The Nucleon Spin Sum

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March 5, 2013

- Generalized parton distributions (GPDs)

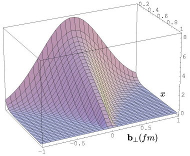
$\hookrightarrow$ 'transverse imaging'

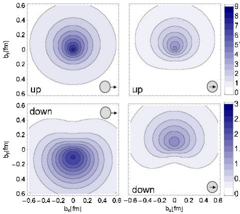
- Chromodynamic lensing and $\perp$ single-spin asymmetries (SSA)

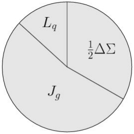
$$\left.\begin{array}{c}\text{transverse distortion of PDFs}\\ +\text{ final state interactions}\end{array}\right\} \Rightarrow$$

$\hookrightarrow$ SSA in $\gamma N \longrightarrow \pi + X$

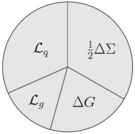
- quark-gluon correlations $\rightarrow \perp$ force on q in DIS
- $\mathcal{L}^q_{JM} - L^q_{Ji} \leftrightarrow$ torque due to FSI in DIS
- Summary



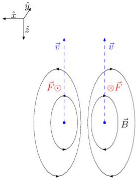




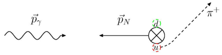
'pizza tre stagioni'

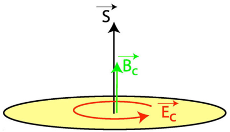


'pizza quattro stagioni'



a.)



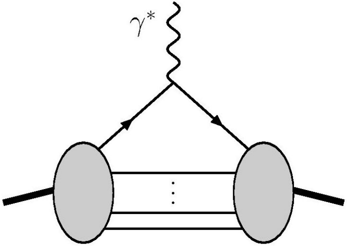


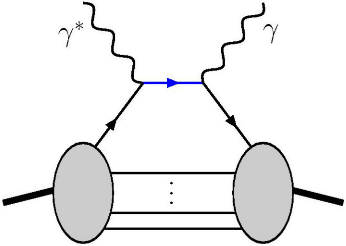
- virtual Compton scattering: $\gamma^* p \longrightarrow \gamma p$ (actually: $e^- p \longrightarrow e^- \gamma p$)
- 'deeply': $-q_\gamma^2 \gg M_p^2, |t| \longrightarrow$ Compton amplitude dominated by (coherent superposition of) Compton scattering off single quarks

$\hookrightarrow$ only difference between form factor (a) and DVCS amplitude (b) is replacement of photon vertex by two photon vertices connected by quark (energy denominator depends on quark momentum fraction x)

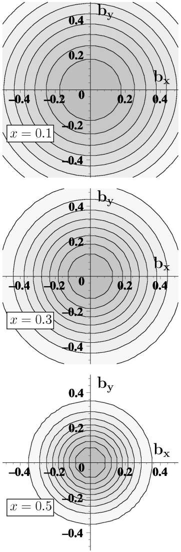
$\hookrightarrow$ DVCS amplitude provides access to momentum-decomposition of form factor = Generalized Parton Distribution (GPDs).

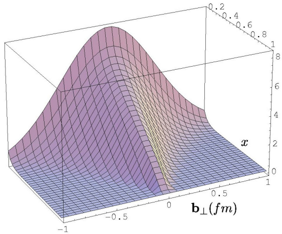
$$\int dx H_q(x,\xi,t) = F_1^q(t) \qquad \int dx E_q(x,\xi,t) = F_2^q(t)$$

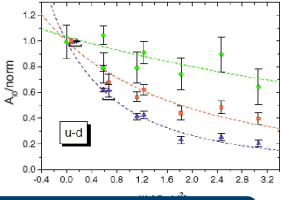




$q(x, \mathbf{b}_\perp)$ for unpol. p



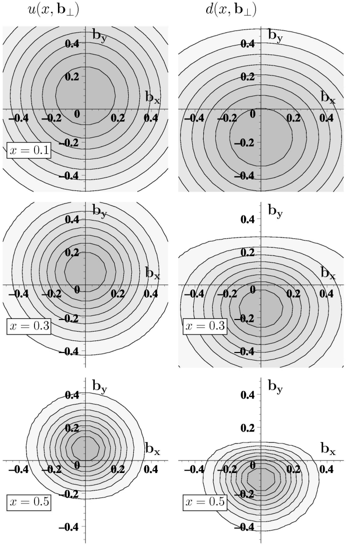


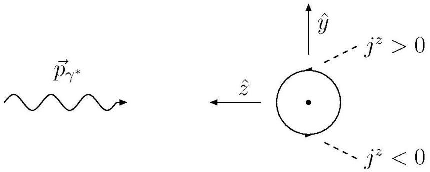


unpolarized proton

- $q(x, \mathbf{b}_\perp) = \int \frac{d^2\Delta_\perp}{(2\pi)^2} H(x, 0, -\boldsymbol{\Delta}_\perp^2) e^{-i\mathbf{b}_\perp \cdot \boldsymbol{\Delta}_\perp}$
- x = momentum fraction of the quark
- $\vec{b} = \perp$ distance of quark from $\perp$ center of momentum
- small x: large 'meson cloud'
- larger x: compact 'valence core'
- $x \to 1$: active quark becomes center of momentum

