

Diffraction studies with EIC

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Outline

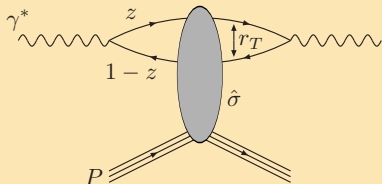
- ▶ Universality of the dipole picture
- ▶ Diffractive structure function
 - Kowalski, T.L. Marquet, Venugopalan 2008
 - ▶ Mapping the A , Q^2 , $x_{\mathbb{P}}$, β –space
- ▶ Coherent and **incoherent** diffraction
 - Exclusive J/Ψ with H. Mäntysaari, 2010 & 2013
 - ▶ t and transverse structure
- ▶ Potential of the EIC

Dipole cross section

DIS at high energy/small x :

dipole cross section.

Given by CGC color field



$$\hat{\sigma}(\mathbf{r}_T) = \int d^2\mathbf{b}_T \frac{1}{N_c} \text{Tr} \left\langle 1 - U^\dagger \left(\mathbf{b}_T + \frac{\mathbf{r}_T}{2} \right) U \left(\mathbf{b}_T - \frac{\mathbf{r}_T}{2} \right) \right\rangle$$

$$U(\mathbf{x}_T) = P \exp \left\{ ig \int dx^- A_{\text{cov}}^+(\mathbf{x}_T, x^-) \right\}$$

Same U — initial condition for Glasma fields in AA collisions:

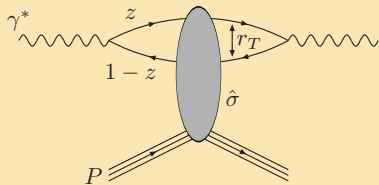
$$A_{(\text{one nucleus})}^i = \frac{i}{g} U(\mathbf{x}_T) \partial_i U^\dagger(\mathbf{x}_T)$$

Universality of the dipole cross section

$$\sigma_{\text{dip}}(X, \mathbf{r}_T) = \int d^2 \mathbf{b}_T \frac{d^2 \sigma_{\text{dip}}}{d^2 \mathbf{b}_T}$$

$$\sigma_{\text{dip}}(X, \mathbf{r}_T, \Delta_T) = \int d^2 \mathbf{b}_T \frac{d^2 \sigma_{\text{dip}}}{d^2 \mathbf{b}_T} e^{i \mathbf{b}_T \cdot \Delta_T}$$

$$\Delta_T^2 = -t$$



From same dipole cross section calculate:

- ▶ Total $\gamma^* p$: $\sigma_{L,T}^{\gamma^* p} = \int d^2 \mathbf{r}_T \int dz \left| \Psi_{L,T}^{\gamma}(Q^2, \mathbf{r}_T, z) \right|^2 \sigma_{\text{dip}}(X, \mathbf{r}_T)$
- ▶ Total diff.: $\frac{\sigma_{L,T}^D}{dt} = \frac{1}{16\pi} \int d^2 \mathbf{r}_T \int dz \left| \Psi_{L,T}^{\gamma}(Q^2, \mathbf{r}_T, z) \right|^2 \sigma_{\text{dip}}^2(X, \mathbf{r}_T, \Delta_T)$
- ▶ X-cl. diff.: $\frac{\sigma_{L,T}^D}{dt} = \frac{1}{16\pi} \left| \int d^2 \mathbf{r}_T \int dz \left(\Psi^{\gamma} \Psi^{*V} \right)_{L,T} \sigma_{\text{dip}}(X, \mathbf{r}_T, \Delta_T) \right|^2$

Use:

- ▶ S-matrix real
- ▶ optical theorem

- ▶ $\Psi^{\gamma}(Q^2, \mathbf{r}_T, z) \sim K_{0,1}(\sqrt{z(1-z)}Q|\mathbf{r}_T|)$
 \implies momentum scale $Q^2 \sim 1/r_T^2$
- ▶ Diffractive: $t \sim$ FT of \mathbf{b}_T .

