complex metric

$$\tilde{\delta}_{AB} = \begin{pmatrix} 0 & \delta_{ab} \\ \delta_{\bar{a}\bar{b}} & 0 \end{pmatrix}$$

so that  $z_a = z^{\bar{a}} = (z^a)^*$ , and  $z_{\bar{a}} = z^a = (z^{\bar{a}})^*$ , where \* denotes complex conjugation.

The Riemann tensor is given by

$$R_{ab\bar{c}\bar{d}} = -h_{ab} h_{\bar{c}\bar{d}} - h_{a\bar{b}} h_{\bar{c}d}$$

and the Ricci tensor is  $R_{ab} = 4h_{ab}$ .

In the case of  $H^6$ , we choose  $h_{AB}$  to have radius of curvature equal to 1, so that

$$R_{ABCD} = h_{Ac} h_{BD} - h_{AD} h_{BC}$$

and the Ricci tensor is  $R_{AB} = 5h_{AB}$ .

Let the inner surface of the thick pipe, where we live, be at  $y = y_1$ , and the outer surface be at  $y = y_2$ .

Then we require

$a_1 \equiv a(y_1)$  = observed de Sitter radius

$$\simeq 1.51 \times 10^{26} \text{ metres} \simeq 0.94 \times 10^{61} \sqrt{G_N}$$

where

$$G_N = 6.7087 \times 10^{-39} \text{ GeV}^{-2}$$

is Newton's constant.

We define  $V(M^6) \equiv \int_{M^6} d^6x \sqrt{h}$ . Then by the generalized Gauss-Bonnet theorem, we find that for smooth compact quotients of  $CH^3$ :

$$V(M^6) = -\frac{\pi^3}{3} \chi(M^6) = -10.3354 \chi(M^6)$$

and for smooth compact quotients of  $H^6$ :

$$V(M^6) = -\frac{8\pi^3}{15} \chi(M^6) = -16.5367 \chi(M^6)$$

where  $\chi(M^6)$  is the Euler number of  $M^6$ , which is an integer  $\leq -1$ .

The value of  $b_1 \equiv b(y_1)$  is fixed by the value of the Yang-Mills fine structure constants in four dimensions at unification,  $\alpha_u = \frac{g_u^2}{4\pi}$ , which will be equal to the value of the QCD fine structure constant at unification, and the magnitude of  $\chi(M^6)$ .

Using the Horava-Witten relation between  $\lambda$  and  $\kappa$ , and reducing the Yang-Mills action at  $y_1$  to four dimensions, we find that for a smooth compact quotient of  $CH^3$ :

$$\alpha_u = \frac{(4\pi\kappa^2)^{2/3}}{2 b_i^6 V(M^6)} = \frac{0.2615}{|X(M^6)|} \left(\frac{\kappa^{2/9}}{b_i}\right)^6$$

And for a smooth compact quotient of  $H^6$ , the coefficient 0.2615 is replaced by 0.1634.

For one of the  $E_8$  breakings I will consider, the value of  $\alpha_u$  is approximately the value of the QCD fine structure constant, evolved in the Standard Model to around 150 TeV, in which case

$$\alpha_u \simeq 0.06 \simeq \frac{1}{17}$$

Taking this as a representative value, we find that for a smooth compact quotient of  $CH^3$ :

$$\frac{b_i}{\kappa^{2/9}} \simeq \frac{1.3}{|X(M^6)|^{1/6}}$$

And from the Giudice, Rattazzi, and Wells estimate of the expansion parameter for graviton loop corrections in 11 dimensions, the condition for perturbation theory to be reliable in the quantum region near the inner surface, is approximately:

$$\frac{b_i}{\kappa^{2/9}} > 0.2$$

Combining these two relations, we find approximately:

$$|\chi(M^6)| < 7 \times 10^4$$

On the other hand, since  $|\chi(M^6)| \geq 1$ , the first of these relations implies approximately

$$\frac{b_1}{K^{2/9}} \leq 1.3$$

Thus we have the approximate bounds:

$$0.2 < \frac{b_1}{K^{2/9}} \leq 1.3$$

I shall consider topologically stabilized breakings of the E8 on the inner surface of the thick pipe to the Standard Model, by vacuum gauge fields that include Abelian vacuum gauge fields, in the Cartan subalgebra of E8, whose field strengths are proportional to Hodge-de Rham harmonic two-forms of  $M^6$ , and are restricted by a Dirac quantization condition to lie on a discrete lattice in the Cartan subalgebra of E8.

