Exclusive B rare decays
A topical example: $B \to K^* \ell^+ \ell^-$ decay at low-q^2

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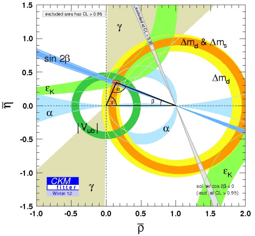
Flavor changing neutral currents (FCNC) in the SM

$$U_i \in \{u, c, t\},\ q_U = +2/3 \qquad D_i \in \{d, s, b\},\ q_D = -1/3$$

$$\mathcal{L}_{\mathrm{c.c.}} = \frac{g_2}{2\sqrt{2}}(\bar{u}, \bar{c}, \bar{t}) \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \gamma^\mu P_L \begin{pmatrix} d \\ s \\ b \end{pmatrix} W^+_\mu + \mathrm{h.c.}$$

- Chiral structure of c.c.: Only the left-handed components of the fields interact

- Complex and Unitary matrix⇒ **3 angles** and **1 phase**



$$V_{CKM} \simeq \begin{pmatrix} 1 - \lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1 - \lambda^2/2 & A\lambda^2 \\ A\lambda^3(1 - \rho - i\eta) & -A\lambda^2 & 1 \end{pmatrix}$$

$$\lambda = 0.2253(7), \quad A = 0.808(22),$$
$$\bar{\rho} = 0.132(22), \quad \bar{\eta} = 0.341(13)$$

- **The structure of the CKM matrix is extremely hierarchical!**

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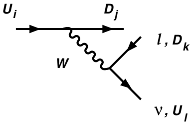
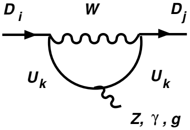
- **CC $U_i \to D_j$: Tree level**



- **FCNC $D_i \to D_j$: Loop**



- $H_1 \to H_2 H_3$
  $\mathcal{M} \sim G_F\, V_{ij} V^*_{kl}$,
  $V_{ij} V^*_{kl}$ can be $\mathcal{O}(1)$

- $H_1 \to H_2 G^0 \to H_2 \{H_3, \gamma, \bar{\ell}\ell\}$
  $\mathcal{M} \sim G_F\, g \sum_k V_{ki} V^*_{kj} f(M_k)$,
  $V_{ki} V^*_{kj} f(M_k)$ is $\mathcal{O}(\lambda^2_{\mathrm{CKM}}) \times$ Loop

- In the SM, FCNCs are suppressed w.r.t. CC interactions: **"Rare" decays!**

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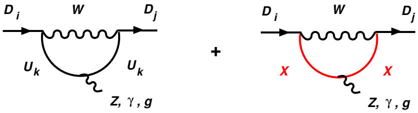
- Chiral structure of c.c.: Only the left-handed components of the fields interact

- **FCNC** $D_i \to D_j$: SM+new particles in the **Loop**



- $H_1 \to H_2 G^0 \to H_2 \{H_3, \gamma, \bar{\ell}\ell\}$

$$\mathcal{M} \sim G_F\, g \left(\sum_k V_{ki} V_{kj}^* f(M_k) + \tilde{V}_{Xi} \tilde{V}_{Xj}^* f(M_X) \right)$$

- FCNCs are sensitive to the effects of virtual new particules!

Experimental situation on "exclusive" rare $b \to s\ell^+\ell^-$ decays

- **"Exclusive decays"**: All the final products detected and identified
  $\bar{B} \to \bar{K}^*\ell^+\ell^-$ ("exclusive") in opposition to $\bar{B} \to X_s\ell^+\ell^-$ ("inclusive")
- Why $b \to s\ell^+\ell^-$ transitions?
  - $b \to d$ transitions are suppressed by an extra $\lambda^2_{\rm CKM}$ factor: **Hard to detect!**
  - $b \to s\ell^+\ell^-$, leptonic and semi-leptonic transitions are theoretically "clean"
- Branching fractions of $\sim 10^{-6}$ and relatively small number of events so far

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# Events	$B^0 \to K^{*0}\ell^+\ell^-$	$B^+ \to K^+\ell^+\ell^-$	$B_s \to \phi\ell^+\ell^-$	$\Lambda_b \to \Lambda\ell^+\ell^-$
BaBar (2012)	137±44	153±41		
Belle (2009)	247±54	162±38		
CDF (2011)	164±15	234±19	49±7	24 ± 5
LHCb (2011)	900±34		77±10	
$10^6 \times \mathcal{B}$	$1.29^{+0.22}_{-0.21}$	0.50±0.04	0.91±0.24	1.7±0.7

Babar 1204.3933, **Belle** 0904.0770, **CDF** 1107.3753 + 1108.0695, **LHCb** LHCb-CONF-2012-0088

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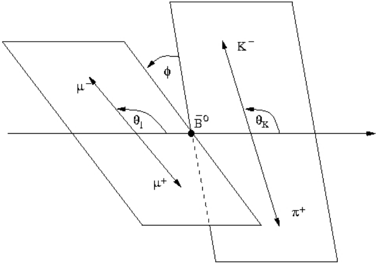
Goal: Analysis of $B \to K^*\ell^+\ell^-$

Exploring BSM scenarios using exclusive $b \to s$ decays at the precision frontier!

