MC@NLO
Combining NLO–Matrix Elements and Parton Showers at HERA

HERA and the LHC workshop
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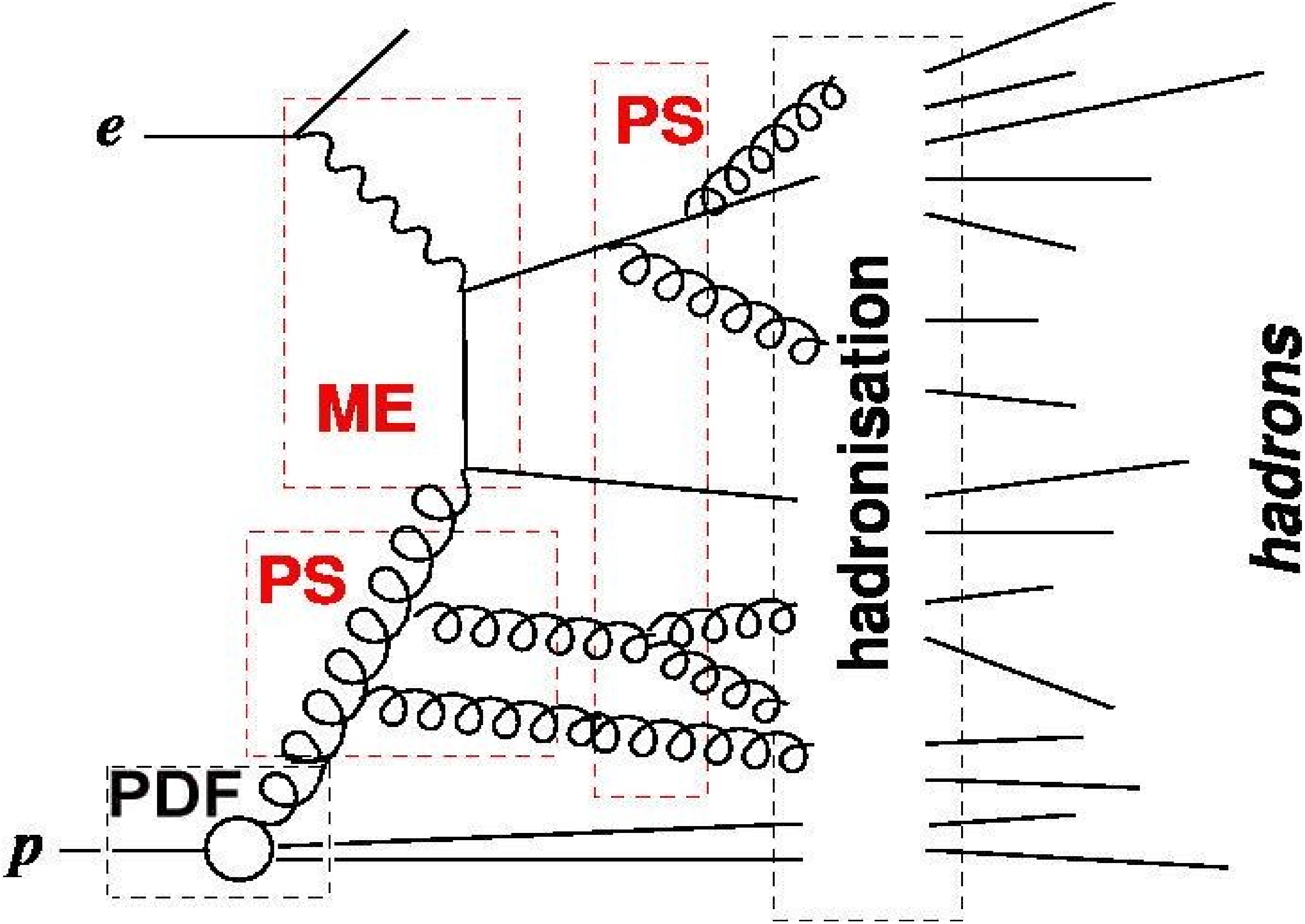
Overview Monte Carlo