IPsat model: protons

Starting point GBW: [Golec-Biernat, Wusthoff 1998](#) $\sigma_{\text{dip}} = \sigma_0 \left(1 - e^{-\frac{\mathbf{r}_T^2}{4Q_s(x)^2}} \right)$

1 possible improvement: [Kowalski & Teaney 2003](#) **IPsat model**

- ▶ DGLAP evolution: improves description large Q^2 .
- ▶ Consistent \mathbf{b}_T dependence for all observables

$$\sigma_{\text{dip}}(x, \mathbf{r}_T) = 2 \int d^2\mathbf{b}_T \left(1 - \exp \left\{ -\frac{\pi^2}{2N_c} \alpha_s(\mu^2) x g(x, \mu^2) T_p(\mathbf{b}_T) \mathbf{r}_T^2 \right\} \right),$$

- ▶ $\mu^2 = \frac{C}{\mathbf{r}_T^2} + \mu_0^2$, Gaussian $T_p(\mathbf{b}_T)$
- ▶ $xg(x, \mu^2)$ evolved with DGLAP, initial condition $\sim x^{\sim 0}$
 - ▶ Consistent with what we expect from small x evolution?
 - ▶ Restricts growth of \mathbf{b}_T -profile at large \mathbf{r}_T : feature or bug?

IPsat: nuclei

Straightforward generalization to nuclei:

$$\frac{d\sigma_{\text{dip}}^A}{d^2\mathbf{b}_T} = 2 \left(1 - \exp \left\{ -\frac{\pi^2}{2N_c} \alpha_s(\mu^2) x g(x, \mu^2) \sum_{i=1}^A T_p(\mathbf{b}_T - \mathbf{b}_{Ti}) r_{Ti}^2 \right\} \right),$$

- ▶ \mathbf{b}_{Ti} : nucleon positions, from Woods-Saxon, average $\langle \cdot \rangle_N$
- ▶ Proton radius $\sim 0.6\text{fm}$, much less than the nucleon-nucleon distance. \implies **lumpy** nucleus \implies visible in **incoherent** diffraction

“Glauber” limit $A \rightarrow \infty$, $R_p \ll R_A$

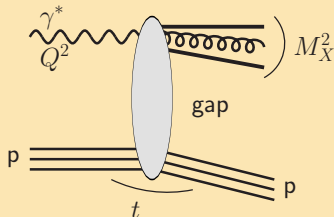
$$\left\langle \frac{d\sigma_{\text{dip}}^A}{d^2\mathbf{b}_T} \right\rangle_N \approx_{A \rightarrow \infty} 2 \left[1 - e^{-\frac{AT_A(\mathbf{b}_T)}{2} \sigma_{\text{dip}}^p} \right]$$

$$\left(\int d^2\mathbf{b}_T T_A(\mathbf{b}_T) \right) = 1 \implies T_A(\mathbf{0}_T) \sim A^{-2/3}$$

Diffractive structure function

$$F_2^{D(3)}(x_{\mathbb{P}}, \beta, Q^2) = \int dt F_2^{D(4)}(x_{\mathbb{P}}, \beta, Q^2, t)$$

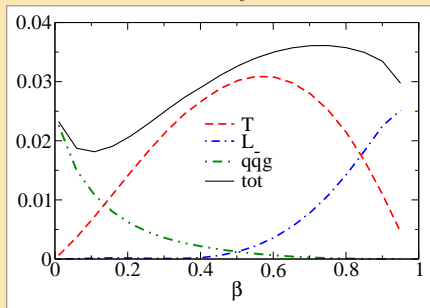
$$x = x_{\mathbb{P}}\beta \quad \beta = \frac{Q^2}{Q^2 + M_X^2}$$



Gap $\sim \ln \frac{1}{x_{\mathbb{P}}}$ rapidity of $X \sim \ln \frac{1}{\beta}$

Essential regimes:

- ▶ Small $\beta \ll 1$: dominated by higher Fock ($q\bar{q}g$ etc.)
- ▶ Medium $\beta \sim 0.5$: dominated by transverse $q\bar{q}$
- ▶ Large $\beta \rightarrow 1$: longitudinal $q\bar{q}$.