$\hookrightarrow$ $\vec{b}_\perp \to 0$ (narrow distribution) for $x \to 1$





proton polarized in $+\hat{x}$ direction

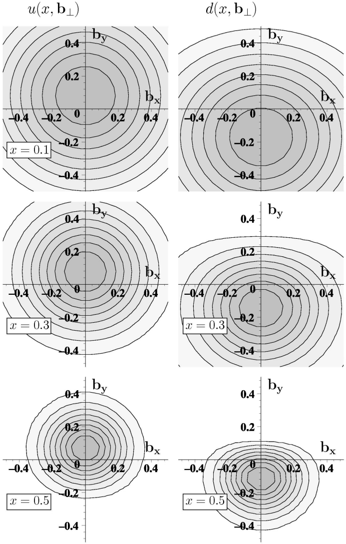
no axial symmetry!

$$q(x,\mathbf{b}_\perp) = \int \frac{d^2\Delta_\perp}{(2\pi)^2} H_q(x,0,-\Delta_\perp^2) e^{-i\mathbf{b}_\perp\cdot\Delta_\perp}$$

$$-\frac{1}{2M}\frac{\partial}{\partial b_y}\int \frac{d^2\Delta_\perp}{(2\pi)^2} E_q(x,0,-\Delta_\perp^2) e^{-i\mathbf{b}_\perp\cdot\Delta_\perp}$$

Physics: relevant density in leading twist DIS is $j^+ \equiv j^0 + j^3$ and left-right asymmetry from j^3

MB, Int. J. Mod. Phys. **A18**, 173 (2002)



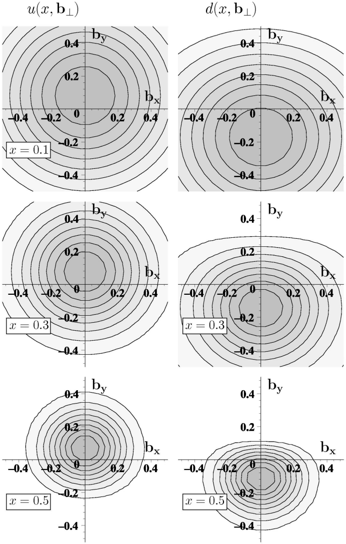
proton polarized in $+\hat{x}$ direction

$$q(x,\mathbf{b}_\perp) = \int \frac{d^2\Delta_\perp}{(2\pi)^2} H_q(x,0,-\Delta_\perp^2) e^{-i\mathbf{b}_\perp\cdot\Delta_\perp} - \frac{1}{2M}\frac{\partial}{\partial b_y}\int \frac{d^2\Delta_\perp}{(2\pi)^2} E_q(x,0,-\Delta_\perp^2) e^{-i\mathbf{b}_\perp\cdot\Delta_\perp}$$

sign & magnitude of the average shift

model-independently related to p/n anomalous magnetic moments:

$$\langle b_y^q \rangle \equiv \int dx \int d^2 b_\perp q(x,\mathbf{b}_\perp) b_y = \frac{1}{2M}\int dx E_q(x,0,0) = \frac{\kappa_q}{2M}$$



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$$\kappa^p = 1.913 = \frac{2}{3}\kappa_u^p - \frac{1}{3}\kappa_d^p + \ldots$$

- u-quarks: $\kappa_u^p = 2\kappa_p + \kappa_n = 1.673$

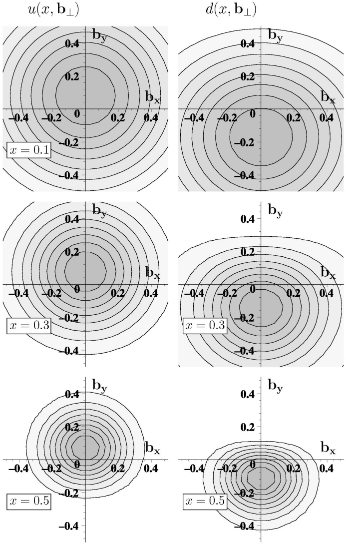
$\hookrightarrow$ shift in $+\hat{y}$ direction

- d-quarks: $\kappa_d^p = 2\kappa_n + \kappa_p = -2.033$

$\hookrightarrow$ shift in $-\hat{y}$ direction

- $\langle b_y^q \rangle = \mathcal{O}(\pm 0.2 fm)$!!!!

Impact parameter dependent quark distributions



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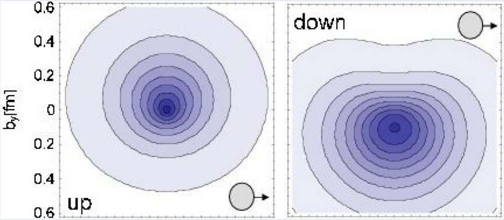
lattice QCD (QCDSF): lowest moment



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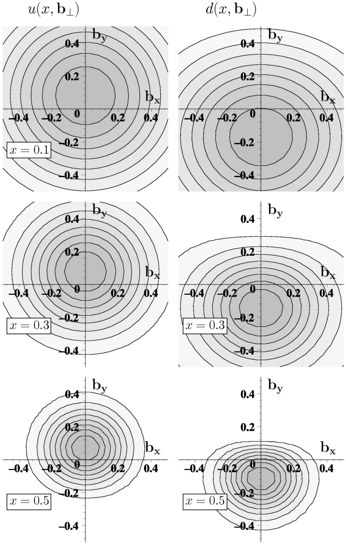


anomalous gravito-magnetic moment = 0

- $\sum_{i\in q,g}\int dx\, xE_i(x,0,0) = 0$ ($\rightarrow$ Brodsky et el.)
- physics: $\perp$ CoM of $\perp$ pol. nucleon does not shift
- appears to be almost saturated by u & d only!

