

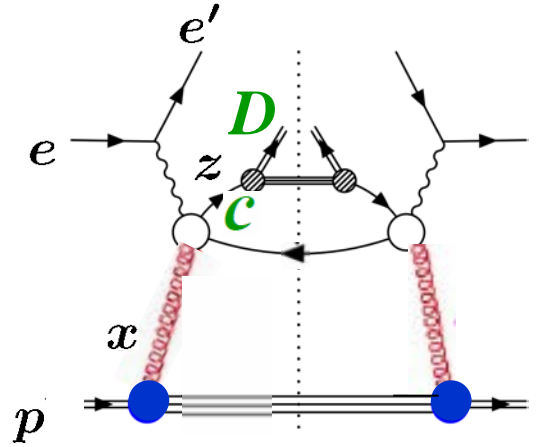
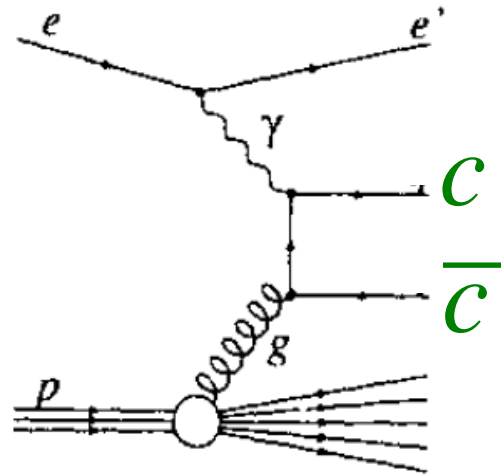
Single spin asymmetry in large P_T
open charm production at EIC and
twist-three multi-gluon correlations
in the nucleon

Kazuhiro Tanaka (Juntendo U)

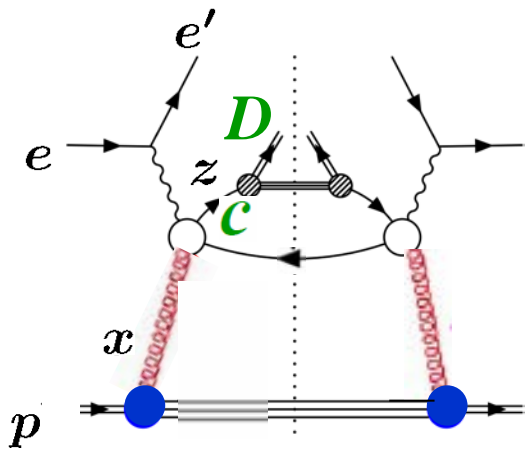
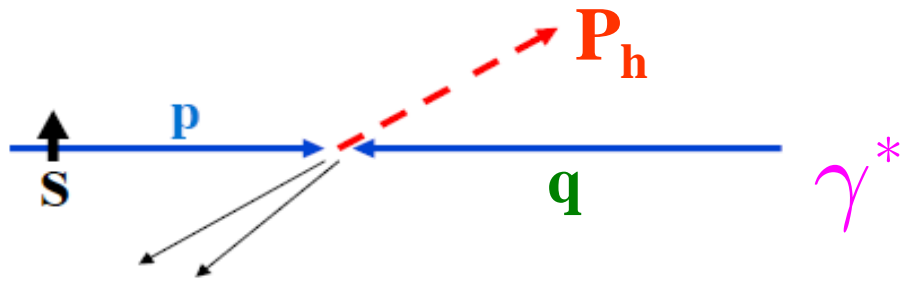
PRD85 ('12) 114026

JHEP1302 ('13) 003

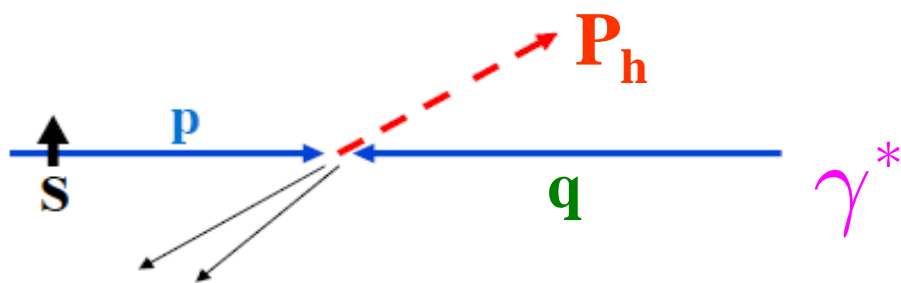
SIDIS $ep \rightarrow eDX$



SSA in **SIDIS** $ep^\uparrow \rightarrow eDX$



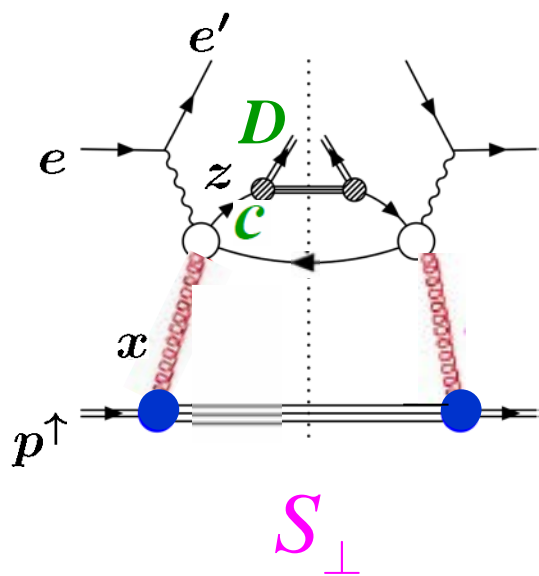
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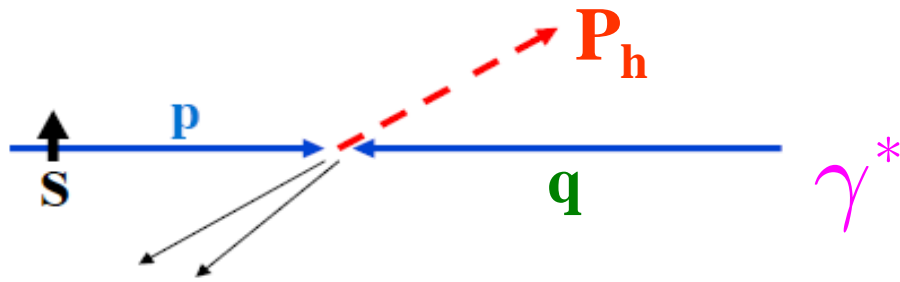
$$d\sigma \sim \vec{S}_\perp \cdot \left(\vec{p} \times \vec{P}_h \right)$$

$$A = \frac{d\sigma^\uparrow - d\sigma^\downarrow}{d\sigma^\uparrow + d\sigma^\downarrow}$$

1. $P_{h\perp} \neq 0$: k_\perp of quark or gluon
2. helicity flip by one unit
3. interaction phase: beyond Born



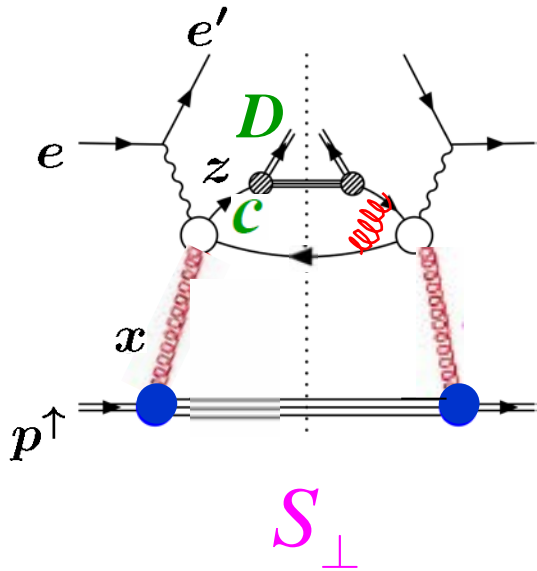
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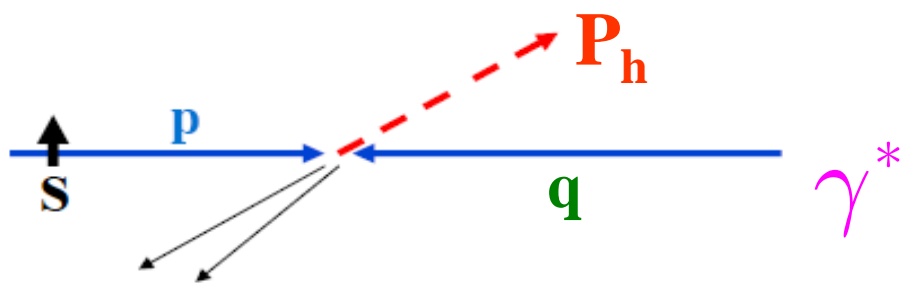
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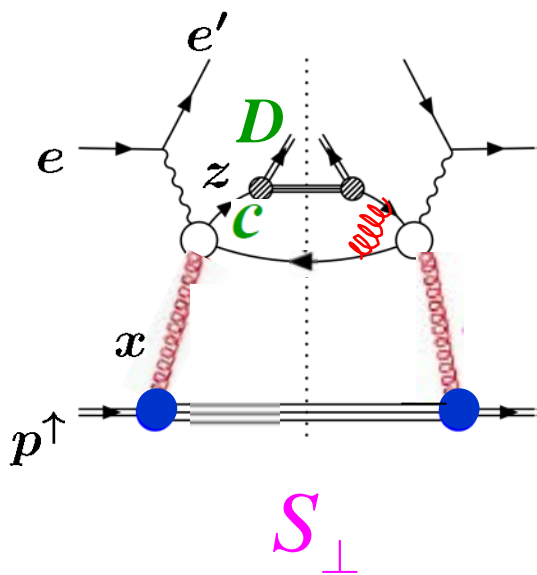


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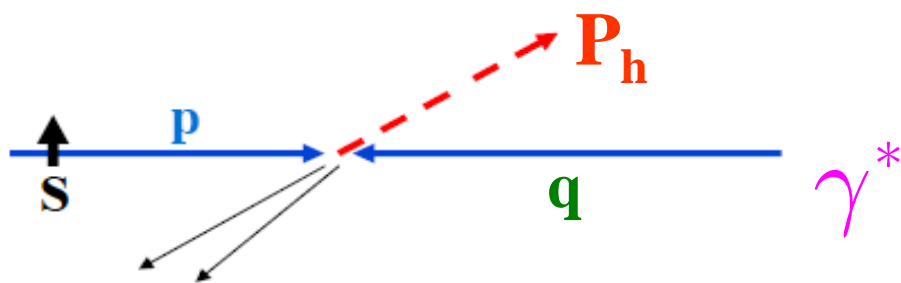
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at twist-2

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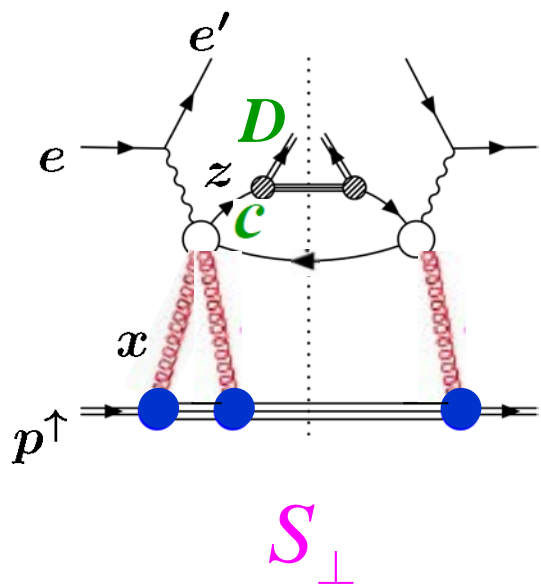


$$d\sigma \sim \vec{S}_\perp \cdot \left(\vec{p} \times \vec{P}_h \right)$$

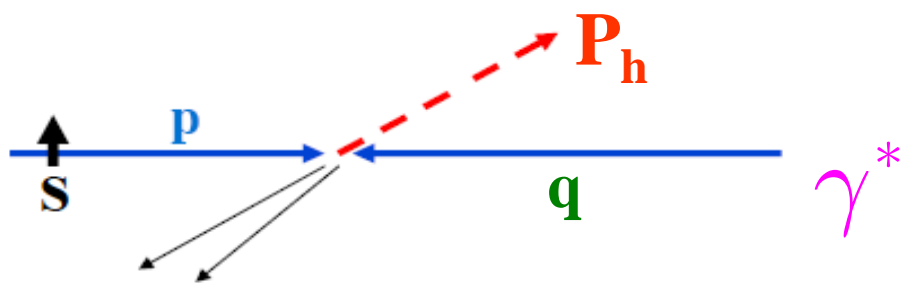
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at twist-3

1. $P_{h\perp} \neq 0$: k_\perp of quark or gluon
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SSA in **SIDIS** $ep^\uparrow \rightarrow eDX$

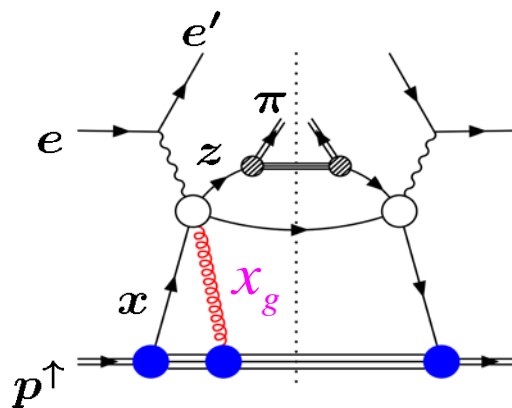
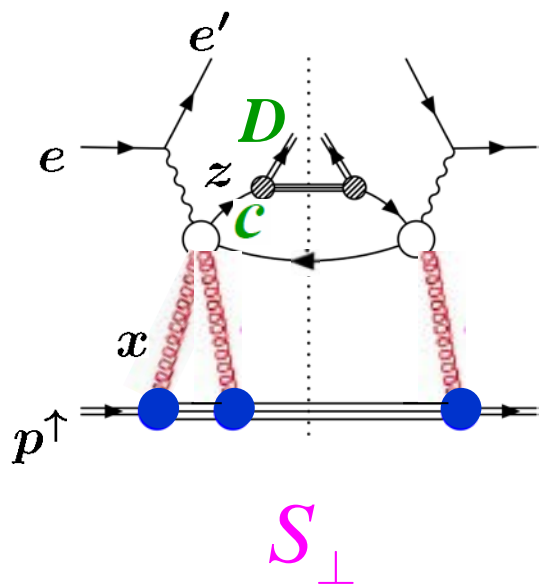


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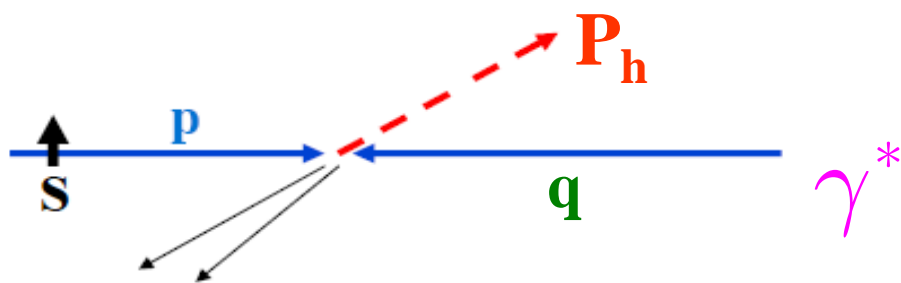
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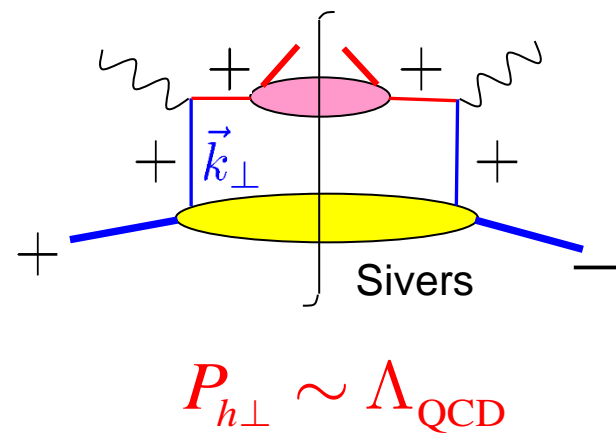
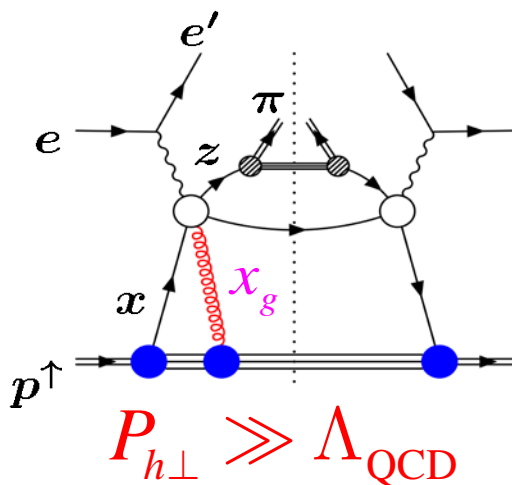
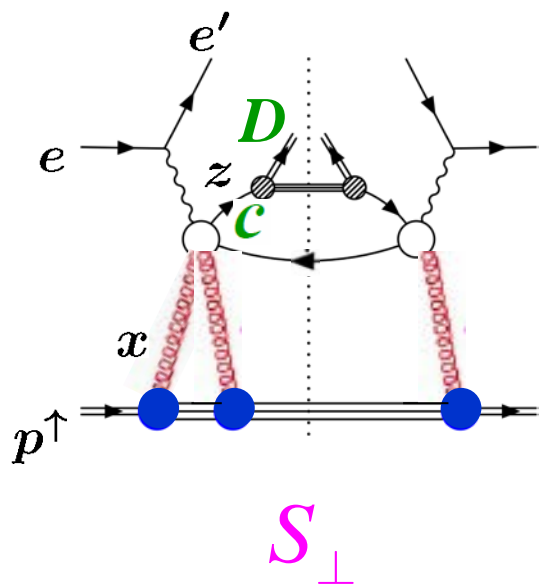


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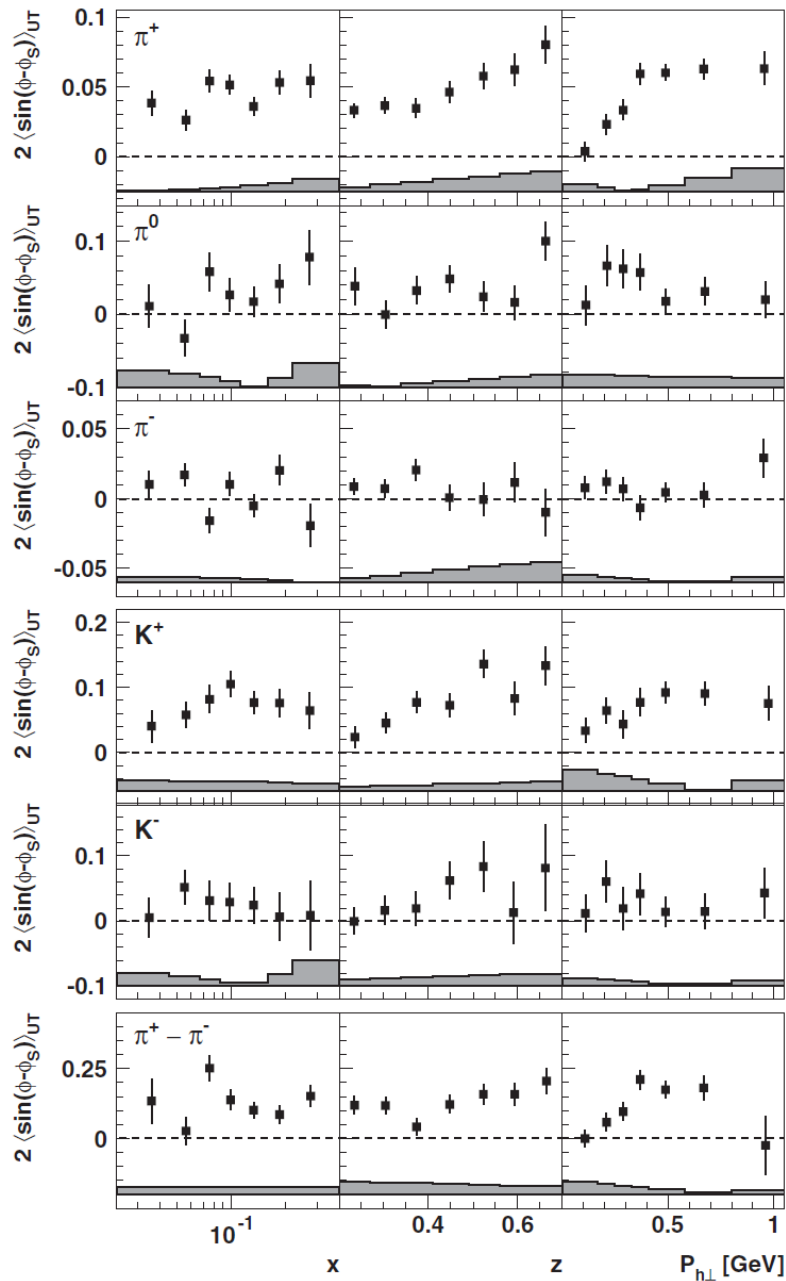
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at twist-3

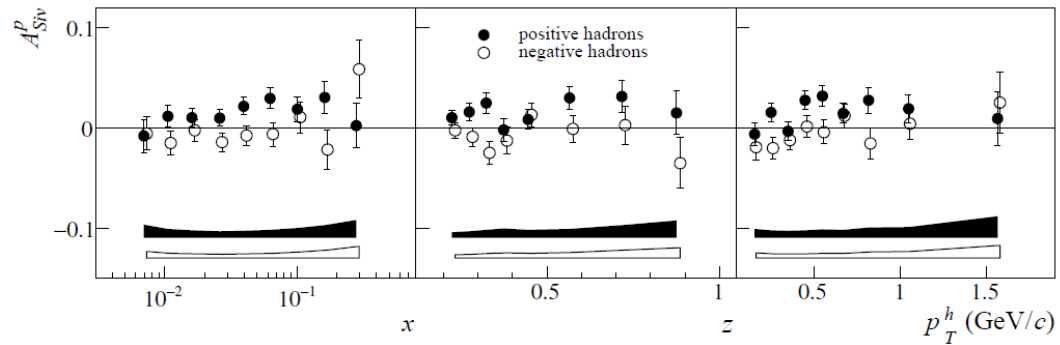
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Sivers asymmetry

