B-meson Observables in the Maximally CP-violating MSSM with Minimal Flavour Violation

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♠ *Motivations* (1/3)

- The SUSY models such as the MSSM contain many possible sources of flavour and CP violation in the soft SUSY-breaking sector:

$\boxed{30 \oplus 33 \oplus 46 = \mathbf{109} \text{ !!!}}$

  - Gaugino mass terms: $3 \oplus 3 = 6$

$$-\mathcal{L}_{\rm soft} \supset \frac{1}{2}(M_3\, \widetilde{g}\widetilde{g} + M_2\, \widetilde{W}\widetilde{W} + M_1\, \widetilde{B}\widetilde{B} + \text{h.c.})$$

  - Trilinear a terms $\mathbf{a}_{\mathbf{f}ij} \equiv \mathbf{h}_{\mathbf{f}ij} \cdot \mathbf{A}_{\mathbf{f}ij}$: $3 \times (3 \oplus 6 \oplus 9) = 54$

$$-\mathcal{L}_{\rm soft} \supset (\widetilde{u}_R^*\, \mathbf{a_u}\, \widetilde{Q} H_2 - \widetilde{d}_R^*\, \mathbf{a_d}\, \widetilde{Q} H_1 - \widetilde{e}_R^*\, \mathbf{a_e}\, \widetilde{L} H_1 + \text{h.c.})$$

  - Sfermion mass terms: $5 \times (3 \oplus 3 \oplus 3) = 45$

$$-\mathcal{L}_{\rm soft} \supset \widetilde{Q}^\dagger\, \mathbf{M}^2_{\widetilde{\mathbf{Q}}}\, \widetilde{Q} + \widetilde{L}^\dagger\, \mathbf{M}^2_{\widetilde{\mathbf{L}}}\, \widetilde{L} + \widetilde{u}_R^*\, \mathbf{M}^2_{\widetilde{\mathbf{u}}}\, \widetilde{u}_R + \widetilde{d}_R^*\, \mathbf{M}^2_{\widetilde{\mathbf{d}}}\, \widetilde{d}_R + \widetilde{e}_R^*\, \mathbf{M}^2_{\widetilde{\mathbf{e}}}\, \widetilde{e}_R$$

  - Higgs mass terms: $3 \oplus 1 = 4$

$$-\mathcal{L}_{\rm soft} \supset M^2_{H_u} H_2^\dagger H_2 + M^2_{H_d} H_1^\dagger H_1 - (m^2_{12} H_1 H_2 + \text{h.c.})$$

♠ *Motivations* $(2/3)$

- How to suppress FCNC and CP violation? $\Longrightarrow$ Minimal Flavour Violation (MFV):
  - Squarks and sleptons are aligned with quarks and leptons
  - All FCNC and CP violation vanish in the limit $\mathbf{V}_{\rm CKM} \to \mathbf{1}$

- FCNC B-meson observables put stringent constraints especially when $\tan\beta$ is large

- For large $\tan\beta$, one-loop threshold corrections to the FCNC Higgs couplings to down-type quarks can be greatly enhanced

♠ *Motivations* $(3/3)$

- Our aim is to study the

$$\boxed{\text{Higgs-mediated FCNC B-meson observables}}$$

  - in the MCPMFV Framework
  - with a *Flavour-Covariant* Effective Lagrangian Formalism

♠ *MCPMFV* $(1/5)$

There can be several variants of MFV

- The scale of MFV ... anywhere between $M_{\rm EW}$ and $M_{\rm GUT}$?

- The "minimal" MFV

$$m_0(M_{\rm MFV})\,,\quad m_{1/2}(M_{\rm MFV})\,,\quad A(M_{\rm MFV})\,;\quad \tan\beta(m_t)\,,\quad M_Z\quad \text{upto sign}(\mu)$$

with real and positive m_0, $m_{1/2}$, and A

- Next to the "minimal" MFV

$$m_0(M_{\rm MFV})\,,\quad m_{1/2}(M_{\rm MFV})\,,\quad A(M_{\rm MFV})\,;\quad \tan\beta(m_t)\,,\quad M_Z$$

with complex $m_{1/2}$ and A

♠ *MCPMFV* $(2/5)$

Then, what is the "maximal" MFV ?

- Consider the unitary flavour rotations U_X:

$$\widehat{Q}' = \mathbf{U}_Q\,\widehat{Q}\,, \qquad \widehat{L}' = \mathbf{U}_L\,\widehat{L}\,,$$
$$\widehat{U}'^C = \mathbf{U}_U^*\,\widehat{U}^C\,, \quad \widehat{D}'^C = \mathbf{U}_D^*\,\widehat{D}^C\,, \quad \widehat{E}'^C = \mathbf{U}_E^*\,\widehat{E}^C\,,$$

- Under the flavour rotations, the interaction Lagrangian remains invariant with the redefinition of the couplings. For example,

$$\widehat{U}^C\,\mathbf{h_u}\,\widehat{Q}\,\widehat{H}_2 \quad \stackrel{\text{F.R.}}{\Longrightarrow} \quad \widehat{U}'^C\,\mathbf{h_u}\,\widehat{Q}'\,\widehat{H}_2 = \widehat{U}^C\,\mathbf{U}_U^\dagger\,\mathbf{h_u}\,\mathbf{U}_Q\,\widehat{Q}\,\widehat{H}_2$$

Flavour rotation is equivalent to the redefinition of: $\mathbf{h}_u \rightarrow \mathbf{U}_U^\dagger\,\mathbf{h}_u\,\mathbf{U}_Q$

♠ *MCPMFV* (3/5)

- Flavour rotations in the MSSM:

$$\mathbf{h}_{u,d} \;\to\; \mathbf{U}^\dagger_{U,D}\,\mathbf{h}_{u,d}\,\mathbf{U}_Q\,, \qquad \mathbf{h}_e \;\to\; \mathbf{U}^\dagger_E\,\mathbf{h}_e\,\mathbf{U}_L\,,$$

$$\widetilde{\mathbf{M}}^2_{Q,L,U,D,E} \;\to\; \mathbf{U}^\dagger_{Q,L,U,D,E}\,\widetilde{\mathbf{M}}^2_{Q,L,U,D,E}\,\mathbf{U}_{Q,L,U,D,E}\,,$$

$$\mathbf{a}_{u,d} \;\to\; \mathbf{U}^\dagger_{U,D}\,\mathbf{a}_{u,d}\,\mathbf{U}_Q\,, \qquad \mathbf{a}_e \;\to\; \mathbf{U}^\dagger_E\,\mathbf{a}_e\,\mathbf{U}_L\,.$$

- $\boxed{\text{MCPMFV}}$ defined as the maximal set of flavour-singlet mass scales:

$$M_{1,2,3}\,, \quad M^2_{H_{u,d}}\,, \quad \widetilde{\mathbf{M}}^2_{Q,L,U,D,E} \;=\; \widetilde{M}^2_{Q,L,U,D,E}\,\mathbf{1}_3\,, \quad \mathbf{A}_{u,d,e} \;=\; A_{u,d,e}\,\mathbf{1}_3$$

$3 \oplus 3$ $\qquad$ 2 $\qquad$ 5 $\qquad$ $3 \oplus 3$

$\boxed{13 \oplus 6 = 19 \text{ Parameters !}}$

♠ *MCPMFV* $(4/5)$

- Flavour non-singlet mass scales:

$$\widetilde{\mathbf{M}}^2_Q(M_X) \;=\; \widetilde{M}^2_Q\,\mathbf{1}_3 \;+\; \widetilde{m}^2_1\,(\mathbf{h}^\dagger_d\mathbf{h}_d) \;+\; \widetilde{m}^2_2\,(\mathbf{h}^\dagger_u\mathbf{h}_u) \;+\; \widetilde{m}^2_3\,(\mathbf{h}^\dagger_d\mathbf{h}_d\,\mathbf{h}^\dagger_u\mathbf{h}_u) \;+\; \ldots$$

  - Flavour non-singlet mass parameters $\widetilde{m}^2_n$ can be as many as 9 including $\widetilde{M}^2_Q$ according to $\dim(\widetilde{\mathbf{M}}^2_Q)$
  - $\widetilde{m}^2_n \neq 0$ can either be introduced by hand and/or induced by RG running
  - With $\widetilde{m}^2_n \ll \widetilde{M}^2_Q$, the MFV solution to the flavour problem is still valid.

♠ *MCPMFV* (5/5)

- The flavour covariance of RGEs:

$$\mathbf{U}_Q^\dagger \frac{d\widetilde{\mathbf{M}}_Q^2}{dt}\mathbf{U}_Q = \frac{1}{16\pi^2}\mathbf{U}_Q^\dagger\Big[-\Big(\frac{1}{15}g_1^2|M_1|^2 + 3g_2^2|M_2|^2 + \frac{16}{3}g_3^2|M_3|^2\Big)\mathbf{1}_3 + \frac{1}{2}\mathbf{h}_u^\dagger\mathbf{h}_u\widetilde{\mathbf{M}}_Q^2 + \frac{1}{2}\widetilde{\mathbf{M}}_Q^2\mathbf{h}_u^\dagger\mathbf{h}_u + \mathbf{h}_u^\dagger\widetilde{\mathbf{M}}_U^2\mathbf{h}_u + M_{H_u}^2\mathbf{h}_u^\dagger\mathbf{h}_u + \underline{\mathbf{a}_u^\dagger\mathbf{a}_u} + \frac{1}{2}\mathbf{h}_d^\dagger\mathbf{h}_d\widetilde{\mathbf{M}}_Q^2 + \frac{1}{2}\widetilde{\mathbf{M}}_Q^2\mathbf{h}_d^\dagger\mathbf{h}_d + \mathbf{h}_d^\dagger\widetilde{\mathbf{M}}_D^2\mathbf{h}_d + M_{H_d}^2\mathbf{h}_d^\dagger\mathbf{h}_d + \underline{\mathbf{a}_d^\dagger\mathbf{a}_d} + \frac{1}{10}g_1^2\,\mathrm{Tr}\,(Y\mathbf{M}^2)\,\mathbf{1}_3\Big]\,\mathbf{U}_Q$$

$$\mathbf{U}_U^\dagger \frac{d\widetilde{\mathbf{M}}_U^2}{dt}\mathbf{U}_U = \frac{1}{16\pi^2}\mathbf{U}_U^\dagger\Big[-\Big(\frac{16}{15}g_1^2|M_1|^2 + \frac{16}{3}g_3^2|M_3|^2\Big)\mathbf{1}_3 + \mathbf{h}_u\mathbf{h}_u^\dagger\widetilde{\mathbf{M}}_U^2 + \widetilde{\mathbf{M}}_U^2\mathbf{h}_u\mathbf{h}_u^\dagger + 2\,\mathbf{h}_u\widetilde{\mathbf{M}}_Q^2\mathbf{h}_u^\dagger + 2\,M_{H_u}^2\mathbf{h}_u\mathbf{h}_u^\dagger + \underline{2\,\mathbf{a}_u\mathbf{a}_u^\dagger} - \frac{2}{5}g_1^2\,\mathrm{Tr}\,(Y\mathbf{M}^2)\,\mathbf{1}_3\Big]\,\mathbf{U}_U$$