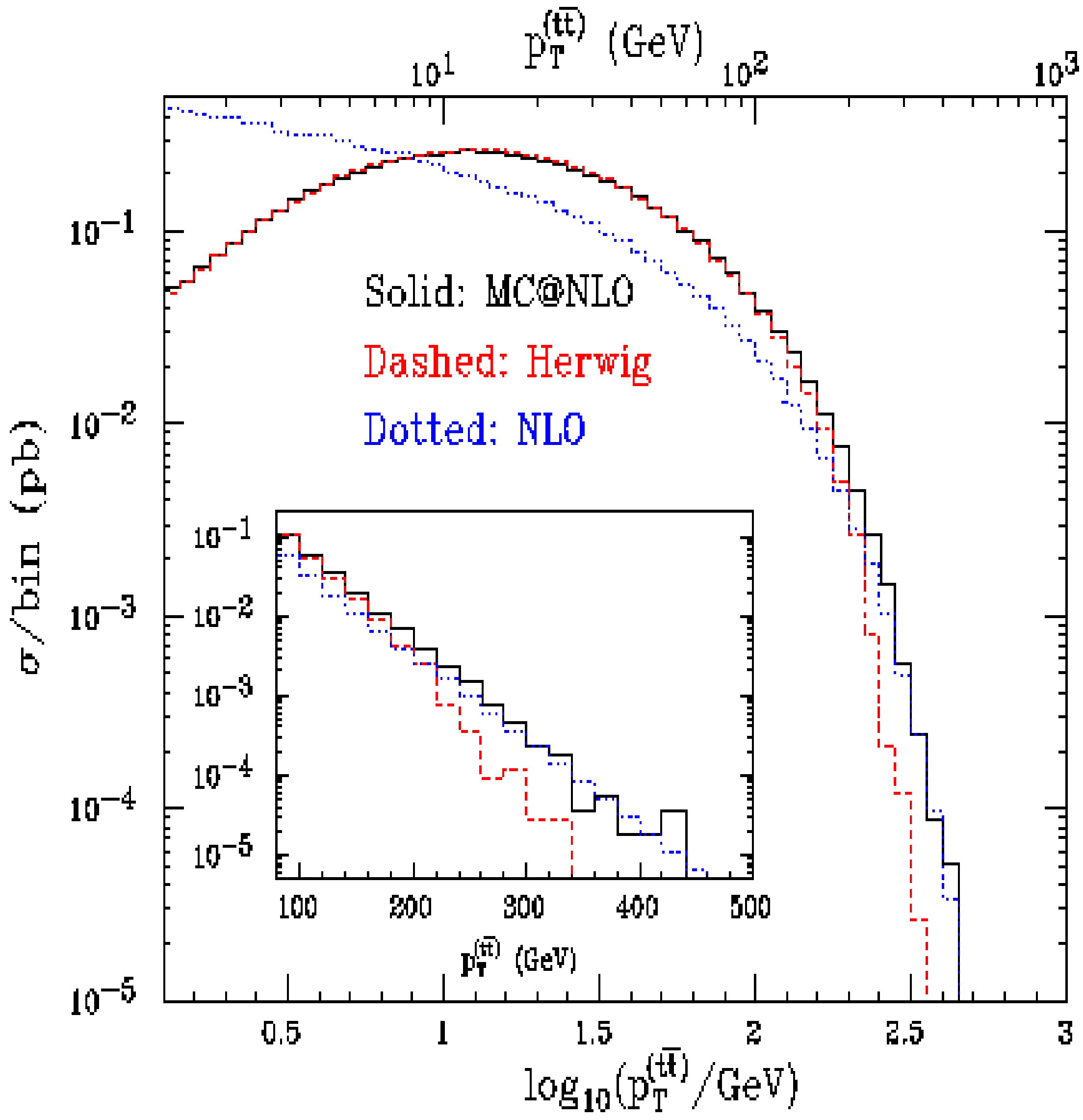


What is an MC@NLO?

