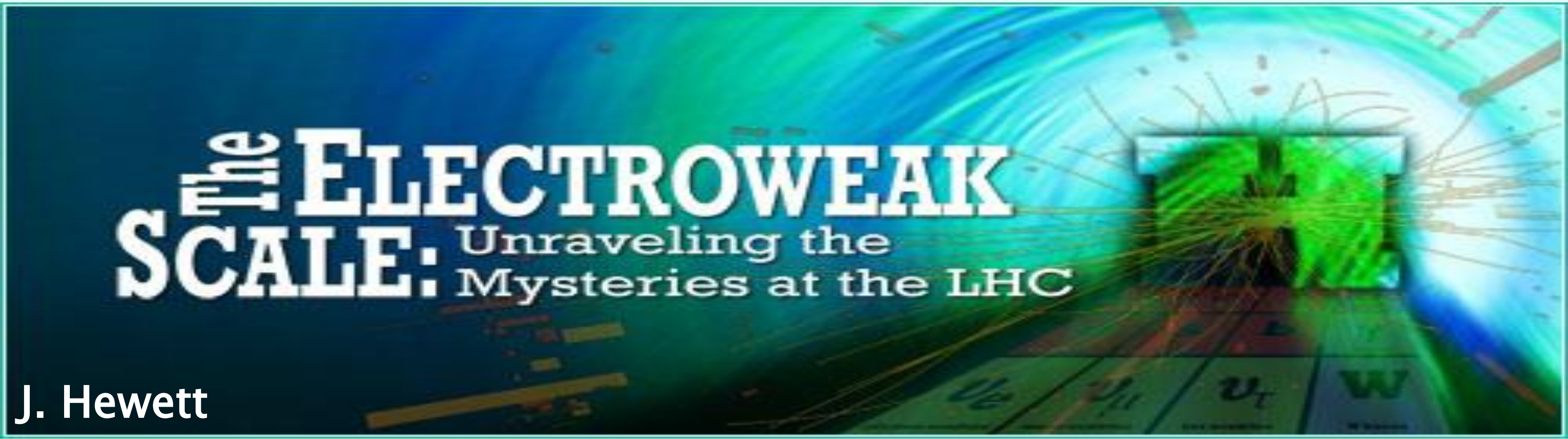
Supersymmetry Basics



J. Hewett

Basic SUSY References

- *A Supersymmetry Primer,* Steve Martin
  hep-ph/9709356

- *Theory and Phenomenology of Sparticles*,
  Manual Drees, Rohini Godbole, Probir Roy
  World Scientific

- *Weak Scale Supersymmetry: From Superfields to Scattering Events*, Howard Baer and Xerxes Tata
  Cambridge University Press

Supersymmetry is a New Symmetry

Symmetries that we know

- Translations, rotations and boosts: Spacetime
- Isospin (approx): Internal symmetry ($\pi^{\pm,0}$, n,p)
- SM Gauge Invariance
- Global Baryon and Lepton number

SYMMETRIES OF NATURE	Exact	Broken
Gauge	$U(1)_{EM,}$ $SU(3)_c$	$SU(2) \times U(1)_Y$
Global	B, L	L_e , L_μ , L_τ
Spacetime	Rotations, Boosts, Translations	SUSY

Supersymmetry is a New Symmetry

- An extension of the Poincare algebra

  P_μ (translations)

  $M_{\mu\nu}$ (rotations and boosts)

  Q_α (SUSY transformation)

$$\{Q_\alpha , Q_\beta \} = \sigma^\mu{}_{\alpha\beta} P_\mu$$

- Supsersymmetry: a translation in Superspace

  Spacetime (x_μ) → Superspace (x_μ, θ)

  SUSY transformation:

  $$x_\mu \rightarrow x'_\mu = x_\mu + i/2\, \bar{\epsilon}\gamma_\mu\, \theta$$

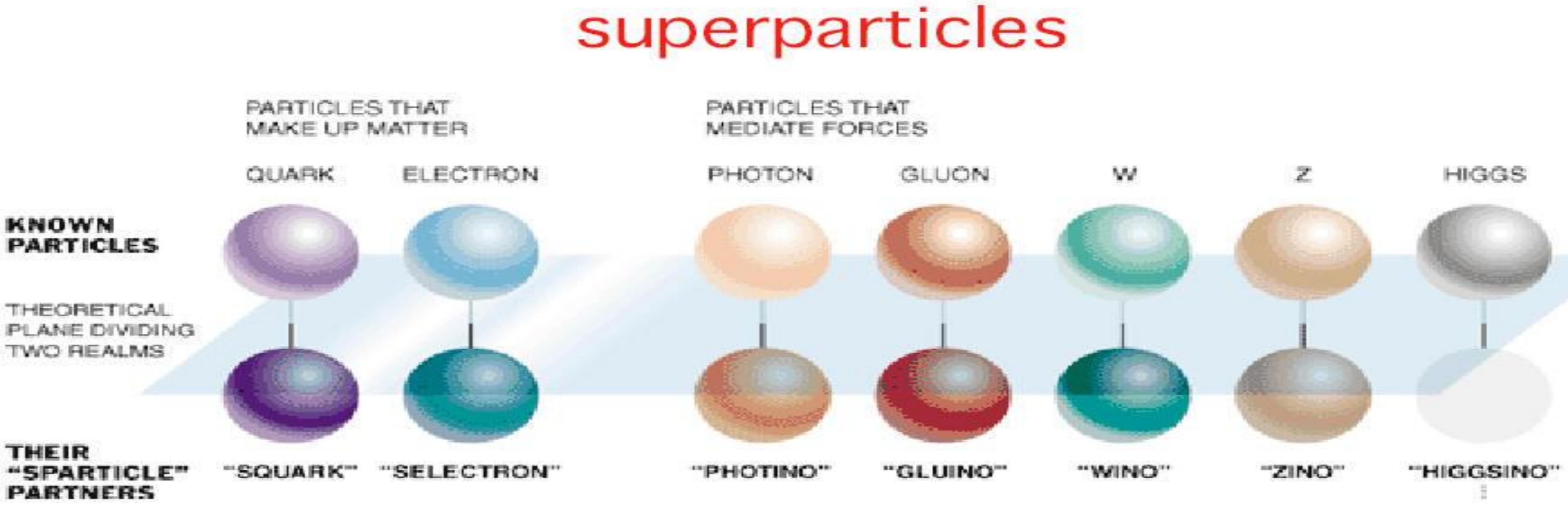
  $$\Theta \rightarrow \theta' = \theta + \epsilon$$

Supersymmetry is a New Symmetry

- Q_α is a fermionic charge that relates particles of different spins

$$Q_\alpha \left| \begin{matrix}\text{Fermion}\\ \text{Boson}\end{matrix} \right\rangle = \left| \begin{matrix}\text{Boson}\\ \text{Fermion}\end{matrix} \right\rangle$$

- Every SM particle has a SUSY partner (of equal mass), identical quantum #'s except for spin



Superpartners

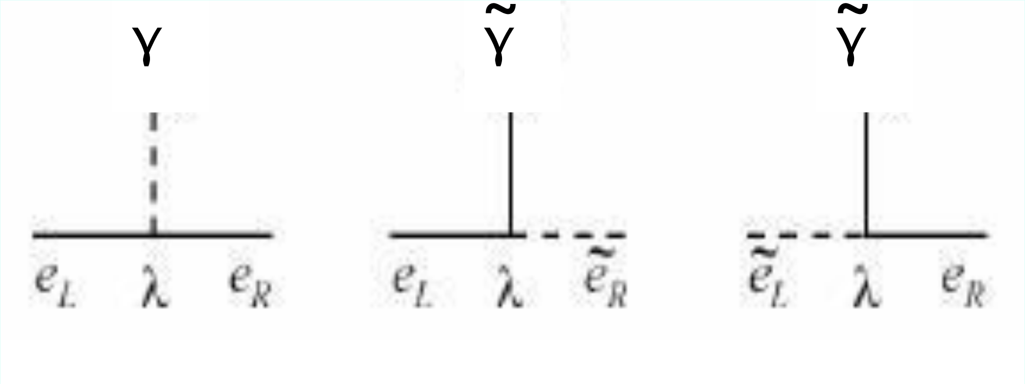
- Translations:

  Particle P at point x → Particle P at point x’

- Supersymmetry:

  Particle P at point x → Particle P̃ at point x

  - P and P̃ differ by spin ½: fermions ↔ bosons
  - P and P̃ are identical in all other ways (mass, couplings….)



Constructing a SUSY Model

Isospin

$$\begin{bmatrix} P \\ n \end{bmatrix}$$

N = nucleon field

Isospin Calculus

Isospin invariant action

Multiplets of the symmetry transform into one another

Supersymmetry

$$\begin{bmatrix} S \\ \psi \end{bmatrix}$$

$\hat{S}$ = Chiral Superfield

Superfield Calculus

SUSY invariant action

This leads to the Superpotential:

$$W = \mu H_1 H_2 - f^e_{ij} H_1 L_i \bar{E}_j - f^d_{ij} H_1 Q_i \bar{D}_j - f^u_{ij} Q_i H_2 \bar{U}_j$$

which describes all interactions

Counting Degrees of Freedom

- Bosonic d.o.f = Fermionic d.o.f

- SM Gauge Sector:

  SM gauge fields A_μ → 2 independent polarizations

  Superpartner gauginos, λ, → 2 d.o.f

  → Majorana spinors

- SM Fermion Sector:

  SM Fermions → 4 component Weyl fields

  Superpartner scalar → 2 scalar fields (Left and Right) for each SM fermion

Supersymmetric Scale

Where is SUSY?

- We know 3 fundamental constants
  - Special Relativity: speed of light, c
  - General Relativity: Newton's constant G
  - Quantum Mechanics: Planck's constant, h

- Together, they form the Planck scale

$$M_{\mathrm{Pl}} = \sqrt{\frac{hc}{G}} \approx 10^{19}\ \mathrm{GeV}$$

- SUSY scale can be anywhere, from 0 up to M_{Pl} !