The Dirac quantization condition is derived by considering Wilson lines in the E8 fundamental/adjoint, formed from trees of hairpins that curl around an arbitrary closed orientable two-surface in  $M^6$  in various ways.

The orientation within  $E_8$  of the Cartan subalgebra itself cannot be completely topologically stabilized, however, so in a mode expansion of the dependence of the Yang-Mills fields on position in  $M^6$ , some of the modes of the extra-dimensional components, tangential to  $M^6$ , will be Goldstone scalars, associated with rotations of the orientation of the Cartan subalgebra within  $E_8$ , which have no energy cost if done globally, or in other words, without dependence on position in the four extended dimensions.

By the Coleman-Weinberg mechanism, as studied recently by Chishtie, Elias, Mann, McKeon, and Steele, for example, some of these Goldstone scalars will get squared negative masses by radiative corrections at one loop, resulting in radiative breaking of the Standard Model at a mass roughly  $\frac{1}{10}$  of the fundamental mass  $b_i^{-1} \sim \kappa^{-2/9}$ .

Thus this framework requires TeV-scale gravity, with  $b_i^{-1} \sim \kappa^{-2/9} \sim \text{TeV}$ .

In other words, the fact that we have scalar fields that are massless at the fundamental mass scale  $b_1^{-1} \sim \kappa^{-2/9}$ , means that there is no possibility of obtaining a large hierarchy by the logarithmic evolution of a scalar mass squared from a positive value at the fundamental mass scale, to a negative value at a much smaller mass scale.

For the simplest topologically stabilized breaking of E8 to the Standard Model considered, the only electrically neutral states, outside the  $SU(3) \times SU(2)$  subgroup, are components of Standard Model Higgs doublets, or their complex conjugates, and Standard Model singlets. Thus in this case, if the generically expected radiative breaking of the Standard Model leaves the photon massless, then it is automatically by a Standard Model Higgs doublet. However, the presence of Standard Model singlet Higgs fields means that we expect spreading and dilution of the Higgs signal (Van der Bij, Wilczek), and in this context we note that there was a partial discovery of a "fractional" Higgs, with mass 95 GeV, at LEP.

In this example, the chiral fermions automatically consist of a number of Standard Model generations, plus a completely independent number of singlet neutrinos, as favoured by neutrino experiments including LSND and MiniBooNE.

In this example, using an  $SU(9)$  basis for  $E_8$ , the vacuum gauge fields are all in the Cartan subalgebra of  $E_8$ , and thus in the Cartan subalgebra of  $SU(9)$ , and in the fundamental representation of  $SU(9)$ , they are diagonal matrices, with diagonal matrix elements

$$(\sigma_1, \sigma_1, \sigma_1, \sigma_2, \sigma_2, \sigma_3, \sigma_4, \sigma_5, \sigma_6)$$

such that:

$$3\sigma_1 + 2\sigma_2 + \sigma_3 + \sigma_4 + \sigma_5 + \sigma_6 = 0$$

$U(1)_4$  is also a diagonal matrix of this form, whose diagonal matrix elements we can take as

$$\frac{1}{3}(-2, -2, -2, 3, 3, 0, 0, 0, 0)$$

For generic values of the  $\sigma_i$ , the subgroup of  $E_8$  that commutes with such a vacuum gauge field would be  $SU(3) \times SU(2) \times (U(1))^5$ . However, the five  $U(1)$  gauge fields do not all remain massless.

Specifically, any of the  $U(1)$ 's that is not perpendicular to all of the vacuum  $U(1)$  gauge fields with nonzero field strength, will become massive by a form of Higgs mechanism that was originally discussed by Witten, in the context of the  $SO(32)$  type I superstring.