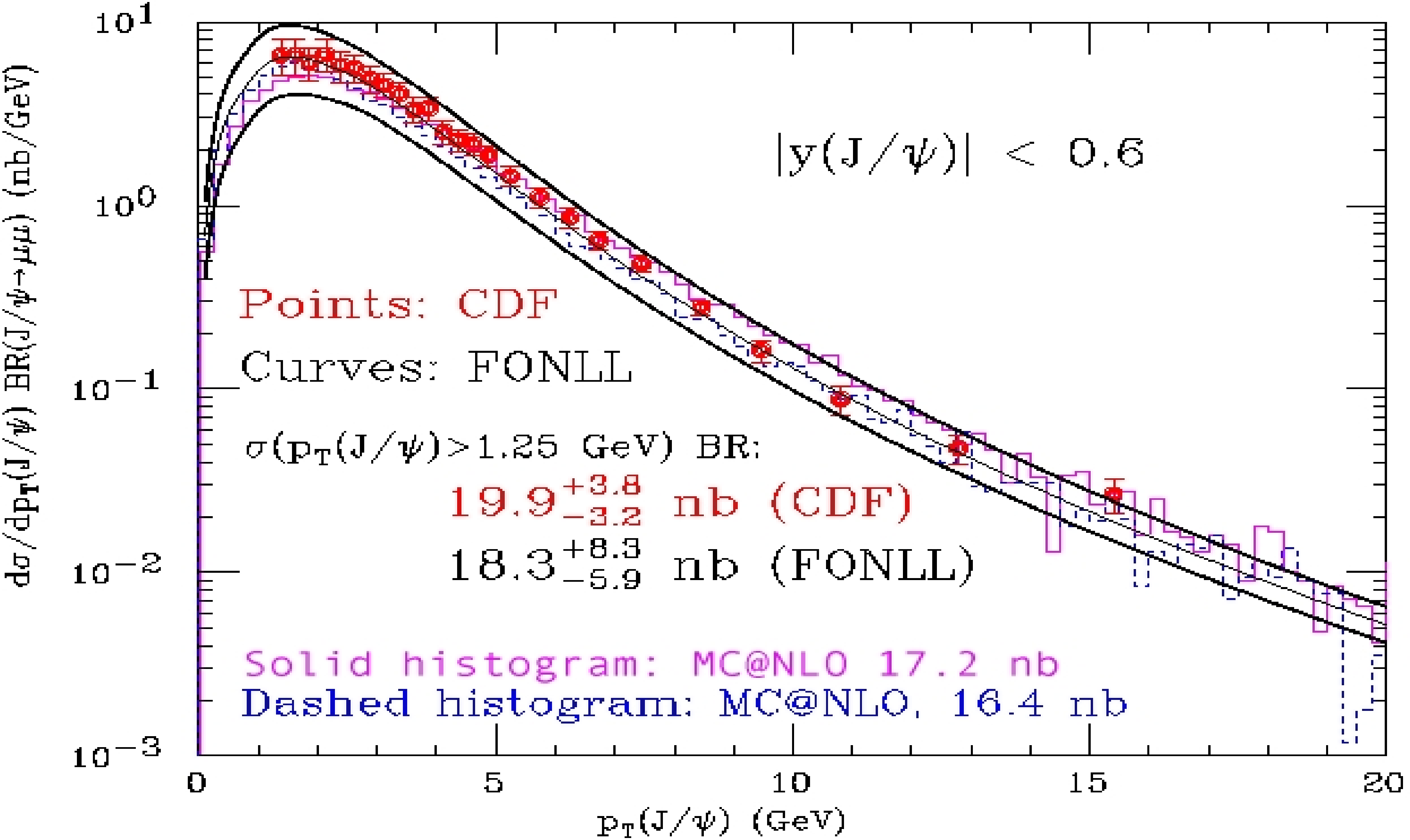
- NLO calculation of ME
- Parton Showers and Hadronisation from MC
- Matching the NLO ME with the PS
- The result is unweighted events
- Here the HERWIG MC is used and the NLO calculations are taken from FMNR.

Why MC@NLO?

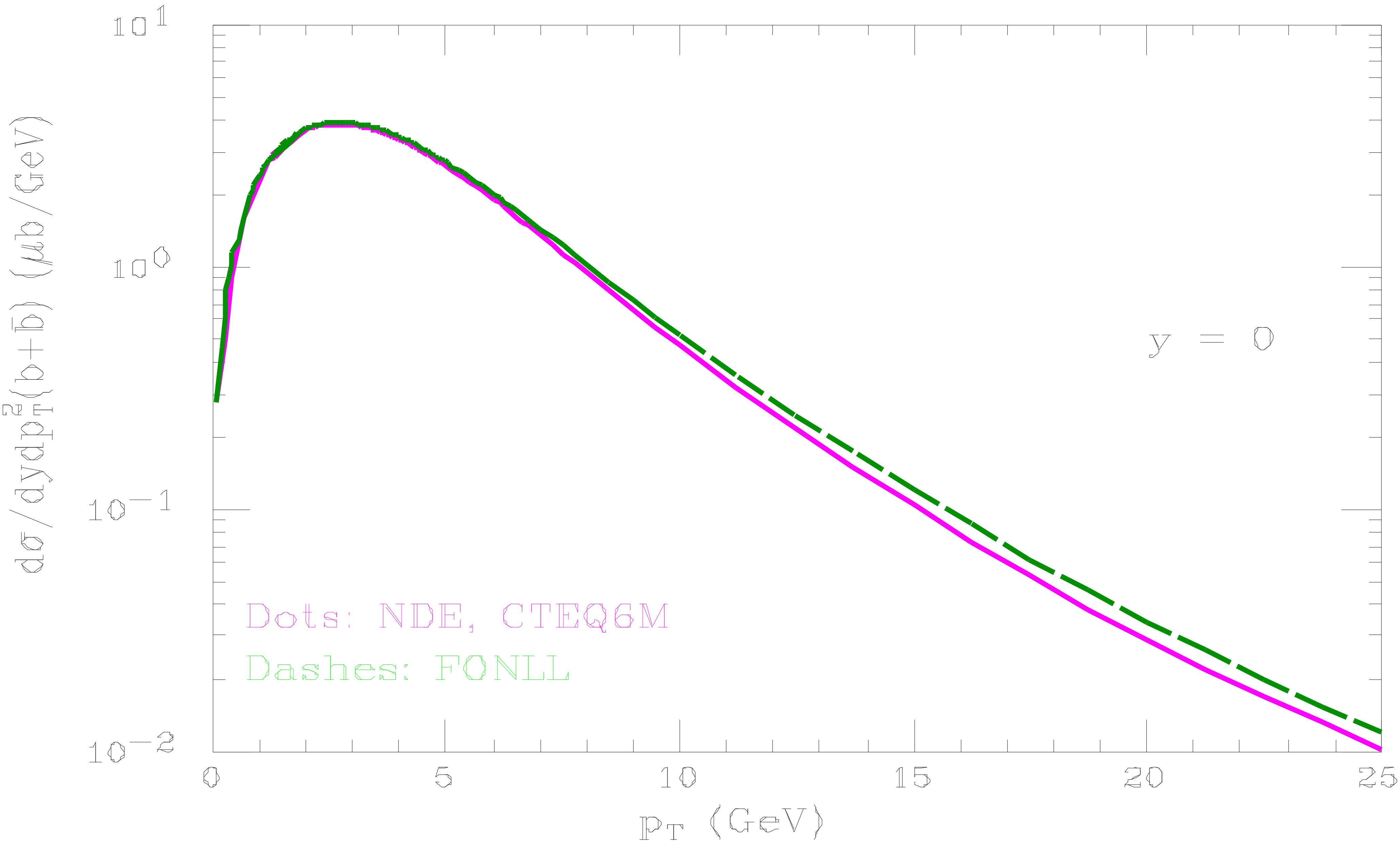
- For large Pt of the top-pair:
  - In parton showers high Pt cannot be described correctly
  - Not a problem for NLO
- For smaller Pt :
  - MC is effectively resumming to all orders through Sudakov form factors and reliably producing soft partons
  - This is impossible for NLO-calculation



