Physics at the LHC, LIP, June 3th, 2013

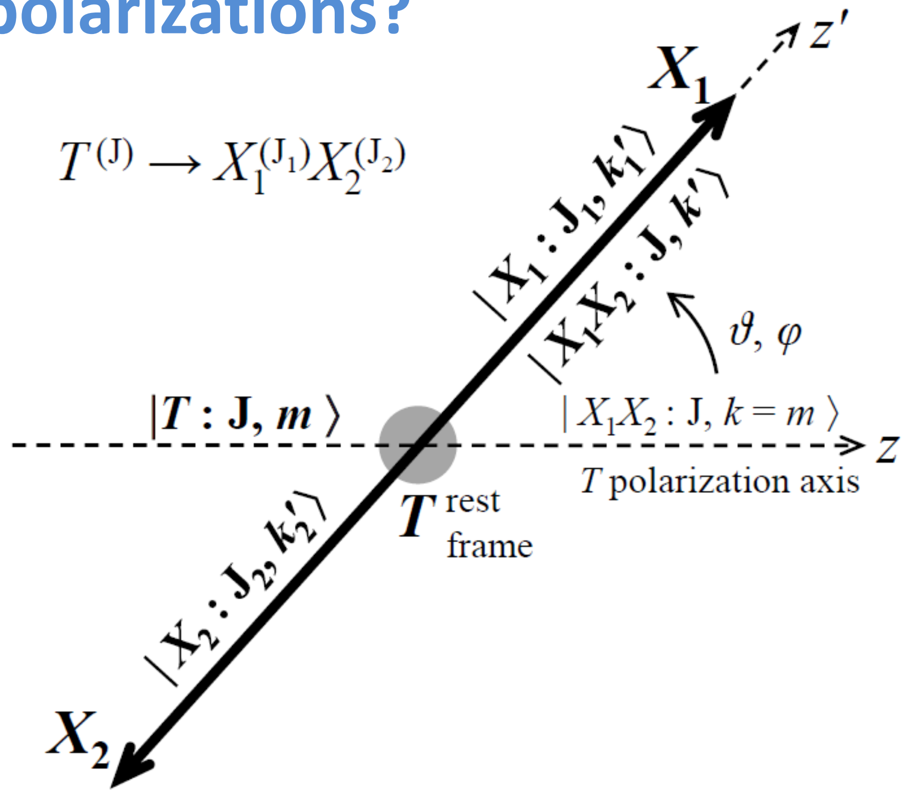
Particle polarizations in LHC physics

Pietro Faccioli

- Motivations
- Basic principles: angular momentum conservation, helicity conservation, parity properties
- Example: dilepton decay distributions of quarkonium and vector bosons
- Reference frames for polarization measurements
- Frame-independent polarization
- Understanding the production mechanisms of vector particles:
  The Lam-Tung relation and its generalizations
- Polarization as a discriminant of physics signals:
  new resonances vs continuum background in the $Z\,Z$ channel

Why do we study particle polarizations?

Measure **polarization** of a particle =
measure the **angular momentum state**
in which the particle is produced,
by studying the **angular distribution**
of its **decay**



Very **detailed** piece of information! Allows us to

- test of perturbative QCD [***Z*** and ***W*** decay distributions]
- constrain universal quantities [**sinθ$_W$** and/or **proton PDFs** from *Z*/*W*/ γ* decays]
- accelerate discovery of new particles or characterize them [**Higgs, *Z'*, anomalous *Z*+γ, graviton**, ...]
- understand the formation of hadrons (non-perturbative QCD)

Example: how are hadron properties generated? A look at quarkonium (J/ψ and Υ) formation

Presently we do not yet understand how/when the observed ***Q*-*Q*bar bound states** (produced at the LHC in gluon-gluon fusion) acquire their quantum numbers.
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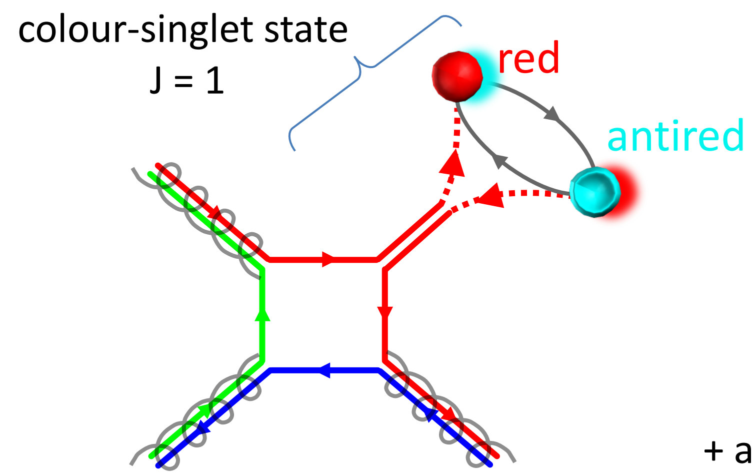
- **Colour-singlet processes**: quarkonia produced directly as observable ***colour-neutral*** *Q*-*Q*bar pairs

purely perturbative



+ analogous colour combinations

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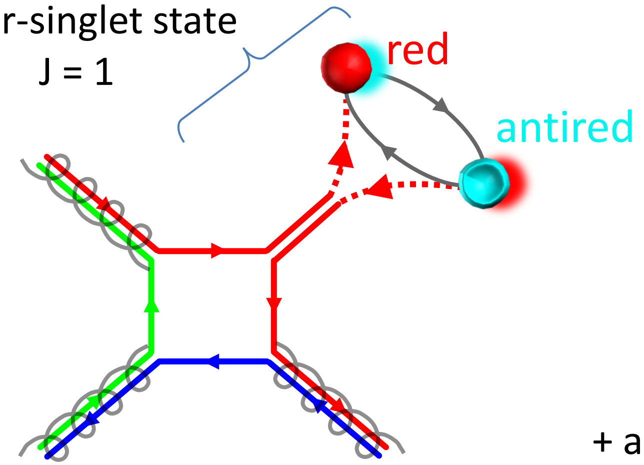

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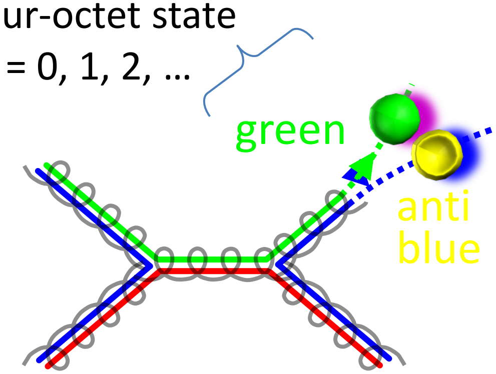
- **Colour-octet processes**: quarkonia are produced through ***coloured*** *Q*-*Q*bar pairs *of any possible quantum numbers*

colour-octet state
J = 0, 1, 2, ...



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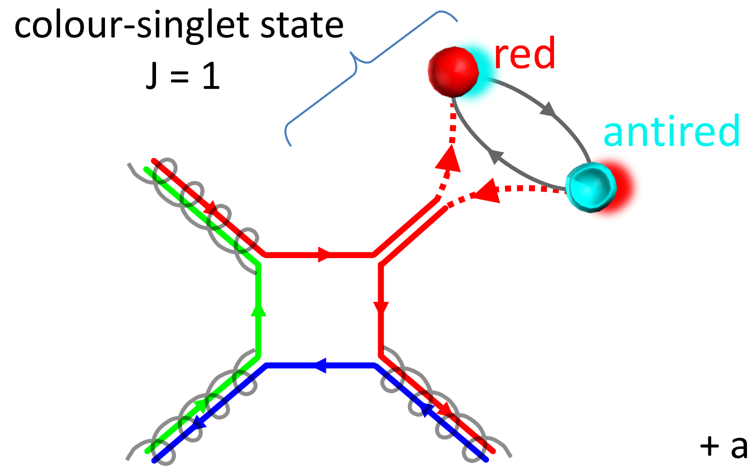

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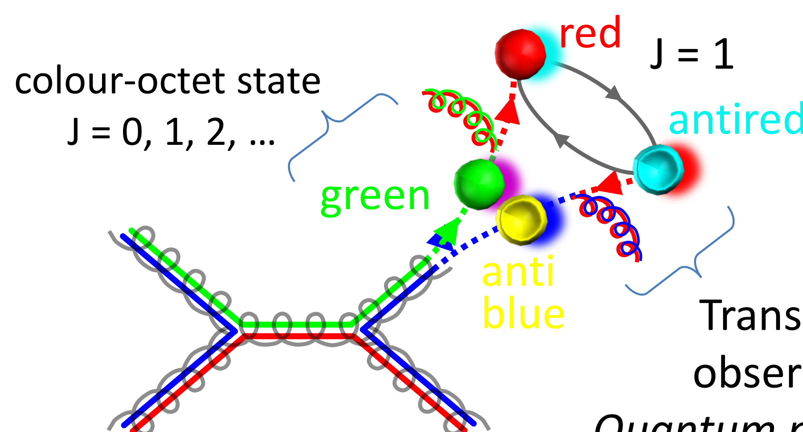
  perturbative
  $\otimes$
  non-perturbative



Transition to the observable state.
Quantum numbers change!
J can change! → ***polarization***!

