Resummation for semi-inclusive hadron production processes

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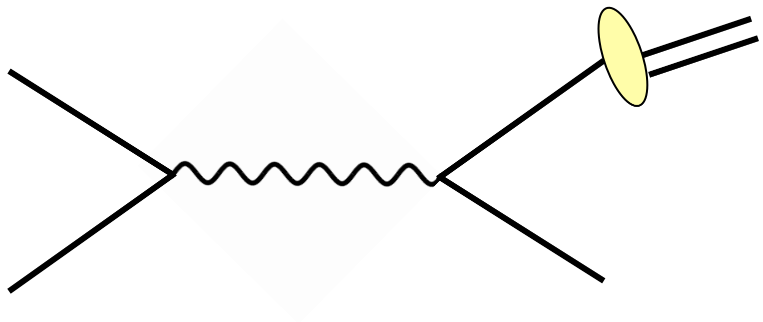
POETIC, 03/05/13

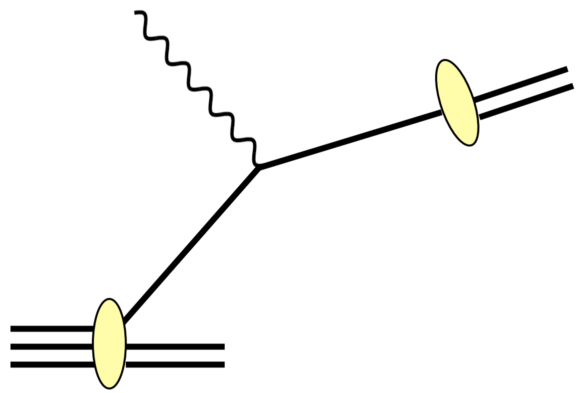
Cross sections with identified hadrons:

- source of information on fragmentation fcts. (and thus hadronization)
- probes of nucleon / nuclear structure (polarized PDFs)
- testbed for QCD calculations

→ focus of this talk: pQCD corrections

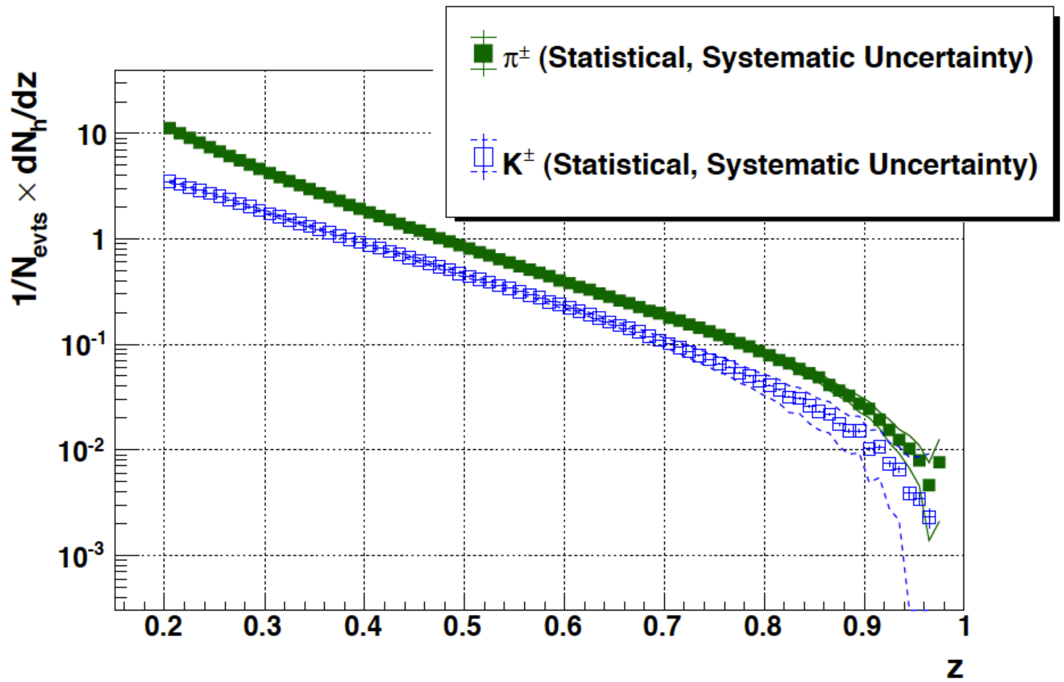


$e^+e^- \rightarrow hX$



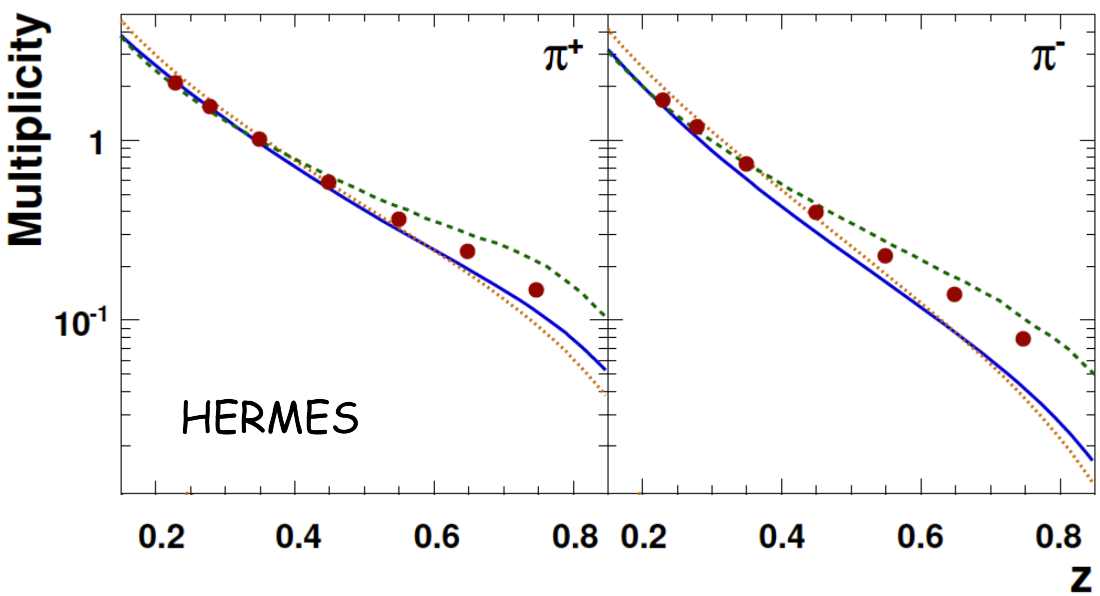
SIDIS

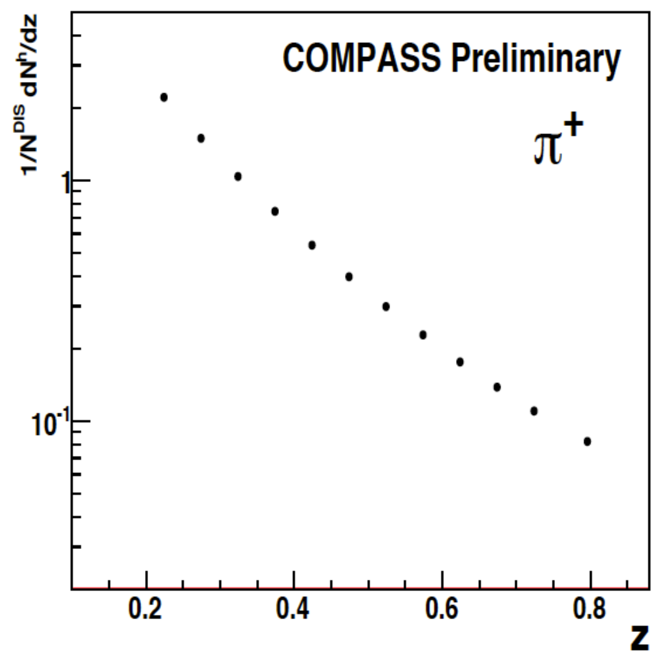
- used in global analyses of FFs (DSS, AKK, HKNS)



BELLE

HERMES
COMPASS
Jlab, EIC...

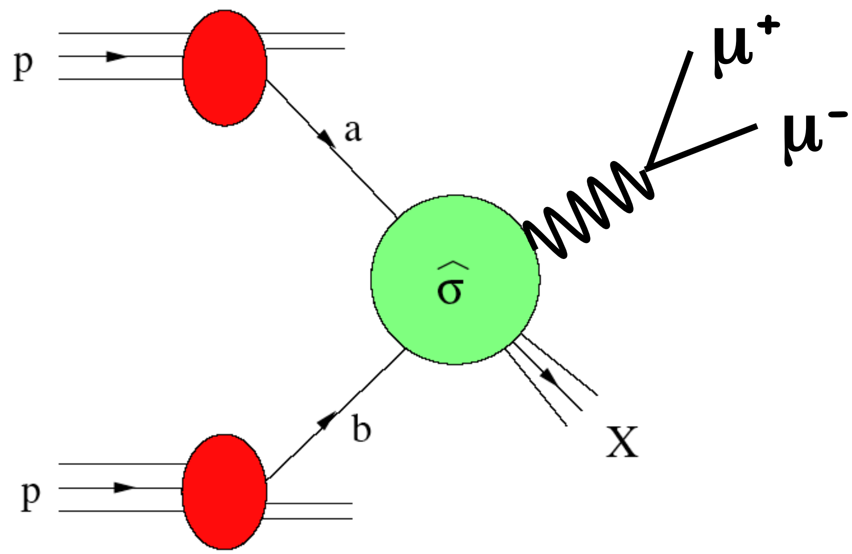




Outline

- The archetype: Drell-Yan
- SIDIS
- e^+e^- annihilation to hadron+X

Drell-Yan process



hard scale Q

$$Q^4 \frac{d\sigma}{dQ^2} = \sum_{ab} \int dx_a dx_b \, f_a(x_a,\mu) \, f_b(x_b,\mu) \, \omega_{ab}\left(z = \frac{Q^2}{\hat{s}}, \alpha_s(\mu), \frac{Q}{\mu} \right) + \ldots$$

universal pdfs

partonic hard scatt.
perturbative QCD

$\mu \sim Q$ fact./ren. scale

$$\omega_{ab} = \omega_{ab}^{(\mathrm{LO})} + \frac{\alpha_s}{2\pi} \omega_{ab}^{(\mathrm{NLO})} + \ldots$$

(up to power corrections $1/Q^2$)

LO:

$$\omega_{q\bar{q}}^{(\mathrm{LO})} \sim \delta(1-z) \qquad\qquad z = \frac{Q^2}{\hat{s}}$$

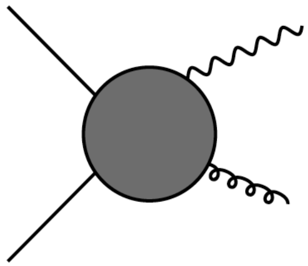
NLO:

$$\omega_{q\bar{q}}^{(\mathrm{NLO})} \sim \frac{\alpha_s}{2\pi} C_F \left[4(1+z^2) \left(\frac{\ln(1-z)}{1-z} \right)_+ - 2\frac{1+z^2}{1-z} \ln z \right.$$

$$\left. + \left(\frac{2}{3}\pi^2 - 8 \right) \delta(1-z) + 2P_{qq}(z) \ln \frac{Q^2}{\mu^2} \right]$$

$$\int_0^1 dz\, f(z) \left(\frac{\ln(1-z)}{1-z} \right)_+ \equiv \int_0^1 dz\, (f(z) - f(1)) \frac{\ln(1-z)}{1-z}$$

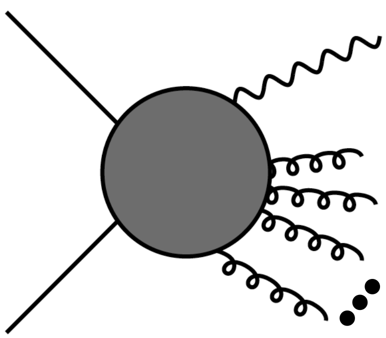
- NLO correction:



$$z \to 1:$$

$$\omega_{q\bar{q}}^{(\mathrm{NLO})} \propto \alpha_s \left(\frac{\log(1-z)}{1-z} \right)_+ + \ldots$$

- higher orders:



$$\omega_{q\bar{q}}^{(\mathrm{N^kLO})} \propto \alpha_s^k \left(\frac{\log^{2k-1}(1-z)}{1-z} \right)_+ + \ldots$$

"threshold logarithms"

- for $z \to 1$ real radiation inhibited

- large logs may spoil perturbative series, unless taken into account to all orders

  = (Threshold) Resummation !