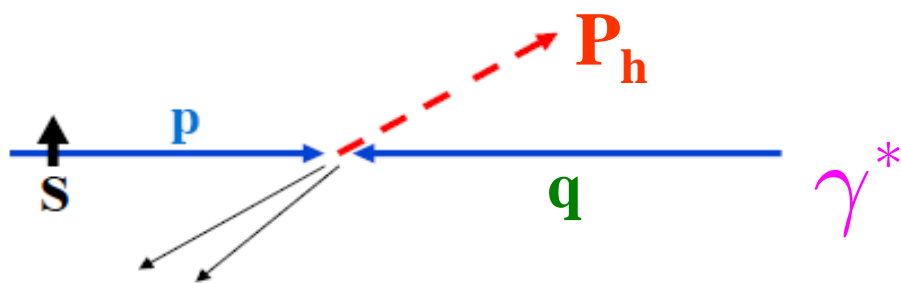


HERMES, PRL103 ('09) 152002



COMPASS, PLB692 ('10) 240

SSA in **SIDIS** $ep^\uparrow \rightarrow eDX$

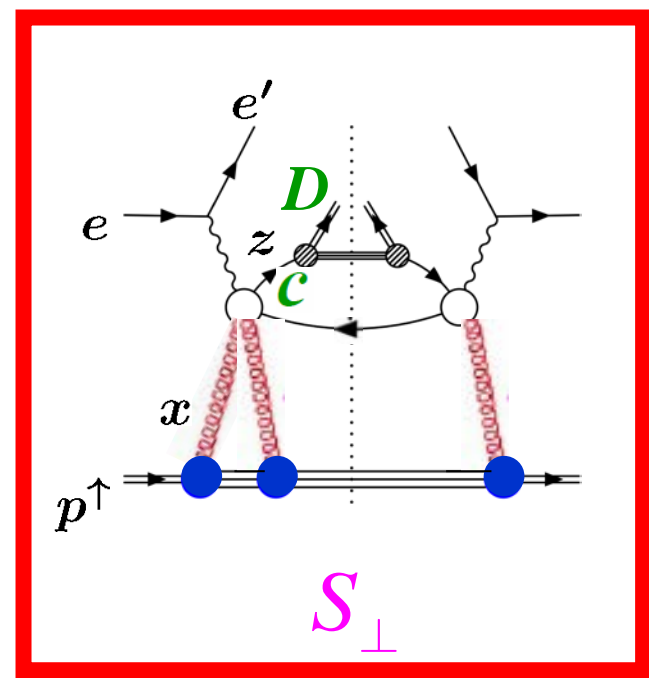


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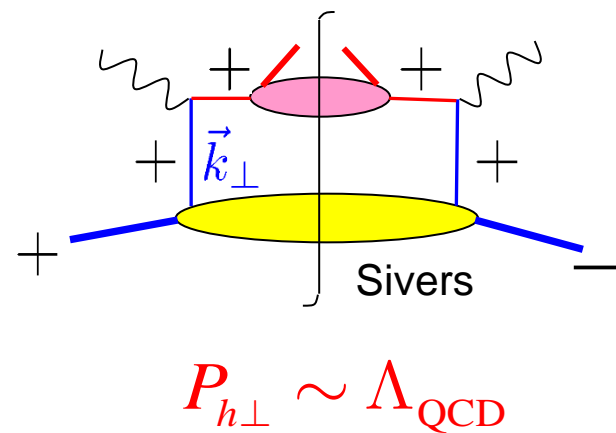
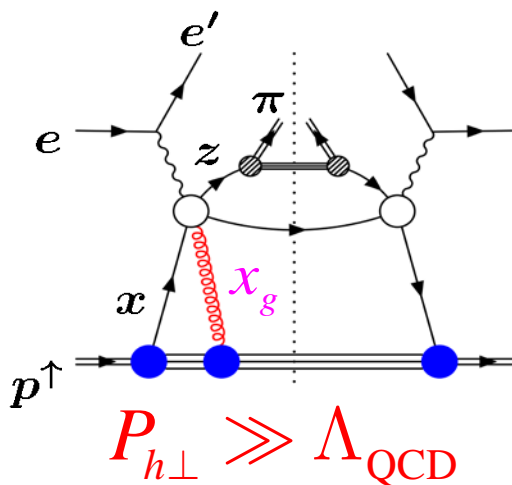
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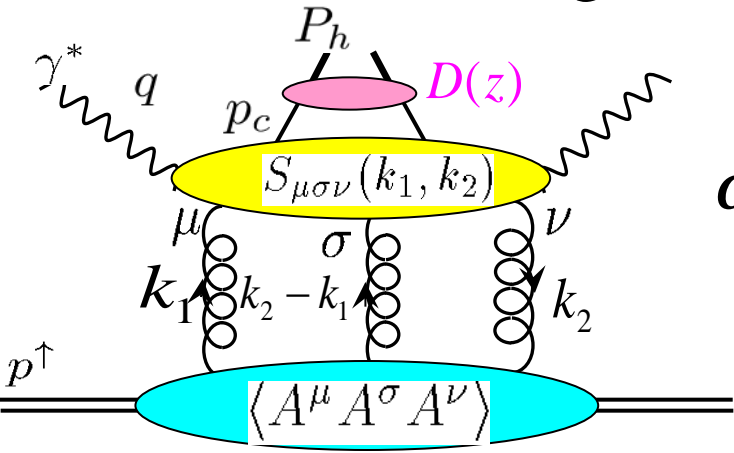


$$m_D \neq 0 \quad m_c \neq 0$$



SSA from multi-gluon correlation in SIDIS at $P_{h\perp} \gg \Lambda_{\text{QCD}}$

Beppu, Koike, K.T., Yoshida, PRD82 ('10) 054005



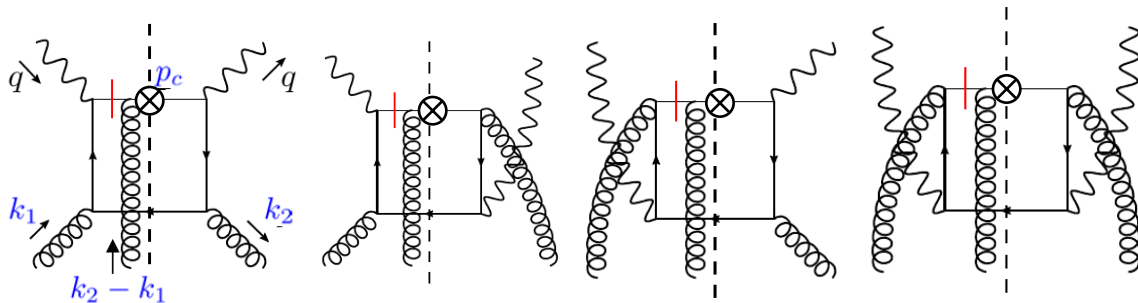
$$d\sigma_{\text{twist-3}} \sim \int dx_1 dx_2 dz \frac{\partial \mathbf{S}_{\mu\beta\nu}(k_1, k_2) p^\beta}{\partial k_{2\perp}^\sigma} \Big|_{k_i=x_i p} \otimes \left\{ \begin{array}{l} N_{\nu\mu\sigma}(x_1, x_2) \\ O_{\nu\mu\sigma}(x_1, x_2) \end{array} \right\} \otimes D(z)$$

$$i \int \frac{d\lambda}{2\pi} \int \frac{d\zeta}{2\pi} e^{i\lambda x_1} e^{i\mu(x_2-x_1)} \langle p \mathbf{S}_\perp | F_a^{\mu n}(0) g F_c^{\sigma n}(\zeta n) F_b^{\nu n}(\lambda n) | p \mathbf{S}_\perp \rangle$$

$$= \frac{1}{24} i f^{abc} N^{\nu\mu\sigma}(x_1, x_2) + \frac{3}{40} d^{abc} O^{\nu\mu\sigma}(x_1, x_2)$$

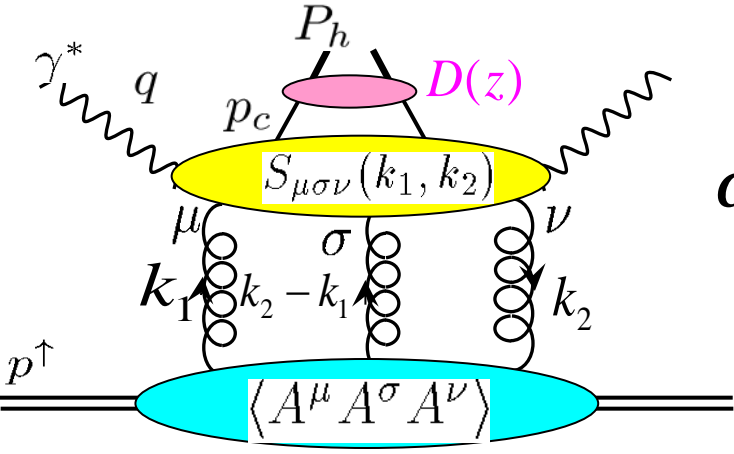
$$i \int \frac{d\lambda}{2\pi} \int \frac{d\zeta}{2\pi} e^{i\lambda x_1} e^{i\mu(x_2-x_1)} \langle p \mathbf{S}_\perp | F^{\mu n}(0) D_\perp^\sigma(\zeta n) F^{\nu n}(\lambda n) | p \mathbf{S}_\perp \rangle$$

$$D_\perp^\sigma = \partial_\perp^\sigma - ig A_\perp^\sigma$$



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Beppu, Koike, K.T., Yoshida, PRD82 ('10) 054005



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$$N^{\nu\mu\sigma}(x_1, x_2) = 2iM_N \left[g_\perp^{\nu\mu} S_{\perp\alpha} \varepsilon^{\sigma p n \alpha} N(x_1, x_2) - g_\perp^{\mu\sigma} S_{\perp\alpha} \varepsilon^{\nu p n \alpha} N(x_2, x_2 - x_1) - g_\perp^{\sigma\nu} S_{\perp\alpha} \varepsilon^{\mu p n \alpha} N(x_1, x_1 - x_2) \right]$$

$$O^{\nu\mu\sigma}(x_1, x_2) = 2iM_N \left[g_\perp^{\nu\mu} S_{\perp\alpha} \varepsilon^{\sigma p n \alpha} O(x_1, x_2) + g_\perp^{\mu\sigma} S_{\perp\alpha} \varepsilon^{\nu p n \alpha} O(x_2, x_2 - x_1) + g_\perp^{\sigma\nu} S_{\perp\alpha} \varepsilon^{\mu p n \alpha} O(x_1, x_1 - x_2) \right]$$

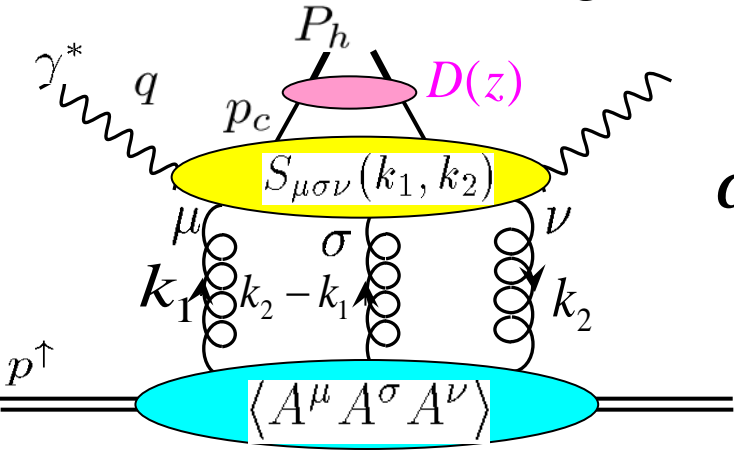
Permutation symmetry

$$g_\perp^{\mu\nu} \equiv g^{\mu\nu} - p^\mu n^\nu - n^\mu p^\nu$$

$$\varepsilon^{\sigma p n \alpha} \equiv \varepsilon^{\sigma \mu \nu \alpha} p_\mu n_\nu = \varepsilon^{\sigma - + \alpha}$$

SSA from multi-gluon correlation in SIDIS at $P_{h\perp} \gg \Lambda_{\text{QCD}}$

Beppu, Koike, K.T., Yoshida, PRD82 ('10) 054005



$$d\sigma_{\text{twist-3}} \sim \int dx_1 dx_2 dz \frac{\partial \mathbf{S}_{\mu\beta\nu}(k_1, k_2) p^\beta}{\partial k_{2\perp}^\sigma} \Big|_{k_i=x_i p} \otimes \begin{Bmatrix} N_{\nu\mu\sigma}(x_1, x_2) \\ O_{\nu\mu\sigma}(x_1, x_2) \end{Bmatrix} \otimes D(z)$$

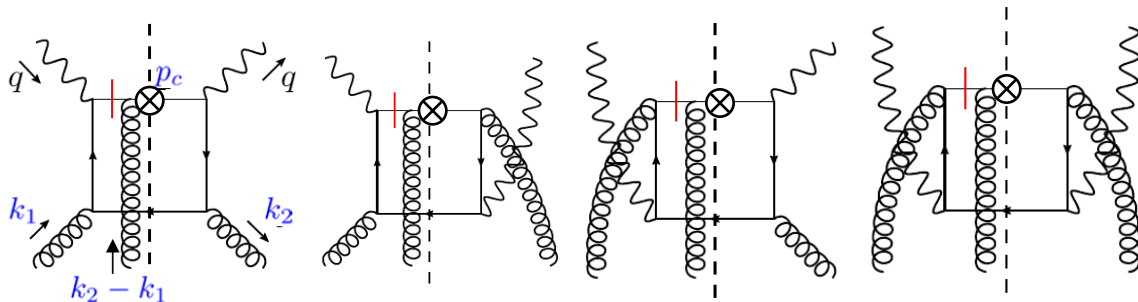
$$i \int \frac{d\lambda}{2\pi} \int \frac{d\zeta}{2\pi} e^{i\lambda x_1} e^{i\mu(x_2-x_1)} \langle p S_\perp | F_a^{\mu n}(0) g F_c^{\sigma n}(\zeta n) F_b^{\nu n}(\lambda n) | p S_\perp \rangle$$

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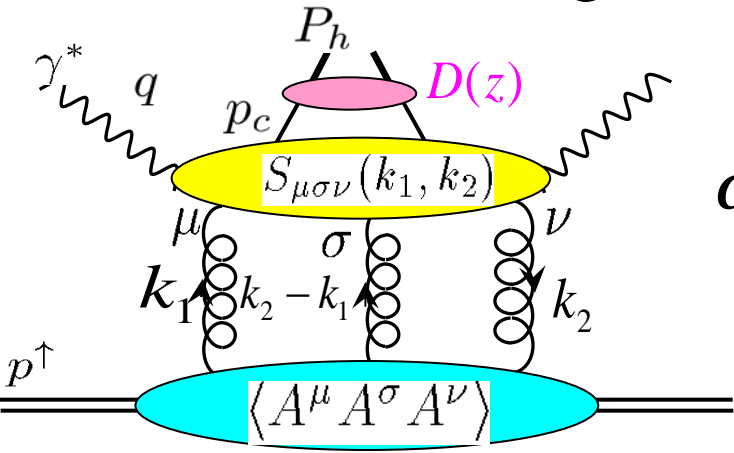


phase from parton propagator:

$$\frac{1}{k^2 - m_c^2 + i\varepsilon} = \text{P} \frac{1}{k^2 - m_c^2} - i\pi \delta(k^2 - m_c^2)$$

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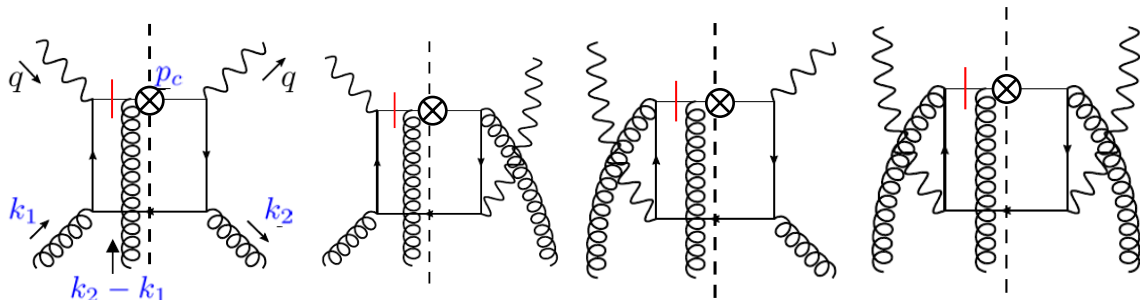
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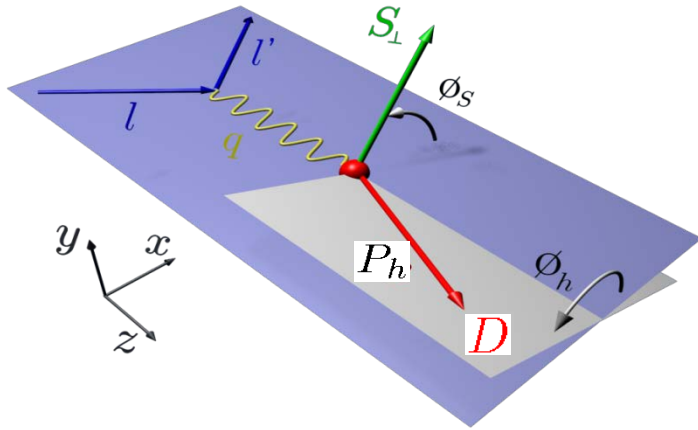
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★ Kinematics for $e(\ell) + p(p, S_{\perp}) \rightarrow e(\ell') + D(P_h) + X$



“Trento convention”

$$S_{ep} = (\ell + p)^2$$

$$q = \ell - \ell'$$

$$x_{bj} = \frac{Q^2}{2p \cdot q}$$

$$z_f = \frac{p \cdot P_h}{p \cdot q}$$

$P_{h\perp}$: \perp -mom. of final D

ϕ_h : azimuth. angle of hadron plane

ϕ_S : azimuth. angle of \vec{S}_{\perp}

● twist-2 unpol. cross section

$$\frac{d^5 \sigma_{\text{twist-2}}^{\text{unpol}}}{dQ^2 dx_{bj} dz_f dP_{h\perp}^2 d\phi_h} = \sigma_1^U + \sigma_2^U \cos(\phi_h) + \sigma_3^U \cos(2\phi_h)$$

Final form of twist-3 SSA for $ep^\uparrow \rightarrow eDX$

$$\frac{d^5 \sigma_{\text{twist-3}}}{dx_{bj} dQ^2 dz_f dP_{h\perp}^2 d\phi_h} = \frac{\alpha_{em}^2 \alpha_s e_c^2 M_N}{8\pi x_{bj}^2 z_f^2 S_{ep}^2 Q^2} \left(\frac{-\pi}{2}\right) \sum_{k=1, \dots, 4, 8, 9} \mathcal{A}_k \mathcal{S}_k \int \frac{dx}{x} \int \frac{dz}{z} \delta \left(\frac{P_{h\perp}^2}{z_f^2 Q^2} - \left(1 - \frac{1}{\hat{x}}\right) \left(1 - \frac{1}{\hat{z}}\right) + \frac{m_c^2}{\hat{z}^2 Q^2} \right)$$

$$\times \sum_{a=c, \bar{c}} D_a(z) \left[\delta_a \left\{ \left(\frac{dO(x, x)}{dx} - \frac{2O(x, x)}{x} \right) \Delta \hat{\sigma}_k^1 + \left(\frac{dO(x, 0)}{dx} - \frac{2O(x, 0)}{x} \right) \Delta \hat{\sigma}_k^2 + \frac{O(x, x)}{x} \Delta \hat{\sigma}_k^3 + \frac{O(x, 0)}{x} \Delta \hat{\sigma}_k^4 \right\} \right.$$

$$\left. + \left\{ \left(\frac{dN(x, x)}{dx} - \frac{2N(x, x)}{x} \right) \Delta \hat{\sigma}_k^1 - \left(\frac{dN(x, 0)}{dx} - \frac{2N(x, 0)}{x} \right) \Delta \hat{\sigma}_k^2 + \frac{N(x, x)}{x} \Delta \hat{\sigma}_k^3 - \frac{N(x, 0)}{x} \Delta \hat{\sigma}_k^4 \right\} \right]$$

$$\mathcal{A}_1 = 1 + \cosh^2 \psi, \quad \mathcal{A}_2 = -2, \quad \mathcal{A}_3 = -\cos \phi_h \sinh 2\psi, \quad \mathcal{A}_4 = \cos 2\phi_h \sinh^2 \psi,$$

$$\mathcal{A}_8 = -\sin \phi_h \sinh 2\psi, \quad \mathcal{A}_9 = \sin 2\phi_h \sinh^2 \psi.$$

$$\mathcal{S}_k = \sin(\phi_h - \phi_S) \quad (k = 1, 2, 3, 4), \quad \mathcal{S}_k = \cos(\phi_h - \phi_S) \quad (k = 8, 9)$$

$$\delta_c = 1, \quad \delta_{\bar{c}} = -1$$

$$\cosh \psi = \frac{2x_{bj} S_{ep} - 1}{Q^2}$$

$$\hat{x} = \frac{x_{bj}}{x}, \quad \hat{z} = \frac{z_f}{z}$$

- Receives contribution from 4 functions $O(x, x)$, $O(x, 0)$, $N(x, x)$ and $N(x, 0)$, and all of them contribute both as derivative and nonderivative terms.

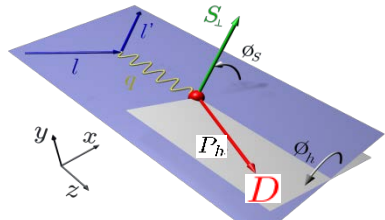
$$\Delta \hat{\sigma}_k^1 \neq \Delta \hat{\sigma}_k^2$$

- 5 structure functions with different azimuthal dependence

$$\Delta \hat{\sigma}_k^3 \neq \Delta \hat{\sigma}_k^4$$

similar as $ep^\uparrow \rightarrow e\pi X$ from quark-gluon correlation

$$= \sin(\phi_h - \phi_S) [\mathcal{F}_1 + \mathcal{F}_2 \cos \phi_h + \mathcal{F}_3 \cos 2\phi_h] + \cos(\phi_h - \phi_S) [\mathcal{F}_4 \sin \phi_h + \mathcal{F}_5 \sin 2\phi_h]$$



$$\Delta\hat{\sigma}_1^1 = \frac{8q_T\hat{x}}{Q^6(1-\hat{z})^3\hat{z}^2} \{Q^4\hat{z}(1-\hat{z})(1-2\hat{z}+2\hat{z}^2-2\hat{x}+2\hat{x}^2+12\hat{x}\hat{z}(1-\hat{x})(1-\hat{z})) \\ + 2m_c^2Q^2\hat{x}(2\hat{z}(1-\hat{z})+\hat{x}(1-8\hat{z}+8\hat{z}^2))-4m_c^4\hat{x}^2\},$$

$$\Delta\hat{\sigma}_2^1 = \frac{64q_T\hat{x}^2}{Q^4(1-\hat{z})^2\hat{z}} \{Q^2\hat{z}(1-\hat{x})(1-\hat{z})-m_c^2\hat{x}\},$$

$$\Delta\hat{\sigma}_3^1 = \frac{16\hat{x}}{Q^5(1-\hat{z})^3\hat{z}^3} (1-2\hat{z})\{Q^2\hat{z}(1-\hat{x})(1-\hat{z})-m_c^2\hat{x}\}\{Q^2\hat{z}(1-2\hat{x})(1-\hat{z})-2m_c^2\hat{x}\},$$

$$\Delta\hat{\sigma}_4^1 = \frac{32q_T\hat{x}^2}{Q^6(1-\hat{z})^3\hat{z}^2} \{Q^2\hat{z}(1-\hat{x})(1-\hat{z})-m_c^2\hat{x}\}\{Q^2\hat{z}(1-\hat{z})+m_c^2\}, \quad \Delta\hat{\sigma}_8^1 = \Delta\hat{\sigma}_9^1 = 0,$$

$$\hat{x} = \frac{x_{bj}}{x}, \quad \hat{z} = \frac{z_f}{z}$$

$$q_T \equiv \frac{P_{h\perp}}{z_f}$$

$$\Delta\hat{\sigma}_1^2 = \frac{8q_T\hat{x}}{Q^6(1-\hat{z})^3\hat{z}^2} \{Q^4\hat{z}(1-\hat{z})(1-2\hat{z}+2\hat{z}^2-4\hat{x}+4\hat{x}^2+24\hat{x}\hat{z}(1-\hat{x})(1-\hat{z})) \\ + 4m_c^2Q^2\hat{x}(2\hat{z}(1-\hat{z})+\hat{x}(1-8\hat{z}+8\hat{z}^2))-8m_c^4\hat{x}^2\},$$

$$\Delta\hat{\sigma}_2^2 = 2\Delta\hat{\sigma}_1^2,$$

$$\Delta\hat{\sigma}_3^2 = 2\Delta\hat{\sigma}_3^1,$$

$$\Delta\hat{\sigma}_4^2 = -\frac{16q_T\hat{x}}{Q^6(1-\hat{z})^3\hat{z}^2} (Q^2\hat{z}(1-2\hat{x})(1-\hat{z})-2m_c^2\hat{x})^2,$$

$$\Delta\hat{\sigma}_8^2 = \frac{16\hat{x}}{Q^3(1-\hat{z})^2\hat{z}^2} (1-2\hat{z})(Q^2\hat{z}(1-\hat{x})(1-\hat{z})-m_c^2\hat{x}),$$

$$\Delta\hat{\sigma}_9^2 = -\frac{16q_T\hat{x}}{Q^4(1-\hat{z})^2\hat{z}} (Q^2\hat{z}(1-2\hat{x})(1-\hat{z})-2m_c^2\hat{x}),$$

$$\Delta\hat{\sigma}_1^3 = \frac{16q_T\hat{x}^2}{Q^6(1-\hat{z})^3\hat{z}^2} (Q^2\hat{z}(1-2\hat{x})(1-\hat{z})-2m_c^2\hat{x})(Q^2(1-6\hat{z}+6\hat{z}^2)-2m_c^2),$$

$$\Delta\hat{\sigma}_2^3 = -\frac{64q_T\hat{x}^2}{Q^4(1-\hat{z})^2\hat{z}} (Q^2\hat{z}(1-2\hat{x})(1-\hat{z})-2m_c^2\hat{x}),$$

$$\Delta\hat{\sigma}_3^3 = -\frac{8\hat{x}}{Q^5(1-\hat{z})^3\hat{z}^3} (1-2\hat{z})\{Q^4\hat{z}^2(1-\hat{z})^2(1-8\hat{x}+8\hat{x}^2)-8m_c^2Q^2\hat{x}\hat{z}(1-2\hat{x})(1-\hat{z})+8m_c^4\hat{x}^2\},$$

$$\Delta\hat{\sigma}_4^3 = -\frac{32q_T\hat{x}^2}{Q^6(1-\hat{z})^3\hat{z}^2} (Q^2\hat{z}(1-\hat{z})+m_c^2)(Q^2\hat{z}(1-2\hat{x})(1-\hat{z})-2m_c^2\hat{x}),$$

$$\Delta\hat{\sigma}_8^3 = -\frac{8\hat{x}}{Q^3(1-\hat{z})^2\hat{z}^2} (1-2\hat{z})(Q^2\hat{z}(1-2\hat{x})(1-\hat{z})-2m_c^2\hat{x}),$$

$$\Delta\hat{\sigma}_9^3 = -\frac{32q_T\hat{x}^2}{Q^4(1-\hat{z})^2\hat{z}} (Q^2\hat{z}(1-\hat{z})+m_c^2),$$

$$\Delta\hat{\sigma}_1^4 = \frac{16q_T\hat{x}^2}{Q^6(1-\hat{z})^3\hat{z}^2} (Q^2\hat{z}(1-4\hat{x})(1-\hat{z})-4m_c^2\hat{x})(Q^2(1-6\hat{z}+6\hat{z}^2)-2m_c^2),$$

$$\Delta\hat{\sigma}_2^4 = -\frac{64q_T\hat{x}^2}{Q^4(1-\hat{z})^2\hat{z}} (Q^2\hat{z}(1-4\hat{x})(1-\hat{z})-4m_c^2\hat{x}),$$

$$\Delta\hat{\sigma}_3^4 = \frac{8\hat{x}}{Q^5(1-\hat{z})^3\hat{z}^3} (1-2\hat{z})\{Q^4\hat{z}^2(1-\hat{z})^2(1+12\hat{x}-16\hat{x}^2)+4m_c^2Q^2\hat{x}\hat{z}(3-8\hat{x})(1-\hat{z})-16m_c^4\hat{x}^2\},$$