$\hookrightarrow$ $\int dx\, xE_g(x,0,0) \stackrel{?}{\approx} 0$ (Brodsky & Gardner)

- DGLAP: CoM conserved

$\hookrightarrow$ expect $\int dx\, xE(x,0,0) \approx 0$ for sea quarks produced in evolution

transverse images ↔ Ji relation for quark angular momentum:

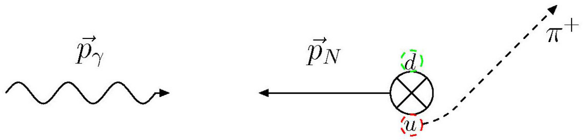
- $J_q^x = m_N \int dx\, x r^y q(x, \mathbf{r}_\perp)$ with $b^y = r^y - \frac{1}{2m_N}$, where $q(x, \mathbf{r}_\perp)$ is distribution relative to CoM of whole nucleon
- recall: $q(x, \mathbf{b}_\perp)$ for nucleon polarized in $+\hat{x}$ direction

$$\begin{aligned} q(x, \mathbf{b}_\perp) &= \int \frac{d^2\Delta_\perp}{(2\pi)^2} H_q(x, 0, -\Delta_\perp^2) e^{-i\mathbf{b}_\perp \cdot \Delta_\perp} \\ &\quad - \frac{1}{2M_N} \frac{\partial}{\partial b_y} \int \frac{d^2\Delta_\perp}{(2\pi)^2} E_q(x, 0, -\Delta_\perp^2) e^{-i\mathbf{b}_\perp \cdot \Delta_\perp} \end{aligned}$$

$$\begin{aligned} \Rightarrow J_q^x &= M_N \int dx\, x r^y q(x, \mathbf{r}_\perp) = \int dx\, x \left(m_N b^y + \frac{1}{2} \right) q(x, \mathbf{r}_\perp) \\ &= \frac{1}{2} \int dx\, x \left[H(x, 0, 0) + E(x, 0, 0) \right] \end{aligned}$$

- X.Ji(1996): rotational invariance ⇒ apply to all components of $\vec{J}_q$
- partonic interpretation exists only for ⊥ components (MB 2005)!

Sivers $f_{1T}^{\perp}$ in semi-inclusive deep-inelastic scattering (SIDIS) $\gamma p \to \pi X$



- u, d distributions in $\perp$ polarized proton have left-right asymmetry in $\perp$ position space (T-even!); sign 'determined' by κ_u & κ_d (M.B.; I.Schmidt; A.Metz; L.Gamberg;...)
- attractive FSI deflects active quark towards the CoM

$\hookrightarrow$ FSI translates position space distortion (before the quark is knocked out) in $+\hat{y}$-direction into momentum asymmetry that favors $-\hat{y}$ direction $\to$ 'chromodynamic lensing'

$\Rightarrow$ $\kappa_p\,,\kappa_n \quad \longleftrightarrow \quad$ sign of SSA!!!!!!!! MB, PRD 69, 074032 (2004)

- confirmed by HERMES (and recent COMPASS) p data; consistent with vanishing isoscalar Sivers (COMPASS)

higher twist in polarized DIS

- $\sigma_{LL} \propto g_1 - \frac{2Mx}{\nu} g_2$
- $g_1 = \frac{1}{2}\sum_q e_q^2 g_1^q$ with $g_1^q = q^\uparrow(x) + \bar{q}^\uparrow(x) - q^\downarrow(x) - \bar{q}^\downarrow(x)$
- g_2 involves quark-gluon correlations

$\hookrightarrow$ no parton interpret. as difference between number densities for g_2

- for $\perp$ pol. target, g_1 & g_2 contribute equally

$$\sigma_{LT} \propto g_T \equiv g_1 + g_2$$

$\hookrightarrow$ 'clean' separation between g_2 and $\frac{1}{Q^2}$ corrections to g_1

What can we learn from g_2?

- $g_2 = g_2^{WW} + \bar{g}_2$ with $g_2^{WW}(x) \equiv -g_1(x) + \int_x^1 \frac{dy}{y} g_1(y)$

$$d_2 \equiv 3\int dx\, x^2 \bar{g}_2(x) = \frac{1}{2MP^{+2}S^x}\left\langle P,S\left|\bar{q}(0)gG^{+y}(0)\gamma^+q(0)\right|P,S\right\rangle$$

$$d_2 \equiv 3\int dx\, x^2 \bar{g}_2(x) = \frac{1}{2M{P^+}^2 S^x}\left\langle P,S\left|\bar{q}(0)gG^{+y}(0)\gamma^+ q(0)\right|P,S\right\rangle$$

$$\sqrt{2}G^{+y} = G^{0y} + G^{zy} = -E^y + B^x$$

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color Lorentz force MB 2008

$$\sqrt{2}G^{+y} = G^{0y} + G^{zy} = -E^y + B^x = -\left(\vec{E} + \vec{v}\times\vec{B}\right)^y \text{ for } \vec{v} = (0,0,-1)$$