- particularly relevant for (lower-energy) fixed-target

- work started in the ‘80s with Drell-Yan process

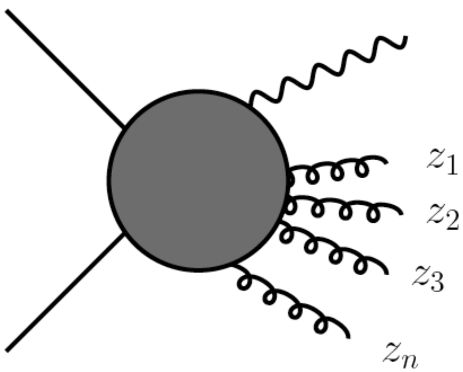
  Sterman; Catani, Trentadue

  various new techniques: Laenen, Sterman, WV
  Forte, Ridolfi; Becher, Neubert
  van Neerven, Smith, Ravindran
  Laenen, Magnea

- resummation can be organized in Mellin-moment space:

$$\int_0^1 dz\, z^{N}\, \alpha_s^k \left(\frac{\log^{2k-1}(1-z)}{1-z}\right)_+ \propto \alpha_s^k \log^{2k}(N)$$

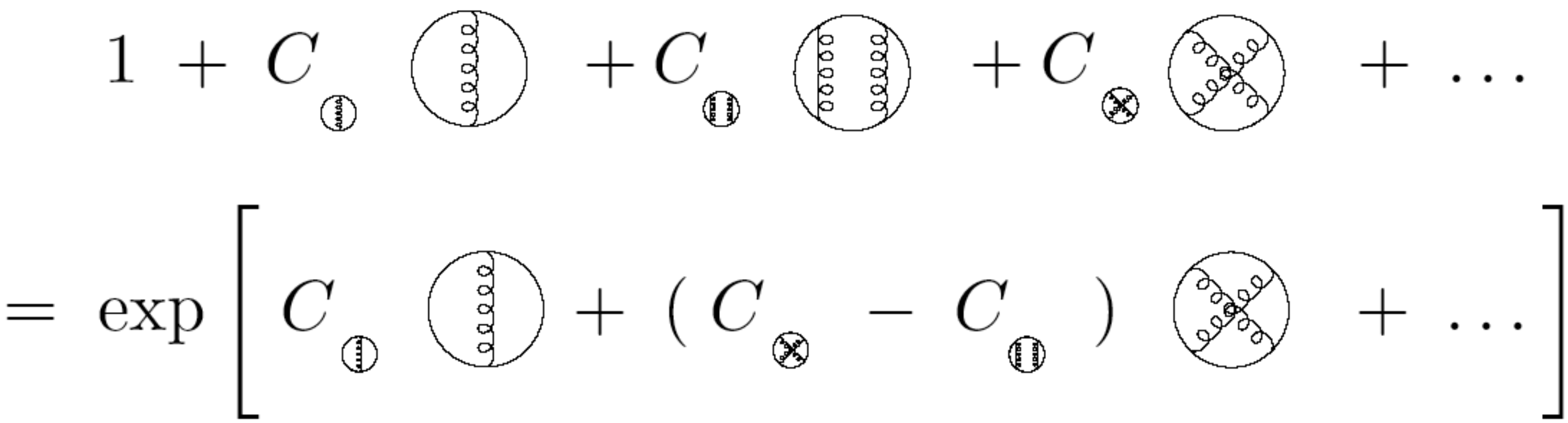
- factorizes gluon phase space:



$$z_i = \frac{2E_i}{\sqrt{\hat{s}}}$$

$$\delta\left(1 - z - \sum_{i=1}^{n} z_i\right) = \frac{1}{2\pi i}\int_C dN\, \mathrm{e}^{N\left(1-z-\sum_{i=1}^{n} z_i\right)}$$

- **exponentiation:** Gatherall; Franklin,Taylor; Sterman



$\alpha_s^k L^{2k}$ $\qquad$ $\alpha_s^k L^{2k-1}$

$$1 + \alpha_s L^2 + \alpha_s^2 L^4 + \ldots + \alpha_s L + \alpha_s^2 L^3 + \ldots$$

$$\leftrightarrow \; \exp\left[\alpha_s L^2 + \alpha_s^2 L^3 + \ldots + \alpha_s L + \alpha_s^2 L^2 + \ldots\right]$$

$\alpha_s^k L^{k+1}$ $\qquad$ $\alpha_s^k L^k$

- find to NLL

Laenen, Sterman, WV

$\xi = \frac{k^+}{Q}$

$\overline{\text{MS}}$ collinear subtraction

$(\bar{N} = N e^{\gamma_E})$

$$\tilde{\omega}_{q\bar{q}}^{(\text{res})}(N)$$

$$\propto \exp\left[\int_0^{Q^2} \frac{dk_\perp^2}{k_\perp^2} A_q\left(\alpha_s(k_\perp^2)\right) \left\{ \int_{\frac{k_\perp^2}{Q^2}}^1 \frac{d\xi}{\xi}\left[e^{-N\left(\xi - \frac{k_\perp^2}{\xi Q^2}\right)} - 1 \right] + 2\ln \bar{N} \right\}\right]$$

$$\approx 2\left[K_0\left(N\frac{2k_\perp}{Q}\right) + \ln\left(\frac{k_\perp}{Q}\bar{N}\right)\right]$$

$$\approx \exp\left[2\int_{\frac{Q^2}{\bar{N}^2}}^{Q^2} \frac{k_\perp^2}{k_\perp^2} A_q\left(\alpha_s(k_\perp^2)\right) \ln\left(\frac{k_\perp \bar{N}}{Q}\right)\right]$$

$$A_q(\alpha_s) = \frac{\alpha_s}{\pi} A_q^{(1)} + \left(\frac{\alpha_s}{\pi}\right)^2 A_q^{(2)} + \ldots$$

$$A_q^{(1)} = C_F, \quad A_q^{(2)} = \frac{1}{2} C_F \left[C_A\left(\frac{67}{18} - \frac{\pi^2}{6}\right) - \frac{5}{9} N_f\right]$$

LL :

$$\tilde{\omega}_{q\bar{q}}^{(\mathrm{res})}(N) \; \propto \; \exp\left[+ \frac{2C_F}{\pi}\, \alpha_s \ln^2 N + \ldots \right]$$

NLL :

Catani, Mangano, Nason, Trentadue

$$\tilde{\omega}_{q\bar{q}}^{(\mathrm{res})}(N) \; \propto \; \exp\left\{ 2 \ln \bar{N} \; h^{(1)}(\lambda) + 2 h^{(2)}\left(\lambda, \frac{Q^2}{\mu^2}\right) \right\}$$

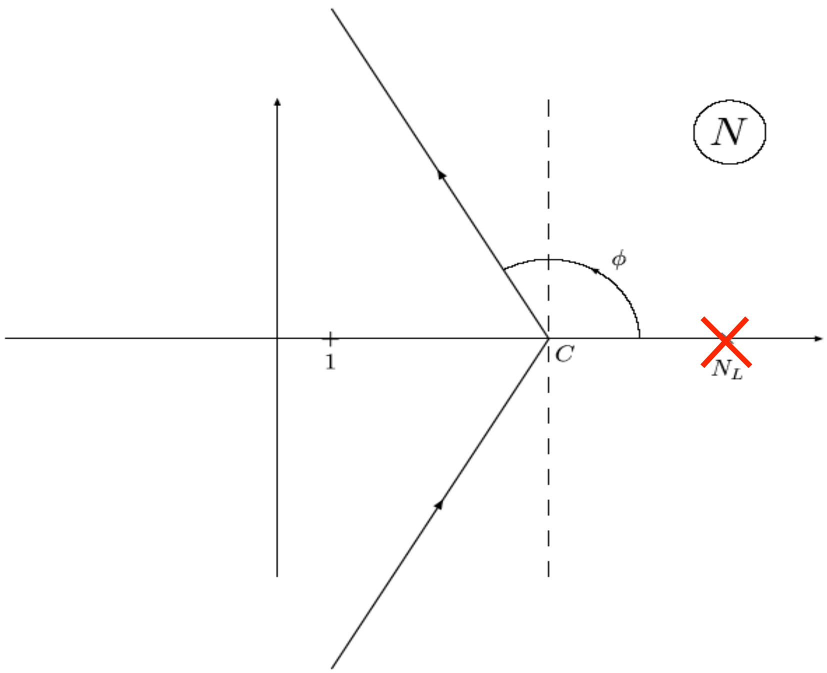
LL NLL

$$\lambda = \alpha_s(\mu^2)\, b_0 \; \log(N \mathrm{e}^{\gamma_E})$$

$$h^{(1)}(\lambda) \; = \; \frac{A_q^{(1)}}{2\pi b_0 \lambda} \left[2\lambda + (1 - 2\lambda) \ln(1 - 2\lambda) \right] \qquad h^{(2)} \; = \; \ldots$$

Inverse transform:

$$\omega^{(\mathrm{res})} = \frac{1}{2\pi i} \int_{C-i\infty}^{C+i\infty} dN \, z^{-N} \, \tilde{\omega}^{(\mathrm{res})}(N)$$



"Minimal prescription"

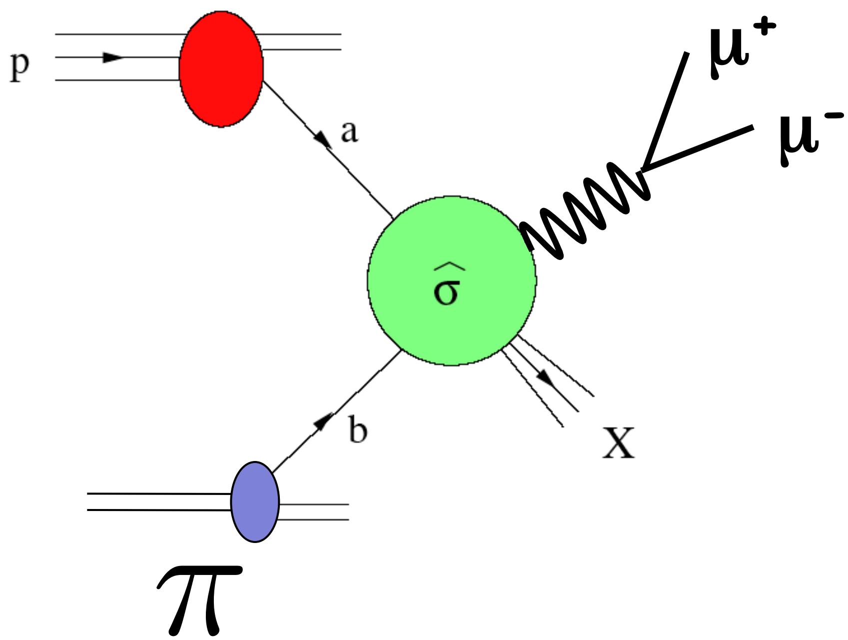
Catani, Mangano, Nason, Trentadue

Drell-Yan process in πN scattering

M. Aicher, A.Schäfer, WV

• Drell-Yan process has been main source of information on pion structure:

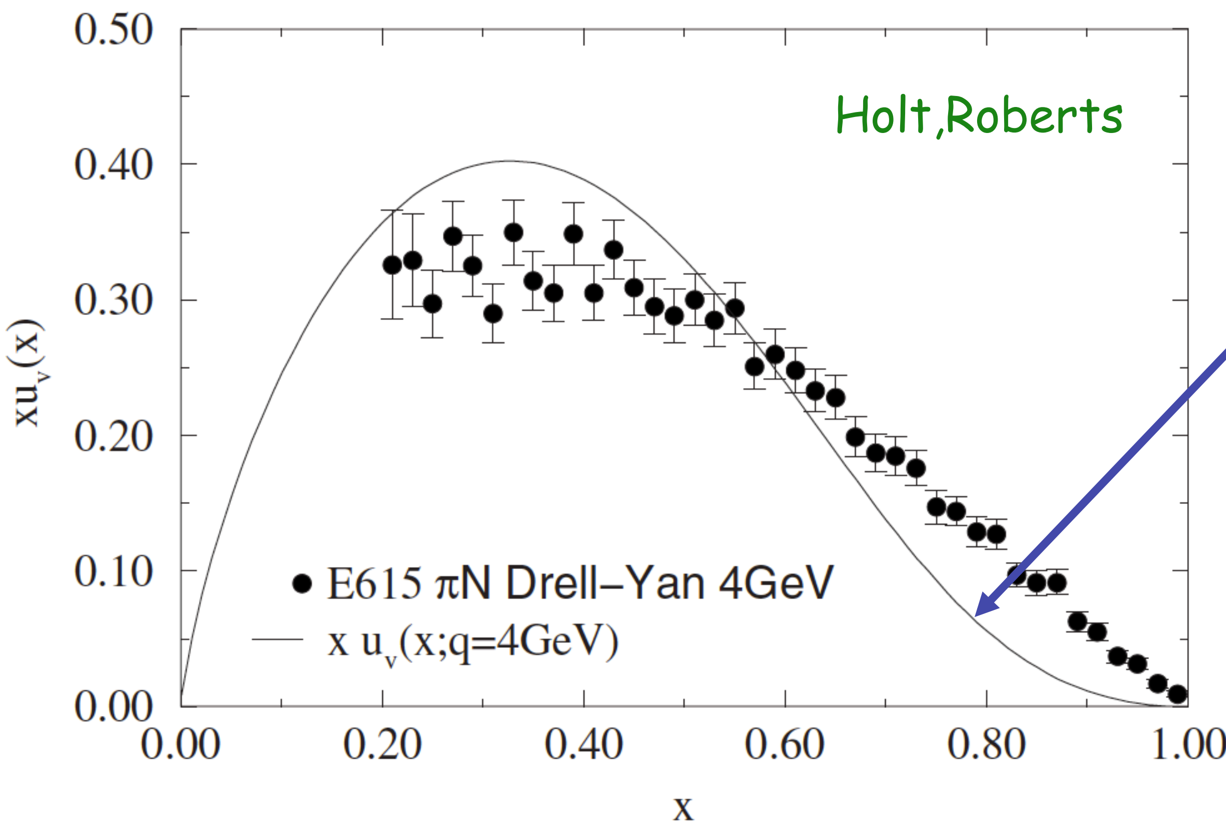
E615, NA10



$$d\sigma = \sum_{ab} \int dx_a \int dx_b \, f_a^{\pi}(x_a, \mu) f_b(x_b, \mu) \, d\hat{\sigma}_{ab}(x_a P_a, x_b P_b, Q, \alpha_s(\mu), \mu)$$

• Kinematics such that data mostly probe valence region: ~200 GeV pion beam on fixed target

- LO extraction of u_v from E615 data: $\sqrt{S} = 21.75\,\mathrm{GeV}$



$\sim\ (1-x)^2$

QCD counting rules

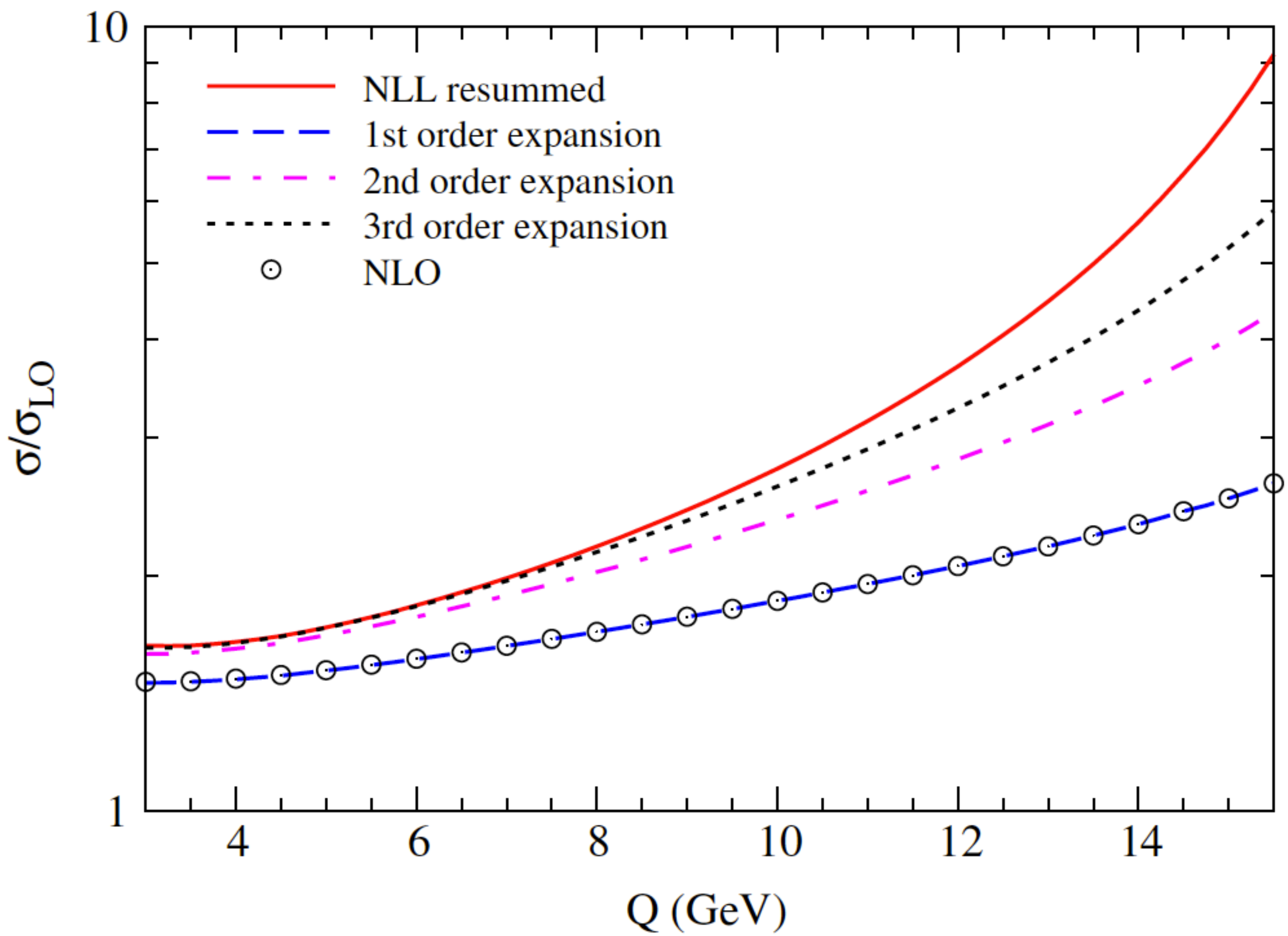
Farrar, Jackson; Berger, Brodsky; Yuan Blankenbecler, Gunion, Nason

Dyson-Schwinger

Hecht et al.

(Compass kinematics)