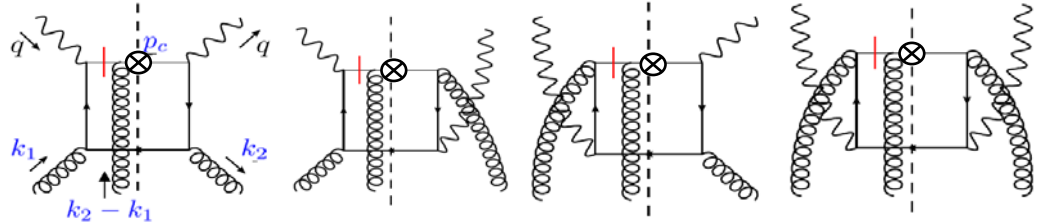
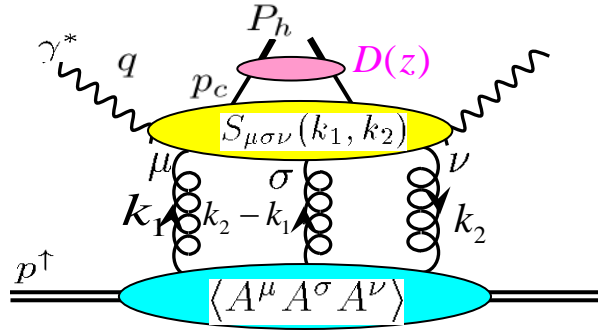
$$\Delta\hat{\sigma}_4^4 = -\frac{32q_T\hat{x}}{Q^6(1-\hat{z})^3\hat{z}^2} (Q^4\hat{z}^2(1-\hat{z})^2(1+\hat{x}-4\hat{x}^2)+m_c^2Q^2\hat{x}\hat{z}(1-8\hat{x})(1-\hat{z})-4m_c^4\hat{x}^2),$$

$$\Delta\hat{\sigma}_8^4 = \frac{8\hat{x}}{Q^3(1-\hat{z})^2\hat{z}^2} (1-2\hat{z})(Q^2\hat{z}(1+2\hat{x})(1-\hat{z})+2m_c^2\hat{x}),$$

$$\Delta\hat{\sigma}_9^4 = -\frac{32q_T\hat{x}}{Q^4(1-\hat{z})^2\hat{z}} (Q^2\hat{z}(1-\hat{z})(1+\hat{x})+m_c^2\hat{x}),$$

$$\left(m_c \rightarrow 0 \text{ for } ep^\uparrow \rightarrow e\pi X \text{ case}\right)$$

$$d\sigma_{\text{twist-3}} \sim \int dx_1 dx_2 dz \left. \frac{\partial S_{\mu\beta\nu}(k_1, k_2) p^\beta}{\partial k_{2\perp}^\sigma} \right|_{k_i = x_i p} \otimes \left\{ \begin{array}{l} N_{\nu\mu\sigma}(x_1, x_2) \\ O_{\nu\mu\sigma}(x_1, x_2) \end{array} \right\} \otimes D(z)$$

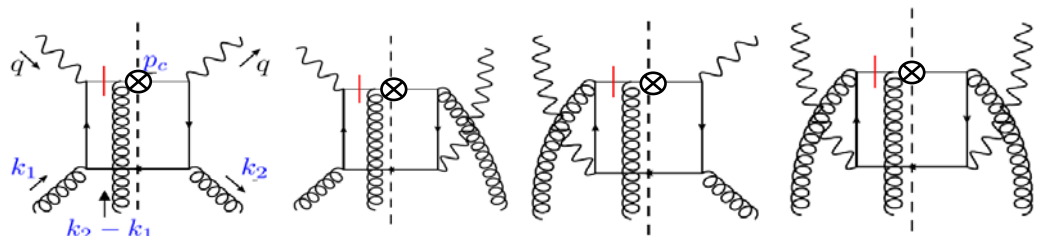
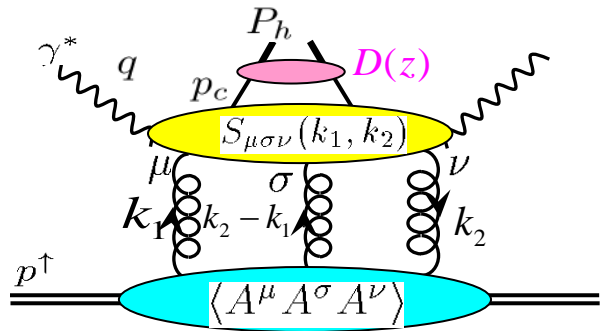


SGP $k_2 - k_1 = 0$

$$N^{\nu\mu\sigma}(x_1, x_2) = 2iM_N \left[g_\perp^{\nu\mu} S_{\perp\alpha} \varepsilon^{\sigma p n \alpha} N(x_1, x_2) - g_\perp^{\mu\sigma} S_{\perp\alpha} \varepsilon^{\nu p n \alpha} N(x_2, x_2 - x_1) - g_\perp^{\sigma\nu} S_{\perp\alpha} \varepsilon^{\mu p n \alpha} N(x_1, x_1 - x_2) \right]$$

$$O^{\nu\mu\sigma}(x_1, x_2) = 2iM_N \left[g_\perp^{\nu\mu} S_{\perp\alpha} \varepsilon^{\sigma p n \alpha} O(x_1, x_2) + g_\perp^{\mu\sigma} S_{\perp\alpha} \varepsilon^{\nu p n \alpha} O(x_2, x_2 - x_1) + g_\perp^{\sigma\nu} S_{\perp\alpha} \varepsilon^{\mu p n \alpha} O(x_1, x_1 - x_2) \right]$$

$$d\sigma_{\text{twist-3}} \sim \int dx_1 dx_2 dz \left. \frac{\partial S_{\mu\beta\nu}(k_1, k_2) p^\beta}{\partial k_{2\perp}^\sigma} \right|_{k_i = x_i p} \otimes \left\{ \begin{array}{l} N_{\nu\mu\sigma}(x_1, x_2) \\ O_{\nu\mu\sigma}(x_1, x_2) \end{array} \right\} \otimes D(z)$$



SGP $k_2 - k_1 = 0$

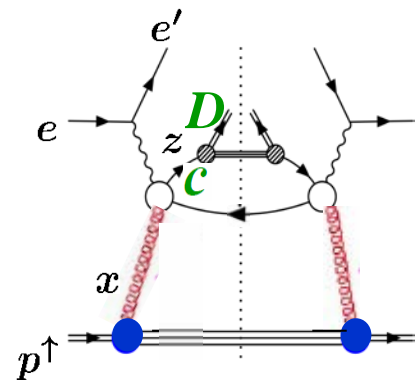
$$N^{\nu\mu\sigma}(x, x) = 2iM_N \left[g_\perp^{\nu\mu} S_{\perp\alpha} \varepsilon^{\sigma p n \alpha} N(x, x) - g_\perp^{\mu\sigma} S_{\perp\alpha} \varepsilon^{\nu p n \alpha} N(x, 0) - g_\perp^{\sigma\nu} S_{\perp\alpha} \varepsilon^{\mu p n \alpha} N(x, 0) \right]$$

$$O^{\nu\mu\sigma}(x, x) = 2iM_N \left[g_\perp^{\nu\mu} S_{\perp\alpha} \varepsilon^{\sigma p n \alpha} O(x, x) + g_\perp^{\mu\sigma} S_{\perp\alpha} \varepsilon^{\nu p n \alpha} O(x, 0) + g_\perp^{\sigma\nu} S_{\perp\alpha} \varepsilon^{\mu p n \alpha} O(x, 0) \right]$$

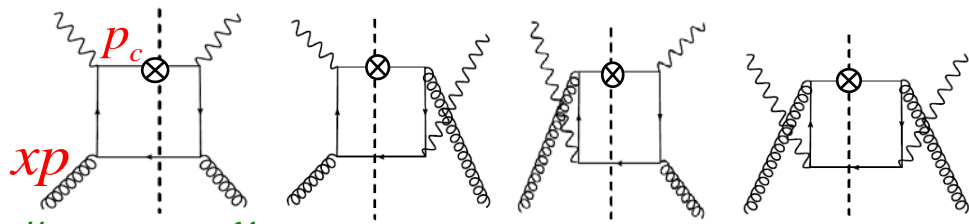
Master formula

Koike, K.T., Yoshida, PRD83('11) 114014

$$d\sigma_{\text{twist-3}} \sim -i\pi \int dx dz \frac{\partial \mathcal{H}_{\mu\nu}(xp, p_c)}{\partial p_c^\sigma} \otimes \left\{ \begin{array}{l} N_{\nu\mu\sigma}(x, x) \\ O_{\nu\mu\sigma}(x, x) \end{array} \right\} \otimes D(z)$$



$$\mathcal{H}_{\mu\nu}(xp, p_c) =$$



2→2 Born subprocess

Final form of twist-3 SSA for $ep^\uparrow \rightarrow eDX$

$$\frac{d^5 \sigma_{\text{twist-3}}}{dx_{bj} dQ^2 dz_f dP_{h\perp}^2 d\phi_h} = \frac{\alpha_{em}^2 \alpha_s e_c^2 M_N}{8\pi x_{bj}^2 z_f^2 S_{ep}^2 Q^2} \left(\frac{-\pi}{2}\right) \sum_{k=1, \dots, 4, 8, 9} \mathcal{A}_k \mathcal{S}_k \int \frac{dx}{x} \int \frac{dz}{z} \delta\left(\frac{P_{h\perp}^2}{z_f^2 Q^2} - \left(1 - \frac{1}{\hat{x}}\right) \left(1 - \frac{1}{\hat{z}}\right) + \frac{m_c^2}{\hat{z}^2 Q^2}\right)$$

$$\times \sum_{a=c, \bar{c}} D_a(z) \left[\delta_a \left\{ \left(\frac{dO(x, x)}{dx} - \frac{2O(x, x)}{x}\right) \Delta \hat{\sigma}_k^1 + \left(\frac{dO(x, 0)}{dx} - \frac{2O(x, 0)}{x}\right) \Delta \hat{\sigma}_k^2 + \frac{O(x, x)}{x} \Delta \hat{\sigma}_k^3 + \frac{O(x, 0)}{x} \Delta \hat{\sigma}_k^4 \right\} \right.$$

$$\left. + \left\{ \left(\frac{dN(x, x)}{dx} - \frac{2N(x, x)}{x}\right) \Delta \hat{\sigma}_k^1 - \left(\frac{dN(x, 0)}{dx} - \frac{2N(x, 0)}{x}\right) \Delta \hat{\sigma}_k^2 + \frac{N(x, x)}{x} \Delta \hat{\sigma}_k^3 - \frac{N(x, 0)}{x} \Delta \hat{\sigma}_k^4 \right\} \right]$$

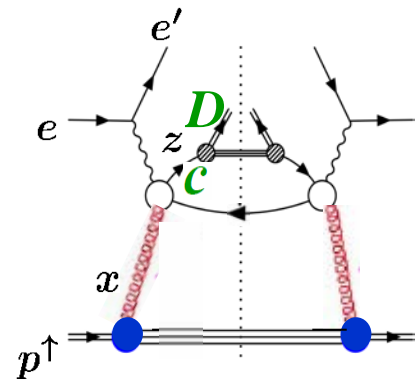
$$\Delta \hat{\sigma}_k^1 = \frac{2P_{h\perp} \hat{x}}{Q^2 z_f (1 - \hat{z})} \hat{\sigma}_k^U, \quad \Delta \hat{\sigma}_k^3 = \frac{2P_{h\perp}}{z_f Q^2} \left(\frac{z_f^2 Q^2}{\hat{z}} \frac{\partial \hat{\sigma}_k^U}{\partial P_{h\perp}^2} - \frac{\hat{x}^2}{1 - \hat{z}} \frac{\partial \hat{\sigma}_k^U}{\partial \hat{x}} \right) - \frac{2^{k-8} z_f \hat{\sigma}_{k-5}^U}{\hat{z} P_{h\perp}}$$

$$\Delta \hat{\sigma}_k^{2,4} \longleftarrow \mathcal{H}_{11}(xp, p_c), \mathcal{H}_{22}(xp, p_c), \mathcal{H}_{12}(xp, p_c) + \mathcal{H}_{21}(xp, p_c)$$

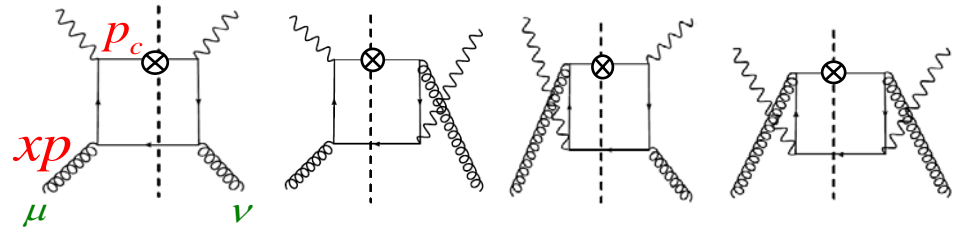
twist-2 unpol. cross section

$$\frac{d^5 \sigma_{\text{twist-2}}^{\text{unpol}}}{dx_{bj} dQ^2 dz_f dP_{h\perp}^2 d\phi_h} = \frac{\alpha_{em}^2 \alpha_s e_c^2}{8\pi x_{bj}^2 z_f^2 S_{ep}^2 Q^2} \frac{1}{4} \sum_{k=1}^4 \mathcal{A}_k \int \frac{dx}{x} \int \frac{dz}{z} \delta\left(\frac{P_{h\perp}^2}{z_f^2 Q^2} - \left(1 - \frac{1}{\hat{x}}\right) \left(1 - \frac{1}{\hat{z}}\right) + \frac{m_c^2}{\hat{z}^2 Q^2}\right)$$

$$\times \sum_{a=c, \bar{c}} D_a(z) g(x) \hat{\sigma}_k^U \quad \hat{\sigma}_k^U \sim g_{\perp}^{\nu\mu} \mathcal{H}_{\mu\nu}(xp, p_c)$$



$$\mathcal{H}_{\mu\nu}(xp, p_c) =$$



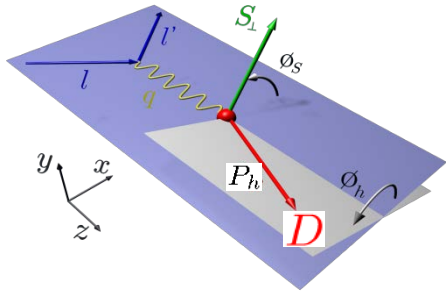
Estimates of twist-3 SSA for $ep^\uparrow \rightarrow eD^0 X$

$$\begin{aligned} \frac{d^5 \sigma_{\text{twist-3}}}{dx_{bj} dQ^2 dz_f dP_{h\perp}^2 d\phi_h} &= \frac{\alpha_{em}^2 \alpha_s e_c^2 M_N}{8\pi x_{bj}^2 z_f^2 S_{ep}^2 Q^2} \left(\frac{-\pi}{2} \right) \sum_{k=1, \dots, 4, 8, 9} \mathcal{A}_k \mathcal{S}_k \int \frac{dx}{x} \int \frac{dz}{z} \delta \left(\frac{P_{h\perp}^2}{z_f^2 Q^2} - \left(1 - \frac{1}{\hat{x}}\right) \left(1 - \frac{1}{\hat{z}}\right) + \frac{m_c^2}{\hat{z}^2 Q^2} \right) \\ &\times \sum_{a=c, \bar{c}} D_a(z) \left[\delta_a \left\{ \left(\frac{dO(x, x)}{dx} - \frac{2O(x, x)}{x} \right) \Delta \hat{\sigma}_k^1 + \left(\frac{dO(x, 0)}{dx} - \frac{2O(x, 0)}{x} \right) \Delta \hat{\sigma}_k^2 + \frac{O(x, x)}{x} \Delta \hat{\sigma}_k^3 + \frac{O(x, 0)}{x} \Delta \hat{\sigma}_k^4 \right\} \right. \\ &\quad \left. + \left\{ \left(\frac{dN(x, x)}{dx} - \frac{2N(x, x)}{x} \right) \Delta \hat{\sigma}_k^1 - \left(\frac{dN(x, 0)}{dx} - \frac{2N(x, 0)}{x} \right) \Delta \hat{\sigma}_k^2 + \frac{N(x, x)}{x} \Delta \hat{\sigma}_k^3 - \frac{N(x, 0)}{x} \Delta \hat{\sigma}_k^4 \right\} \right] \end{aligned}$$

$$= \sin(\phi_h - \phi_S) [\mathcal{F}_1 + \mathcal{F}_2 \cos \phi_h + \mathcal{F}_3 \cos 2\phi_h] + \cos(\phi_h - \phi_S) [\mathcal{F}_4 \sin \phi_h + \mathcal{F}_5 \sin 2\phi_h]$$

$$\mathbf{A}_N : \frac{\mathcal{F}_1}{\sigma_1^U}, \frac{\mathcal{F}_{2,3,4,5}}{2\sigma_1^U} \left(\frac{d^5 \sigma_{\text{twist-2}}^{\text{unpol}}}{dQ^2 dx_{bj} dz_f dP_{h\perp}^2 d\phi_h} = \sigma_1^U + \sigma_2^U \cos \phi_h + \sigma_3^U \cos 2\phi_h \right)$$

$$\langle 1 \rangle, \langle \cos \phi \rangle, \langle \cos 2\phi \rangle, \langle \sin \phi \rangle, \langle \sin 2\phi \rangle$$



Estimates of twist-3 SSA for $ep^\uparrow \rightarrow eD^0 X$

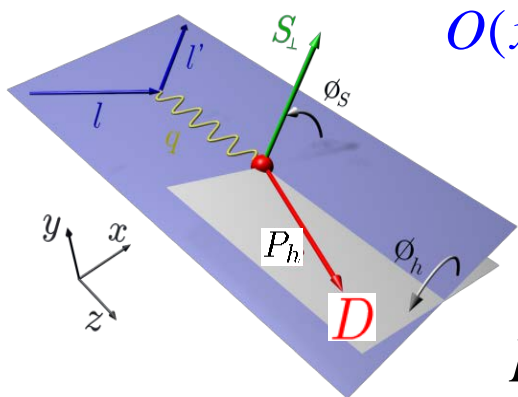
$$\frac{d^5 \sigma_{\text{twist-3}}}{dx_{bj} dQ^2 dz_f dP_{h\perp}^2 d\phi_h} = \frac{\alpha_{em}^2 \alpha_s e_c^2 M_N}{8\pi x_{bj}^2 z_f^2 S_{ep}^2 Q^2} \left(\frac{-\pi}{2}\right) \sum_{k=1, \dots, 4, 8, 9} \mathcal{A}_k \mathcal{S}_k \int \frac{dx}{x} \int \frac{dz}{z} \delta\left(\frac{P_{h\perp}^2}{z_f^2 Q^2} - \left(1 - \frac{1}{\hat{x}}\right) \left(1 - \frac{1}{\hat{z}}\right) + \frac{m_c^2}{\hat{z}^2 Q^2}\right)$$

$$\times \sum_{a=c, \bar{c}} D_a(z) \left[\delta_a \left\{ \left(\frac{dO(x, x)}{dx} - \frac{2O(x, x)}{x}\right) \Delta \hat{\sigma}_k^1 + \left(\frac{dO(x, 0)}{dx} - \frac{2O(x, 0)}{x}\right) \Delta \hat{\sigma}_k^2 + \frac{O(x, x)}{x} \Delta \hat{\sigma}_k^3 + \frac{O(x, 0)}{x} \Delta \hat{\sigma}_k^4 \right\} \right.$$

$$\left. + \left\{ \left(\frac{dN(x, x)}{dx} - \frac{2N(x, x)}{x}\right) \Delta \hat{\sigma}_k^1 - \left(\frac{dN(x, 0)}{dx} - \frac{2N(x, 0)}{x}\right) \Delta \hat{\sigma}_k^2 + \frac{N(x, x)}{x} \Delta \hat{\sigma}_k^3 - \frac{N(x, 0)}{x} \Delta \hat{\sigma}_k^4 \right\} \right]$$

$$= \sin(\phi_h - \phi_S) [\mathcal{F}_1 + \mathcal{F}_2 \cos \phi_h + \mathcal{F}_3 \cos 2\phi_h] + \cos(\phi_h - \phi_S) [\mathcal{F}_4 \sin \phi_h + \mathcal{F}_5 \sin 2\phi_h]$$

$$A_N : \frac{\mathcal{F}_1}{\sigma_1^U}, \frac{\mathcal{F}_{2,3,4,5}}{2\sigma_1^U} \left(\frac{d^5 \sigma_{\text{twist-2}}^{\text{unpol}}}{dQ^2 dx_{bj} dz_f dP_{h\perp}^2 d\phi_h} = \sigma_1^U + \sigma_2^U \cos \phi_h + \sigma_3^U \cos 2\phi_h \right)$$