Proton, $Q^2 = 5\text{GeV}^2$, $x_{\mathbb{P}} = 10^{-3}$
(IPsat)

Computing F_2^D

The structure function is

$$x_{\mathbb{P}} F_2^D = \underbrace{x_{\mathbb{P}} F_{T,q\bar{q}}^D}_{\alpha_s^0} + \underbrace{x_{\mathbb{P}} F_{L,q\bar{q}}^D}_{\alpha_s} + \underbrace{x_{\mathbb{P}} F_{T,q\bar{q}g}^D}_{\alpha_s^2} + \text{higher Fock states}$$

$$x_{\mathbb{P}} F_{T,q\bar{q}}^D = \frac{N_c Q^4}{16\pi^3 \beta} \sum_f e_f^2 \int_{z_0}^{1/2} dz z (1-z) \left[\varepsilon^2 (z^2 + (1-z)^2) \Phi_1 + m_f^2 \Phi_0 \right]$$

$$x_{\mathbb{P}} F_{L,q\bar{q}}^D = \frac{N_c Q^6}{4\pi^3 \beta} \sum_f e_f^2 \int_{z_0}^{1/2} dz z^3 (1-z)^3 \Phi_0$$

$$\Phi_n = \int d^2 \mathbf{b}_T \left[\int_0^\infty dr r K_n(\varepsilon r) J_n(kr) \frac{d\sigma_{\text{dip}}}{d^2 \mathbf{b}_T}(\mathbf{b}_T, \mathbf{r}_T, x_{\mathbb{P}}) \right]^2,$$

- ▶ $\int_{-\infty}^0 dt \implies \int d^2 \mathbf{b}_T$: easy to evaluate
- ▶ Differential in t : tougher numerically

Interpolate between large Q^2 , finite β (“GBW”) and $\beta \rightarrow 0$ (“MS”):

$$x_{\mathbb{P}} F_2^D(x_{\mathbb{P}}, \beta, Q^2) = \frac{x_{\mathbb{P}} F_{T,q\bar{q}g}^D(\text{GBW})(x_{\mathbb{P}}, \beta, Q^2) \times x_{\mathbb{P}} F_{T,q\bar{q}g}^D(\text{MS})(x_{\mathbb{P}}, Q^2)}{x_{\mathbb{P}} F_{T,q\bar{q}g}^D(\text{GBW})(x_{\mathbb{P}}, \beta = 0, Q^2)}.$$

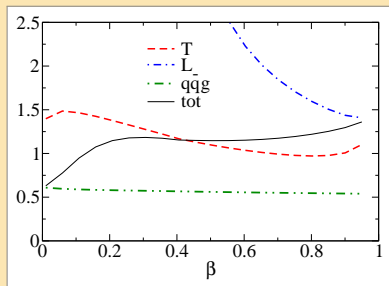
$$x_{\mathbb{P}} F_{T,q\bar{q}g}^D(\text{GBW}) = \frac{\alpha_s \beta}{8\pi^4} \sum_f e_f^2 \int d^2 \mathbf{b}_T \int_{\beta}^1 dz \left[\left(1 - \frac{\beta}{z}\right)^2 + \left(\frac{\beta}{z}\right)^2 \right] \int_0^{Q^2} dk^2 k^4 \ln \frac{Q^2}{k^2} \\ \times \left[\int_0^{\infty} dr r K_2(\sqrt{z}kr) J_2(\sqrt{1-z}kr) \frac{d\tilde{\sigma}_{\text{dip}}}{d^2 \mathbf{b}_T}(\mathbf{b}_T, \mathbf{r}_T, x_{\mathbb{P}}) \right]^2.$$

$$x_{\mathbb{P}} F_{T,q\bar{q}g}^D(\text{MS}) = \frac{C_F \alpha_s Q^2}{4\pi^4 \alpha_{\text{em}}} \int d^2 \mathbf{b}_T d^2 \mathbf{r}_T d^2 \mathbf{r}_{T'} \int_0^1 dz \left| \Psi_T^{\gamma^*}(r, Q, z) \right|^2 \frac{\mathbf{r}_T^2}{(\mathbf{r}_{T'})^2 (\mathbf{r}_T - \mathbf{r}_{T'})^2} \\ \left[\mathcal{N}(\mathbf{b}_T, \mathbf{r}_{T'}, x_{\mathbb{P}}) + \mathcal{N}(\mathbf{r}_T - \mathbf{r}_{T'}) - \mathcal{N}(\mathbf{r}_T) - \mathcal{N}(\mathbf{r}_{T'}) \mathcal{N}(\mathbf{r}_T - \mathbf{r}_{T'}) \right]^2$$

Nuclear effects: β -dependence

Essential regimes:

- ▶ Small $\beta \ll 1$: $q\bar{q}g$ strongly suppressed (black disk limit)
- ▶ Medium $\beta \sim 0.5$: transverse $q\bar{q}$ enhanced.
- ▶ Large $\beta \rightarrow 1$: longitudinal $q\bar{q}$ very much enhanced.