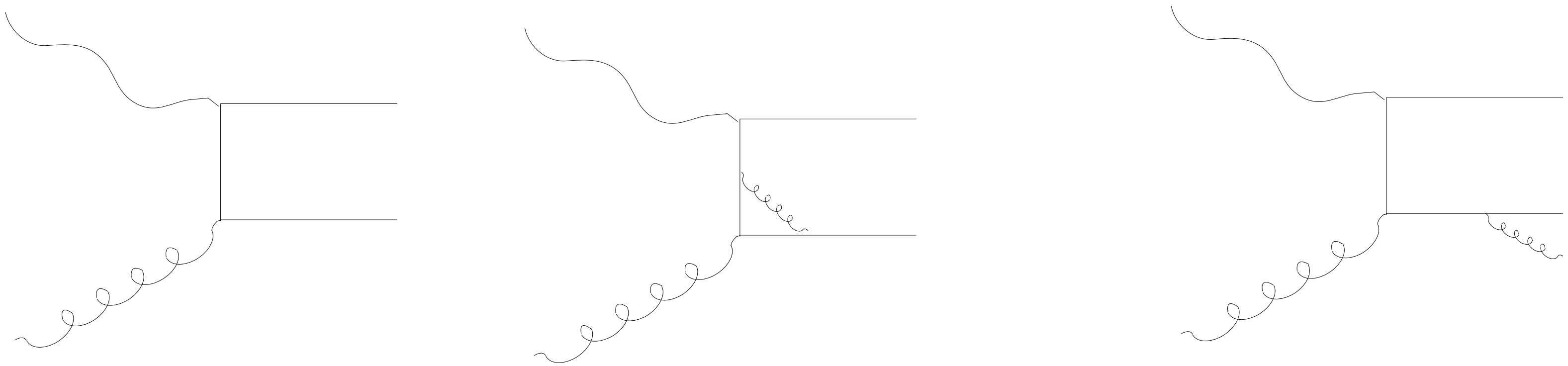
Comparisons with b-production Data



Comparisons with b-production models



Next to Leading Order (NLO) MEs for BGF



B V R

- Amplitudes for Born and Virtual Corrections interfere:

$$|A_m|^2 = \underbrace{B^*B}_{\propto\,\alpha_s} + \underbrace{(B^*V + V^*B)}_{\propto\,\alpha_s^2} + \underbrace{V^*V}_{\propto\,\alpha_s^4}$$

$$|A_{m+1}|^2 = \underbrace{R^*R}_{\propto\,\alpha_s^2}$$

NLO calculation

$$\sigma = \int_{m+1} \left(\mathrm{d}\sigma^{\mathrm{Real}}\right) + \int_{m} \left(\mathrm{d}\sigma^{\mathrm{Born}} + \mathrm{d}\sigma^{\mathrm{Virtual}}\right)$$

Divergent Divergent

- This is the general form of the NLO calculation
- However, there are two separate divergent integrals.