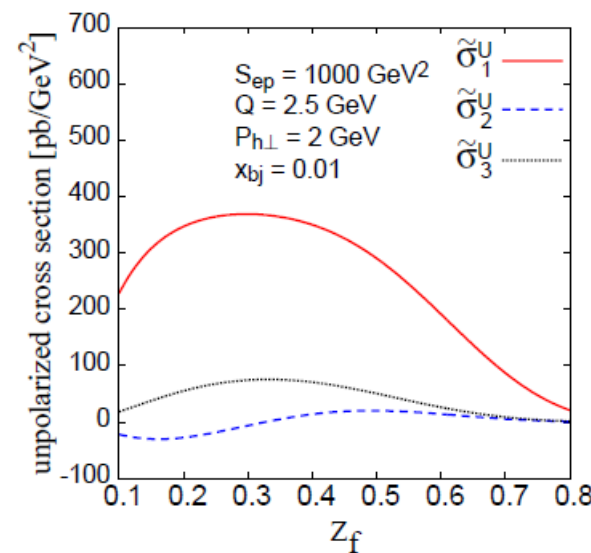
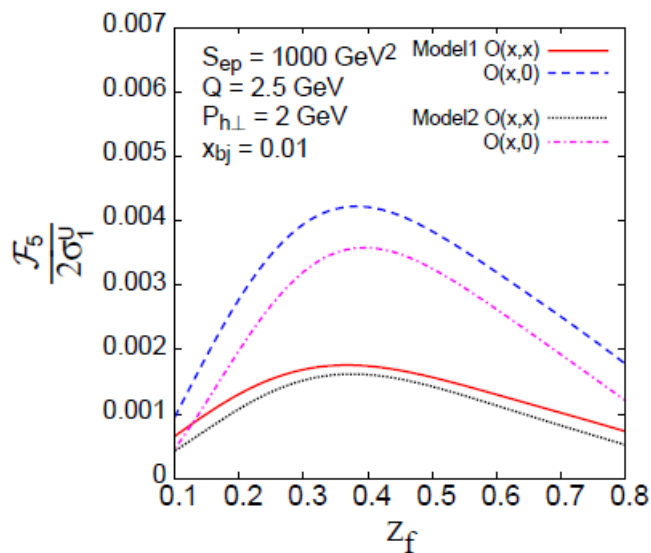
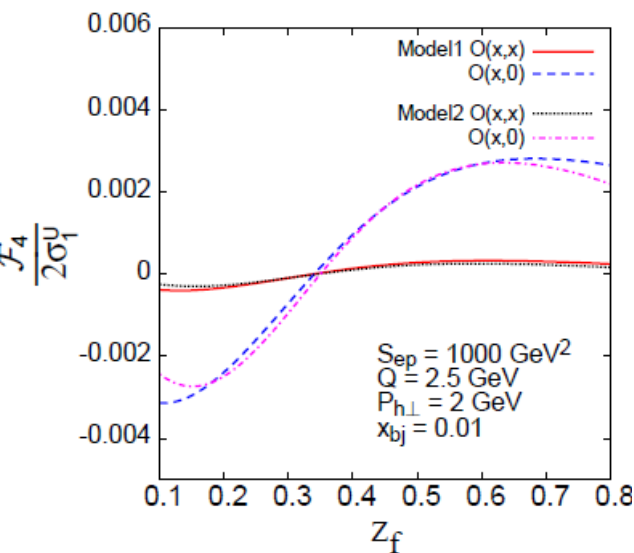
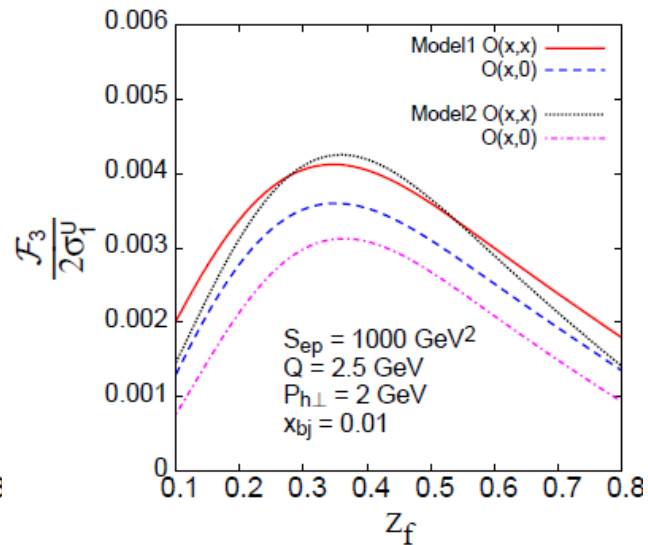
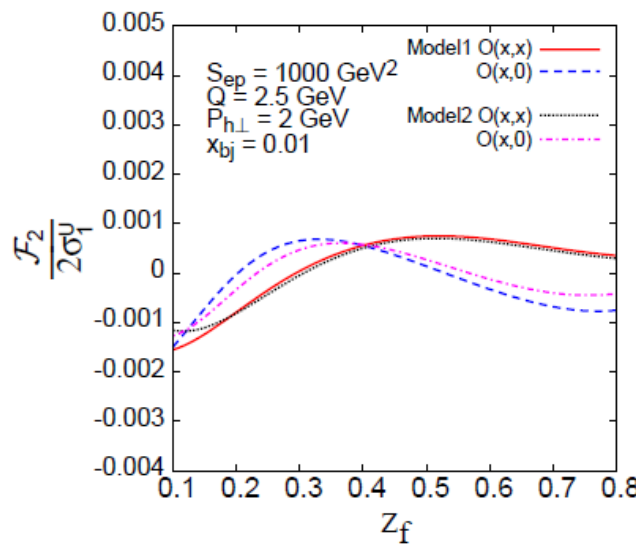
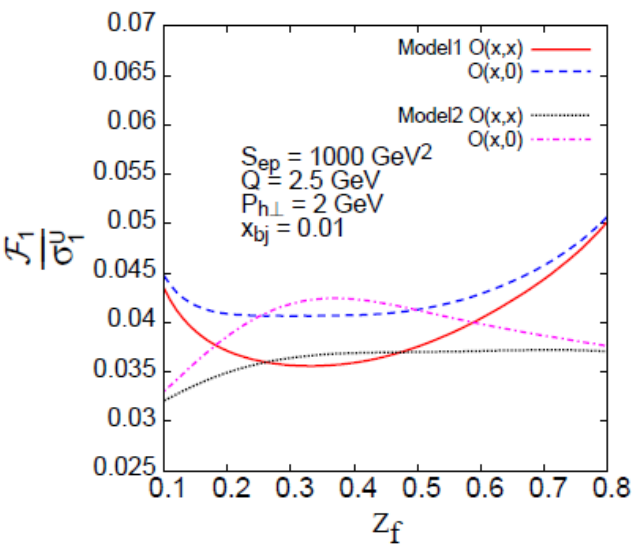


$$O(x, x) = O(x, 0) = \begin{cases} 0.004 x g(x) & \text{Model1} \\ 0.001 \sqrt{x} g(x) & \text{Model2} \end{cases}$$

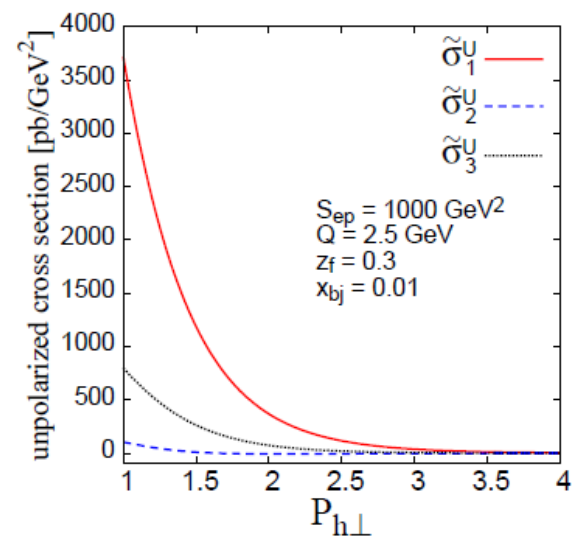
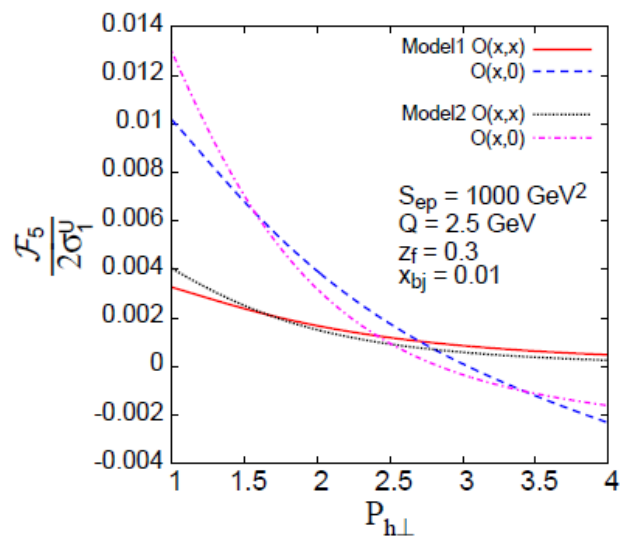
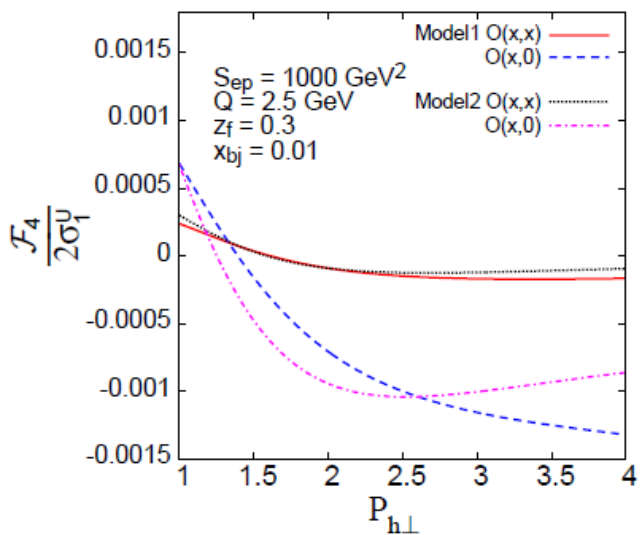
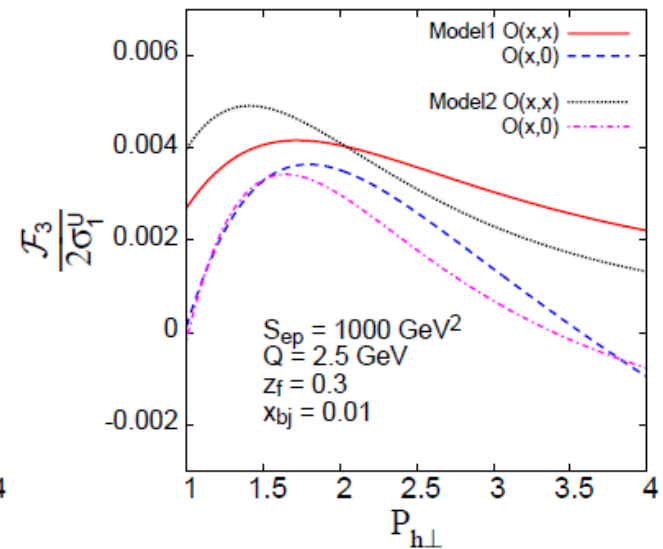
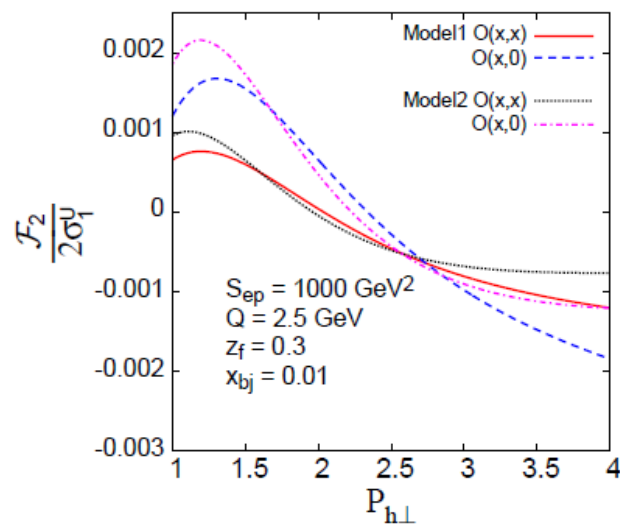
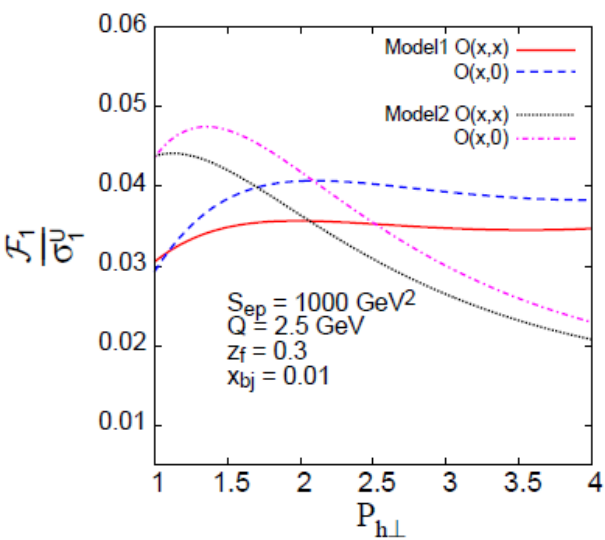
$$g(x, \mu) : \text{CTEQ6L} \quad \mu = \sqrt{Q^2 + m_c^2 + P_{h\perp}^2}$$

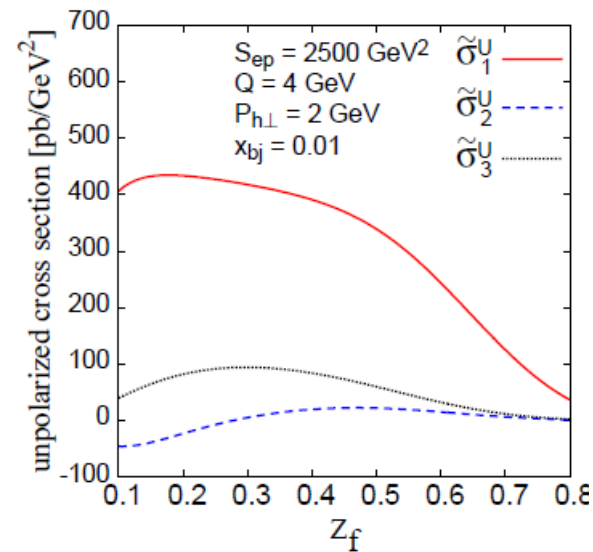
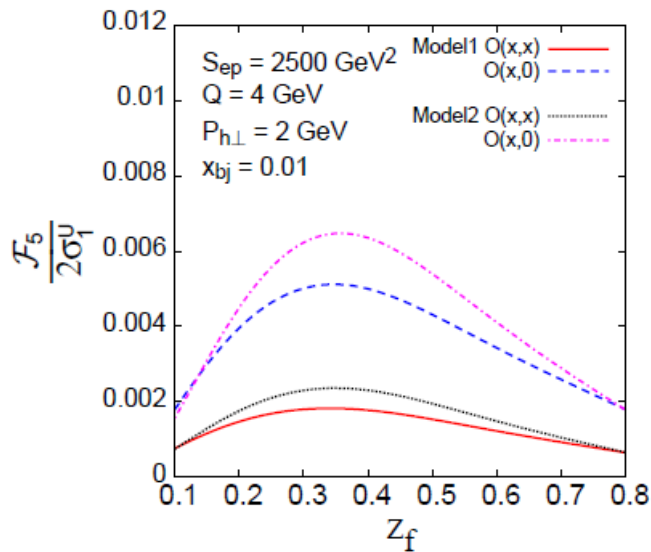
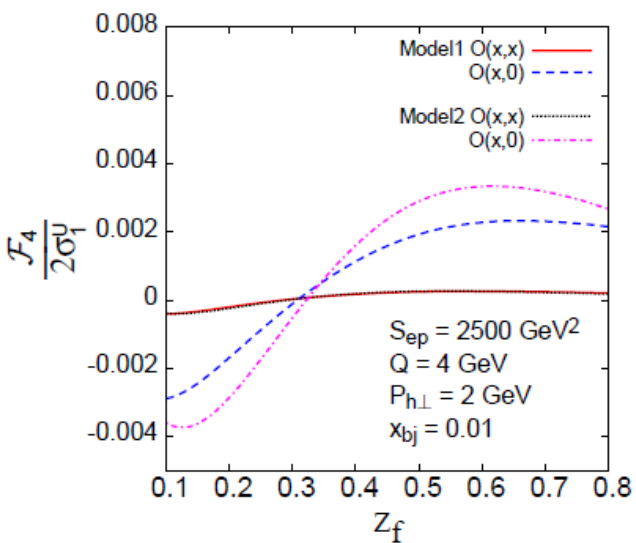
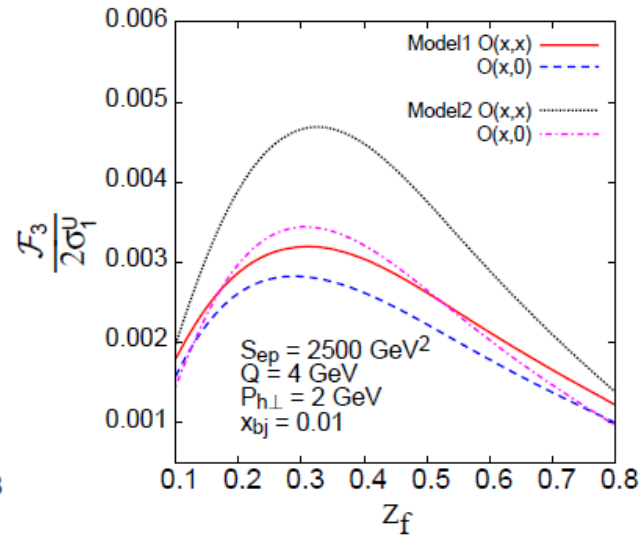
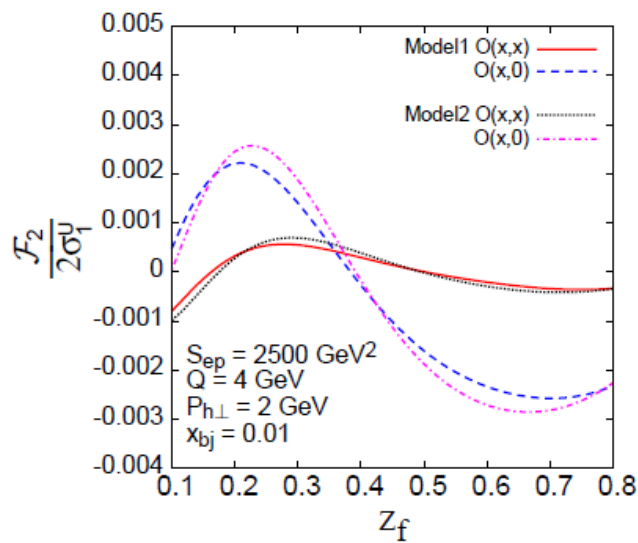
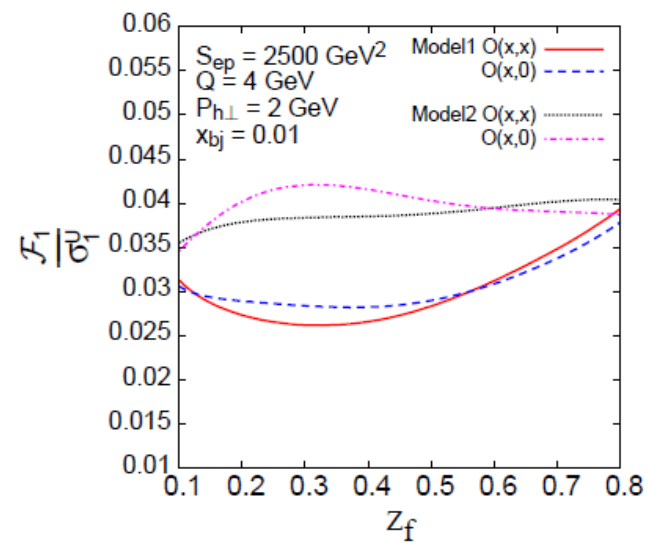
$D_c(z)$: Knesch-Kniehl-Kramer-Schienbein08

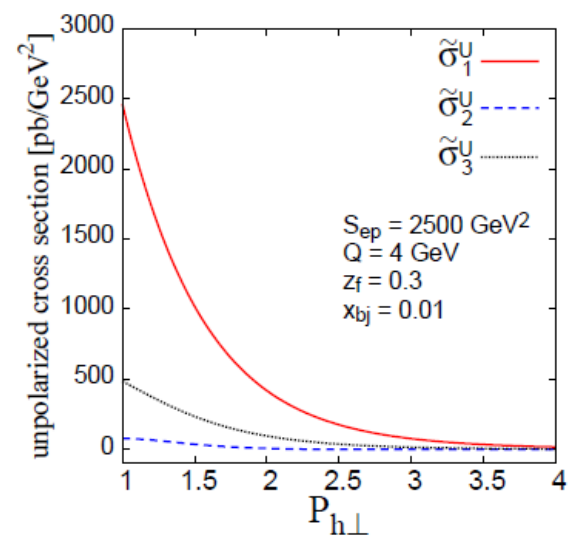
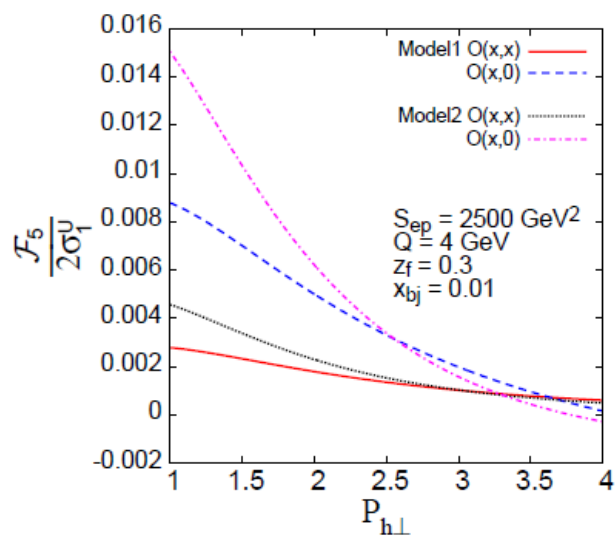
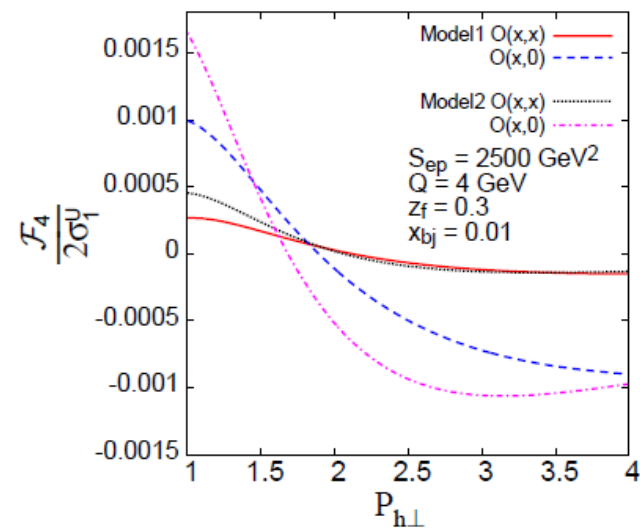
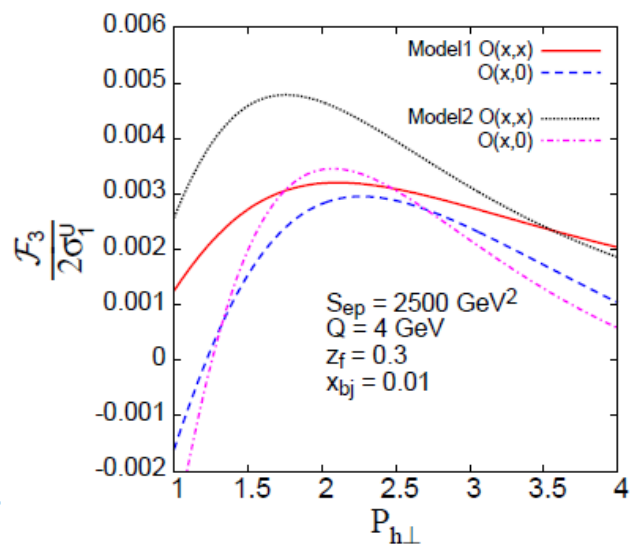
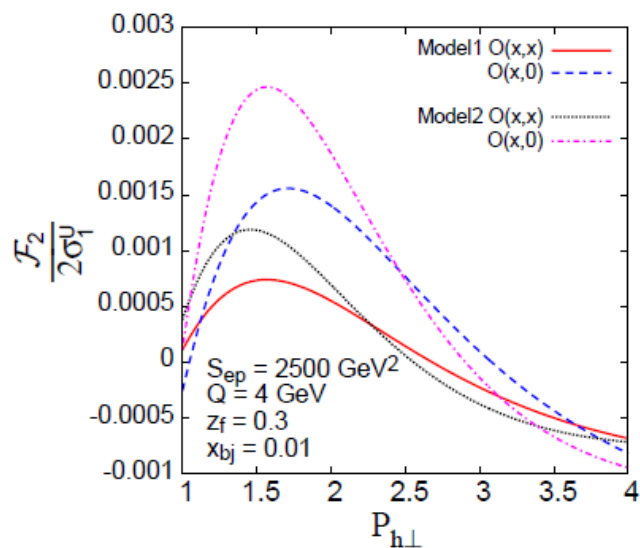
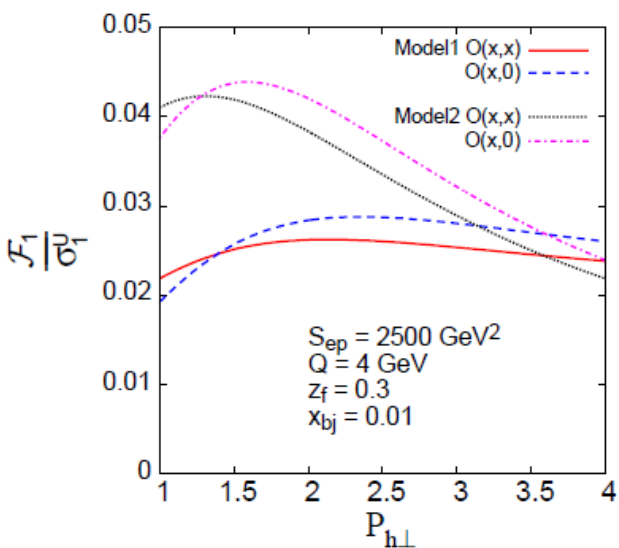
guided by
 $pp^\uparrow \rightarrow DX$
 at RHIC
 Koike, Yoshida,
 PRD84 ('11)
 014026