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$\hookrightarrow$ $d_2 \leftrightarrow$ average **color Lorentz force** acting on quark moving with $v = c$ in $-\hat{z}$ direction in the instant after being struck by γ^*

$$\langle F^y\rangle = -2M^2 d_2 = -\frac{M}{P^{+2}S^x}\left\langle P,S\left|\bar{q}(0)gG^{+y}(0)\gamma^+q(0)\right|P,S\right\rangle$$

cf. Qiu-Sterman matrix element $\langle k_\perp^y\rangle \equiv \int_0^1 dx \int d^2k_\perp\, k_\perp^2 f_{1T}^\perp(x,k_\perp^2)$

$$\langle k_\perp^y\rangle = -\frac{1}{2p^+}\left\langle P,S\left|\bar{q}(0)\int_0^\infty dx^- gG^{+y}(x^-)\gamma^+q(0)\right|P,S\right\rangle$$

semi-classical interpretation: average $k_\perp$ in SIDIS obtained by correlating the quark density with the transverse impulse acquired from (color) Lorentz force acting on struck quark along its trajectory to (light-cone) infinity

matrix element defining d_2 $\leftrightarrow$ 1^{st} integration point in QS-integral

$$d_2 \equiv 3\int dx\, x^2 \bar{g}_2(x) = \frac{1}{2MP^{+2}S^x}\left\langle P,S\left|\bar{q}(0)gG^{+y}(0)\gamma^+q(0)\right|P,S\right\rangle$$

color Lorentz force — MB 2008

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sign of $d_2 \leftrightarrow \perp$ imaging

- $\kappa_q/p \longrightarrow$ sign of deformation

↪ direction of average force

↪ $d_2^u > 0,\ d_2^d < 0$

- *cf.* $f_{1T}^{\perp u} < 0,\ f_{1T}^{\perp u} < 0$

lattice (Göckeler et al., 2005)

$d_2^u \approx 0.010,\ d_2^d \approx -0.0056$

magnitude of d_2

- $\langle F^y\rangle = -2M^2 d_2 = -10\frac{GeV}{fm}d_2$
- expect partial cancellation of forces in SSA

↪ $|\langle F^y\rangle| \ll \sigma \approx 1\frac{GeV}{fm}$

↪ $d_2 = \mathcal{O}(0.01)$

transverse images $\leftrightarrow$ Ji relation for quark angular momentum:

- $J_q^x = m_N \int dx\, x r^y q(x,\mathbf{r}_\perp)$ with $b^y = r^y - \frac{1}{2m_N}$, where $q(x,\mathbf{r}_\perp)$ is distribution relative to CoM of whole nucleon
- recall: $q(x,\mathbf{b}_\perp)$ for nucleon polarized in $+\hat{x}$ direction

$$
\begin{aligned}
q(x,\mathbf{b}_\perp) &= \int \frac{d^2\Delta_\perp}{(2\pi)^2} H_q(x,0,-\Delta_\perp^2) e^{-i\mathbf{b}_\perp\cdot\Delta_\perp} \\
&\quad -\frac{1}{2M_N}\frac{\partial}{\partial b_y}\int \frac{d^2\Delta_\perp}{(2\pi)^2} E_q(x,0,-\Delta_\perp^2) e^{-i\mathbf{b}_\perp\cdot\Delta_\perp}
\end{aligned}
$$

$$
\begin{aligned}
\Rightarrow J_q^x &= M_N \int dx\, x r^y q(x,\mathbf{r}_\perp) = \int dx\, x \left(m_N b^y + \frac{1}{2} \right) q(x,\mathbf{r}_\perp) \\
&= \frac{1}{2}\int dx\, x \left[H(x,0,0) + E(x,0,0) \right]
\end{aligned}
$$

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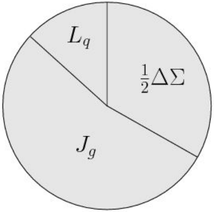
- partonic interpretation exists only for $\perp$ components (MB 2005)!

Ji decomposition



'pizza tre stagioni'

$$\frac{1}{2} = \sum_q \frac{1}{2}\Delta q + L_q + J_g$$

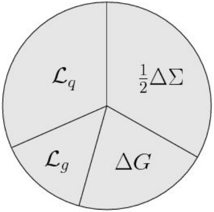
$$\frac{1}{2}\Delta q = \frac{1}{2}\int d^3x \, \langle P,S|\, q^\dagger(\vec{x})\Sigma^3 q(\vec{x})\,|P,S\rangle$$

$$L_q = \int d^3x \langle P,S| q^\dagger(\vec{x})\left(\vec{x}\times i\vec{D}\right)^3 q(\vec{x})|P,S\rangle$$

$$J_g = \int d^3x \, \langle P,S| \left[\vec{x}\times\left(\vec{E}\times\vec{B}\right)\right]^3 |P,S\rangle$$