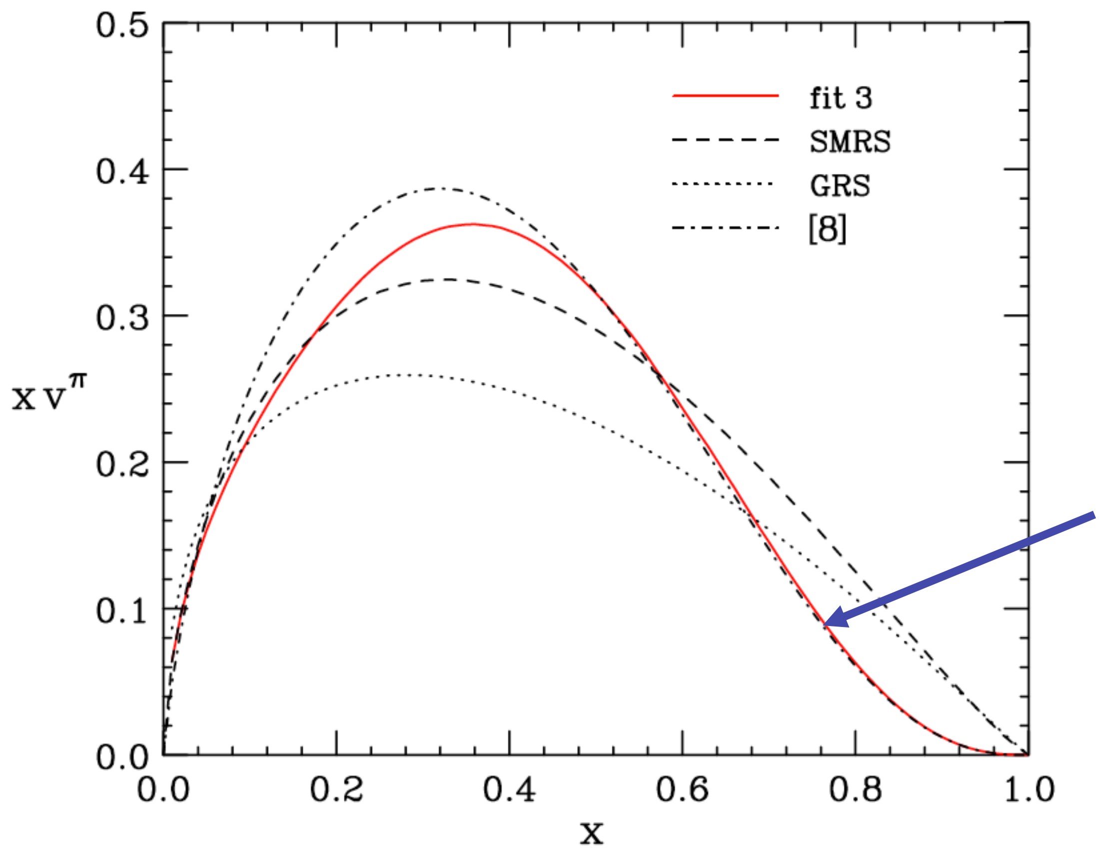
$\sqrt{S} = 19\,\mathrm{GeV}$



Aicher, Schäfer, WV

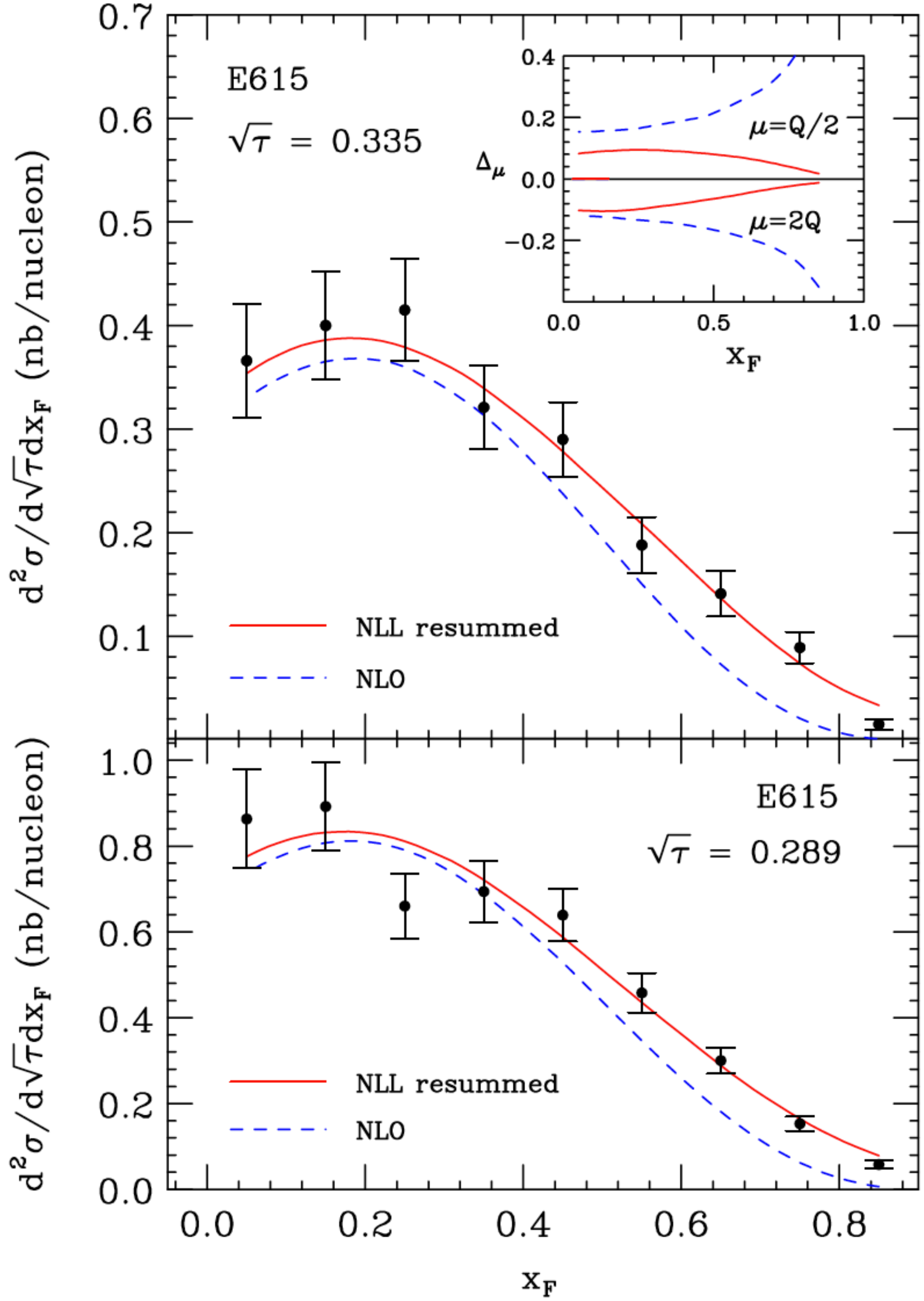
$$xv^{\pi}(x, Q_0^2) = N_v x^{\alpha}(1-x)^{\beta}(1+\gamma x^{\delta})$$

Fit	$2\langle xv^{\pi}\rangle$	α	β	γ	K	χ^2 (no. of points)
1	0.55	0.15 ± 0.04	1.75 ± 0.04	89.4	0.999 ± 0.011	82.8 (70)
2	0.60	0.44 ± 0.07	1.93 ± 0.03	25.5	0.968 ± 0.011	80.9 (70)
3	0.65	0.70 ± 0.07	2.03 ± 0.06	13.8	0.919 ± 0.009	80.1 (70)
4	0.7	1.06 ± 0.05	2.12 ± 0.06	6.7	0.868 ± 0.009	81.0 (70)



$Q = 4$ GeV

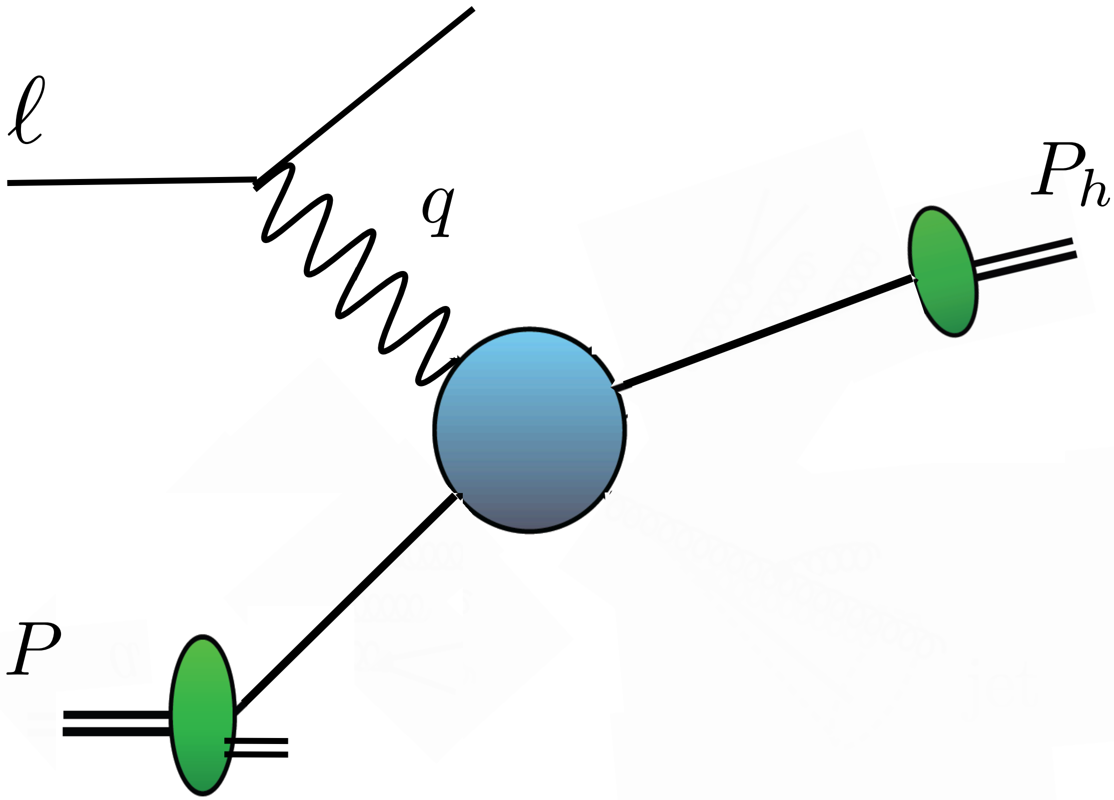
$\sim (1-x)^{2.34}$



E615
$\sqrt{\tau}$ = 0.335
Δ_μ
μ=Q/2
μ=2Q
0.4
0.2
0.0
−0.2
0.0
0.5
1.0
x_F
NLL resummed
NLO
$d^2\sigma/d\sqrt{\tau}dx_F$ (nb/nucleon)
0.7
0.6
0.5
0.4
0.3
0.2
0.1
E615
$\sqrt{\tau}$ = 0.289
NLL resummed
NLO
$d^2\sigma/d\sqrt{\tau}dx_F$ (nb/nucleon)
1.0
0.8
0.6
0.4
0.2
0.0
0.0
0.2
0.4
0.6
0.8
x_F

Resummation in SIDIS

D. Anderle, F. Ringer, WV



$$x \equiv \frac{Q^2}{2P\cdot q}$$

$$y \equiv \frac{P\cdot q}{P\cdot \ell}$$

$$z \equiv \frac{P\cdot P_h}{P\cdot q}$$

$$\frac{d^3\sigma^h}{dx\,dy\,dz} = \frac{4\,\pi\alpha^2}{Q^2}\left[\frac{1+(1-y)^2}{2y}\mathcal{F}_T^h(x,z,Q^2) + \frac{1-y}{y}\mathcal{F}_L^h(x,z,Q^2)\right]$$

LO: $$\mathcal{F}_T^h = \sum_q e_q^2\, q(x,\mu^2)\, D_q^h(z,\mu^2) \qquad \mathcal{F}_L^h = 0$$

Collinear factorization:

$$\mathcal{F}_i^h(x,z,Q^2) \;=\; \sum_{f,f'} \int_x^1 \frac{d\hat{x}}{\hat{x}} \int_z^1 \frac{d\hat{z}}{\hat{z}} f\left(\frac{x}{\hat{x}},\mu^2\right) D_{f'}^h\left(\frac{z}{\hat{z}},\mu^2\right)\; \mathcal{C}_{f'f}^i\left(\hat{x},\hat{z},\frac{Q^2}{\mu^2},\alpha_s(\mu^2)\right)$$

$$\mathcal{C}_{f'f}^i \;=\; C_{f'f}^{i,(0)} + \frac{\alpha_s(\mu^2)}{2\pi} C_{f'f}^{i,(1)} + \mathcal{O}(\alpha_s^2)$$

$$C_{qq}^{T,(0)}(\hat{x},\hat{z}) \;=\; e_q^2\, \delta(1-\hat{x})\delta(1-\hat{z})$$

NLO, as $\hat{x},\hat{z} \;\rightarrow\; 1:$

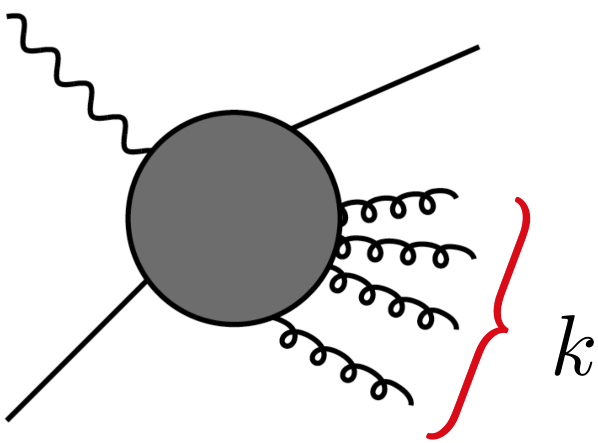
Altarelli et al;
de Florian, Stratmann, WV

$$C_{qq}^{T,(1)}(\hat{x},\hat{z}) \sim e_q^2\, C_F \Bigg[-8\,\delta(1-\hat{x})\,\delta(1-\hat{z}) + 2\,\delta(1-\hat{x})\left(\frac{\ln(1-\hat{z})}{1-\hat{z}}\right)_+ + 2\,\delta(1-\hat{z})\left(\frac{\ln(1-\hat{x})}{1-\hat{x}}\right)_+ + \frac{2}{(1-\hat{x})_+(1-\hat{z})_+} \Bigg]$$