Polarization of vector particles

***J* = 1** → three J_z eigenstates $|\,1, +1\,\rangle$, $|\,1, 0\,\rangle$, $|\,1, -1\,\rangle$ wrt a certain z

Measure polarization = measure (average) angular momentum composition

Method: study the **angular distribution of the particle decay** in its rest frame

The decay **into a fermion-antifermion pair** is an especially clean case to be studied

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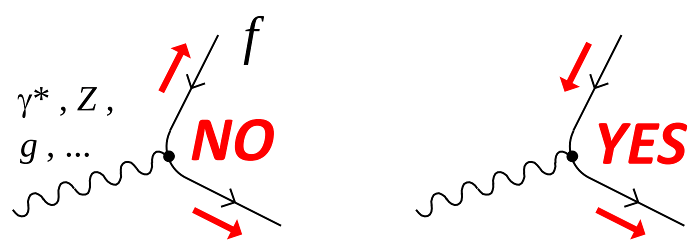

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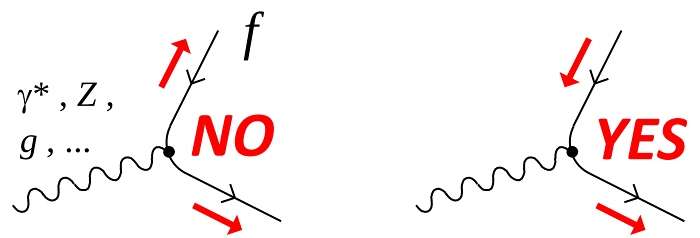
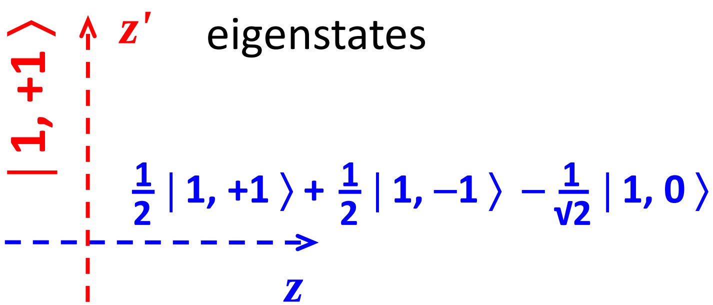


2) rotational covariance of angular momentum eigenstates



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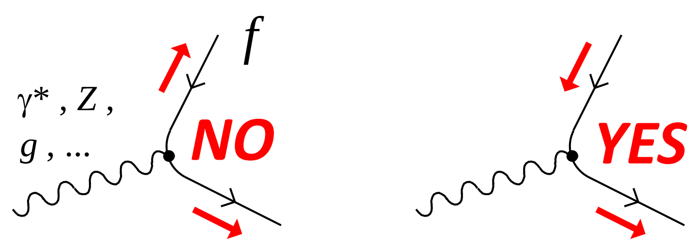

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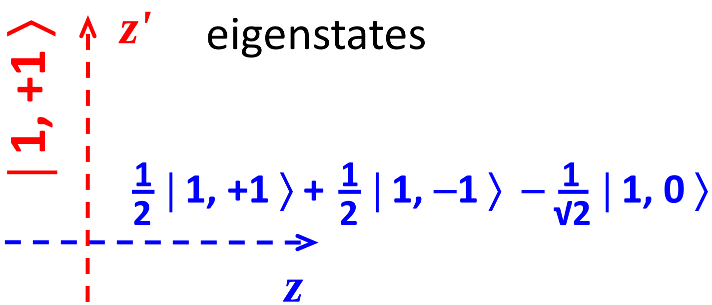
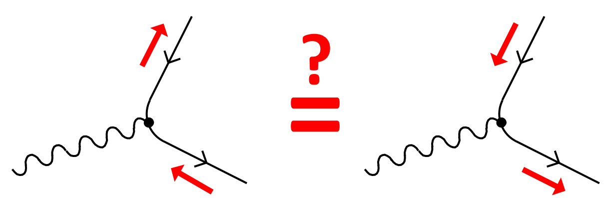
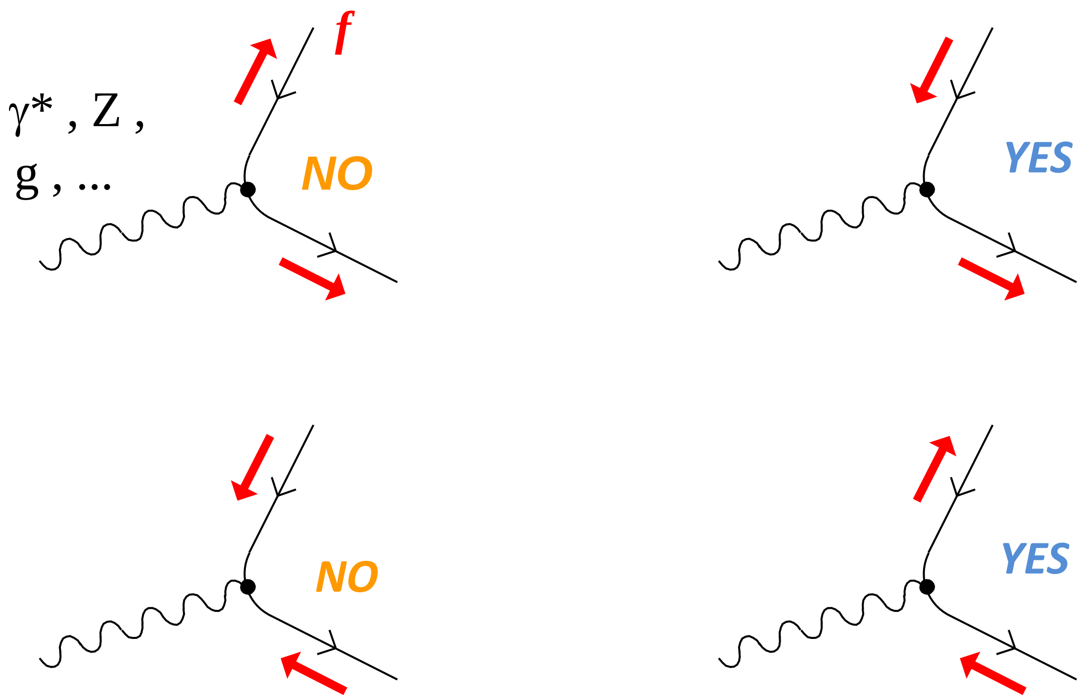


3) parity properties

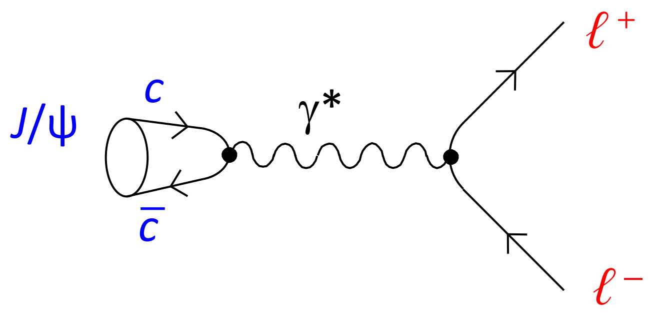


1: helicity conservation

EW and strong forces preserve the *chirality* (L/R) of fermions.
In the relativistic (massless) limit, *chirality* = ***helicity* = spin-momentum alignment**
→ the **fermion spin never flips** in the coupling to gauge bosons:



example: dilepton decay of J/ψ



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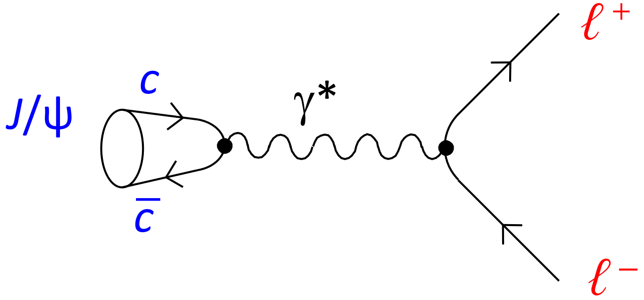