In the case of Horava-Witten theory, this arises from the redefinition of  $G_{UVW}$  to include a term  $\frac{\kappa^2}{\sqrt{2}\lambda^2} \delta(y-y_1) \omega_{UVW}^{[1]}$ , and an analogous term involving  $\delta(y-y_2)$ , in the course of coupling the SUSY Yang-Mills multiplets on the orbifold fixed-point hyperplanes, to the  $d=11$  supergravity multiplet in the bulk, in a locally supersymmetric manner. Here  $\omega_{UVW}^{[1]}$  is the Chern-Simons form constructed from the E8 gauge fields at  $y_1$ :

$$\omega_{UVW}^{[1]} = \text{tr} \left( A_U^{[1]} (\partial_V A_W^{[1]} - \partial_W A_V^{[1]}) + \frac{2}{3} A_U^{[1]} [A_V^{[1]}, A_W^{[1]}] + \text{cyclic permutations} \right) \text{ of } U, V, W$$

This contains a term  $2 \text{tr}(A_\mu^{[1]} F_{AB}^{[1]})$ , and when  $F_{AB}^{[1]}$  has a vacuum expectation value in the Cartan subalgebra of E8, this leads, through the kinetic term  $G_{IJKL} G^{IJKL}$  of the three-form gauge field, to a mass term for the corresponding gauge field in the Cartan subalgebra.

However, when  $G_{UVW}$  is redefined as above, the resulting  $\omega_{UVW} \omega_{UVW}^{[1]}$  term in the action is formally infinite, being proportional to  $\delta(0)$ . This should be corrected to a finite nonzero term in Moss's improved form of Horava-Witten theory, in which the  $\delta(0)$  terms are absent.

In the particular case of the E8 breaking mentioned above, the requirement that  $U(1)_Y$  does not become massive by Witten's Higgs mechanism means that the topologically stabilized vacuum gauge fields in the Cartan subalgebra of E8, that have nonzero field strength, have to have  $\sigma_1 = \sigma_2$ , so for generic values of the  $\sigma_i$  subject to this constraint, the subgroup of E8 that commutes with the vacuum gauge fields that have nonzero field strength, is  $SU(5) \times (U(1))^4$ .

Thus in this case, it is necessary to complete the topologically stabilized breaking of E8 to the Standard Model, by the inclusion of a topologically stabilized Hosotani vacuum gauge field, proportional to  $U(1)_Y$ , with vanishing field strength. The topologically stabilized Hosotani mechanism is usually associated with a compact manifold whose fundamental group has torsion in the sense of discrete group theory, or in other words, a nontrivial finite subgroup. However the fundamental group of a smooth compact quotient of  $C\mathbb{H}^3$  or  $H^6$  necessarily has no torsion.

However to obtain a topologically stabilized Hosotani mechanism, it is sufficient for the first homology group to have torsion, and examples in three and four dimensions show that this is possible. For example the first homology group of the Weeks manifold is  $\mathbb{Z}/5 + \mathbb{Z}/5$ .

The E8 breakings considered have a mechanism for suppressing proton decay that is related to the Aranda-Carone mechanism, and which is optimized in the example mentioned above.

We also note that to explain the phenomenological observation that the mass ratios of the quarks in the up quark sector are very roughly the squares of the corresponding mass ratios in the down quark sector, there must be some kind of systematic difference between the up quark and down quark sectors, beyond the difference in their weak hypercharges. There is indeed such a difference in the example mentioned above, because the E8 fundamental contains twice as many d states as u states, even though, as I mentioned above, the chiral fermions automatically consist of a number of Standard Model generations, plus an independent number of singlet neutrinos.

Horava-Witten theory is M-theory on the orbifold  $M^0 \times S/\mathbb{Z}_2$ , where  $M^0$  is a ten-dimensional manifold.

M-theory is an unknown theory in eleven dimensions, whose defining properties are that it is the strong coupling limit of type IIA superstring theory, and its low energy limit is supergravity in eleven dimensions.

HW suggested M-theory would have a built-in short-distance cutoff, but left open the question of whether that is connected to the  $d=11$  supermembrane.