Supersymmetric Scale: What we know

- SUSY is required by string theory to help relate quantum mechanics to gravity

  ➔ $M_{SUSY} < M_{string}$

- SUSY @ the EW scale provides Naturalness, Grand Unification, and a Dark Matter candidate

  ➔ $M_{SUSY} \sim M_{EW}$

The Hierarchy Problem

Energy (GeV)

10^{19} Planck

10^{16} GUT

desert

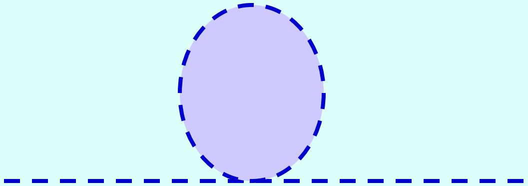
LHC → 10
3
Weak

All of known physics

10^{-18} Solar System Gravity

Quantum Corrections:

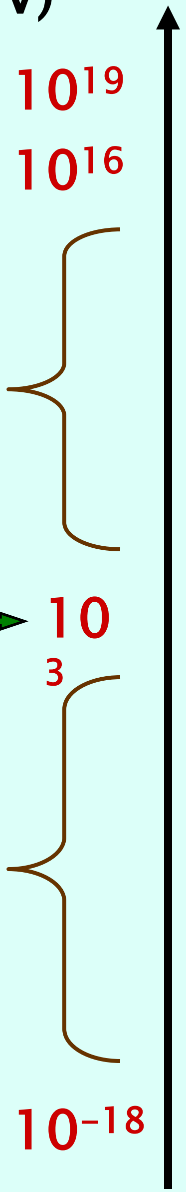
Virtual Effects drag Weak Scale to M_{Pl}



$\delta m_H^2 \sim$ ~ M_{Pl}^2

The Hierarchy Problem: Supersymmetry

Energy (GeV)



10^{19} Planck

10^{16} GUT

desert

LHC → 10^{3} Weak

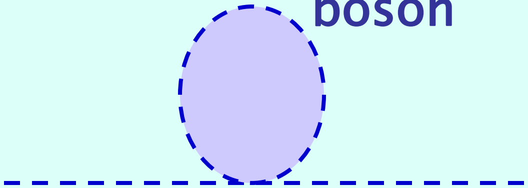
All of known physics

10^{-18} Solar System Gravity

Quantum Corrections:

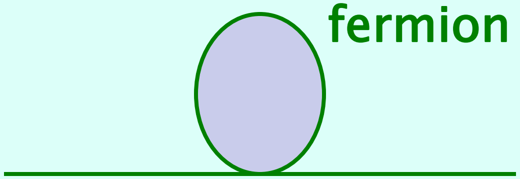
Virtual Effects drag Weak Scale to M_{Pl}

$\delta m_H^2 \sim$



$\sim M_{Pl}^2$

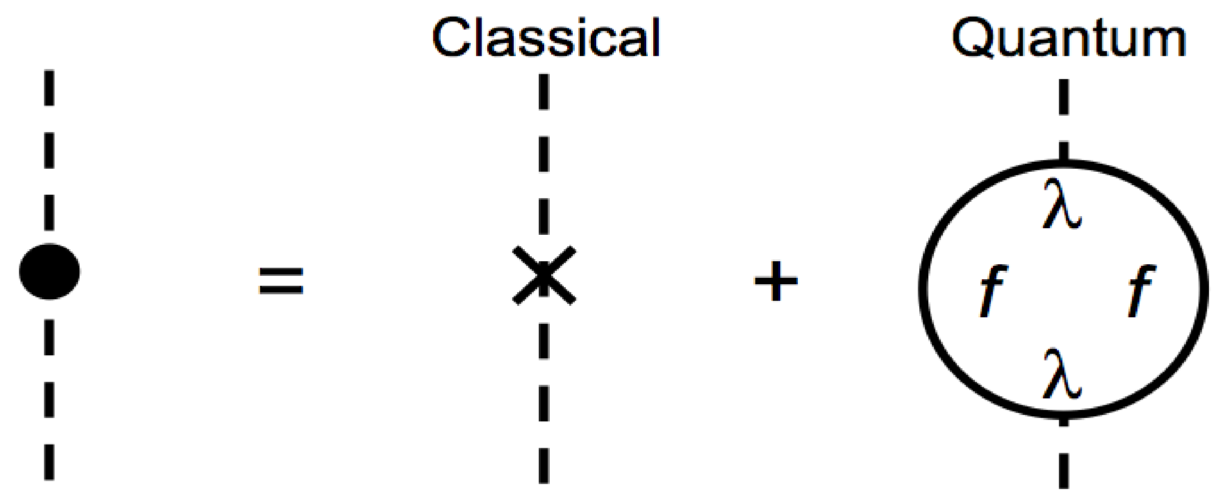
$\delta m_H^2 \sim$



$\sim - M_{Pl}^2$

Large virtual effects cancel order by order in perturbation theory

The Hierarchy Problem and Naturalness



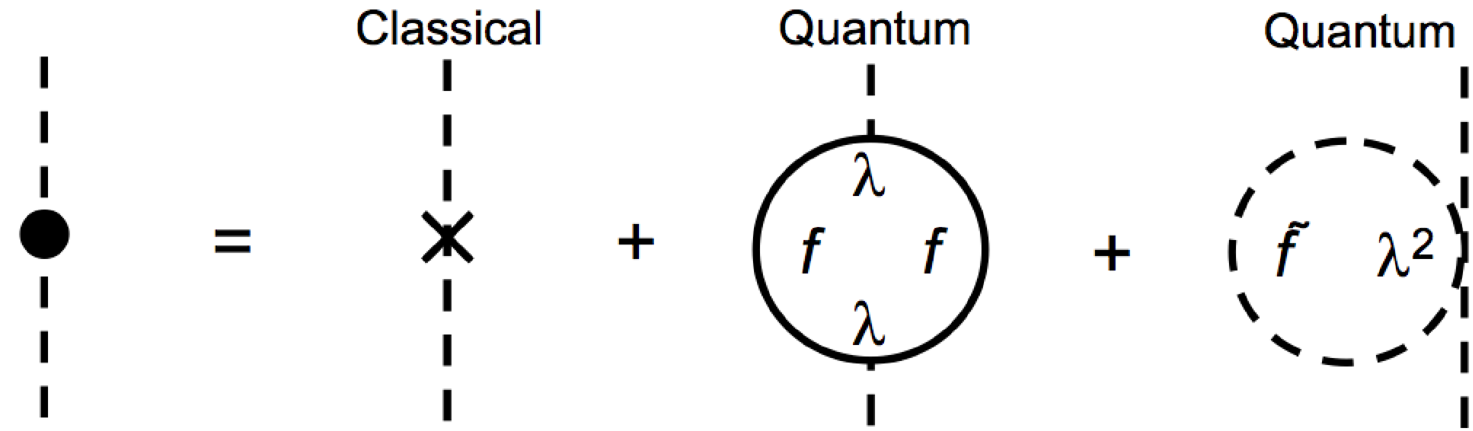
$$m_h^2 = (m_h^2)_0 - \frac{1}{16\pi^2}\lambda^2\Lambda^2$$

In the SM, m_h is naturally ~ Λ (= M_{Pl}) the highest energy scale

With $m_h = 125$ GeV, $M_{Pl} = 10^{19}$ GeV,

➔ requires cancellation in one part to 10^{34} !

Supersymmetry and Naturalness

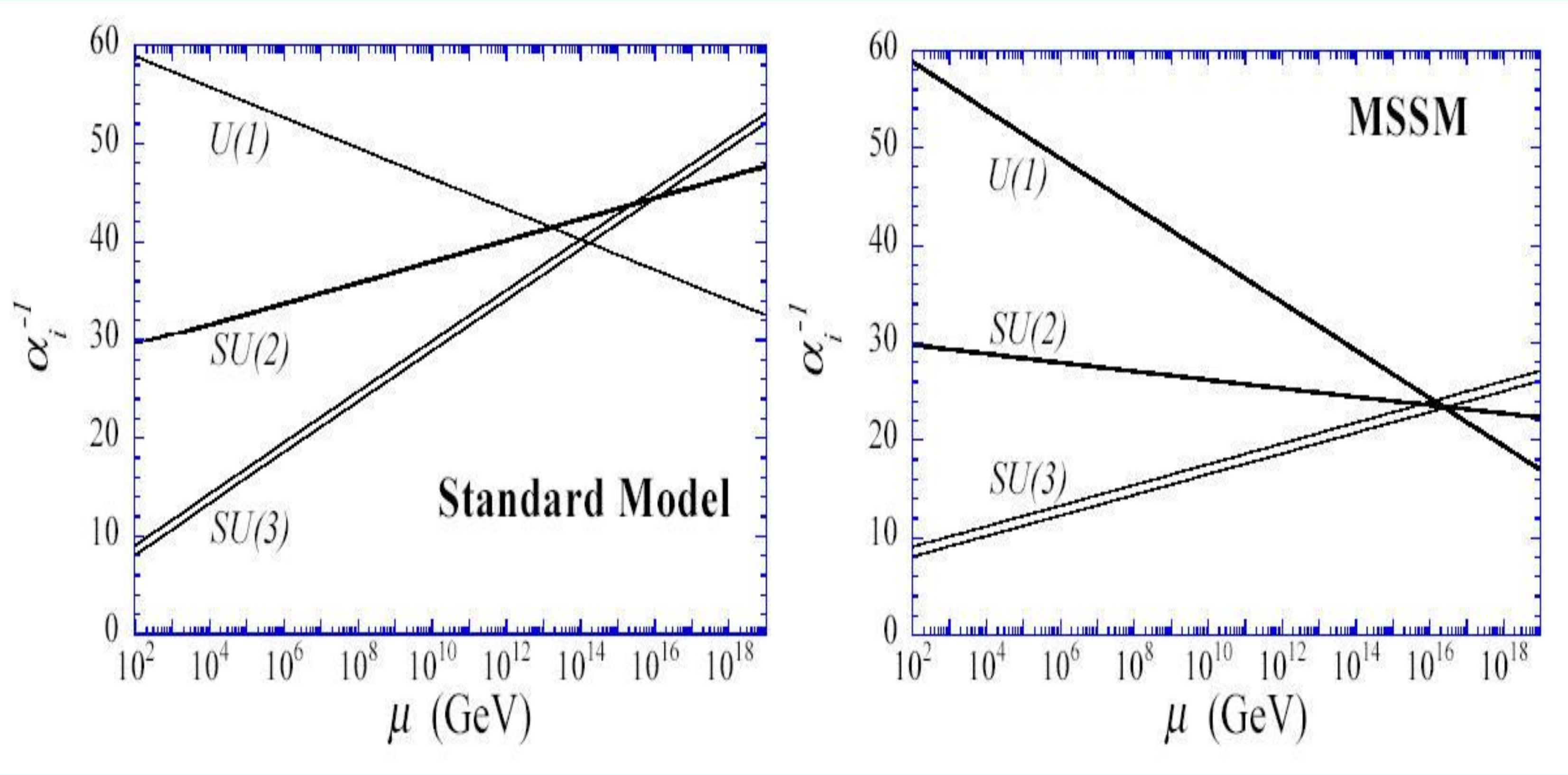


$$m_h^2 = (m_h^2)_0 - \underbrace{\frac{1}{16\pi^2}\lambda^2\Lambda^2 + \frac{1}{16\pi^2}\lambda^2\Lambda^2}_{} + \frac{1}{16\pi^2}\lambda^2(m_{\tilde{f}}^2 - m_f^2)\,\ln(\Lambda/m_h)$$

- Dependence on Λ is softened to a logarithm
- SUSY solves the hierarchy problem, as long as sparticle masses are at the EW scale

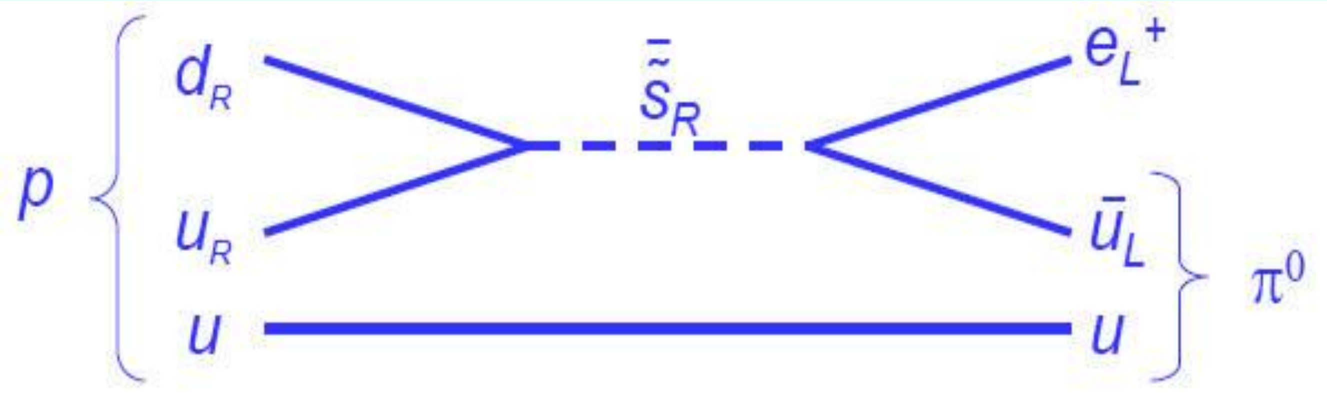
Telescope to Gauge Unification

- Superpartners modify the scale dependence of couplings
- With TeV superpartners, the forces are unified!
- Unification scale ~ 10^{16} GeV



R–Parity: New Quantum Number

- A BIG problem: proton decay occurs very rapidly!