- k^th order of pert. theory:

$$\alpha_s^k \left(\frac{\ln^{2k-1}(1-\hat{x})}{1-\hat{x}} \right)_+ \delta(1-\hat{z})$$

$$\alpha_s^k \left(\frac{\ln^{m}(1-\hat{x})}{1-\hat{x}} \right)_+ \left(\frac{\ln^{n}(1-\hat{z})}{1-\hat{z}} \right)_+ \qquad (m+n \leq 2k-2)$$



$$(1-\hat{x}) + (1-\hat{z}) \approx \frac{2k_0}{Q}$$

Mellin moments:

$$\tilde{\mathcal{F}}_i^h(N,M,Q^2) \equiv \int_0^1 dx\, x^{N-1} \int_0^1 dz\, z^{M-1}\, \mathcal{F}_i^h(x,z,Q^2)$$

$$= \sum_{f,f'} \tilde{f}^N(\mu^2)\, \tilde{D}_{f'}^{h,M}(\mu^2) \times \tilde{C}_{f'f}^i\left(N, M, \frac{Q^2}{\mu^2}, \alpha_s(\mu^2)\right)$$

Large $\hat{x}, \hat{z}$ → large N, M : $\bar{N} \equiv N e^{\gamma_E},\ \bar{M} \equiv M e^{\gamma_E}$

$$\tilde{C}_{qq}^{T,(1)}(N,M) \sim e_q^2 C_F \left[-8 + \frac{\pi^2}{3} + \left(\ln \bar{N} + \ln \bar{M}\right)^2 \right]$$

k[th] order: $\alpha_s^k \ln^n \bar{N} \ln^m \bar{M}$ $\quad n + m \leq 2k$

$$\xi = \frac{k^+}{Q}$$

$$\zeta = \frac{k^-}{Q} \approx \frac{k_\perp^2}{\xi Q^2}$$

$$\mathcal{C}_{qq}^{T,\mathrm{res}}(N, M, \alpha_s(Q^2))$$

$$\propto \; \exp\left[\int_0^{Q^2} \frac{dk_\perp^2}{k_\perp^2} A_q\left(\alpha_s(k_\perp^2)\right) \left\{ \int_{\frac{k_\perp^2}{Q^2}}^1 \frac{d\xi}{\xi} \left[\mathrm{e}^{-N\xi - M\frac{k_\perp^2}{\xi Q^2}} - 1 \right] + \ln \bar{N} + \ln \bar{M} \right\} \right]$$

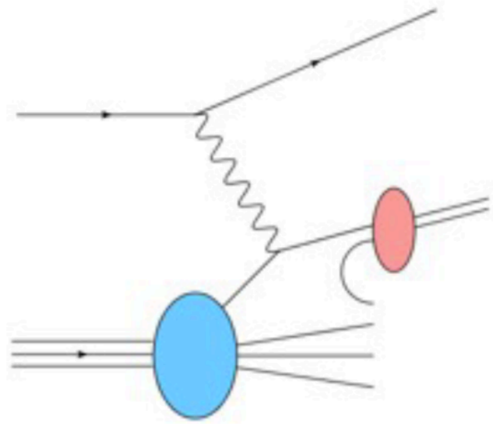
collinear subtraction

$$A_q(\alpha_s) = \frac{\alpha_s}{\pi} A_q^{(1)} + \left(\frac{\alpha_s}{\pi}\right)^2 A_q^{(2)} + \ldots$$

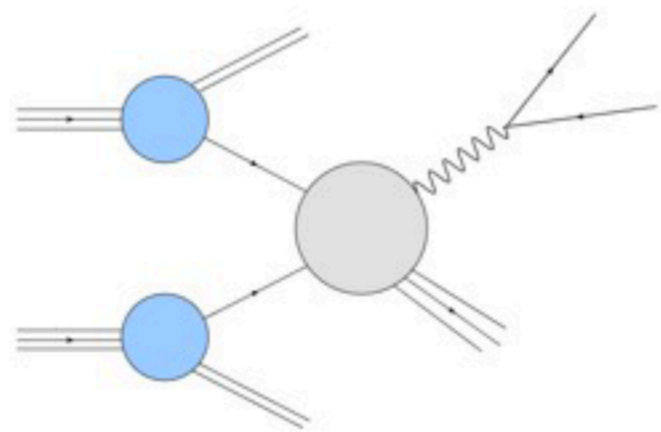
$$A_q^{(1)} = C_F, \quad A_q^{(2)} = \frac{1}{2} C_F \left[C_A \left(\frac{67}{18} - \frac{\pi^2}{6} \right) - \frac{5}{9} N_f \right]$$

(see also Catani, Cacciari; Sterman, WV)

SIDIS



Drell-Yan



$$\int_{\frac{k_\perp^2}{Q^2}}^1 \frac{d\xi}{\xi}\left[\mathrm{e}^{-N\xi - M\frac{k_\perp^2}{\xi Q^2}} - 1\right] + \ln \bar{N} + \ln \bar{M}$$

$$\approx 2\left[K_0\left(\sqrt{NM}\,\frac{2k_\perp}{Q}\right) + \ln\left(\frac{k_\perp}{Q}\sqrt{\bar{N}\bar{M}}\right)\right]$$

$$\int_{\frac{k_\perp^2}{Q^2}}^1 \frac{d\xi}{\xi}\left[\mathrm{e}^{-N\left(\xi - \frac{k_\perp^2}{\xi Q^2}\right)} - 1\right] + 2\ln \bar{N}$$

$$\approx 2\left[K_0\left(N\,\frac{2k_\perp}{Q}\right) + \ln\left(\frac{k_\perp}{Q}\bar{N}\right)\right]$$

$$\longleftarrow$$

$$\bar{N} \to \sqrt{\bar{N}\bar{M}}$$

Leading-Log structure:

SIDIS: $$\exp\left[\frac{\alpha_s}{2\pi}C_F\left(\ln\bar{N}+\ln\bar{M}\right)^2\right]$$

DY: $$\exp\left[\frac{\alpha_s}{2\pi}4C_F\ln^2\bar{N}\right]$$

incl. DIS:

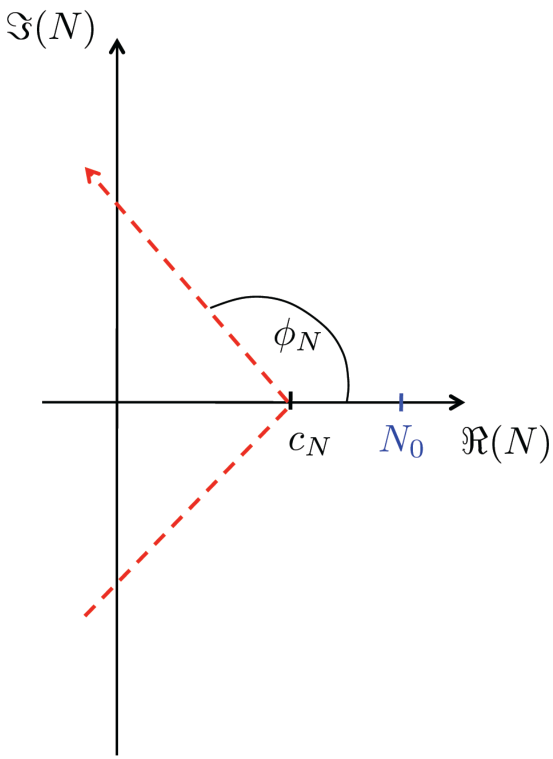
$e^+e^- \to h\,X$

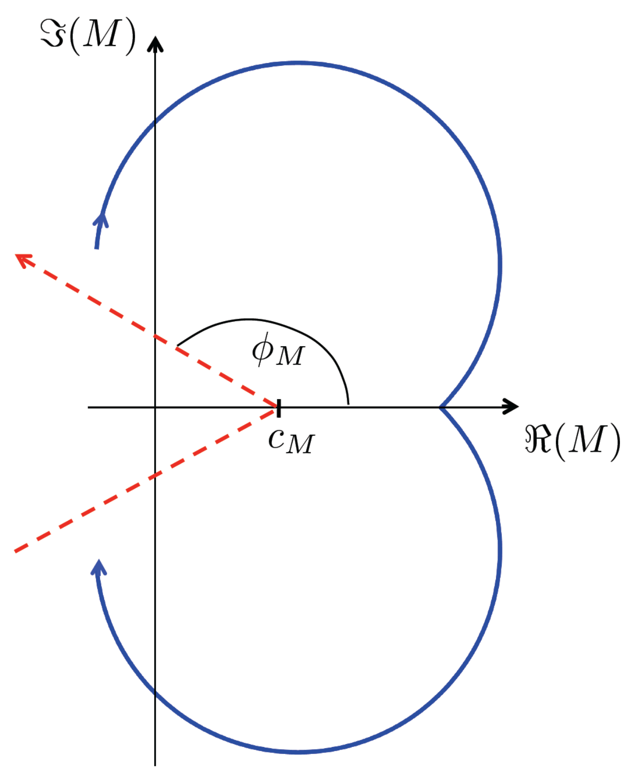
$$\left.\begin{matrix}\text{incl. DIS} \\ e^+e^-\to h\,X\end{matrix}\right\}\quad \exp\left[\frac{\alpha_s}{2\pi}C_F\ln^2\bar{N}\right]$$

Expansion to NLL (SIDIS):

$$\mathcal{C}_{qq}^{T,\mathrm{res}}(N,M,\alpha_s(Q^2)) \;\propto\; \exp\left[2\int_{\frac{Q^2}{\bar{N}\bar{M}}}^{Q^2}\frac{dk_\perp^2}{k_\perp^2}A_q\left(\alpha_s(k_\perp^2)\right)\ln\left(\frac{k_\perp}{Q}\sqrt{\bar{N}\bar{M}}\right)\right]$$

Landau pole at $\quad N\,M = \mathrm{e}^{1/(\alpha_s b_0)-2\gamma_E} \equiv L_0$





$\mu p \quad \sqrt{s} = 17.4\ \mathrm{GeV}$ (COMPASS)

MSTW08 / DSS

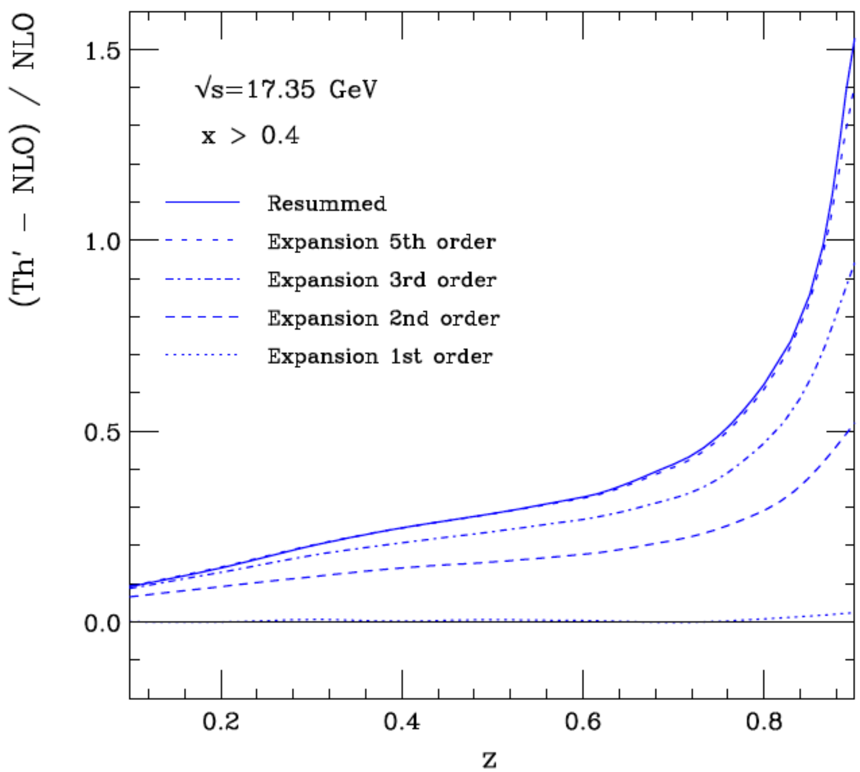
(fit to get moments)