- Careful study of the theoretical uncertainties
- High-precision of experimental data needed

$\bar{B} \to \bar{K}^{*0}(\to K^-\pi^+)\ell^+\ell^-$ decay rate

- 4-body decay with K^* assumed to be on-shell
  - $m_{\ell\ell}^2 = q^2$, $4m_\ell^2 \leq q^2 \leq (m_B - m_{K^*})^2$
  - θ_k angle between $\vec{p}_{K-}$ and $\vec{p}_B$ in the (πK) CM
  - θ_ℓ angle between $\vec{p}_{\bar{B}}$ and $\vec{p}_{\ell+}$ in the $(\ell\ell)$ CM
  - ϕ angle between $\vec{p}_{K-} \times \vec{p}_{\pi^+}$, $\vec{p}_{\ell+} \times \vec{p}_{\ell-}$ in $\bar{B}$ CM



$$\frac{d^{(4)}\Gamma}{dq^2\, d(\cos\theta_l) d(\cos\theta_k) d\phi} = \frac{9}{32\pi}\big(I_1^s \sin^2\theta_k + I_1^c \cos^2\theta_k + (I_2^s \sin^2\theta_k + I_2^c \cos^2\theta_k)\cos 2\theta_l$$
$$+ \quad I_3 \sin^2\theta_k \sin^2\theta_l \cos 2\phi + I_4 \sin 2\theta_k \sin 2\theta_l \cos\phi + I_5 \sin 2\theta_k \sin\theta_l \cos\phi + I_6 \sin^2\theta_k \cos\theta_l$$
$$+ \quad I_7 \sin 2\theta_k \sin\theta_l \sin\phi + I_8 \sin 2\theta_k \sin 2\theta_l \sin\phi + I_9 \sin^2\theta_k \sin^2\theta_l \sin 2\phi\big)$$

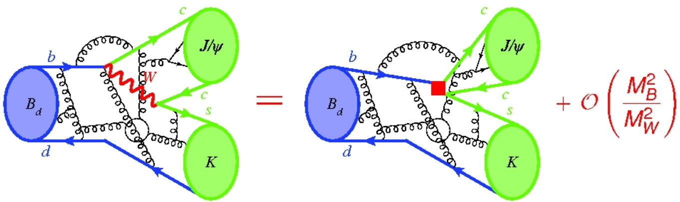
$\bar{B} \to \bar{K}^{*0}(\to K^-\pi^+)\ell^+\ell^-$ decay has a very rich phenomenology

- $I_i(q^2)$ **11 q^2-dependent observables** in the SM
- The CP-partner decay add other 11 independent observables
- A total of **22** observables per lepton mode!!

The theory of the exclusive decays I: The weak Hamiltonian

- A theoretical treatment of the B meson decay starts with a separation of the different scales

Weak scale	B-meson mass, external momenta	"Long-distance" hadronic effects
$m_W,\, m_t,\, m_X \sim \mathcal{O}(100)$ GeV	$m_B \sim \mathcal{O}(5)$ GeV	$\Lambda_{QCD} \sim \mathcal{O}(0.5)$ GeV



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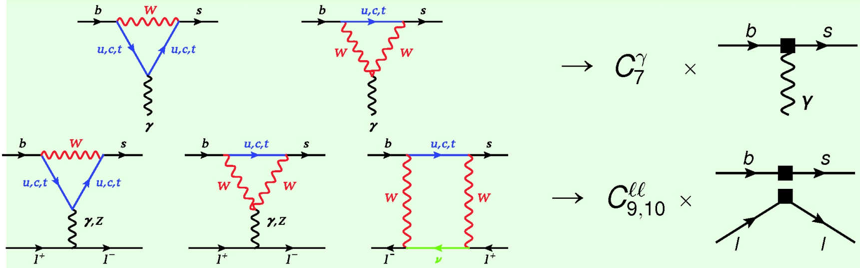
- At the hadronic scales interactions involving $\mathcal{O}(m_W)$ are approximately local (short-range)

The weak Hamiltonian $\mathcal{H}_w$

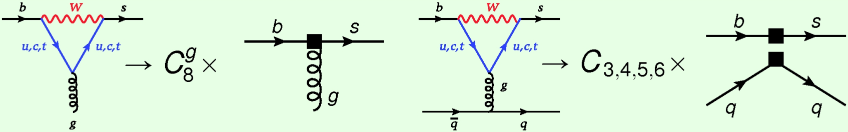
Integrate out $\mathcal{O}(m_W)$ DOF and construct a low-energy EFT