$$\mathbf{U}_D^\dagger \frac{d\widetilde{\mathbf{M}}_D^2}{dt}\mathbf{U}_D = \frac{1}{16\pi^2}\mathbf{U}_D^\dagger\Big[-\Big(\frac{4}{15}g_1^2|M_1|^2 + \frac{16}{3}g_3^2|M_3|^2\Big)\mathbf{1}_3 + \mathbf{h}_d\mathbf{h}_d^\dagger\widetilde{\mathbf{M}}_D^2 + \widetilde{\mathbf{M}}_D^2\mathbf{h}_d\mathbf{h}_d^\dagger + 2\,\mathbf{h}_d\widetilde{\mathbf{M}}_Q^2\mathbf{h}_d^\dagger + 2\,M_{H_d}^2\mathbf{h}_d\mathbf{h}_d^\dagger + \underline{2\,\mathbf{a}_d\mathbf{a}_d^\dagger} + \frac{1}{5}g_1^2\,\mathrm{Tr}\,(Y\mathbf{M}^2)\,\mathbf{1}_3\Big]\,\mathbf{U}_D$$

where $t = \ln(Q^2/M_{\rm GUT}^2)$

♠ *Flavour-covariant Effective Lagrangian Formalism* $(1/5)$

- Flavour- and Gauge-Covariant Effective Lagrangian for the down-type quarks ($H_u = \Phi_2$ and $H_d = i\tau_2\Phi_1^*$):

$$-\mathcal{L}^d_{\rm eff}[\Phi_1,\Phi_2] \;=\; \overline{d^0_{R\,\alpha}}\,(\,\mathbf{h}_d\,\Phi_1^\dagger \;+\; \Delta\mathbf{h}_d[\Phi_1,\Phi_2]\,)_{\alpha\beta}\,Q^0_{L\,\beta} \;+\; \text{h.c.},$$

where the superscript '0' indicates weak eigenstate fields and

– $\Delta\mathbf{h}_d[\Phi_1,\Phi_2]$ is a Coleman-Weinberg-type field-dependent effective functional of the background Higgs doublets $\Phi_{1,2}$

– $\Delta\mathbf{h}_d[\Phi_1,\Phi_2]$ flavour- and gauge-transforms as $\mathbf{h}_d\,\Phi_1^\dagger$

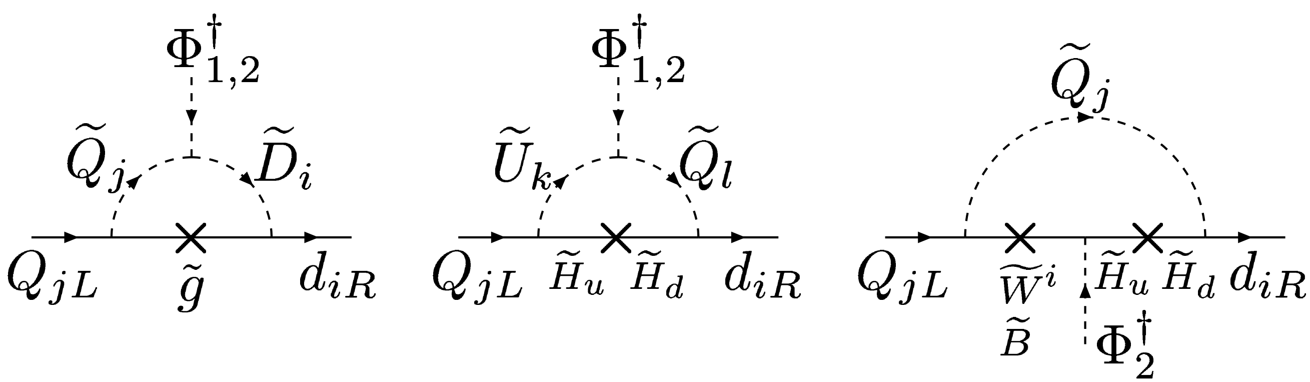
Convention:

$$\Phi_i = \begin{pmatrix} \phi_i^+ \\ \phi_i^0 \end{pmatrix} = \begin{pmatrix} \phi_i^+ \\ \frac{1}{\sqrt{2}}(v_i + \phi_i + i a_i) \end{pmatrix} \;\;\text{with}\;\; \langle\Phi_{1,2}\rangle = \begin{pmatrix} 0 \\ \frac{1}{\sqrt{2}}\,v_{1,2} \end{pmatrix}$$

♠ *Flavour-covariant Effective Lagrangian Formalism* (2/5)

- The analytic form of $\Delta\mathbf{h}_d[\Phi_1, \Phi_2]$ may be calculated via (gauge couplings are suppressed)

$$
\begin{aligned}
(\Delta\mathbf{h}_d)_{ij} \quad = & \int \frac{d^n k}{(2\pi)^n\, i} \Big[P_L \frac{M_3^*}{k^2 - |M_3^2|} \Big(\frac{1}{k^2 \mathbf{1}_{12} - \widetilde{\mathbf{M}}^2} \Big)_{\widetilde{D}_i \widetilde{Q}_j^\dagger} \\
& + P_L \Big(\frac{1}{\not{k} \mathbf{1}_8 - \mathbf{M}_C P_L - \mathbf{M}_C^\dagger P_R} \Big)_{\widetilde{H}_u \widetilde{H}_d} P_L \, (\mathbf{h}_d)_{il} \Big(\frac{1}{k^2 \mathbf{1}_{12} - \widetilde{\mathbf{M}}^2} \Big)_{\widetilde{Q}_l \widetilde{U}_k^\dagger} (\mathbf{h}_u)_{kj} \\
& + P_L \Big(\frac{1}{\not{k} \mathbf{1}_8 - \mathbf{M}_C P_L - \mathbf{M}_C^\dagger P_R} \Big)_{\widetilde{H}_d \widetilde{W}^i, \widetilde{H}_d \widetilde{B}} P_L \, (\mathbf{h}_d)_{ij} \Big(\frac{1}{k^2 \mathbf{1}_{12} - \widetilde{\mathbf{M}}^2} \Big)_{\widetilde{Q}_j \widetilde{Q}_j^\dagger} \Big]
\end{aligned}
$$



♠ *Flavour-covariant Effective Lagrangian Formalism* $(3/5)$

- The 12×12 Higgs-field dependent squark mass matrix $\widetilde{\mathbf{M}}^2[\Phi_1, \Phi_2]$:

$$\widetilde{\mathbf{M}}^2[\Phi_1, \Phi_2] \;=\; \begin{pmatrix} (\widetilde{\mathbf{M}}^2)_{\widetilde{Q}^\dagger \widetilde{Q}} & (\widetilde{\mathbf{M}}^2)_{\widetilde{Q}^\dagger \widetilde{U}} & (\widetilde{\mathbf{M}}^2)_{\widetilde{Q}^\dagger \widetilde{D}} \\ (\widetilde{\mathbf{M}}^2)_{\widetilde{U}^\dagger \widetilde{Q}} & (\widetilde{\mathbf{M}}^2)_{\widetilde{U}^\dagger \widetilde{U}} & (\widetilde{\mathbf{M}}^2)_{\widetilde{U}^\dagger \widetilde{D}} \\ (\widetilde{\mathbf{M}}^2)_{\widetilde{D}^\dagger \widetilde{Q}} & (\widetilde{\mathbf{M}}^2)_{\widetilde{D}^\dagger \widetilde{U}} & (\widetilde{\mathbf{M}}^2)_{\widetilde{D}^\dagger \widetilde{D}} \end{pmatrix}_{ij}$$

$$\begin{aligned}
(\widetilde{\mathbf{M}}^2)_{\widetilde{Q}^\dagger_i \widetilde{Q}_j} \;&=\; (\widetilde{\mathbf{M}}^2_Q)_{ij}\, \mathbf{1}_2 \;+\; (\mathbf{h}_d^\dagger \mathbf{h}_d)_{ij}\, \Phi_1 \Phi_1^\dagger \;+\; (\mathbf{h}_u^\dagger \mathbf{h}_u)_{ij}\, (\Phi_2^\dagger \Phi_2\, \mathbf{1}_2 \;-\; \Phi_2 \Phi_2^\dagger) \\
&\quad -\, \frac{1}{2}\, g^2\, \delta_{ij}\, (\Phi_1 \Phi_1^\dagger \;-\; \Phi_2 \Phi_2^\dagger) \;+\; \delta_{ij} \left(\frac{1}{4}\, g^2 \;-\; \frac{1}{12}\, g'^2 \right) (\Phi_1^\dagger \Phi_1 \;-\; \Phi_2^\dagger \Phi_2)\, \mathbf{1}_2\,, \\
(\widetilde{\mathbf{M}}^2)_{\widetilde{U}^\dagger_i \widetilde{Q}_j} \;&=\; (\widetilde{\mathbf{M}}^2)^\dagger_{\widetilde{Q}^\dagger_j \widetilde{U}_i} \;=\; -\,(\mathbf{a}_u)_{ij}\, \Phi_2^T i\tau_2 \;+\; (\mathbf{h}_u)_{ij}\, \mu^*\, \Phi_1^T i\tau_2\,, \\
(\widetilde{\mathbf{M}}^2)_{\widetilde{D}^\dagger_i \widetilde{Q}_j} \;&=\; (\widetilde{\mathbf{M}}^2)^\dagger_{\widetilde{Q}^\dagger_j \widetilde{D}_i} \;=\; (\mathbf{a}_d)_{ij}\, \Phi_1^\dagger \;-\; (\mathbf{h}_d)_{ij}\, \mu^*\, \Phi_2^\dagger\,, \\
(\widetilde{\mathbf{M}}^2)_{\widetilde{U}^\dagger_i \widetilde{U}_j} \;&=\; (\widetilde{\mathbf{M}}^2_U)_{ij} \;+\; (\mathbf{h}_u \mathbf{h}_u^\dagger)_{ij}\, \Phi_2^\dagger \Phi_2 \;+\; \frac{1}{3}\, \delta_{ij}\, g'^2\, (\Phi_1^\dagger \Phi_1 \;-\; \Phi_2^\dagger \Phi_2)\,, \\
(\widetilde{\mathbf{M}}^2)_{\widetilde{D}^\dagger_i \widetilde{D}_j} \;&=\; (\widetilde{\mathbf{M}}^2_D)_{ij} \;+\; (\mathbf{h}_d \mathbf{h}_d^\dagger)_{ij}\, \Phi_1^\dagger \Phi_1 \;-\; \frac{1}{6}\, \delta_{ij}\, g'^2\, (\Phi_1^\dagger \Phi_1 \;-\; \Phi_2^\dagger \Phi_2)\,, \\
(\widetilde{\mathbf{M}}^2)_{\widetilde{U}^\dagger_i \widetilde{D}_j} \;&=\; (\widetilde{\mathbf{M}}^2)^\dagger_{\widetilde{D}^\dagger_j \widetilde{U}_i} \;=\; (\mathbf{h}_u \mathbf{h}_d^\dagger)_{ij}\, \Phi_1^T i\tau_2 \Phi_2
\end{aligned}$$

