Gauge/Gravity Duality:
New methods from string theory for strongly coupled systems

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Work with M. Ammon, M. Kaminski, P. Kerner, S. Müller, A. O'Bannon, J. Shock, H. Zeller

Motivation

Physics is the science of matter and its interactions.

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Two approaches:

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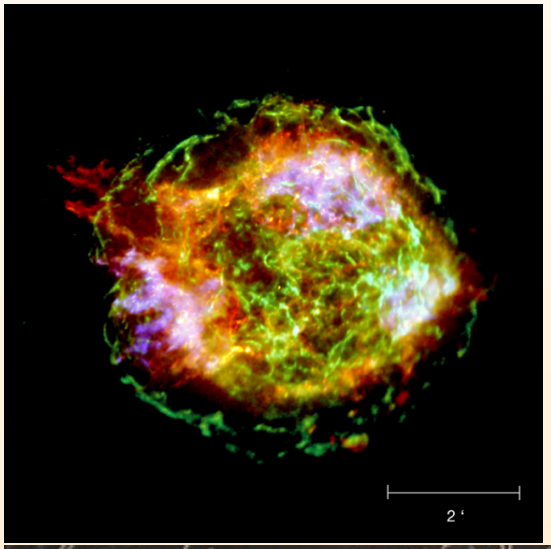
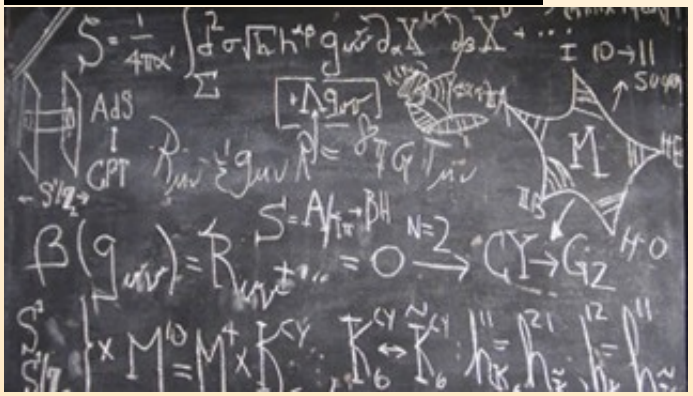
Two approaches:

I. Fundamental approach



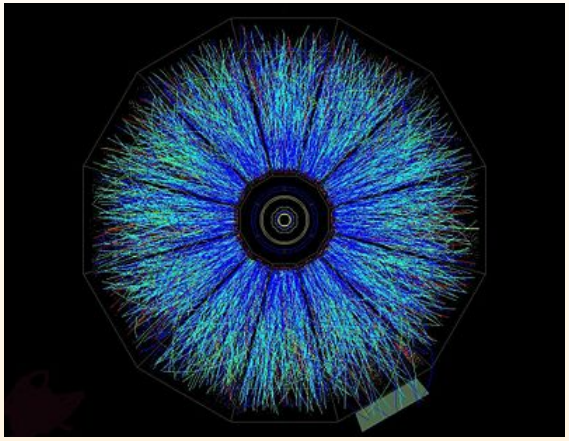
Find a unified theory of all known interactions:

Electromagnetism, Weak force, Strong force
$\Leftrightarrow$ Gravity



Challenge: Quantization of gravity

II. Applied approach



Describe observables and processes
in systems with a given interaction

Challenge: Strongly coupled interactions

Example: Yang-Mills theory

$$\mathcal{L} = -\frac{1}{2g^2}\,\mathrm{Tr}\,F^{\mu\nu}F_{\mu\nu}$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\mu A_\nu - ig[A_\mu, A_\nu]$$

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Recent development in theoretical physics:

Fundamental and applied approach much more closely related than we thought!

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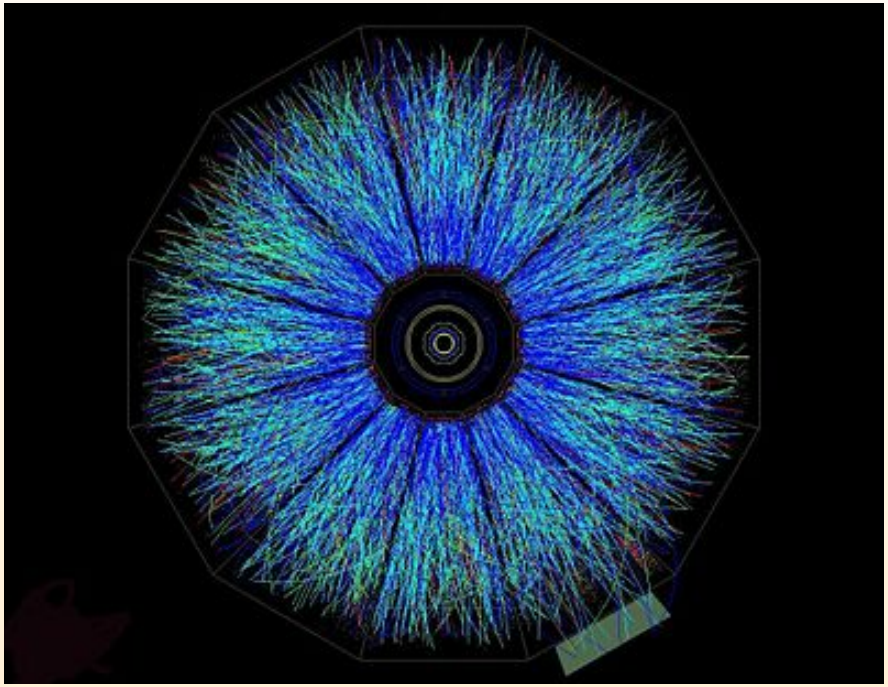
Gauge/gravity duality

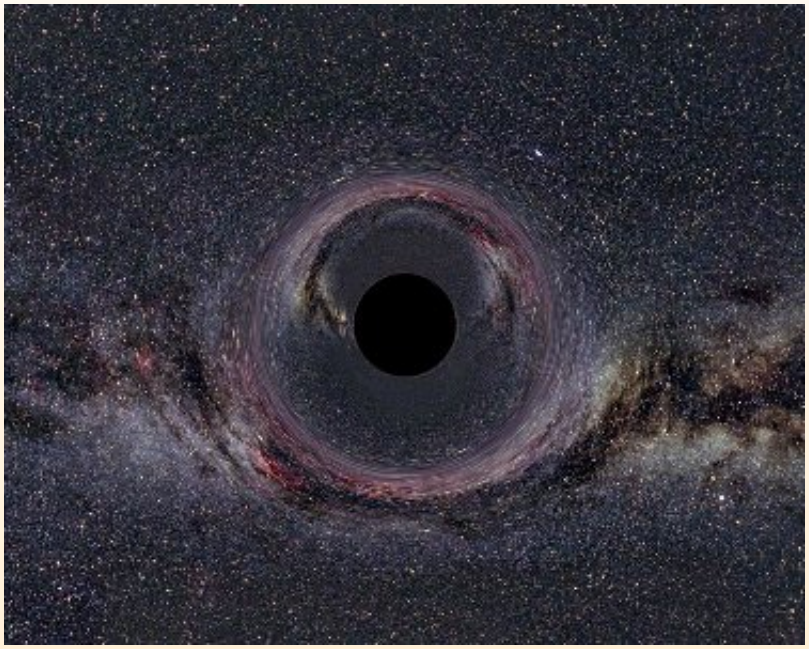
Dualities between quantum field theories and gravitation

Map: Quantum field theory $\Leftrightarrow$ Classical theory of Gravitation



Event at RHIC accelerator

$\Leftrightarrow$



Black hole geometry

Dualities between quantum field theories and gravitation

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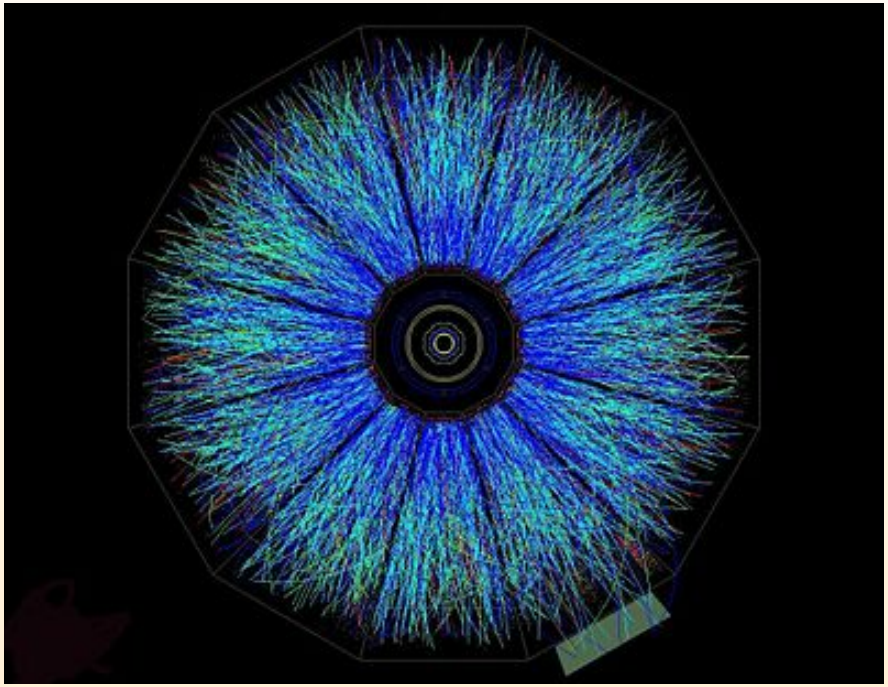

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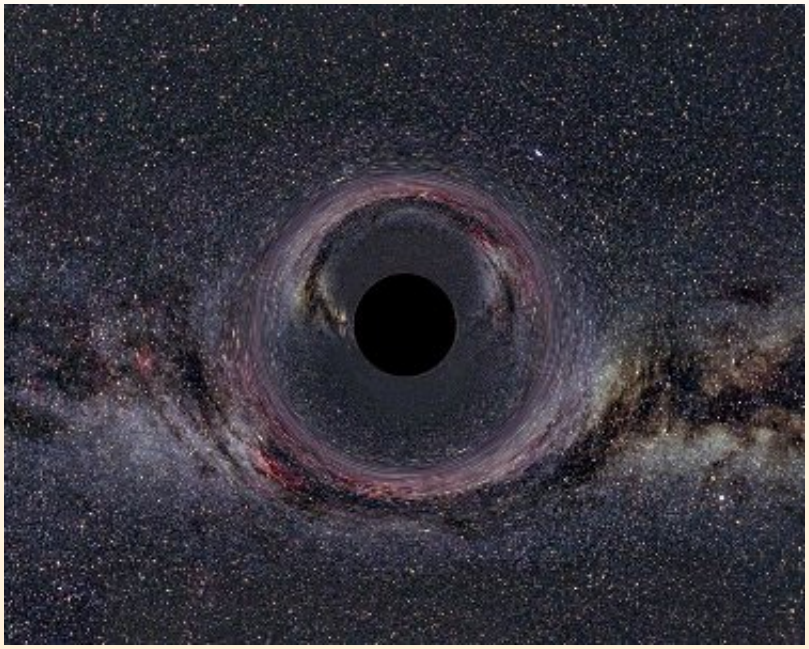


Black hole geometry

Practical Significance:

- Map: Strongly coupled quantum field theory
  $\Leftrightarrow$ Weakly coupled classical theory of gravitation (Solvable)

Gauge/gravity duality

- Conjecture which follows from a low-energy limit of string theory

- Duality:

  Quantum field theory at strong coupling

  $\Leftrightarrow$ Theory of gravitation at weak coupling

- Holography:

  Quantum field theory in four dimensions

  $\Leftrightarrow$ Gravitational theory in five dimensions

Gauge/gravity duality

A very active area in theoretical physics!