Au at $Q^2 = 5 \text{ GeV}^2$ and $x_P = 10^{-3}$

Note: plotting $\frac{F_{2A,(x)}^D}{AF_{2,(x)}^D}$ with $x = L, T, q\bar{q}g, \text{tot}$.

Nuclear effects: Q^2 -dependence

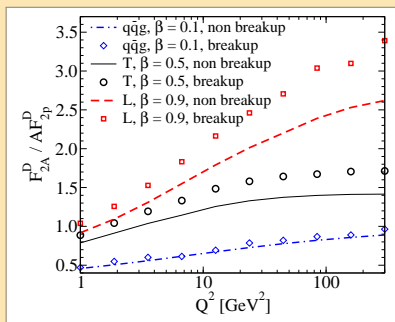
Nuclei have smaller Q^2/Q_s^2 at same $x_{\mathbb{P}}$, Q^2



Nuclear enhancement of F_2^D 's grows with Q^2

Non breakup = coherent

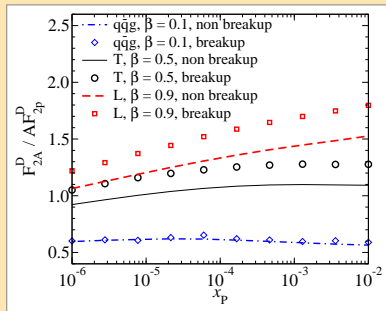
Breakup = coherent + incoherent



Au at $x_{\mathbb{P}} = 10^{-3}$

Nuclear effects: $x_{\mathbb{P}}$ -dependence

At smaller $x_{\mathbb{P}}$ also proton closer to black disk limit \Rightarrow smaller enhancement in $q\bar{q}$ -components.

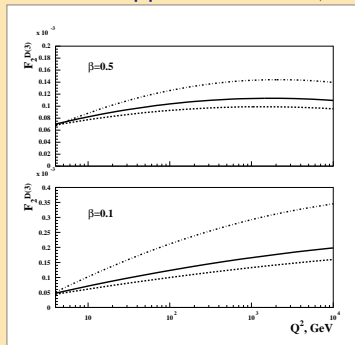


Au at $Q^2 = 5\text{GeV}^2$

Nuclear effects: $x_{\mathbb{P}}$ -dependence

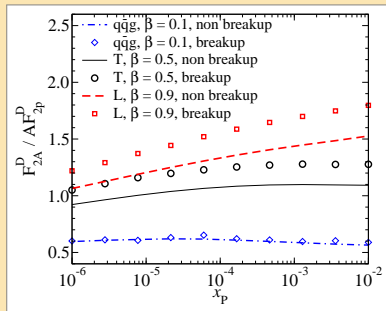
At smaller $x_{\mathbb{P}}$ also proton closer to black disk limit \Rightarrow smaller enhancement in $q\bar{q}$ -components.

Contrast with leading twist:
Nuclear suppression at all β



Frankfurt, Guzey, Strikman, 2003,

Dot-dash is proton

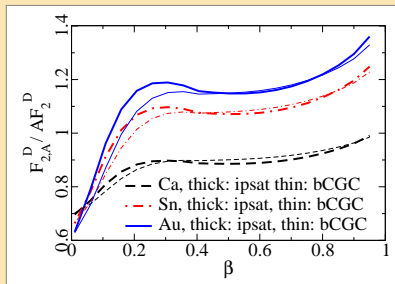
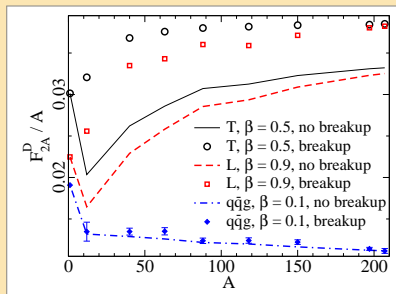