- Introduce R–parity: $R_p = (-1)^{3(B-L)+2S}$
- New multiplicative, conserved quantum number
  - P has $R_p = +1$; $\tilde{P}$ has $R_p = -1$
  - Requires 2 superpartners in each interaction
- Consequence: the Lightest Supersymmetric Particle (LSP) is stable and cosmologically significant

R–Parity Violation

- RPV allows for new terms in the superpotential and thus allows for new interactions
- $W = \lambda_{ijk} L_i L_j E^c_k + \lambda'_{ijk} L_i Q_j D^c_k + \lambda''_{ijk} U^c_i D^c_j D^c_k$
- Cannot simultaneously have lepton and baryon number violating terms!
- RPV leads to new collider search strategies and new limits. Strong restrictions on 1st and 2nd generation RPV couplings from flavor processes
- From here on, assume that R–Parity is conserved.

Neutral SUSY Particles: LSP Candidates

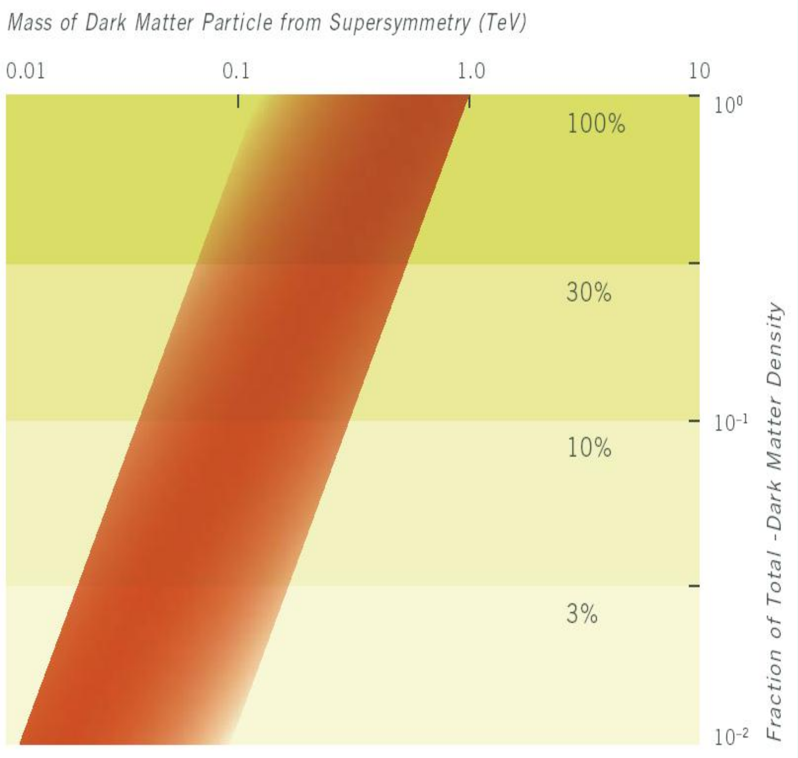
Spin	U(1) M_1	SU(2) M_2	Up-type μ	Down-type μ	$m_{\tilde{\nu}}$	$m_{3/2}$
2						G graviton
3/2		Neutralinos: $\{\chi \equiv \chi_1, \chi_2, \chi_3, \chi_4\}$				$\tilde{G}$ gravitino
1	B	W^0				
1/2	$\tilde{B}$ Bino	$\tilde{W}^0$ Wino	$\tilde{H}_u$ Higgsino	$\tilde{H}_d$ Higgsino	ν	
0			H_u	H_d	$\tilde{\nu}$ sneutrino	

The LSP and Dark Matter

- The amount of dark matter relic density is inversely proportional to the annihilation cross section:

$$\Omega_{DM} \sim \langle \sigma_A v \rangle^{-1}$$

$$\sigma_A \sim \alpha^2 / m^2$$



HEPAP 2006 LHC/ILC Subpanel

Remarkable "coincidence":

Ω_{DM} ~ 0.1 for m ~ 100 GeV – 1 TeV!

Supersymmetry independently predicts particles with about the right density to be dark matter !

Higgs Doubling

- SUSY requires 2 Higgs doublets to cancel anomalies and to give mass to both up- and down-type particles in a gauge and SUSY invariant way
- Anomaly cancellation requires $\Sigma\, Y^3 = 0$, where Y is hypercharge and the sum is over all fermions
- SUSY adds an extra fermion with $Y = -1$

$$\begin{pmatrix} h^0 \\ h^- \end{pmatrix} \equiv \begin{pmatrix} h_d^0 \\ h_d^- \end{pmatrix} \Rightarrow \begin{pmatrix} \tilde{H}_d^0 \\ \tilde{H}_d^- \end{pmatrix}$$

- To cancel this anomaly, we add another Higgs doublet with $Y = +1$

$$\begin{pmatrix} h_u^+ \\ h_u^0 \end{pmatrix} \Rightarrow \begin{pmatrix} \tilde{H}_u^+ \\ \tilde{H}_u^0 \end{pmatrix}$$

Supersymmetry is Broken

- SUSY is not an exact symmetry: otherwise would have 511 keV slectrons! This is excluded experimentally
- Terms that break SUSY w/o introducing new Λ^2 divergences are called soft-breaking terms
- We don't know how SUSY is broken, but soft SUSY breaking effects can be parameterized in the Lagrangian

$$
\begin{aligned}
\mathcal{L}_{soft} = -\frac{1}{2}(M_3\tilde{g}\tilde{g} + M_2\tilde{W}\tilde{W} + M_1\tilde{B}\tilde{B}) \\
-m_Q^2\tilde{Q}^\dagger\tilde{Q} - m_U^2\tilde{U}^\dagger\tilde{U} - m_D^2\tilde{D}^\dagger\tilde{D} - m_L^2\tilde{L}^\dagger\tilde{L} - m_E^2\tilde{E}^\dagger\tilde{E} \\
-m_{H_1}^2 H_1^* H_1 - m_{H_2}^2 H_2^* H_2 - (\mu \boxed{B} H_1 H_2 + cc.) \\
-(\underline{A_u h_u \tilde{U}\tilde{Q}H_2 + A_d h_d \tilde{D}\tilde{Q}H_1 + A_l h_l \tilde{E}\tilde{L}H_1)} + c.c.
\end{aligned}
$$

  - A-terms result in L-R sfermion mixing, proportional to fermion Yukawa
  - B-term is SUSY-breaking parameter

Parameterized SUSY Breaking

There are over 100 parameters!

Most of these are new flavor violation parameters or CP violating phases

Causes difficulties in the flavor sector

Need some simplifying assumptions

There are many, many models of SUSY breaking….
Each with their own characteristics leading to some different signatures!

Supersymmetric Parameters

SUSY breaking introduces many unknown parameters. These are

- Masses for sleptons and squarks: $m^2_{f\,ij}$
- Masses for gauginos: M_1 , M_2 , M_3
- Trilinear scalar couplings (similar to Yukawa couplings): A^f_{ij}
- Mass for the 2 Higgsinos: $\mu\, \tilde{H}_u\, \tilde{H}_d$
- Masses for the 2 neutral Higgs bosons: $B\, H_u H_d + m^2_{Hu}\, |H_u|^2 + m^2_{Hd}\, |H_d|^2$
- The 2 neutral Higgs bosons both contribute to electroweak symmetry breaking:

$$v^2 = (174\text{ GeV})^2 \rightarrow v_u^2 + v_d^2 = (174\text{ GeV})^2$$

The extra degree of freedom is called $\tan\beta = v_u/v_d$

Minimal Supersymmetric Standard Model

- Minimal number of new SUSY particles
- Contains R-parity conservation
  - Superpartners are produced in pairs
  - Heavier Superpartners decay to the Lightest
  - Lightest Superpartner is stable
- Soft SUSY-breaking implemented, with many possible models