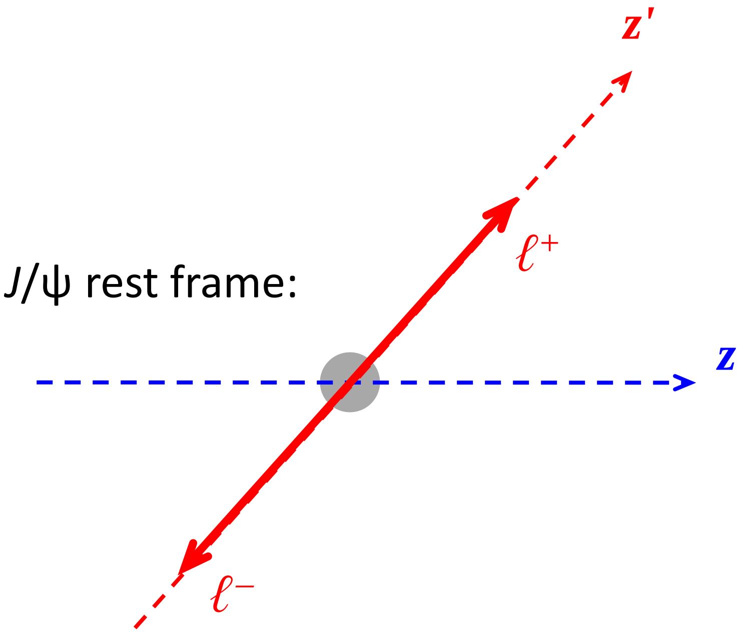

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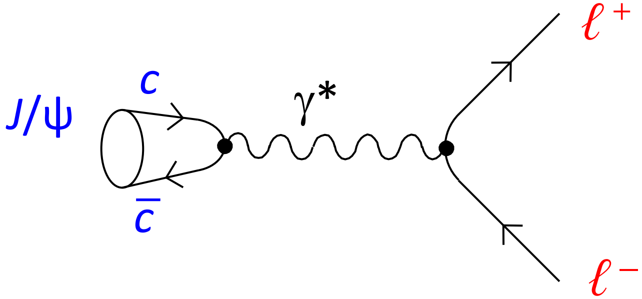
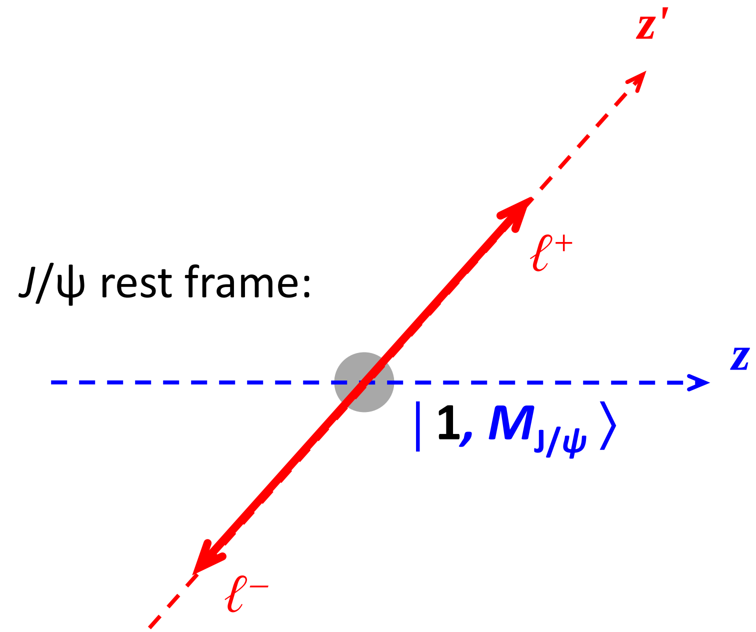
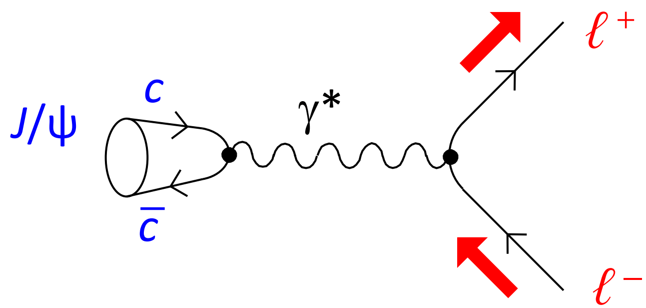


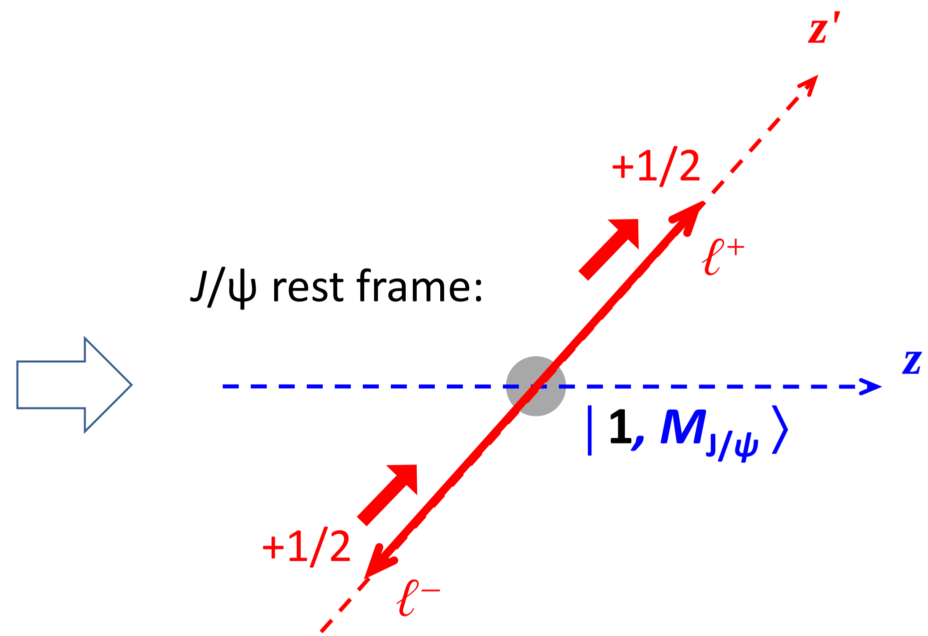


***J*/ψ** angular momentum component along the polarization axis ***z*:**

$M_{J/\psi}$ = -1, 0, +1 (determined by *production mechanism*)

example: dilepton decay of *J/ψ*





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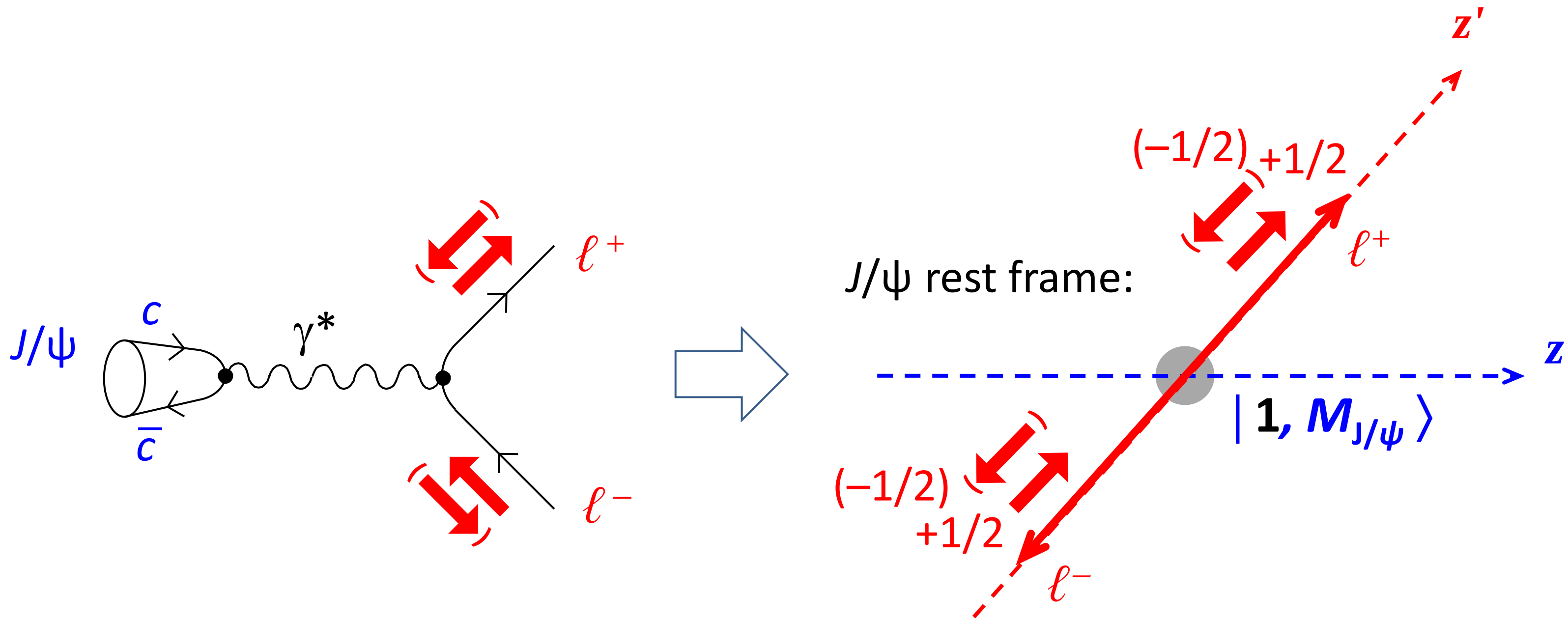

