Lepton Flavour Violation

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The SM flavor sector (at the ren. pert. level)

- No lepton or baryon number violation
- No individual lepton number or CP violation in the lepton sector
- All flavor and CP violating effects (neglecting ϑ_{QCD}) lie in the quark charged current and are encoded in the CKM matrix V

Is that so?

- No hints of violation of the unitarity of V
  - in CP-conserving quantities: $|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1$
  - or CP-violating: $V_{ud}V^*_{ub} + V_{cd}V^*_{cb} + V_{td}V^*_{tb} = 0$ (UT)
- Lepton sector
  - no $BR(\mu \to e\gamma) < 1.2 \times 10^{-11}$
  - no $d_e < 1.6 \times 10^{-27}$ e cm $\approx 10^{-11}$ μ_B
  - but $\nu_{ei} \leftrightarrow \nu_{ej}$ $(i \neq j)$

The SM as an effective theory

- No evidence of remnants from TeV

- Only convincing evidence from neutrino masses (compelling but not unique interpretation)

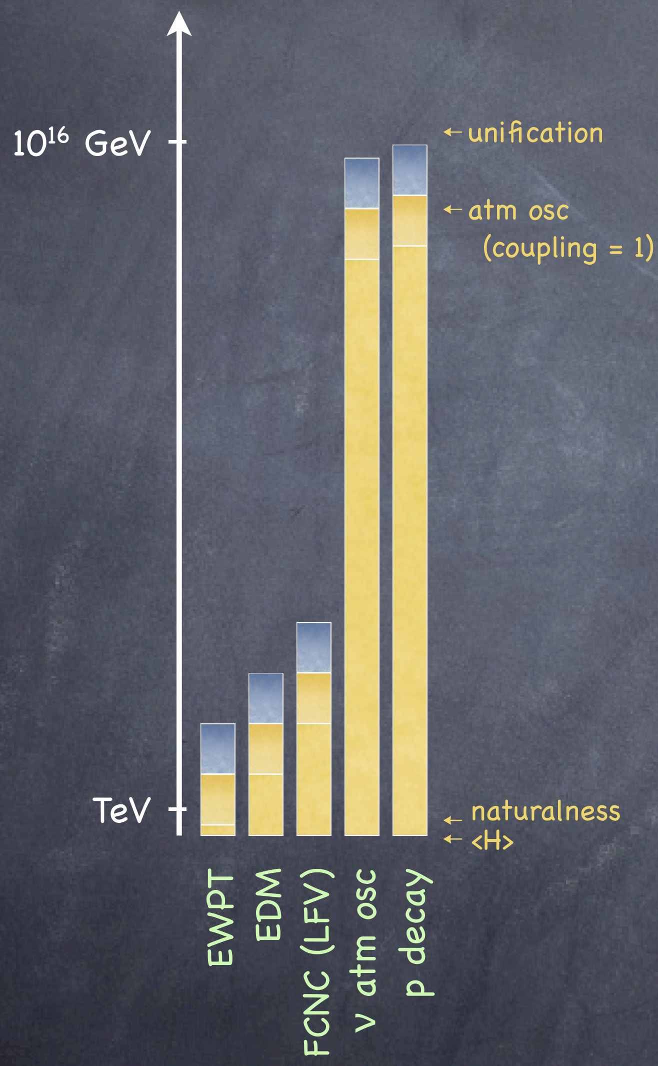
$$\mathcal{L}^{\rm eff}_{E\ll\Lambda} = \mathcal{L}^{\rm ren}_{\rm SM} + \mathcal{L}^{\rm NR}_{\rm SM}$$
$$= \mathcal{L}^{\rm ren}_{\rm SM} + \frac{h_{ij}}{2\Lambda}(HL_i)(HL_j) + \dots$$

$$m^{E,D,U}_{ij} = \lambda^{E,D,U}_{ij} v \qquad m^\nu_{ij} = h_{ij} v \times \frac{v}{\Lambda}$$

$$\Lambda \sim 0.5 \times 10^{15}\,{\rm GeV} h \left(\frac{0.05\,{\rm eV}}{m_\nu}\right)$$