Gauge/gravity duality

A very active area in theoretical physics!

Decisive new input for the description of strongly coupled systems in

Elementary particle physics, heavy ion physics, condensed matter physics

Outline

I. Foundations

1. Dualities between quantum field theories and gravity theories

   AdS/CFT correspondence (Maldacena 1997)

2. String theory origin

II. Applications

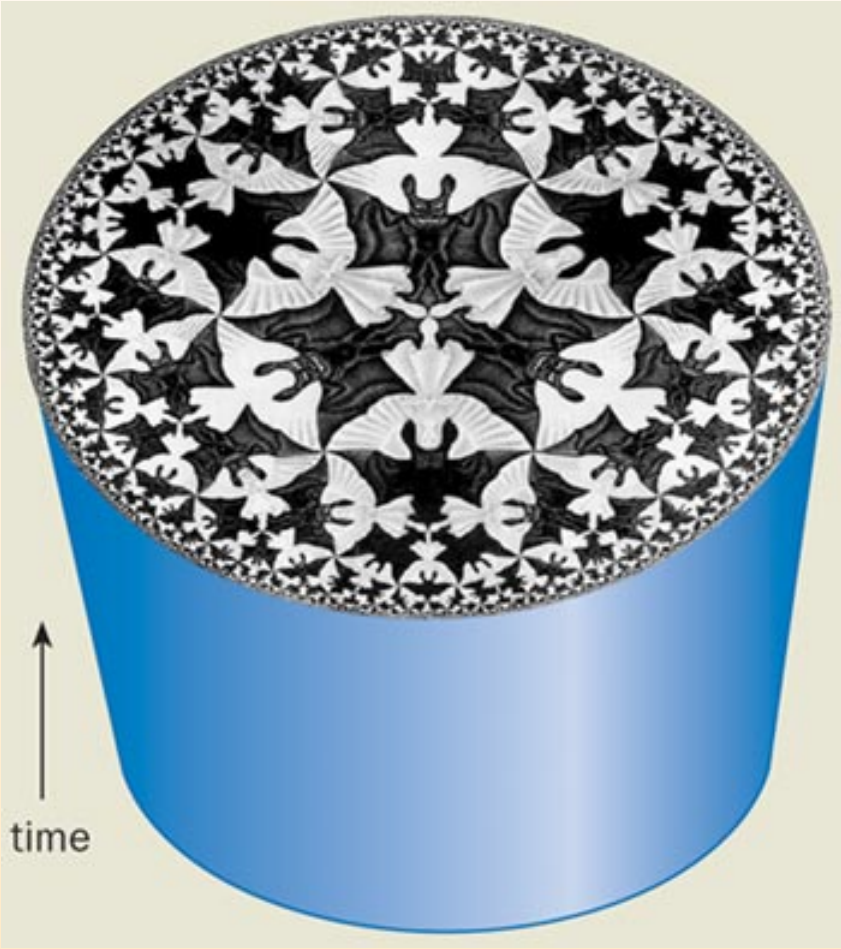
1. Theory of Strong Interactions (QCD)
2. Quark-gluon plasma and black holes
3. Superfluidity and superconductivity

Anti-de Sitter Space

Space of constant negative curvature, has a boundary

$ds^2 = e^{2r/L} dx_\mu dx^\mu + dr^2$

Figure source: Institute of Physics, Copyright: C. Escher



Quantum field theory

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Conformal coordinate transformations: preserve angles locally

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Conformal coordinate transformations: preserve angles locally

$\Rightarrow$ Correlation functions are determined up to a small number of parameters

Search for a

Unified Theory of fundamental interactions

- Quantum theory of gravitation
- Description of standard model of elementary particles and gravity in a unified framework
- Reduction of the number of free parameters in the standard model

Promising candidate: String theory

Introduction: String theory

Quantum theory of gravitation and unification of all four interactions:

Locality is abandoned at very short distances

Natural cut-off: String length

$$l_s \sim \frac{1}{M_{Planck}}, \quad l_s = \sqrt{\frac{\hbar G}{c^3}} = 1.616 \times 10^{-35} m$$

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String theory provides framework for gauge/gravity duality

String theory

Open strings : Gauge degrees of freedom of the Standard Model

Closed Strings: Gravitation

Two types of degrees of freedom: open and closed strings

Higher oscillation modes are excited

Quantization:

Supersymmetric string theory well-defined in $9 + 1$ dimensions

(No tachyons, no anomalies)

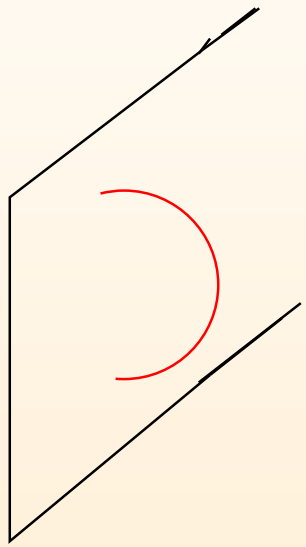
What is the significance of the additional dimensions?

1. Compactification

2. D-branes

D-Branes

D-branes are hypersurfaces embedded into 9+1 dimensional space



D3-Branes: (3+1)-dimensional hypersurfaces

Open Strings may end on these hypersurfaces $\Leftrightarrow$ Dynamics

D-Branes

Low-energy limit (Strings point-like) $\Rightarrow$

Open Strings $\Leftrightarrow$ Dynamics of gauge fields on the brane

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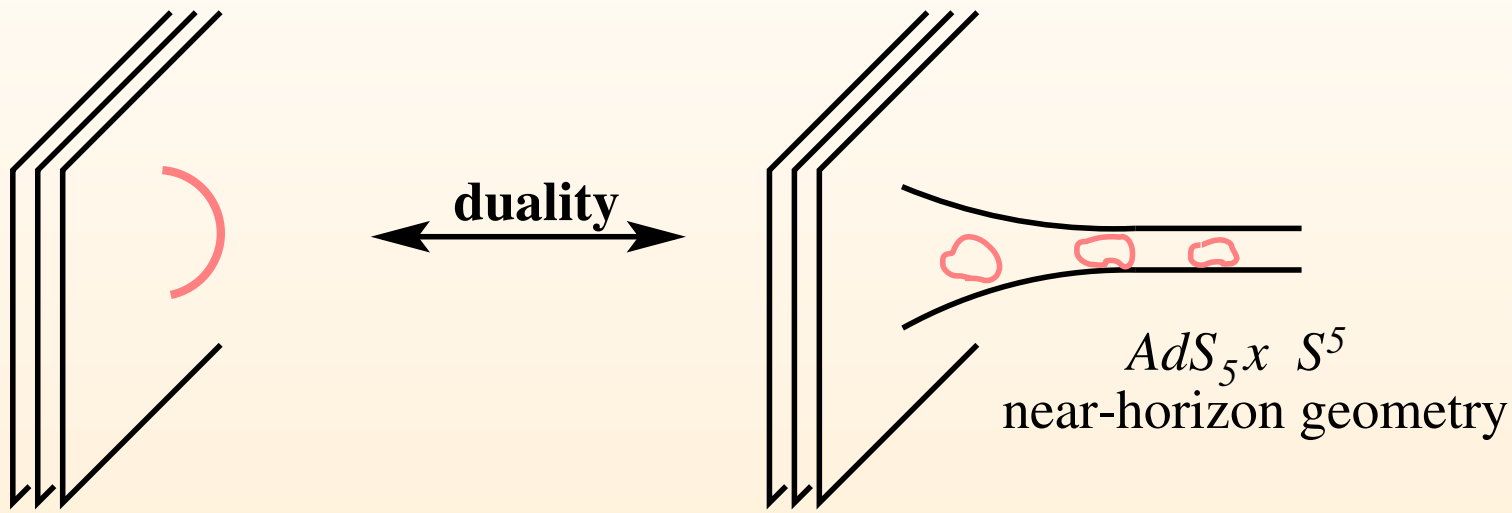
Second interpretation of D-branes:

Solitonic solutions of ten-dimensional supergravity

Heavy objects which curve the space around them

String theory origin of the AdS/CFT correspondence

D3 branes in 10d

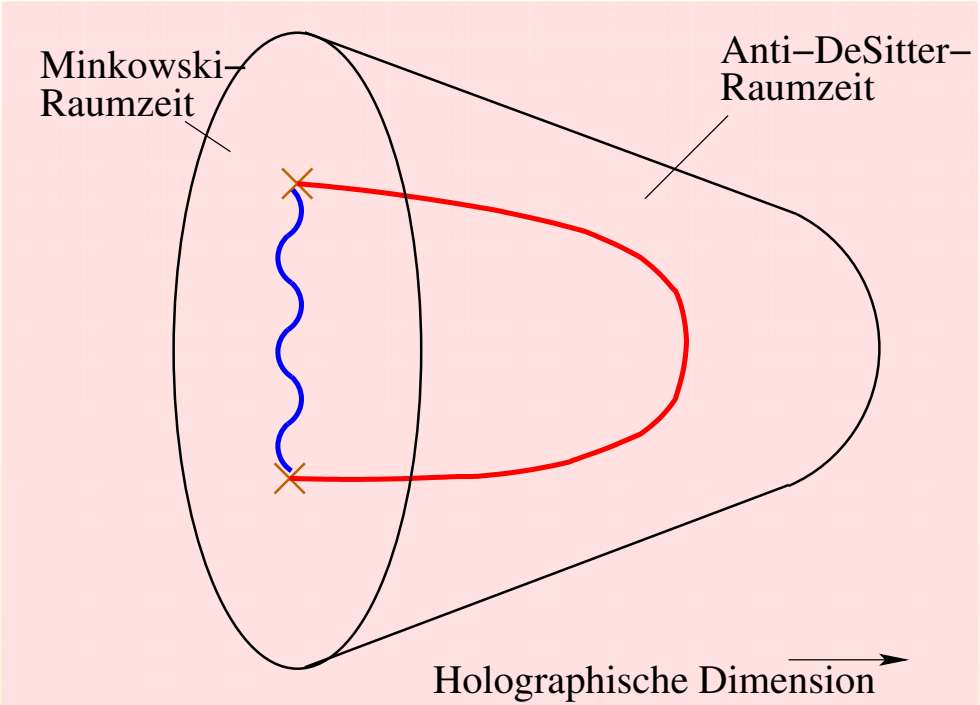


$\Downarrow$ Low energy limit

Supersymmetric $SU(N)$ gauge theory in four dimensions ($N \to \infty$)

Supergravity on the space $AdS_5 \times S^5$

AdS/CFT correspondence



'Dictionary' Field theory operators $\Leftrightarrow$ Classical fields in gravity theory

Symmetry properties coincide

Test: (e.g.) Calculation of correlation functions

AdS/CFT correspondence

- Field-operator correspondence:

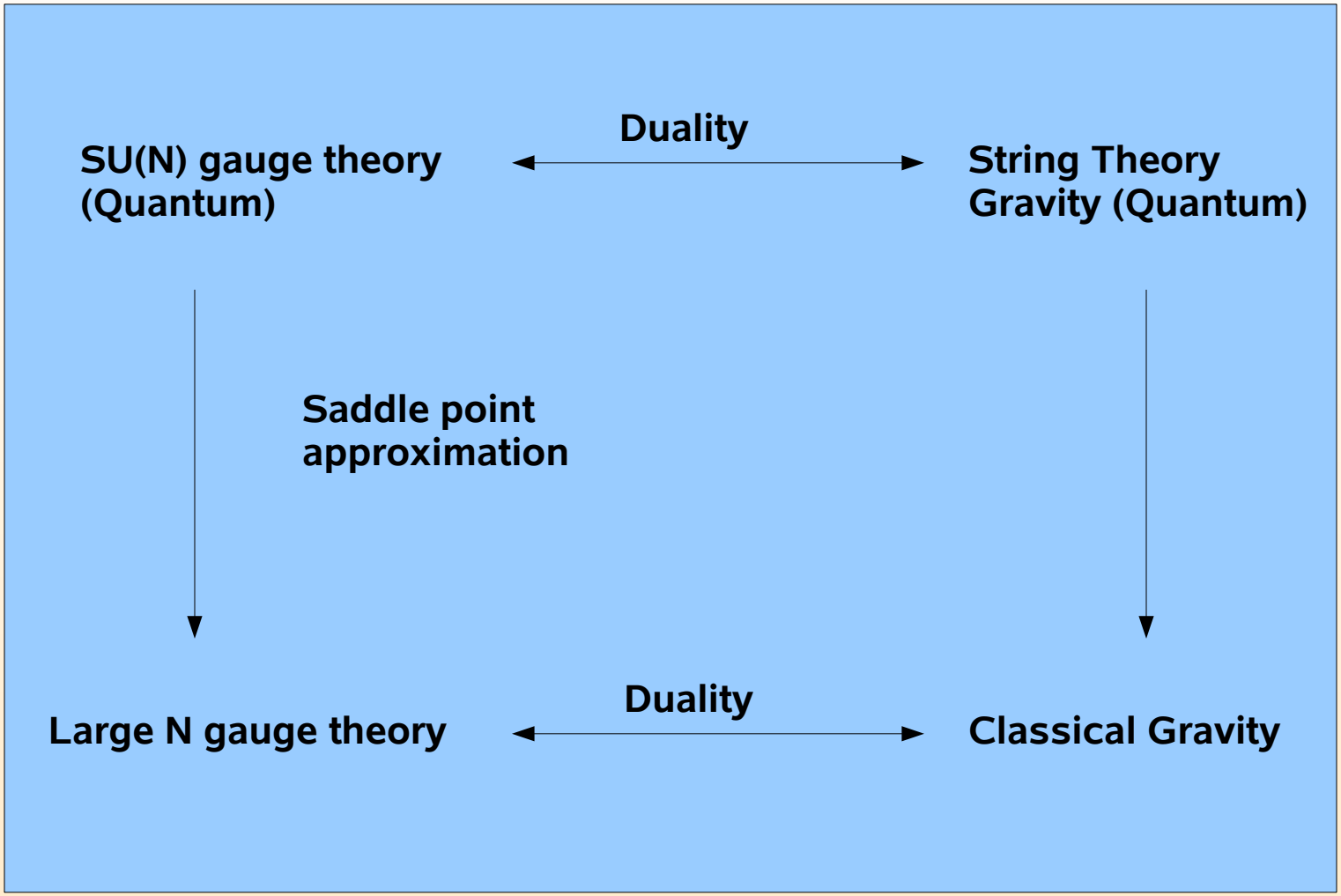
$$\langle e^{\int d^d x\, \phi_0(\vec{x}) \mathcal{O}(\vec{x})} \rangle_{CFT} = Z_{sugra}\Big|_{\phi(0,\vec{x})=\phi_0(\vec{x})}$$

Generating functional for correlation functions of particular composite operators in the quantum field theory

coincides with

Classical tree diagram generating functional in supergravity

AdS/CFT correspondence (Maldacena 1997)



Applications

1. Strong interactions - Quarks and Gluons
2. Finite temperature: Quark-Gluon Plasma
3. Superfluidity and Superconductivity

Generalizations:

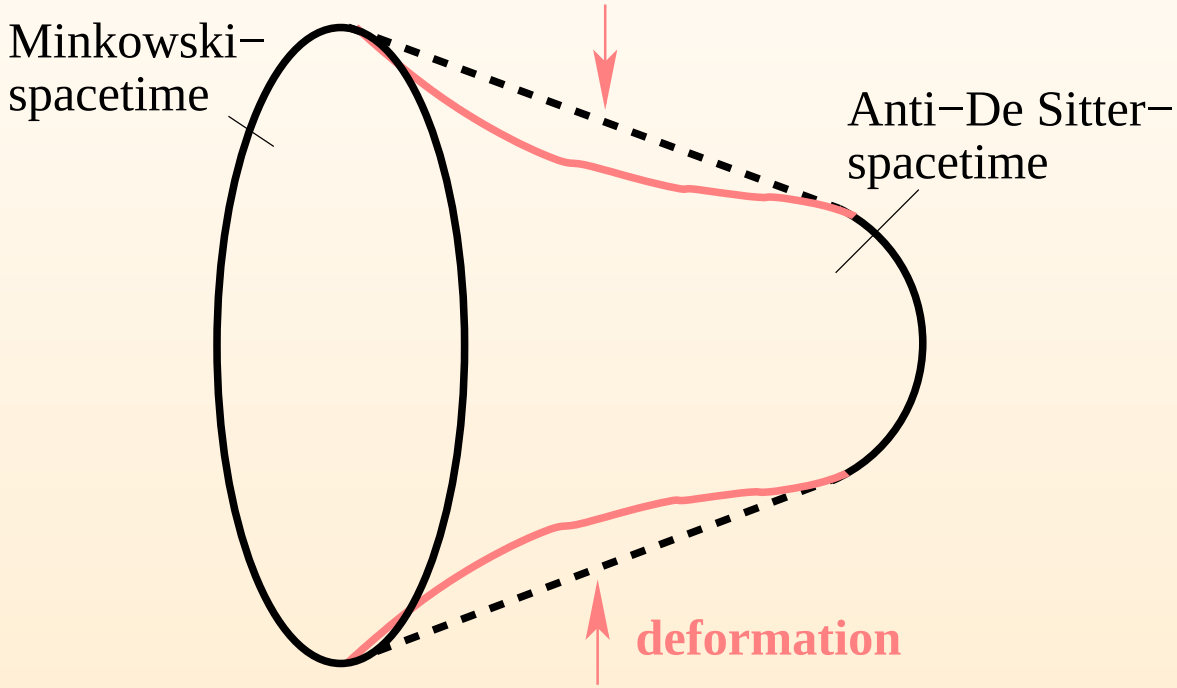
1. Symmetry requirements are relaxed in a controlled way

   $\Rightarrow$ RG flows

   $\Rightarrow$ Theories with confinement

2. More degrees of freedom are added (Example: quarks, electrons)

Deformations of AdS_5 and S^5



Fifth Dimension $\Leftrightarrow$ Energy scale

Supersymmetry breaking by deforming the sphere S^5

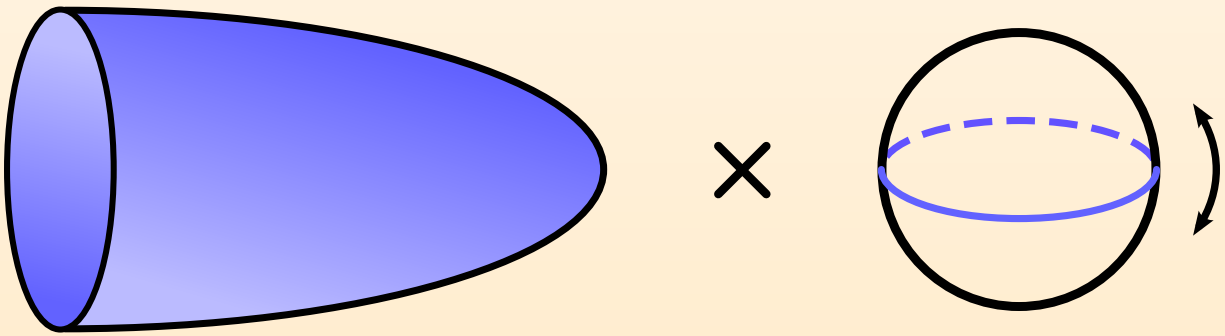
Quarks in the AdS/CFT correspondence

Add D7-Branes (Hypersurfaces)

	0	1	2	3	4	5	6	7	8	9
N D3	X	X	X	X						
1,2 D7	X	X	X	X	X	X	X	X		

Quarks: Low energy limit of open strings between D3- and D7-Branes

Meson masses from fluctuations of the hypersurface (D7-Brane):



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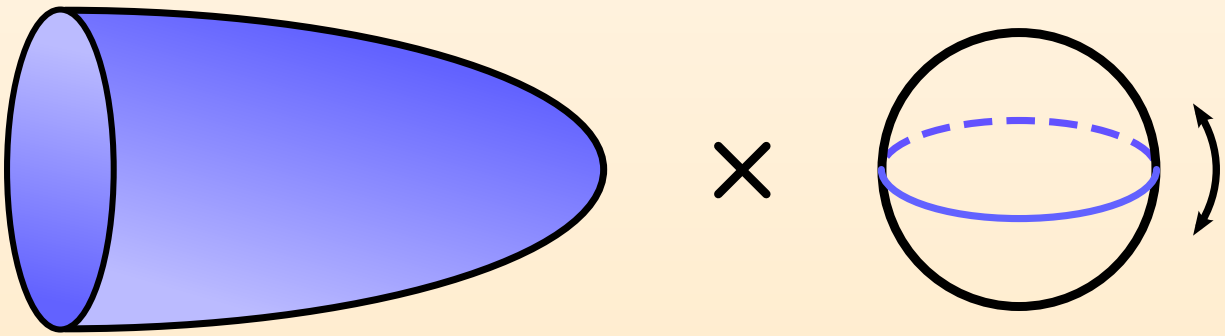


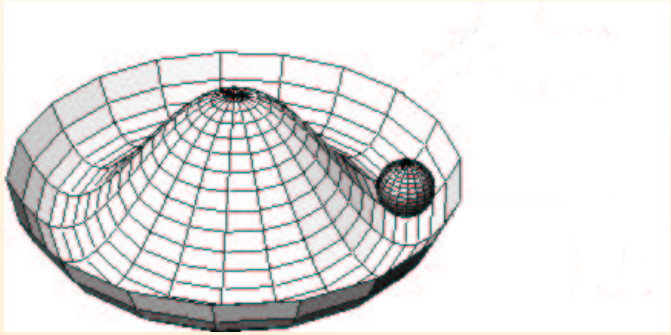
Confinement: $\mathrm{AdS}_5 \times S^5$-metric has to be deformed