$b \to s + \gamma$ AND $b \to s + \ell^+\ell^-$



$$\mathcal{O}_7 = m_b[\bar{s}\,\sigma^{\mu\nu}P_R\,b]F_{\mu\nu} \qquad \mathcal{O}_{9,10} = [\bar{s}\,\gamma^{\mu}P_L\,b][\bar{\ell}\,\gamma_{\mu}(1,\,\gamma_5)\,\ell]$$

$b \to s + gluon$ AND $b \to s + \bar{q}q$



$$\mathcal{O}_8 = m_b[\bar{s}\,\sigma^{\mu\nu}P_R T^a\,b]G^a_{\mu\nu} \qquad \mathcal{O}_{3,4} = [\bar{s}\,\gamma^{\mu}(1,\,T^a)P_L\,b]\sum_q[\bar{q}\,\gamma_{\mu}(1,\,T^a)\,q]$$

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The weak Hamiltonian for $b \to s$ transitions

- In the SM we have

$$\mathcal{H}_W = \frac{G_F}{\sqrt{2}} \sum_{p=u,\,c} \lambda_p \left[C_1 \mathcal{O}_1^p + C_2 \mathcal{O}_2^p + \sum_{i=3,\,10} C_i \mathcal{O}_i \right] ,$$

$$\mathcal{O}_1^p = (\bar{p}b)_{V-A}(\bar{s}p)_{V-A}\,, \qquad \mathcal{O}_2^p = (\bar{p}_i b_j)_{V-A}(\bar{s}_j p_i)_{V-A}\,,$$

$$\mathcal{O}_3 = (\bar{s}b)_{V-A} \sum\nolimits_q (\bar{q}q)_{V-A}\,, \qquad \mathcal{O}_4 = (\bar{s}_i b_j)_{V-A} \sum\nolimits_q (\bar{q}_j q_i)_{V-A}\,,$$

$$\mathcal{O}_5 = (\bar{s}b)_{V-A} \sum\nolimits_q (\bar{q}q)_{V+A}\,, \qquad \mathcal{O}_6 = (\bar{s}_i b_j)_{V-A} \sum\nolimits_q (\bar{q}_j q_i)_{V+A}\,,$$

$$\mathcal{O}_7 = \frac{e}{4\pi^2}\, \hat{m}_b\, \bar{s}\sigma_{\mu\nu} P_R F^{\mu\nu}\, b\,, \qquad \mathcal{O}_8 = \frac{g_s}{4\pi^2}\, \hat{m}_b\, \bar{s}\sigma_{\mu\nu} P_R G^{\mu\nu}\, b\,,$$

$$\mathcal{O}_9 = \frac{\alpha_{\rm em}}{2\pi} (\bar{s}b)_{V-A}(\bar{l}l)_V\,, \qquad \mathcal{O}_{10} = \frac{\alpha_{\rm em}}{2\pi} (\bar{s}b)_{V-A}(\bar{l}l)_A.$$

- **Information on interactions/DOFs at $\Lambda \sim \mathcal{O}(m_W)$ stored in the Wilson coeffs. $C_i(\mu)$'s**

Table: Wilson coefficients of the SM at $\mu = 4.8$ GeV.

C_1	C_2	C_3	C_4	C_5	C_6	$C_7^{\rm eff}$	$C_8^{\rm eff}$	C_9	C_{10}
-0.144	1.060	0.011	-0.034	0.010	-0.040	-0.305	-0.168	4.24	-4.312

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$$\mathcal{O}_7 = \frac{e}{4\pi^2} \hat{m}_b \, \bar{s}\sigma_{\mu\nu} P_R F^{\mu\nu} b , \qquad \mathcal{O}_8 = \frac{g_s}{4\pi^2} \hat{m}_b \, \bar{s}\sigma_{\mu\nu} P_R G^{\mu\nu} b ,$$

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- **Information on interactions/DOFs at $\Lambda \sim \mathcal{O}(m_W)$ stored in the Wilson coeffs. $C_i(\mu)$'s**
- Physics BSM manifest at the operator level through...
  - Different values of the Wilson coefficients $C_i^{\rm expt.} = C_i^{\rm SM} + \delta C_i$
  - New operators absent or very suppressed in the SM

Chirally-flipped operators

$$\mathcal{O}_7' = \frac{e}{4\pi^2} \hat{m}_b \, \bar{s}\sigma_{\mu\nu} P_L F^{\mu\nu} b$$

The theory of the exclusive decays II: The hadronic uncertainties

- We can't access the short range $b \to s$ operators "directly" in experiment! Instead, we can access them convoluted with hadronic effects in B-decays

Realistic theoretical analysis!