♠ *Flavour-covariant Effective Lagrangian Formalism* $(4/5)$

- The 8×8 Higgs-field dependent chargino-neutralino mass matrix $\mathbf{M}_C[\Phi_1, \Phi_2]$:

$$\mathbf{M}_C[\Phi_1, \Phi_2] \;=\; \begin{pmatrix} M_1 & 0 & -\frac{1}{\sqrt{2}}\, g'\,\Phi_2^\dagger & \frac{1}{\sqrt{2}}\, g'\,\Phi_1^T\,(i\tau_2) \\ 0 & M_2\,\mathbf{1}_3 & \frac{1}{\sqrt{2}}\, g\,\Phi_2^\dagger\,\tau_i & -\frac{1}{\sqrt{2}}\, g\,\Phi_1^T\,(i\tau_2)\,\tau_i \\ -\,\frac{1}{\sqrt{2}}\, g'\,\Phi_2^* & \frac{1}{\sqrt{2}}\, g\tau_i^T\,\Phi_2^* & \mathbf{0}_2 & \mu\,(i\tau_2) \\ -\,\frac{1}{\sqrt{2}}\,(i\tau_2)\,g'\,\Phi_1 & \frac{1}{\sqrt{2}}\, g\tau_i^T\,(i\tau_2)\Phi_1 & -\mu\,(i\tau_2) & \mathbf{0}_2 \end{pmatrix}$$

in the Weyl basis, $(\widetilde{B},\ \widetilde{W}^{1,2,3},\ \widetilde{H}_u,\ \widetilde{H}_d)$, with $\widetilde{H}_u = (\tilde{h}_u^+, \tilde{h}_u^0)$ and $\widetilde{H}_d = (\tilde{h}_d^0, \tilde{h}_d^-)$

♠ *Flavour-covariant Effective Lagrangian Formalism* $(5/5)$

- Flavour covariance of the functional $\Delta\mathbf{h}_d$:

$$(\Delta\mathbf{h}_d)_{ij} = \int \frac{d^n k}{(2\pi)^n\, i} \Big[P_L \frac{M_3^*}{k^2 - |M_3^2|} \Big(\frac{1}{k^2 \mathbf{1}_{12} - \widetilde{\mathbf{M}}^2} \Big)_{\widetilde{D}_i \widetilde{Q}_j^\dagger}$$
$$+ P_L \Big(\frac{1}{\not{k}\mathbf{1}_8 - \mathbf{M}_C P_L - \mathbf{M}_C^\dagger P_R} \Big)_{\widetilde{H}_u \widetilde{H}_d} P_L\, (\mathbf{h}_d)_{il} \Big(\frac{1}{k^2 \mathbf{1}_{12} - \widetilde{\mathbf{M}}^2} \Big)_{\widetilde{Q}_l \widetilde{U}_k^\dagger} (\mathbf{h}_u)_{kj}$$
$$+ P_L \Big(\frac{1}{\not{k}\mathbf{1}_8 - \mathbf{M}_C P_L - \mathbf{M}_C^\dagger P_R} \Big)_{\widetilde{H}_d \widetilde{W}^i, \widetilde{H}_d \widetilde{B}} P_L\, (\mathbf{h}_d)_{ij} \Big(\frac{1}{k^2 \mathbf{1}_{12} - \widetilde{\mathbf{M}}^2} \Big)_{\widetilde{Q}_j \widetilde{Q}_j^\dagger} \Big]$$

For example,

$$\mathbf{h}_d \Big(\frac{1}{k^2 \mathbf{1}_{12} - \widetilde{\mathbf{M}}^2} \Big)_{\widetilde{Q}\widetilde{U}^\dagger} \mathbf{h}_u \;\to\; \Big[\mathbf{U}_D^\dagger\, \mathbf{h}_d\, \mathbf{U}_Q\Big] \Big[\mathbf{U}_Q^\dagger \Big(\frac{1}{k^2 \mathbf{1}_{12} - \widetilde{\mathbf{M}}^2} \Big)_{\widetilde{Q}\widetilde{U}^\dagger} \mathbf{U}_U\Big] \Big[\mathbf{U}_U^\dagger \mathbf{h}_u\, \mathbf{U}_Q\Big]$$

N.B. $\widetilde{\mathbf{M}}^2{}_{\widetilde{Q}^\dagger \widetilde{U}} = \mathbf{a}_u^\dagger\, i\tau_2\, \Phi_2^* - \mathbf{h}_u^\dagger\, \mu\, i\tau_2\, \Phi_1^*$

Flavour covariance: $\Delta\mathbf{h}_d \to \mathbf{U}_D^\dagger\, \Delta\mathbf{h}_d\, \mathbf{U}_Q$ under flavour rotations

♠ *Higgs-mediated Flavour-changing Effective Lagrangian* (1/8)

- Rotate the (weak) chiral states into their mass eigenstates:

$$
\begin{aligned}
u^0_{L\,\alpha} &= \left(\mathcal{U}^Q_L\right)_{\alpha\, i} u_{L\, i} \;\; ; \;\; d^0_{L\,\alpha} = \left(\mathcal{U}^Q_L\, V_{\rm CKM}\right)_{\alpha\, i} d_{L\, i} \\
u^0_{R\,\alpha} &= (\mathcal{U}^u_R)_{\alpha\, i}\, u_{R\, i} \;\; ; \;\; d^0_{R\,\alpha} = (\mathcal{U}^d_R)_{\alpha\, i}\, d_{R\, i}
\end{aligned}
$$

In terms of the mass eigenstates, the effective Lagrangian becomes

$$
\begin{aligned}
-\mathcal{L}^d_{\rm eff}[\Phi_1, \Phi_2] \;=\; & \overline{d_{R\,i}}\,\left(\mathcal{U}^{d\,\dagger}_R\, \mathbf{h}_d\right)_{i\alpha} \Big\{ \left(\phi^-_1, \phi^{0*}_1\right)\, \delta_{\alpha\beta} + \left(\mathbf{h}_d^{-1} \Delta\mathbf{h}_d[\Phi_1, \Phi_2]\right)_{\alpha\beta} \Big\} \\
& \times \begin{pmatrix} \left(\mathcal{U}^Q_L\right)_{\beta\, j} u_{L\, j} \\ \left(\mathcal{U}^Q_L\, V_{\rm CKM}\right)_{\beta\, j} d_{L\, j} \end{pmatrix} + \ \text{h.c.}
\end{aligned}
$$

♠ *Higgs-mediated Flavour-changing Effective Lagrangian* $(2/8)$

- The functional $\mathbf{h}_d^{-1}\Delta\mathbf{h}_d[\Phi_1,\Phi_2]$ might be expanded as:

$$\mathbf{h}_d^{-1}\Delta\mathbf{h}_d[\Phi_1,\Phi_2] = (0\,,\Delta_d) + \sum_{i=1,2}\left(\Delta_d^{\phi_i^-}\,\phi_i^-\,,\ \frac{\Delta_d^{\phi_i}}{\sqrt{2}}\,\phi_i + \frac{\Delta_d^{a_i}}{i\,\sqrt{2}}\,a_i\right) + \cdots$$

where Δ_d and the 6 coefficients are

$$\Delta_d = \langle\mathbf{h}_d^{-1}\Delta\mathbf{h}_d[\Phi_1,\Phi_2]\rangle \quad ; \quad \Delta_d^{\phi_i^-} = \left\langle\frac{\delta\,\mathbf{h}_d^{-1}\Delta\mathbf{h}_d[\Phi_1,\Phi_2]}{\delta\phi_i^-}\right\rangle$$

$$\frac{\Delta_d^{\phi_i}}{\sqrt{2}} = \left\langle\frac{\delta\,\mathbf{h}_d^{-1}\Delta\mathbf{h}_d[\Phi_1,\Phi_2]}{\delta\phi_i}\right\rangle \quad ; \quad \frac{\Delta_d^{a_i}}{i\,\sqrt{2}} = \left\langle\frac{\delta\,\mathbf{h}_d^{-1}\Delta\mathbf{h}_d[\Phi_1,\Phi_2]}{\delta a_i}\right\rangle$$

suppressing the vanishing iso-doublet components on the RHSs

♠ *Higgs-mediated Flavour-changing Effective Lagrangian* $(3/8)$

- Mass terms:

$$\left(-\,\mathcal{L}^{d}_{\rm eff}[\Phi_1,\Phi_2]\right)^{\rm Mass} = \overline{d_{R\,i}}\;\left[\mathcal{U}^{d\,\dagger}_{R}\,\mathbf{h}_d\left(\frac{1}{\sqrt{2}}v_1+\Delta_d\right)\,\mathcal{U}^{Q}_{L}\,V_{\rm CKM}\right]_{ij} d_{L\,j} = \overline{d_R}\,\widehat{M}_d\,d_L$$

Therefore, we have

$$\widehat{M}_d = {\rm diag}(m_d,m_s,m_b) = \mathcal{U}^{d\,\dagger}_{R}\,\mathbf{h}_d\left(\frac{1}{\sqrt{2}}v_1+\Delta_d\right)\,\mathcal{U}^{Q}_{L}\,V_{\rm CKM}$$

In other words, the Yukawa-coupling matrix is given by

$$\boxed{\mathcal{U}^{d\,\dagger}_{R}\,\mathbf{h}_d\,\mathcal{U}^{Q}_{L} = \frac{\sqrt{2}}{v_1}\widehat{M}_d\,V^{\dagger}_{\rm CKM}\,R^{-1}_d \quad \text{with} \quad R_d \equiv \mathcal{U}^{Q\,\dagger}_{L}\left(\mathbf{1}+\frac{\sqrt{2}}{v_1}\Delta_d\right)\mathcal{U}^{Q}_{L}}$$

As we will see, the Higgs-mediated FCNC is propotional to $V^{\dagger}_{\rm CKM}\,R^{-1}_d V_{\rm CKM}$ in the SHI approximation

♠ *Higgs-mediated Flavour-changing Effective Lagrangian* $(4/8)$

- The interaction of the charged Higgs bosons:

$$\left(-\mathcal{L}^{d}_{\rm eff}\right)^{H^\pm} = \frac{g}{\sqrt{2}M_W}\,\overline{d}\,\widehat{M}_d\,\mathbf{g}^L_{H^-\bar{d}u}\,P_L\,u\,H^- + \mathrm{h.c.}$$

where

$$\mathbf{g}^L_{H^-\bar{d}u} \equiv V^\dagger_{\rm CKM}\,R_d^{-1}\,\mathcal{U}_L^{Q\,\dagger}\left[-t_\beta(\mathbf{1}+\Delta_d^{\phi_1^-})+\Delta_d^{\phi_2^-}\right]\mathcal{U}_L^Q$$

Here we have used:

$$\phi_1 = O_{1i}\,H_i,\quad a_1 = c_\beta\,G^0 - s_\beta\,O_{3i}\,H_i,\quad \phi_1^- = c_\beta\,G^- - s_\beta\,H^-,$$
$$\phi_2 = O_{2i}\,H_i,\quad a_2 = s_\beta\,G^0 + c_\beta\,O_{3i}\,H_i,\quad \phi_2^- = s_\beta\,G^- + c_\beta\,H^-.$$