Light mesons

Babington, J.E., Evans, Guralnik, Kirsch PRD 2004

Gravitational realization of

Spontaneous chiral symmetry breaking

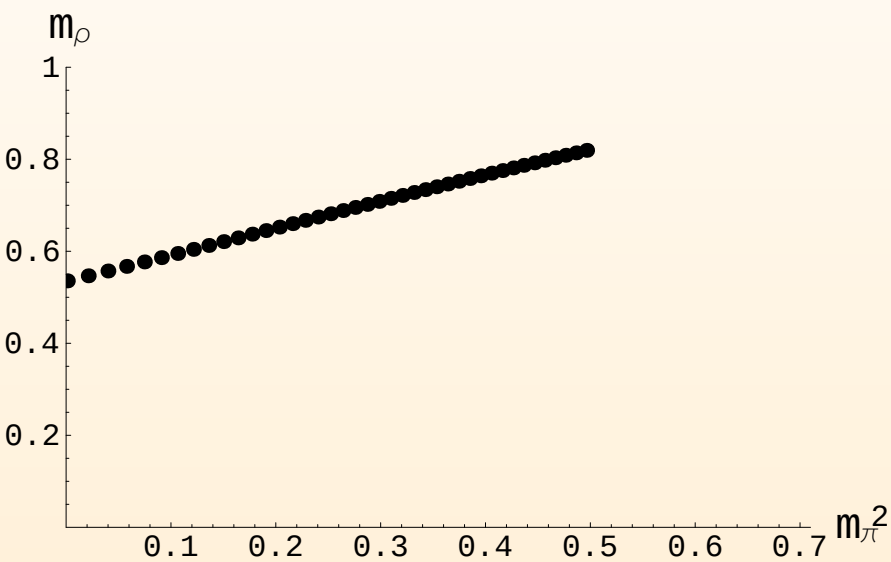


New ground state given by quark condensate $\langle \bar{\psi}\psi \rangle$

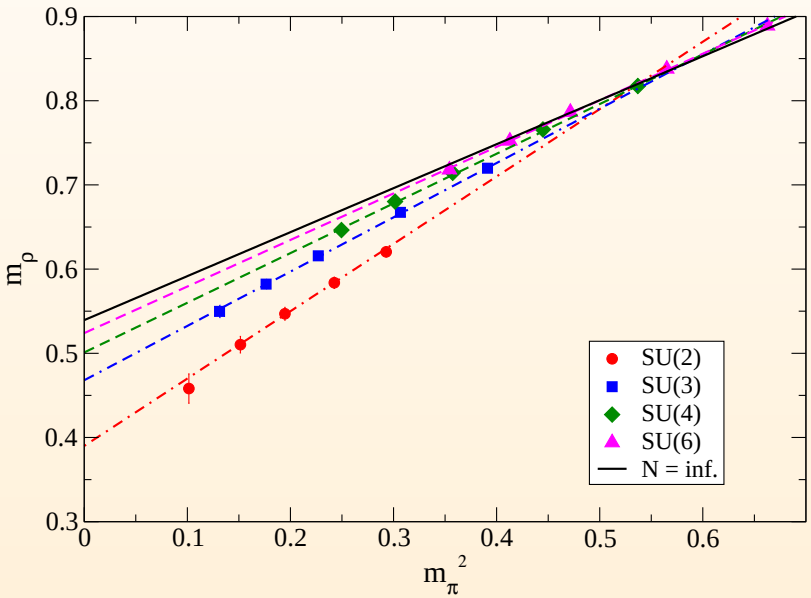
Spontaneous symmetry breaking $\rightarrow$ Goldstone bosons (Mesons)

Comparison to lattice gauge theory

Mass of ρ meson as function of π meson mass^2 (for $N \to \infty$)



J.E., Evans, Kirsch, Threlfall '07, review EPJA



Lattice: Lucini, Del Debbio, Patella, Pica '07

AdS/CFT result:

$$\frac{m_\rho(m_\pi)}{m_\rho(0)} = 1 + 0.307 \left(\frac{m_\pi}{m_\rho(0)} \right)^2$$

Lattice result (from Bali, Bursa '08): slope 0.341 ± 0.023

Prime example: Phase diagram of strongly interacting matter

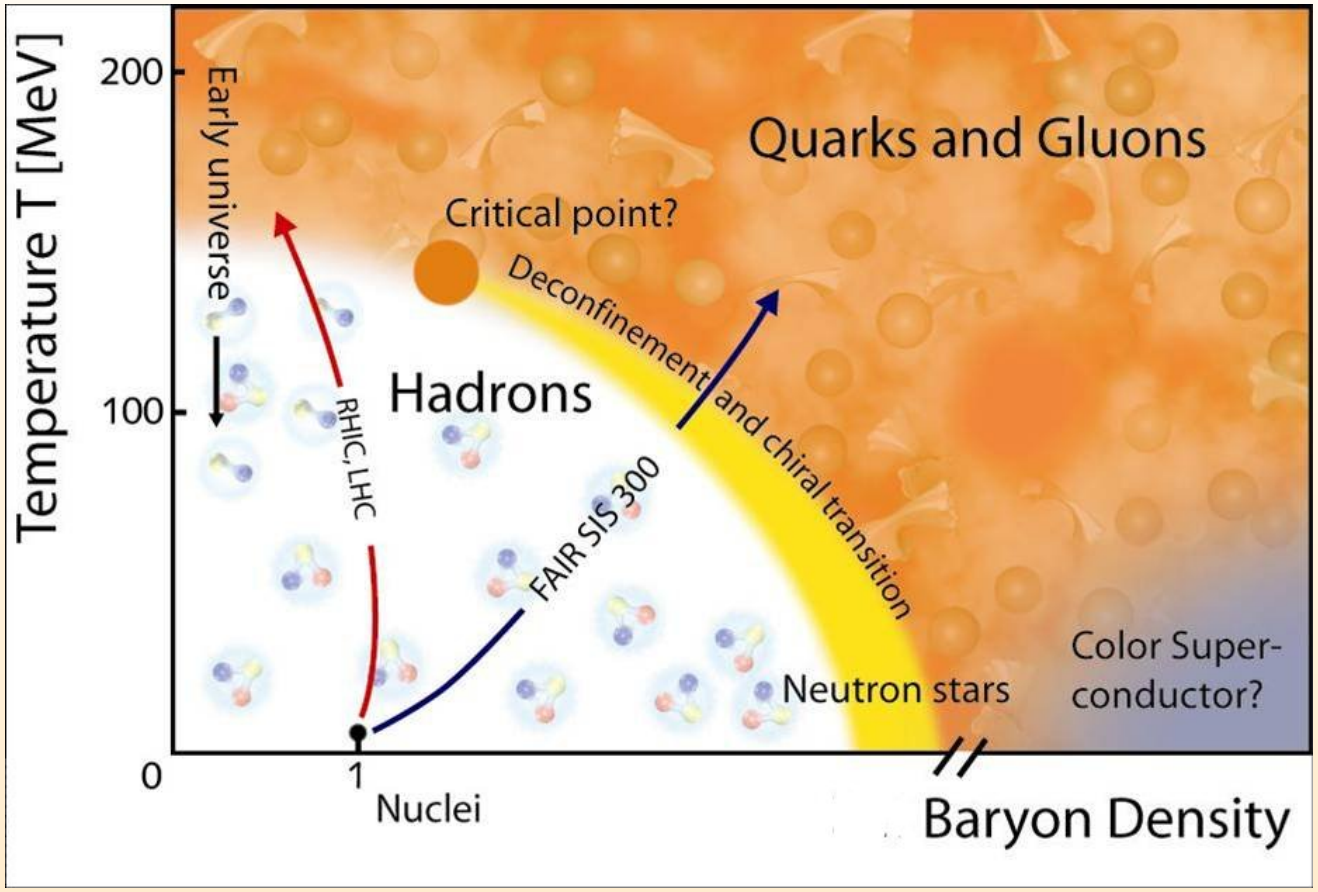


Bild: CBM @ FAIR, GSI

Quark-gluon plasma:

Strongly coupled state of matter above deconfinement temperature T_d

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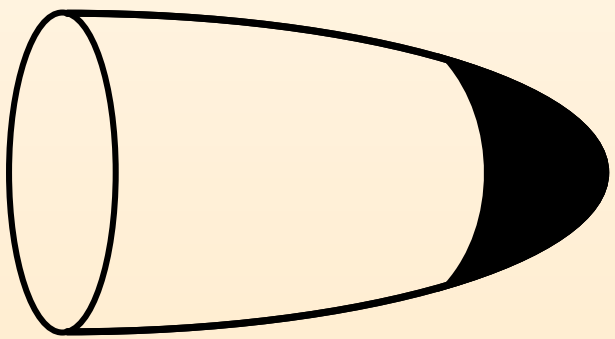
AdS/CFT dual of field theory at finite temperature: AdS black hole

(Witten 1998)



Hawking temperature $\Leftrightarrow$ temperature in the dual quantum field theory

Universal result:

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Ratio of shear viscosity and entropy

$$\frac{\eta}{s} = \frac{\hbar}{k_B}\frac{1}{4\pi}$$

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Calculated using AdS/CFT at finite temperature

using linear response formalism and graviton propagation through AdS

Policastro, Son, Starinets 2001; Kovtun, Son, Starinets 2004

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Lower bound: smallest possible value

Hydrodynamics

Long-distance expansion

$$T^{\mu\nu} = (\epsilon + P)u^\mu u^\nu + P g^{\mu\nu} - P^{\mu\alpha}P^{\nu\beta}\left[\eta\left(\partial_\alpha u_\beta + \partial_\beta u_\alpha - \frac{2}{3}g_{\alpha\beta}\partial_\lambda u^\lambda\right) + \zeta g_{\alpha\beta}\partial_\lambda u^\lambda\right]$$

$$P^{\mu\nu} = g^{\mu\nu} + u^\mu u^\nu$$

Shear viscosity η from energy-momentum tensor $T_{\mu\nu}$

Kubo formula

$$\eta = \lim_{\omega\to 0}\frac{1}{2\omega}\int dt d^3x\, e^{i\omega t}\,\langle[T_{xy}(t,x)T_{xy}(0,0)]\rangle$$

$\langle[T_{xy}(t,x)T_{xy}(0,0)]\rangle$ obtained from propagation of graviton through AdS space

Policastro, Son, Starinets 2001

Chiral vortex effect

J.E., Haack, Kaminski, Yarom 0809.2488, JHEP 2008.

Action of $\mathcal{N} = 2$, $d = 5$ Supergravity:
From compactification of $d = 11$ supergravity on a Calabi-Yau manifold

$$S = -\frac{1}{16\pi G_5} \int \left[\sqrt{-g} \left(R + 12 - \frac{1}{4} F^2 \right) - \frac{1}{2\sqrt{3}} A \wedge F \wedge F \right] d^5 x$$

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Chern-Simons term leads to axial anomaly for boundary field theory:

$$\partial^\mu J_\mu = \frac{1}{16\pi^2} \varepsilon_{\mu\nu\rho\sigma} F^{\mu\nu} F^{\rho\sigma}$$

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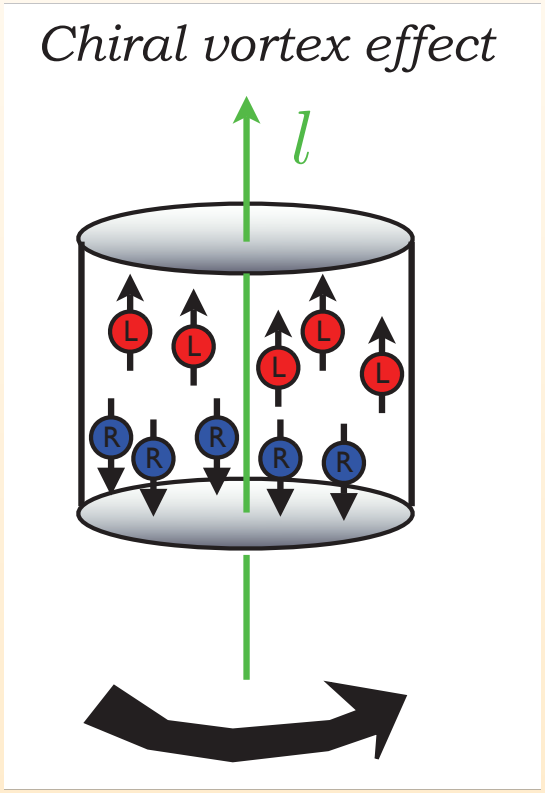
Contribution to relativistic hydrodynamics, proportional to angular momentum:

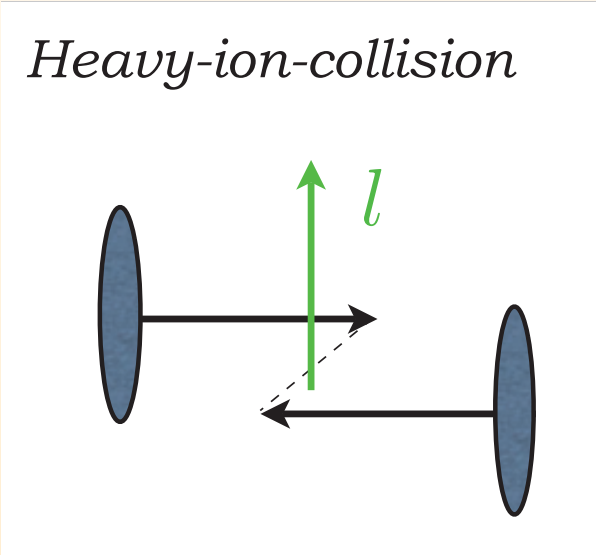
$J_\mu = \rho u_\mu + \xi\omega_\mu$, $\quad \omega_\mu = \frac{1}{2}\epsilon_{\mu\nu\sigma\rho}u^\nu\partial^\sigma u^\rho$, in fluid rest frame $\vec{J} = \frac{1}{2}\xi\nabla\times\vec{v}$

Chiral vortex effect

Prediction for quark-gluon plasma:

Chiral separation: In a volume of rotating quark matter, quarks of opposite helicity move in opposite directions. (Son, Surowka 2009)





Proposal for experimental confirmation (Oz, Keren-Zur 2010):
Enhanced production of spin-excited hadrons along rotation axis

Chiral separation: Relativistic quantum effect

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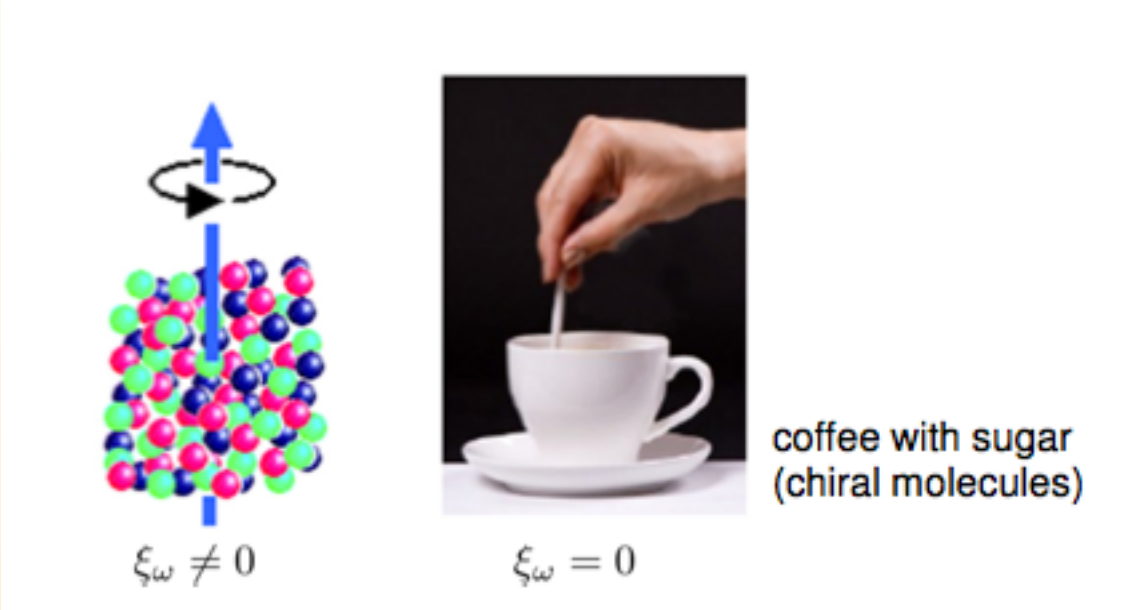
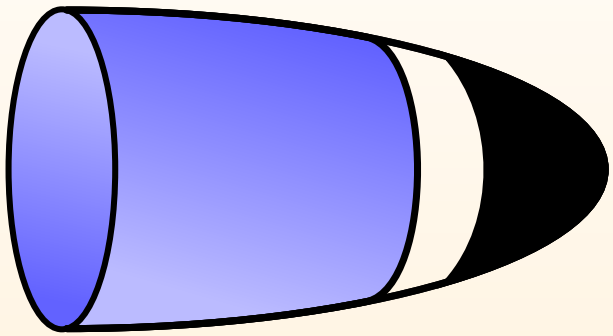
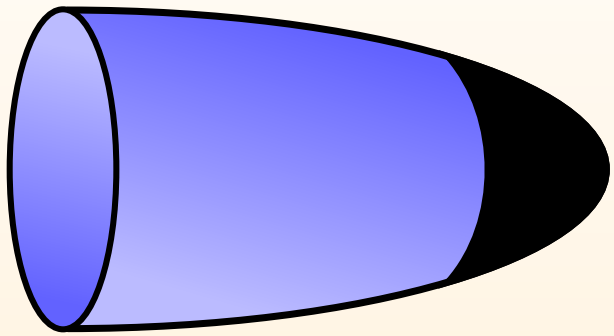


Quarks and mesons at finite temperature

Babington, J.E., Evans, Guralnik, Kirsch 2004; Mateos, Myers, Thomson 2006





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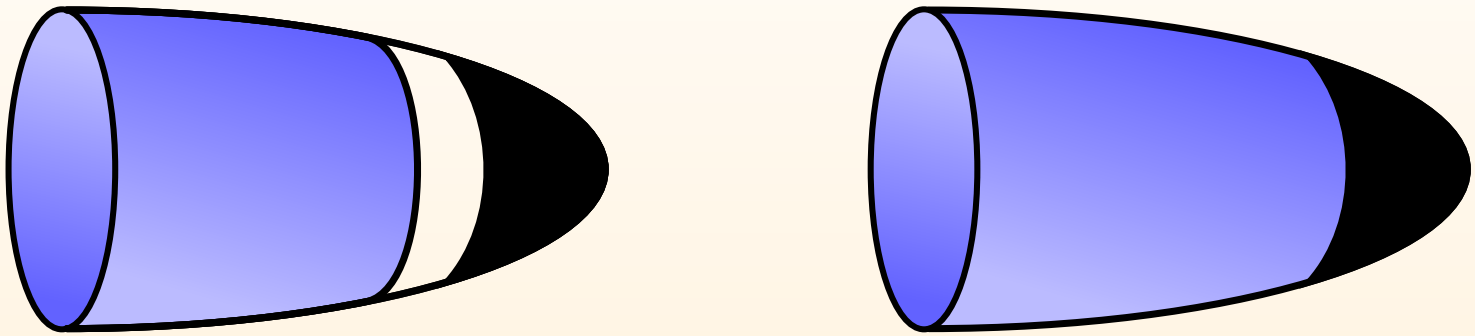

Prediction:

Mesons survive deconfinement if quark mass m_q sufficiently large

1st order fundamental phase transition at $T = T_f$ where mesons dissociate

$T_f \sim 0.12 M_{\text{meson at } T=0}$

Masses and decay widths of mesons - Spectral functions

Standard procedure in D3/D7: Mateos, Myers et al 2003

Meson masses calculated from linearized fluctuations of D7 embedding

Fluctuations: $\delta w(x,\rho) = f(\rho) e^{i(\vec{k}\cdot\vec{x}-\omega t)}$, $M^2 = -k^2$

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For black hole embeddings, ω develops negative imaginary part

$\Rightarrow$ damping $\Rightarrow$ decay width

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Make contact with hydrodynamics: Starinets, Kovtun ....

Spectral function determined by poles of retarded Green function

Quasinormal modes

Identify mesons with resonances in spectral function

Landsteiner, Hoyos, Montero 2006

Spectral functions

Spectral function for flavour current from

imaginary part of retarded Green function

$$G^R{}_{\mu\nu}(\omega, \boldsymbol{k}) = -i \int d^4x \, e^{i\,\vec{k}\vec{x}} \, \theta(x^0) \, \langle [J_\mu(\vec{x}), J_\nu(0)] \rangle$$

Correlator calculated from propagation through AdS black hole space

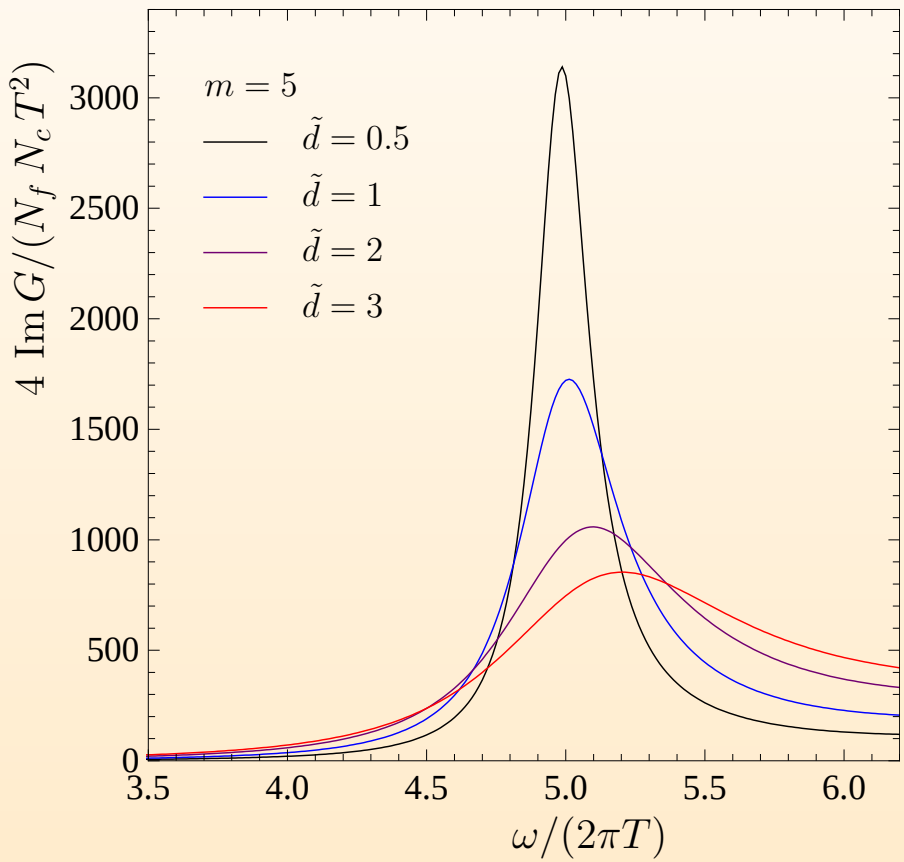
Here: J_μ: flavour current dual to gauge field on D7 brane

Spectral function $\Rightarrow$ Quasiparticle spectrum (vector mesons; ρ)

Finite baryon chemical potential

Easy to introduce: Non-trivial for time component of gauge field: $A_t \sim \mu + \frac{d}{r^2}$

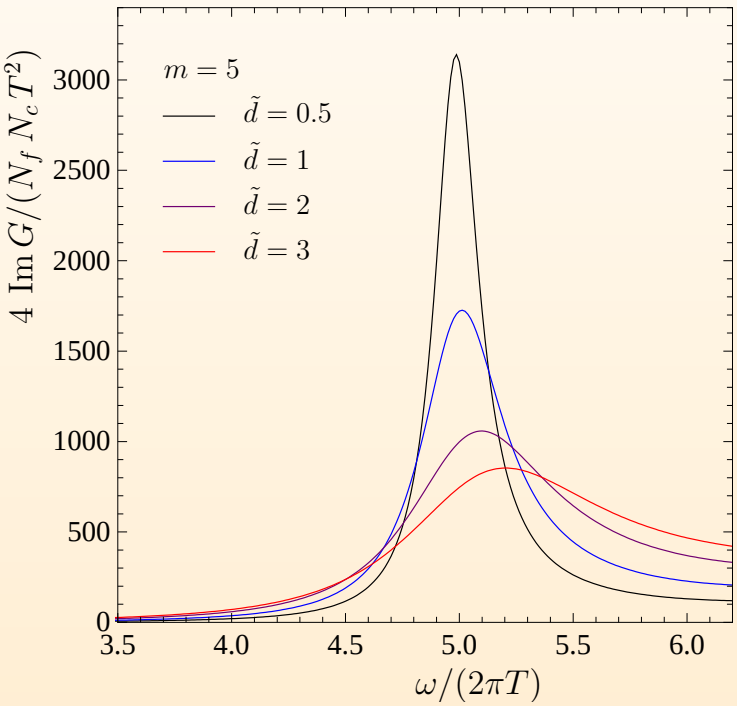
ρ vector meson spectral function in dense hadronic medium