In fact the  $d=11$  supermembrane has a continuous spectrum (de Wit, Lüscher, Nicolai), and has been reinterpreted as a second quantized theory (de Wit). It is possible that the supermembrane mass spectrum (in flat space) corresponds simply to the single particle and multi-particle states of  $d=11$  supergravity (de Wit). It is thus possible that the supermembrane, on a smooth background in eleven uncompactified dimensions, is a kind of second quantized description of supergravity in eleven dimensions, dual to the standard Cremmer-Julia-Scherk component description.

In that case, there is then no known physical effect to provide the basis for any difference, at the quantum level, on a smooth background, in eleven uncompactified dimensions, between M-theory, and the CJS theory of  $d=11$  supergravity.

This is consistent with the conclusion of Hull and Townsend, that M-theory cannot contain a separate fundamental supermembrane, distinct from the solitonic membrane of  $d=11$  supergravity.

It is also consistent with the fact that the full dynamics of type IIA superstring theory arises from the solitonic membrane of the CJS theory, on compactification on a circle to ten dimensions (reviewed in sec. 2.3.3).

The predictions of  $d=11$  supergravity can be calculated in the framework of effective field theory (BPHZ renormalization). But they will depend on one free parameter for each nontrivial, linearly independent, higher derivative local multinomial in the CJS fields and their derivatives, whose CJS variations vanish when the CJS field equations are satisfied.

Such locally-supersymmetric-on-shell higher derivative counterterms certainly do exist for maximal supergravities in 10 or fewer dimensions, because examples are present in the effective action obtained, as an expansion in powers of  $\alpha'$ , by integrating out the massive superstring single particle states. The first such counterterms in ten dimensions involve four Riemann tensors ( $R^4$ ).

However in  $d=11$  supergravity there is no extra parameter such as  $\alpha'$ , and there are no known massive single particle states, on a smooth uncompactified background, to integrate out.

Parts of candidate locally-supersymmetric-on-shell higher derivative counterterms have been constructed for  $d=11$  by Peeters, Vanhove, and Westerberg, by lifting corresponding counterterm from  $d=10$ , and by Deser and Seminara, and by Metsaev, who constructed linearized dimension 8 counterterms, but these have not yet been completed to full counterterms.

There are also two different superspace constructions of  $d=11$  counterterms, by Duff and Toms, and by Howe and Tsimpis, but these will not lead to CJS counterterms unless the gauge completion mapping of the CJS theory into superspace can be completed to higher orders in  $\theta$ , and at present it has not been fully completed through order  $\theta^2$ .

The possibility therefore exists that there might be an obstruction that prevents the geometrical transformations in superspace from matching the CJS supersymmetry variations for a general solution of the CJS field equations beyond a certain power of  $\theta$ . This would mean that the superspace counterterms do not result in locally supersymmetric deformations of the CJS theory, so that it might be possible to calculate the predictions of the CJS theory in the framework of effective field theory, without the occurrence of undetermined parameters connected with the short distance completion of the theory.

There is partial evidence for the existence of such an obstruction in a discrepancy between a component framework calculation of Hyakutake and Ogushi, and the superspace construction of Howe and Tsimpis. Howe and Tsimpis found that there should be an independent on-shell superinvariant for each independent Chern-Simons term, of which there are two at dimension 8,  $C \wedge \text{tr}(R \wedge R \wedge R \wedge R)$  and  $C \wedge \text{tr}(R \wedge R) \wedge \text{tr}(R \wedge R)$ . But Hyakutake and Ogushi found that only one linear combination of these two terms might occur in a superinvariant, namely the combination which occurs in the bulk Green-Schwarz term. However Hyakutake and Ogushi's ansatz did not include all possible terms.

The fact that the coefficient of the bulk Green-Schwarz term, in the M-theory quantum effective action, has been fixed by consideration of chiral anomaly cancellation on fivebranes, suggests that this term is also part of a combination that cancels a potential local supersymmetry anomaly, rather than being part of a locally-supersymmetric-on-shell higher derivative counterterm.