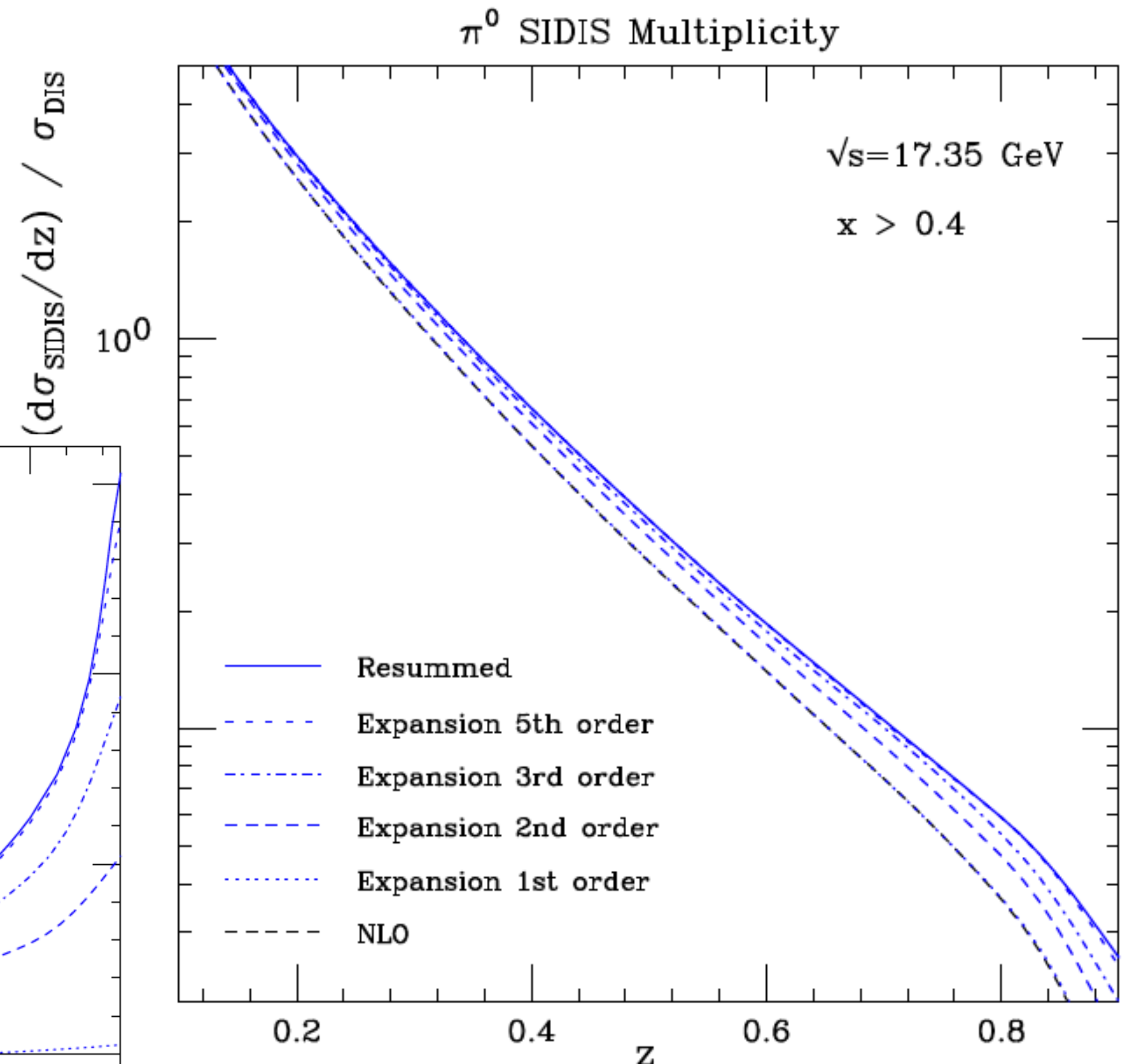
$0.4 < x < 0.7$

$0.1 < y < 0.9$

$Q^2 > 1\,\mathrm{GeV}^2$

$W > 7\,\mathrm{GeV}$





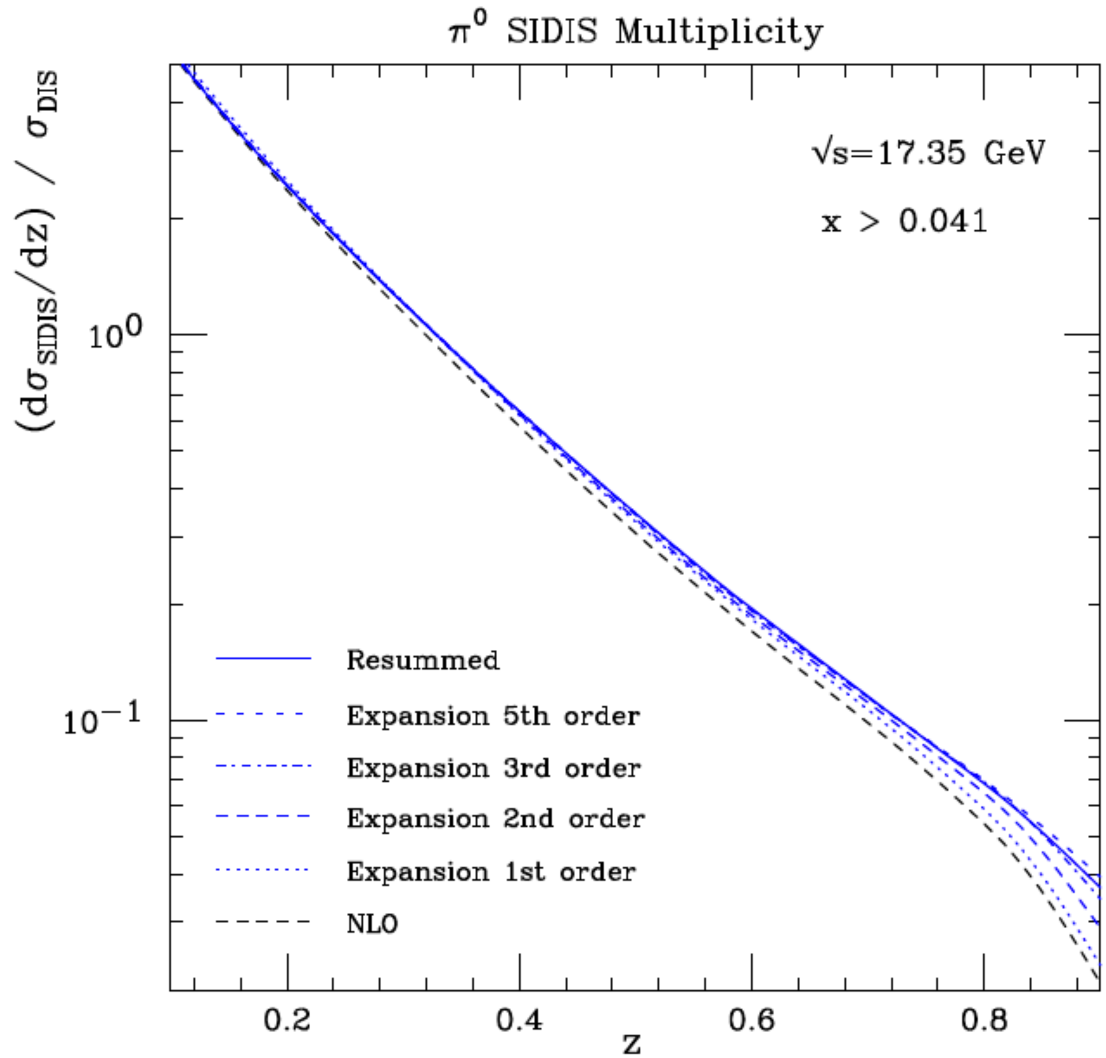
$$\mu p \quad \sqrt{s} = 17.4 \text{ GeV}$$