- Due to WIs relating $W^\pm ud$ to $G^\pm ud$, the interaction of $G^\pm$ is as in the SM:

$$-\mathcal{L}^{G^\pm}_{\rm eff} = \frac{g}{\sqrt{2}M_W}\,G^-\,\bar{d}\,(\,\widehat{\mathbf{M}}_d\,\mathbf{V}^\dagger P_L\;-\;\mathbf{V}^\dagger\,\widehat{\mathbf{M}}_u\,P_R\,)\,u\;+\;\mathrm{H.c.}$$

♠ *Higgs-mediated Flavour-changing Effective Lagrangian* $(5/8)$

- The interaction of the neutral Higgs bosons:

$$\left(-\mathcal{L}^{d}_{\rm eff}\right)^{H} = \frac{g}{2M_W}\,\overline{d}\,\widehat{M}_d\,\mathbf{g}^{L}_{H_i\bar{d}d}\,P_L\,d\,H_i + {\rm h.c.}$$

where

$$\begin{aligned}\mathbf{g}^{L}_{H_i\bar{d}d} \;&\equiv\; \frac{O_{1i}}{c_\beta}\,V^{\dagger}_{\rm CKM}\,R_d^{-1}\,\mathcal{U}^{Q\,\dagger}_L\left(\mathbf{1}+\Delta_d^{\phi_1}\right)\mathcal{U}^{Q}_L\,V_{\rm CKM}\\ &+\; \frac{O_{2i}}{c_\beta}\,V^{\dagger}_{\rm CKM}\,R_d^{-1}\,\mathcal{U}^{Q\,\dagger}_L\,\Delta_d^{\phi_2}\,\mathcal{U}^{Q}_L\,V_{\rm CKM}\\ &+\; i\,O_{3i}\,t_\beta\,V^{\dagger}_{\rm CKM}\,R_d^{-1}\,\mathcal{U}^{Q\,\dagger}_L\,(\mathbf{1}+\Delta_d^{a_1}-\Delta_d^{a_2}/t_\beta)\,\mathcal{U}^{Q}_L\,V_{\rm CKM}\end{aligned}$$

and $R_d = \mathcal{U}^{Q\,\dagger}_L\left(\mathbf{1}+\frac{\sqrt{2}}{v_1}\,\Delta_d\right)\mathcal{U}^{Q}_L$

♠ *Higgs-mediated Flavour-changing Effective Lagrangian* $(6/8)$

- The functional $\mathbf{h}_d^{-1}\Delta\mathbf{h}_d[\Phi_1,\Phi_2]$ should have the following form:

$$\mathbf{h}_d^{-1}\Delta\mathbf{h}_d[\Phi_1,\Phi_2] \;=\; \Phi_1^\dagger\,\mathbf{f_1}(\Phi_i^\dagger\Phi_j) \;+\; \Phi_2^\dagger\,\mathbf{f_2}(\Phi_i^\dagger\Phi_j)$$

In the Single Higgs Insertion (SHI) approximation, the 3×3-dimensional functionals $\mathbf{f_1}$ and $\mathbf{f_2}$ are only functions of vevs, $\mathbf{f_1} = \langle\mathbf{f_1}\rangle$ and $\mathbf{f_2} = \langle\mathbf{f_2}\rangle$, and one can obtain

$$\Delta_d = \frac{1}{\sqrt{2}}\left(\langle\mathbf{f_1}\rangle\, v_1 + \langle\mathbf{f_2}\rangle\, v_2\right)$$

$$\Delta_d^{\phi_1^-} = \Delta_d^{a_1} = \Delta_d^{\phi_1} = \langle\mathbf{f_1}\rangle \quad ; \quad \Delta_d^{\phi_2^-} = \Delta_d^{a_2} = \Delta_d^{\phi_2} = \langle\mathbf{f_2}\rangle$$

In the limit $v_1 \to 0$ and neglecting $\mathbf{f_1}$, $\langle\mathbf{f_1}\rangle = \mathbf{0}$, we have

$$\Delta_d^{\phi_1^-} = \Delta_d^{a_1} = \Delta_d^{\phi_1} = 0 \quad ; \quad \Delta_d^{\phi_2^-} = \Delta_d^{a_2} = \Delta_d^{\phi_2} = \frac{\sqrt{2}}{v_2}\,\Delta_d = \frac{\mathcal{U}_L^Q\,(R_d - \mathbf{1})\,\mathcal{U}_L^{Q\,\dagger}}{t_\beta}$$

♠ *Higgs-mediated Flavour-changing Effective Lagrangian* (7/8)

- In the $\mathcal{U}_R^d = \mathcal{U}_L^Q = \mathbf{1}_3$ basis,
  - $\mathbf{h}_d = \frac{\sqrt{2}}{v_1}\widehat{M}_d\, V_{\rm CKM}^\dagger\, R_d^{-1}$
  - $R_d = \mathbf{1} + \frac{\sqrt{2}}{v_1}\Delta_d = \mathbf{1} + \tan\beta\left(\frac{\sqrt{2}}{v_2}\Delta_d\right)$
  - $\left(\mathbf{g}^L_{H^-\bar{d}u}\right)_{\rm SHI} = V_{\rm CKM}^\dagger\,\frac{1}{t_\beta}\left(\mathbf{1} - \frac{R_d^{-1}}{c_\beta^2}\right)$
  - $$\left(\mathbf{g}^L_{H_i\bar{d}d}\right)_{\rm SHI} = \frac{O_{1i}}{c_\beta}V_{\rm CKM}^\dagger\, R_d^{-1}\, V_{\rm CKM} + \frac{O_{2i}}{s_\beta}(\mathbf{1} - V_{\rm CKM}^\dagger\, R_d^{-1}\, V_{\rm CKM}) - i\,\frac{O_{3i}}{t_\beta}\left(\mathbf{1} - \frac{1}{c_\beta^2}V_{\rm CKM}^\dagger\, R_d^{-1}\, V_{\rm CKM}\right)$$

In agreement with A. Dedes and A.P., PRD**67** (2003) 015012 [arXiv:hep-ph/0209306]

All we need to know is Δ_d !

N.B. In the SHI approximation, $\left(V_{\rm CKM}^\dagger\, R_d^{-1}\, V_{\rm CKM}\right)_{\bar{d}d'} \propto (V_{\rm CKM})^*_{td}(V_{\rm CKM})_{td'}\,\tan\beta$

♠ *Higgs-mediated Flavour-changing Effective Lagrangian* (8/8)

- Finally,

$$\boxed{\Delta_d \equiv \Delta_d^{\widetilde{g}} + \Delta_d^{\widetilde{H}} + \cdots}$$

$$\begin{aligned}
\frac{\sqrt{2}}{v_2}(\Delta_d^{\widetilde{g}})_{ij} &= \frac{2\alpha_3}{3\pi}\,\mu^* M_3^*\,(\mathbf{h_d}^{-1})_{ik}\,(\mathbf{h_d})_{kj}\,I(M^2_{\widetilde{D}_k}, M^2_{\widetilde{Q}_j}, |M_3|^2)\,,\\
\frac{\sqrt{2}}{v_2}(\Delta_d^{\widetilde{H}})_{ij} &= \frac{1}{16\pi^2}\,\mu^*\,(\mathbf{a_u}^\dagger)_{ik}\,(\mathbf{h_u})_{kj}\,I(M^2_{\widetilde{U}_k}, M^2_{\widetilde{Q}_i}, |\mu|^2)\,,
\end{aligned}$$

where the flavour off-diagonal elements of $M^2_{\widetilde{Q},\widetilde{U},\widetilde{D}}$ are neglected

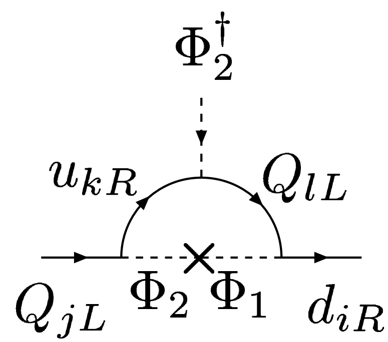
If, furthermore, the squark mass matrices are universal,

$$\begin{aligned}
\frac{\sqrt{2}}{v_2}\left(\Delta_d^{\widetilde{g}}\right)_{ij} &= \frac{2\alpha_3}{3\pi}\,\mu^* M_3^*\,\delta_{ij}\,I\left(M^2_{\widetilde{D}}, M^2_{\widetilde{Q}}, |M_3|^2\right)\,,\\
\frac{\sqrt{2}}{v_2}\left(\Delta_d^{\widetilde{H}}\right)_{ij} &= \frac{1}{16\pi^2}\,\mu^*\left(\mathbf{a_u}^\dagger\,\mathbf{h_u}\right)_{ij}\,I\left(M^2_{\widetilde{U}}, M^2_{\widetilde{Q}}, |\mu|^2\right)
\end{aligned}$$

For universal A terms, $\mathbf{a_u} = A_u\,\mathbf{h_u}$

♠ *Higgs-mediated Flavour-changing Effective Lagrangian* $(8'/8)$

- The subleading Higgs-mediated contribution Δ_d^{2HDM}



$$\frac{\sqrt{2}}{v_2}\left(\boldsymbol{\Delta}_d^{\mathrm{2HDM}}\right)_{ij} = \frac{(\mathbf{h}_u^\dagger \mathbf{h}_u)_{ij}}{16\,\pi^2}\,\frac{B^*\mu^*}{M_{H_d}^2 - M_{H_u}^2}\,\ln\left|\frac{M_{H_d}^2 + |\mu|^2}{M_{H_u}^2 + |\mu|^2}\right|$$

♠ *Numerical Examples* $(1/11)$

- For numerical analysis,
  - Develop RG program for the MCPMFV framework
  - Take the GUT scale to be the MFV scale
  - Use the code CPsuperH for the Higgs mass spectrum and mixing matrix
  - Take the SHI approximation limit of $v_1 \to 0$
  - Keep only leading contributions to Δ_d at the SUSY scale, by neglecting
    (i) flavour off-diagonal elements in the squark mass matrices;
    (ii) EW corrections.
  - Calculate: (i) ΔM_{B_s}; (ii) ΔM_{B_d}; (iii) $B(B_s \to \mu^+\mu^-)$; (iv) $B(B_u^- \to \tau^-\nu)$; (v) $B(b \to X_s\gamma)$; (vi) $\mathcal{A}_{\rm CP}^{\rm dir}(b \to X_s\gamma)$.
  - Work in progress: (i) EDMs; (ii) K-meson observables; (iii) More B-meson observables; (iv) $\cdots$

♠ *Numerical Examples* $(2/11)$

- Input parameters
  - At M_Z: Three gauge couplings $\alpha_1(M_Z)$, $\alpha_2(M_Z)$, and $\alpha_3(M_Z)$
  - At $m_t^{\rm pole}$: Quark and Lepton masses $m_{q,l}(m_t^{\rm pole})$ and $V_{\rm CKM}(m_t^{\rm pole})$
  - At $M_{\rm SUSY}$: $\tan\beta(M_{\rm SUSY})$
  - At $M_{\rm MFV} = M_{\rm GUT}$: 19 MCPMFV Parameters

$$|M_{1,2,3}|\, {\rm e}^{i\,\Phi_{1,2,3}}\,, \qquad |A_{u,d,e}|\, {\rm e}^{i\,\Phi_{A_{u,d,e}}}\,, \qquad \widetilde{M}^2_{Q,U,D,L,E}\,, \qquad M^2_{H_{u,d}}$$