One has to know how far does he understand the hadronic effects

- In the $B \to K^* \ell^+ \ell^+$ we have the following contributions

$$\mathcal{M} \propto C_9 \, \langle K^*(k)|(\bar{s}\gamma^\mu P_L b)|B(p)\rangle \times \langle \ell^+\ell^-|\bar{\ell}\gamma_\mu \ell|0\rangle$$

$$\mathcal{M} \propto \frac{1}{q^2} C_7 \, \langle K^*(k)|(\bar{s}\sigma^{\mu\nu} q_\nu P_R b)|B(p)\rangle \times \langle \ell^+\ell^-|\bar{\ell}\gamma_\mu \ell|0\rangle$$

Hadronic matrix elements parameterized by **4+3** q^2 dependent functions: **form factors**
  - At large $q^2 \simeq (m_B - m_{K^*})^2$ the form factors can be calculated in **LQCD**
  - At low q^2 the form factors can be calculated in **LCSRs** and models
- The FFs are the major source of uncertainty in the treatment of exclusive B decays

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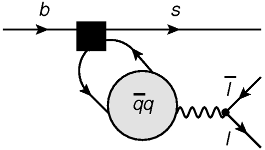
- Non-local contribution from the contraction of 4-quark operators $\mathcal{O}^{(c)}_{1-6}$ with the EM current



$$\mathcal{A}^{(\mathrm{had})} \propto \int d^4y e^{iq\cdot y} \langle \bar{K}^* | j^{\mathrm{em,had},\mu}(y) \mathcal{H}^{\mathrm{had}}(0) | \bar{B} \rangle$$

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- This object is untractable in a model-independent manner in certain regions of q^2
- At $\sqrt{q^2} \sim m_{J/\Psi}$ it describes the decay through the charmonium resonances

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Two q^2 regions considered clean

- Low q^2, large recoil of the K^*: $q^2 < 6$ GeV^2
  Treated in **QCD factorization** and/or **SCET**$\Rightarrow$ Expansion in Λ/E_{K^*}
- High q^2, low recoil of the K^*: $q^2 > 15$ GeV^2
  Treated using **OPE+HQET**

- Much work has already been done on this decay
  Kruger, Sinha and Sinha '98, Beneke, Feldmann and Seidel'01'05, Kruger and Matias'05, Bobeth et al.'08, Altmannshofer et al.'08, Egede et al.'09'10, Beylich et al.'11, Becirevic et al.'11, Matias and Virto'12,. . .

Connecting the theory to experiment: The helicity amplitudes

- Helicity decomposition

$$\mathcal{A} = -\sum_{\lambda=\pm1,0} \mathcal{L}_V(\lambda) H_V(\lambda) - \sum_{\lambda=\pm1,0} \mathcal{L}_A(\lambda) H_A(\lambda) + L_P H_P$$

$$L_V^\mu = \langle \ell^+\ell^- | \bar{l}\gamma^\mu l | 0 \rangle,\ L_A^\mu = \langle \ell^+\ell^- | \bar{l}\gamma^\mu\gamma^5 l | 0 \rangle,\ L_P = \langle \ell^+\ell^- | \bar{l}\gamma^5 l | 0 \rangle$$

- Helicity amplitudes $\lambda = \pm1, 0$

$$\begin{aligned} H_V(\lambda) &= -iN\Big\{ C_{9V}\,\tilde{V}_{L\lambda} + C'_{9V}\,\tilde{V}_{R\lambda} - \frac{m_B^2}{q^2}\Big[\frac{2\,\hat{m}_b}{m_B}(C_{7\gamma}\,\tilde{T}_{L\lambda} + C'_{7\gamma}\,\tilde{T}_{R\lambda}) - 16\pi^2 h_\lambda\Big]\Big\}, \\ H_A(\lambda) &= -iN(C_{10A}\,\tilde{V}_{L\lambda} + C'_{10A}\,\tilde{V}_{R\lambda}), \\ H_P &= iN\frac{2\,m_l\hat{m}_b}{q^2}\,C_{10A}\Big(\tilde{S}_L + \frac{m_s}{m_b}\tilde{S}_R\Big) \end{aligned}$$

- **Short-range:** Wilson coefficients
- **Long-range (QCD):** FFs and the non-local piece h_λ

$$\begin{aligned} -im_B\,\tilde{V}_{L(R)\lambda}(q^2) &= \langle M(\lambda)|\bar{s}\not{\epsilon}^*(\lambda)P_{L(R)}b|\bar{B}\rangle, \\ m_B^2\,\tilde{T}_{L(R)\lambda}(q^2) &= \epsilon^{*\mu}(\lambda)q^\nu\langle M(\lambda)|\bar{s}\sigma_{\mu\nu}P_{R(L)}b|\bar{B}\rangle, \\ im_B\,\tilde{S}_{L(R)}(q^2) &= \langle M(\lambda=0)|\bar{s}P_{R(L)}b|\bar{B}\rangle. \end{aligned}$$