- $i\vec{D} = i\vec{\partial} - g\vec{A}$

Jaffe-Manohar decomposition



'pizza quattro stagioni'

light-cone framework & gauge $A^+ = 0$

$$\frac{1}{2} = \sum_q \frac{1}{2}\Delta q + \mathcal{L}_q + \Delta G + \mathcal{L}_g$$

$$\mathcal{L}_q = \int d^3r \langle P,S|\, \bar{q}(\vec{r})\gamma^+\left(\vec{r}\times i\vec{\partial}\right)^z q(\vec{r})|P,S\rangle$$

$$\Delta G = \varepsilon^{+-ij}\int d^3r \, \langle P,S|\, \mathrm{Tr} F^{+i}A^j \,|P,S\rangle$$

$$\mathcal{L}_g = 2\int d^3r \langle P,S| \mathrm{Tr} F^{+j}\left(\vec{x}\times i\vec{\partial}\right)^z A^j|P,S\rangle$$

manifestly gauge invariant definition for each term exists

Ji decomposition

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manifestly gauge invariant definitions for each term exist

- GPDs $\longrightarrow L^q$
- $\vec{p}\overleftarrow{p} \longrightarrow \Delta G \longrightarrow \mathcal{L} \equiv \sum_{i \in q,g} \mathcal{L}^i$
- $L^q \neq \mathcal{L}^q$ [QED: M.B. + Hikmat BC, PRD **79**, 071501 (2009)]
- $\mathcal{L}^q - L^q = ?$
  - can we calculate/predict the difference?
  - what does it represent?

Wigner Functions (Belitsky, Ji, Yuan; Metz et al.)

$$W(x,\vec{b}_\perp,\vec{k}_\perp) \equiv \int \frac{d^2\vec{q}_\perp}{(2\pi)^2} \int \frac{d^2\xi_\perp d\xi^-}{(2\pi)^3} e^{ik\cdot\xi} e^{-i\vec{q}_\perp\cdot\vec{b}_\perp} \langle P'S'|\bar{q}(0)\gamma^+ q(\xi)|PS\rangle.$$

- (quasi) probabilty distribution for $\mathbf{b}_\perp$ and $\mathbf{k}_\perp$
- $f(x,\mathbf{k}_\perp) = \int d^2\mathbf{b}_\perp W(x,\vec{b}_\perp,\vec{k}_\perp)$
- $q(x,\mathbf{b}_\perp) = \int d^2\mathbf{k}_\perp W(x,\vec{b}_\perp,\vec{k}_\perp)$

OAM from Wigner (Lorcé et al.)

$$L_z = \int dx \int d^2\mathbf{b}_\perp \int d^2\mathbf{k}_\perp W(x,\vec{b}_\perp,\vec{k}_\perp)(b_x k_y - b_y k_x)$$
$$= \int d^3r \langle P,S|\,\bar{q}(\vec{r})\gamma^+ \left(\vec{r}\times i\vec{\partial}\right)^z q(\vec{r})|P,S\rangle = \mathcal{L}^q$$

Gauge Invariance?

need to include Wilson-line gauge link to connect 0 and ξ (Ji, Yuan; Hatta)

Wigner Functions with gauge link $\mathcal{U}_{0\xi}$ (Ji, Yuan; Hatta)

$$W(x,\vec{b}_\perp,\vec{k}_\perp) \equiv \int\frac{d^2\vec{q}_\perp}{(2\pi)^2}\int\frac{d^2\xi_\perp d\xi^-}{(2\pi)^3}e^{ik\cdot\xi}e^{-i\vec{q}_\perp\cdot\vec{b}_\perp}\langle P'S'|\bar{q}(0)\gamma^+\mathcal{U}_{0\xi}q(\xi)|PS\rangle.$$

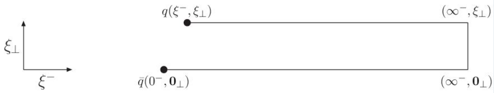
$\langle\vec{k}_\perp\rangle \equiv \int dx \int d^2\mathbf{b}_\perp \int d^2\mathbf{k}_\perp W(x,\vec{b}_\perp,\vec{k}_\perp)\vec{k}_\perp$ depends on choice of path!

straight-line gauge link

$\langle\vec{k}_\perp\rangle = \langle P,S|\bar{q}(\vec{x})\gamma^+ i\vec{D}q(\vec{x})|P,S\rangle$

- $i\vec{D} = i\vec{\partial} - g\vec{A}(\vec{x})$
- $\langle\vec{k}_\perp\rangle = 0$ (T-odd !)

light-cone staple



- correct choice for $\mathbf{k}_\perp$ distributions relevant for SIDIS

$\langle\vec{\mathcal{K}}_\perp\rangle = \langle P,S|\bar{q}(\vec{x})\gamma^+ i\vec{\mathcal{D}}q(\vec{x})|P,S\rangle$