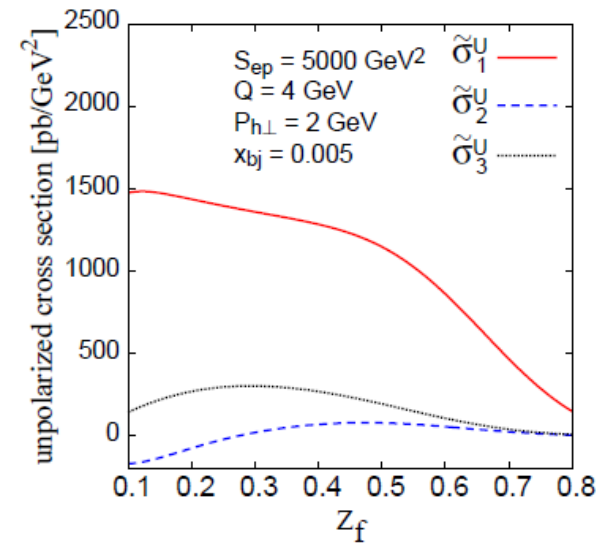
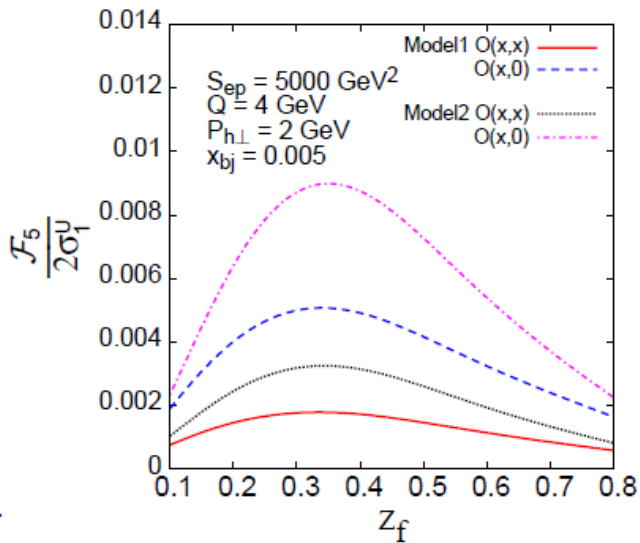
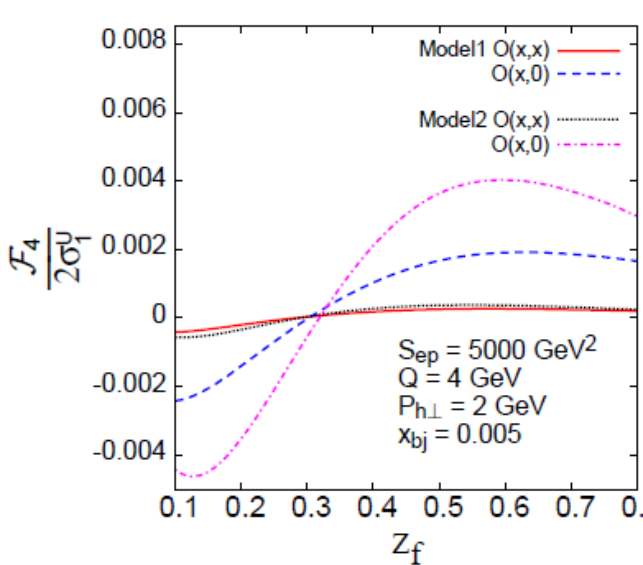
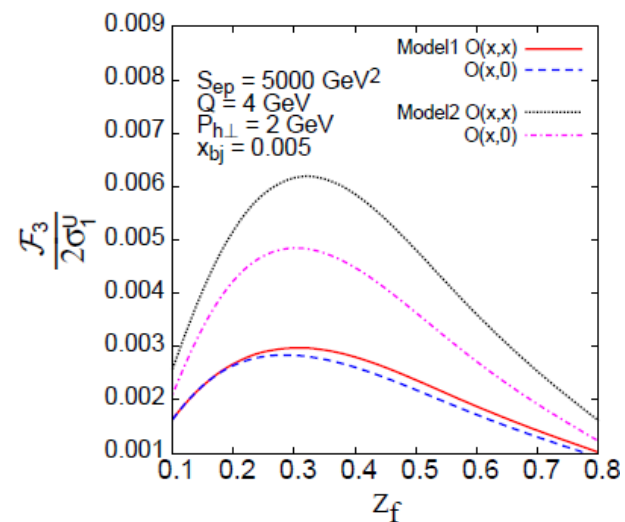
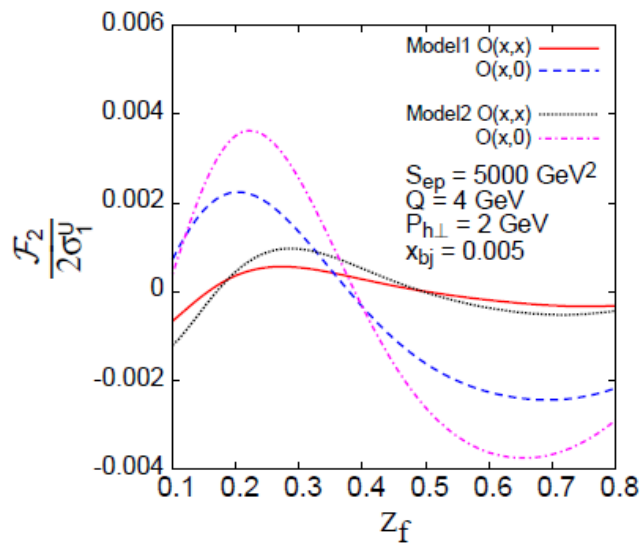
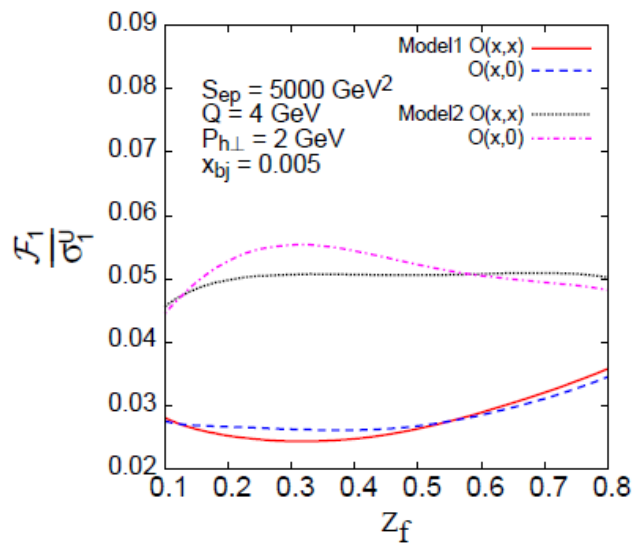


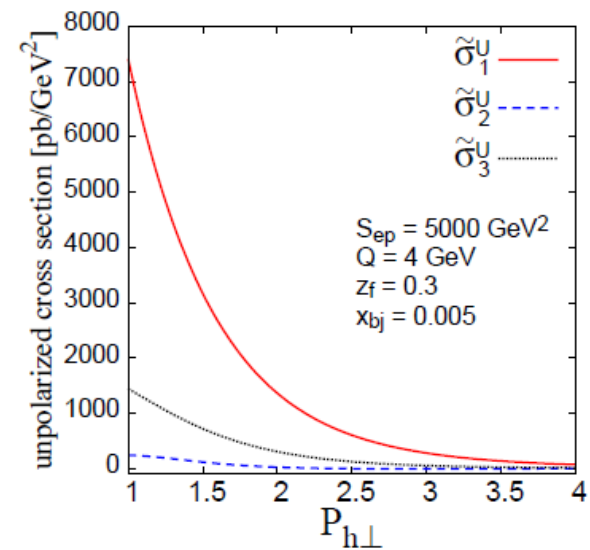
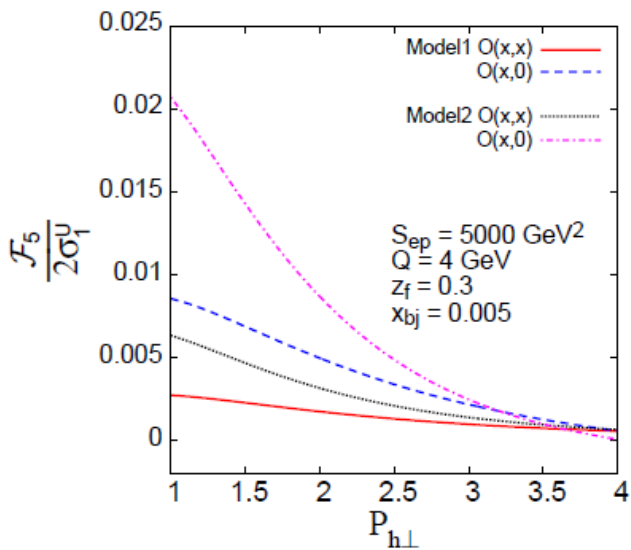
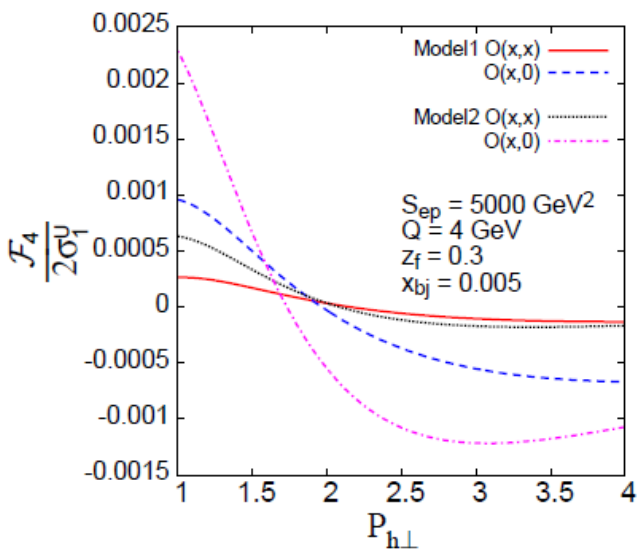
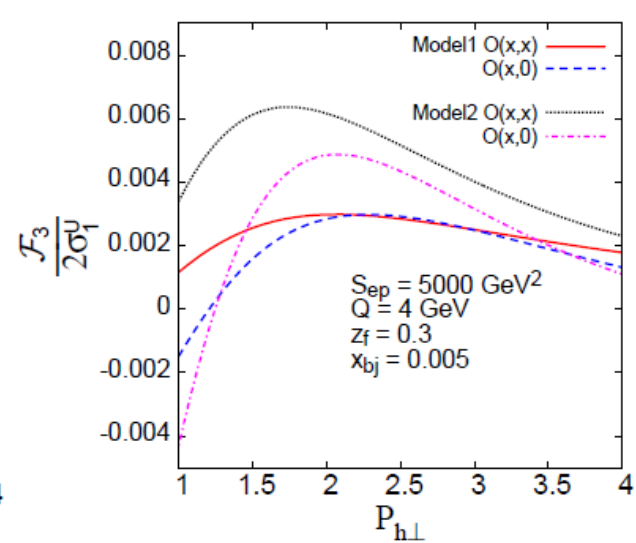
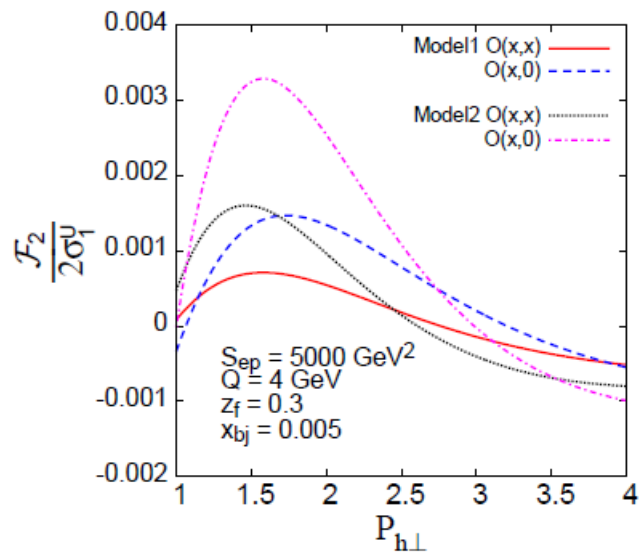
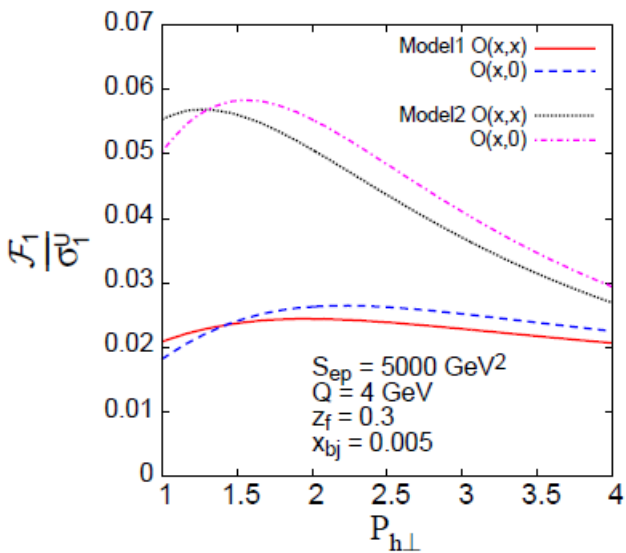
$$\frac{d^5 \sigma_{\text{twist-2}}^{\text{unpol}}}{dx_{\text{bj}} dy dz_f dP_{h\perp}^2 d\phi_h} = \tilde{\sigma}_1^U + \tilde{\sigma}_2^U \cos \phi_h + \tilde{\sigma}_3^U \cos 2\phi_h$$





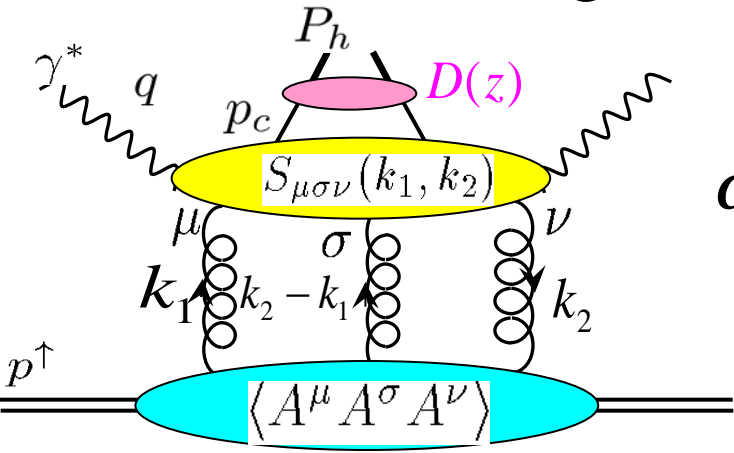






SSA from multi-gluon correlation in SIDIS at $P_{h\perp} \gg \Lambda_{\text{QCD}}$

Beppu, Koike, K.T., Yoshida, PRD82 ('10) 054005



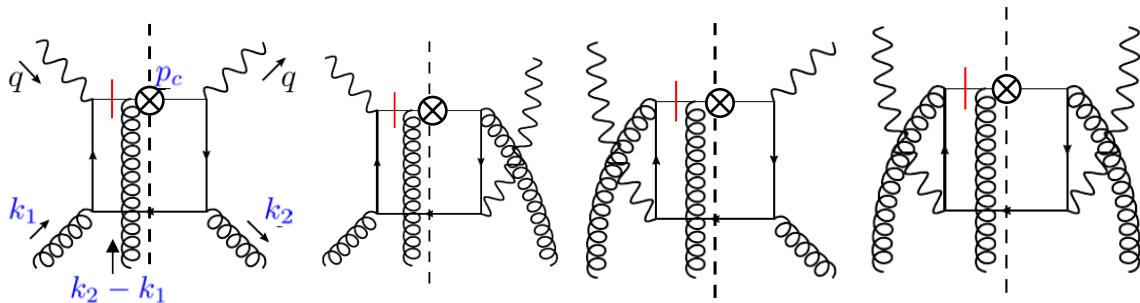
$$d\sigma_{\text{twist-3}} \sim \int dx_1 dx_2 dz \frac{\partial \mathbf{S}_{\mu\beta\nu}(k_1, k_2) p^\beta}{\partial k_{2\perp}^\sigma} \Big|_{k_i = x_i p} \otimes \left\{ \begin{array}{l} N_{\nu\mu\sigma}(x_1, x_2) \\ O_{\nu\mu\sigma}(x_1, x_2) \end{array} \right\} \otimes D(z)$$

$$i \int \frac{d\lambda}{2\pi} \int \frac{d\zeta}{2\pi} e^{i\lambda x_1} e^{i\mu(x_2 - x_1)} \langle p \mathbf{S}_\perp | F_a^{\mu n}(0) g F_c^{\sigma n}(\zeta n) F_b^{\nu n}(\lambda n) | p \mathbf{S}_\perp \rangle$$

$$= \frac{1}{24} i f^{abc} N^{\nu\mu\sigma}(x_1, x_2) + \frac{3}{40} d^{abc} O^{\nu\mu\sigma}(x_1, x_2)$$

$$i \int \frac{d\lambda}{2\pi} \int \frac{d\zeta}{2\pi} e^{i\lambda x_1} e^{i\mu(x_2 - x_1)} \langle p \mathbf{S}_\perp | F^{\mu n}(0) D_\perp^\sigma(\zeta n) F^{\nu n}(\lambda n) | p \mathbf{S}_\perp \rangle$$

$$D_\perp^\sigma = \partial_\perp^\sigma - ig A_\perp^\sigma$$



$$\begin{aligned}
& i \int \frac{d\lambda}{2\pi} \int \frac{d\zeta}{2\pi} e^{i\lambda x_1} e^{i\mu(x_2-x_1)} \langle p \mathbf{S}_\perp | F_a^{\mu n}(0) g F_c^{\sigma n}(\zeta n) F_b^{\nu n}(\lambda n) | p \mathbf{S}_\perp \rangle \\
& \qquad \qquad \qquad = \frac{1}{24} i f^{abc} N^{\nu\mu\sigma}(x_1, x_2) + \frac{3}{40} d^{abc} O^{\nu\mu\sigma}(x_1, x_2)
\end{aligned}$$

$$\begin{aligned}
& -i \int \frac{d\lambda}{2\pi} \int \frac{d\zeta}{2\pi} e^{i\lambda x_1} e^{i\mu(x_2-x_1)} \langle p \mathbf{S}_\perp | F^{\mu n}(0) D_\perp^\sigma(\zeta n) F^{\nu n}(\lambda n) | p \mathbf{S}_\perp \rangle \\
& \qquad \qquad \qquad = M^{\nu\mu\sigma}(x_1, x_2)
\end{aligned}$$

$$\begin{aligned}
& i \int \frac{d\lambda}{2\pi} \int \frac{d\zeta}{2\pi} e^{i\lambda x_1} e^{i\mu(x_2-x_1)} \langle p \mathbf{S}_\perp | F_a^{\mu n}(0) g F_c^{\sigma n}(\zeta n) F_b^{\nu n}(\lambda n) | p \mathbf{S}_\perp \rangle \\
& = \frac{1}{24} i f^{abc} N^{\nu\mu\sigma}(x_1, x_2) + \frac{3}{40} d^{abc} O^{\nu\mu\sigma}(x_1, x_2)
\end{aligned}$$

$$\begin{aligned}
& -i \int \frac{d\lambda}{2\pi} \int \frac{d\zeta}{2\pi} e^{i\lambda x_1} e^{i\mu(x_2-x_1)} \langle p \mathbf{S}_\perp | F^{\mu n}(0) D_\perp^\sigma(\zeta n) F^{\nu n}(\lambda n) | p \mathbf{S}_\perp \rangle \\
& = M^{\nu\mu\sigma}(x_1, x_2)
\end{aligned}$$

$$\begin{aligned}
& i \int \frac{d\lambda}{2\pi} \int \frac{d\zeta}{2\pi} e^{i\lambda x_1} e^{i\mu(x_2-x_1)} \langle p \mathbf{S}_\perp | F_a^{\mu n}(0) \mathbf{g} F_c^{\sigma n}(\zeta n) F_b^{\nu n}(\lambda n) | p \mathbf{S}_\perp \rangle \\
& = \frac{1}{24} i f^{abc} N^{\nu\mu\sigma}(x_1, x_2) + \frac{3}{40} d^{abc} O^{\nu\mu\sigma}(x_1, x_2)
\end{aligned}$$

$$N^{\nu\mu\sigma}(x_1, x_2) = 2iM_N \left[\mathbf{g}_\perp^{\nu\mu} \mathbf{S}_{\perp\alpha} \varepsilon^{\sigma p n \alpha} N(x_1, x_2) - \mathbf{g}_\perp^{\mu\sigma} \mathbf{S}_{\perp\alpha} \varepsilon^{\nu p n \alpha} N(x_2, x_2 - x_1) - \mathbf{g}_\perp^{\sigma\nu} \mathbf{S}_{\perp\alpha} \varepsilon^{\mu p n \alpha} N(x_1, x_1 - x_2) \right]$$

$$\begin{aligned}
& -i \int \frac{d\lambda}{2\pi} \int \frac{d\zeta}{2\pi} e^{i\lambda x_1} e^{i\mu(x_2-x_1)} \langle p \mathbf{S}_\perp | F^{\mu n}(0) \mathbf{D}_\perp^\sigma(\zeta n) F^{\nu n}(\lambda n) | p \mathbf{S}_\perp \rangle \\
& = M^{\nu\mu\sigma}(x_1, x_2)
\end{aligned}$$

$$M^{\nu\mu\sigma}(x_1, x_2) = 2iM_N \left[\mathbf{g}_\perp^{\nu\mu} \mathbf{S}_{\perp\alpha} \varepsilon^{\sigma p n \alpha} M_1(x_1, x_2) + \mathbf{g}_\perp^{\mu\sigma} \mathbf{S}_{\perp\alpha} \varepsilon^{\nu p n \alpha} M_2(x_1, x_2) - \mathbf{g}_\perp^{\sigma\nu} \mathbf{S}_{\perp\alpha} \varepsilon^{\mu p n \alpha} M_2(x_2, x_1) \right]$$

$$\begin{aligned}
& i \int \frac{d\lambda}{2\pi} \int \frac{d\zeta}{2\pi} e^{i\lambda x_1} e^{i\mu(x_2-x_1)} \langle p \mathbf{S}_\perp | F_a^{\mu n}(0) \mathbf{g} F_c^{\sigma n}(\zeta n) F_b^{\nu n}(\lambda n) | p \mathbf{S}_\perp \rangle \\
& = \frac{1}{24} i f^{abc} N^{\nu\mu\sigma}(x_1, x_2) + \frac{3}{40} d^{abc} O^{\nu\mu\sigma}(x_1, x_2)
\end{aligned}$$

$$N^{\nu\mu\sigma}(x_1, x_2) = 2iM_N \left[\mathbf{g}_\perp^{\nu\mu} \mathbf{S}_{\perp\alpha} \varepsilon^{\sigma p n \alpha} N(x_1, x_2) - \mathbf{g}_\perp^{\mu\sigma} \mathbf{S}_{\perp\alpha} \varepsilon^{\nu p n \alpha} N(x_2, x_2 - x_1) - \mathbf{g}_\perp^{\sigma\nu} \mathbf{S}_{\perp\alpha} \varepsilon^{\mu p n \alpha} N(x_1, x_1 - x_2) \right]$$