- Additional strong hint of physics beyond the EW scale: $M_{GUT} \approx 2 \times 10^{16}$ GeV (no remnants yet)

Large scale indirect probes



Origin of soft masses (flavour and CP blind) $m_0^2 \tilde{L}_i^\dagger \tilde{L}_i + m_0^2 \tilde{e}_{R_i}^\dagger \tilde{e}_{Ri} + \dots$

Sources of LFV (see-saw, unif, flavour dynamics)

$\tilde{m}$

$$(m_L^2)_{ij} \tilde{L}_i^\dagger \tilde{L}_j + (m_{e_R}^2)_{ij} \tilde{e}_{Ri}^\dagger \tilde{e}_{Rj} + \dots$$

$$\frac{1}{\Lambda_{\mathrm{LFV}}} \to \frac{1}{\tilde{m}_{\mathrm{SUSY}}}$$

Info on the source of LFV (to be reinforced)

Info on the origin of SUSY breaking

- MSSM with SUSY messengers at Λ_{SUSY} (e.g. sugra: $\Lambda_{SUSY} \approx M_{Pl}$) + source of LFV at Λ_{LFV} (e.g. ν_R: $\Lambda_{LFV} \approx M_R$)
  - $\Lambda_{SUSY} > \Lambda_{LFV}$: LFV interactions affect soft terms (through loop or tree level effects)
  - $\Lambda_{SUSY} < \Lambda_{LFV}$: only NR remnants of LFV interactions at Λ_{SUSY}, effects suppressed by $(\Lambda_{LFV}/\Lambda_{SUSY})^n$
- MSSM as above + source of flavour ($U(3)^5$) breaking at Λ_F (e.g. flavour symmetry $G_F \subseteq U(3)^5$ broken and mediated at $\approx \Lambda_F$)
  - $\Lambda_{SUSY} > \Lambda_F$: the source of flavour breaking enters the soft terms (at tree or loop level, in minimal or non minimal way)
  - $\Lambda_{SUSY} < \Lambda_F$: the soft terms are only affected by effects suppressed by $(\Lambda_{LFV}/\Lambda_{SUSY})^n$

Info on the origin of flavour mixing, if $\Lambda_{SUSY} > \Lambda_{LFV}, \Lambda_F$:

Example: the large atmospheric neutrino angle

- Can originate from m_ν or m_E:

$$U = U_e U_\nu^\dagger \qquad \begin{aligned} m_\nu &= U_\nu^T m_\nu^{\text{diag}} U_\nu \\ m_E &= U_{e^c}^T m_E^{\text{diag}} U_e \end{aligned}$$

- From m_E: $m_E \propto \begin{pmatrix} & & \\ & & \\ & A & 1 \end{pmatrix} \qquad A = 1.0 \pm 0.3$

Leads to large charged lepton-slepton mixing, barring alignment (which may arise if $\Lambda_{SUSY} < \Lambda_{LFV}, \Lambda_F$)

In the see-saw context:

- From m_E: misalignment of m_E and m_N → sizeable slepton mixing

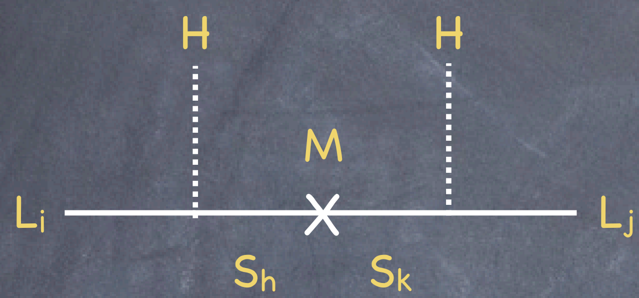
$$\delta^L_{\mu\tau} \sim -\frac{3}{(4\pi)^2}\left(\lambda_N^\dagger \log\frac{M_0^2}{MM^\dagger}\lambda_N\right)_{\mu\tau} \sim -\lambda^2_{\nu_3}\sin 2\theta_{\mathrm{ATM}}\frac{3}{2(4\pi)^2}\left(\log\frac{M_0^2}{MM^\dagger}\right)_{33}$$