Au at $Q^2 = 5 \text{ GeV}^2$

Results: A-dependence

Small nuclei are more dilute than proton: coherent diffraction suppressed

Different components at $x_P = 10^{-3}$ and $Q^2 = 5\text{GeV}^2$ \Rightarrow



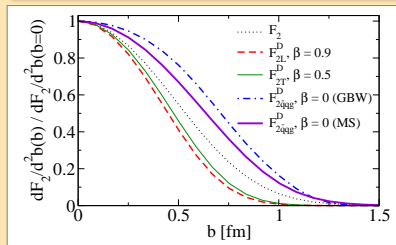
\Leftarrow Different nuclei vs β

Note: b_T -dependence of $q\bar{q}$, inclusive, $q\bar{q}g$

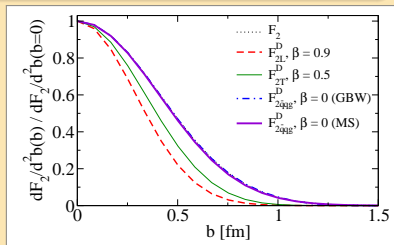
Plots here for proton, same would be for coherent diffraction

Dominant impact parameters different:

$$b^{\text{diff}}(q\bar{q}) < b^{\text{incl}} < b^{\text{diff}}(q\bar{q}g)$$



Different components at
 $x_P = 10^{-3}$ and $x_P = 10^{-3}$
 $Q^2 = 1 \text{ GeV}^2$



Different components at
 $x_P = 10^{-3}$ and $x_P = 10^{-3}$
 $Q^2 = 100 \text{ GeV}^2$

b -dependence beyond Glauber

Denote $\frac{d\sigma_{\text{dip}}}{d^2\mathbf{b}_T}(\mathbf{r}_T, \mathbf{b}_T) = 2(1 - S(\mathbf{r}_T, \mathbf{b}_T))$.

- ▶ Glauber independent scatterings:

$S_A(\mathbf{r}_T, \mathbf{b}_T) = \prod_{i=1}^A S_\rho(\mathbf{r}_T, \mathbf{b}_T - \mathbf{b}_{Ti})$, \implies for IPsat equivalent to

$Q_s^2(\mathbf{b}_T) \sim T_\rho(\mathbf{b}_T) \rightarrow \sum_{i=1}^A T_\rho(\mathbf{b}_T - \mathbf{b}_{Ti})$,

because $S_A(\mathbf{r}_T, \mathbf{b}_T) \sim e^{-r_T^2 Q_s^2/4}$

$$\sigma_{\gamma^*A} \sim A^{2/3} \left(1 - e^{-r_T^2 Q_s^2 A^{1/3}/4} \right) \sim A \text{ for large } Q^2$$

- ▶ Not so with BK etc. evolution, when $S_A(\mathbf{r}_T, \mathbf{b}_T) \sim e^{-(r_T^2 Q_s^2)^\gamma/4}$

$$\sigma_{\gamma^*A} \sim A^{2/3} \left(1 - e^{-(r_T^2 Q_s^2 A^{1/3})^\gamma/4} \right) \sim A^{2/3+\gamma/3}$$

To combine BK with realistic nuclear geometry need b -dependent BK evolution \implies not (really) doable ...

What would be the best parametrization?

More on t -dependence

Quasielastic = coherent + incoherent

$$\frac{d\sigma^{\gamma^* A \rightarrow VA^*/A}}{dt} = \frac{1}{16\pi} \langle |\mathcal{A}(x_{\mathbb{P}}, Q^2, \mathbf{\Delta}_T)|^2 \rangle_N$$

Coherent

$$\frac{d\sigma^{\gamma^* A \rightarrow VA}}{dt} = \frac{1}{16\pi} |\langle \mathcal{A}(x_{\mathbb{P}}, Q^2, \mathbf{\Delta}_T) \rangle_N|^2$$

Incoherent is fluctuations:

$$\frac{d\sigma^{\gamma^* A \rightarrow VA^*}}{dt} \sim \langle |\mathcal{A}(x_{\mathbb{P}}, Q^2, \mathbf{\Delta}_T)|^2 \rangle_N - |\langle \mathcal{A}(x_{\mathbb{P}}, Q^2, \mathbf{\Delta}_T) \rangle_N|^2$$

Here average over nucleon positions denoted as

$$\langle \mathcal{O}(\{\mathbf{b}_{Ti}\}) \rangle_N \equiv \int \prod_{i=1}^A [d^2\mathbf{b}_{Ti} T_A(\mathbf{b}_{Ti})] \mathcal{O}(\{\mathbf{b}_{Ti}\}).$$