$$\begin{aligned}
& -i \int \frac{d\lambda}{2\pi} \int \frac{d\zeta}{2\pi} e^{i\lambda x_1} e^{i\mu(x_2-x_1)} \langle p \mathbf{S}_\perp | F^{\mu n}(0) \mathbf{D}_\perp^\sigma(\zeta n) F^{\nu n}(\lambda n) | p \mathbf{S}_\perp \rangle \\
& = M^{\nu\mu\sigma}(x_1, x_2) \underbrace{\hspace{10em}}_{W_{\zeta,\lambda} \equiv \text{Pexp} \left(ig \int_\lambda^\zeta dt n_\mu A^\mu(tn) \right)}
\end{aligned}$$

$$M^{\nu\mu\sigma}(x_1, x_2) = 2iM_N \left[\mathbf{g}_\perp^{\nu\mu} \mathbf{S}_{\perp\alpha} \varepsilon^{\sigma p n \alpha} M_1(x_1, x_2) + \mathbf{g}_\perp^{\mu\sigma} \mathbf{S}_{\perp\alpha} \varepsilon^{\nu p n \alpha} M_2(x_1, x_2) - \mathbf{g}_\perp^{\sigma\nu} \mathbf{S}_{\perp\alpha} \varepsilon^{\mu p n \alpha} M_2(x_2, x_1) \right]$$

$$\begin{aligned}
& i \int \frac{d\lambda}{2\pi} \int \frac{d\zeta}{2\pi} e^{i\lambda x_1} e^{i\mu(x_2-x_1)} \langle p \mathbf{S}_\perp | F_a^{\mu n}(0) \mathbf{g} F_c^{\sigma n}(\zeta n) F_b^{\nu n}(\lambda n) | p \mathbf{S}_\perp \rangle \\
& = \frac{1}{24} i f^{abc} N^{\nu\mu\sigma}(x_1, x_2) + \frac{3}{40} d^{abc} O^{\nu\mu\sigma}(x_1, x_2)
\end{aligned}$$

$$N^{\nu\mu\sigma}(x_1, x_2) = 2iM_N \left[\mathbf{g}_\perp^{\nu\mu} \mathbf{S}_{\perp\alpha} \varepsilon^{\sigma p n \alpha} N(x_1, x_2) - \mathbf{g}_\perp^{\mu\sigma} \mathbf{S}_{\perp\alpha} \varepsilon^{\nu p n \alpha} N(x_2, x_2 - x_1) - \mathbf{g}_\perp^{\sigma\nu} \mathbf{S}_{\perp\alpha} \varepsilon^{\mu p n \alpha} N(x_1, x_1 - x_2) \right]$$

$$\begin{aligned}
& -i \int \frac{d\lambda}{2\pi} \int \frac{d\zeta}{2\pi} e^{i\lambda x_1} e^{i\mu(x_2-x_1)} \langle p \mathbf{S}_\perp | F^{\mu n}(0) D_\perp^\sigma(\zeta n) F^{\nu n}(\lambda n) | p \mathbf{S}_\perp \rangle \\
& = M^{\nu\mu\sigma}(x_1, x_2) \underbrace{\hspace{10em}}_{W_{\zeta,\lambda} \equiv \text{Pexp} \left(ig \int_\lambda^\zeta dt n_\mu A^\mu(tn) \right)}
\end{aligned}$$

$$M^{\nu\mu\sigma}(x_1, x_2) = 2iM_N \left[\mathbf{g}_\perp^{\nu\mu} \mathbf{S}_{\perp\alpha} \varepsilon^{\sigma p n \alpha} M_1(x_1, x_2) + \mathbf{g}_\perp^{\mu\sigma} \mathbf{S}_{\perp\alpha} \varepsilon^{\nu p n \alpha} M_2(x_1, x_2) - \mathbf{g}_\perp^{\sigma\nu} \mathbf{S}_{\perp\alpha} \varepsilon^{\mu p n \alpha} M_2(x_2, x_1) \right]$$

$$D_\perp^\sigma(\zeta n) W_{\zeta,\lambda} = i \int_\lambda^\zeta d\mu W_{\zeta,\mu} \mathbf{g} F^{\sigma n}(\mu n) W_{\mu,\lambda} + W_{\zeta,\lambda} D_\perp^\sigma(\lambda n)$$

$$\begin{aligned}
& i \int \frac{d\lambda}{2\pi} \int \frac{d\zeta}{2\pi} e^{i\lambda x_1} e^{i\mu(x_2-x_1)} \langle p \mathbf{S}_\perp | F_a^{\mu n}(0) \mathbf{g} F_c^{\sigma n}(\zeta n) F_b^{\nu n}(\lambda n) | p \mathbf{S}_\perp \rangle \\
& = \frac{1}{24} i f^{abc} N^{\nu\mu\sigma}(x_1, x_2) + \frac{3}{40} d^{abc} O^{\nu\mu\sigma}(x_1, x_2)
\end{aligned}$$

$$N^{\nu\mu\sigma}(x_1, x_2) = 2iM_N \left[\mathbf{g}_\perp^{\nu\mu} \mathbf{S}_{\perp\alpha} \varepsilon^{\sigma p n \alpha} N(x_1, x_2) - \mathbf{g}_\perp^{\mu\sigma} \mathbf{S}_{\perp\alpha} \varepsilon^{\nu p n \alpha} N(x_2, x_2 - x_1) - \mathbf{g}_\perp^{\sigma\nu} \mathbf{S}_{\perp\alpha} \varepsilon^{\mu p n \alpha} N(x_1, x_1 - x_2) \right]$$

$$\begin{aligned}
& -i \int \frac{d\lambda}{2\pi} \int \frac{d\zeta}{2\pi} e^{i\lambda x_1} e^{i\mu(x_2-x_1)} \langle p \mathbf{S}_\perp | F^{\mu n}(0) D_\perp^\sigma(\zeta n) F^{\nu n}(\lambda n) | p \mathbf{S}_\perp \rangle \\
& = M^{\nu\mu\sigma}(x_1, x_2) \quad \underbrace{\hspace{10em}}_{W_{\zeta,\lambda} \equiv \text{Pexp} \left(ig \int_\lambda^\zeta dt n_\mu A^\mu(tn) \right)}
\end{aligned}$$

$$M^{\nu\mu\sigma}(x_1, x_2) = 2iM_N \left[\mathbf{g}_\perp^{\nu\mu} \mathbf{S}_{\perp\alpha} \varepsilon^{\sigma p n \alpha} M_1(x_1, x_2) + \mathbf{g}_\perp^{\mu\sigma} \mathbf{S}_{\perp\alpha} \varepsilon^{\nu p n \alpha} M_2(x_1, x_2) - \mathbf{g}_\perp^{\sigma\nu} \mathbf{S}_{\perp\alpha} \varepsilon^{\mu p n \alpha} M_2(x_2, x_1) \right]$$

$$D_\perp^\sigma(\zeta n) W_{\zeta,\lambda} = i \int_\lambda^\zeta d\mu W_{\zeta,\mu} \mathbf{g} F^{\sigma n}(\mu n) W_{\mu,\lambda} + W_{\zeta,\lambda} D_\perp^\sigma(\lambda n)$$

$$M_1(x_1, x_2) = \mathcal{P} \frac{N(x_1, x_2)}{x_2 - x_1} \quad M_2(x_1, x_2) = \mathcal{P} \frac{N(x_2, x_2 - x_1)}{x_1 - x_2} + \delta(x_1 - x_2) \tilde{g}(x_1)$$

$$i \int \frac{d\lambda}{2\pi} \int \frac{d\zeta}{2\pi} e^{i\lambda x_1} e^{i\mu(x_2-x_1)} \langle p S_{\perp} | F_a^{\mu n}(0) g F_c^{\sigma n}(\zeta n) F_b^{\nu n}(\lambda n) | p S_{\perp} \rangle$$

$$= \frac{1}{24} i f^{abc} N^{\nu\mu\sigma}(x_1, x_2) + \frac{3}{40} d^{abc} O^{\nu\mu\sigma}(x_1, x_2)$$

$$N^{\nu\mu\sigma}(x_1, x_2) = 2iM_N \left[g_{\perp}^{\nu\mu} S_{\perp\alpha} \varepsilon^{\sigma p n \alpha} N(x_1, x_2) - g_{\perp}^{\mu\sigma} S_{\perp\alpha} \varepsilon^{\nu p n \alpha} N(x_2, x_2 - x_1) - g_{\perp}^{\sigma\nu} S_{\perp\alpha} \varepsilon^{\mu p n \alpha} N(x_1, x_1 - x_2) \right]$$

$$-i \int \frac{d\lambda}{2\pi} \int \frac{d\zeta}{2\pi} e^{i\lambda x_1} e^{i\mu(x_2-x_1)} \langle p S_{\perp} | F^{\mu n}(0) D_{\perp}^{\sigma}(\zeta n) F^{\nu n}(\lambda n) | p S_{\perp} \rangle$$

$$= M^{\nu\mu\sigma}(x_1, x_2) \quad \underbrace{\hspace{10em}}_{W_{\zeta, \lambda} \equiv \text{P exp} \left(ig \int_{\lambda}^{\zeta} dt n_{\mu} A^{\mu}(tn) \right)}$$

$$M^{\nu\mu\sigma}(x_1, x_2) = 2iM_N \left[g_{\perp}^{\nu\mu} S_{\perp\alpha} \varepsilon^{\sigma p n \alpha} M_1(x_1, x_2) + g_{\perp}^{\mu\sigma} S_{\perp\alpha} \varepsilon^{\nu p n \alpha} M_2(x_1, x_2) - g_{\perp}^{\sigma\nu} S_{\perp\alpha} \varepsilon^{\mu p n \alpha} M_2(x_2, x_1) \right]$$

$$D_{\perp}^{\sigma}(\zeta n) W_{\zeta, \lambda} = i \int_{\lambda}^{\zeta} d\mu W_{\zeta, \mu} g F^{\sigma n}(\mu n) W_{\mu, \lambda} + W_{\zeta, \lambda} D_{\perp}^{\sigma}(\lambda n)$$

$$M_1(x_1, x_2) = \mathcal{P} \frac{N(x_1, x_2)}{x_2 - x_1} \quad M_2(x_1, x_2) = \mathcal{P} \frac{N(x_2, x_2 - x_1)}{x_1 - x_2} + \delta(x_1 - x_2) \tilde{g}(x_1)$$

$$\int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle p S | F^{\mu n}(0) \left(D_{\perp}^{\sigma}(\lambda n) + ig \int d\mu \frac{\epsilon(\mu - \lambda)}{2} F^{\sigma n}(\mu n) \right) F^{\nu n}(\lambda n) | p S \rangle = 2\epsilon^{np\mu\nu} S_{\perp}^{\sigma} \tilde{g}(x)$$

$$\int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle p S | F^{\mu n}(0) F^{\nu n}(\lambda n) | p S \rangle = -\frac{x}{2} \left[G(x) g_{\perp}^{\mu\nu} + \Delta G(x) i\epsilon^{\mu\nu\rho n} (S \cdot n) M_N + 2\mathcal{G}_T(x) i\epsilon^{\mu\nu\alpha n} S_{\perp\alpha} M_N + \dots \right]$$

$$\int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle p S | \bar{\psi}(0) \gamma^{\sigma} \gamma_5 \psi(\lambda n) | p S \rangle = 2M_N \left[\Delta g(x) (S \cdot n) p^{\sigma} + \mathbf{g}_T(x) S_{\perp}^{\sigma} + \dots \right]$$

$$\int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle p S | F^{\mu n}(0) F^{\nu n}(\lambda n) | p S \rangle = -\frac{x}{2} \left[G(x) g_{\perp}^{\mu\nu} + \Delta G(x) i\epsilon^{\mu\nu\rho n} (S \cdot n) M_N + 2\mathcal{G}_T(x) i\epsilon^{\mu\nu\alpha n} S_{\perp\alpha} M_N + \dots \right]$$

$$\int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle p S | \bar{\psi}(0) \gamma^{\sigma} \gamma_5 \psi(\lambda n) | p S \rangle = 2M_N \left[\Delta g(x) (S \cdot n) p^{\sigma} + g_T(x) S_{\perp}^{\sigma} + \dots \right]$$

$$\int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle p S_{\perp} | F^{\mu+}(0) F^{-+}(\lambda n) | p S_{\perp} \rangle = x \mathcal{G}_T(x) i\epsilon^{\mu-+\alpha} S_{\perp\alpha} M_N$$

$$F^{-+} = -\partial^+ A^- = -\frac{1}{\partial_-} \left(D_{\perp j} F^{j+} + g \bar{\psi} t^a \gamma^+ \psi t^a \right) \qquad D_{\nu} F^{\mu\nu} = g \bar{\psi} t^a \gamma^{\mu} \psi t^a$$

$$\int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle p S | F^{\mu n}(0) F^{\nu n}(\lambda n) | p S \rangle = -\frac{x}{2} \left[G(x) g_{\perp}^{\mu\nu} + \Delta G(x) i\epsilon^{\mu\nu pn} (S \cdot n) M_N + 2\mathcal{G}_T(x) i\epsilon^{\mu\nu\alpha n} S_{\perp\alpha} M_N + \dots \right]$$

$$\int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle p S | \bar{\psi}(0) \gamma^{\sigma} \gamma_5 \psi(\lambda n) | p S \rangle = 2M_N \left[\Delta g(x) (S \cdot n) p^{\sigma} + \mathbf{g}_T(x) S_{\perp}^{\sigma} + \dots \right]$$

$$\int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle p S_{\perp} | F^{\mu+}(0) F^{-+}(\lambda n) | p S_{\perp} \rangle = x \mathcal{G}_T(x) i\epsilon^{\mu-+\alpha} S_{\perp\alpha} M_N$$

$$F^{-+} = -\partial^+ A^- = -\frac{1}{\partial_-} \left(D_{\perp j} F^{j+} + g \bar{\psi} t^a \gamma^+ \psi t^a \right) \quad D_{\nu} F^{\mu\nu} = g \bar{\psi} t^a \gamma^{\mu} \psi t^a$$

$$-i \int \frac{d\lambda}{2\pi} \int \frac{d\zeta}{2\pi} e^{i\lambda x_1} e^{i\mu(x_2-x_1)} \langle p S_{\perp} | F^{\mu n}(0) D_{\perp}^{\sigma}(\zeta n) F^{\nu n}(\lambda n) | p S_{\perp} \rangle = M^{\nu\mu\sigma}(x_1, x_2)$$

$$M^{\nu\mu\sigma}(x_1, x_2) = 2iM_N \left[g_{\perp}^{\nu\mu} S_{\perp\alpha} \varepsilon^{\sigma pn\alpha} M_1(x_1, x_2) + g_{\perp}^{\mu\sigma} S_{\perp\alpha} \varepsilon^{\nu pn\alpha} M_2(x_1, x_2) - g_{\perp}^{\sigma\nu} S_{\perp\alpha} \varepsilon^{\mu pn\alpha} M_2(x_2, x_1) \right]$$

$$\int \frac{d\lambda}{2\pi} \int \frac{d\mu}{2\pi} e^{i\lambda x_1} e^{i\mu(x_2-x_1)} \langle p S_{\perp} | \bar{\psi}(0) \gamma^{\mu} g F^{\alpha n}(\mu n) \psi(\lambda n) | p S_{\perp} \rangle = M_N p^{\mu} \epsilon^{\alpha pn S_{\perp}} G_F(x_1, x_2)$$

$$\int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle p S | F^{\mu n}(0) F^{\nu n}(\lambda n) | p S \rangle = -\frac{x}{2} \left[G(x) g_{\perp}^{\mu\nu} + \Delta G(x) i\epsilon^{\mu\nu pn} (S \cdot n) M_N + 2\mathcal{G}_T(x) i\epsilon^{\mu\nu\alpha n} S_{\perp\alpha} M_N + \dots \right]$$

$$\int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle p S | \bar{\psi}(0) \gamma^{\sigma} \gamma_5 \psi(\lambda n) | p S \rangle = 2M_N \left[\Delta g(x) (S \cdot n) p^{\sigma} + g_T(x) S_{\perp}^{\sigma} + \dots \right]$$

$$\int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle p S_{\perp} | F^{\mu+}(0) F^{-+}(\lambda n) | p S_{\perp} \rangle = x \mathcal{G}_T(x) i\epsilon^{\mu+ \alpha} S_{\perp\alpha} M_N$$

$$F^{-+} = -\partial^+ A^- = -\frac{1}{\partial_-} \left(D_{\perp j} F^{j+} + g \bar{\psi} t^a \gamma^+ \psi t^a \right) \quad D_{\nu} F^{\mu\nu} = g \bar{\psi} t^a \gamma^{\mu} \psi t^a$$

$$-i \int \frac{d\lambda}{2\pi} \int \frac{d\zeta}{2\pi} e^{i\lambda x_1} e^{i\mu(x_2-x_1)} \langle p S_{\perp} | F^{\mu n}(0) D_{\perp}^{\sigma}(\zeta n) F^{\nu n}(\lambda n) | p S_{\perp} \rangle = M^{\nu\mu\sigma}(x_1, x_2)$$

$$M^{\nu\mu\sigma}(x_1, x_2) = 2iM_N \left[g_{\perp}^{\nu\mu} S_{\perp\alpha} \varepsilon^{\sigma pn\alpha} M_1(x_1, x_2) + g_{\perp}^{\mu\sigma} S_{\perp\alpha} \varepsilon^{\nu pn\alpha} M_2(x_1, x_2) - g_{\perp}^{\sigma\nu} S_{\perp\alpha} \varepsilon^{\mu pn\alpha} M_2(x_2, x_1) \right]$$

$$\int \frac{d\lambda}{2\pi} \int \frac{d\mu}{2\pi} e^{i\lambda x_1} e^{i\mu(x_2-x_1)} \langle p S_{\perp} | \bar{\psi}(0) \gamma^{\mu} g F^{\alpha n}(\mu n) \psi(\lambda n) | p S_{\perp} \rangle = M_N p^{\mu} \epsilon^{\alpha pn} S_{\perp} G_F(x_1, x_2)$$

$$\mathcal{G}_T(x) = -\frac{1}{2x^2} \int dx' \left(8M_2(x, x') - 4M_2(x', x) - 4M_1(x', x) + G_F(x' + x, x') + G_F(x' - x, x') \right)$$

$$\int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle p S | F^{\mu n}(0) F^{\nu n}(\lambda n) | p S \rangle = -\frac{x}{2} \left[G(x) g_{\perp}^{\mu\nu} + \Delta G(x) i\epsilon^{\mu\nu\rho n} (S \cdot n) M_N + 2\mathcal{G}_T(x) i\epsilon^{\mu\nu\alpha n} S_{\perp\alpha} M_N + \dots \right]$$

$$\int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle p S | F^{\mu n}(0) F^{\nu n}(\lambda n) | p S \rangle = -\frac{x}{2} \left[G(x) g_{\perp}^{\mu\nu} + \Delta G(x) i\epsilon^{\mu\nu\rho n} (S \cdot n) M_N + 2\mathcal{G}_T(x) i\epsilon^{\mu\nu\alpha n} S_{\perp\alpha} M_N + \dots \right]$$

$$\begin{aligned} & z^{\nu} \frac{\partial}{\partial z^{\nu}} F^{\mu z}(0) \tilde{F}_{\mu}^{\rho}(z) - z^{\nu} \frac{\partial}{\partial z_{\rho}} F^{\mu z}(0) \tilde{F}_{\mu\nu}(z) \quad z^2 \rightarrow 0 \\ &= i \int_0^1 dt F^{\mu z}(0) t g F^{\rho z}(tz) \tilde{F}_{\mu}^z(z) - i \int_0^1 dt F^{\mu z}(0) g F_{\mu}^z(tz) \tilde{F}^{\rho z}(z) + \int_0^1 du \left(i \int_0^1 dt F^{\mu z}(0) g u F^{\rho z}(tuz) \tilde{F}_{\mu}^z(uz) \right. \\ &\quad \left. - i \int_0^1 dt F^{\mu z}(0) g u F_{\mu}^z(tuz) \tilde{F}^{\rho z}(uz) - i \int_0^1 dt F^{\rho z}(0) g u F^{\mu z}(tuz) \tilde{F}_{\mu}^z(uz) \right. \\ &\quad \left. - F^{\mu z}(0) \epsilon_{\mu z \sigma}^{\rho} D_{\lambda} F^{\lambda \sigma}(uz) + F^{\mu z}(0) \tilde{D}_{\mu} \tilde{F}^{\rho z}(uz) + F^{\rho z}(0) D^{\mu} \tilde{F}_{\mu}^z(uz) \right) - F^{\mu z}(0) \epsilon_{\mu z \sigma}^{\rho} D_{\lambda} F^{\lambda \sigma}(z) + F^{\mu z}(0) \tilde{D}_{\mu} \tilde{F}^{\rho z}(z) \end{aligned}$$

$$\int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle p S | F^{\mu n}(0) F^{\nu n}(\lambda n) | p S \rangle = -\frac{x}{2} \left[G(x) g_{\perp}^{\mu\nu} + \Delta G(x) i\epsilon^{\mu\nu\rho n} (S \cdot n) M_N + 2\mathcal{G}_T(x) i\epsilon^{\mu\nu\alpha n} S_{\perp\alpha} M_N + \dots \right]$$

$$\begin{aligned} & z^{\nu} \frac{\partial}{\partial z^{\nu}} F^{\mu z}(0) \tilde{F}_{\mu}^{\rho}(z) - z^{\nu} \frac{\partial}{\partial z_{\rho}} F^{\mu z}(0) \tilde{F}_{\mu\nu}(z) \quad z^2 \rightarrow 0 \\ &= i \int_0^1 dt F^{\mu z}(0) t g F^{\rho z}(tz) \tilde{F}_{\mu}^z(z) - i \int_0^1 dt F^{\mu z}(0) g F_{\mu}^z(tz) \tilde{F}^{\rho z}(z) + \int_0^1 du \left(i \int_0^1 dt F^{\mu z}(0) g u F^{\rho z}(tuz) \tilde{F}_{\mu}^z(uz) \right. \\ &\quad \left. - i \int_0^1 dt F^{\mu z}(0) g u F_{\mu}^z(tuz) \tilde{F}^{\rho z}(uz) - i \int_0^1 dt F^{\rho z}(0) g u F^{\mu z}(tuz) \tilde{F}_{\mu}^z(uz) \right. \\ &\quad \left. - F^{\mu z}(0) \epsilon_{\mu z \sigma}^{\rho} D_{\lambda} F^{\lambda \sigma}(uz) + F^{\mu z}(0) \tilde{D}_{\mu} \tilde{F}^{\rho z}(uz) + F^{\rho z}(0) D^{\mu} \tilde{F}_{\mu}^z(uz) \right) - F^{\mu z}(0) \epsilon_{\mu z \sigma}^{\rho} D_{\lambda} F^{\lambda \sigma}(z) + F^{\mu z}(0) \tilde{D}_{\mu} \tilde{F}^{\rho z}(z) \end{aligned}$$

$$\begin{aligned} i \int \frac{d\lambda}{2\pi} \int \frac{d\zeta}{2\pi} e^{i\lambda x_1} e^{i\mu(x_2 - x_1)} \langle p S_{\perp} | F_a^{\mu n}(0) g F_c^{\sigma n}(\zeta n) F_b^{\nu n}(\lambda n) | p S_{\perp} \rangle &= \frac{1}{24} i f^{abc} N^{\nu\mu\sigma}(x_1, x_2) + \dots \\ N^{\nu\mu\sigma}(x_1, x_2) &= 2i M_N \left[g_{\perp}^{\nu\mu} S_{\perp\alpha} \epsilon^{\sigma\rho n\alpha} N(x_1, x_2) - g_{\perp}^{\mu\sigma} S_{\perp\alpha} \epsilon^{\nu\rho n\alpha} N(x_2, x_2 - x_1) - g_{\perp}^{\sigma\nu} S_{\perp\alpha} \epsilon^{\mu\rho n\alpha} N(x_1, x_1 - x_2) \right] \end{aligned}$$

$$\int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle p S | F^{\mu n}(0) F^{\nu n}(\lambda n) | p S \rangle = -\frac{x}{2} \left[G(x) g_{\perp}^{\mu\nu} + \Delta G(x) i\epsilon^{\mu\nu\rho n} (S \cdot n) M_N + 2\mathcal{G}_T(x) i\epsilon^{\mu\nu\alpha n} S_{\perp\alpha} M_N + \dots \right]$$