Considering now the case of Horava-Witten theory, Horava and Witten did not include a Gibbons-Hawking term  $\frac{2}{K^2} \int_{M^{10}} \sqrt{-g} K d^{10}x$  in their boundary action, where  $K$  is the scalar extrinsic curvature of the boundary. However it is known that it is necessary to include a Gibbons-Hawking term to obtain a well-defined variational problem for the action, such that a stationary action can be obtained fixing only the metric on the boundary, and not also its normal derivatives.

Moss introduced a supersymmetrized Gibbons-Hawking term into the Horava-Witten boundary action, and was able to avoid the  $\delta(0)$  terms in the action that Horava and Witten found at higher orders in  $K$ . This suggests that it might also be possible to calculate the predictions of Moss's form of Horava-Witten theory in the framework of effective field theory, without the occurrence of undetermined parameters connected with the short-distance completion of the theory.

## The quantum corrected field equations and boundary conditions

- The Riemann tensor components for the ansatz metric

$$ds_{\mu\nu}^2 = G_{IJ} dx^I dx^J = a(y)^2 g_{\mu\nu} dx^\mu dx^\nu + b(y)^2 h_{AB} dx^A dx^B + dy^2$$

are

$$R_{\mu\nu\sigma}^{\tau} = R_{\mu\nu\sigma}^{\tau}(g) + \frac{\dot{a}^2}{a^2} (G_{\mu\sigma} \delta_{\nu}^{\tau} - G_{\nu\sigma} \delta_{\mu}^{\tau})$$

$$R_{ABC}^D = R_{ABC}^D(h) + \frac{\dot{b}^2}{b^2} (G_{AC} \delta_B^D + G_{BC} \delta_A^D)$$

$$R_{\mu A \nu}^B = \frac{\dot{a} \dot{b}}{ab} G_{\mu\nu} \delta_A^B, \quad R_{A \mu B}^{\nu} = \frac{\dot{a} \dot{b}}{ab} G_{AB} \delta_{\mu}^{\nu}$$

$$R_{\mu y \nu}^y = \ddot{\frac{a}{a}} G_{\mu\nu}, \quad R_{yy \mu}^{\nu} = \ddot{\frac{a}{a}} \delta_{\mu}^{\nu}$$

$$R_{AyB}^y = \frac{\ddot{b}}{b} G_{AB}, \quad R_{yAy}^B = \frac{\ddot{b}}{b} \delta_A^B$$

where a dot denotes differentiation with respect to  $y$ ,  $R_{\mu\nu\sigma}^{\tau}(g)$  denotes the Riemann tensor calculated from the four-dimensional metric  $g_{\mu\nu}$ , and  $R_{ABC}^D(h)$  denotes the Riemann tensor calculated from the six-dimensional metric  $h_{AB}$ .

The non-vanishing Ricci tensor components, in eleven dimensions, are:

$$R_{\mu\nu} = R_{\mu\nu}(g) + \left( \frac{\ddot{a}}{a} + 3 \frac{\dot{a}^2}{a^2} + 6 \frac{\dot{a}\dot{b}}{ab} \right) G_{\mu\nu} = \left( \frac{\ddot{a}}{a} + 3 \frac{\dot{a}^2}{a^2} + 6 \frac{\dot{a}\dot{b}}{ab} - \frac{3}{a^2} \right) G_{\mu\nu}$$

$$R_{AB} = R_{AB}(h) + \left( \frac{\ddot{b}}{b} + 5 \frac{\dot{b}^2}{b^2} + 4 \frac{\dot{a}\dot{b}}{ab} \right) G_{AB} = \left( \frac{\ddot{b}}{b} + 5 \frac{\dot{b}^2}{b^2} + 4 \frac{\dot{a}\dot{b}}{ab} + \frac{4}{b^2} \right) G_{AB}$$

$$R_{yy} = 4 \frac{\ddot{a}}{a} + 6 \frac{\ddot{b}}{b}$$

where I used the relations  $R_{\mu\nu}(g) = -3g_{\mu\nu}$ , and, for smooth compact quotients of  $CH^3$ ,  $R_{AB}(h) = 4h_{AB}$ , from above. Thus the final equality in the second line above is specific to  $CH^3$ , and I shall consider the case of  $CH^3$  from now on.