Collider signatures dependent on R-Parity and on model of SUSY breaking

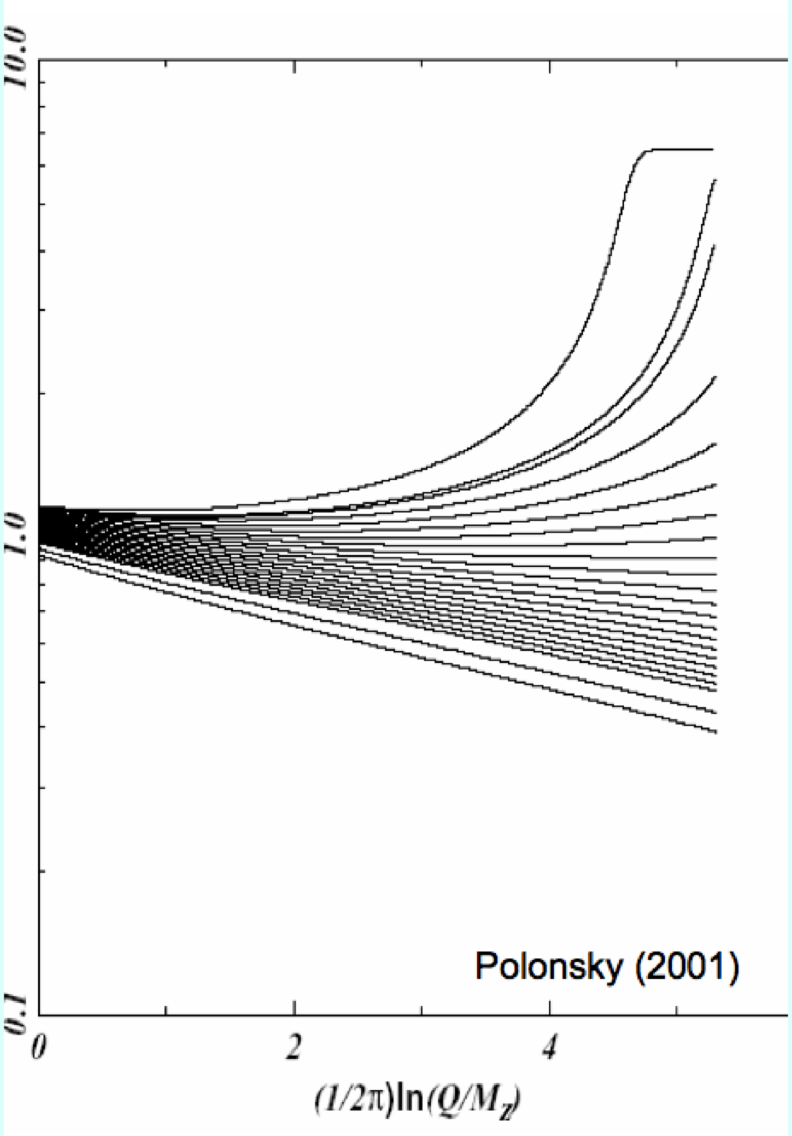
SUSY and the top-quark Mass

- Force unification suggests that extrapolation to very high scales is possible

- All parameters have scale dependence

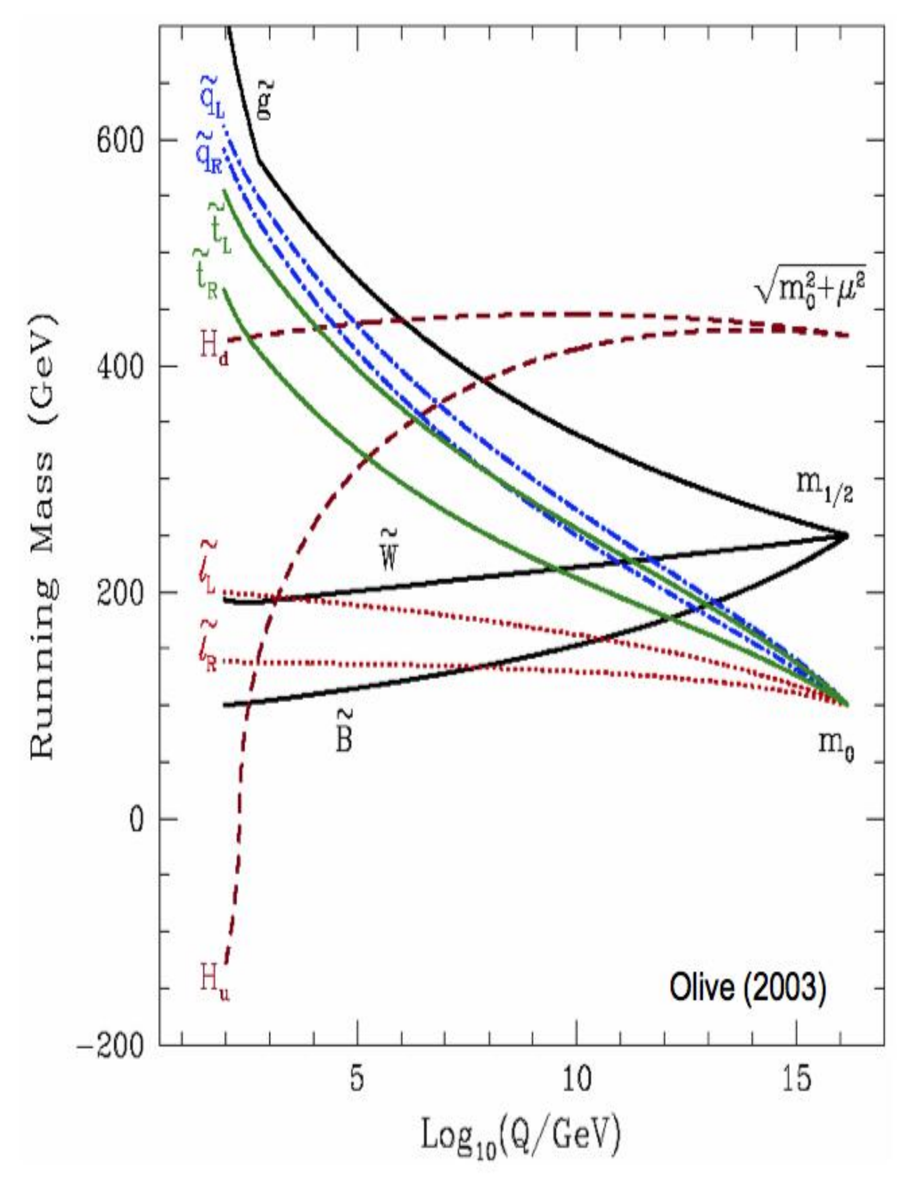
- Top-quark yukawa has a quasi-fixed point near its Measured value

- SUSY predicts large top-quark mass!



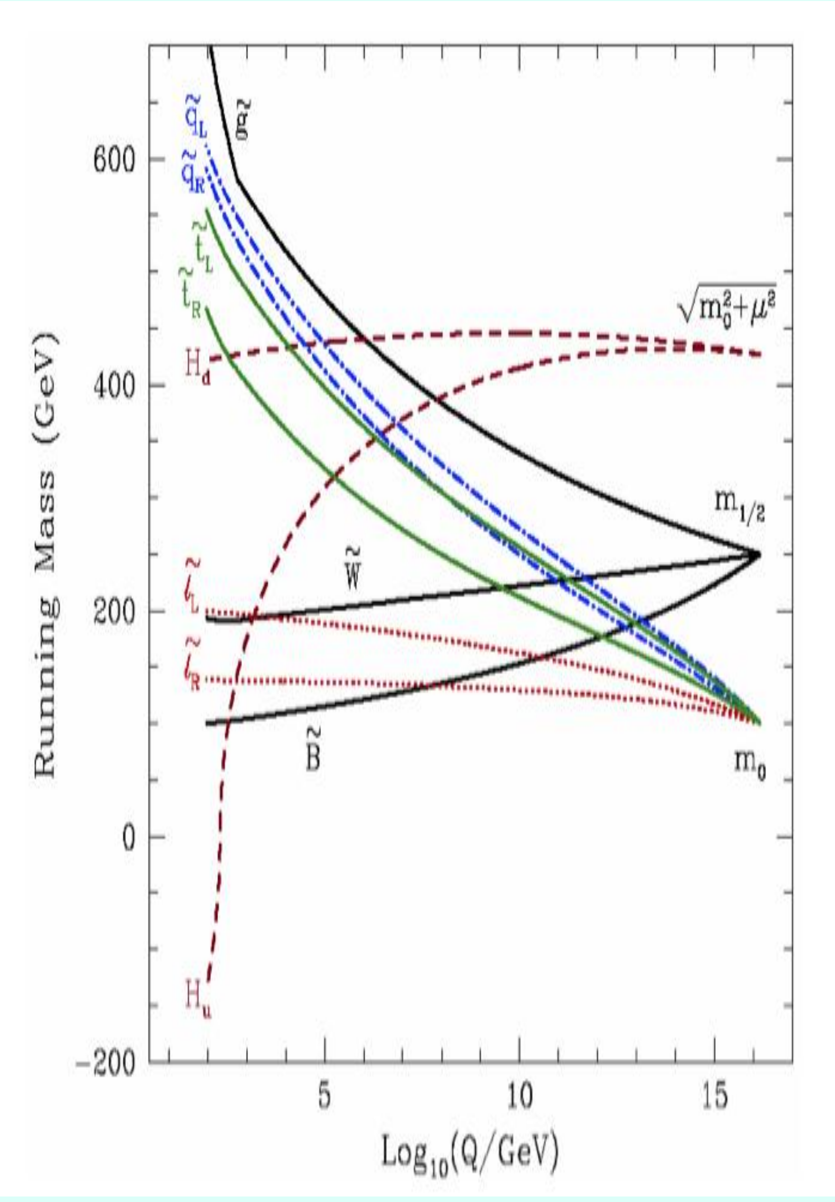
Evolution of Scalar Masses

- How do scalar masses change with scale?
- Gauge couplings increase mass, Yukawa couplings decrease mass
- H_u is the lightest sparticle at the EW scale, by far!
- EWSB requires $m_{Hu}^2 < 0$
- SUSY "explains" why SU(2) is broken



Sneutrino and Higgsino Masses

- Lightest physical scalars are typically the right-handed sleptons
- Sneutrinos are typically heavier and are disfavored as LSP's

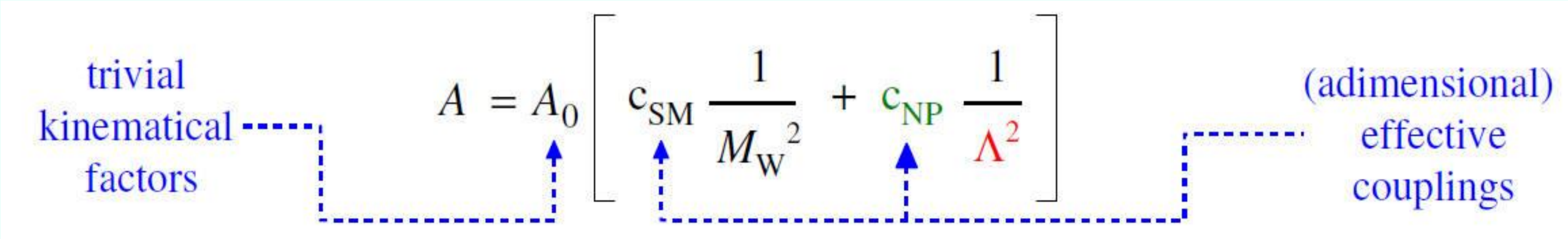


SUSY and Flavor Changing Neutral Currents

- FCNC’s provide strong constraints on SUSY

- There are strong connections between LHC results and flavor physics

Generic amplitude for flavor process



$$A = A_0 \left[c_{SM} \frac{1}{M_W{}^2} + c_{NP} \frac{1}{\Lambda^2} \right]$$

SUSY and Flavor Changing Neutral Currents

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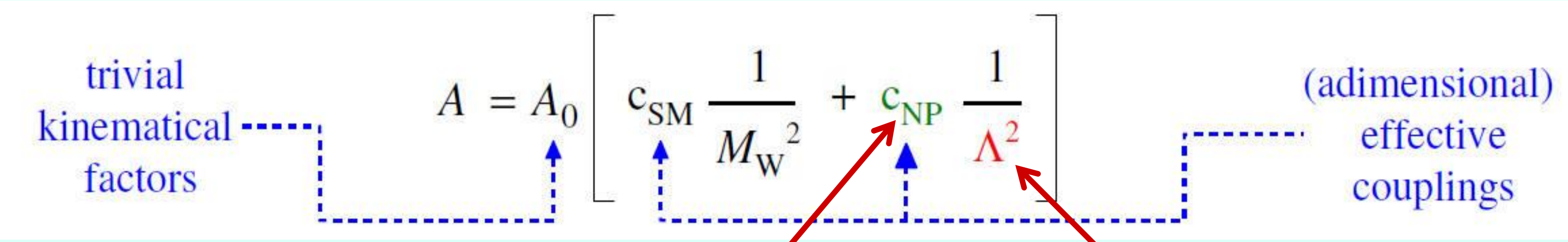
Generic amplitude for flavor process