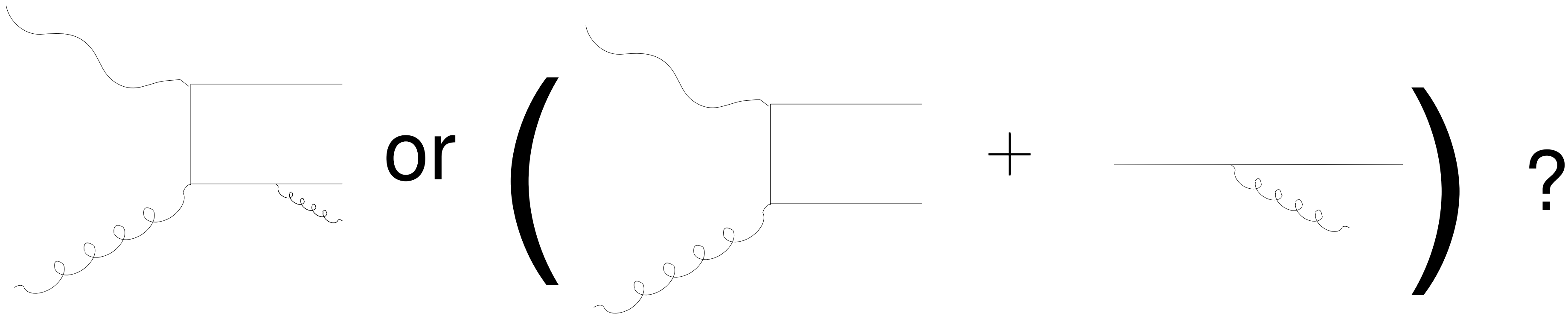
NLO calculation Subtraction

$$\sigma = \int_{m+1} \left(d\sigma^{\mathrm{Real}} - d\sigma^{\mathrm{subtr}}_{m+1} \right) + \int_{m} \left(d\sigma^{\mathrm{Born}} + d\sigma^{\mathrm{Virtual}} + d\sigma^{\mathrm{subtr}}_{m} \right)$$

Event Counter Event

- A term is subtracted and added which exactly cancels the divergencies
- The second term includes the virtual corrections. These yield a negative contribution to the cross-section.
- To cope with the numerical instabilities the negative weights may give rise to, for each event in the calculation the EVENT and the COUNTER EVENT are calculated separately and *then* added.
- This turns out to be a good (necessary) quality for MC@NLO!

MC@NLO Double Counting & neg. weights



- For real emissions there is a problem with double counting when combining the NLO ME with parton-showers
- There is also a problem with negative weights from the NLO-ME. This is not possible to have in a probabilistic MC.

Modified Subtraction

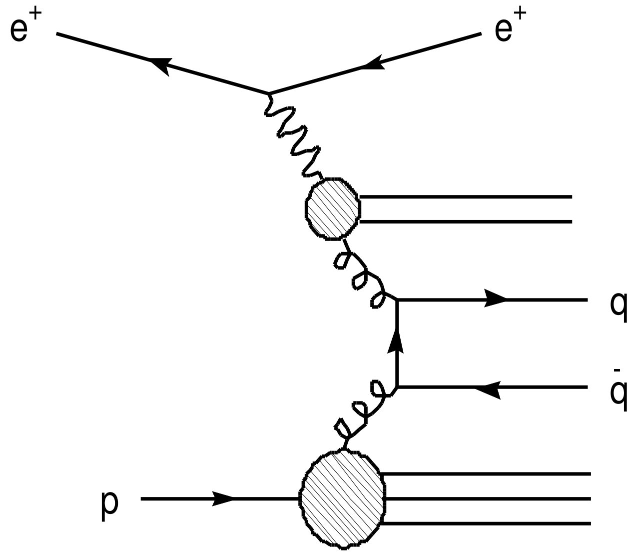
$$\sigma = \int_{m+1} \left(\mathrm{d}\sigma^{\mathrm{Real}} - \mathrm{d}\sigma^{\mathrm{subtr}}_{m+1} - \mathrm{d}\sigma_{MC}\right) +$$