- From m_ν: misalignment of m_ν and m_N → small slepton mixing

$$\delta^L_{\mu\tau} \sim -\frac{3}{(4\pi)^2}\left(\lambda_N^\dagger \log\frac{M_0^2}{MM^\dagger}\lambda_N\right)_{\mu\tau} \sim -\lambda^2_{\nu_3}\sin 2\theta_{\mathrm{CKM}}\frac{3}{2(4\pi)^2}\left(\log\frac{M_0^2}{MM^\dagger}\right)_{33}$$

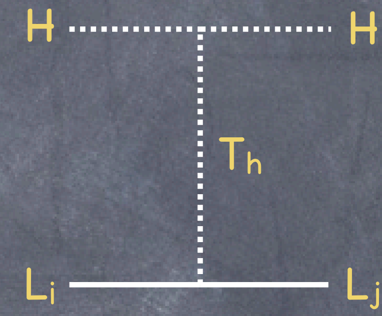
See-saw induced LFV

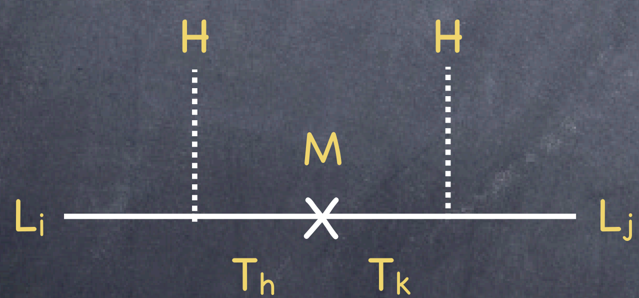
$$\mathcal{L}^{\rm eff}_{E\ll\Lambda} = \mathcal{L}^{\rm ren}_{\rm SM} + \frac{h_{ij}}{\Lambda}(HL_i)(HL_j) + \ldots$$