(For interpretation coherent=mean, incoherent=fluctuations see T. Toll's talk)

Case study: incoherent J/ψ

Approximations used in T.L. & Mäntysaari, 2010, 2013

- ▶ Factorized b -dependence: **Note** $\sigma_0 = 4\pi B_p$ for gaussian $T_p(\mathbf{b}_T)$.

$$\frac{d\sigma_{\text{dip}}^p}{d^2\mathbf{b}_T}(\mathbf{r}_T, \mathbf{b}_T, x) = 2T_p(\mathbf{b}_T)\mathcal{N}(\mathbf{r}_T, x)$$

- ▶ “fIPsat” $\mathcal{N}(\mathbf{r}_T, x) = 1 - \exp\left(-r^2 \frac{1}{2\pi B_p} \frac{\pi^2}{2N_c} \alpha_s(\mu(r^2)) xg(x, \mu(r^2))\right)$
Really Bartels, Golec-Biernat, Kowalski 2002
- ▶ Also use IIM Iancu, Itakura, Munier for proton, generalize to nuclei with:

$$S_A(\mathbf{r}_T, \mathbf{b}_T, x) = \prod_{i=1}^A S_p(\mathbf{r}_T, \mathbf{b}_T - \mathbf{b}_{Ti}, x).$$

- ▶ Real part $1 + \beta^2$ and skewness corrections R_g as in Kowalski, Motyka, Watt 2006, depend on $\lambda \equiv \frac{dA}{d \ln 1/x}$

Notation:

$$\frac{d\sigma^{\gamma^* A \rightarrow V A^*}}{dt} = \frac{R_g^2(1 + \beta^2)}{16\pi} \int \frac{dz}{4\pi} \frac{dz'}{4\pi} d^2\mathbf{r}_T d^2\mathbf{r}'_T$$

$$\times [\Psi_V^* \Psi](r, z, Q) [\Psi_V^* \Psi](r', z', Q) \left\langle |A_{q\bar{q}}|^2(x_{\mathbb{P}}, r, r', \mathbf{\Delta}_T) \right\rangle_N.$$

Equation for incoherent

Incoherent at large enough t :

$$\begin{aligned} \langle |A_{q\bar{q}}|^2(x_{\mathbb{P}}, r, r', \Delta_T) \rangle_N &= A(4\pi B_p)^2 e^{-B_p \Delta_T^2} \mathcal{N}(r) \mathcal{N}(r') \int d^2 \mathbf{b}_T T_A(b) \\ &\times \exp \{ -2\pi B_p (A - 1) T_A(b) [\mathcal{N}(r) + \mathcal{N}(r')] \} \end{aligned}$$

Interpretation:

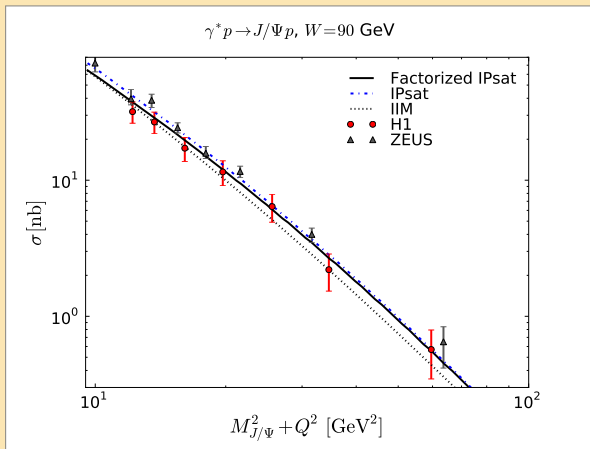
- ▶ $\sim A \times$ squared amplitude for proton
- ▶ nuclear attenuation factor: must *not* scatter inelastically off the other $A - 1$ nucleons (otherwise the interaction would not be diffractive) .