$$A = A_0 \left[c_{SM} \frac{1}{M_W{}^2} + c_{NP} \frac{1}{\Lambda^2} \right]$$

trivial kinematical factors

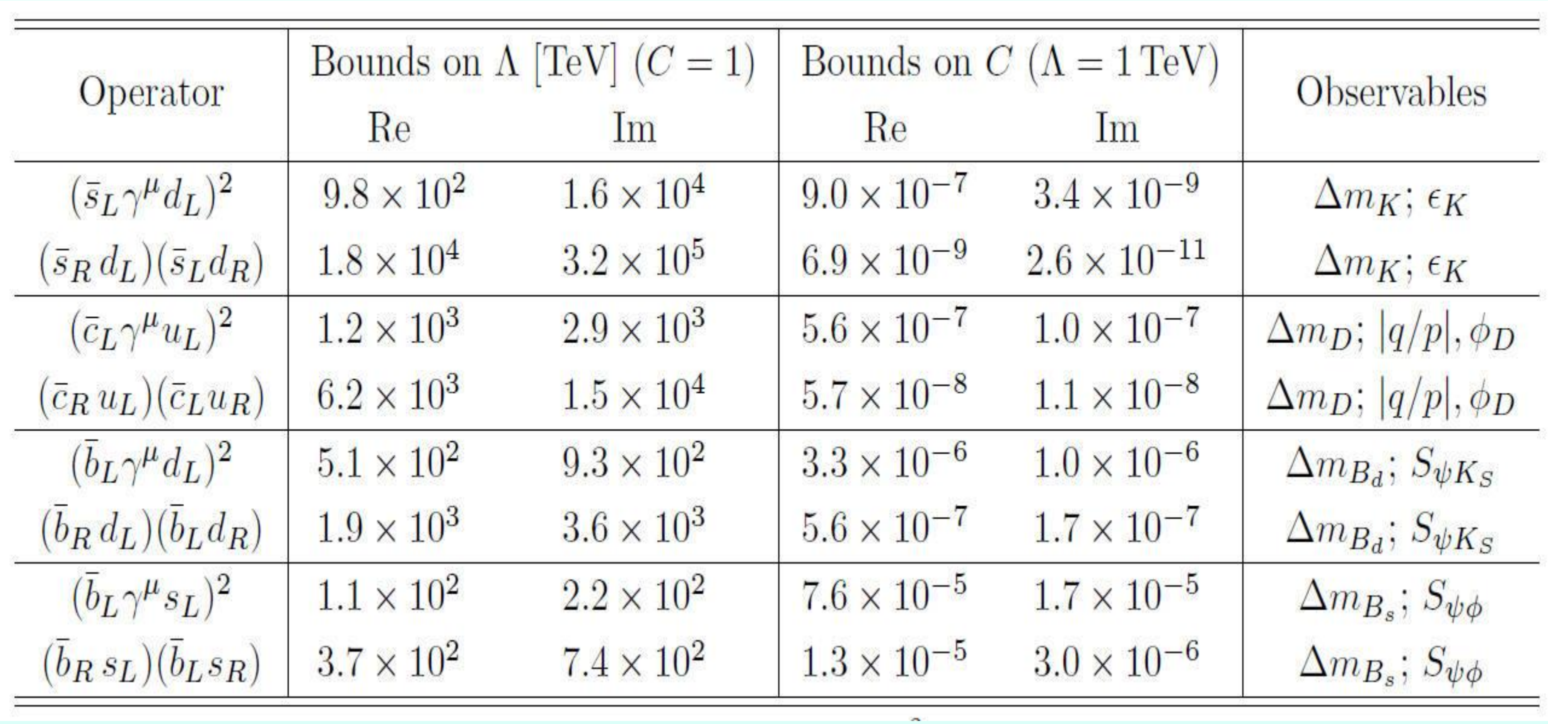
(adimensional) effective couplings

Flavor non-diagonal measured in LFV and heavy quark physics

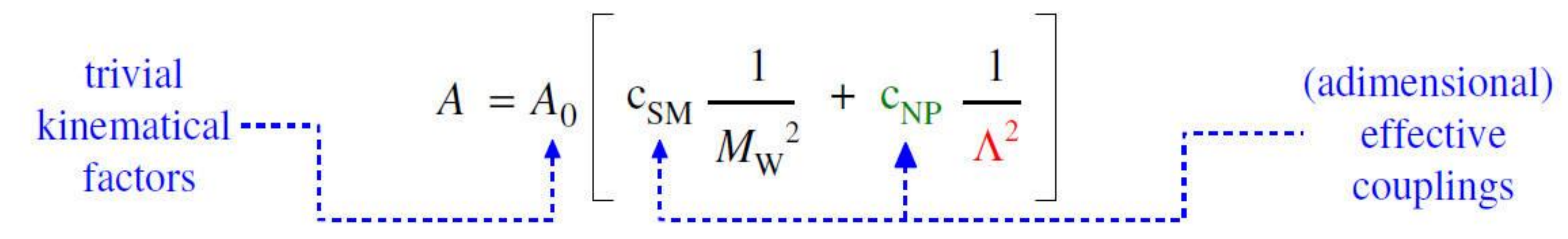
LHC measures this!

Flavor Bounds on New Physics

ΔF=2 processes



Operator	Bounds on Λ [TeV] ($C = 1$)		Bounds on C ($\Lambda = 1$ TeV)		Observables
	Re	Im	Re	Im	
$(\bar{s}_L \gamma^\mu d_L)^2$	9.8×10^2	1.6×10^4	9.0×10^{-7}	3.4×10^{-9}	Δm_K; ϵ_K
$(\bar{s}_R\, d_L)(\bar{s}_L d_R)$	1.8×10^4	3.2×10^5	6.9×10^{-9}	2.6×10^{-11}	Δm_K; ϵ_K
$(\bar{c}_L \gamma^\mu u_L)^2$	1.2×10^3	2.9×10^3	5.6×10^{-7}	1.0×10^{-7}	Δm_D; $\lvert q/p \rvert, \phi_D$
$(\bar{c}_R\, u_L)(\bar{c}_L u_R)$	6.2×10^3	1.5×10^4	5.7×10^{-8}	1.1×10^{-8}	Δm_D; $\lvert q/p \rvert, \phi_D$
$(\bar{b}_L \gamma^\mu d_L)^2$	5.1×10^2	9.3×10^2	3.3×10^{-6}	1.0×10^{-6}	Δm_{B_d}; $S_{\psi K_S}$
$(\bar{b}_R\, d_L)(\bar{b}_L d_R)$	1.9×10^3	3.6×10^3	5.6×10^{-7}	1.7×10^{-7}	Δm_{B_d}; $S_{\psi K_S}$
$(\bar{b}_L \gamma^\mu s_L)^2$	1.1×10^2	2.2×10^2	7.6×10^{-5}	1.7×10^{-5}	Δm_{B_s}; $S_{\psi\phi}$
$(\bar{b}_R\, s_L)(\bar{b}_L s_R)$	3.7×10^2	7.4×10^2	1.3×10^{-5}	3.0×10^{-6}	Δm_{B_s}; $S_{\psi\phi}$



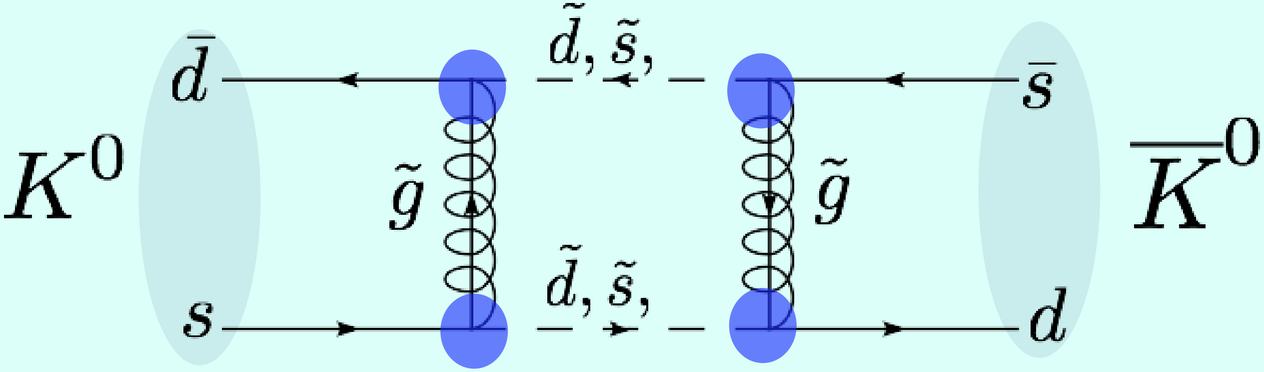
trivial kinematical factors

$$A = A_0 \left[c_{SM} \frac{1}{M_W{}^2} + c_{NP} \frac{1}{\Lambda^2} \right]$$

(adimensional) effective couplings

SUSY Effects in FCNC: Kaon Mixing

Rate exceeds experimental value by ~1000!



SUSY GIM mechanism invoked:

Rate ~ $\Sigma\, V_{CKM}\, (m_{\tilde{f}i}^2 - m_{\tilde{f}j}^2)$

One Solution: 1st 2 generation scalar particles are approximately degenerate!

Muon g−2 Anomaly

$a_\mu = (g_\mu - 2)/2$

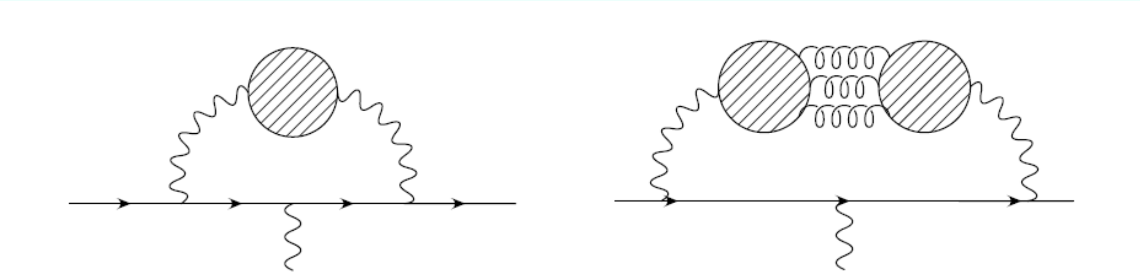
$a_\mu(\text{Expt}) = 116592089(54)(33) \times 10^{-11}$ (BNL 821)

$a_\mu(\text{SM}) = 116591802(42)(26)(02) \times 10^{-11}$

$\Delta a_\mu = 287(80) \times 10^{-11}$ 3.6σ discrepancy!!

New FNAL exp't: reduce exp't error by factor of 2−3

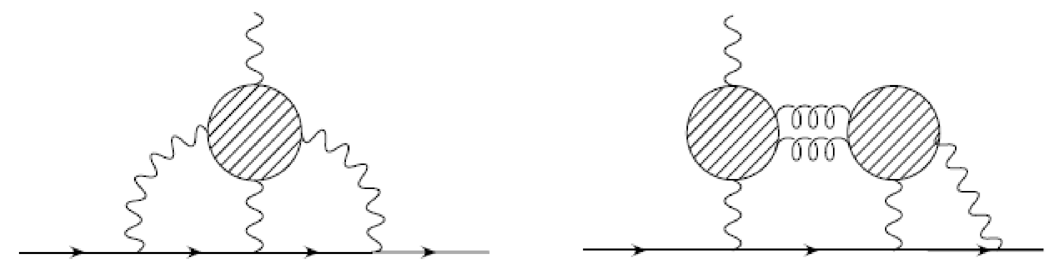
Major theory uncertainty in hadronic vacuum polarization



$a_\mu(\text{HVP}) = (692.3 \pm 4.2) \times 10^{-10}$

$= (701.5 \pm 4.7) \times 10^{-10}$

(e^+e^-, τ data)

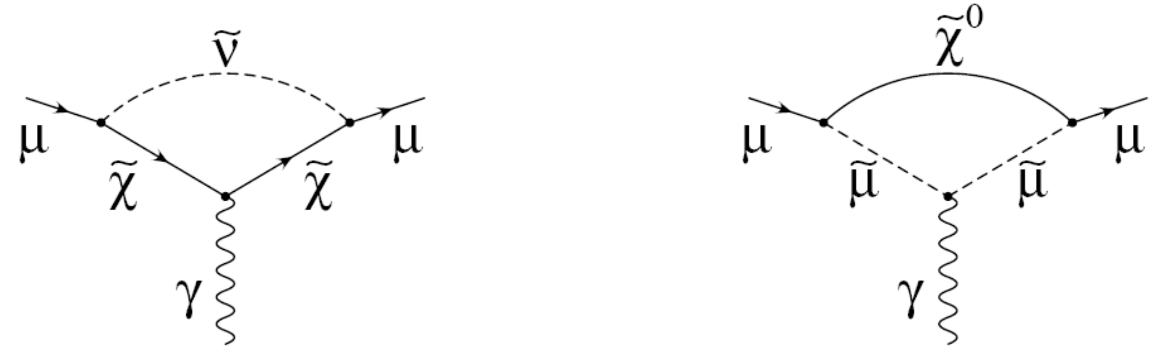


$a_\mu(\text{LbL}) = 105(26) \times 10^{-11}$

Lattice calculation underway!