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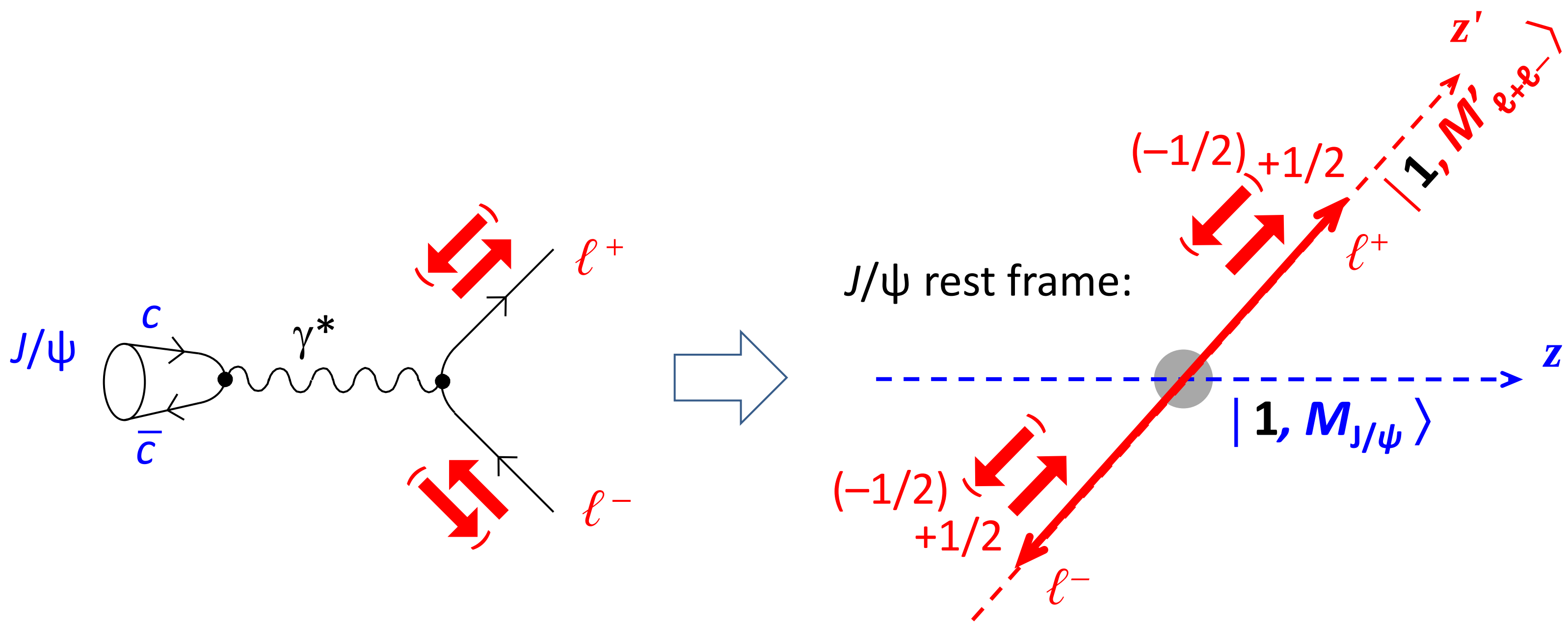

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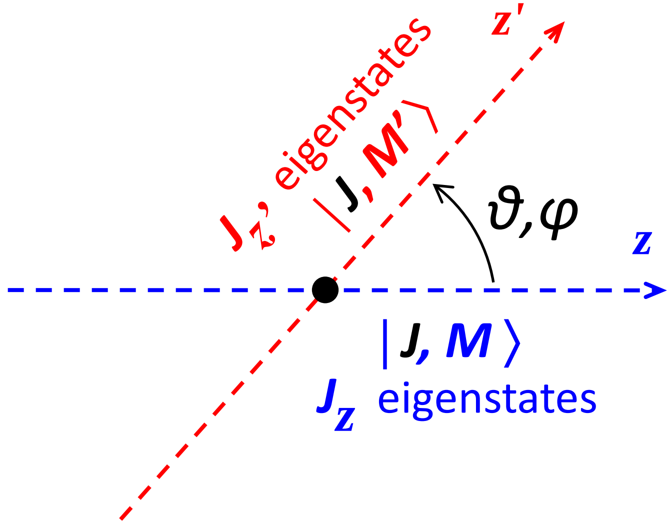
The **two leptons** can only have total angular momentum component

$\boxed{M'_{\ell^+\ell^-} = +1 \text{ or } -1}$ along their common direction ***z′***

0 is forbidden

2: rotation of angular momentum eigenstates



change of quantization frame:

$R(\vartheta,\varphi)$: $z \rightarrow z'$
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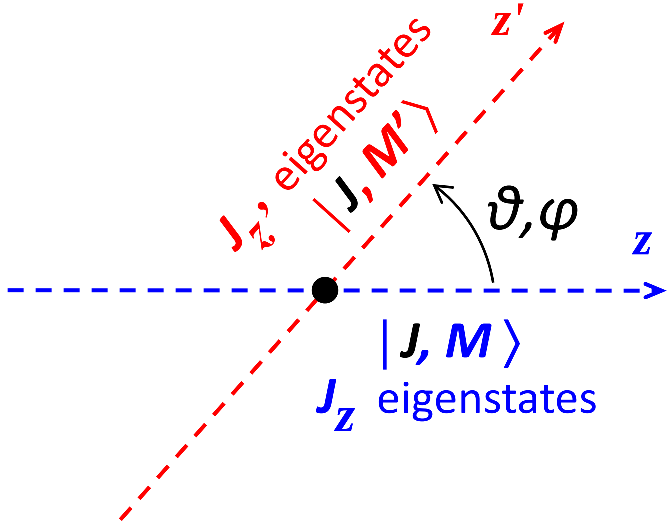

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Wigner D-matrices

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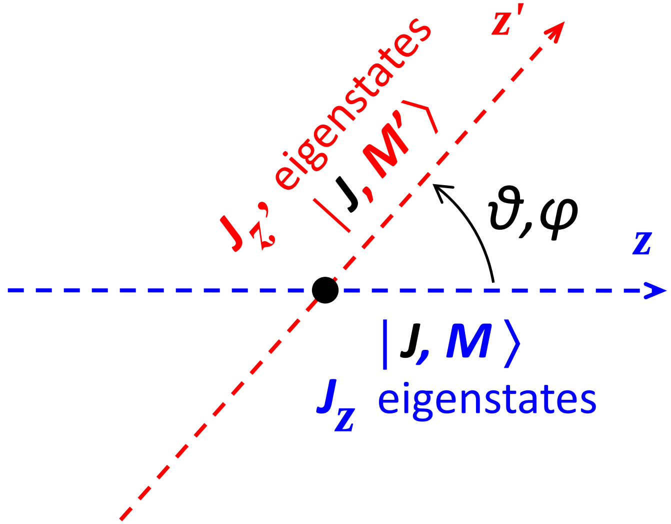