Specifically, we have taken the parameter set:

$$|M_{1,2,3}| = 250 \;\; {\rm GeV}$$

$$M^2_{H_u} = M^2_{H_d} = \widetilde{M}^2_Q = \widetilde{M}^2_U = \widetilde{M}^2_D = \widetilde{M}^2_L = \widetilde{M}^2_E = (100 \;\; {\rm GeV})^2$$

$$|A_u| = |A_d| = |A_e| = 100 \;\; {\rm GeV}$$

This parameter set is equivalent to SPS1a when $\Phi_{A_{u,d,e}} = 180^\circ$ and $\Phi_{1,2,3} = 0^\circ$ if $M_{\rm SUSY} = m_t^{\rm pole}$ and $\tan\beta = 10$

♠ *Numerical Examples* $(3/11)$

- For CP phases, we vary three types of them:

$$\boxed{\Phi_{12}\,, \qquad \Phi_3\,, \qquad \Phi_A^{\rm GUT}}$$

  - We adopt the convention $\Phi_\mu = 0^\circ$

    Note that the phase of μ does not change during RG running: $\Phi_\mu(M_{\rm SUSY}) = \Phi_\mu(M_{\rm GUT})$

  - For simplicity, we take a common phase $\Phi_{12} \equiv \Phi_1 = \Phi_2$

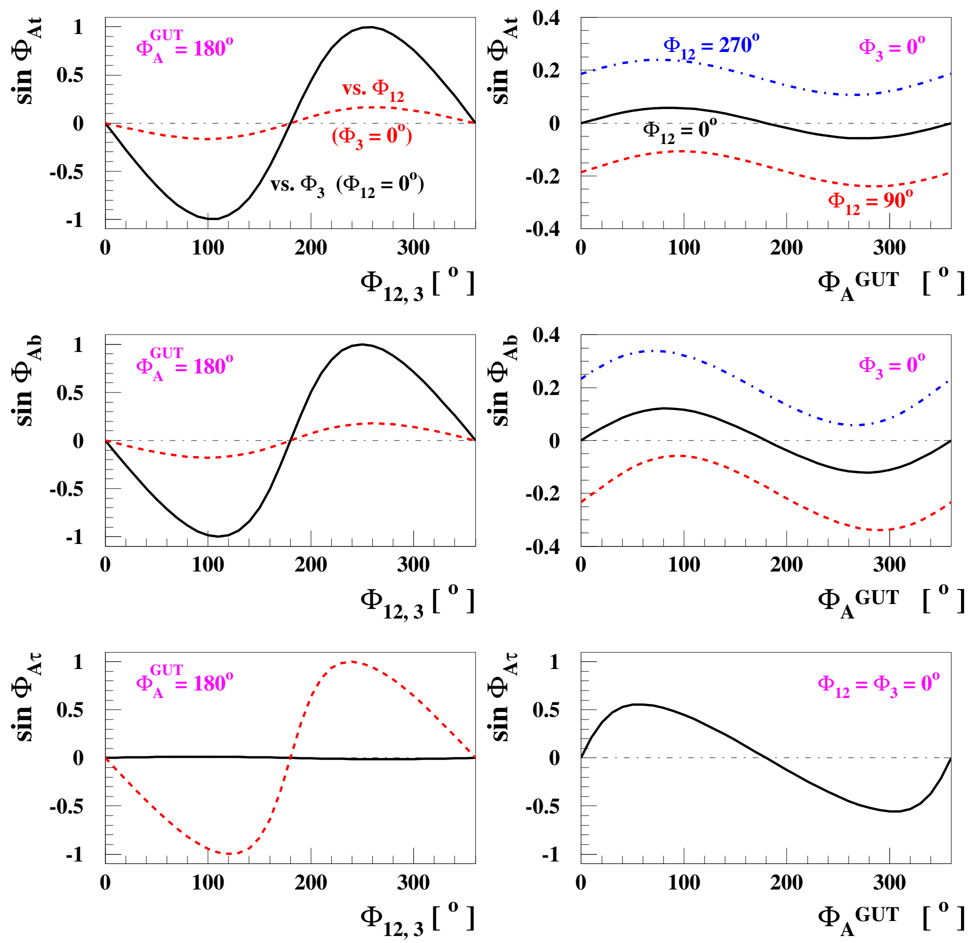
    Again the three phases of the gaugino mass parameters $\Phi_{1,2,3}$ remain same: $\Phi_{1,2,3}(M_{\rm SUSY}) = \Phi_{1,2,3}(M_{\rm GUT})$

  - Again, for simplicity, we take a common phase for A terms:

$$\Phi_A^{\rm GUT} \equiv \Phi_{A_u} = \Phi_{A_d} = \Phi_{A_e}$$

♠ *Numerical Examples* $(4/11)$

- $A_f(M_{\rm SUSY}) \equiv (\mathbf{a}_f)_{33}/(\mathbf{h}_f)_{33}$ at $M_{\rm SUSY}$ with $f = t, b, \tau$ $\tan\beta(M_{\rm SUSY}) = 10$ with $M_{\rm SUSY} \sim 535$ GeV



We have found $A_f(M_{\rm SUSY})$ can be written as:

$$\boxed{A_f(M_{\rm SUSY}) \approx C_f^{A_f}\, A_f^{\rm GUT} - C_f^{M_j}\, M_j^{\rm GUT}}$$

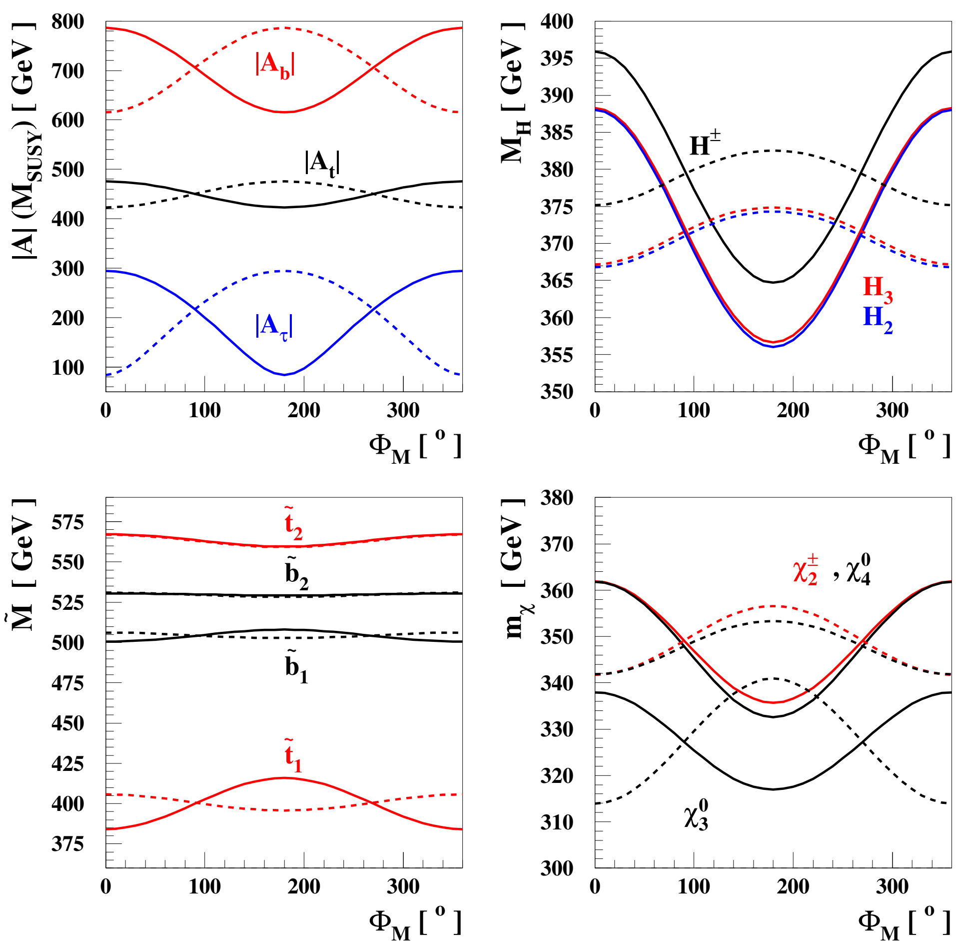
T. Goto, Y. Y. Keum, T. Nihei, Y. Okada and Y. Shimizu, PLB**460** (1999) 333, [arXiv:hep-ph/9812369]

Some comments:

- ▼ $C_{t,b}^{A_{t,b}} < C_{t,b}^{M_{1,2}} \ll C_{t,b}^{M_3}$
- ▼ $C_t^{A_t} < C_b^{A_b}$
- ▼ $C_\tau^{A_\tau} < C_\tau^{M_{1,2}}$
- ▼ $C_\tau^{M_3} \sim 0$
- ▼ For large $\tan\beta$, $|C_\tau^{M_3}|$ becomes significant and $C_b^{A_b}$ becomes smaller

♠ *Numerical Examples* (5/11)

- Masses as functions of $\Phi_M \equiv \Phi_1 = \Phi_2 = \Phi_3$ $\tan\beta(M_{\rm SUSY}) = 10$



$$\boxed{|A_f(M_{\rm SUSY})|^2 \approx \alpha_f - \beta_f \cos(\Phi_A^{\rm GUT} - \Phi_M)}$$

$\alpha_f\,,\beta_f\ > 0$

<u>Some comments</u>:

- Solid : $\Phi_A^{\rm GUT} = 180^\circ$; Dashed : $\Phi_A^{\rm GUT} = 0^\circ$
- Strong correlation between $|A_f(M_{\rm SUSY})|$ and the particle masses mainly due to the CP-phase dependent term ${\rm Tr}(\mathbf{a}_u^\dagger \mathbf{a}_u)$ in RGEs:

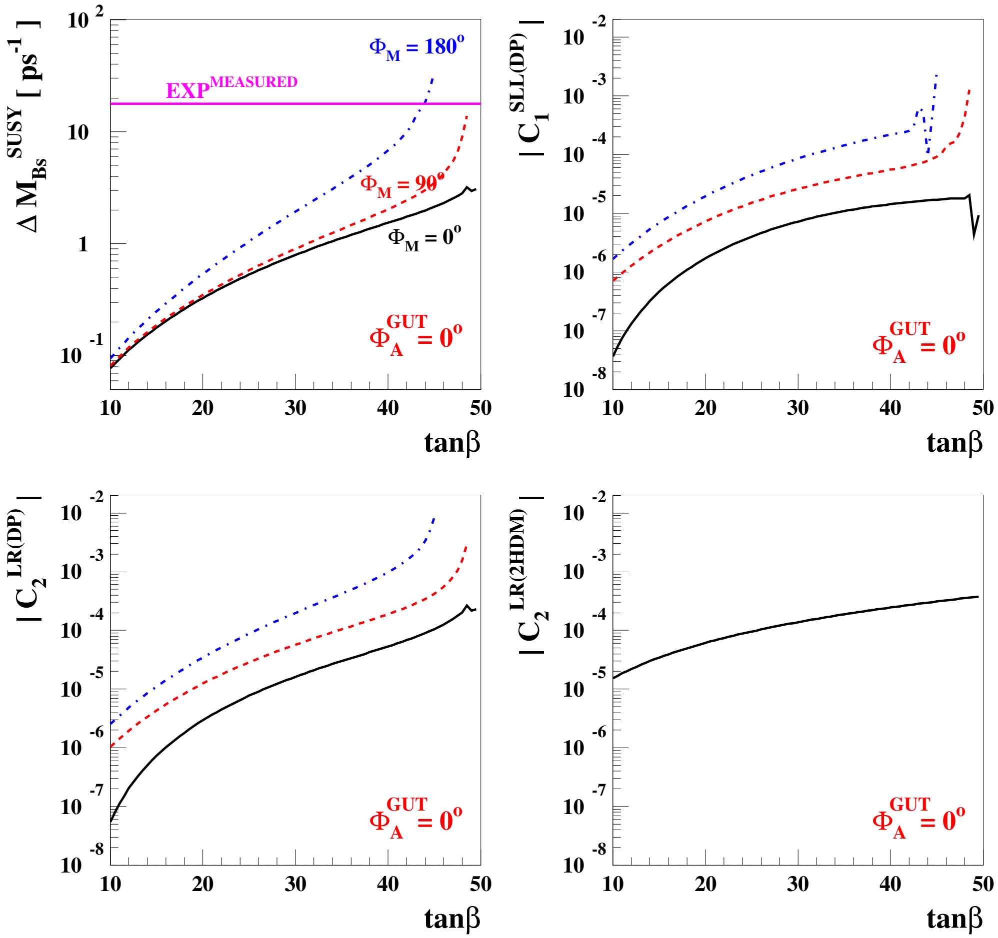
  - ${\rm Tr}(\mathbf{a}_u^\dagger \mathbf{a}_u) \Uparrow \longrightarrow |M^2_{H_u}| \Uparrow \longrightarrow M_{H^\pm} \Uparrow$
  - ${\rm Tr}(\mathbf{a}_u^\dagger \mathbf{a}_u) \Uparrow \longrightarrow \widetilde{M}^2_{U,Q} \Downarrow$
  - ${\rm Tr}(\mathbf{a}_u^\dagger \mathbf{a}_u) \Uparrow \longrightarrow |M^2_{H_u}| \Uparrow \longrightarrow |\mu| \Uparrow$

♠ *Numerical Examples* (6/11)

- $\Delta M_{B_s}^{\rm SUSY}$ as functions of $\tan\beta(M_{\rm SUSY})$ for three values of $\Phi_M \equiv \Phi_1 = \Phi_2 = \Phi_3$

$\widetilde{M}_{L,E} = 200$ GeV and $\Phi_A^{\rm GUT} = 0^\circ$

$$\boxed{\Delta M_{B_s}^{\rm SUSY} \;=\; 2\,|\langle \bar{B}_s^0|\,H_{\rm eff}^{\Delta B=2}\,|B_s^0\rangle_{\rm SUSY}|}$$



$$\Delta M_{B_s}^{\rm EXP} \;=\; 17.77 \pm 0.10(stat) \pm 0.07(syst)\,{\rm ps}^{-1}$$

H. G. Evans, arXiv:0705.4598v1 [hep-ex]

$$\langle \bar{B}_s^0|\,H_{\rm eff}^{\Delta B=2}\,|B_s^0\rangle_{\rm SUSY} = \frac{2310}{\rm ps}\left(\frac{\hat{B}_{B_s}^{1/2}\,F_{B_s}}{265\ {\rm MeV}}\right)^2\left(\frac{\eta_B}{0.55}\right)$$

$$\times\left[0.88\left(C_{2\,{\rm LR}}^{\rm (DP)} + C_{2\,{\rm LR}}^{\rm 2HDM}\right) - 0.52\left(C_{1\,{\rm SLL}}^{\rm (DP)} + C_{1\,{\rm SRR}}^{\rm (DP)}\right)\right]$$

$$C_{1\,{\rm SLL}}^{\rm (DP)} \;=\; -\,\frac{16\pi^2 m_b^2}{\sqrt{2}\,G_F M_W^2}\sum_{i=1}^{3}\frac{g_{H_i\bar{b}s}^L\,g_{H_i\bar{b}s}^L}{M_{H_i}^2}$$

$$C_{1\,{\rm SRR}}^{\rm (DP)} \;=\; -\,\frac{16\pi^2 m_s^2}{\sqrt{2}\,G_F M_W^2}\sum_{i=1}^{3}\frac{g_{H_i\bar{b}s}^R\,g_{H_i\bar{b}s}^R}{M_{H_i}^2}$$

$$C_{2\,{\rm LR}}^{\rm (DP)} \;=\; -\,\frac{32\pi^2 m_b m_s}{\sqrt{2}\,G_F M_W^2}\sum_{i=1}^{3}\frac{g_{H_i\bar{b}s}^L\,g_{H_i\bar{b}s}^R}{M_{H_i}^2}$$

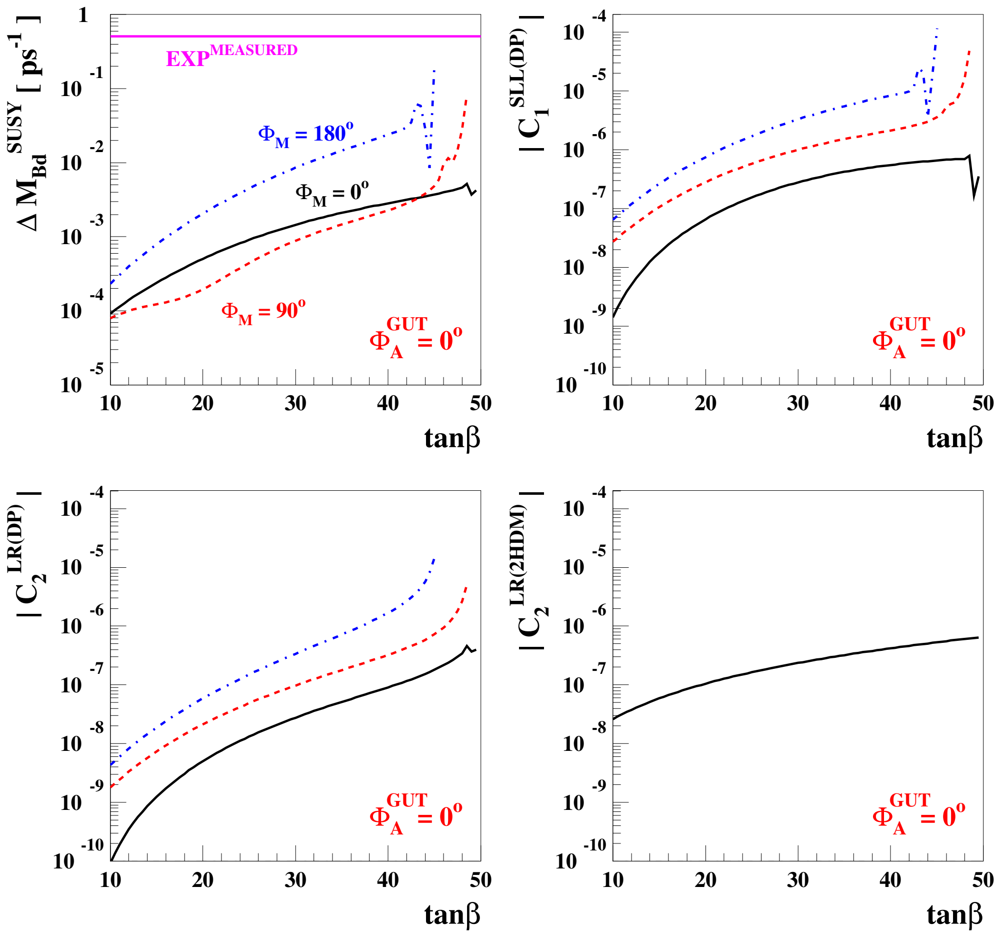
$$C_{2\,{\rm LR}}^{\rm (2HDM)} \;\approx\; -\,\frac{2m_b m_s}{M_W^2}\,(V_{tb}^* V_{ts})^2\,\tan^2\beta$$

♠ *Numerical Examples* (7/11)

- $\Delta M_{B_d}^{\rm SUSY}$ as functions of $\tan\beta(M_{\rm SUSY})$ for three values of $\Phi_M \equiv \Phi_1 = \Phi_2 = \Phi_3$

$\widetilde{M}_{L,E} = 200$ GeV and $\Phi_A^{\rm GUT} = 0^\circ$

$$\Delta M_{B_d}^{\rm SUSY} = 2\,|\langle \bar{B}_d^0|\, H_{\rm eff}^{\Delta B=2}\, |B_d^0\rangle_{\rm SUSY}|$$



$$\Delta M_{B_d}^{\rm EXP} = 0.507 \pm 0.005\,{\rm ps}^{-1} \qquad \text{PDG2006}$$

$$\langle \bar{B}_d^0|\, H_{\rm eff}^{\Delta B=2}\, |B_d^0\rangle_{\rm SUSY} = \frac{1711}{\rm ps}\left(\frac{\hat{B}_{B_d}^{1/2}\, F_{B_d}}{230\ {\rm MeV}}\right)^2 \left(\frac{\eta_B}{0.55}\right)$$

$$\times\left[0.88\left(C_{2\,{\rm LR}}^{\rm (DP)} + C_{2\,{\rm LR}}^{\rm 2HDM}\right) - 0.52\left(C_{1\,{\rm SLL}}^{\rm (DP)} + C_{1\,{\rm SRR}}^{\rm (DP)}\right)\right]$$

$$C_{1\,{\rm SLL}}^{\rm (DP)} = -\frac{16\pi^2 m_b^2}{\sqrt{2}\,G_F M_W^2}\sum_{i=1}^{3}\frac{{\rm g}^L_{H_i\bar{b}d}\,{\rm g}^L_{H_i\bar{b}d}}{M_{H_i}^2}$$