AdS/CFT result (J.E., Kaminski, Kerner, Rust 2008)

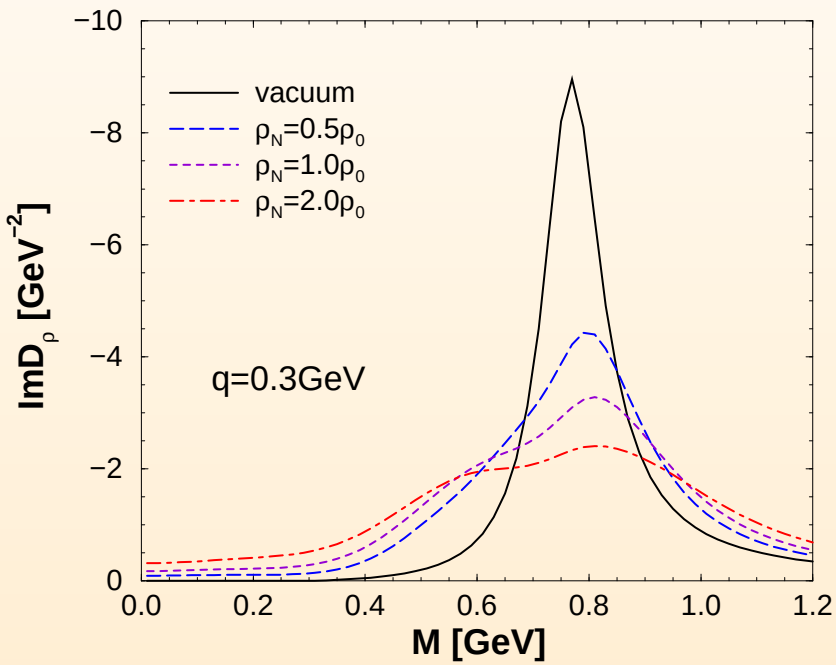
Spectral function at finite baryon density

ρ vector meson spectral function in dense hadronic medium



AdS/CFT result

(J.E., Kaminski, Kerner, Rust 2008)

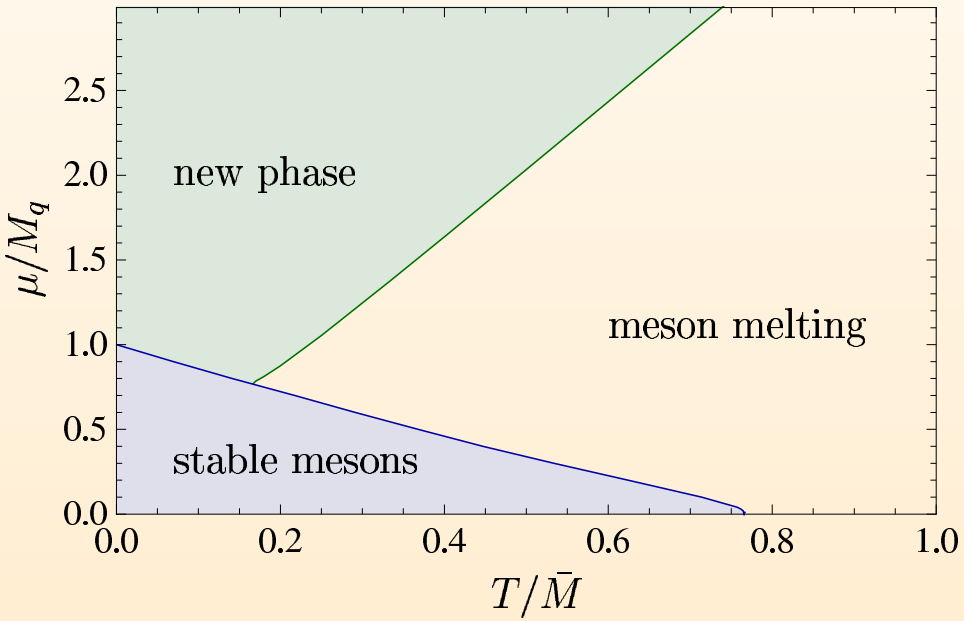


Field theory (Rapp, Wambach 2000)

Chemical Potential and Finite Density for Isospin ($SU(2)$)

u- and d-Quarks

Phase diagram



Instability!

3. Superfluidity and Superconductivity

New solution to the equations of motion with lower free energy

Finite SU(2) isospin chemical potential

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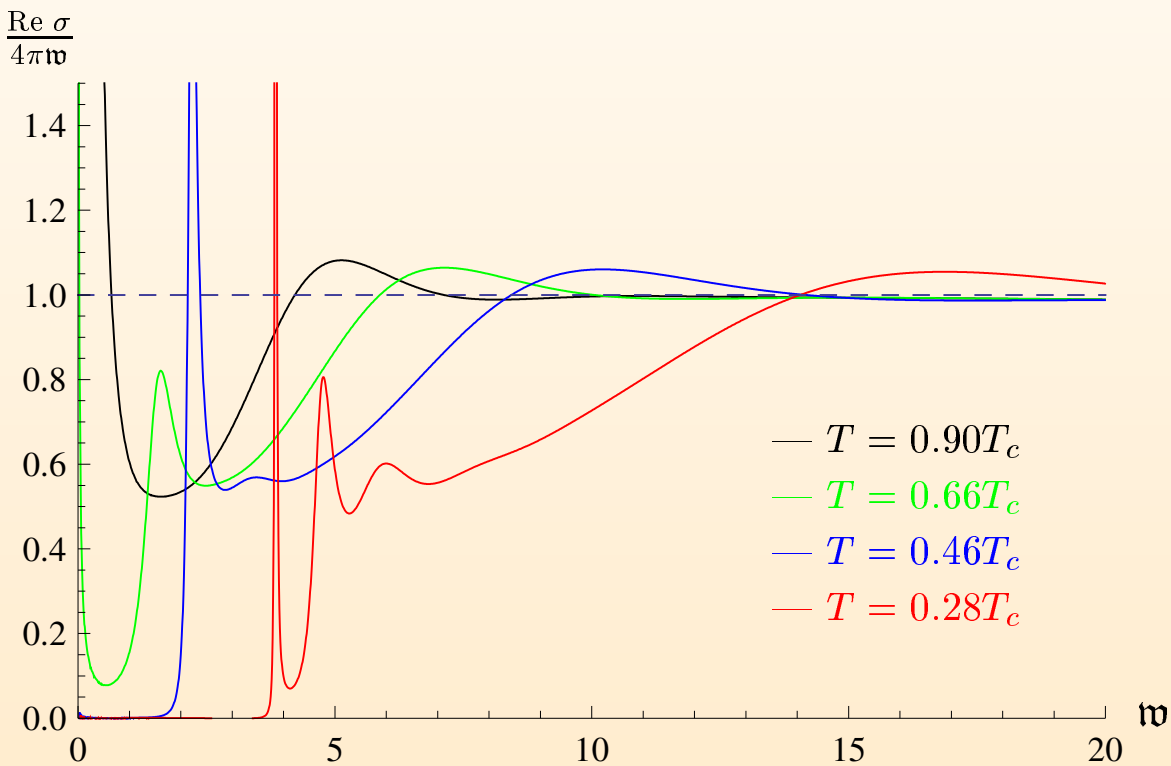
ρ meson condensate (p-wave, triplet pairing)

Ammon, J.E., Kaminski, Kerner 2008

The new ground state is a superfluid.

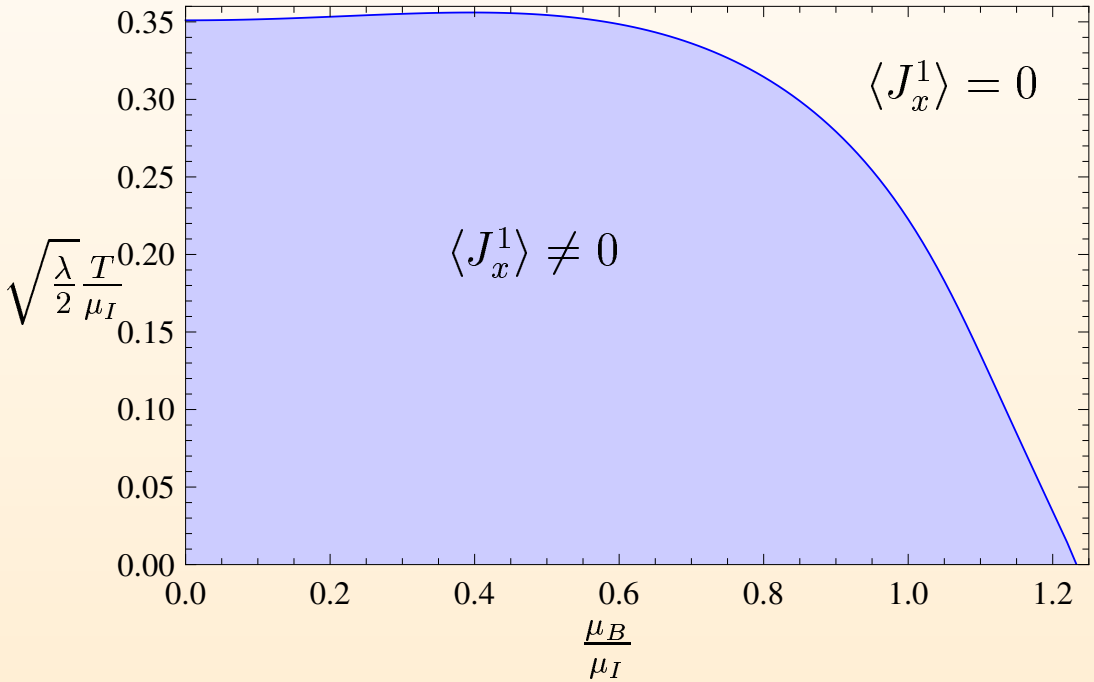
Frequency-dependent conductivity from spectral function



$\mathfrak{w} = \omega/(2\pi T)$

Prediction: Frictionless motion of mesons through the plasma

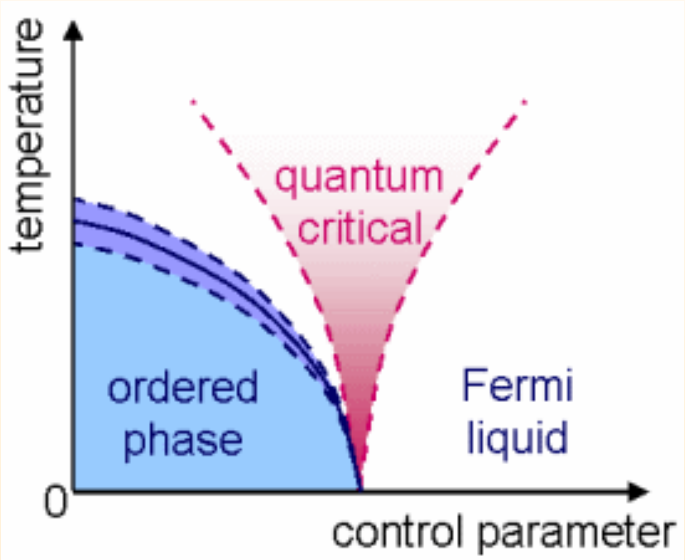
Two chemical potentials: Isospin and Baryon Chemical Potential