$$0.041 < x < 0.7$$

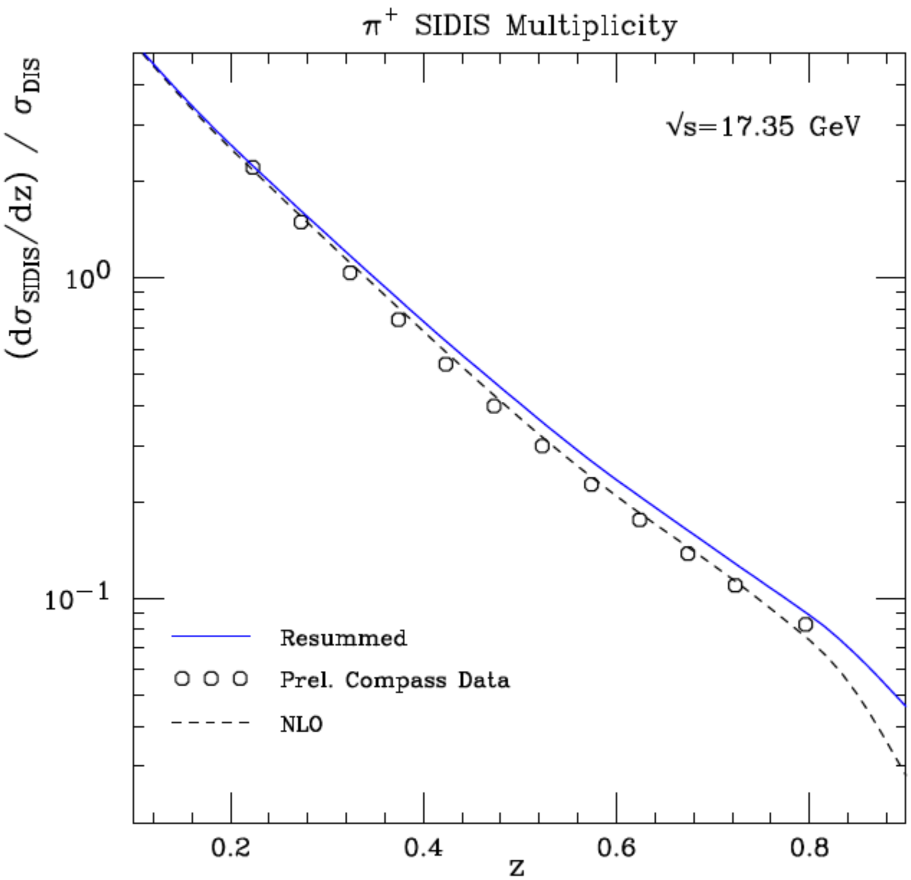
$$0.1 < y < 0.9$$

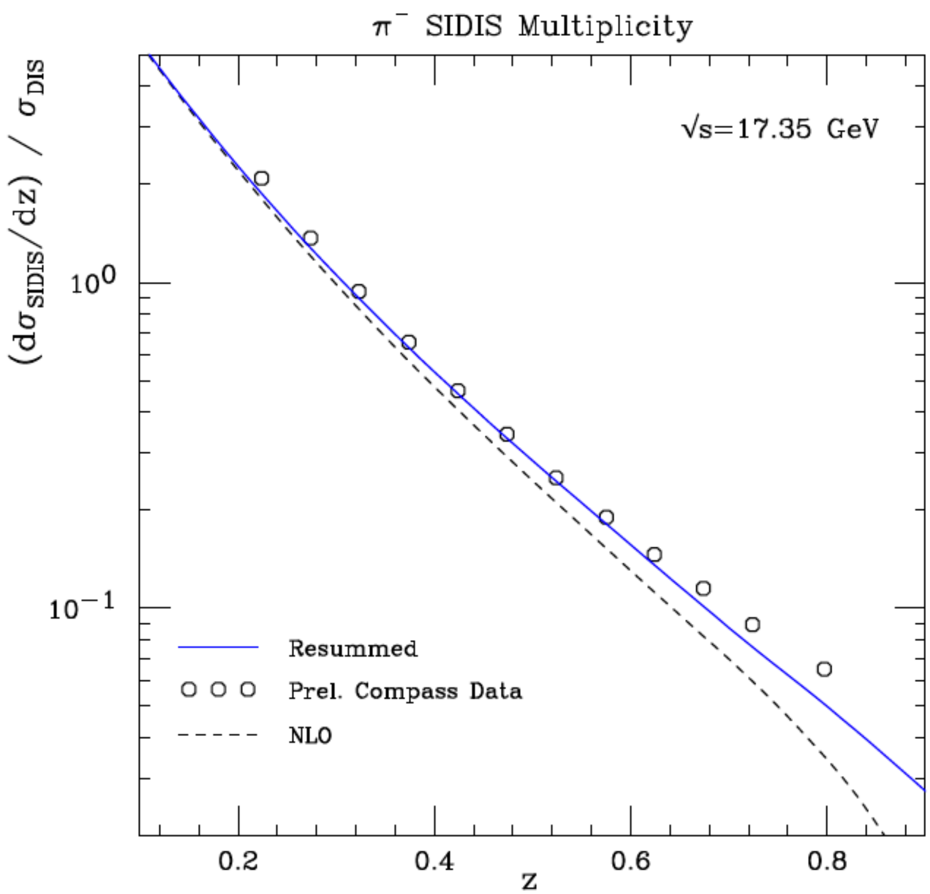
$$Q^2 > 1\,\text{GeV}^2$$

$$W > 7\,\text{GeV}$$

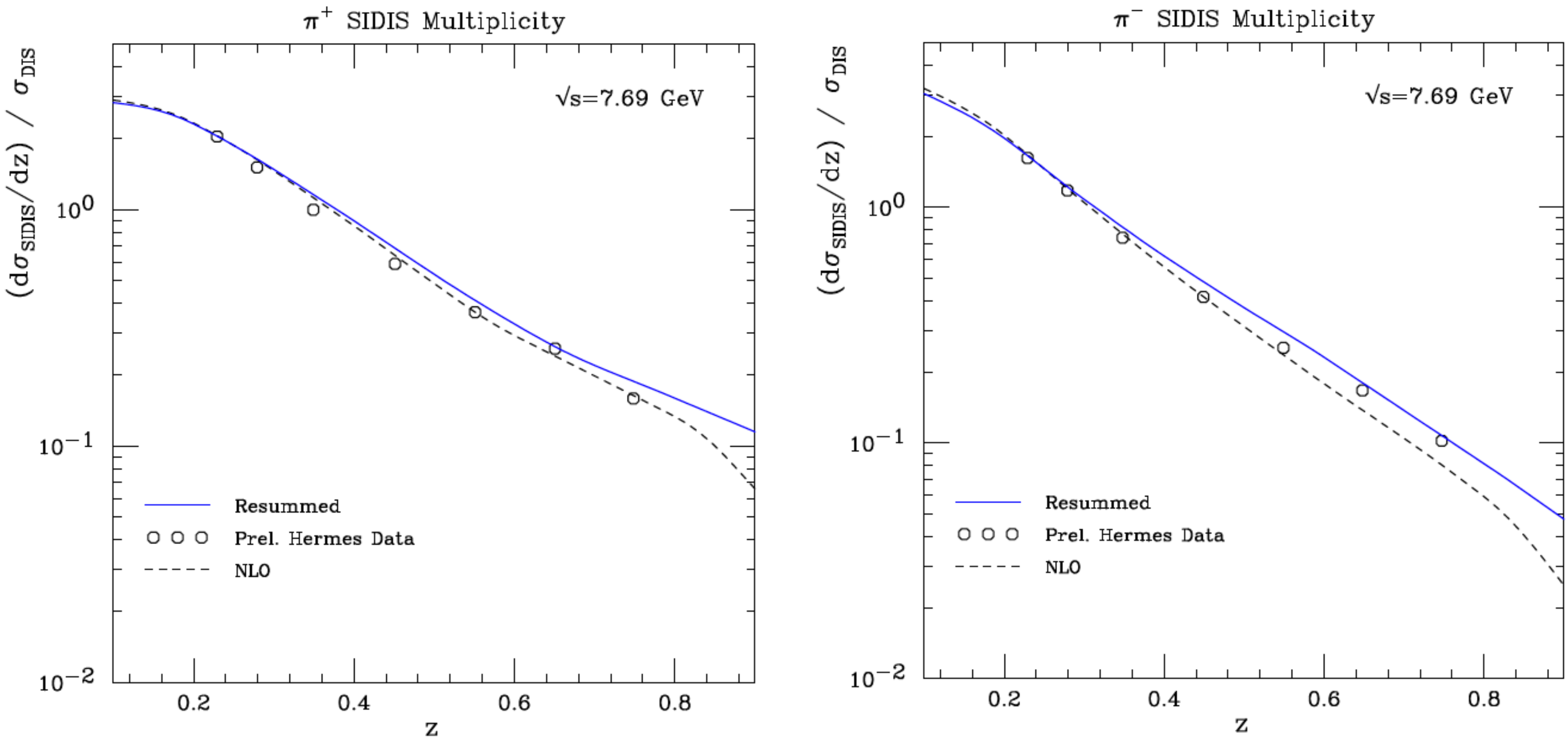


COMPASS (prel.)

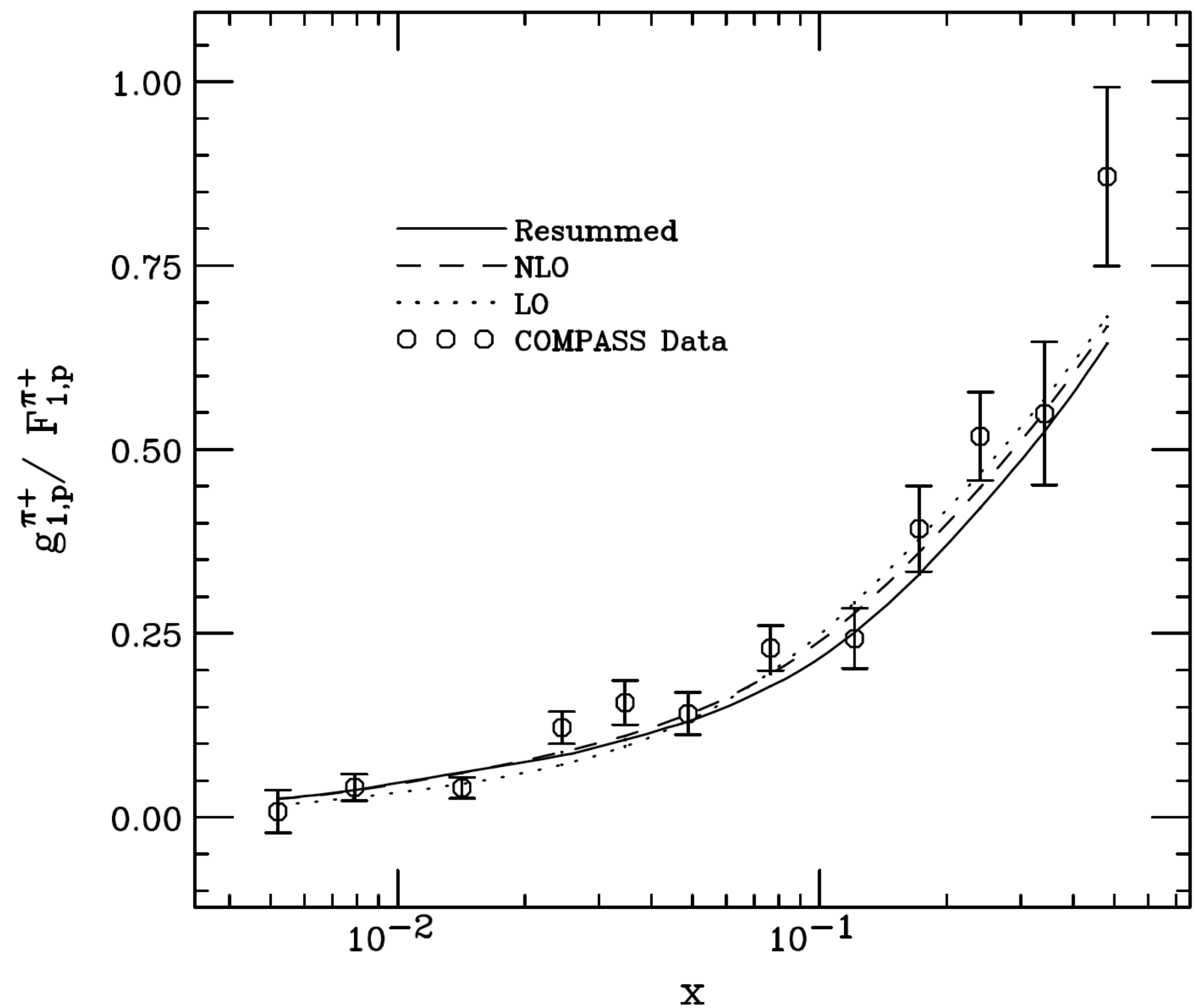




HERMES (prel.)