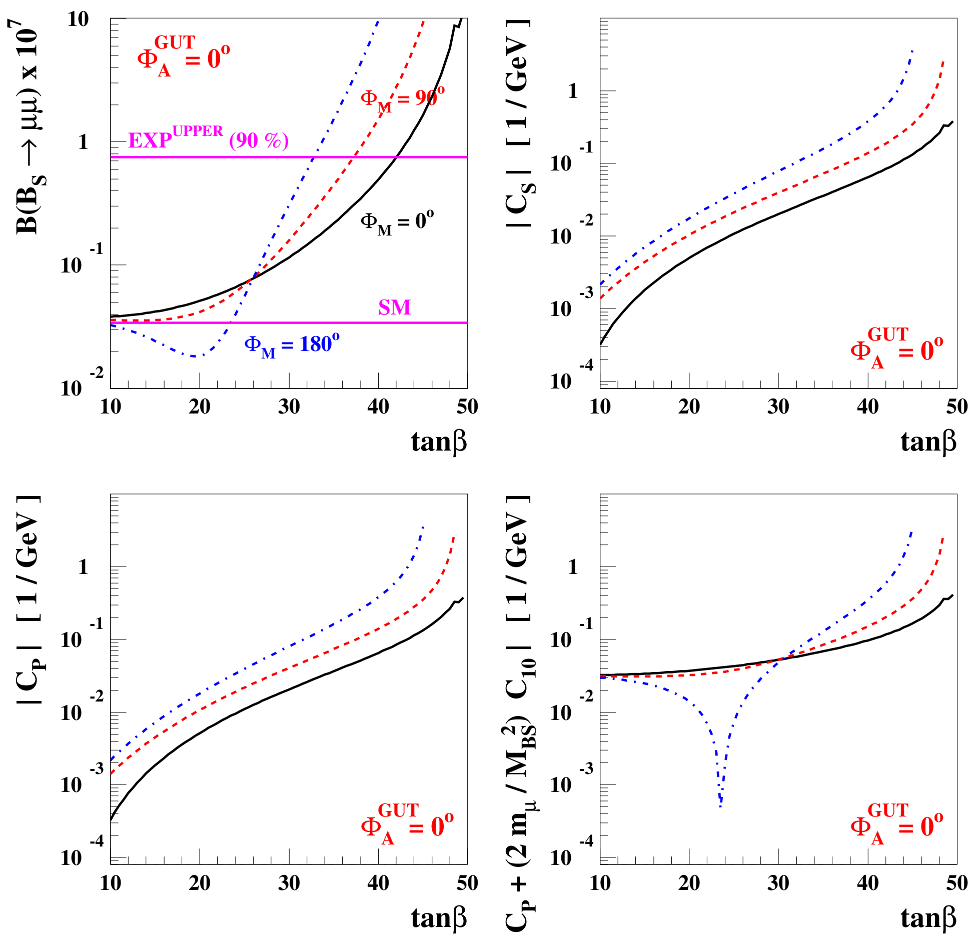
$$C_{1\,{\rm SRR}}^{\rm (DP)} = -\frac{16\pi^2 m_d^2}{\sqrt{2}\,G_F M_W^2}\sum_{i=1}^{3}\frac{{\rm g}^R_{H_i\bar{b}d}\,{\rm g}^R_{H_i\bar{b}d}}{M_{H_i}^2}$$

$$C_{2\,{\rm LR}}^{\rm (DP)} = -\frac{32\pi^2 m_b m_d}{\sqrt{2}\,G_F M_W^2}\sum_{i=1}^{3}\frac{{\rm g}^L_{H_i\bar{b}d}\,{\rm g}^R_{H_i\bar{b}d}}{M_{H_i}^2}$$

$$C_{2\,{\rm LR}}^{\rm (2HDM)} \approx -\frac{2m_b m_d}{M_W^2}\,(V_{tb}^* V_{td})^2\,\tan^2\beta$$

♠ *Numerical Examples* (8/11)

- $B(\bar{B}_s^0 \to \mu^+\mu^-)$ as functions of $\tan\beta(M_{\rm SUSY})$ for three values of $\Phi_M \equiv \Phi_1 = \Phi_2 = \Phi_3$ $\widetilde{M}_{L,E} = 200$ GeV and $\Phi_A^{\rm GUT} = 0^\circ$



$$\boxed{B(\bar{B}_s^0 \to \mu^+\mu^-)} =$$

$$\frac{G_F^2\alpha_{\rm em}^2}{16\pi^3} M_{B_s}\tau_{B_s} |V_{tb}V_{ts}^*|^2 \sqrt{1-\frac{4m_\mu^2}{M_{B_s}^2}}$$

$$\times \left[\left(1-\frac{4m_\mu^2}{M_{B_s}^2}\right) |F_S|^2 + |F_P + 2m_\mu F_A|^2 \right]$$

$$F_{S,P} = -\frac{i}{2} M_{B_s}^2 F_{B_s} \frac{m_b}{m_b+m_s} C_{S,P}$$

$$F_A = -\frac{i}{2} F_{B_s} C_{10}^{\rm SM}$$

where $C_{10}^{\rm SM} = -4.221$ and, with $\mathbf{g}^R_{H_i\bar{d}d} = \left(\mathbf{g}^L_{H_i\bar{d}d}\right)^\dagger$,

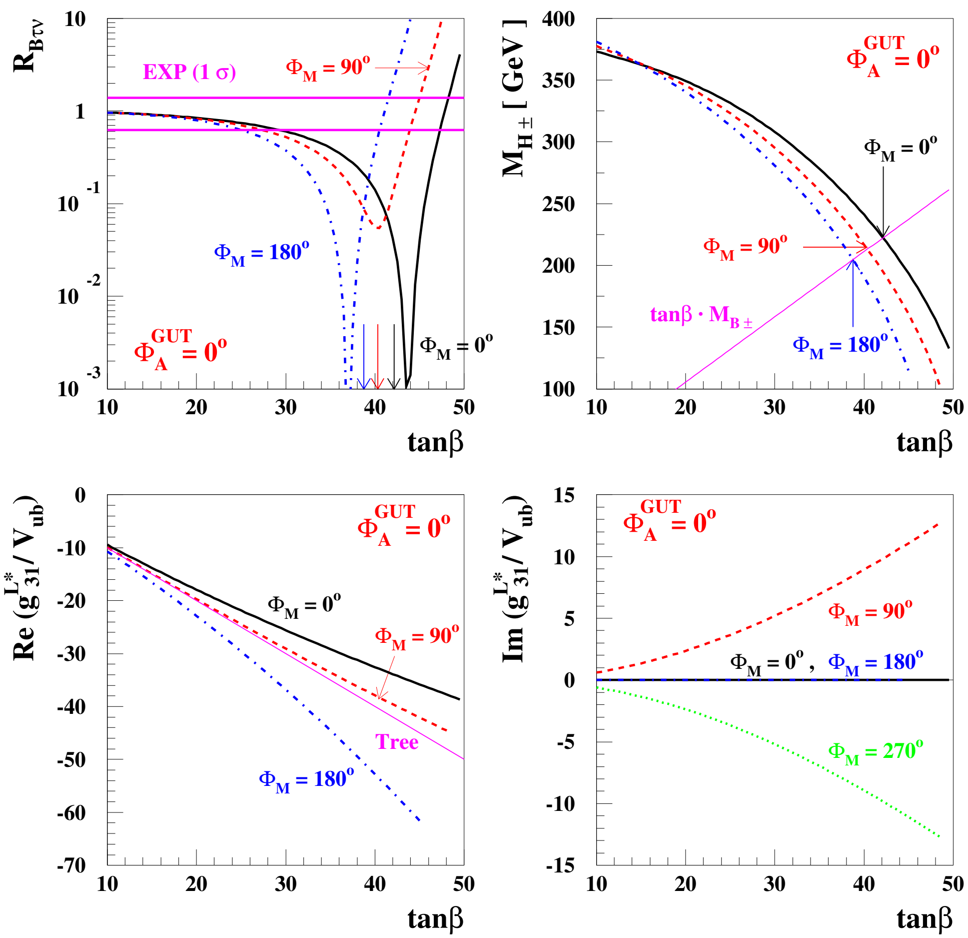
$$C_{S(P)} = (i)\frac{2\pi m_\mu}{\alpha_{\rm em}} \frac{1}{V_{tb}V_{ts}^*} \sum_{i=1}^{3} \frac{\mathbf{g}^R_{H_i\bar{s}b}\, g^{S(P)}_{H_i\bar{\mu}\mu}}{M_{H_i}^2} ,$$

$$g^S_{H_i\bar{\mu}\mu} = \frac{O_{1i}}{\cos\beta} , \qquad g^P_{H_i\bar{\mu}\mu} = -\tan\beta\, O_{3i}$$

Note $|C_P| \sim |C_S|$ since $H_1 \sim \phi_2$ and $M_{H_2} \sim M_{H_3}$

♠ *Numerical Examples* (9/11)

- The ratio $R_{B\tau\nu}$ as functions of $\tan\beta(M_{\rm SUSY})$ for three values of $\Phi_M \equiv \Phi_1 = \Phi_2 = \Phi_3$ $\widetilde{M}_{L,E} = 200$ GeV and $\Phi_A^{\rm GUT} = 0^\circ$



$$\boxed{R_{B\tau\nu} \equiv \frac{B(B^-\to\tau^-\nu)}{B^{\rm SM}(B^-\to\tau^-\nu)}}$$

$$= \left| 1 + \tan\beta \frac{\left(\mathbf{g}^{L\,\dagger}_{H^-\bar{d}u}\right)_{13}}{(\mathbf{V}_{\rm CKM})_{13}} \left(\frac{M_{B^\pm}}{M_{H^\pm}}\right)^2 \right|^2$$

At tree level, $\mathbf{g}^{L}_{H^-\bar{d}u} = -\tan\beta\, \mathbf{V}^\dagger_{\rm CKM}$

$$\boxed{R^{\rm EXP}_{B\tau\nu} = 1.00 \pm 0.38}$$

$$B(B^- \to \tau^-\bar{\nu}) \times 10^4 \;=\; 1.79^{+0.56}_{-0.49}\,({\rm stat})^{+0.46}_{-0.51}\,({\rm syst})$$

BELLE, PRL97(2006)251802

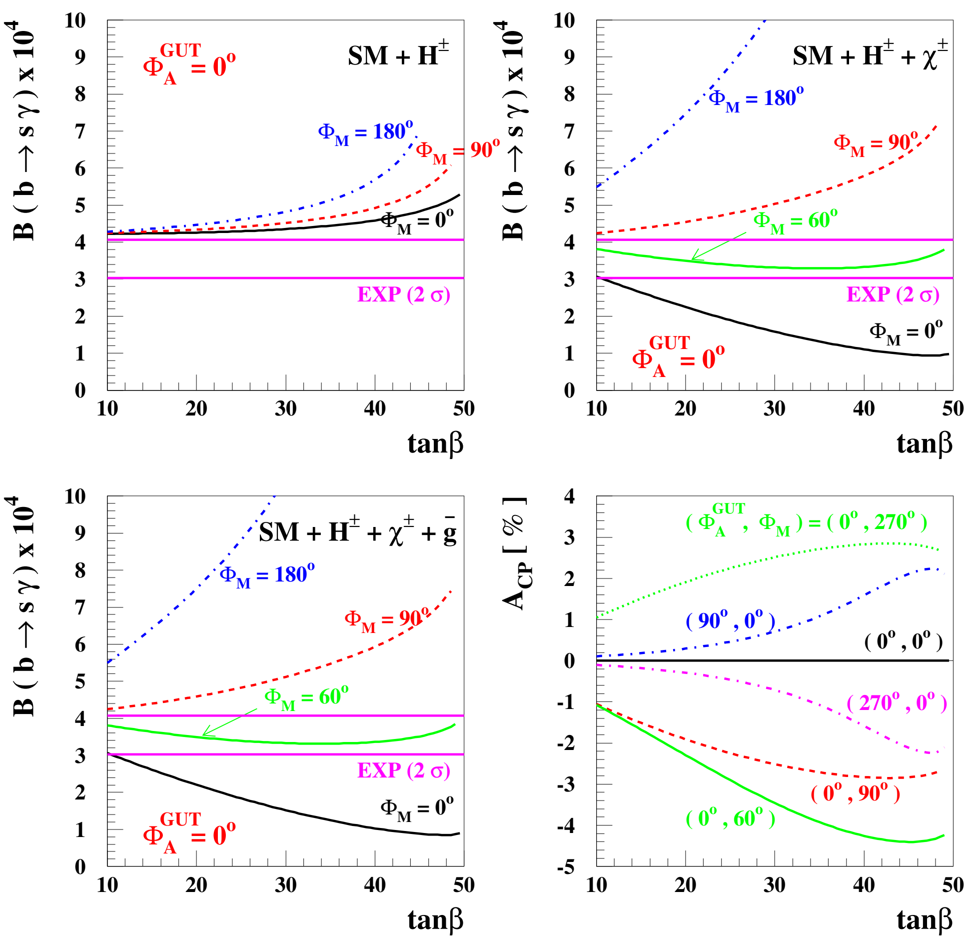
$$B(B^- \to \tau^-\bar{\nu}) \times 10^4 \;=\; 1.2 \pm 0.4\,({\rm stat}) \pm 0.3\,({\rm syst}_{\rm bkg}) \pm 0.2\,({\rm syst}')$$