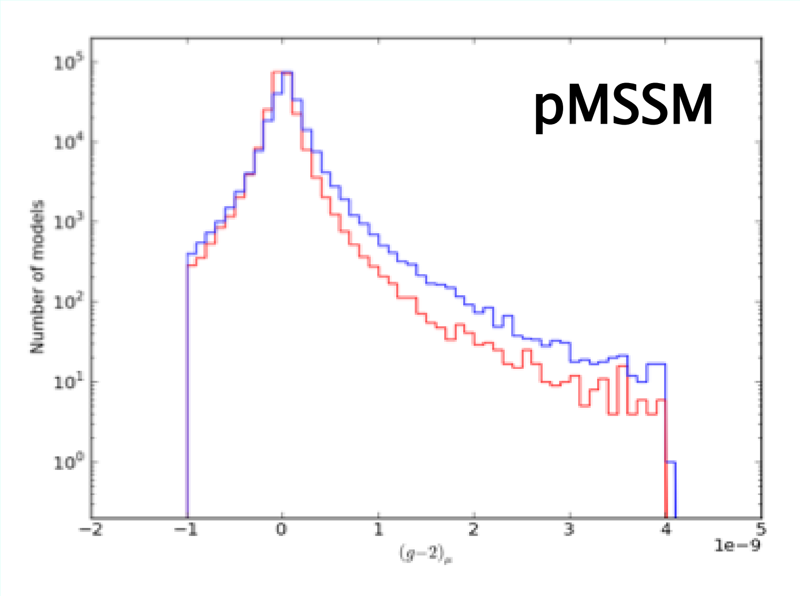
Muon g−2 Anomaly and New Physics

Supersymmetric Contributions



$$|a_\mu(\mathrm{SUSY})| \simeq 130 \times 10^{-11} \left(\frac{100\ \mathrm{GeV}}{\tilde{m}}\right)^2 \tan\beta$$

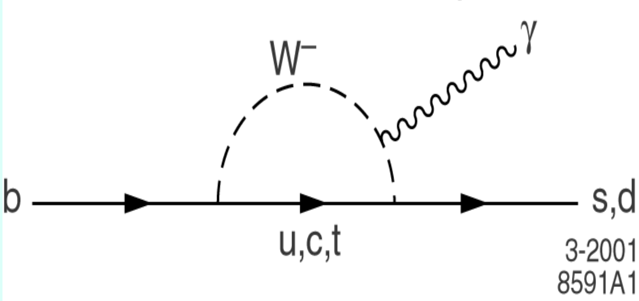
Large tanβ preferred
In constrained SUSY models

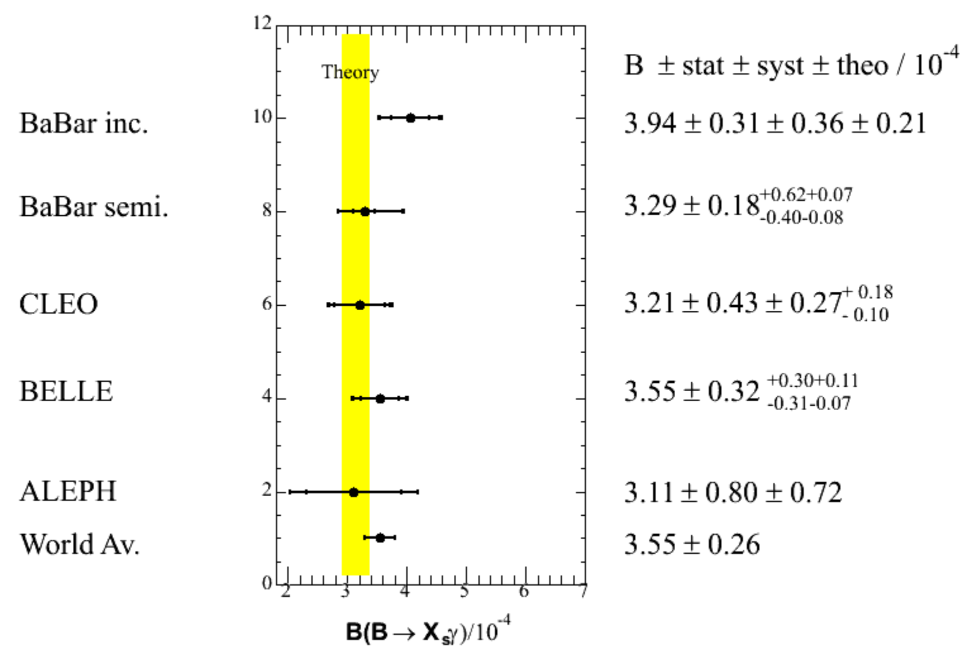


b →sγ in SUSY

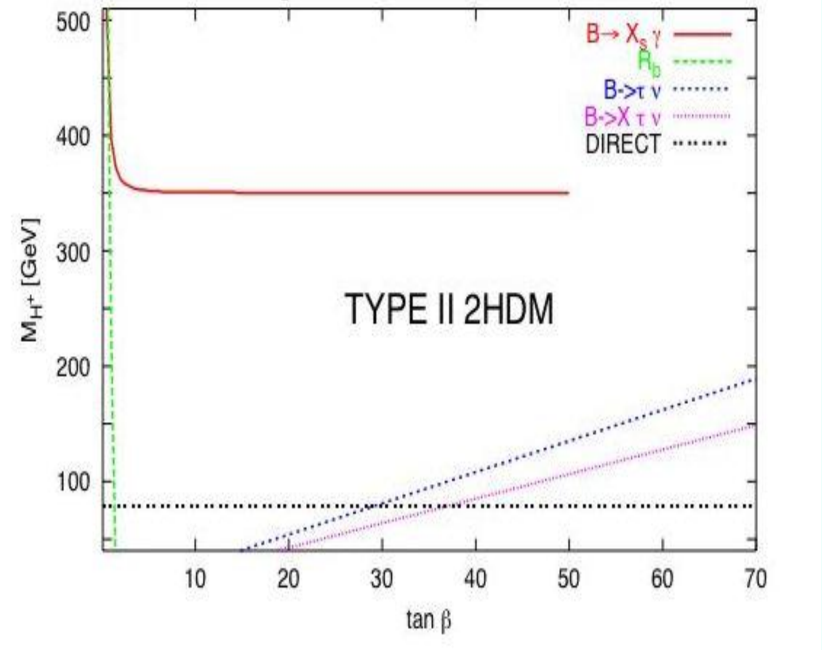
- This rare decay gives strong constraints on SUSY contributions
- There are several SUSY contributions: charged Higgs, stop/chargino, gluino/sbottom being most important