$$\frac{d^{(4)}\Gamma}{dq^2\, d(\cos\theta_l) d(\cos\theta_k) d\phi} = \frac{9}{32\pi}(I_1^s \sin^2\theta_k + I_1^c \cos^2\theta_k + (I_2^s \sin^2\theta_k + I_2^c \cos^2\theta_k)\cos 2\theta_l$$

$$+ \quad I_3 \sin^2\theta_k \sin^2\theta_l \cos 2\phi + I_4 \sin 2\theta_k \sin 2\theta_l \cos\phi + I_5 \sin 2\theta_k \sin\theta_l \cos\phi + I_6 \sin^2\theta_k \cos\theta_l$$

$$+ \quad I_7 \sin 2\theta_k \sin\theta_l \sin\phi + I_8 \sin 2\theta_k \sin 2\theta_l \sin\phi + I_9 \sin^2\theta_k \sin^2\theta_l \sin 2\phi)$$

$$I_1^c = \frac{1}{2}\left(|H_V^0|^2 + |H_A^0|^2\right) + 4|H_P|^2 + \frac{2m_\ell^2}{q^2}\left(|H_V^0|^2 - |H_A^0|^2\right),$$

$$I_1^s = \frac{\beta^2+2}{8}\Big(|H_V^+|^2 + |H_V^-|^2 + (V \to A)\Big) + \frac{m_\ell^2}{q^2}\Big(|H_V^+|^2 + |H_V^-|^2 - (V \to A)\Big),$$

$$I_2^c = -\frac{\beta^2}{2}\left(|H_V^0|^2 + |H_A^0|^2\right), \quad I_2^s = \frac{\beta^2}{8}\Big(|H_V^+|^2 + |H_V^-|^2\Big) + (V \to A),$$

$$I_3 = -\frac{1}{2}\mathrm{Re}\left[H_V^+ (H_V^-)^*\right] + (V \to A), \quad I_4 = \frac{\beta^2}{4}\mathrm{Re}\left[(H_V^- + H_V^+)\,(H_V^0)^*\right] + (V \to A),$$

$$I_5 = \frac{\beta}{2}\mathrm{Re}\left[(H_V^- - H_V^+)\,(H_A^0)^*\right] + (V \leftrightarrow A), \quad I_6 = \beta\mathrm{Re}\left[H_V^- (H_A^-)^* - H_V^+ (H_A^+)^*\right],$$

$$I_7 = \frac{\beta}{2}\mathrm{Im}\left[\Big(H_A^+ + H_A^-\Big)(H_V^0)^*\right] + (V \leftrightarrow A),$$

$$I_8 = \frac{\beta^2}{4}\mathrm{Im}\left[(H_V^- - H_V^+)(H_V^0)^*\right] + (V \to A), \quad I_9 = \frac{\beta^2}{2}\mathrm{Im}\left[H_V^+ (H_V^-)^*\right] + (V \to A).$$

$$\frac{d^{(4)}\Gamma}{dq^2\, d(\cos\theta_l) d(\cos\theta_k) d\phi} = \frac{9}{32\pi}\Big(I_1^s \sin^2\theta_k + I_1^c \cos^2\theta_k + (I_2^s \sin^2\theta_k + I_2^c \cos^2\theta_k)\cos 2\theta_l$$

$$+ \quad I_3 \sin^2\theta_k \sin^2\theta_l \cos 2\phi + I_4 \sin 2\theta_k \sin 2\theta_l \cos\phi + I_5 \sin 2\theta_k \sin\theta_l \cos\phi + I_6 \sin^2\theta_k \cos\theta_l$$

$$+ \quad I_7 \sin 2\theta_k \sin\theta_l \sin\phi + I_8 \sin 2\theta_k \sin 2\theta_l \sin\phi + I_9 \sin^2\theta_k \sin^2\theta_l \sin 2\phi\Big)$$

- Hadronic parameters (FFs and h_λ) + Wilson coefficients(model) $\Rightarrow$ Prediction 22 observables vs. Expt.
- Hadronic parameters + Expt. $\Rightarrow$ Wilson coefficients $\Rightarrow$ Constraints onto BSMs
- Note that each observable I_i can have different sensitivity to different operators

$$I_3 \propto Re[C_7 (C_7')^*] \text{ at } q^2 \simeq 0$$

Analysis of the hadronic uncertainties at low-q^2

- At low $q^2 \simeq 0$, h_λ contain light-resonances $\rightarrow q^2 < 1$ GeV^2 cut-off from analyses
- The contribution of $\mathcal{O}_7^{(\prime)}$ is enhanced by $1/q^2$

Sensitivity to "wrong-helicity" photons at $q^2 \simeq 0$

$$\mathcal{O}_7' = \frac{e}{4\pi^2}\, \hat{m}_b\, \bar{s}\sigma_{\mu\nu} P_L F^{\mu\nu} b$$

vs.