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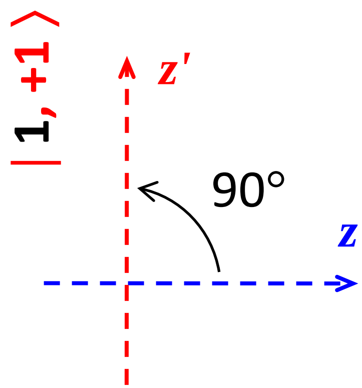

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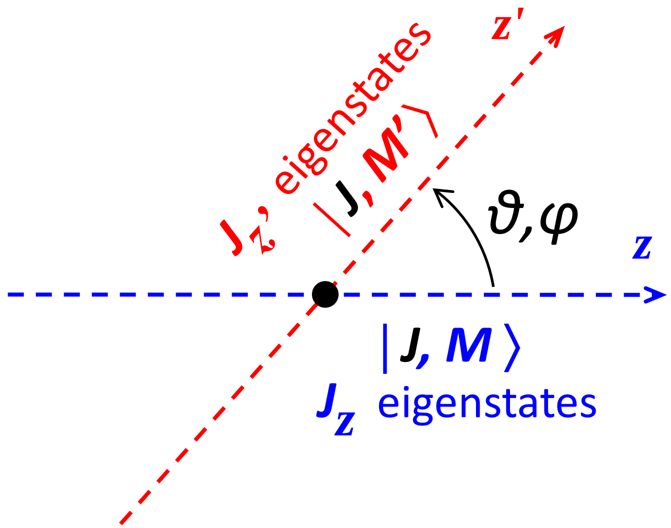

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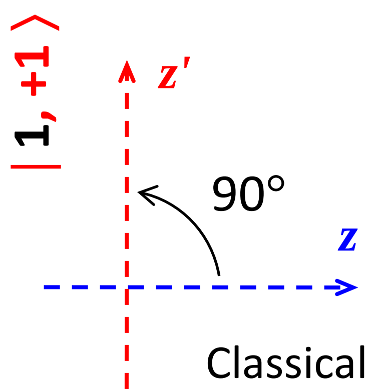

Classically, we would expect $|1, 0\rangle$

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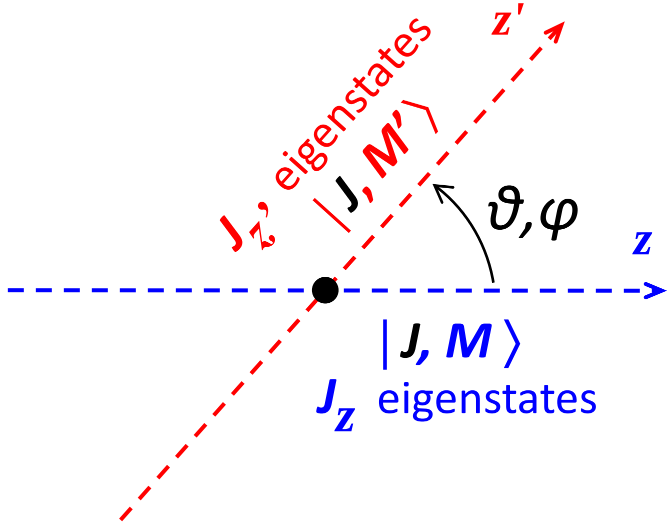

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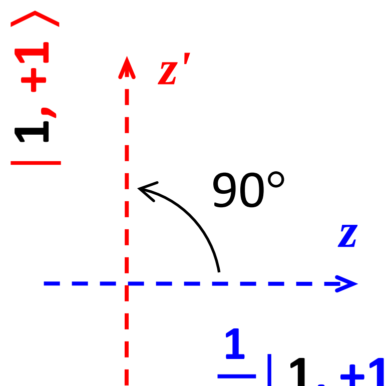
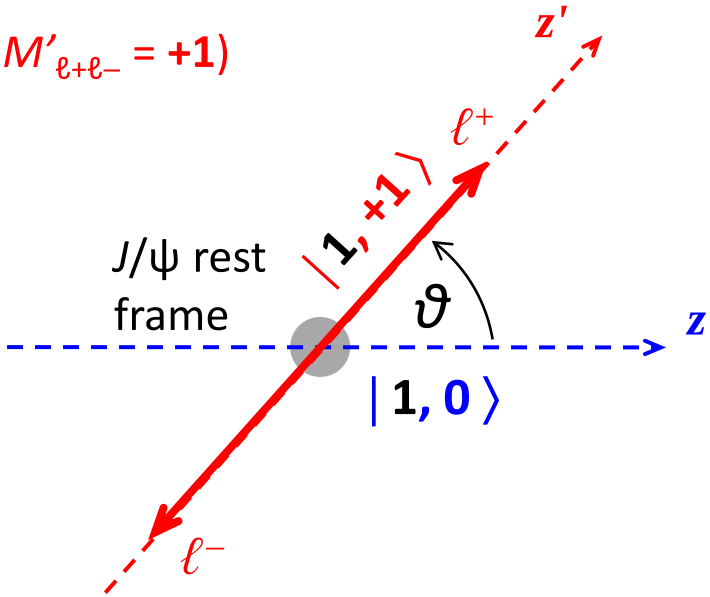


$$\frac{1}{2}\,|1, +1\rangle + \frac{1}{2}\,|1, -1\rangle - \frac{1}{\sqrt{2}}\,|1, 0\rangle$$

example: $M = 0$

J/ψ ($M_{J/\psi} = \mathbf{0}$) → $\ell^+\ell^-(M'_{\ell+\ell-} = \mathbf{+1})$



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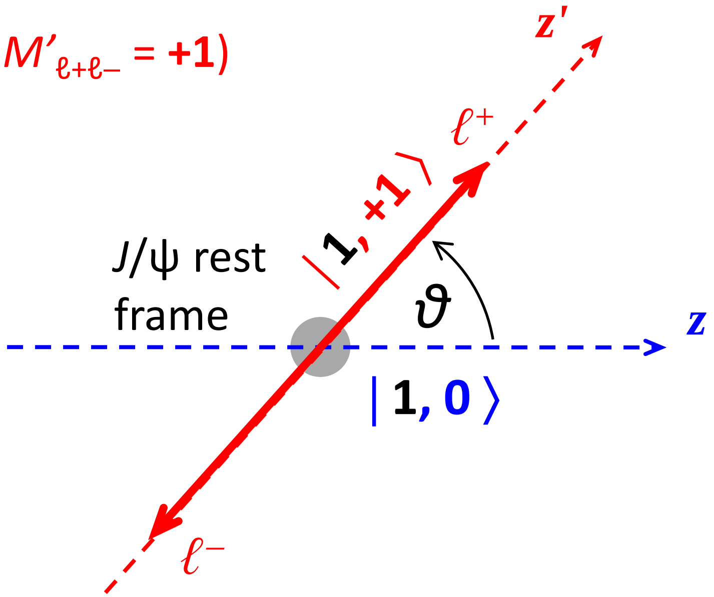

$$|\,1, +1\,\rangle = D^1_{-1,+1}(\vartheta,\varphi)\ |\,1, -1\,\rangle + D^1_{0,+1}(\vartheta,\varphi)\ |\,1, 0\,\rangle + D^1_{+1,+1}(\vartheta,\varphi)\ |\,1, +1\,\rangle$$

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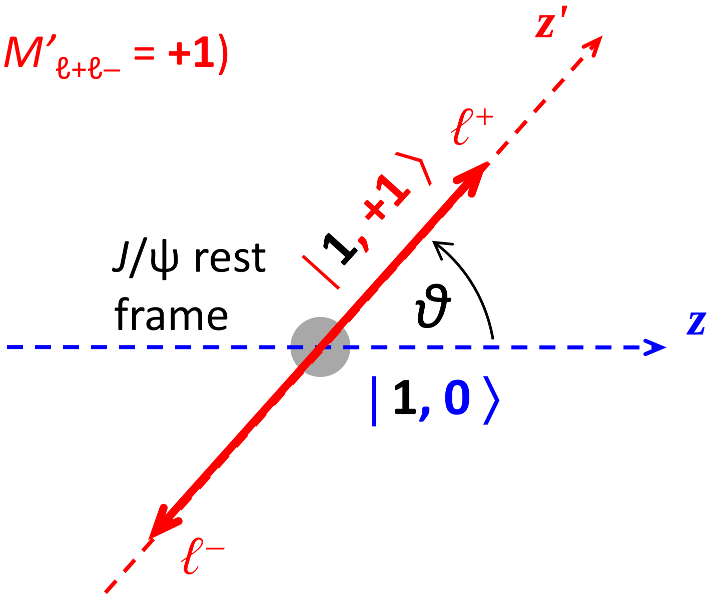