See-saw type I

See-saw type II

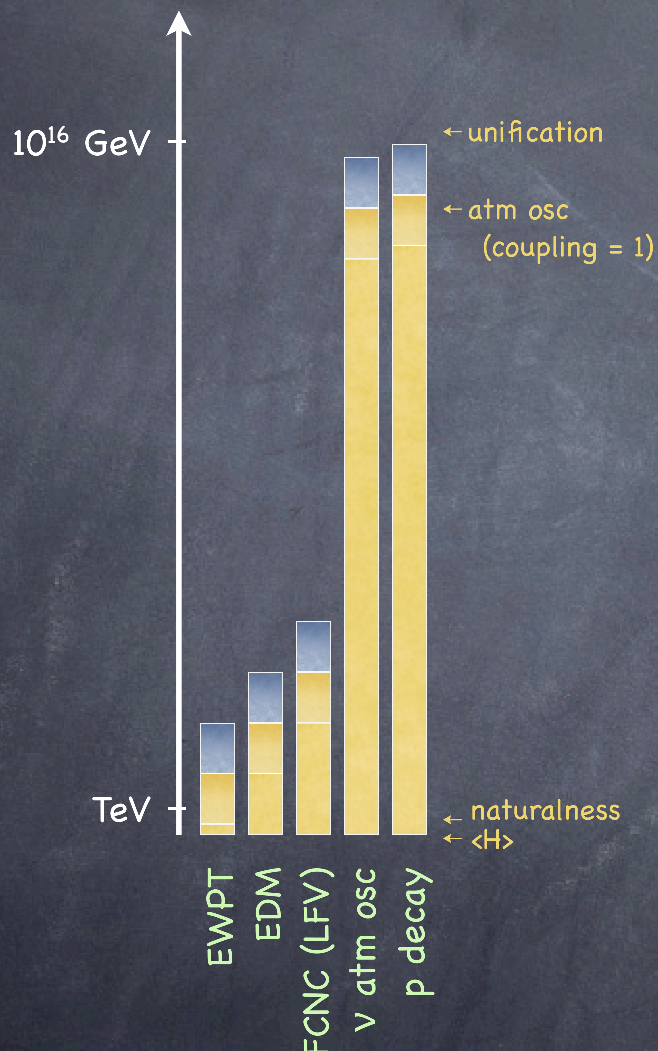




See-saw type III

(Any number of S_h, T_h)

See-saw (Type I) induced LFV



M_0: $m_L^2 = m_0^2 \mathbf{1}$

Borzumati Masiero 86
Hisano Moroi Tobe Yamaguchi Yanagida 95

Source of LFV: $\lambda^E_{ij} e^c_i L_j H_d + \lambda^N_{ij} \nu^c_i L_j H_u + \frac{M_{ij}}{2} \nu^c_i \nu^c_j$

$\tilde{m}$: $\delta^L_{e_i e_j} \equiv \frac{(m_L^2)_{e_i e_j}}{m^2_{\tilde{e}}} \sim -\frac{3}{(4\pi)^2} \left(\lambda_N^\dagger \log \frac{M_0^2}{M M^\dagger} \lambda_N \right)_{e_i e_j}$

$$\mathrm{BR}(e_i \to e_j \gamma) \sim \frac{\alpha^3}{G_F^2} \frac{|\delta^L_{ij}|^2}{m^4_{\tilde{l}}} \tan^2\beta$$

Model dependence

[Ellis Gomez Leontaris Lola Nanopoulos 99
Lavignac Masina Savoy 01
Casas Ibarra 01
Masiero Vempati Vives 02]

- Overall size of neutrino Yukawa couplings

$$\begin{array}{l} \lambda_N \to k\,\lambda_N \\ M \to k^2 M \end{array} \Rightarrow \begin{array}{l} m_\nu \to m_\nu \\ \mathrm{BR}(e_i \to e_j\gamma) \to k^4 \log k\, \mathrm{BR}(e_i \to e_j\gamma) \end{array}$$

- Unknown flavour structure

$$v_d\lambda_E,\ v_u\lambda_N,\ M$$

21 physical parameters

$$m_e\, m_\mu\, m_\tau,\ m_{\nu_1}\, m_{\nu_2}\, m_{\nu_3},\ U$$

12 known or measurable parameters

e.g. $v_u\lambda_N = v_u\lambda_N^{\mathrm{diag}} V_N$ or M^{diag},

$$R = 1/\sqrt{M^{\mathrm{diag}}}\, v_u\lambda_N\, U^\dagger/\sqrt{M^{\mathrm{diag}}}$$

9 unknowns = 3 masses + 3 angles + 3 phases

A predictive Type II option

[Frigerio Hosteins Lavignac R]

- SO(10): 10_i[SU(5)] ⊆ 16_i[SO(10)], $\bar{5}_i$[SU(5)] ⊆ 10_i[SO(10)]

- $\mathcal{L} \supseteq y_{ij} 16_i 16_j 10 + h_{ij} 16_i 10_j 16 + f_{ij} 10_i 10_j 54 + \sigma 10\, 10\, 54$

<16> pairs up the spare components in 16_i 10_i

- $$\begin{cases} m^U_{ij} = v_u\, y_{ij} \\ m^E_{ij} = v_d\, h_{ij} \\ m^\nu_{ij} = \sigma \dfrac{v_u^2}{M_{54}} f_{ij} \end{cases}$$

$$\frac{\delta \tilde{m}^2_L}{\tilde{m}^2} \supseteq -\frac{a}{(4\pi)^2} f^\dagger \log\left(\frac{M_0^2}{v_{16}^2 h h^\dagger}\right) f = -\frac{a}{(4\pi)^2} \frac{M_{54}^2}{\sigma^2 v_u^4} m_\nu^\dagger \log\left(\frac{M_0^2 v_d^2}{v_{16}^2 m_E m_E^\dagger}\right) m_\nu$$