$$\mathcal{O}_7 = \frac{e}{4\pi^2}\, \hat{m}_b\, \bar{s}\sigma_{\mu\nu} P_R F^{\mu\nu} b$$

$$\begin{aligned}\mathcal{A}(\bar{B} \rightarrow V\ell^-\ell^+) \quad &= \quad \sum_i C_i \langle \ell^-\ell^+ | \bar{l}\Gamma_i l | 0 \rangle \langle V | \bar{s}\Gamma_i' b | \bar{B} \rangle \\ &\quad + \frac{e^2}{q^2} \langle \ell^-\ell^+ | \bar{l}\gamma^\mu l | 0 \rangle F.T. \langle V | T(j_{\mu,\mathrm{em}}^{\mathrm{had}}(x) \mathcal{H}_W^{\mathrm{had}}(0)) | \bar{B} \rangle\end{aligned}$$

- At low q^2 the amplitude lend itself to a systematic treatment at leading power of Λ/m_b

QCD factorization

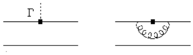
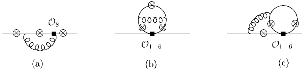
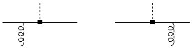
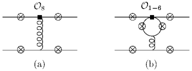
QCD factorization in a nut-shell

- The $\bar{B} \to K^* \gamma^*$ amplitude can be factorized as (Beneke, Feldmann and Seidel '01)

$$\mathcal{T}_a = \xi_a \left(C_a^{(0)} + \frac{\alpha_s C_F}{4\pi} C_a^{(1)} \right) + \frac{\pi^2}{N_c} \frac{f_B f_{K^*,a}}{m_B} \Xi_a \sum_{\pm} \int \frac{d\omega}{\omega} \Phi_{B,\pm}(\omega) \int_0^1 du \, \Phi_{K^*,a}(u) \, T_{a,\pm}(u,\omega) + \mathcal{O}(\Lambda/m_b),$$

for $a = \parallel, \perp$.

- **Non-perturbative objects:** ξ_a soft form factors, f_M decay constants and Φ_M LCDAs
- **Perturbative kernels:** $C_a^{(i)}$ and $T_a^{(i)}$

	"Factorizable" (QCD FFs)	"Non-factorizable"
Vertex		
Hard Scattering		

- QCD factorization is valid as long the amplitudes are IR-safe!

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Estimate Λ/m_b corrections

- Factorizable power corrections: Relate QCD form factors with soft-form factors
- Non-perturbative charm-quark loop (non-local!) effects
- Non-perturbative light-quatk loop (non-local!) effects

Form Factors

- In the heavy-quark and large-recoil (K^*) limit:
  Charles et al. '99
  The **7** $B \to K^*$ FFs to **2** soft form factors $\xi_\parallel(q^2)$ and $\xi_\perp(q^2)$:

$$T_- = V_- = \frac{2E}{m_B}\xi_\perp, \quad T_+ = V_+ = 0,$$
$$T_0 = V_0 = A_0 = \frac{E}{m_{K^*}}\xi_\parallel$$

- These relations receive calculable α_s corrections and unknown Λ/m_b power corrections
  Beneke et al. '01
- We use

$$\xi_\perp(0) = T_1(0) = 0.275(26), \qquad \xi_\parallel(0) = \frac{2m_{K^*}}{m_B}A_0(0) = 0.09(2)$$

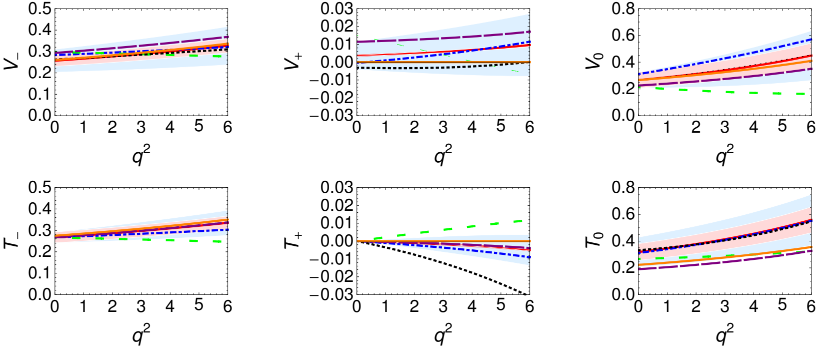
- We fix (for numerics) $\xi_\perp(0)$ with $\mathcal{B}(\bar{B}^0 \to K^{*0}\gamma)_{\rm expt}$ and $C_7^{\rm SM}$ (BFS'01)
- We fix $\xi_\parallel(0)$ using (normalized) theoretical predictions on A_0
  - **Light-cone SRs** (Ball&Zwicky'05, Khodjamirian *et al.*'10)
  - **QCD SRs** (Colangelo *et al.*'96)
  - **Dyson-Schwinger** (Ivanov *et al.*'07)