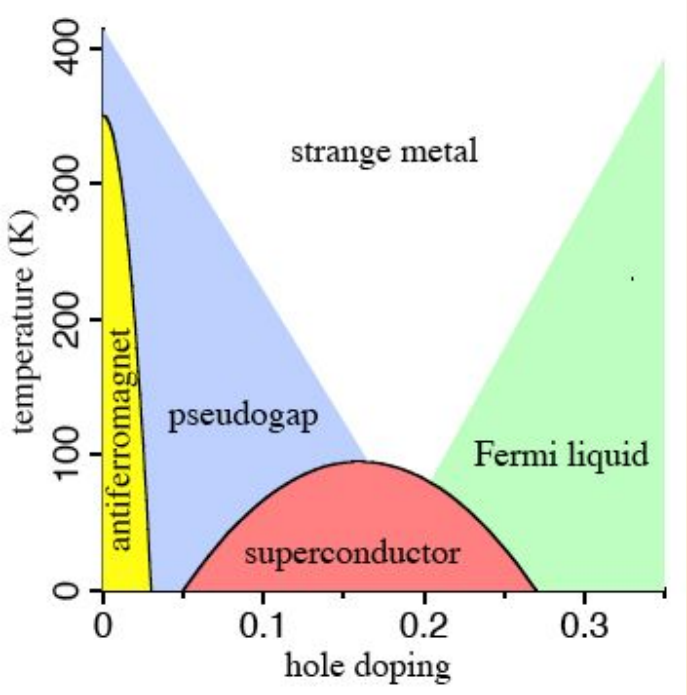
J.E., Grass, Kerner, Ngo 2010

Example for Quantum Phase Transition

Phase diagrams



Quantum phase transition



Superconductor

Shear viscosity in presence of ρ meson condensate

Universal result:

$$\frac{\eta}{s} = \frac{1}{4\pi}\frac{\hbar}{k_B}$$

Relies on space-time isotropy

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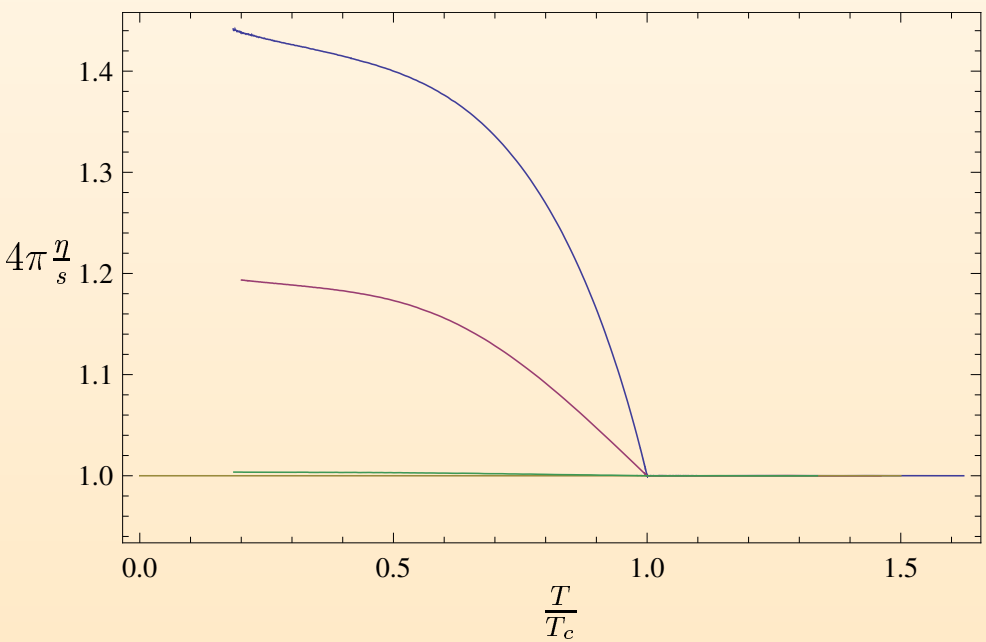
Relies on space-time isotropy

ρ meson condensate: Anisotropy, Lorentz symmetry broken
Shear viscosity becomes tensor;
one of the components becomes temperature dependent (non-universal)



J.E., Kerner, Zeller
PLB 2010, JHEP 2011

External electromagnetic fields

A magnetic field leads to

ρ meson condensation and superconductivity in the QCD vacuum

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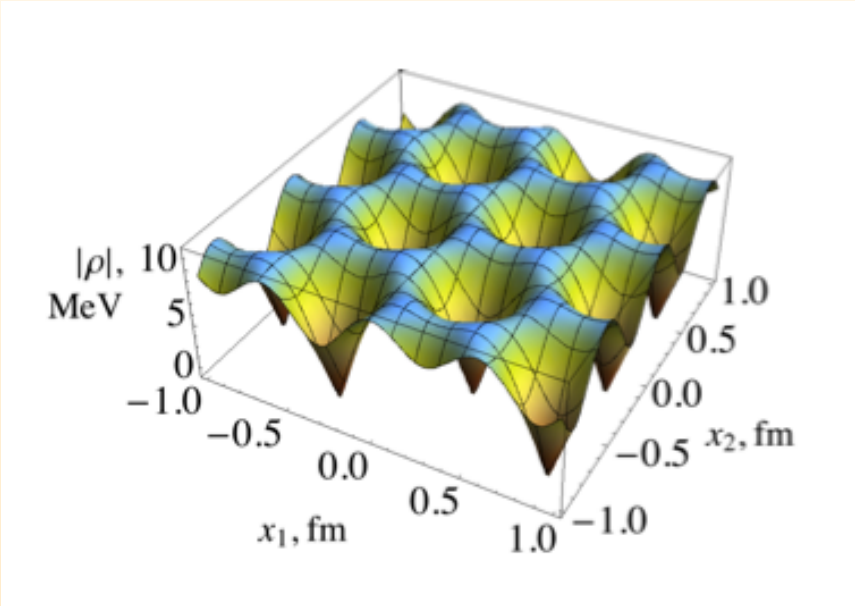
Effective field theory:
(Chernodub)



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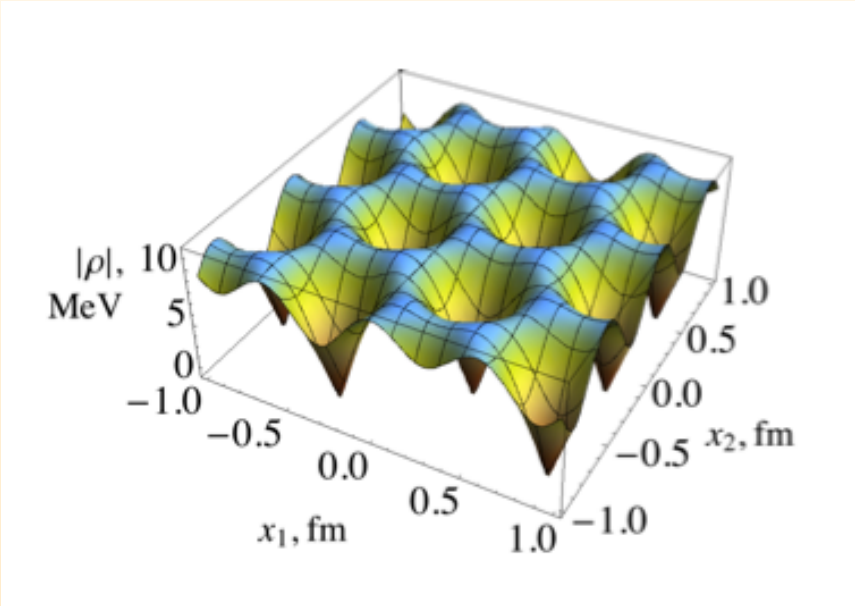
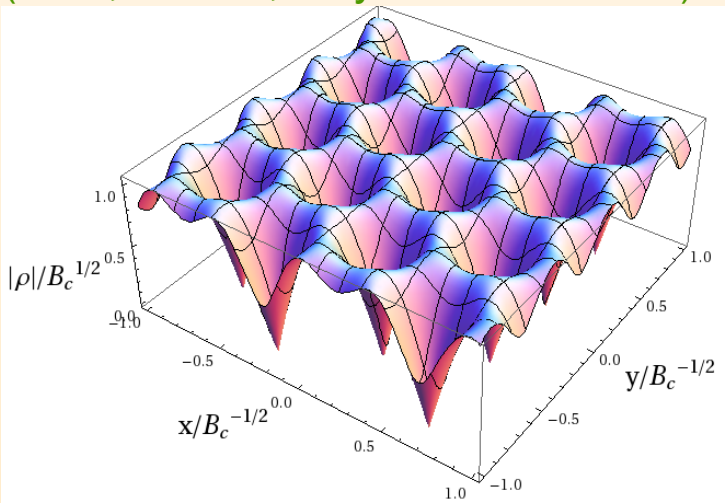


Gauge/gravity duality
magnetic field in black hole supergravity background
(J.E., Kerner, Strydom PLB 2011)



Finite B field in gauge/gravity duality

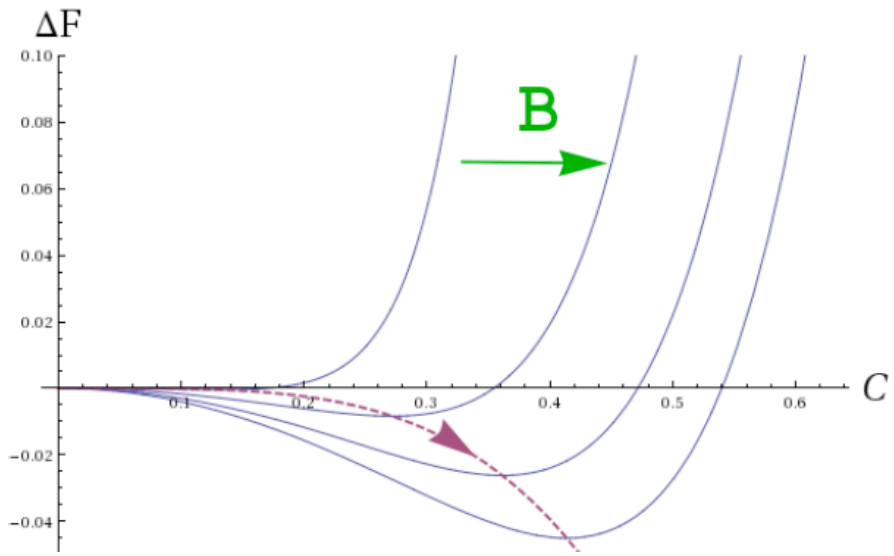
Easy to realize conceptually: $A_t \to A_x$

The model:

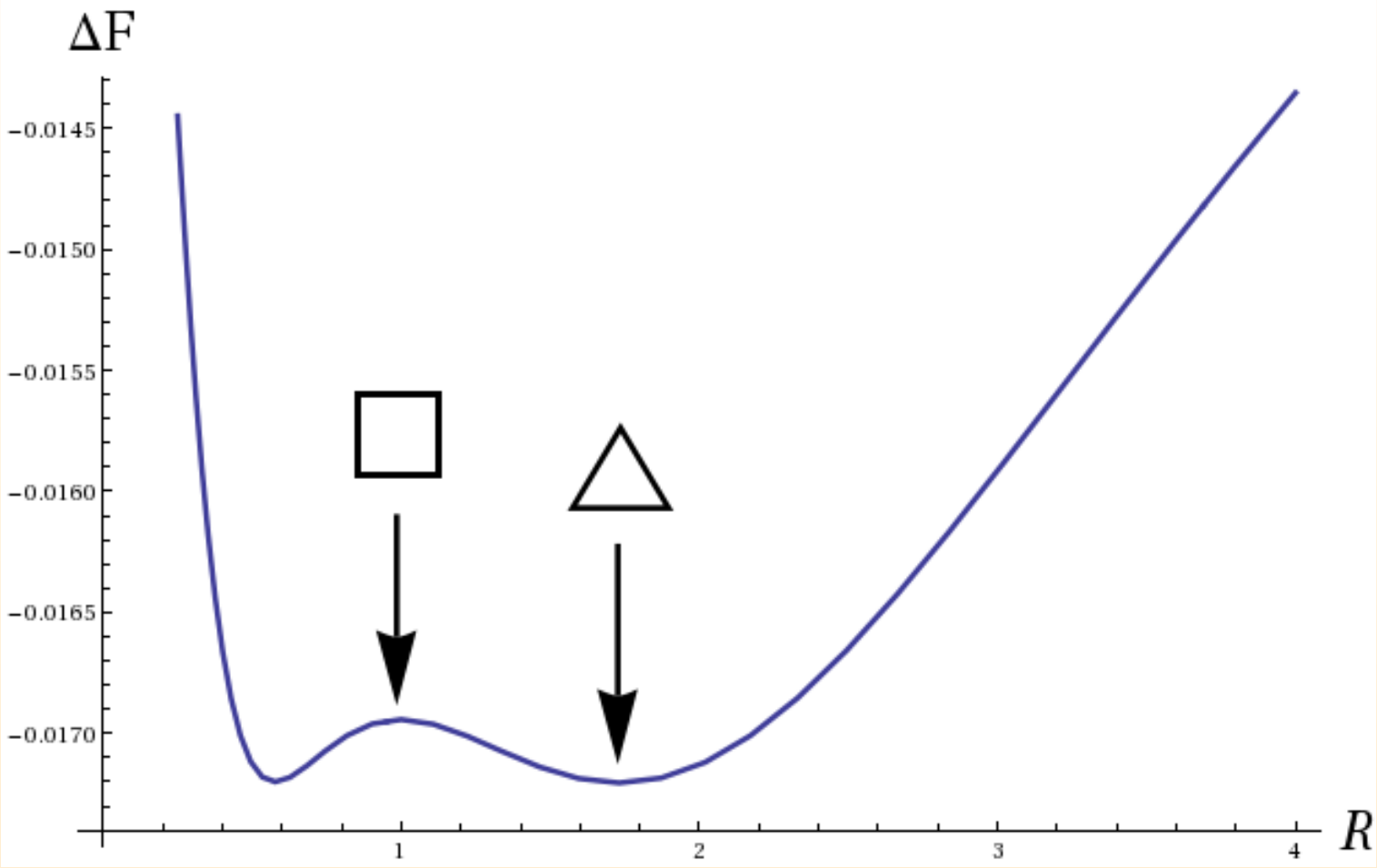
$$S = \int d^5x\sqrt{-g}\left[\frac{1}{16\pi G_N}\left(R + \frac{12}{L^2}\right) - \frac{1}{4\hat{g}^2}\mathrm{tr}F_{\mu\nu}F^{\mu\nu}\right] + S_{\mathrm{bdy}}$$

Subleading term in E_x: condensate

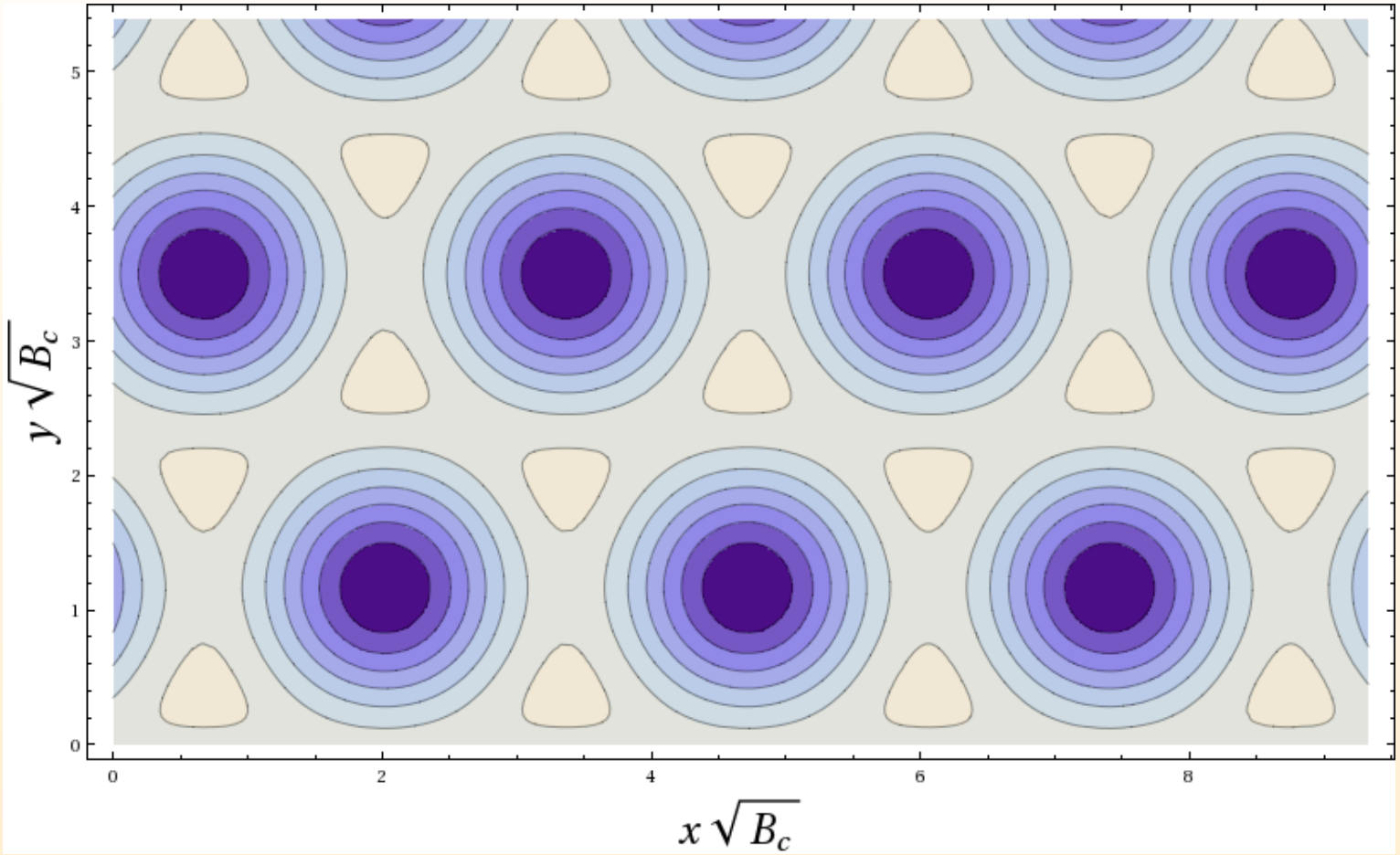
Free energy



Free energy for different lattices



Contour plot of triangular lattice



Outlook: Universality and Homes' Law

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Universal result from gauge/gravity duality for hydrodynamics:

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$$n_s(T=0) = C\sigma(T_c)T_c$$

n_s: Superfluid density; σ: Conductivity

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Found experimentally to great accuracy

Outlook: Homes' Law and Universality

Relation between η/s and Homes' Law:

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J.E., Kerner, Müller 1206.5305, JHEP 2012

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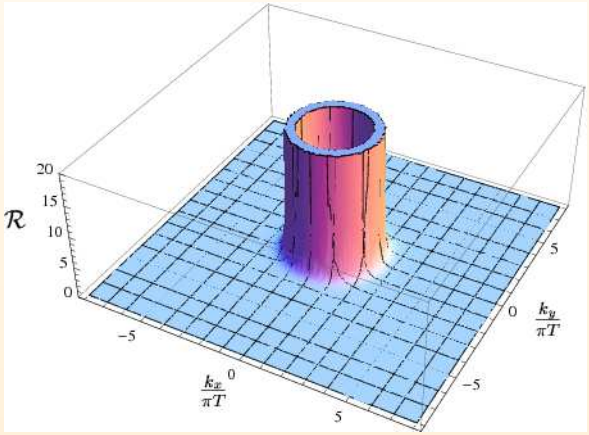
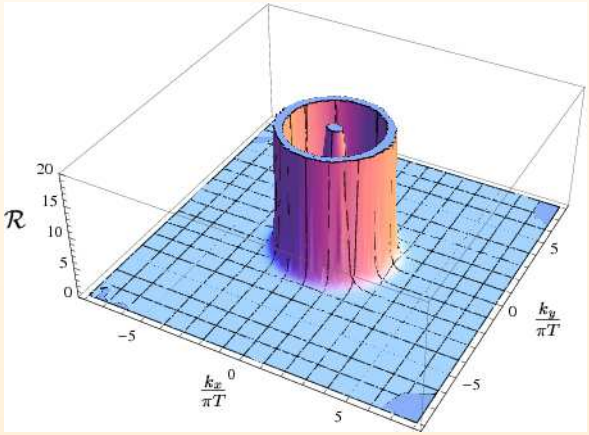
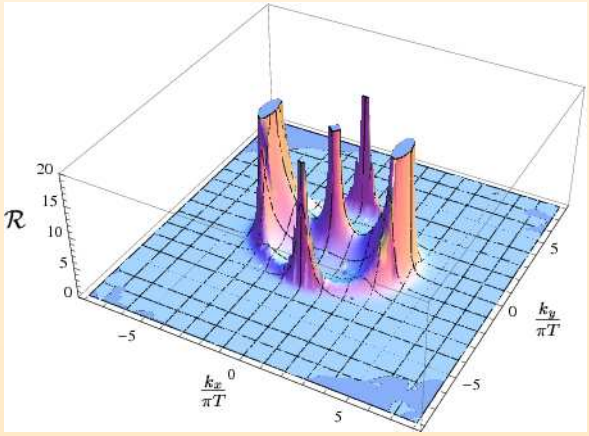
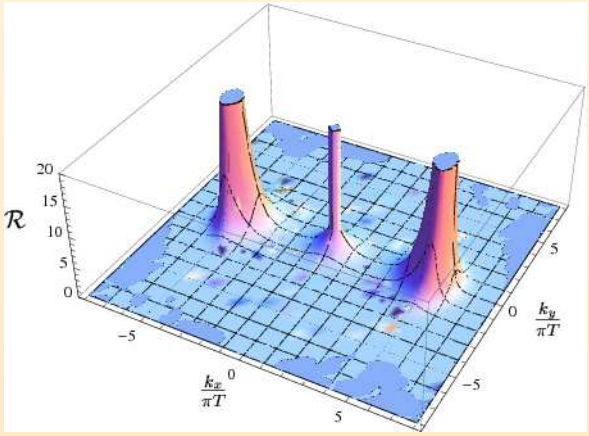
Sum rule ${\omega_P}^2 = 8 \int\limits_0^\infty \mathrm{Re}\sigma(\omega)$

Implement lattice for Drude peak









Fermionic Excitations in p-wave Superconductors

Ammon, J.E., Kaminski, O'Bannon 2010

Conclusion and Outlook

Generalized AdS/CFT correspondence: Gauge/Gravity Duality

- New relation between fundamental and applied aspects of theoretical physics

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Generalized AdS/CFT correspondence: Gauge/Gravity Duality

- New relation between fundamental and applied aspects of theoretical physics

- Identification of Universal Behaviour
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- Comparison and combination with other approaches
- New relations between different areas of physics