- Correlation among LFV effects

- Leptogenesis also predictive

LFV and colliders

Arkani-Hamed Cheng Feng Hall 96
Hisano Nojiri Shimizu Tanaka 98
Hinchliffe Paige 00
Deppish Kalinowski Päs Redelbach Rückl
Carvalho Ellis Gomez Lola Romao 05
Bartl Hidaka Hohenwarter-Sodek
Kernreiter Majerotto Porod 05

Interplay of LFV at low E and colliders

- The possibility of large lepton-slepton mixing may lead to the observation of charged LFV in slepton decays at colliders
- Low E LFV constraints are only partially relevant, as they become ineffective for more and more degenerate sleptons
- The $e_i \to e_j \gamma$ rates, controlled by the size of the mass insertions, are GIM-suppressed; e.g.:

$$\delta_{\mu\tau} \sim \sin 2\tilde{\theta} \, \frac{\Delta m^2_{\tilde{\mu}\tilde{\tau}}}{2\tilde{m}^2}$$

Neglecting 1-[23] mixing

- On the other hand, for large mixing

$$P(\chi_2 \to (\tilde{e}_i e_j)_L \to \mu^\pm \tau^\mp \chi_1) \sim P(\chi_2 \to (\tilde{e}_i e_j)_L \to \mu^\pm \mu^\mp \chi_1)$$

independently of $\Delta m^2_{\tilde{\mu}\tilde{\tau}}$, provided that $\dfrac{\Delta m^2_{\tilde{\mu}\tilde{\tau}}}{2\tilde{m}^2} \gtrsim \dfrac{\Gamma_{\tilde{\mu},\tilde{\tau}}}{\tilde{m}} \sim 0.01$

[Arkani-Hamed Cheng Feng Hall 96]

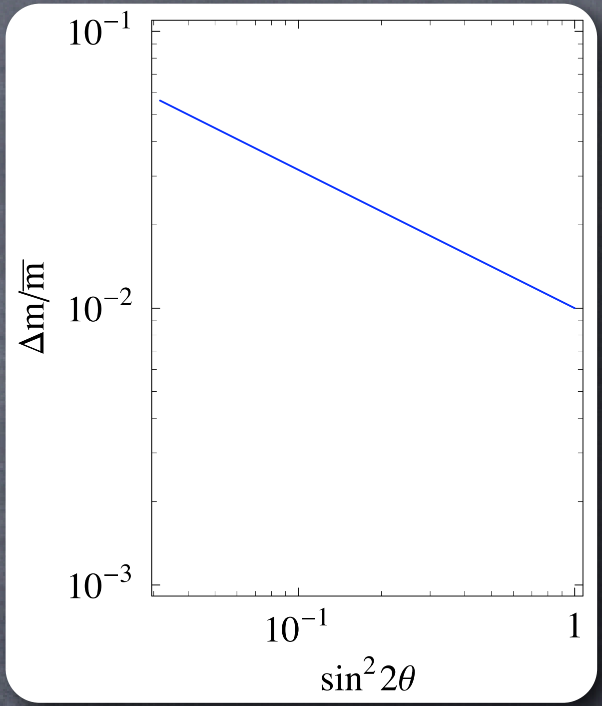
- BR($e_i \to e_j \gamma$)

$$\propto |\delta_{ij}|^2 \approx \sin^2 2\tilde{\theta} \left(\frac{\Delta m_{ij}^2}{2\tilde{m}^2} \right)^2$$

$$\tilde{m} = \frac{\tilde{m}_i - \tilde{m}_j}{2}$$

$$\frac{\Delta m_{ij}^2}{2\tilde{m}^2} = \frac{\Delta m_{ij}}{\tilde{m}}$$

$$\begin{cases} |\delta_{e\mu}^L| < 3 \times 10^{-4} \\ |\delta_{\mu\tau}^L| < 0.09 \qquad (m_0 = 400\,\mathrm{GeV}) \\ |\delta_{e\tau}^L| < 0.09 \end{cases}$$