Factorizable power-corrections

- Power corrections to the HQ-LE relations

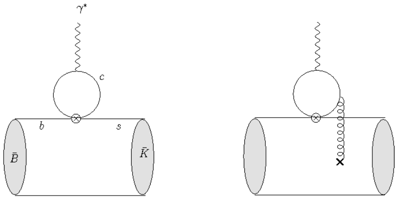
$$F^{\mathrm{p.c.},\pm} = \pm a_F \pm b_F \frac{q^2}{m_B^2}$$

a_F and $b_F \equiv$spread of th.predictions



$T_+(q^2)$ has a negligible uncertainty at low q^2 ($a_{T_+} \equiv 0!$)

Non-local charm-loop contribution



- The LHS diagram and α_s corrections are treated in **QCDf** (BFS'01)
- Soft-gluon contributions: $\delta H_- \sim 8\% C_7^{eff}$ (Khodjamirian *et al.*'10)
- For the numerics, our NF charm-loop uncertainty is

$$\delta H_- = (0.1 \times C_7^{eff}) e^{i\phi_-}, \qquad \delta H_+ = (0.1 \times C_7^{eff} \times \Lambda/m_b) e^{i\phi_+}$$

Recent discussion in Becirevic *et al.*'12 and JMC and Jäger, to appear

Non-local light-quark contribution

$$a_\mu^{\rm had,\,l-q} = \int d^4x\, e^{-iq\cdot x}\langle K^*|\,T\{j_\mu^{\rm em,\,l-q}(x),\, H_W^{\rm had}(0)\}|B\rangle$$

Probing the hadronic structure of the photon! BFS'01

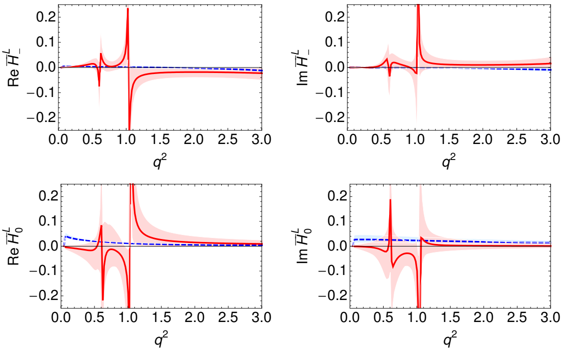
$$a_\mu^{\rm had,\,l-q} \equiv \int d^4x\, e^{-iq\cdot x}\sum_{P,P'}\langle 0|j_\mu^{\rm em,\,l-q}(x)|P'\rangle\langle P'(x)|P(0)\rangle\langle K^* P|\,H_W^{\rm had}(0)|B\rangle$$

- We assume **Vector-Meson Dominance** $P,\ P' \equiv \rho^0,\ \omega,\ \phi$ (Korchin *et al.*'10)
  - $\langle 0|j_\mu^{\rm em,\,l-q}(x)|P'\rangle \equiv f_V$
  - $\langle P'(x)|P(0)\rangle \equiv$(Dressed) V propagator
  - $\langle K^* P|\,H_W^{\rm had}(0)|B\rangle \equiv B \to VK^{*0}$ computed in **QCDf** (Beneke *et al.*'06*)
- We treat these contributions as **uncertainties**

* **QCDf** predictions are consistent within errors with experimental data

$$H^{0,\pm}_{\mathrm{sl},\,L,R} = \frac{\alpha_{\mathrm{em}}\; G_F\; \lambda_t}{2\sqrt{2}} \overbrace{\frac{8\pi\; Q_V\; f_{K^*}\; f_V}{\left(q^2 - m_V^2 + i m_V \Gamma_V\right)}}^{F(q^2)} \left(\frac{m_B}{m_V}\right) H_V^{0,\pm}$$

- $H_V^{0,\pm}$ *CKM* suppressed or hadronic-penguin dominated: $\lambda \sim \mathcal{O}(0.01)$
- However in $\sqrt{q^2} \sim m_V$ and $\Gamma_{\phi,\omega} \sim 1$ MeV$\rightarrow$ $F(q^2)$ is $\sim \mathcal{O}(100)$



SM predictions on the angular coefficients I_i for the μ-mode

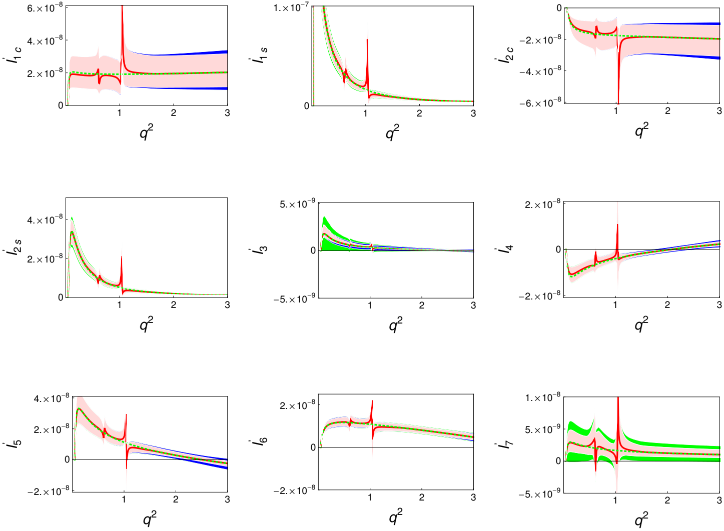


Figure: Red: soft-form factors; blue: factorizable power corrections; green : non-local charm-loop.