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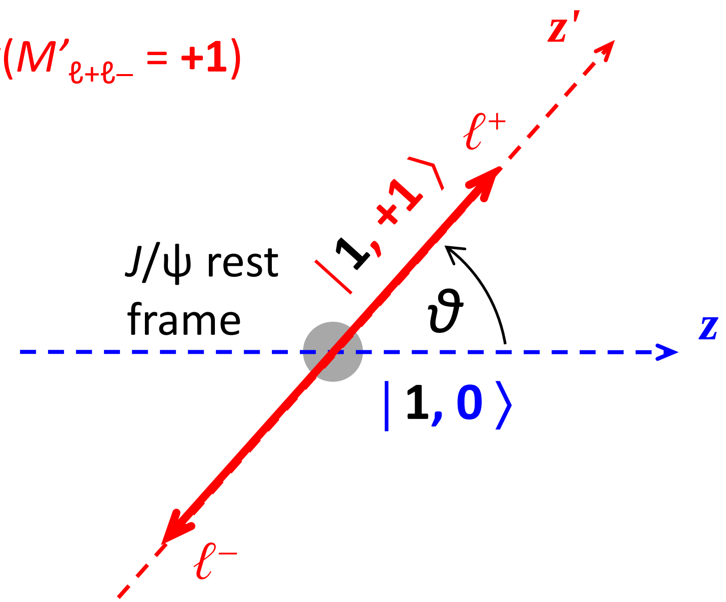

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$$\underbrace{|\langle\, 1, +1\,|\,\mathcal{O}\,|\,1, 0\,\rangle|^2}_{\ell^+\ell^- \;\leftarrow\; J/\psi} \;\propto\; |D^{1*}_{0,+1}(\vartheta,\varphi)|^2 \;=\; \frac{1}{2}\,(\,1 - \cos^2\vartheta\,)$$

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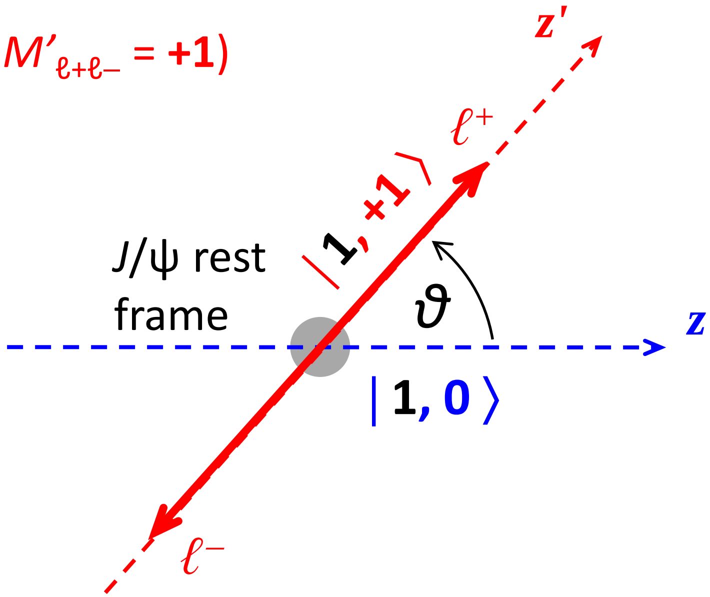

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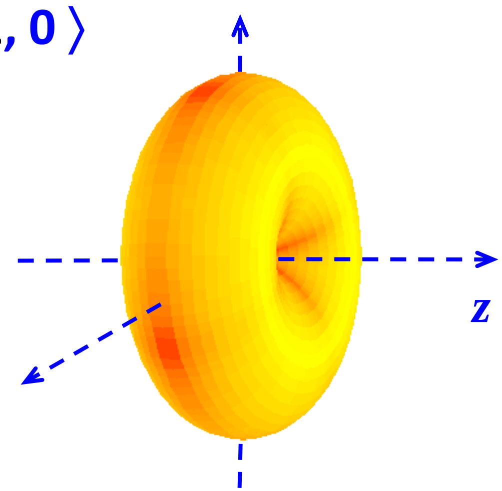
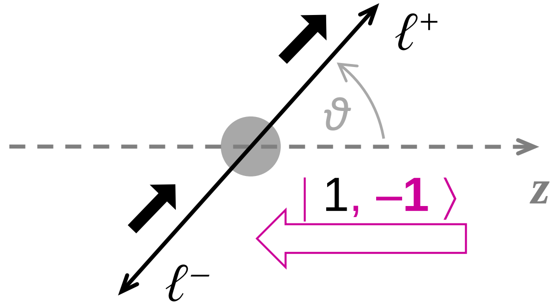


3: parity



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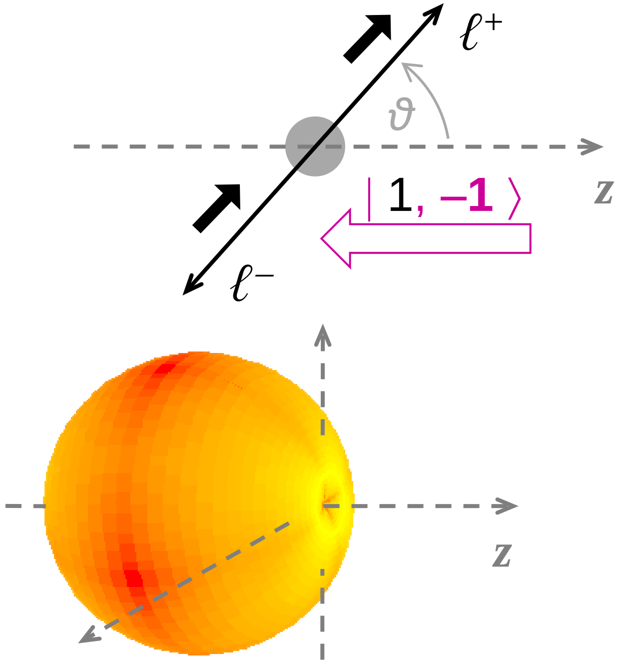

$$\frac{dN}{d\Omega} \propto |D^{1*}_{-1,+1}(\vartheta,\varphi)|^2 \;\propto\; 1 \;+\; \cos^2\vartheta - \mathbf{2\cos\vartheta}$$

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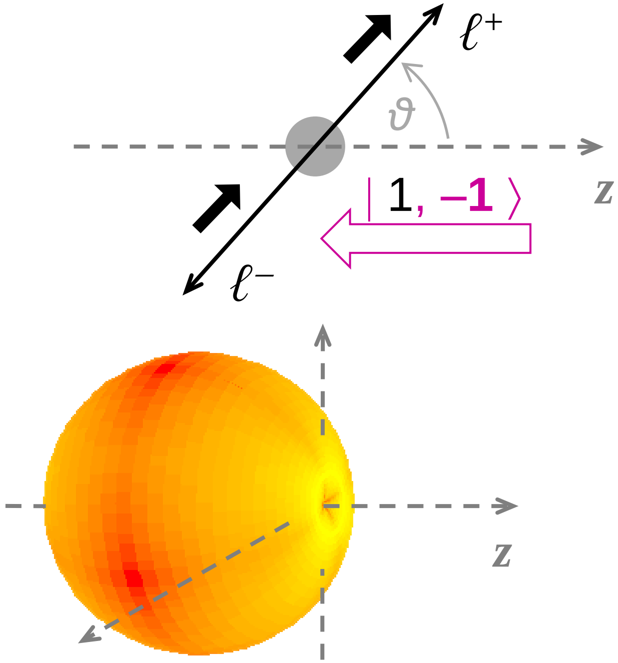