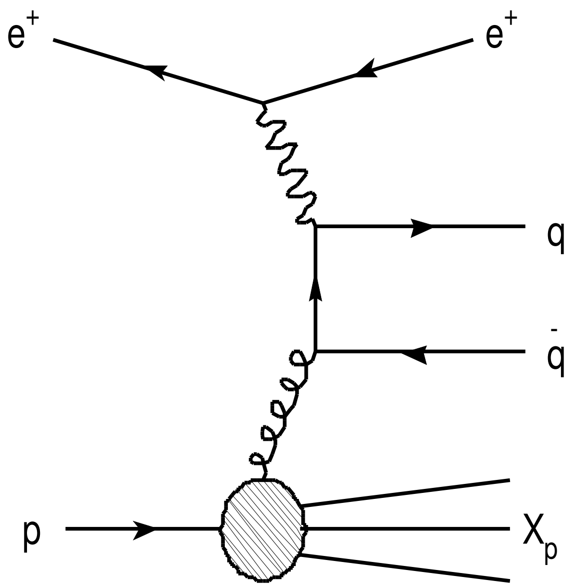
$$+ \int_{m} \left(\mathrm{d}\sigma^{\mathrm{Born}} + \mathrm{d}\sigma^{\mathrm{Virtual}} + \mathrm{d}\sigma^{\mathrm{subtr}}_{m} + \mathrm{d}\sigma_{MC}\right)$$

- To avoid double counting the contribution from the MC parton shower has to be calculated and subtracted from the contribution from the NLO ME.
- These terms in MC@NLO are called MC-subtraction terms
- These yields one integral with positive weights, and one with negative. This way the probabilistic approach of an MC still holds.
- The terms are MC-generator dependent.
- MC@NLO is using Herwig as MC and FMNR for the NLO calculation.

MC-subtraction terms for ep-scattering

- In ep-scattering two parts have to be considered separately:
  - the hadronic (resolved)
  - the direct interaction





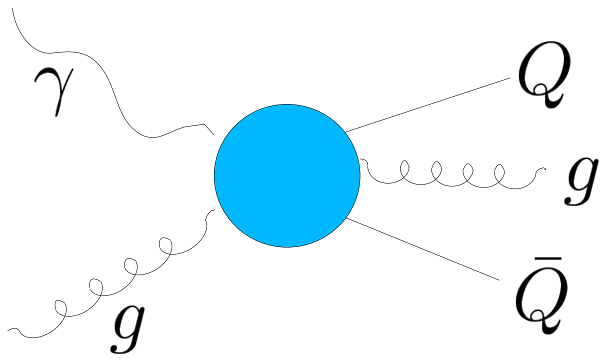
- Hadronic Case exists: PDF of P is replaced to electron PDF
- For direct case MC subtraction terms need to be calculated

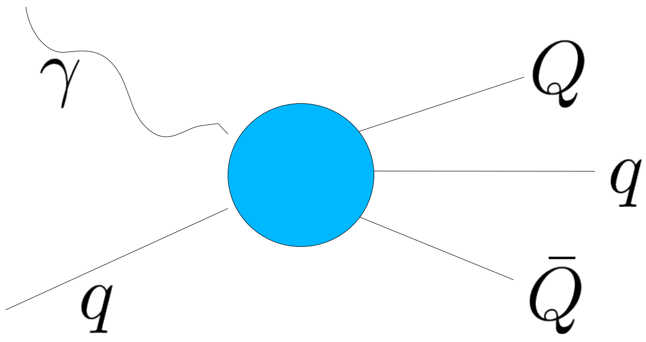
Calculation of MC-subtr. terms direct case

- The MC subtraction terms for HERWIG all have the same general structure:

$$\sigma_{\mathrm{MC}} = \sigma_{\mathrm{Born}}^{(s,t,u)} \times P_{\mathrm{A.P.}}(z) \times \{\mathrm{PhaseSpace, couplings\ etc.}\}$$

- Two main processes at Born level for direct case:
  - Photon-Gluon scattering
  - Photon-Quark/Antiquark scattering