See e.g. Kopeliovich, Nemchik, Schaefer, Tarasov 2001

For comparison: coherent in similar approximation

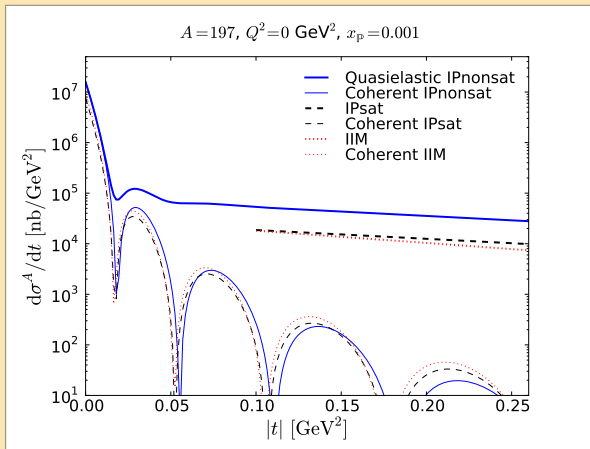
$$\begin{aligned} \langle A(x_{\mathbb{P}}, Q^2, \Delta_T) \rangle_N &= \int \frac{dz}{4\pi} d^2 \mathbf{r}_T d^2 \mathbf{b}_T e^{-i\mathbf{b}_T \cdot \Delta_T} [\psi_V^* \psi](r, Q^2) \\ &\times 2 [1 - \exp \{ -2\pi B_p A T_A(b) \mathcal{N}(r, x_{\mathbb{P}}) \}]. \end{aligned}$$

Check HERA



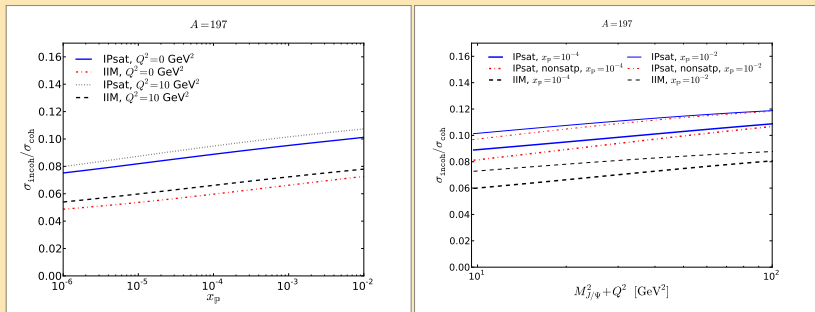
Works, skewness and real part correction $R_g^2(1 + \beta^2)$ essential.

Coherent and incoherent cross sections



IPnonsat: linearized in r , [Caldwell, Kowalski 2009], explicitly $A \times \sigma_p$.
Factor of 3 suppression from $A\sigma_p$ in incoherent.

Nuclear suppression, dependence on x and Q^2



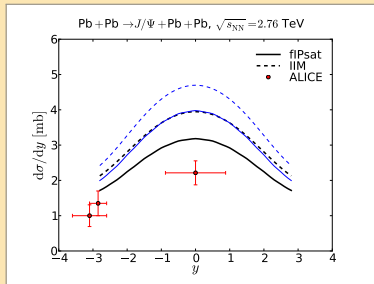
- ▶ Not very striking differences in shape of $x_{\mathbb{P}}$, Q^2 -dependence.
- ▶ Constraint comes from different σ_0 , related to B_p , i.e:
 1. Different fits to inclusive data require different normalizations σ_0/B_p
 2. Influences incoherent and coherent diffraction in different ways
 3. Inclusive, coherent and incoherent constrain parametrizations differently.

UPC, comparison to LHC

T.L., Mäntysaari arXiv:1301.4095

Coherent:

Different J/ψ wavefunctions, σ_{dip} ,



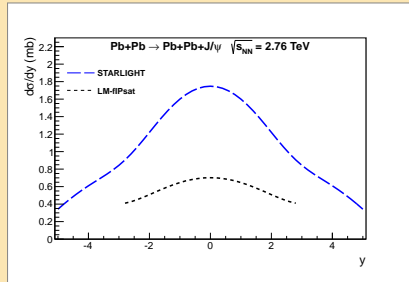
- ▶ R_g^2 is large; $1 + \beta^2$ also.
 - ▶ ep data requires (?) these
 - ▶ γA (UPC) better without
- ▶ Wavefunction differences significant for $Q^2 = 0$

(PHENIX $y = 0$ UPC J/ψ : measurement $76 \pm 35 \mu\text{b}$,

Calculation $108 \mu\text{b}$. Compare $119 \mu\text{b}$ Toll, Ullrich with 116% correction $R_g^2(1 + \beta^2)$)

Incoherent

Dipole model vs. STARLIGHT



Plot Nystrand

- ▶ Big difference vs. STARLIGHT
- ▶ Suppression vs $A \times \sigma_{\gamma p}$ clear prediction of dipole picture.