BABAR, arXiv : 0708.2260[hep − ex]

$$B(B^- \to \tau^-\bar{\nu})^{\rm SM} \times 10^4 \;=\; 1.41 \pm 0.33$$

♠ *Numerical Examples* (10/11)

- $B(B \to X_s\gamma)$ and $\mathcal{A}_{\rm CP}^{\rm dir}(B \to X_s\gamma)$ as functions of $\tan\beta(M_{\rm SUSY})$ for several values of $\Phi_M \equiv \Phi_1 = \Phi_2 = \Phi_3$ and $\Phi_A^{\rm GUT}$ $\widetilde{M}_{L,E} = 200$ GeV



$$B^{\rm EXP} \times 10^4 = 3.55 \pm 0.24^{+0.09}_{-0.10} \pm 0.03$$

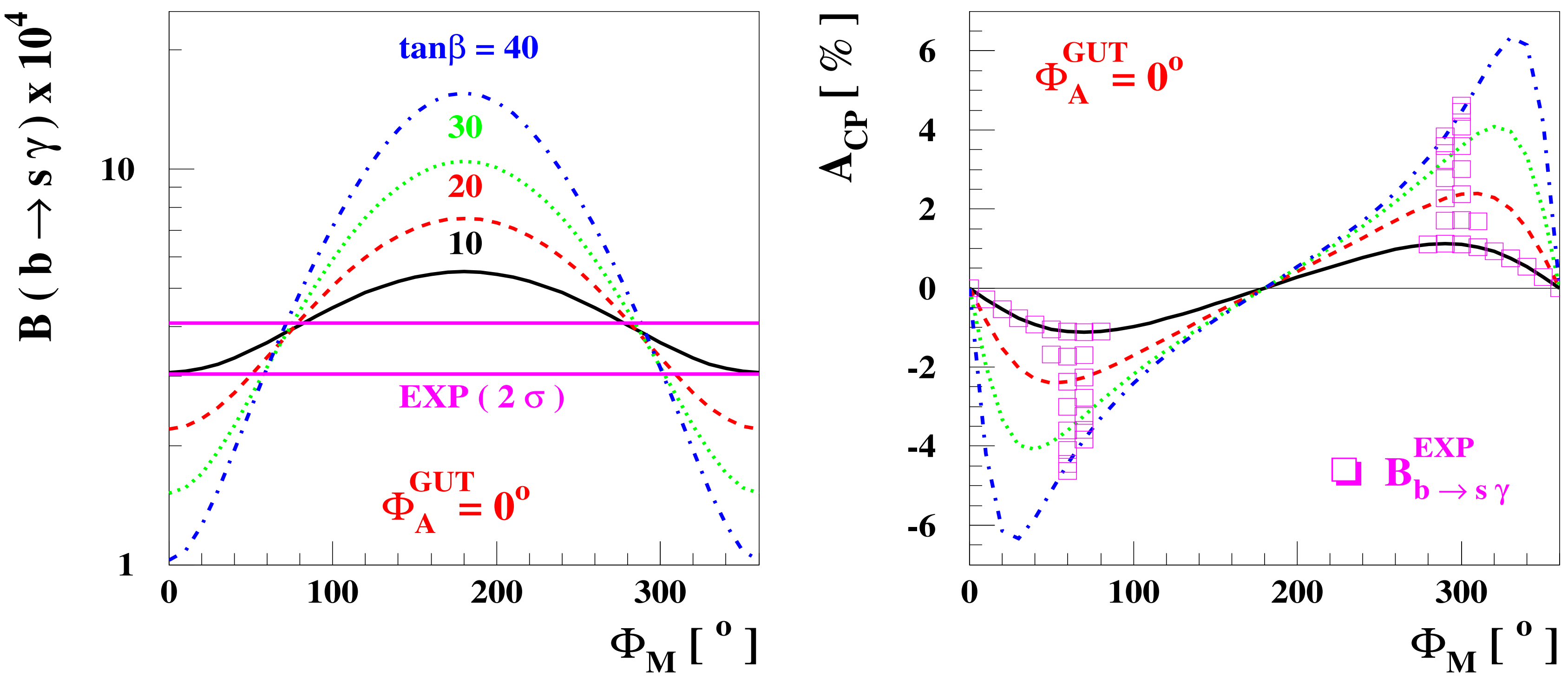
HFAG, `arXiv:0704.3575[hep-ex]`

Some comments:

- ▼ The charged-Higgs contribution is larger: $\tan\beta \Uparrow \to M_{H^\pm} \Downarrow$
- ▼ The chargino contribution:
$$C_{7,8}^{\chi^\pm} \propto \sim -\Re\text{e}\left(\text{e}^{i\,\Phi_{A_t}} / \cos_\beta\right)$$
$$\sim \cos\Phi_M / \cos_\beta$$
▽ N.B. $\cos(\Phi_{A_t}) \sim -\cos(\Phi_M)$
- ▼ The gluino contribution is negligible
- ▼ The case of $\Phi_M = 60^\circ \to \Phi_M$ dependence

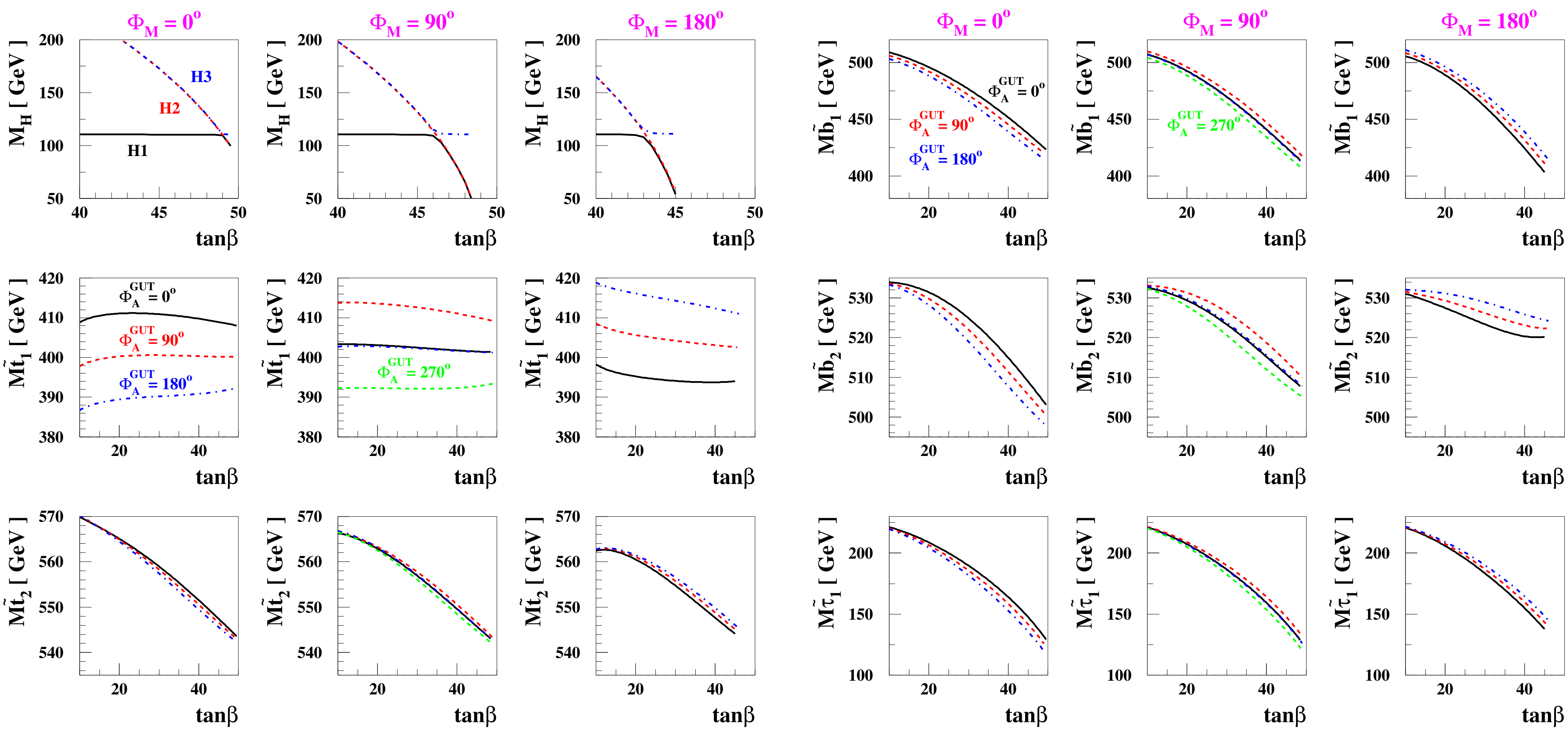
♠ *Numerical Examples* (11/11)

- $B(B \to X_s\gamma)$ and $\mathcal{A}_{\rm CP}^{\rm dir}(B \to X_s\gamma)$ as functions of Φ_M for four values of $\tan\beta(M_{\rm SUSY}) = 10\,, 20\,, 30\,, 40$ $\widetilde{M}_{L,E} = 200$ GeV and $\Phi_A^{\rm GUT} = 0^\circ$



♠ *Numerical Examples* $(11'/11)$

- (BACKUP) Masses as functions of $\tan\beta(M_{\rm SUSY})$ for three values of $\Phi_M \equiv \Phi_1 = \Phi_2 = \Phi_3$ $\widetilde{M}_{L,E} = 200$ GeV and $\Phi_A^{\rm GUT} = 0^\circ$



♠ *Summary and Future Directions* $(1/1)$

- We introduced the MSSM with Maximal CP and Minimal Flavour Violation, MCPMFV
- We presented a Flavour-Covariant Effective Lagrangian formalism, from which we derived the Higgs-mediated flavour-changing effective couplings
- For high $\tan\beta$, FCNC B-meson observables set severe limits on parameters.
- EDM constraints need be implemented.
- Further Improvements on a Flavour-Covariant Effective Lagrangian without approximations are under way.
- Release of the new version CPsuperH2.0.

  [J.S. Lee, M. Carena, J. Ellis, A.P., C.E.M. Wagner, arXiv:0712.nnnn]