$$\begin{aligned} & z^{\nu} \frac{\partial}{\partial z^{\nu}} F^{\mu z}(0) \tilde{F}_{\mu}^{\rho}(z) - z^{\nu} \frac{\partial}{\partial z^{\rho}} F^{\mu z}(0) \tilde{F}_{\mu\nu}(z) \quad z^2 \rightarrow 0 \\ &= i \int_0^1 dt F^{\mu z}(0) t g F^{\rho z}(tz) \tilde{F}_{\mu}^z(z) - i \int_0^1 dt F^{\mu z}(0) g F_{\mu}^z(tz) \tilde{F}^{\rho z}(z) + \int_0^1 du \left(i \int_0^1 dt F^{\mu z}(0) g u F^{\rho z}(tuz) \tilde{F}_{\mu}^z(uz) \right. \\ &\quad \left. - i \int_0^1 dt F^{\mu z}(0) g u F_{\mu}^z(tuz) \tilde{F}^{\rho z}(uz) - i \int_0^1 dt F^{\rho z}(0) g u F^{\mu z}(tuz) \tilde{F}_{\mu}^z(uz) \right. \\ &\quad \left. - F^{\mu z}(0) \epsilon_{\mu z \sigma}^{\rho} D_{\lambda} F^{\lambda \sigma}(uz) + F^{\mu z}(0) \bar{D}_{\mu} \tilde{F}^{\rho z}(uz) + F^{\rho z}(0) D^{\mu} \tilde{F}_{\mu}^z(uz) \right) - F^{\mu z}(0) \epsilon_{\mu z \sigma}^{\rho} D_{\lambda} F^{\lambda \sigma}(z) + F^{\mu z}(0) \bar{D}_{\mu} \tilde{F}^{\rho z}(z) \end{aligned}$$

$$i \int \frac{d\lambda}{2\pi} \int \frac{d\zeta}{2\pi} e^{i\lambda x_1} e^{i\mu(x_2-x_1)} \langle p S_{\perp} | F_a^{\mu n}(0) g F_c^{\sigma n}(\zeta n) F_b^{\nu n}(\lambda n) | p S_{\perp} \rangle = \frac{1}{24} i f^{abc} N^{\nu\mu\sigma}(x_1, x_2) + \dots$$

$$N^{\nu\mu\sigma}(x_1, x_2) = 2i M_N \left[g_{\perp}^{\nu\mu} S_{\perp\alpha} \epsilon^{\sigma\rho n\alpha} N(x_1, x_2) - g_{\perp}^{\mu\sigma} S_{\perp\alpha} \epsilon^{\nu\rho n\alpha} N(x_2, x_2 - x_1) - g_{\perp}^{\sigma\nu} S_{\perp\alpha} \epsilon^{\mu\rho n\alpha} N(x_1, x_1 - x_2) \right]$$

$$D_{\nu} F^{\mu\nu} = g \bar{\psi} t^a \gamma^{\mu} \psi t^a \quad D_{\mu} \tilde{F}^{\mu\nu} = 0$$

$$\int \frac{d\lambda}{2\pi} \int \frac{d\mu}{2\pi} e^{i\lambda x_1} e^{i\mu(x_2-x_1)} \langle p S_{\perp} | \bar{\psi}(0) \gamma^{\mu} g F^{\alpha n}(\mu n) \psi(\lambda n) | p S_{\perp} \rangle = M_N p^{\mu} \epsilon^{\alpha\rho n S_{\perp}} G_F(x_1, x_2)$$

$$\int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle p S | F^{\mu n}(0) F^{\nu n}(\lambda n) | p S \rangle = -\frac{x}{2} \left[G(x) g_{\perp}^{\mu\nu} + \Delta G(x) i\epsilon^{\mu\nu\rho n} (S \cdot n) M_N + 2\mathcal{G}_T(x) i\epsilon^{\mu\nu\alpha n} S_{\perp\alpha} M_N + \dots \right]$$

$$\begin{aligned} & z^{\nu} \frac{\partial}{\partial z^{\nu}} F^{\mu z}(0) \tilde{F}_{\mu}^{\rho}(z) - z^{\nu} \frac{\partial}{\partial z^{\rho}} F^{\mu z}(0) \tilde{F}_{\mu\nu}(z) \quad z^2 \rightarrow 0 \\ &= i \int_0^1 dt F^{\mu z}(0) t g F^{\rho z}(tz) \tilde{F}_{\mu}^z(z) - i \int_0^1 dt F^{\mu z}(0) g F_{\mu}^z(tz) \tilde{F}^{\rho z}(z) + \int_0^1 du \left(i \int_0^1 dt F^{\mu z}(0) g u F^{\rho z}(tuz) \tilde{F}_{\mu}^z(uz) \right. \\ &\quad \left. - i \int_0^1 dt F^{\mu z}(0) g u F_{\mu}^z(tuz) \tilde{F}^{\rho z}(uz) - i \int_0^1 dt F^{\rho z}(0) g u F^{\mu z}(tuz) \tilde{F}_{\mu}^z(uz) \right. \\ &\quad \left. - F^{\mu z}(0) \epsilon_{\mu z \sigma}^{\rho} D_{\lambda} F^{\lambda \sigma}(uz) + F^{\mu z}(0) \bar{D}_{\mu} \tilde{F}^{\rho z}(uz) + F^{\rho z}(0) D^{\mu} \tilde{F}_{\mu}^z(uz) \right) - F^{\mu z}(0) \epsilon_{\mu z \sigma}^{\rho} D_{\lambda} F^{\lambda \sigma}(z) + F^{\mu z}(0) \bar{D}_{\mu} \tilde{F}^{\rho z}(z) \end{aligned}$$

$$i \int \frac{d\lambda}{2\pi} \int \frac{d\zeta}{2\pi} e^{i\lambda x_1} e^{i\mu(x_2-x_1)} \langle p S_{\perp} | F_a^{\mu n}(0) g F_c^{\sigma n}(\zeta n) F_b^{\nu n}(\lambda n) | p S_{\perp} \rangle = \frac{1}{24} i f^{abc} N^{\nu\mu\sigma}(x_1, x_2) + \dots$$

$$N^{\nu\mu\sigma}(x_1, x_2) = 2i M_N \left[g_{\perp}^{\nu\mu} S_{\perp\alpha} \epsilon^{\sigma\rho n\alpha} N(x_1, x_2) - g_{\perp}^{\mu\sigma} S_{\perp\alpha} \epsilon^{\nu\rho n\alpha} N(x_2, x_2 - x_1) - g_{\perp}^{\sigma\nu} S_{\perp\alpha} \epsilon^{\mu\rho n\alpha} N(x_1, x_1 - x_2) \right]$$

$$D_{\nu} F^{\mu\nu} = g \bar{\psi} t^a \gamma^{\mu} \psi t^a \quad D_{\mu} \tilde{F}^{\mu\nu} = 0$$

$$\int \frac{d\lambda}{2\pi} \int \frac{d\mu}{2\pi} e^{i\lambda x_1} e^{i\mu(x_2-x_1)} \langle p S_{\perp} | \bar{\psi}(0) \gamma^{\mu} g F^{\alpha n}(\mu n) \psi(\lambda n) | p S_{\perp} \rangle = M_N p^{\mu} \epsilon^{\alpha\rho n S_{\perp}} G_F(x_1, x_2)$$

$$\begin{aligned} 2x^2 \frac{\partial}{\partial x} \mathcal{G}_T(x) + x \Delta G(x) &= \int dx' P \frac{1}{x-x'} \left(\frac{\partial}{\partial x} - \frac{\partial}{\partial x'} \right) (2N(x, x-x') - 2N(x', x'-x)) \\ &\quad + \int dx' P \frac{1}{x-x'} \left(\frac{\partial}{\partial x} + \frac{\partial}{\partial x'} \right) (4N(x, x') - 6N(x', x'-x) - 6N(x, x-x')) \\ &\quad + \int dx' P \frac{1}{x-x'} P \frac{1}{x} (8N(x', x'-x) + 8N(x, x-x')) + \int dx' \left(P \frac{1}{x} - \frac{\partial}{\partial x} \right) (G_F(x'+x, x') + G_F(x'-x, x')) \end{aligned}$$

$$\mathcal{G}_T(x) = -\frac{1}{2x^2} \int dx' (8M_2(x, x') - 4M_2(x', x) - 4M_1(x', x) + G_F(x' + x, x') + G_F(x' - x, x'))$$

$$\begin{aligned} \mathcal{G}_T(x) &= \frac{1}{2} \int_x^{\epsilon(x)} dx' \frac{\Delta G(x')}{x'} \\ &\quad - \int_x^{\epsilon(x)} dx' \int dx'' \frac{1}{2x'^2} \left[P \frac{1}{x' - x''} \left(\frac{\partial}{\partial x'} - \frac{\partial}{\partial x''} \right) (2N(x', x' - x'') - 2N(x'', x'' - x')) \right. \\ &\quad \left. + P \frac{1}{x' - x''} \left(\frac{\partial}{\partial x'} + \frac{\partial}{\partial x''} \right) (4N(x', x'') - 6N(x'', x'' - x') - 6N(x', x' - x'')) \right. \\ &\quad \left. + P \frac{1}{x' - x''} P \frac{1}{x'} (8N(x'', x'' - x') + 8N(x', x' - x'')) + \left(P \frac{1}{x'} - \frac{\partial}{\partial x'} \right) (G_F(x'' + x', x'') + G_F(x'' - x', x'')) \right] \end{aligned}$$

$$\epsilon(x) = \frac{x}{|x|}$$

$$\int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle p S | F^{\mu n}(0) F^{\nu n}(\lambda n) | p S \rangle = -\frac{x}{2} \left[G(x) g_{\perp}^{\mu\nu} + \Delta G(x) i\epsilon^{\mu\nu pn} (S \cdot n) M_N + 2\mathcal{G}_T(x) i\epsilon^{\mu\nu\alpha n} S_{\perp\alpha} M_N + \dots \right]$$

$$\int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle p S | \bar{\psi}(0) \gamma^{\sigma} \gamma_5 \psi(\lambda n) | p S \rangle = 2M_N \left[\Delta g(x) (S \cdot n) p^{\sigma} + \mathbf{g}_T(x) S_{\perp}^{\sigma} + \dots \right]$$

$$\int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle p S_{\perp} | F^{\mu+}(0) F^{-+}(\lambda n) | p S_{\perp} \rangle = x \mathcal{G}_T(x) i\epsilon^{\mu+ \alpha} S_{\perp\alpha} M_N$$

$$F^{-+} = -\partial^+ A^- = -\frac{1}{\partial_-} \left(D_{\perp j} F^{j+} + g \bar{\psi} t^a \gamma^+ \psi t^a \right) \quad D_{\nu} F^{\mu\nu} = g \bar{\psi} t^a \gamma^{\mu} \psi t^a$$

$$-i \int \frac{d\lambda}{2\pi} \int \frac{d\zeta}{2\pi} e^{i\lambda x_1} e^{i\mu(x_2-x_1)} \langle p S_{\perp} | F^{\mu n}(0) D_{\perp}^{\sigma}(\zeta n) F^{\nu n}(\lambda n) | p S_{\perp} \rangle = M^{\nu\mu\sigma}(x_1, x_2)$$

$$M^{\nu\mu\sigma}(x_1, x_2) = 2iM_N \left[\mathbf{g}_{\perp}^{\nu\mu} S_{\perp\alpha} \varepsilon^{\sigma pn\alpha} M_1(x_1, x_2) + \mathbf{g}_{\perp}^{\mu\sigma} S_{\perp\alpha} \varepsilon^{\nu pn\alpha} M_2(x_1, x_2) - \mathbf{g}_{\perp}^{\sigma\nu} S_{\perp\alpha} \varepsilon^{\mu pn\alpha} M_2(x_2, x_1) \right]$$

$$\int \frac{d\lambda}{2\pi} \int \frac{d\mu}{2\pi} e^{i\lambda x_1} e^{i\mu(x_2-x_1)} \langle p S_{\perp} | \bar{\psi}(0) \gamma^{\mu} g F^{\alpha n}(\mu n) \psi(\lambda n) | p S_{\perp} \rangle = M_N p^{\mu} \epsilon^{\alpha pn} S_{\perp} G_F(x_1, x_2)$$

$$\mathcal{G}_T(x) = -\frac{1}{2x^2} \int dx' \left(8M_2(x, x') - 4M_2(x', x) - 4M_1(x', x) + G_F(x' + x, x') + G_F(x' - x, x') \right)$$

$$\int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle p S | F^{\mu n}(0) F^{\nu n}(\lambda n) | p S \rangle = -\frac{x}{2} \left[G(x) g_{\perp}^{\mu\nu} + \Delta G(x) i\epsilon^{\mu\nu\rho n} (S \cdot n) M_N + 2\mathcal{G}_T(x) i\epsilon^{\mu\nu\alpha n} S_{\perp\alpha} M_N + \dots \right]$$

$$\begin{aligned} & z^{\nu} \frac{\partial}{\partial z^{\nu}} F^{\mu z}(0) \tilde{F}_{\mu}^{\rho}(z) - z^{\nu} \frac{\partial}{\partial z^{\rho}} F^{\mu z}(0) \tilde{F}_{\mu\nu}(z) \quad z^2 \rightarrow 0 \\ &= i \int_0^1 dt F^{\mu z}(0) t g F^{\rho z}(tz) \tilde{F}_{\mu}^z(z) - i \int_0^1 dt F^{\mu z}(0) g F_{\mu}^z(tz) \tilde{F}^{\rho z}(z) + \int_0^1 du \left(i \int_0^1 dt F^{\mu z}(0) g u F^{\rho z}(tuz) \tilde{F}_{\mu}^z(uz) \right. \\ &\quad \left. - i \int_0^1 dt F^{\mu z}(0) g u F_{\mu}^z(tuz) \tilde{F}^{\rho z}(uz) - i \int_0^1 dt F^{\rho z}(0) g u F^{\mu z}(tuz) \tilde{F}_{\mu}^z(uz) \right. \\ &\quad \left. - F^{\mu z}(0) \epsilon_{\mu z \sigma}^{\rho} D_{\lambda} F^{\lambda \sigma}(uz) + F^{\mu z}(0) \bar{D}_{\mu} \tilde{F}^{\rho z}(uz) + F^{\rho z}(0) D^{\mu} \tilde{F}_{\mu}^z(uz) \right) - F^{\mu z}(0) \epsilon_{\mu z \sigma}^{\rho} D_{\lambda} F^{\lambda \sigma}(z) + F^{\mu z}(0) \bar{D}_{\mu} \tilde{F}^{\rho z}(z) \end{aligned}$$

$$i \int \frac{d\lambda}{2\pi} \int \frac{d\zeta}{2\pi} e^{i\lambda x_1} e^{i\mu(x_2-x_1)} \langle p S_{\perp} | F_a^{\mu n}(0) g F_c^{\sigma n}(\zeta n) F_b^{\nu n}(\lambda n) | p S_{\perp} \rangle = \frac{1}{24} i f^{abc} N^{\nu\mu\sigma}(x_1, x_2) + \dots$$

$$N^{\nu\mu\sigma}(x_1, x_2) = 2i M_N \left[g_{\perp}^{\nu\mu} S_{\perp\alpha} \epsilon^{\sigma\rho n\alpha} N(x_1, x_2) - g_{\perp}^{\mu\sigma} S_{\perp\alpha} \epsilon^{\nu\rho n\alpha} N(x_2, x_2 - x_1) - g_{\perp}^{\sigma\nu} S_{\perp\alpha} \epsilon^{\mu\rho n\alpha} N(x_1, x_1 - x_2) \right]$$

$$D_{\nu} F^{\mu\nu} = g \bar{\psi} t^a \gamma^{\mu} \psi t^a \quad D_{\mu} \tilde{F}^{\mu\nu} = 0$$

$$\int \frac{d\lambda}{2\pi} \int \frac{d\mu}{2\pi} e^{i\lambda x_1} e^{i\mu(x_2-x_1)} \langle p S_{\perp} | \bar{\psi}(0) \gamma^{\mu} g F^{\alpha n}(\mu n) \psi(\lambda n) | p S_{\perp} \rangle = M_N p^{\mu} \epsilon^{\alpha\rho n S_{\perp}} G_F(x_1, x_2)$$

$$\begin{aligned} 2x^2 \frac{\partial}{\partial x} \mathcal{G}_T(x) + x \Delta G(x) &= \int dx' P \frac{1}{x-x'} \left(\frac{\partial}{\partial x} - \frac{\partial}{\partial x'} \right) (2N(x, x-x') - 2N(x', x'-x)) \\ &\quad + \int dx' P \frac{1}{x-x'} \left(\frac{\partial}{\partial x} + \frac{\partial}{\partial x'} \right) (4N(x, x') - 6N(x', x'-x) - 6N(x, x-x')) \\ &\quad + \int dx' P \frac{1}{x-x'} P \frac{1}{x} (8N(x', x'-x) + 8N(x, x-x')) + \int dx' \left(P \frac{1}{x} - \frac{\partial}{\partial x} \right) (G_F(x'+x, x') + G_F(x'-x, x')) \end{aligned}$$

$$\mathcal{G}_T(x) = -\frac{1}{2x^2} \int dx' (8M_2(x, x') - 4M_2(x', x) - 4M_1(x', x) + G_F(x' + x, x') + G_F(x' - x, x'))$$

$$\begin{aligned} \mathcal{G}_T(x) &= \frac{1}{2} \int_x^{\epsilon(x)} dx' \frac{\Delta G(x')}{x'} \\ &\quad - \int_x^{\epsilon(x)} dx' \int dx'' \frac{1}{2x'^2} \left[P \frac{1}{x' - x''} \left(\frac{\partial}{\partial x'} - \frac{\partial}{\partial x''} \right) (2N(x', x' - x'') - 2N(x'', x'' - x')) \right. \\ &\quad \left. + P \frac{1}{x' - x''} \left(\frac{\partial}{\partial x'} + \frac{\partial}{\partial x''} \right) (4N(x', x'') - 6N(x'', x'' - x') - 6N(x', x' - x'')) \right. \\ &\quad \left. + P \frac{1}{x' - x''} P \frac{1}{x'} (8N(x'', x'' - x') + 8N(x', x' - x'')) + \left(P \frac{1}{x'} - \frac{\partial}{\partial x'} \right) (G_F(x'' + x', x'') + G_F(x'' - x', x'')) \right] \end{aligned}$$

$$\epsilon(x) = \frac{x}{|x|}$$

$$i \int \frac{d\lambda}{2\pi} \int \frac{d\zeta}{2\pi} e^{i\lambda x_1} e^{i\mu(x_2-x_1)} \langle p S_{\perp} | F_a^{\mu n}(0) g F_c^{\sigma n}(\zeta n) F_b^{\nu n}(\lambda n) | p S_{\perp} \rangle$$

$$= \frac{1}{24} i f^{abc} N^{\nu\mu\sigma}(x_1, x_2) + \frac{3}{40} d^{abc} O^{\nu\mu\sigma}(x_1, x_2)$$

$$N^{\nu\mu\sigma}(x_1, x_2) = 2iM_N \left[g_{\perp}^{\nu\mu} S_{\perp\alpha} \varepsilon^{\sigma p n \alpha} N(x_1, x_2) - g_{\perp}^{\mu\sigma} S_{\perp\alpha} \varepsilon^{\nu p n \alpha} N(x_2, x_2 - x_1) - g_{\perp}^{\sigma\nu} S_{\perp\alpha} \varepsilon^{\mu p n \alpha} N(x_1, x_1 - x_2) \right]$$

$$-i \int \frac{d\lambda}{2\pi} \int \frac{d\zeta}{2\pi} e^{i\lambda x_1} e^{i\mu(x_2-x_1)} \langle p S_{\perp} | F^{\mu n}(0) D_{\perp}^{\sigma}(\zeta n) F^{\nu n}(\lambda n) | p S_{\perp} \rangle$$

$$= M^{\nu\mu\sigma}(x_1, x_2) \quad \underbrace{\hspace{10em}}_{W_{\zeta, \lambda} \equiv \text{P exp} \left(ig \int_{\lambda}^{\zeta} dt n_{\mu} A^{\mu}(tn) \right)}$$

$$M^{\nu\mu\sigma}(x_1, x_2) = 2iM_N \left[g_{\perp}^{\nu\mu} S_{\perp\alpha} \varepsilon^{\sigma p n \alpha} M_1(x_1, x_2) + g_{\perp}^{\mu\sigma} S_{\perp\alpha} \varepsilon^{\nu p n \alpha} M_2(x_1, x_2) - g_{\perp}^{\sigma\nu} S_{\perp\alpha} \varepsilon^{\mu p n \alpha} M_2(x_2, x_1) \right]$$

$$D_{\perp}^{\sigma}(\zeta n) W_{\zeta, \lambda} = i \int_{\lambda}^{\zeta} d\mu W_{\zeta, \mu} g F^{\sigma n}(\mu n) W_{\mu, \lambda} + W_{\zeta, \lambda} D_{\perp}^{\sigma}(\lambda n)$$

$$M_1(x_1, x_2) = \mathcal{P} \frac{N(x_1, x_2)}{x_2 - x_1} \quad M_2(x_1, x_2) = \mathcal{P} \frac{N(x_2, x_2 - x_1)}{x_1 - x_2} + \delta(x_1 - x_2) \tilde{g}(x_1)$$

$$\int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle p S | F^{\mu n}(0) \left(D_{\perp}^{\sigma}(\lambda n) + ig \int d\mu \frac{\epsilon(\mu - \lambda)}{2} F^{\sigma n}(\mu n) \right) F^{\nu n}(\lambda n) | p S \rangle = 2\epsilon^{np\mu\nu} S_{\perp}^{\sigma} \tilde{g}(x)$$

$$\mathcal{G}_T(x) = -\frac{1}{2x^2} \int dx' (8M_2(x, x') - 4M_2(x', x) - 4M_1(x', x) + G_F(x' + x, x') + G_F(x' - x, x'))$$

$$\begin{aligned} \mathcal{G}_T(x) &= \frac{1}{2} \int_x^{\epsilon(x)} dx' \frac{\Delta G(x')}{x'} \\ &\quad - \int_x^{\epsilon(x)} dx' \int dx'' \frac{1}{2x'^2} \left[P \frac{1}{x' - x''} \left(\frac{\partial}{\partial x'} - \frac{\partial}{\partial x''} \right) (2N(x', x' - x'') - 2N(x'', x'' - x')) \right. \\ &\quad \left. + P \frac{1}{x' - x''} \left(\frac{\partial}{\partial x'} + \frac{\partial}{\partial x''} \right) (4N(x', x'') - 6N(x'', x'' - x') - 6N(x', x' - x'')) \right. \\ &\quad \left. + P \frac{1}{x' - x''} P \frac{1}{x'} (8N(x'', x'' - x') + 8N(x', x' - x'')) + \left(P \frac{1}{x'} - \frac{\partial}{\partial x'} \right) (G_F(x'' + x', x'') + G_F(x'' - x', x'')) \right] \end{aligned}$$

$$\epsilon(x) = \frac{x}{|x|}$$

$$M_1(x_1, x_2) = \mathcal{P} \frac{N(x_1, x_2)}{x_2 - x_1}$$

$$M_2(x_1, x_2) = \mathcal{P} \frac{N(x_2, x_2 - x_1)}{x_1 - x_2} + \delta(x_1 - x_2) \tilde{g}(x_1)$$

$$\mathcal{G}_T(x) = -\frac{1}{2x^2} \int dx' (8M_2(x, x') - 4M_2(x', x) - 4M_1(x', x) + G_F(x' + x, x') + G_F(x' - x, x'))$$

$$\begin{aligned} \mathcal{G}_T(x) &= \frac{1}{2} \int_x^{\epsilon(x)} dx' \frac{\Delta G(x')}{x'} \\ &\quad - \int_x^{\epsilon(x)} dx' \int dx'' \frac{1}{2x'^2} \left[P \frac{1}{x' - x''} \left(\frac{\partial}{\partial x'} - \frac{\partial}{\partial x''} \right) (2N(x', x' - x'') - 2N(x'', x'' - x')) \right. \\ &\quad \left. + P \frac{1}{x' - x''} \left(\frac{\partial}{\partial x'} + \frac{\partial}{\partial x''} \right) (4N(x', x'') - 6N(x'', x'' - x') - 6N(x', x' - x'')) \right. \\ &\quad \left. + P \frac{1}{x' - x''} P \frac{1}{x'} (8N(x'', x'' - x') + 8N(x', x' - x'')) + \left(P \frac{1}{x'} - \frac{\partial}{\partial x'} \right) (G_F(x'' + x', x'') + G_F(x'' - x', x'')) \right] \end{aligned}$$