Physics at EIC $F_2^{D(3)}$

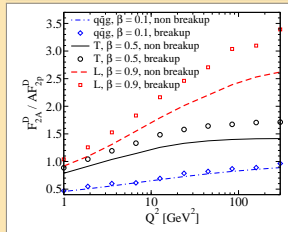
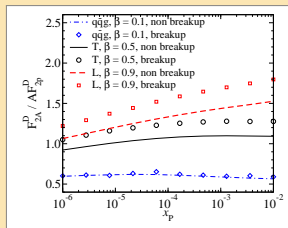
Assume 130 GeV on 30 GeV:

$x \gtrsim 7 \times 10^{-5}$ for $Q^2 = 1$ or

$Q^2 \gtrsim 140 \text{ GeV}^2$ for $x = 0.01$.

t -integrated:

- ▶ Basic features of nuclear enhancement/suppression vs β should be seen \implies validation of dipole picture
- ▶ Some structure in Q^2
- ▶ Energy dependence not very conclusive

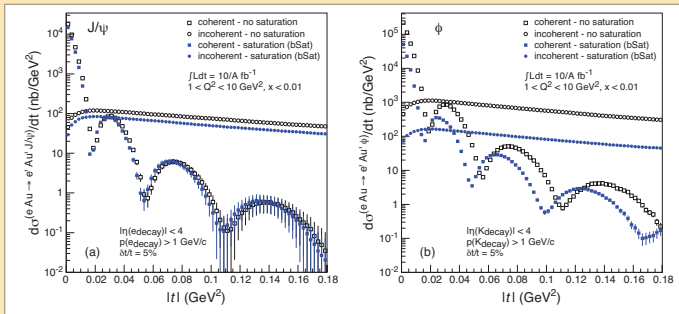


Kowalski, T.L., Marquet, Venugopalan

Physics at EIC: exclusive vector mesons

Transverse structure

- ▶ What is the resolution in t ?
- ▶ Also information in the incoherent/diffractive ep t range
- ▶ Lighter mesons (larger r) more sensitive to saturation, but also bigger ambiguities in LC-wavefunction.

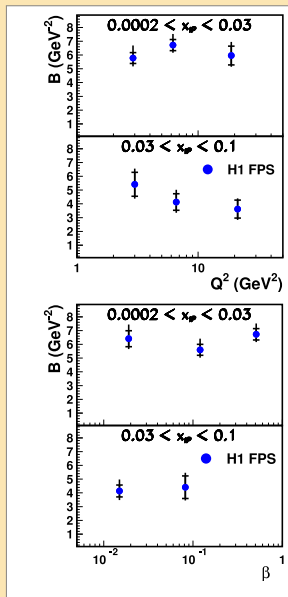


Toll, Ullrich

Physics at EIC: $F_2^{D(4)}$

No calculations to present here ...

- ▶ Measure t -slope B in
 - ▶ The proton/incoherent region
– $t \sim 0.5\text{GeV}^2$: easy?
 - ▶ The coherent region
– $t \sim 0.02\text{GeV}^2$: too hard?
- ▶ Look at M_X and Q^2 dependence in B .
⇒ Beyond factorized \mathbf{b}_T -profiles
- ▶ Can one improve on HERA? ⇒
- ▶ Same for nuclei (incoherent)



Conclusion

- ▶ Universality of dipole picture: lot of predictive power, in principle.
- ▶ Impact parameter dependence
 - ▶ IPsat nice: fit to $e p \implies$ unambiguous prediction for $e A$
 - ▶ \nexists real b -dependent BK/JIMWLK calculation!
- ▶ Normalization: should avoid proliferation of K factors (cf. hadron-hadron phenomenology)
- ▶ EIC:
 - ▶ Limited sensitivity to intricacies of evolution in x
 - ▶ Good resolution in t for nuclei and protons would be very valuable
 - ▶ **Measure also incoherent diffraction**
 - t similar to $e p$, and nontrivial nuclear effects!
 - Probe fluctuations in the gluon density

POETIC IV in Jyväskylä

Stellenbosch — Bloomington — Valparaíso —

University of Jyväskylä Department of Physics

~ **August 26 – 30, 2013**



Organizing committee:

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