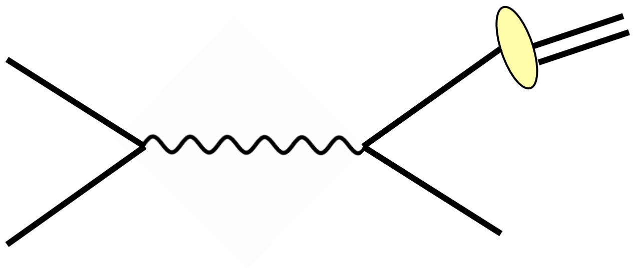

π+ SIDIS Multiplicity
√s=7.69 GeV
(dσSIDIS/dz) / σDIS
10^0
10^−1
10^−2
0.2
0.4
0.6
0.8
z
Resummed
Prel. Hermes Data
NLO
π− SIDIS Multiplicity
√s=7.69 GeV
(dσSIDIS/dz) / σDIS
10^0
10^−1
10^−2
0.2
0.4
0.6
0.8
z
Resummed
Prel. Hermes Data
NLO

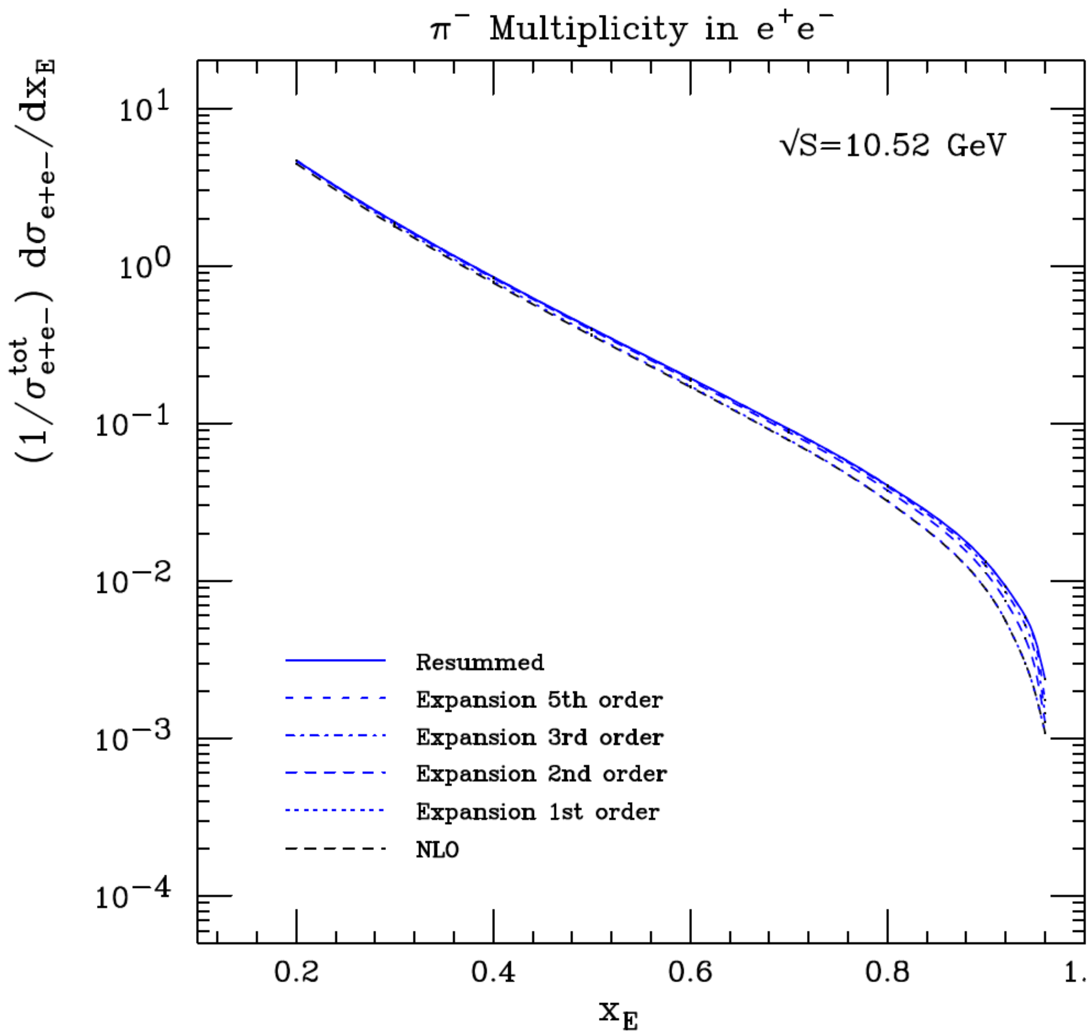
SIDIS spin asymmetry



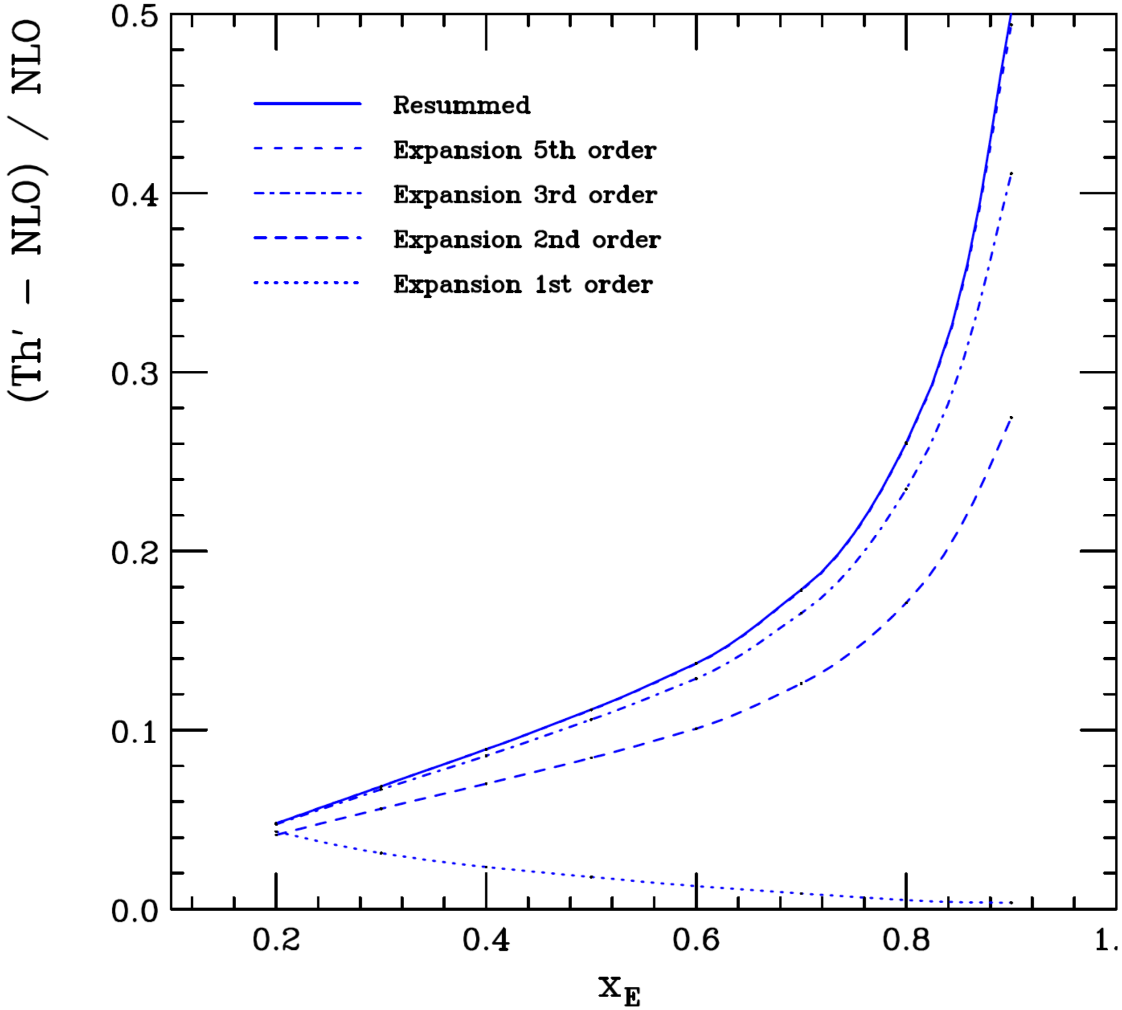
Resummation in $e^+e^- \rightarrow hX$

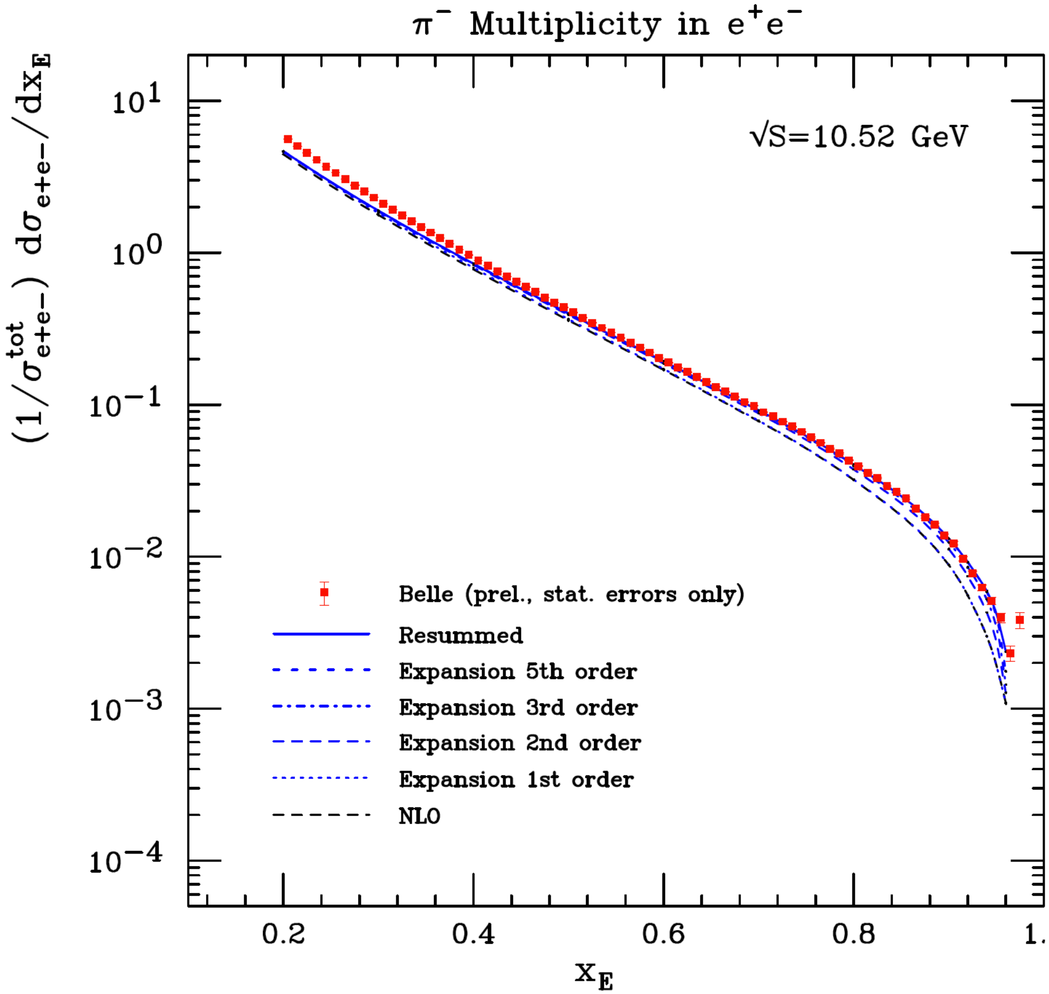
D. Anderle, F. Ringer, WV





$$x_E \equiv \frac{2P_h \cdot q}{Q^2}$$


Resummed
Expansion 5th order
Expansion 3rd order
Expansion 2nd order
Expansion 1st order
(Th' − NLO) / NLO
0.0
0.1
0.2
0.3
0.4
0.5
0.2
0.4
0.6
0.8
1.
x_E



π^- Multiplicity in e^+e^-
$(1/\sigma^{tot}_{e+e-})\ d\sigma_{e+e-}/dx_E$
$\sqrt{S}$=10.52 GeV
Belle (prel., stat. errors only)
Resummed
Expansion 5th order
Expansion 3rd order
Expansion 2nd order
Expansion 1st order
NLO
10^1
10^0
10^{-1}
10^{-2}
10^{-3}
10^{-4}
0.2
0.4
0.6
0.8
1.
x_E

Conclusions:

- although generally modest beyond-NLO effects, precision of data warrants inclusion in global fits

- interplay with power corrections, TMC

- many applications at EIC !