- BR($e_i \rightarrow e_j \gamma$)

$$\propto |\delta_{ij}|^2 \approx \sin^2 2\tilde{\theta} \left(\frac{\Delta m^2_{ij}}{2\tilde{m}^2} \right)^2 \qquad \tilde{m} = \frac{\tilde{m}_i - \tilde{m}_j}{2}$$

$$\frac{\Delta m^2_{ij}}{2\tilde{m}^2} = \frac{\Delta m_{ij}}{\tilde{m}}$$

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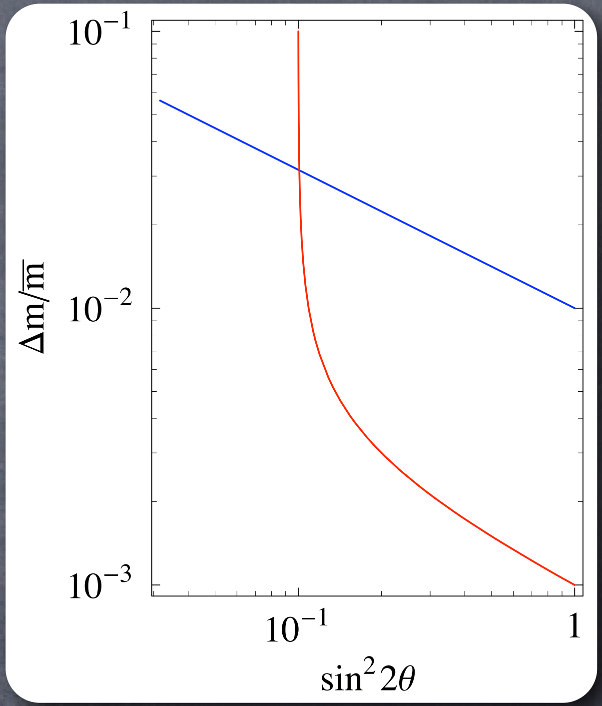

- $\Gamma(\tilde{e}_i \rightarrow e_j \chi^0, \chi^0{}_2 \rightarrow e_i e_j\, \chi^0{}_1)$

$$\propto \sin^2 2\tilde{\theta}\, \frac{(\Delta m^2_{ij}/2\tilde{m}^2)^2}{(\Gamma/\tilde{m})^2 + (\Delta m^2_{ij}/2\tilde{m}^2)^2}$$

(oscillations with L → $1/(\Gamma/2)$, $\Delta m^2/E \rightarrow \Delta m$)

- BR($e_i \to e_j \gamma$)

$$\propto |\delta_{ij}|^2 \approx \sin^2 2\tilde\theta \left(\frac{\Delta m^2_{ij}}{2\tilde m^2}\right)^2 \qquad \tilde m = \frac{\tilde m_i - \tilde m_j}{2}$$