- $i\vec{\mathcal{D}} = i\vec{\partial} - g\vec{A}(x^- = \infty, \mathbf{x}_\perp)$ $A^+ = 0$
- $\langle\vec{\mathcal{K}}_\perp\rangle \neq 0$ (FSI! Brodsky,Hwang,Schmidt)

difference $\langle \vec{\mathcal{K}}_\perp^q \rangle - \langle \vec{k}_\perp^q \rangle$

$$\langle \vec{\mathcal{K}}_\perp^q \rangle - \langle \vec{k}_\perp^q \rangle = -g \int d^3x \langle P,S|\bar{q}(\vec{x})\gamma^+[\mathbf{A}_\perp(\infty,\mathbf{x}_\perp) - \mathbf{A}_\perp(\vec{x})]q(\vec{x})|P,S\rangle$$
$$\mathbf{A}_\perp(\infty,\mathbf{x}_\perp) - \mathbf{A}_\perp(\vec{x}) = \int_{x^-}^{\infty} dr^- F^{+\perp}(r^-,\mathbf{x}_\perp)$$

$$\sqrt{2}F^{+y} = F^{0y} + F^{zy} = -E^y + B^x$$

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$$\langle \vec{k}_\perp \rangle = \langle P,S|\bar{q}(\vec{x})\gamma^+ i\vec{D} q(\vec{x})|P,S\rangle$$

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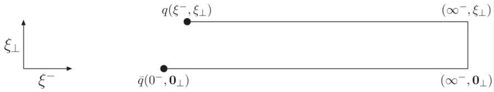
light-cone staple



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color Lorentz Force acting on ejected quark (Qiu, Sterman)

$$\sqrt{2}F^{+y} = F^{0y} + F^{zy} = -E^y + B^x = -\left(\vec{E} + \vec{v} \times \vec{B}\right)^y \text{ for } \vec{v} = (0,0,-1)$$

Impulse due to FSI

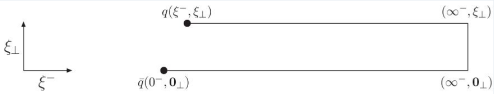
$\Delta \vec{k}^q_\perp \equiv \langle \vec{\mathcal{K}}^q_\perp \rangle - \langle \vec{k}^q_\perp \rangle$
= (average) change in $\perp$ momentum due to FSI!

straight-line gauge link

$$\langle \vec{k}_\perp \rangle = \langle P,S|\bar{q}(\vec{x})\gamma^+ i\vec{D} q(\vec{x})|P,S\rangle$$

- $i\vec{D} = i\vec{\partial} - g\vec{A}(\vec{x})$
- $\langle \vec{k}_\perp \rangle = 0$ (T-odd !)

light-cone staple



- correct choice for $\mathbf{k}_\perp$ distributions relevant for SIDIS

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Wigner Functions with gauge link $\mathcal{U}_{0\xi}$ (Ji, Yuan; Hatta)

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W and thus $L_z = \int dx \int d^2\mathbf{b}_\perp \int d^2\mathbf{k}_\perp W(x,\vec{b}_\perp,\vec{k}_\perp)(b_x k_y - b_y k_x)$ may depend on choice of path!

straight line (Ji et al.)

straigth Wilson line from 0 to ξ yields

$$L^q = \int d^3x \langle P,S| q^\dagger(\vec{x}) \left(\vec{x} \times i\vec{D}\right)^3 q(\vec{x}) |P,S\rangle$$

- $i\vec{D} = i\vec{\partial} - g\vec{A}(\vec{x})$
- same as Ji-OAM
- $f(x,\mathbf{k}_\perp) = \int d^2\mathbf{b}_\perp W(x,\vec{b}_\perp,\vec{k}_\perp)$ not the TMDs relevant for SIDIS (missing FSI!)

Wigner Functions with gauge link $\mathcal{U}_{0\xi}$ (Ji, Yuan; Hatta)

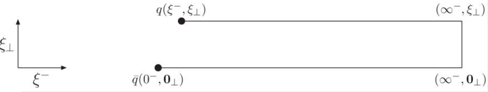
$$W(x,\vec{b}_\perp,\vec{k}_\perp) \equiv \int \frac{d^2\vec{q}_\perp}{(2\pi)^2}\int\frac{d^2\xi_\perp d\xi^-}{(2\pi)^3} e^{ik\cdot\xi}e^{-i\vec{q}_\perp\cdot\vec{b}_\perp}\langle P'S'|\bar{q}(0)\gamma^+\mathcal{U}_{0\xi}q(\xi)|PS\rangle.$$

W and thus $L_z = \int dx \int d^2\mathbf{b}_\perp \int d^2\mathbf{k}_\perp W(x,\vec{b}_\perp,\vec{k}_\perp)(b_x k_y - b_y k_x)$ may depend on choice of path!

Light-Cone Staple for $\mathcal{U}^{\pm LC}_{0\xi}$ (Hatta)

- want Wigner function that yields TMDs relevant for SIDIS when integrated $d^2\mathbf{b}_\perp$

$\hookrightarrow$ path for gauge link $\longrightarrow$ 'light-cone staple' $\longrightarrow \mathcal{U}^{+LC}_{0\xi}$



$$\mathcal{L}^q_+ = \int d^3x \langle P,S|\bar{q}(\vec{x})\gamma^+\left(\vec{x}\times i\vec{\mathcal{D}}\right)^3 q(\vec{x})|P,S\rangle$$

- $i\vec{\mathcal{D}} = i\vec{\partial} - g\vec{A}(x^- = \infty, \mathbf{x}_\perp)$
- staple at $x^- = -\infty \quad \mathcal{L}^q_-$
- PT $\Rightarrow \quad \mathcal{L}^q_- = \mathcal{L}^q_+ = \mathcal{L}^q$
- $A_\perp(\infty,\mathbf{x}_\perp) = -A_\perp(-\infty,\mathbf{x}_\perp) \Rightarrow$ $\mathcal{L}^q_+ = \mathcal{L}^q_{JM}$

$\hookrightarrow$ link at $x^- = \pm\infty$ no role for OAM!