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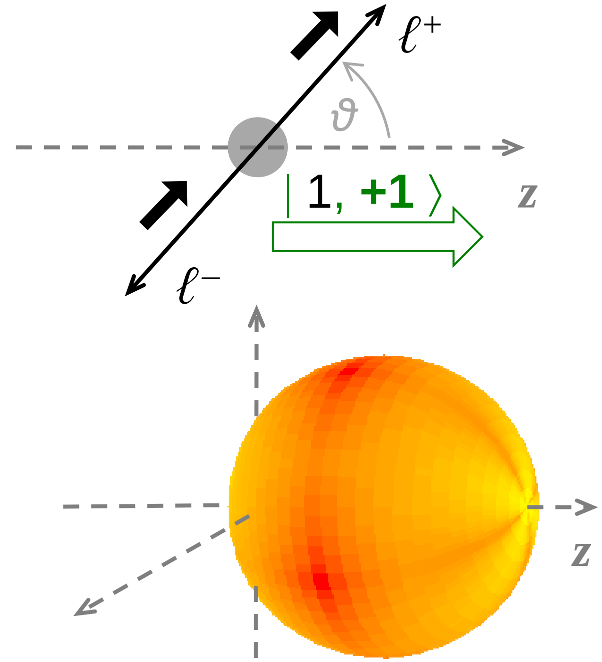

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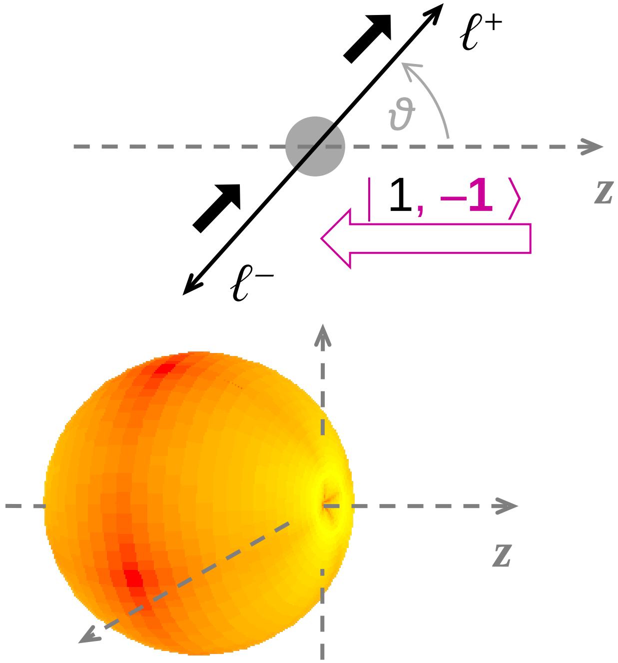

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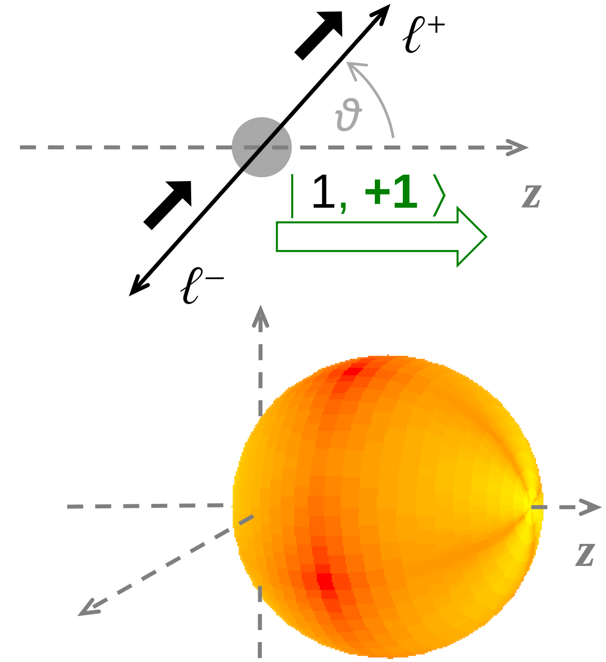

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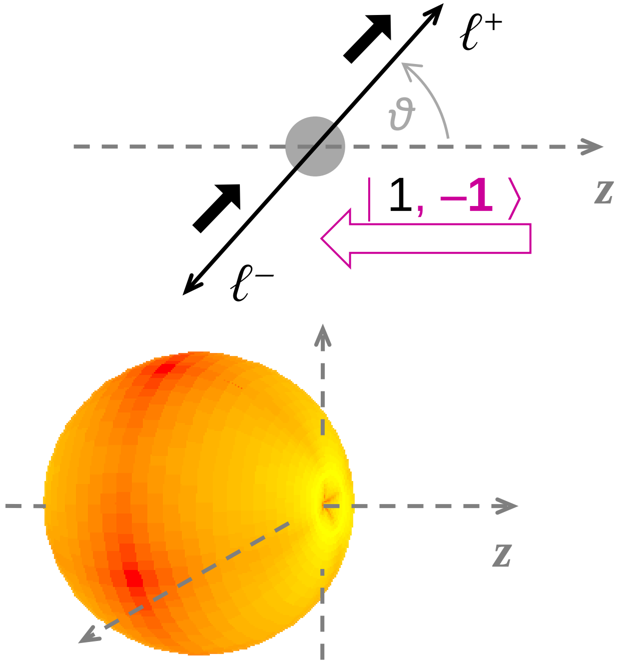

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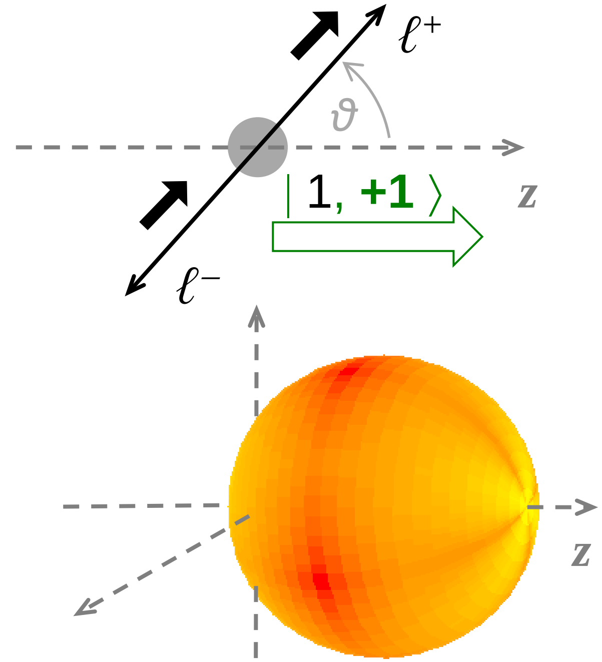

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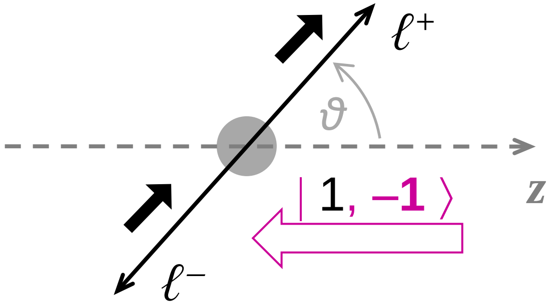
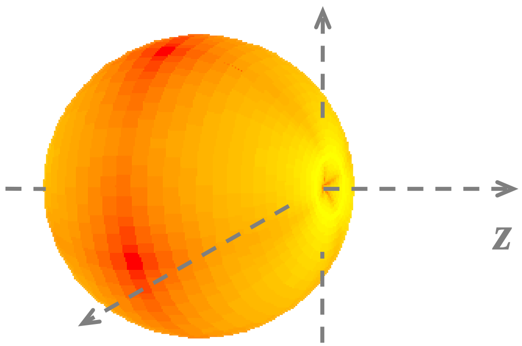
Are they equally probable?