$$\frac{\Delta m^2_{ij}}{2\tilde m^2} = \frac{\Delta m_{ij}}{\tilde m}$$

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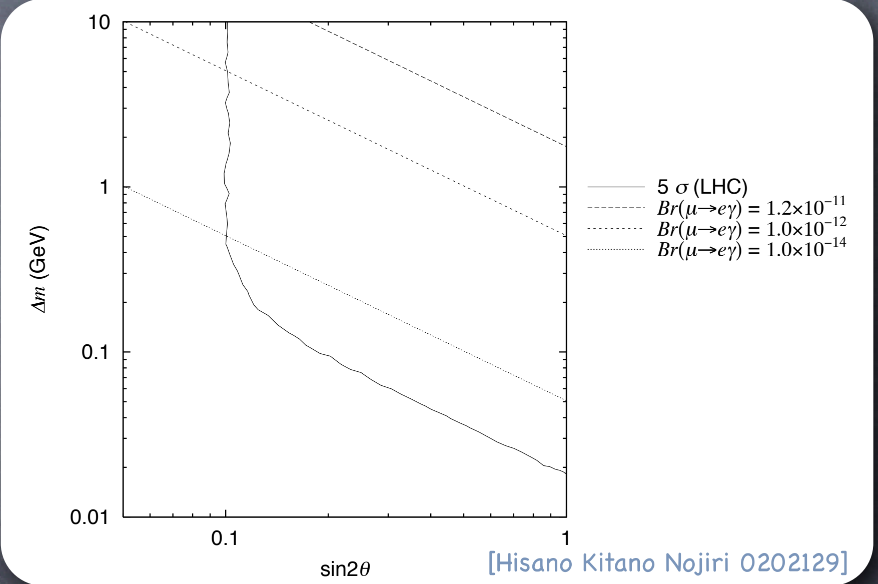
- $\Gamma(\tilde e_i \to e_j \chi^0,\ \chi^0{}_2 \to e_i e_j\ \chi^0{}_1)$

$$\propto \sin^2 2\tilde\theta\ \frac{(\Delta m^2_{ij}/2\tilde m^2)^2}{(\Gamma/\tilde m)^2 + (\Delta m^2_{ij}/2\tilde m^2)^2}$$

(oscillations with L → 1/(Γ/2), Δm²/E → Δm)