$\hookrightarrow$ manifestly gauge invariant definition for $\mathcal{L}^q_{JM}$

straight line (→Ji)

$$L_q = \int d^3x \langle P,S|\bar{q}(\vec{x})\gamma^+\left(\vec{x}\times i\vec{D}\right)^z q(\vec{x})|P,S\rangle$$

- $i\vec{D} = i\vec{\partial} - g\vec{A}$

light-cone staple (→ Jaffe-Manohar)

$$\mathcal{L}^q = \int d^3x \langle P,S|\bar{q}(\vec{x})\gamma^+\left(\vec{x}\times i\vec{\mathcal{D}}\right)^z q(\vec{x})|P,S\rangle$$

- $i\vec{\mathcal{D}} = i\vec{\partial} - g\vec{A}(x^- = \infty, \mathbf{x}_\perp)$

difference $\mathcal{L}^q - L^q$

$$\mathcal{L}^q - L^q = -g\int d^3x \langle P,S|\bar{q}(\vec{x})\gamma^+[\vec{x}\times(\mathbf{A}_\perp(\infty,\mathbf{x}_\perp) - \mathbf{A}_\perp(\vec{x}))]^z q(\vec{x})|P,S\rangle$$

$$\mathbf{A}_\perp(\infty,\mathbf{x}_\perp) - \mathbf{A}_\perp(\vec{x}) = \int_{x^-}^{\infty} dr^- F^{+\perp}(r^-,\mathbf{x}_\perp)$$

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straight line ($\to$Ji)

$$L_q = \int d^3x \langle P,S|\bar{q}(\vec{x})\gamma^+\left(\vec{x}\times i\vec{D}\right)^z q(\vec{x})|P,S\rangle$$

- $i\vec{D} = i\vec{\partial} - g\vec{A}$

light-cone staple ($\to$ Jaffe-Manohar)

$$\mathcal{L}^q = \int d^3x \langle P,S|\bar{q}(\vec{x})\gamma^+\left(\vec{x}\times i\vec{\mathcal{D}}\right)^z q(\vec{x})|P,S\rangle$$

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color Lorentz Force acting on ejected quark (MB: arXiv:08103589)

$$\sqrt{2}F^{+y} = F^{0y} + F^{zy} = -E^y + B^x = -\left(\vec{E} + \vec{v}\times\vec{B}\right)^y \text{ for } \vec{v} = (0,0,-1)$$

straight line (→Ji)

$$L_q = \int d^3x \langle P,S|\bar{q}(\vec{x})\gamma^+\left(\vec{x}\times i\vec{D}\right)^z q(\vec{x})|P,S\rangle$$

- $i\vec{D} = i\vec{\partial} - g\vec{A}$

light-cone staple (→ Jaffe-Manohar)

$$\mathcal{L}^q = \int d^3x \langle P,S|\bar{q}(\vec{x})\gamma^+\left(\vec{x}\times i\vec{\mathcal{D}}\right)^z q(\vec{x})|P,S\rangle$$

- $i\vec{\mathcal{D}} = i\vec{\partial} - g\vec{A}(x^- = \infty, \mathbf{x}_\perp)$

difference $\mathcal{L}^q - L^q$

$$\mathcal{L}^q - L^q = -g\int d^3x \langle P,S|\bar{q}(\vec{x})\gamma^+[\vec{x}\times(\mathbf{A}_\perp(\infty,\mathbf{x}_\perp) - \mathbf{A}_\perp(\vec{x}))]^z q(\vec{x})|P,S\rangle$$

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Torque along the trajectory of q

$$T^z = \left[\vec{x}\times\left(\vec{E} - \hat{\vec{z}}\times\vec{B}\right)\right]^z$$

Change in OAM

$$\Delta L^z = \int_{x^-}^\infty dr^- \left[\vec{x}\times\left(\vec{E} - \hat{\vec{z}}\times\vec{B}\right)\right]^z$$

straight line (→Ji)

$$L_q = \int d^3x \langle P,S|\bar{q}(\vec{x})\gamma^+\left(\vec{x}\times i\vec{D}\right)^z q(\vec{x})|P,S\rangle$$

- $i\vec{D} = i\vec{\partial} - g\vec{A}(\vec{x})$

light-cone staple (→ Jaffe-Manohar)

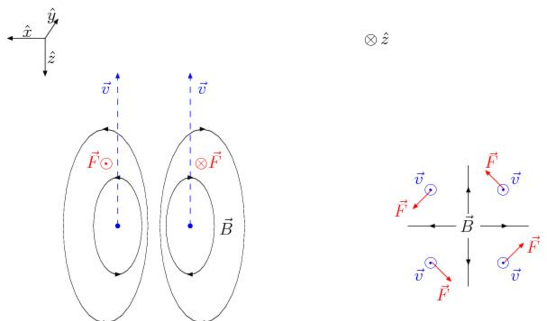
$$\mathcal{L}^q = \int d^3x \langle P,S|\bar{q}(\vec{x})\gamma^+\left(\vec{x}\times i\vec{\mathcal{D}}\right)^z q(\vec{x})|P,S\rangle$$