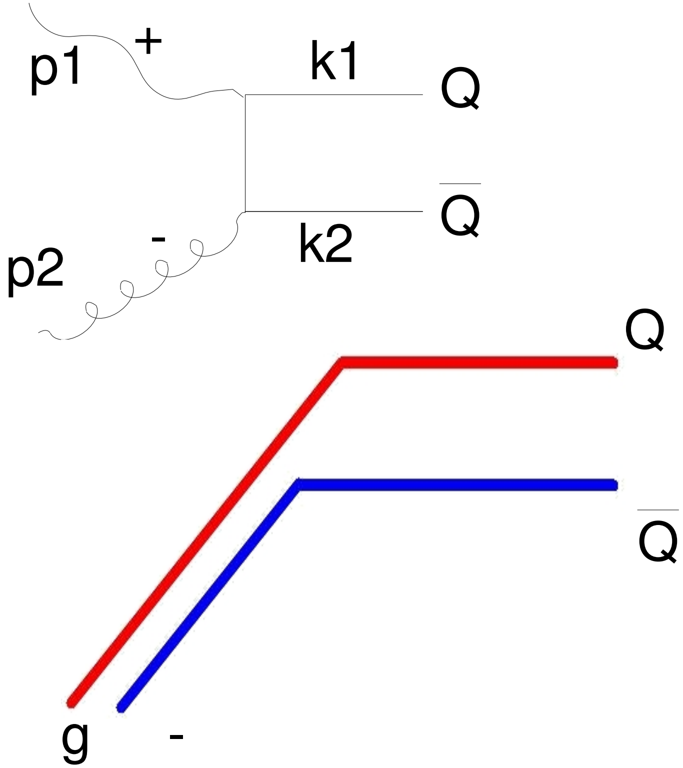


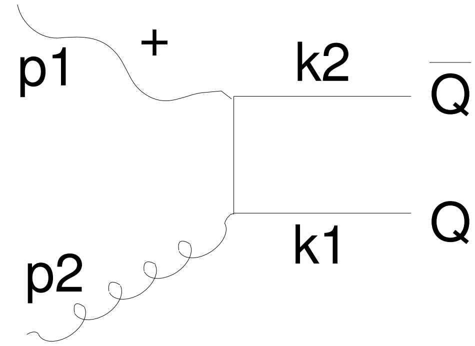


MC subtraction terms γg-scattering

- Two diagrams at Born level
- Emissions possible from “–“ leg and from outgoing legs (no QCD-emission from photon)

MC subtraction terms γg-scattering



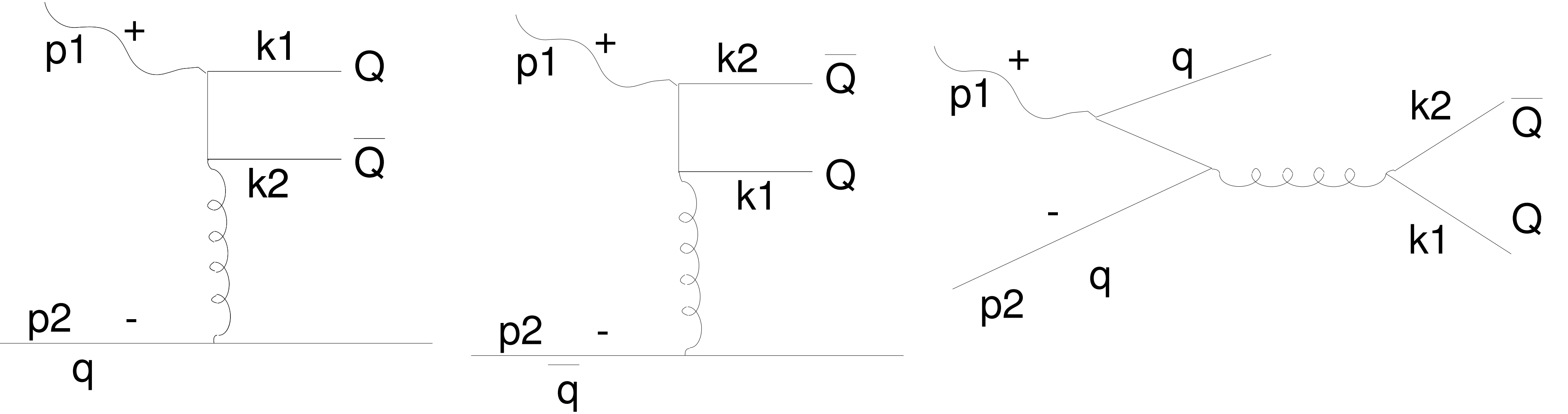


Initial state radiation: “-” leg: $d\sigma_{\gamma g}\left[\frac{1}{2}t+\frac{1}{2}u\right]\cdot P_{gg}$

Final state radiation: “Q” leg: $d\sigma_{\gamma g}\left[u\right]\cdot P_{qq}$

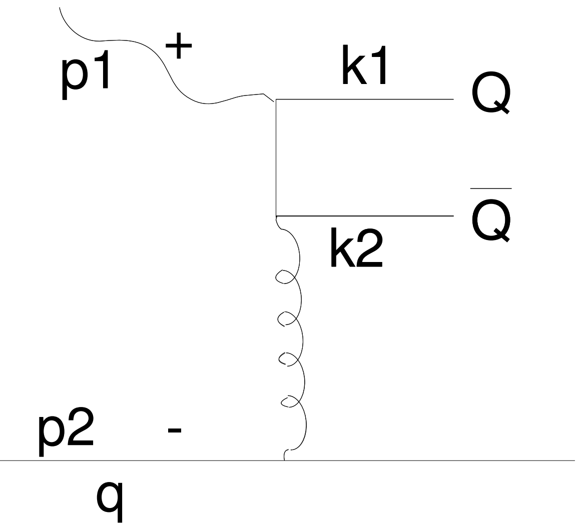
“$\bar{Q}$” leg: $d\sigma_{\gamma g}\left[t\right]\cdot P_{qq}$