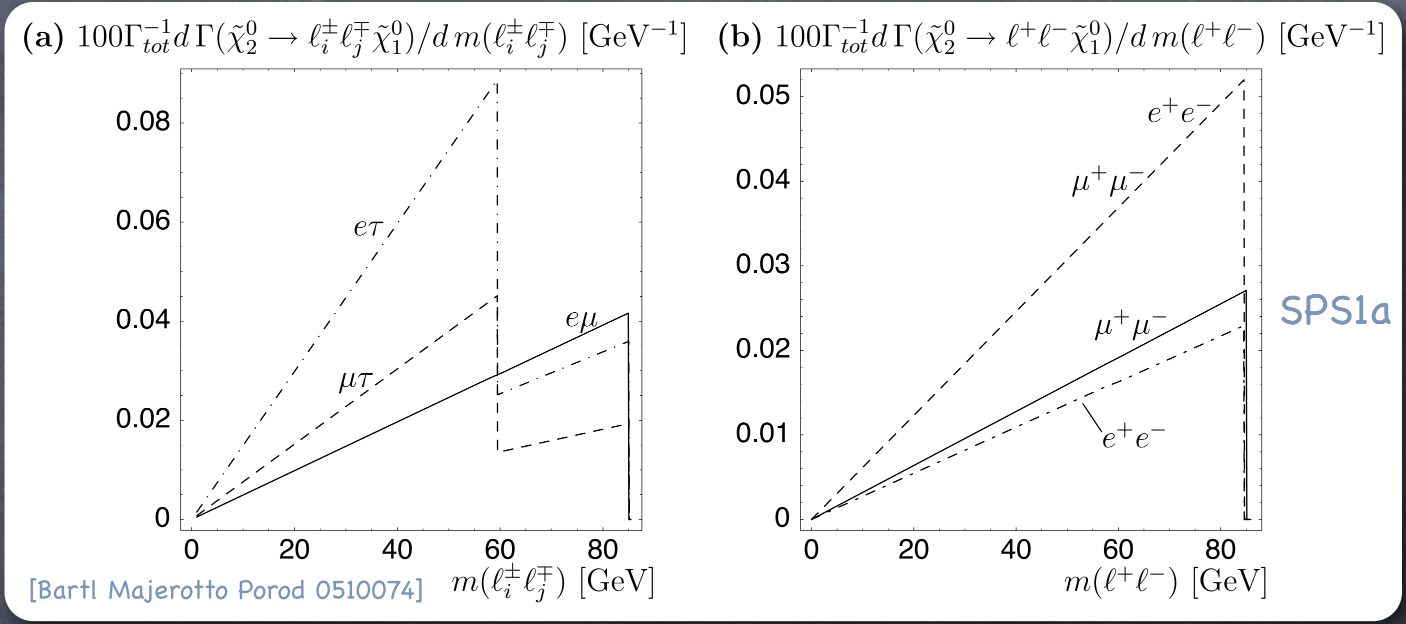


[Hisano Kitano Nojiri 0202129]

$m_H \neq m_0$, $\mu = M_2$ $\tan\beta = 10$

$m_0 = 100\,\mathrm{GeV}$, $M_{1/2} = 300\,\mathrm{GeV}$

- χ_2^0 from squark, gluino decay, $\chi_2^0 \to l_i^\pm \tilde{l}_k^\mp \to l_i^\pm l_j^\mp \chi_1^0$
- Typically ≥2 leptons (not all SF) + ≥2 jets + E_{miss}
- Bckg (SM and SUSY) does not exhibit edge structure



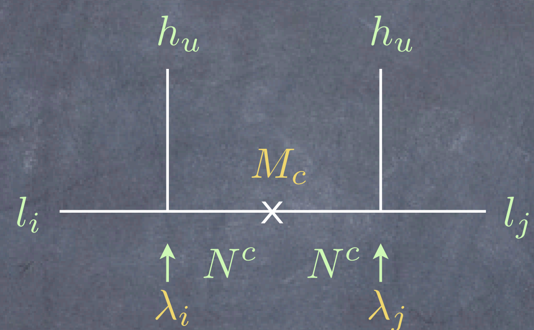
- Plenty of room for further studies

  E.g. chirality of intermediate slepton

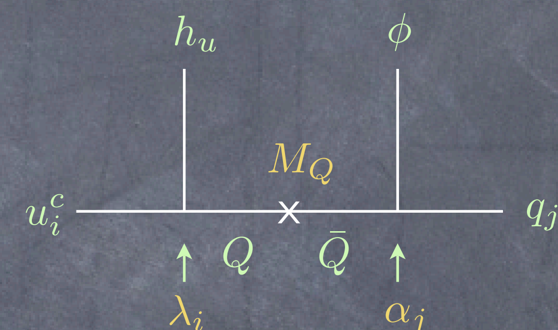
Flavour violation and chirality

- SU(5): FV in l_R, d_L
- See-saw: FV in l_L
- Flavour, an example motivated by neutrinos: l_R d_R

[Ferretti King R 0609047
Calibbi Ferretti R Ziegler, in prep]



$$m^\nu_{ij} = -\lambda_i \lambda_j \frac{\langle h_u \rangle^2}{M_c}$$



$$\lambda^U_{ij} = -\lambda_i \alpha_j \frac{\langle \phi \rangle}{M_Q}$$

Single N^c: $m_3 \gg m_2 = 0$ hierarchy

$\lambda_2 \approx \lambda_3$: $\tan \vartheta_{23} \approx 1$ large ϑ_{23}

Single Q: $m_3 \gg m_2 = 0$ hierarchy

L-handed: $\tan \vartheta_{23} = 0$ large ϑ_{23}

$3 \gg 2$ from $M_L \ll M_R$ → FV in l_R d_R

Conclusions

- LFV studies at colliders are well motivated: large mixing angles exist in the lepton sector and may well contaminate charged lepton-slepton mixing + better flavour identification
- LFV effects at colliders and low E provide info on the physics underlying the effects, the origin of flavour mixing, the supersymmetry breaking scale...
- The interplay of collider and low E measurements may help disentangling the origin of the effect