$$\epsilon(x) = \frac{x}{|x|}$$

$$\begin{aligned} \tilde{g}(x) &= x^2 \int_x^{\epsilon(x)} dx' \int dx'' \frac{1}{x'^3} P \frac{2x' - x''}{(x' - x'')^2} (N(x', x' - x'') - N(x'', x'' - x')) \\ &\quad + x^2 \int_x^{\epsilon(x)} dx' \int dx'' \frac{1}{x'^3} P \frac{1}{x' - x''} (2N(x', x'') - N(x'', x'' - x') - N(x', x' - x'')) \\ &\quad - \frac{1}{2} x^2 \int_x^{\epsilon(x)} dx' \int dx'' \frac{1}{2x'^3} (G_F(x'' + x', x'') + G_F(x'' - x', x'')) - \frac{1}{4} x^2 \int_x^{\epsilon(x)} dx' \frac{\Delta G(x')}{x'} \end{aligned}$$

$$i \int \frac{d\lambda}{2\pi} \int \frac{d\zeta}{2\pi} e^{i\lambda x_1} e^{i\mu(x_2-x_1)} \langle p S_{\perp} | F_a^{\mu n}(0) g F_c^{\sigma n}(\zeta n) F_b^{\nu n}(\lambda n) | p S_{\perp} \rangle$$

$$= \frac{1}{24} i f^{abc} N^{\nu\mu\sigma}(x_1, x_2) + \frac{3}{40} d^{abc} O^{\nu\mu\sigma}(x_1, x_2)$$

$$N^{\nu\mu\sigma}(x_1, x_2) = 2iM_N \left[g_{\perp}^{\nu\mu} S_{\perp\alpha} \varepsilon^{\sigma p n \alpha} N(x_1, x_2) - g_{\perp}^{\mu\sigma} S_{\perp\alpha} \varepsilon^{\nu p n \alpha} N(x_2, x_2 - x_1) - g_{\perp}^{\sigma\nu} S_{\perp\alpha} \varepsilon^{\mu p n \alpha} N(x_1, x_1 - x_2) \right]$$

$$-i \int \frac{d\lambda}{2\pi} \int \frac{d\zeta}{2\pi} e^{i\lambda x_1} e^{i\mu(x_2-x_1)} \langle p S_{\perp} | F^{\mu n}(0) D_{\perp}^{\sigma}(\zeta n) F^{\nu n}(\lambda n) | p S_{\perp} \rangle$$

$$= M^{\nu\mu\sigma}(x_1, x_2) \quad \underbrace{\hspace{10em}}_{W_{\zeta, \lambda} \equiv \text{P exp} \left(ig \int_{\lambda}^{\zeta} dt n_{\mu} A^{\mu}(tn) \right)}$$

$$M^{\nu\mu\sigma}(x_1, x_2) = 2iM_N \left[g_{\perp}^{\nu\mu} S_{\perp\alpha} \varepsilon^{\sigma p n \alpha} M_1(x_1, x_2) + g_{\perp}^{\mu\sigma} S_{\perp\alpha} \varepsilon^{\nu p n \alpha} M_2(x_1, x_2) - g_{\perp}^{\sigma\nu} S_{\perp\alpha} \varepsilon^{\mu p n \alpha} M_2(x_2, x_1) \right]$$

$$D_{\perp}^{\sigma}(\zeta n) W_{\zeta, \lambda} = i \int_{\lambda}^{\zeta} d\mu W_{\zeta, \mu} g F^{\sigma n}(\mu n) W_{\mu, \lambda} + W_{\zeta, \lambda} D_{\perp}^{\sigma}(\lambda n)$$

$$M_1(x_1, x_2) = \mathcal{P} \frac{N(x_1, x_2)}{x_2 - x_1} \quad M_2(x_1, x_2) = \mathcal{P} \frac{N(x_2, x_2 - x_1)}{x_1 - x_2} + \delta(x_1 - x_2) \tilde{g}(x_1)$$

$$\int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle p S | F^{\mu n}(0) \left(D_{\perp}^{\sigma}(\lambda n) + ig \int d\mu \frac{\epsilon(\mu - \lambda)}{2} F^{\sigma n}(\mu n) \right) F^{\nu n}(\lambda n) | p S \rangle = 2\epsilon^{np\mu\nu} S_{\perp}^{\sigma} \tilde{g}(x)$$

$$\langle pS_{\perp} | \bar{\psi}(0) \mathbf{g} F^{\mu+}(\zeta n) \psi(\lambda n) | pS_{\perp} \rangle = \frac{M_N}{4} \not{p} S_{\perp \alpha} P_{\beta} \varepsilon^{\alpha\beta\mu+} G_F + i \frac{M_N}{4} \gamma_5 \not{p} S_{\perp}^{\mu} \tilde{G}_F$$

$$D_{\perp}^{\mu}(\zeta n) \qquad \qquad \qquad G_D \qquad \qquad \qquad \tilde{G}_D$$

$$G_D(x_1, x_2) = P \frac{1}{x_1 - x_2} G_F(x_1, x_2),$$

$$\tilde{G}_D(x_1, x_2) = \delta(x_1 - x_2) \tilde{g}(x_1) + P \frac{1}{x_1 - x_2} \tilde{G}_F(x_1, x_2)$$

$$\tilde{g}(x) = -x \int_x^{\epsilon(x)} dx_1 \left[\frac{2\Delta q(x_1)}{x_1} + \frac{1}{x_1^2} P \int_{-1}^1 dx_2 \left\{ \frac{G_F(x_1, x_2)}{x_1 - x_2} + (3x_1 - x_2) \frac{\tilde{G}_F(x_1, x_2)}{(x_1 - x_2)^2} \right\} \right]$$

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$$\mathcal{G}_T(x) = -\frac{1}{2x^2} \int dx' (8M_2(x, x') - 4M_2(x', x) - 4M_1(x', x) + G_F(x' + x, x') + G_F(x' - x, x'))$$

$$\begin{aligned} \mathcal{G}_T(x) &= \frac{1}{2} \int_x^{\epsilon(x)} dx' \frac{\Delta G(x')}{x'} \\ &\quad - \int_x^{\epsilon(x)} dx' \int dx'' \frac{1}{2x'^2} \left[P \frac{1}{x' - x''} \left(\frac{\partial}{\partial x'} - \frac{\partial}{\partial x''} \right) (2N(x', x' - x'') - 2N(x'', x'' - x')) \right. \\ &\quad \left. + P \frac{1}{x' - x''} \left(\frac{\partial}{\partial x'} + \frac{\partial}{\partial x''} \right) (4N(x', x'') - 6N(x'', x'' - x') - 6N(x', x' - x'')) \right. \\ &\quad \left. + P \frac{1}{x' - x''} P \frac{1}{x'} (8N(x'', x'' - x') + 8N(x', x' - x'')) + \left(P \frac{1}{x'} - \frac{\partial}{\partial x'} \right) (G_F(x'' + x', x'') + G_F(x'' - x', x'')) \right] \end{aligned}$$

$$\epsilon(x) = \frac{x}{|x|}$$

$$\begin{aligned} \tilde{g}(x) &= x^2 \int_x^{\epsilon(x)} dx' \int dx'' \frac{1}{x'^3} P \frac{2x' - x''}{(x' - x'')^2} (N(x', x' - x'') - N(x'', x'' - x')) \\ &\quad + x^2 \int_x^{\epsilon(x)} dx' \int dx'' \frac{1}{x'^3} P \frac{1}{x' - x''} (2N(x', x'') - N(x'', x'' - x') - N(x', x' - x'')) \\ &\quad - \frac{1}{2} x^2 \int_x^{\epsilon(x)} dx' \int dx'' \frac{1}{2x'^3} (G_F(x'' + x', x'') + G_F(x'' - x', x'')) - \frac{1}{4} x^2 \int_x^{\epsilon(x)} dx' \frac{\Delta G(x')}{x'} \end{aligned}$$

$$\begin{aligned}
\mathcal{G}_T(x) &= \frac{1}{2} \int_x^{\epsilon(x)} dx' \frac{\Delta G(x')}{x'} \\
&\quad - \int_x^{\epsilon(x)} dx' \int dx'' \frac{1}{2x'^2} \left[P \frac{1}{x' - x''} \left(\frac{\partial}{\partial x'} - \frac{\partial}{\partial x''} \right) (2N(x', x' - x'') - 2N(x'', x'' - x')) \right. \\
&\quad + P \frac{1}{x' - x''} \left(\frac{\partial}{\partial x'} + \frac{\partial}{\partial x''} \right) (4N(x', x'') - 6N(x'', x'' - x') - 6N(x', x' - x'')) \\
&\quad \left. + P \frac{1}{x' - x''} P \frac{1}{x'} (8N(x'', x'' - x') + 8N(x', x' - x'')) + \left(P \frac{1}{x'} - \frac{\partial}{\partial x'} \right) (G_F(x'' + x', x'') + G_F(x'' - x', x'')) \right] \\
\int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle p S | F^{\mu n}(0) F^{\nu n}(\lambda n) | p S \rangle &= -\frac{x}{2} \left[G(x) g_{\perp}^{\mu\nu} + \Delta G(x) i\epsilon^{\mu\nu\rho n} (S \cdot n) M_N + 2\mathcal{G}_T(x) i\epsilon^{\mu\nu\alpha n} S_{\perp\alpha} M_N + \dots \right]
\end{aligned}$$

$$\begin{aligned}
\mathcal{G}_T(x) &= \frac{1}{2} \int_x^{\epsilon(x)} dx' \frac{\Delta G(x')}{x'} \\
&\quad - \int_x^{\epsilon(x)} dx' \int dx'' \frac{1}{2x'^2} \left[P \frac{1}{x' - x''} \left(\frac{\partial}{\partial x'} - \frac{\partial}{\partial x''} \right) (2N(x', x' - x'') - 2N(x'', x'' - x')) \right. \\
&\quad + P \frac{1}{x' - x''} \left(\frac{\partial}{\partial x'} + \frac{\partial}{\partial x''} \right) (4N(x', x'') - 6N(x'', x'' - x') - 6N(x', x' - x'')) \\
&\quad \left. + P \frac{1}{x' - x''} P \frac{1}{x'} (8N(x'', x'' - x') + 8N(x', x' - x'')) + \left(P \frac{1}{x'} - \frac{\partial}{\partial x'} \right) (G_F(x'' + x', x'') + G_F(x'' - x', x'')) \right]
\end{aligned}$$

$$\int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle p S | F^{\mu n}(0) F^{\nu n}(\lambda n) | p S \rangle = -\frac{x}{2} \left[G(x) g_{\perp}^{\mu\nu} + \Delta G(x) i\epsilon^{\mu\nu\rho n} (S \cdot n) M_N + 2\mathcal{G}_T(x) i\epsilon^{\mu\nu\alpha n} S_{\perp\alpha} M_N + \dots \right]$$

$$\int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle p S | \bar{\psi}(0) \gamma^{\sigma} \gamma_5 \psi(\lambda n) | p S \rangle = 2M_N \left[\Delta g(x) (S \cdot n) p^{\sigma} + g_T(x) S_{\perp}^{\sigma} + \dots \right]$$

$$g_T(x) = \int_x^{\epsilon(x)} \frac{dx_1}{x_1} \left[\Delta q(x_1) + \frac{1}{2} P \int_{-1}^1 dx_2 \frac{1}{x_1 - x_2} \left\{ \left(\frac{\partial}{\partial x_1} + \frac{\partial}{\partial x_2} \right) G_F(x_1, x_2) + \left(\frac{\partial}{\partial x_1} - \frac{\partial}{\partial x_2} \right) \tilde{G}_F(x_1, x_2) \right\} \right]$$

WW

genuine twist-3

$$\begin{aligned}
\mathcal{G}_T(x) &= \frac{1}{2} \int_x^{\epsilon(x)} dx' \frac{\Delta G(x')}{x'} \quad \text{WW} \\
&\quad - \int_x^{\epsilon(x)} dx' \int dx'' \frac{1}{2x'^2} \left[P \frac{1}{x' - x''} \left(\frac{\partial}{\partial x'} - \frac{\partial}{\partial x''} \right) (2N(x', x' - x'') - 2N(x'', x'' - x')) \right. \\
&\quad + P \frac{1}{x' - x''} \left(\frac{\partial}{\partial x'} + \frac{\partial}{\partial x''} \right) (4N(x', x'') - 6N(x'', x'' - x') - 6N(x', x' - x'')) \quad \text{genuine twist-3} \\
&\quad \left. + P \frac{1}{x' - x''} P \frac{1}{x'} (8N(x'', x'' - x') + 8N(x', x' - x'')) + \left(P \frac{1}{x'} - \frac{\partial}{\partial x'} \right) (G_F(x'' + x', x'') + G_F(x'' - x', x'')) \right]
\end{aligned}$$

$$\int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle p S | F^{\mu n}(0) F^{\nu n}(\lambda n) | p S \rangle = -\frac{x}{2} \left[G(x) g_{\perp}^{\mu\nu} + \Delta G(x) i\epsilon^{\mu\nu\rho n} (S \cdot n) M_N + 2\mathcal{G}_T(x) i\epsilon^{\mu\nu\alpha n} S_{\perp\alpha} M_N + \dots \right]$$

$$\int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle p S | \bar{\psi}(0) \gamma^{\sigma} \gamma_5 \psi(\lambda n) | p S \rangle = 2M_N \left[\Delta g(x) (S \cdot n) p^{\sigma} + g_T(x) S_{\perp}^{\sigma} + \dots \right]$$

$$g_T(x) = \int_x^{\epsilon(x)} \frac{dx_1}{x_1} \left[\Delta q(x_1) + \frac{1}{2} P \int_{-1}^1 dx_2 \frac{1}{x_1 - x_2} \left\{ \left(\frac{\partial}{\partial x_1} + \frac{\partial}{\partial x_2} \right) G_F(x_1, x_2) + \left(\frac{\partial}{\partial x_1} - \frac{\partial}{\partial x_2} \right) \tilde{G}_F(x_1, x_2) \right\} \right]$$

WW

genuine twist-3

$$\begin{aligned}
\mathcal{G}_T(x) &= \frac{1}{2} \int_x^{\epsilon(x)} dx' \frac{\Delta G(x')}{x'} \quad \text{WW} \\
&\quad - \int_x^{\epsilon(x)} dx' \int dx'' \frac{1}{2x'^2} \left[P \frac{1}{x' - x''} \left(\frac{\partial}{\partial x'} - \frac{\partial}{\partial x''} \right) (2N(x', x' - x'') - 2N(x'', x'' - x')) \right. \\
&\quad \left. + P \frac{1}{x' - x''} \left(\frac{\partial}{\partial x'} + \frac{\partial}{\partial x''} \right) (4N(x', x'') - 6N(x'', x'' - x') - 6N(x', x' - x'')) \right] \quad \text{genuine twist-3} \\
&\quad + P \frac{1}{x' - x''} P \frac{1}{x'} (8N(x'', x'' - x') + 8N(x', x' - x'')) + \left(P \frac{1}{x'} - \frac{\partial}{\partial x'} \right) (G_F(x'' + x', x'') + G_F(x'' - x', x'')) \quad \left. \right]
\end{aligned}$$

$$\begin{aligned}
\int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle p S | F^{\mu n}(0) F^{\nu n}(\lambda n) | p S \rangle &= -\frac{x}{2} \left[G(x) g_{\perp}^{\mu\nu} + \Delta G(x) i\epsilon^{\mu\nu\rho n} (S \cdot n) M_N + 2\mathcal{G}_T(x) i\epsilon^{\mu\nu\alpha n} S_{\perp\alpha} M_N + \dots \right] \\
\int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle p S | \bar{\psi}(0) \gamma^{\sigma} \gamma_5 \psi(\lambda n) | p S \rangle &= 2M_N \left[\Delta g(x) (S \cdot n) p^{\sigma} + g_T(x) S_{\perp}^{\sigma} + \dots \right]
\end{aligned}$$

$$g_T(x) = \int_x^{\epsilon(x)} \frac{dx_1}{x_1} \left[\Delta q(x_1) + \frac{1}{2} P \int_{-1}^1 dx_2 \frac{1}{x_1 - x_2} \left\{ \left(\frac{\partial}{\partial x_1} + \frac{\partial}{\partial x_2} \right) G_F(x_1, x_2) + \left(\frac{\partial}{\partial x_1} - \frac{\partial}{\partial x_2} \right) \tilde{G}_F(x_1, x_2) \right\} \right]$$

WW
genuine twist-3

$$\int dx g_T(x) = \int dx \Delta q(x) \equiv \Delta q \qquad \int_{-1}^1 dx \mathcal{G}_T(x) = \int_0^1 dx \Delta G(x) = a_0 \equiv \Delta G$$

$$\langle p \mathbf{S}_\perp | -\int d\lambda \epsilon(\lambda) F^{\beta n}(0) F^{\alpha n}(\lambda n) | p \mathbf{S}_\perp \rangle = 2\epsilon^{n\alpha\beta\sigma} S_{\perp\sigma} \int dx \mathcal{G}_T(x) = 2\epsilon^{n\alpha\beta\sigma} S_{\perp\sigma} \Delta G$$

$$\begin{aligned}
\langle p \mathbf{S}_\perp | -\int d\lambda \epsilon(\lambda) F^{\beta n}(0) F^{\alpha n}(\lambda n) | p \mathbf{S}_\perp \rangle &= 2\epsilon^{n\alpha\beta\sigma} \mathbf{S}_{\perp\sigma} \int dx \mathcal{G}_T(x) = 2\epsilon^{n\alpha\beta\sigma} \mathbf{S}_{\perp\sigma} \Delta G \\
&\sim \langle p \mathbf{S}_\perp | \mathcal{M}_{g\text{-spin}}^{+\alpha\beta} | p \mathbf{S}_\perp \rangle
\end{aligned}$$

$$\mathcal{M}_{g\text{-spin}}^{\mu\alpha\beta} = F^{\mu\beta} A^\alpha - F^{\mu\alpha} A^\beta$$

$$A^\alpha(0) = \frac{1}{2} \int d\lambda \epsilon(\lambda) F^{\alpha n}(\lambda n)$$

$$\langle p \mathbf{S}_\perp | -\int d\lambda \epsilon(\lambda) F^{\beta n}(0) F^{\alpha n}(\lambda n) | p \mathbf{S}_\perp \rangle = 2\epsilon^{n\alpha\beta\sigma} S_{\perp\sigma} \int dx \mathcal{G}_T(x) = 2\epsilon^{n\alpha\beta\sigma} S_{\perp\sigma} \Delta G$$

$$\sim \langle p \mathbf{S}_\perp | \mathcal{M}_{g\text{-spin}}^{+\alpha\beta} | p \mathbf{S}_\perp \rangle \quad \mathcal{M}_{g\text{-spin}}^{\mu\alpha\beta} = F^{\mu\beta} A^\alpha - F^{\mu\alpha} A^\beta$$

$$\frac{1}{2} = L + \frac{1}{2} \Delta\Sigma + \Delta G$$

transverse spin sum rule

$$A^\alpha(0) = \frac{1}{2} \int d\lambda \epsilon(\lambda) F^{\alpha n}(\lambda n)$$

$$\langle p \mathbf{S}_\perp | -\int d\lambda \epsilon(\lambda) F^{\beta n}(0) F^{\alpha n}(\lambda n) | p \mathbf{S}_\perp \rangle = 2\epsilon^{n\alpha\beta\sigma} S_{\perp\sigma} \int dx \mathcal{G}_T(x) = 2\epsilon^{n\alpha\beta\sigma} S_{\perp\sigma} \Delta G$$

$$\sim \langle p \mathbf{S}_\perp | \mathcal{M}_{g\text{-spin}}^{+\alpha\beta} | p \mathbf{S}_\perp \rangle \quad \mathcal{M}_{g\text{-spin}}^{\mu\alpha\beta} = F^{\mu\beta} A^\alpha - F^{\mu\alpha} A^\beta$$

$$\frac{1}{2} = L + \frac{1}{2} \Delta\Sigma + \Delta G \quad \text{transverse spin sum rule}$$

$$A^\alpha(0) = \frac{1}{2} \int d\lambda \epsilon(\lambda) F^{\alpha n}(\lambda n)$$

$$L \stackrel{?}{=} L_q + L_g$$

$$\langle p \mathbf{S}_\perp | -\int d\lambda \epsilon(\lambda) F^{\beta n}(0) F^{\alpha n}(\lambda n) | p \mathbf{S}_\perp \rangle = 2\epsilon^{n\alpha\beta\sigma} \mathbf{S}_{\perp\sigma} \int dx \mathcal{G}_T(x) = 2\epsilon^{n\alpha\beta\sigma} \mathbf{S}_{\perp\sigma} \Delta G$$

$$\sim \langle p \mathbf{S}_\perp | \mathcal{M}_{g\text{-spin}}^{+\alpha\beta} | p \mathbf{S}_\perp \rangle \quad \mathcal{M}_{g\text{-spin}}^{\mu\alpha\beta} = F^{\mu\beta} A^\alpha - F^{\mu\alpha} A^\beta$$

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$$J_{q,g} = \frac{1}{2} (A_{q,g} + B_{q,g}) \quad \text{Ji, Xiong, Yuan, PLB717 ('12) 214}$$

$$\mathcal{M}_{q,g}^{\lambda\mu\nu} = x^\mu T_{q,g}^{\lambda\nu} - x^\nu T_{q,g}^{\lambda\mu} \quad \bar{p}^\mu = \frac{p^\mu + p'^\mu}{2} \quad \Delta^\mu = p'^\mu - p^\mu$$

$$\langle p' S' | T_{q,g}^{\mu\nu} | p S \rangle = \bar{u}(p', S') \left[A_{q,g} \gamma^{(\mu} \bar{p}^{\nu)} + B_{q,g} \frac{\bar{p}^{(\mu} i\sigma^{\nu)\alpha} \Delta_\alpha}{2M} + C_{q,g} \frac{\Delta^\mu \Delta^\nu - g^{\mu\nu} \Delta^2}{M} + \bar{C}_{q,g} M g^{\mu\nu} \right] u(p, S)$$

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$$J_{q,g} = \frac{1}{2} (A_{q,g} + B_{q,g}) + \frac{p^3}{2(p^0 + M_N)} \bar{C}_{q,g} \quad \text{Hatta, K.T., Yoshida, JHEP1302 ('13) 003}$$

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$$L = L_q + L_g \quad \left(\frac{1}{2} = J_q + J_g \right) \quad \text{frame dependent!}$$

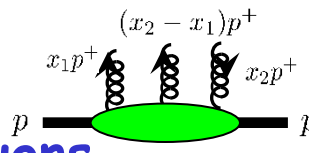
Summary

SSA in SIDIS $ep^\uparrow \rightarrow eDX$ at $P_{h\perp} \gg \Lambda_{\text{QCD}}$

Twist-3 mechanism from three-gluon correlation inside the nucleon
 photon-gluon fusion

Factorization formula for twist-3 SSA

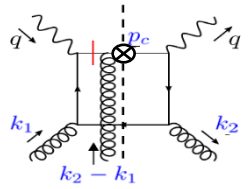
gauge invariance permutation symmetry for three gluons



partonic hard parts from **SGP** contributions

convoluted with

$$\begin{cases} N(x,x), O(x,x), \frac{dN(x,x)}{dx}, \frac{dO(x,x)}{dx} \\ N(x,0), O(x,0), \frac{dN(x,0)}{dx}, \frac{dO(x,0)}{dx} \end{cases}$$



5 different azimuthal dependences

Numerical estimates of A_N : $\langle 1 \rangle$, $\langle \sin 2\phi \rangle$ ~ % level

good chance to access multi-gluon effects at EIC

Exact relation between $\langle p S_\perp | F^{+\perp} F^{+\perp} F^{+\perp} | p S_\perp \rangle$ & $\langle p S_\perp | F^{+\perp} D^\perp F^{+\perp} | p S_\perp \rangle$

$$\mathcal{G}_T(x) = \frac{1}{2} \int_x^{\epsilon(x)} dx' \frac{\Delta G(x')}{x'} + [\text{genuine tw.3}] \quad g_T(x) = \int_x^{\epsilon(x)} \frac{dx'}{x'} \Delta q(x') + [\text{genuine tw.3}]$$

$$\int_{-1}^1 dx \mathcal{G}_T(x) = \int_0^1 dx \Delta G(x) = a_0 \equiv \Delta G, \quad \int dx g_T(x) = \int dx \Delta q(x) \equiv \Delta q \quad \Rightarrow \text{transverse spin SR}$$