SM diagram



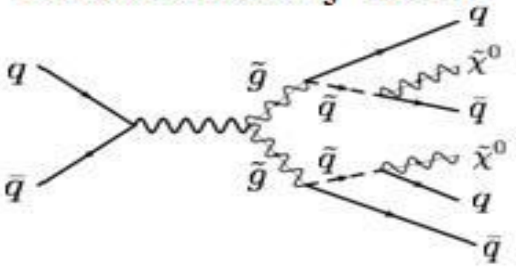


Strong H^+ constraints w/o other sparticles

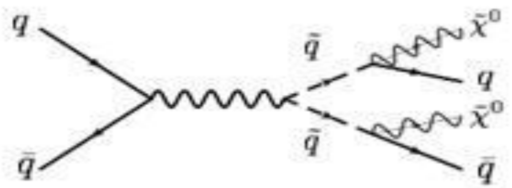


Supersymmetry at the LHC

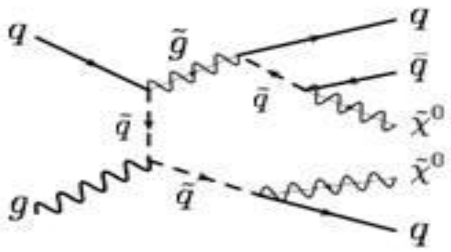
Gluino Pairs: 4j +MET

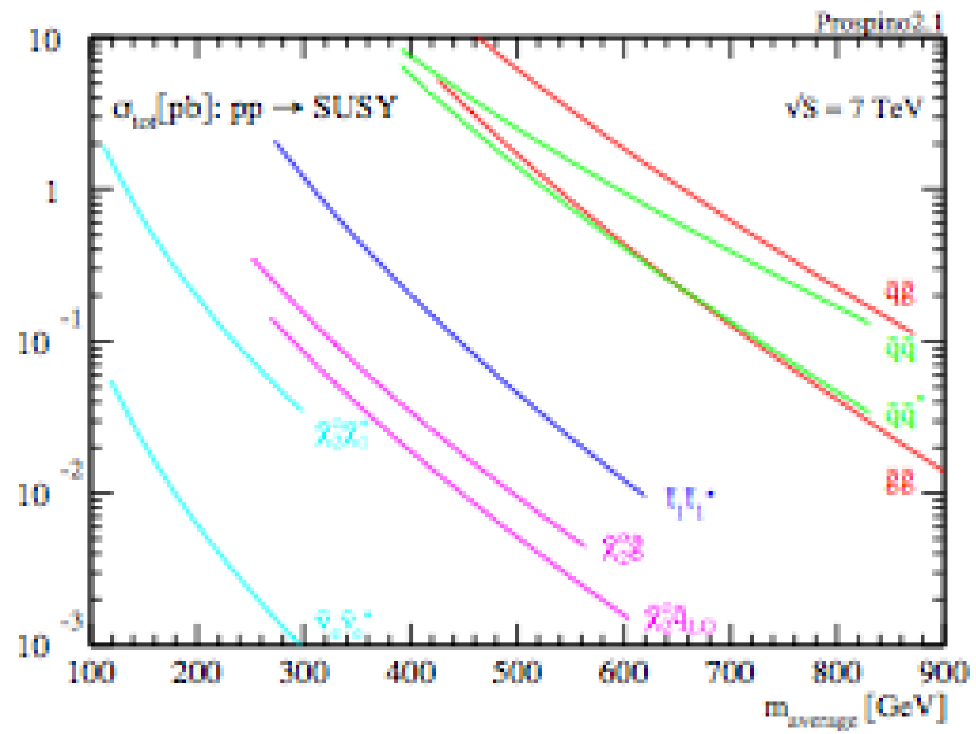


Squark Pairs: 2j +MET



Squark-Gluino Pairs: 3j +MET



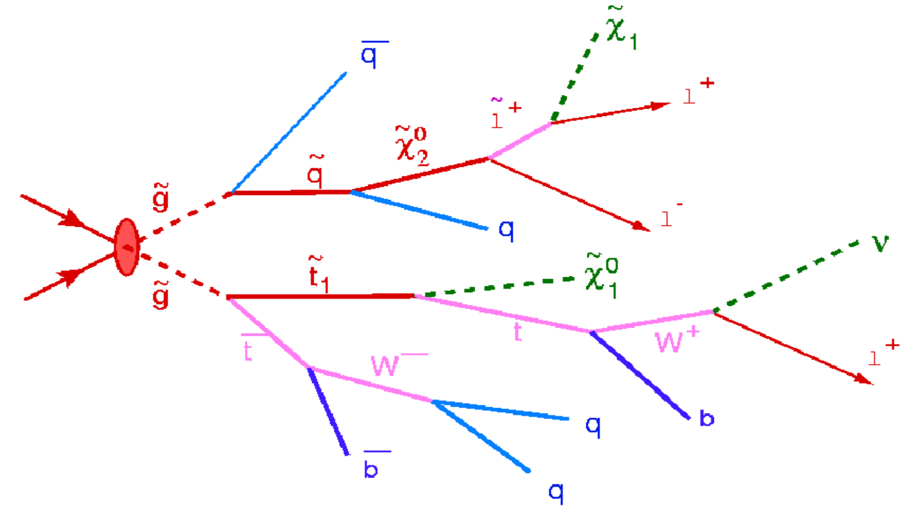


Colored sparticles have strong production cross sections @ LHC

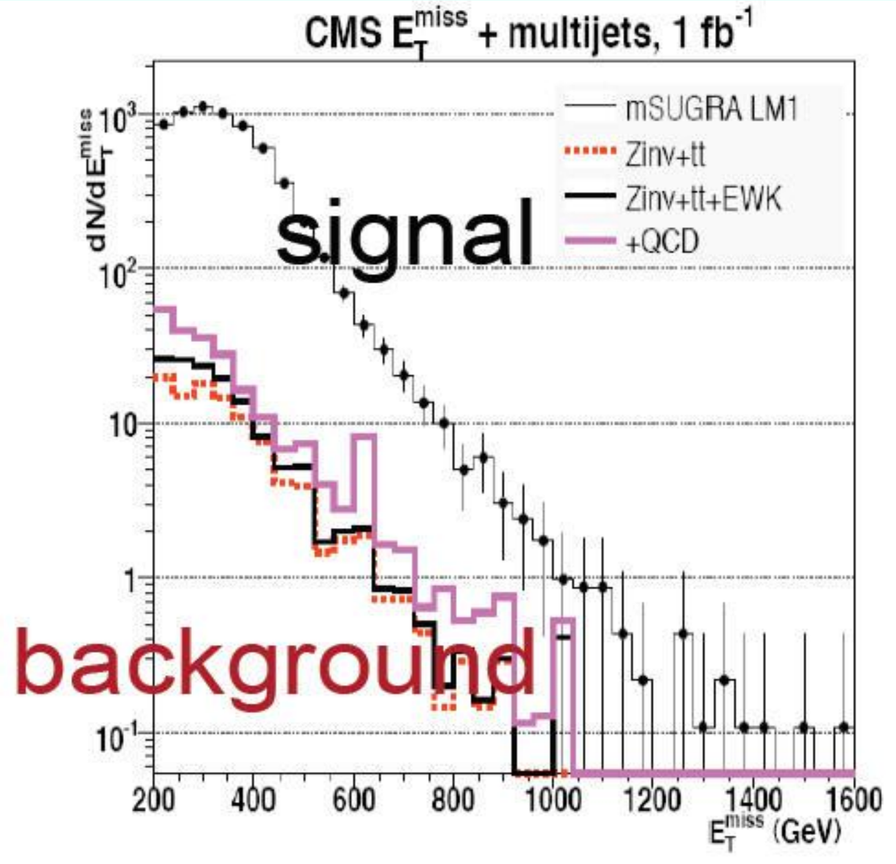
Decay to Jets + MET should give large MET signal over SM!

Supersymmetry at the LHC

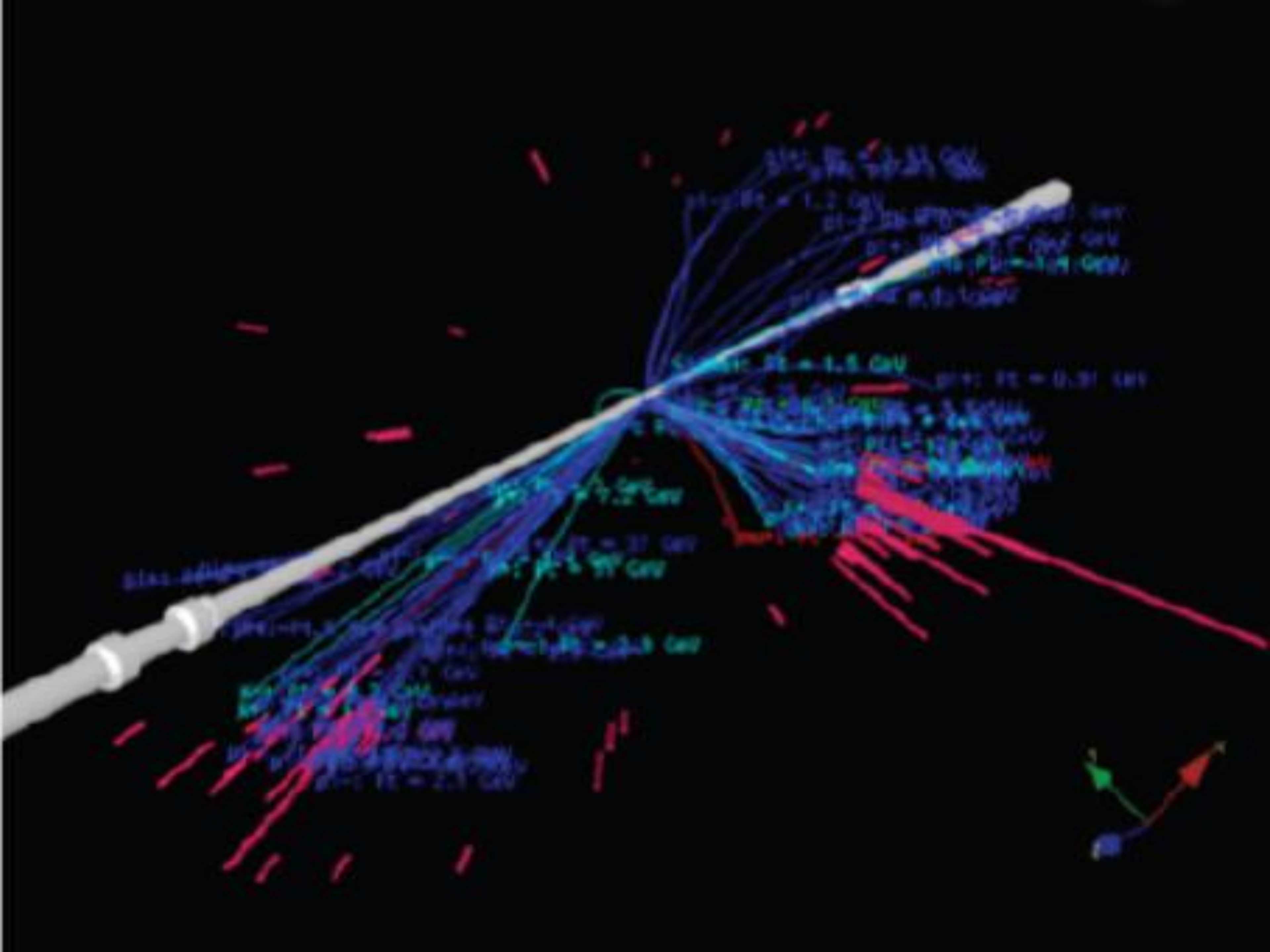
SUSY discovery supossedly ‘easy’ at LHC



Short or long cascade decay
Chains lead to large MET



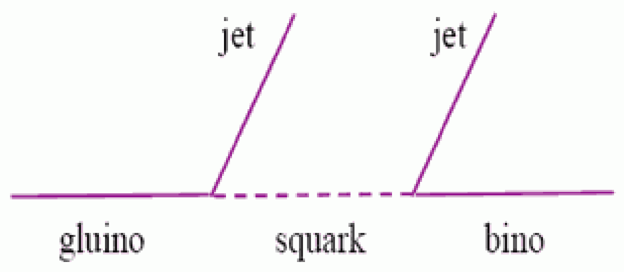
Cut: $E_T^{miss} > 300$ GeV
MC before LHC run



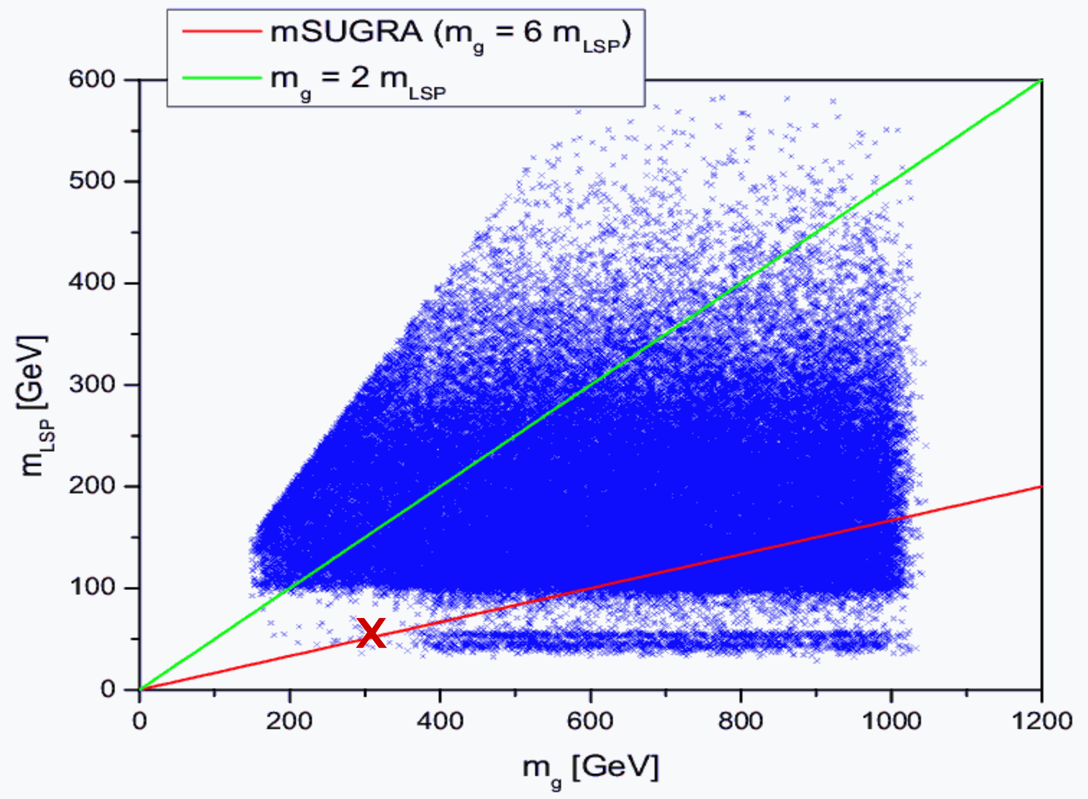
Effects of compressed spectra: Tevatron

- Tevatron gluino/squark analyses performed for constant ratio $m_{gluino} : m_{Bino} \simeq 6 : 1$