A "clean" set of observables

- One can use ratios of $I_i' s$ to reduce theoretical uncertainties
  Kruger et al '05
- The P-basis is composed by the combinations
  Matias et al '12

$$P_1 = \frac{I_3}{2I_{2s}}, \qquad P_2 = \frac{I_6}{8I_{2s}}, \qquad P_3 = -\frac{I_9}{4I_{2s}},$$

$$P_4' = \frac{I_4}{\sqrt{-I_{2s}I_{2c}}}, \qquad P_5' = \frac{I_5}{2\sqrt{-I_{2s}I_{2c}}}, \qquad P_6' = -\frac{I_7}{2\sqrt{-I_{2s}I_{2c}}}.$$

plus

$$\Gamma' = \frac{d\Gamma + d\bar{\Gamma}}{dq^2} = \frac{1}{4}\left((3I_{1c} - I_{2c}) + 2\,(3I_{1s} - I_{2s})\right)$$

$$F_L = \frac{3I_{1c} - I_{2c}}{4\Gamma'},$$

- These observables are independent and the P's are defined such that $\xi_\perp$ and $\xi_\|$ factor out

SM predictions for the P-basis and the μ-mode

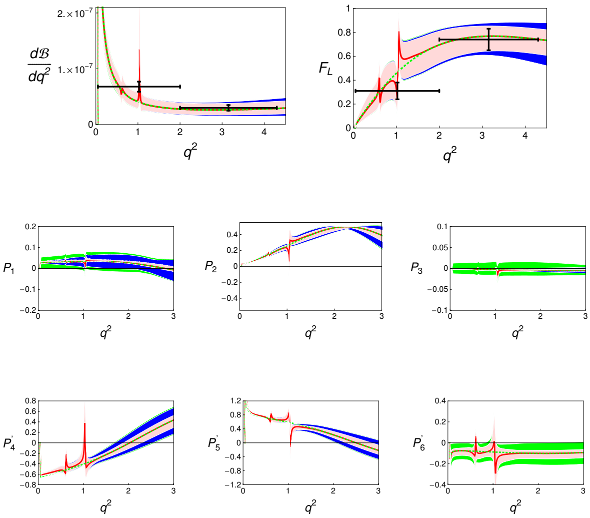
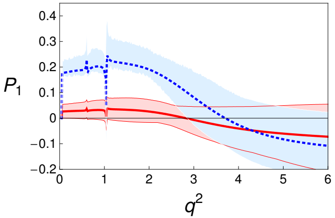
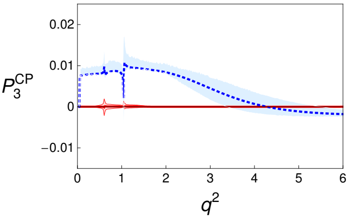


Figure: Red: soft-form factors; blue: factorizable power corrections; green : non-local charm-loop. Data from LHCb-CONF-2012-008

Physics potential on constraining C_7'





- The observables I_3 and I_9 are proportional to

$$I_3 \propto \mathrm{Re}\left(C_7\, C_7'^*\right), \qquad I_9 \propto \mathrm{Im}\left(C_7\, C_7'^*\right),$$

so they vanish unless $C_7' \neq 0$!!

- To study the sensitivity take the "clean" versions P_1 and P_3^{CP} respectively
  - BSM 1: Take $C_7' = 0.1 C_7^{\mathrm{SM}}$ (left panel)
  - BSM 2: Take $C_7' = 0.01 \times i \times C_7^{\mathrm{SM}}$ (right panel)

These observables are very sensitive to BSMs contributions surfacing in C_7' for $q^2 < 3$ GeV^2

Conclusions

- Rare B decays provides and empirical ground to test the SM and to explore NPs effects
- One has access to powerful tools dealing with the different scales in the problem
  - Weak hamiltonians integrate out $\Lambda \sim m_W$ vs.m_b
  - QCD factorization integrate out m_b vs. Λ_{QCD}
- The $B \to K^*(\to K\pi)\ell^+\ell^-$ is phenomenologically very rich
  Total of **22 observables** per leptonic mode
- Sound conclusions can be derived from the phenomenology only when the hadronic uncertainties are carefully tackled
- **Exciting times ahead!**
  LHCb 2011+2012 results on $B_s \to \ell^+\ell^-$ appeared earlier this week!
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