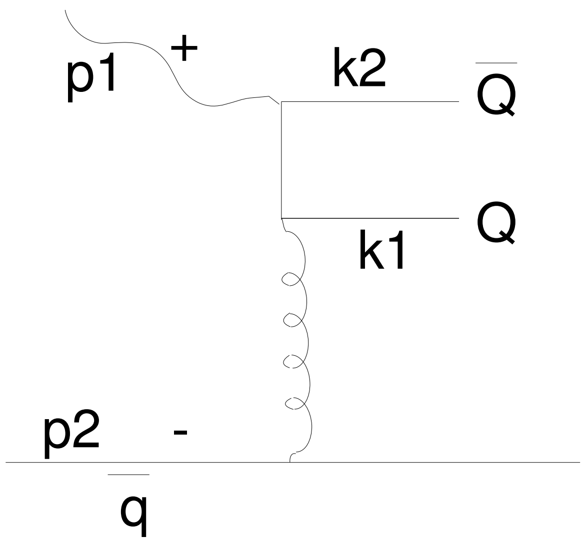
MC subtraction terms γq-scattering

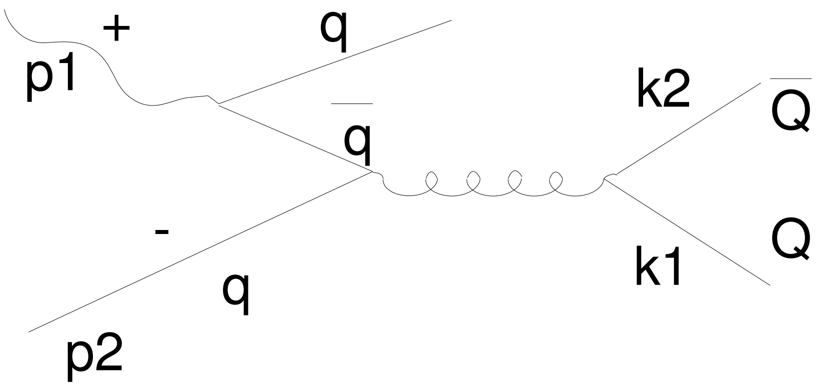


- In the first two diagrams only radiation from the “–“ leg. Born is photon-gluon.
- In the third diagram only radiation from “+” leg. Here electromagnetic “splitting function” has to be used. Born is quark-antiquark

MC subtraction terms γq-scattering







"-" leg: $d\sigma_{\gamma g}\left[\frac{1}{2}t+\frac{1}{2}u\right]\cdot P_{gq}$

"+" leg: $d\sigma_{\bar{q}q}\left[u\right]\cdot P_{\gamma\to q\bar{q}}$

$$P_{\gamma\to q\bar{q}}=\frac{T_f}{N_C}\cdot P_{qg}$$

Internal Consistency Test

$$\frac{MC_{\mathrm{subtr.}}(\cos\theta \to -1)}{MC_{\mathrm{subtr.}}(\cos\theta = -1)}$$

Each iteration closer to the limit. Example collinear - : $\cos(\theta) = -1 + 0.1^n$

**** Collinear- limit	**** Soft limit
0.911385659	1.0209289
0.963471603	0.999248357
0.988041227	0.999897717
0.996217561	0.999989494
0.998803796	0.999998902
0.9999995	1.00000232
0.99999995	1.00000023
0.999999995	1.00000002

Comparison with ME

$$\frac{MC_{\text{subtr.}}(\cos\theta \to -1)}{ME_{2\to3}(\cos\theta = -1)}$$

$$\frac{MC_{\text{subtr.}}(x \to 0)}{ME_{2\to3}(x = 0)}$$

**** Collinear- limit

0.974061006
1.004122
1.00254968
1.00093199
1.00030953
0.999999501
0.99999995
0.999999995

**** Soft limit

1.00412524
0.998628304
0.999845295
0.999984345
0.999995661
0.999996662
0.999999666
0.999999967

This is the important test!!

Present Status: Implementation

- Comparison of NLO-rates between MC@NLO and FMNR. When implementation is correct these should be equal

- Charm production [nb]

	Total Rate	Ratio
MC@NLO	0.8204 +/- .003	0.974 +/- 0.005
FMNR	0.8423 +/- .003	

- Beauty production [pb]

	Total Rate	Ratio
MC@NLO	5081 +/-30	0.974 +/- 0.006
FMNR	5215 +/-10	

Summary and Outlook

- When combining NLO ME and Parton showers there is a problem with double counting.
- In MC@NLO this problem is solved with process dependent MC Subtraction Terms.
- I have successfully constructed and tested MC Subtraction Terms for Heavy Quark production in eP collisions
- The subtraction terms are successfully implemented in the NLO-calculation
- Now I'm working with the Event Generation
- Hopefully there will be an MC@NLO for Hera within a couple of months.