- $i\vec{\mathcal{D}} = i\vec{\partial} - g\vec{A}(x^- = \infty, \mathbf{x}_\perp)$

difference $\mathcal{L}^q - L^q$ (→ Wakamatsu: $-L^q_{pot}$)

$\mathcal{L}^q - L^q = \Delta L^q_{FSI}$ = change in OAM as quark leaves nucleon

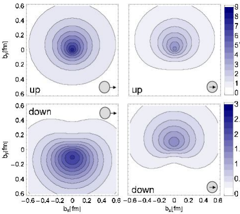
example: torque in magnetic dipole field

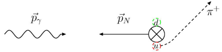


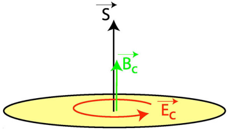
- Deeply Virtual Compton Scattering $\longrightarrow$ GPDs

$\hookrightarrow$ impact parameter dependent PDFs $q(x, \mathbf{b}_\perp)$

- $E^q(x, 0, -\mathbf{\Delta}_\perp^2) \leftrightarrow \kappa_{q/p}$ (contribution from quark flavor q to anomalous magnetic moment)
- $E^q(x, 0, -\mathbf{\Delta}_\perp^2) \longrightarrow \perp$ deformation of PDFs for $\perp$ polarized target
- parton interpretation for Ji-relation
- higher-twist $(\int dx\, x^2 \bar{g}_2(x),\ \int dx\, x^2 \bar{e}(x)) \leftrightarrow \perp$ force in DIS
- $\perp$ deformation $\leftrightarrow$ (sign of) quark-gluon correlations $(\int dx\, x^2 \bar{g}_2(x),\ \int dx\, x^2 \bar{e}(x)\ )$
- $\mathcal{L}_{JM}^q - L_{Ji}^q = \Delta L_{FSI}$ change in OAM of ejected quark due to FSI

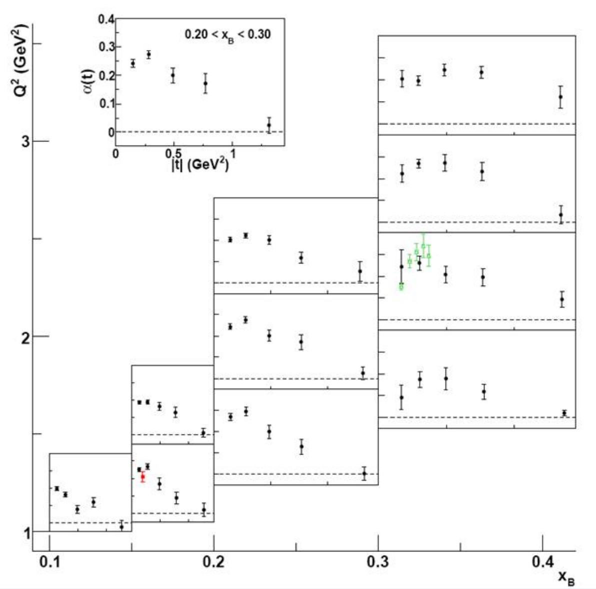






combine complementary information from deeply-virtual Compton scattering, semi-inclusive DIS & Drell-Yan to study orbital angular momentum and map the 3-d structure of hadrons

Q^2 scaling for Compton form factor (JLab)



scalar diquark model

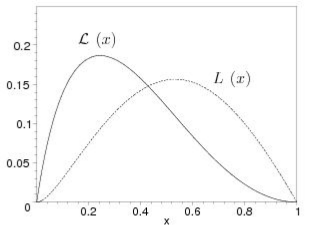
- LC wave functions $\psi_s^S(x, \mathbf{k}_\perp)$

$\hookrightarrow$ $\mathcal{L}_q$ from $|\psi_s^S(x, \mathbf{k}_\perp)|^2$

- GPDs from overlap integrals of $\psi^\dagger \psi$

$\hookrightarrow$ L_q from Ji

- $L_q = \mathcal{L}_q$.
  No surprise since
  $L_q - \mathcal{L}_q \sim \langle q^\dagger \vec{r} \times \vec{A} q \rangle$ and no $\vec{A}$ in scalar diquark model
- $L_q(x) \neq \mathcal{L}_q(x)$



scalar diquark model

- interpretation of
  $J_q(x) \equiv \frac{x}{2}[q(x) + E^q(x, 0, 0)]$
  not that of distribution of AM in x
- FT of
  $J(t) \equiv \frac{x}{2}[q(x) + E^q(x, 0, 0)]$
  not distribution of J_q^z in $\mathbf{b}_\perp$

M.B. + Hikmat BC,
PRD **79**, 071501 (2009)

QED for dressed e^- in QED

- LC wave functions $\psi_{sh}^S(x, \mathbf{k}_\perp)$

$\hookrightarrow$ $\mathcal{L}_q$ from $|\psi_{sh}^S(x, \mathbf{k}_\perp)|^2$

- GPDs from overlap integrals of $\psi^\dagger \psi$

$\hookrightarrow$ L_q from Ji

- $\mathcal{L}_e = L_e + \frac{\alpha}{4\pi} \neq L_e$