Now our vacuum Yang-Mills fields, on the boundaries, will not, in general, be covariantly constant, and the three-form gauge fields they induce in the bulk, via the Horava-Witten boundary condition

$$G_{uvwx}|_{y=y_+} = -\frac{3}{\sqrt{2}} \frac{k^2}{\lambda^2} \text{tr} F_{[uv}^{[i]} F_{wx]}^{i]}$$

in "downstairs" picture, on the eleven-manifold with boundary, will also not be covariantly constant on  $M^6$ . However we can expand the fields and the energy-momentum tensor in the harmonic expansion on  $M^6$  introduced by Lukas, Ovrut, and Waldram, and I shall from now on work to lowest order in this expansion. We expect the relative importance of the omitted higher harmonic terms to decrease as we move away from the boundaries into the bulk. 22

I assume that the leading term in the harmonic expansion of the energy-momentum tensor in the bulk has the form:

$$T_{\mu\nu} = t^{(1)}(y) G_{\mu\nu}, \quad T_{AB} = t^{(2)}(y) G_{AB}, \quad T_{yy} = t^{(3)}(y)$$

The conservation equation,  $D_I T^{IJ} = 0$ , then reduces to

$$0 = D_I T^{Iy} = \partial_y t^{(3)} + \left( 4 \frac{a}{a} + 6 \frac{b}{b} \right) t^{(3)} - 4 \frac{a}{a} t^{(1)} - 6 \frac{b}{b} t^{(2)}$$

We define  $T^{IJ}$  to include the classical energy-momentum tensor of the three-form gauge field, plus all the quantum corrections from the one-loop and higher loop terms in the quantum effective action,  $\Gamma$ . We note that  $\Gamma$ , the generating functional of proper vertices, will be calculated in a specific gauge in the gauge-fixed theory, and that varying  $\Gamma$  will result in field equations that are quantum corrected versions of the classical field equations, plus quantum corrected versions of the gauge-fixing conditions, plus quantum corrected field equations for the FP ghosts and any other auxiliary fields. We set the FP ghosts and any other auxiliary fields to zero, and perform a gauge transformation, as necessary, to bring the metric back to the Gaussian normal coordinates ansatz used above.

The quantum-corrected Einstein equations, written for convenience in the form

$$R_{IJ} + \kappa^2 \left( T_{IJ} - \frac{1}{9} G_{IJ} G^{KL} T_{KL} \right) = 0$$

are then:

$$\ddot{\frac{a}{a}} + 3 \frac{\dot{a}^2}{a^2} + 6 \frac{\dot{a}\dot{b}}{ab} - \frac{3}{a^2} + \frac{\kappa^2}{9} (5t^{(1)}(y) - 6t^{(2)}(y) - t^{(3)}(y)) = 0$$

$$\ddot{\frac{b}{b}} + 5 \frac{\dot{b}^2}{b^2} + \frac{4\dot{a}\dot{b}}{ab} + \frac{4}{b^2} + \frac{\kappa^2}{9} (-4t^{(1)}(y) + 3t^{(2)}(y) - t^{(3)}(y)) = 0$$

$$4 \frac{\ddot{a}}{a} + 6 \frac{\ddot{b}}{b} + \frac{\kappa^2}{9} (-4t^{(1)}(y) - 6t^{(2)}(y) + 8t^{(3)}(y)) = 0$$

where the  $t^{(i)}(y)$  satisfy the conservation equation above. The three Einstein equations reduce to

$$\frac{\dot{a}}{a} = -2 \frac{\dot{b}}{b} \pm \frac{1}{2} \sqrt{6 \frac{\dot{b}^2}{b^2} - \frac{8}{b^2} + \frac{4}{a^2} + \frac{2}{3} \kappa^2 t^{(3)}}$$

$$\frac{\ddot{b}}{b} - 3 \frac{\dot{b}^2}{b^2} \pm 2 \frac{\dot{b}}{b} \sqrt{6 \frac{\dot{b}^2}{b^2} - \frac{8}{b^2} + \frac{4}{a^2} + \frac{2}{3} \kappa^2 t^{(3)}} + \frac{4}{b^2} + \frac{\kappa^2}{9} (-4t^{(1)} + 3t^{(2)} - t^{(3)}) = 0$$