Gluino-Bino mass ratio determines kinematics

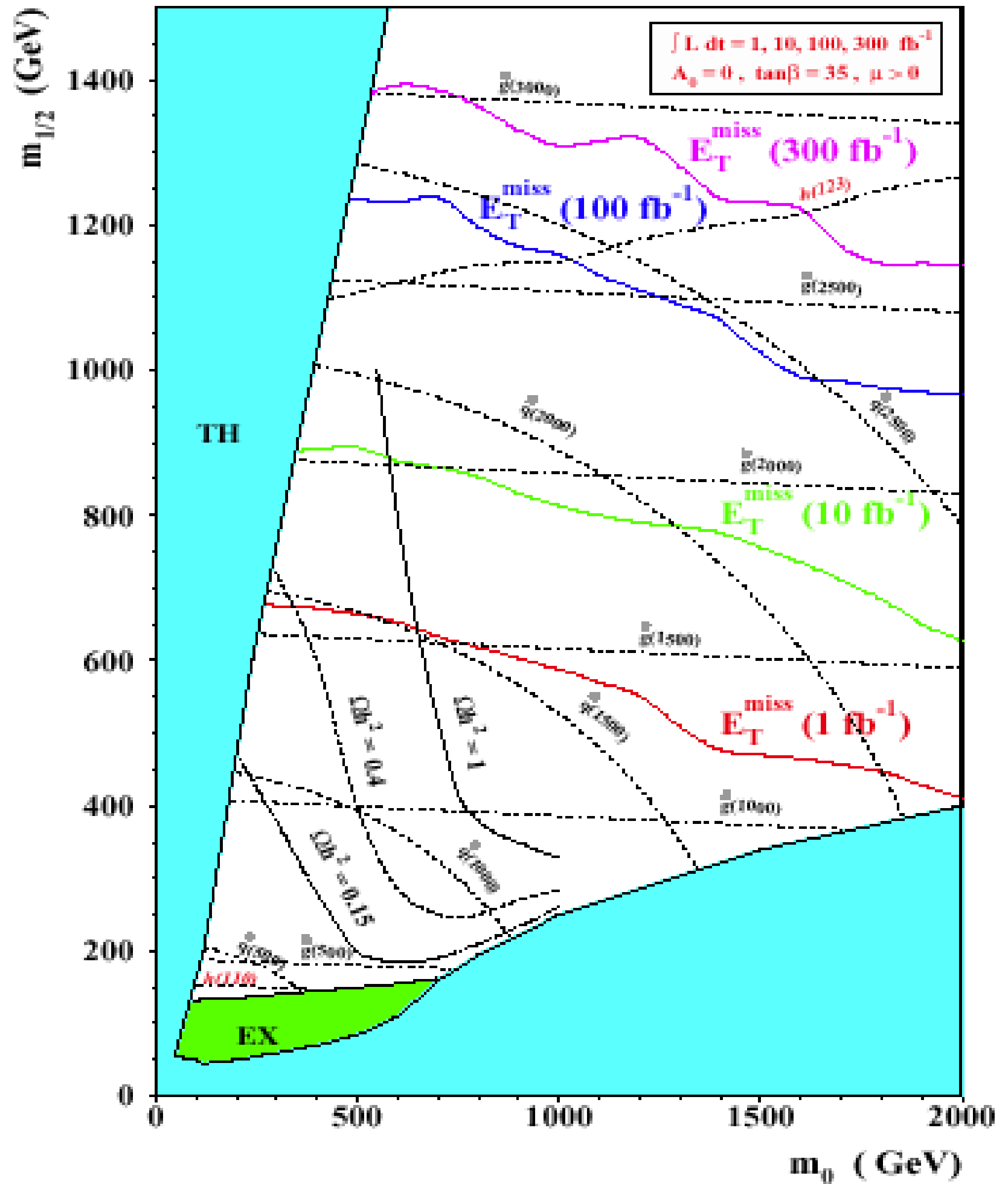


Distribution of Gluino Masses



Berger, Gainer, JLH, Rizzo 0812.0980

LHC Supersymmetry Discovery Reach

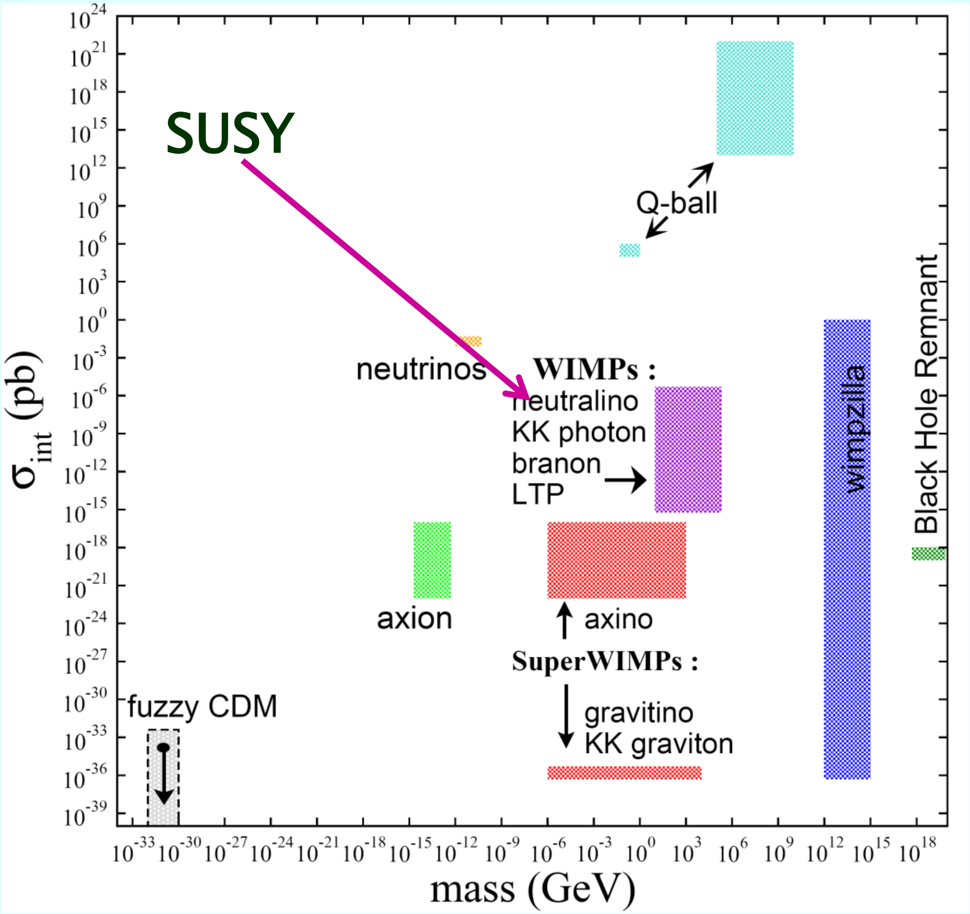


mSUGRA – Model where gravity mediates SUSY breaking – 5 free parameters at high energies

Squark and Gluino mass reach is 2.5-3.0 TeV @ 300 fb^{-1} at 14 TeV

Some Dark Matter Candidates

- The observational constraints are no match for the creativity of theorists
- Masses and interaction strengths span many, many orders of magnitude, but not all candidates are equally motivated
- Weakly Interacting Massive Particle (WIMP)



HEPAP/AAAC DMSAG Subpanel (2007)

The WIMP ‘Miracle’

(1) Assume a new (heavy) particle χ is initially in thermal equilibrium:

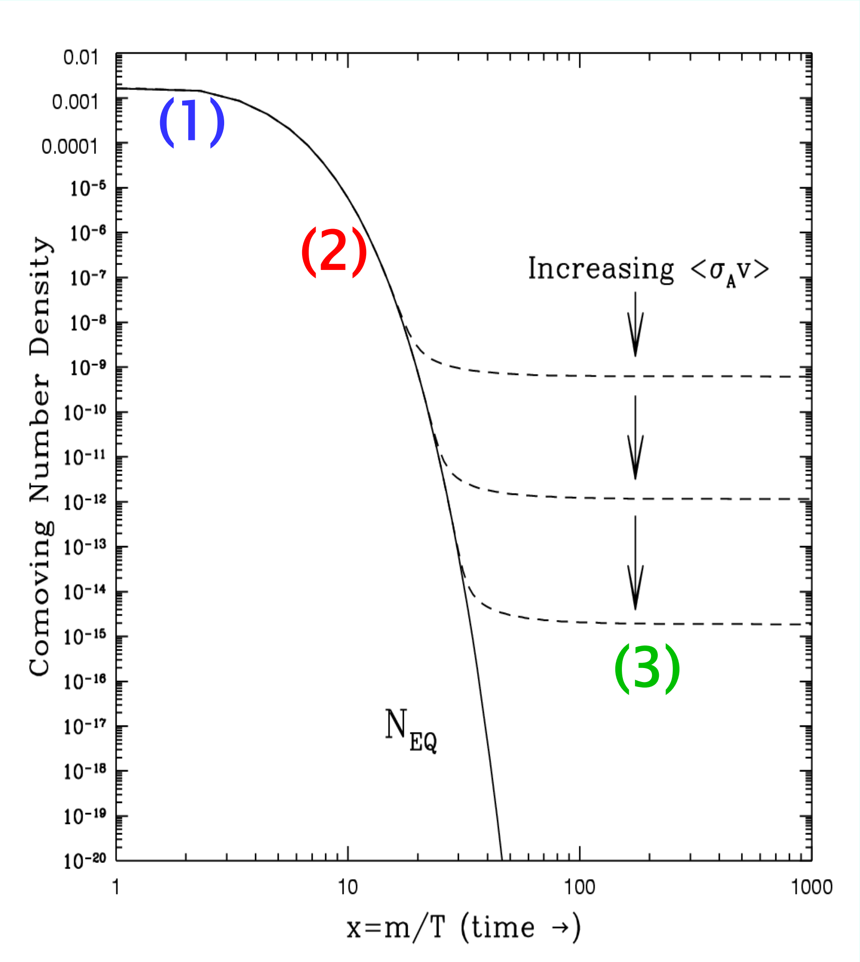
$$\chi\chi \;\leftrightarrow\; f\,f$$

(2) **Universe cools**:

$$\chi\chi \;\underset{\cancel{\leftarrow}}{\rightarrow}\; f\,f$$

(3) χ□s **“freeze out”**:

$$\chi\chi \;\underset{\cancel{\leftarrow}}{\cancel{\rightarrow}}\; f\,f$$



Zeldovich et al. (1960s)

Techniques to observe Dark Matter

$\chi\chi \rightarrow$ photons, positrons , anti-protons…. 'in the sky' right now may be seen by FERMI & other experiments

$\chi N \rightarrow \chi N$ (elastic) scattering may be detected on earth in deep underground experiments

If χ is really a WIMP it may be directly produced at the LHC !

Of course, χ does not come by itself in any new physics model & there is usually a significant accompanying edifice of other interesting particles & interactions with many other observational predictions

So this general picture can be tested in many ways….

Two MSSM Model Frameworks

- The constrained MSSM (CMSSM)
  - Based on mSUGRA: Gravity mediated SUSY breaking
  - Common masses & couplings at the GUT scale
  - m_0, $m_{1/2}$, A_0, $\tan\beta = v_2/v_1$, sign μ

- The phenomenological MSSM (pMSSM)
  - 19 real, weak-scale parameters

scalars:

m_{Q_1}, m_{Q_3}, m_{u_1}, m_{d_1}, m_{u_3}, m_{d_3}, m_{L_1}, m_{L_3}, m_{e_1}, m_{e_3}

gauginos: M_1, M_2, M_3

tri-linear couplings: A_b, A_t, A_τ

Higgs/Higgsino: μ, M_A, $\tan\beta$

Tomorrow's lecture will be based on these 2 models

Lecture 1 Summary

- Supersymmetry is a new symmetry allowed by Nature
- Contains many new parameters!
- Dimensionless couplings are fixed
- Dimensionful parameters are allowed (soft breaking), but should be at the EW scale

Analogy	Soap Bubble	SM
Large Parameter	Length L Height H	M_{Pl}
Small Parameter	L - H	m_h
Symmetry explanation	Rotational invariance	SUSY
Symmetry breaking	Gravity	M_{SUSY}
Natural if	Gravity weak	M_{SUSY} small