An arbitrary solution of these two equations satisfies all three Einstein equations, provided the square root

$$\sqrt{6 \frac{\dot{b}^2}{b^2} - \frac{8}{b^2} + \frac{4}{a^2} + \frac{2}{3} \kappa^2 t^{(3)}} \text{ does not vanish identically.}$$

We take either the upper sign in both eqns. or the lower sign in both eqns., and find that for a thick pipe geometry, the lower sign choice is not acceptable near the inner surface, so we choose the upper signs.

Now we require  $a$  to equal the observed de Sitter radius  $\sim 10^{26}$  metres near the inner surface, while  $b$  is required to be  $\sim 10^{-19}$  metres near the inner surface, so we first neglect the  $\frac{4}{a^2}$  term in the square root and consider a region where the  $t^{(i)}$  are negligible. We define  $c = \frac{db}{dy}$ . The second equation then reduces to:

$$\frac{dc}{db} = -\frac{1}{b} \sqrt{3c^2 + 4} \left( 2\sqrt{2} - \sqrt{3 - \frac{4}{c^2}} \right)$$

We require  $c \geq \sqrt{\frac{4}{3}}$ , in order for the square root to be real. Considering a region of the  $(b, c)$  plane where  $c \gg \sqrt{\frac{4}{3}}$ , the equation reduces to

$$\frac{dc}{db} = -(2\sqrt{6} - 3) \frac{c}{b} \approx -1.8990 \frac{c}{b}$$

which has general solution

$$c = \frac{db}{dy} \approx \left( \frac{B}{b} \right)^{1.8990}$$

To obtain a thick pipe geometry we need the boundary region near the inner surface to meet us on one of these trajectories with a large value of  $\frac{B}{k^{219}}$ .

The required value of  $\frac{B}{k^{219}}$  depends on the way in which the boundary conditions at the outer surface of the thick pipe are satisfied, but is never less than around  $10^5$  for TeV-scale gravity.

Three qualitatively different ways of satisfying the boundary conditions at the outer surface are considered in the paper, and in the simplest of them, where the full burden of stiffening the universe occurs at the inner surface, the required value of  $\frac{B}{K^{2/9}}$  is around  $10^{13}$ .

The other two ways of satisfying the boundary conditions at the outer surface "spread the load" between the inner surface and the outer surface, and allow the required value of  $\frac{B}{K^{2/9}}$  to be as small as  $10^5$ .

A perturbative mechanism by which a large value of  $\frac{B}{K^{2/9}}$  could occur is identified in subsection 2.4.2 of the paper. In essence, the bulk power law  $\frac{db}{dy} \approx \left(\frac{B}{b}\right)^{1.8990}$  holds only for  $\frac{b}{K^{2/9}} > \left(\frac{B}{K^{2/9}}\right)^{0.6551}$ , which is greater than around  $10^3$  when  $\frac{B}{K^{2/9}}$  is  $\approx 10^5$ , while for  $b_i \approx K^{2/9} < b < \left(\frac{B}{K^{2/9}}\right)^{0.6551} K^{2/9}$ , we find self-consistently that

$$\frac{db}{dy} \approx \frac{b}{K^{2/9}}$$

when the Casimir energy density corrections are taken into account.

Thus there is a self-consistent quantum region near the inner surface, in which  $b$  and  $a$  depend exponentially on  $y$ . The larger the thickness of this quantum region, the greater the value of  $\frac{B}{k^{2/9}}$  in the eventual classical region will be.

The self-consistent trajectory  $\frac{db}{dy} \sim \frac{b}{k^{2/9}}$ , in the quantum region  $B$  unique, and if the boundary conditions at the inner surface do not set the system very close to this trajectory, the system will make a rapid deviation from this trajectory into the classical region, with a resulting smaller value of  $\frac{B}{k^{2/9}}$ .

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Some details not covered in the talk, in particular the fits of Newton's constant and the cosmological constant, can be found in the paper arXiv:0704.1476.