3: parity





$|\,1, -1\,\rangle$ and $|\,1, +1\,\rangle$
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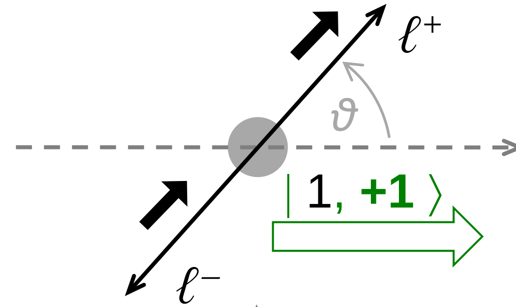

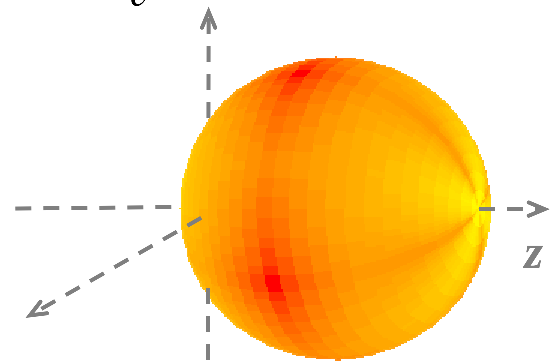

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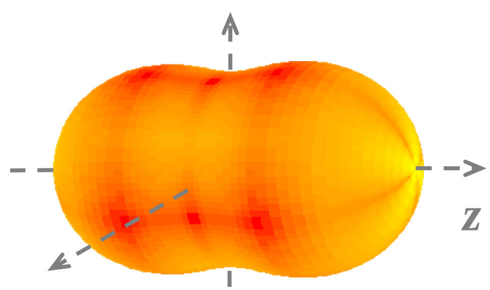

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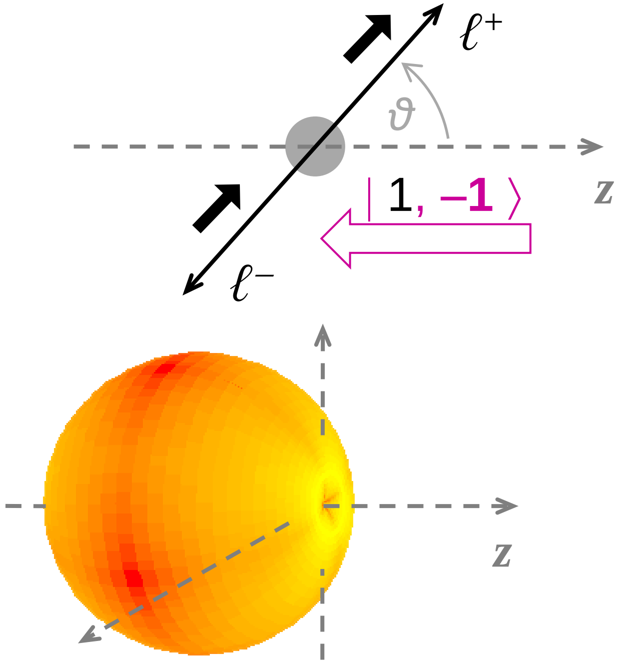

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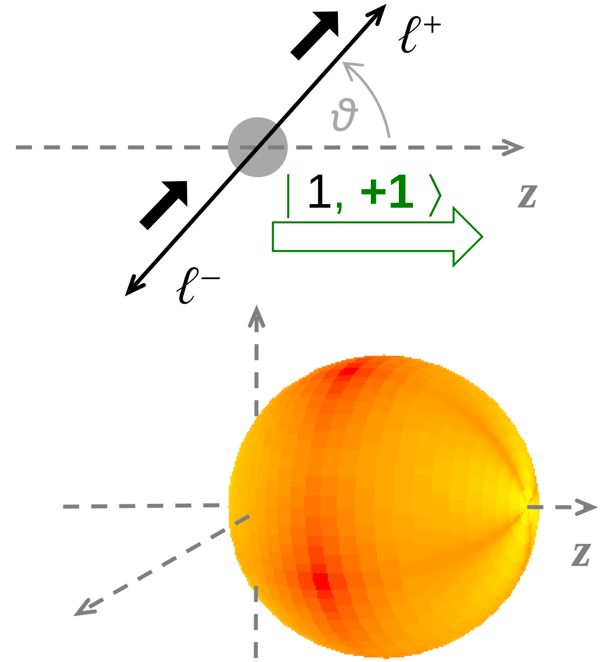

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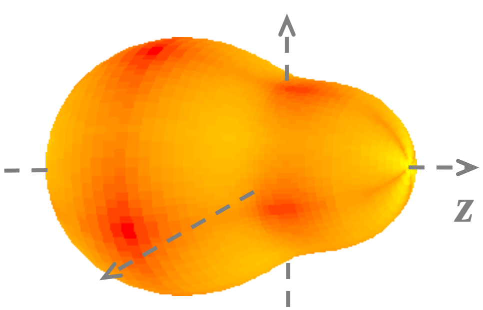

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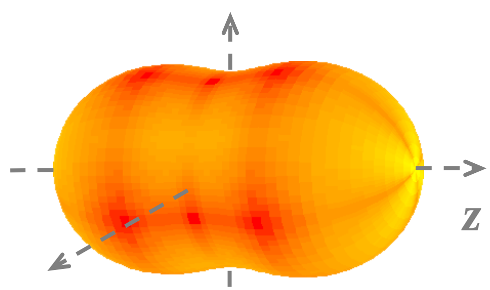

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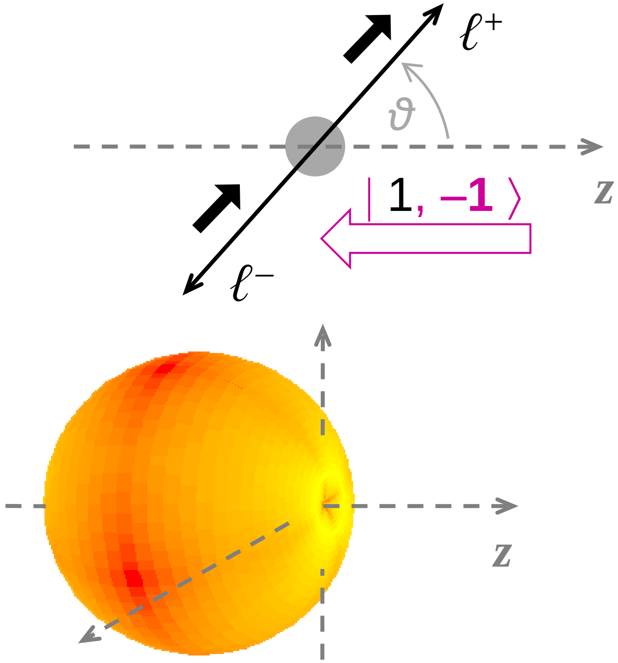

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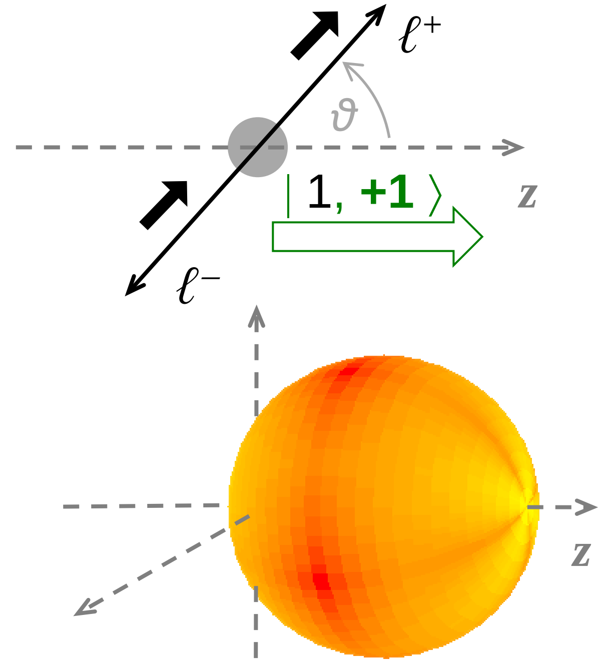

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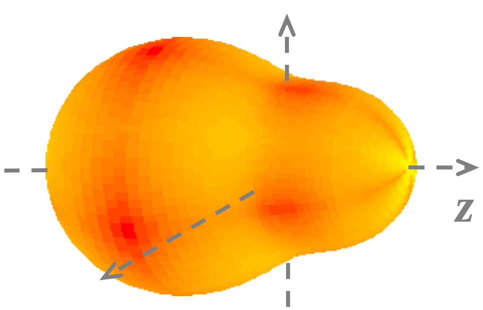

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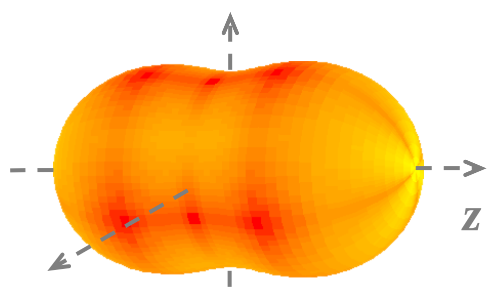

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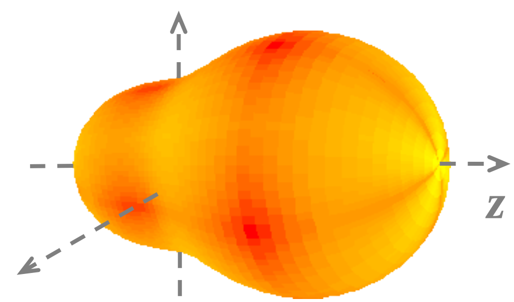

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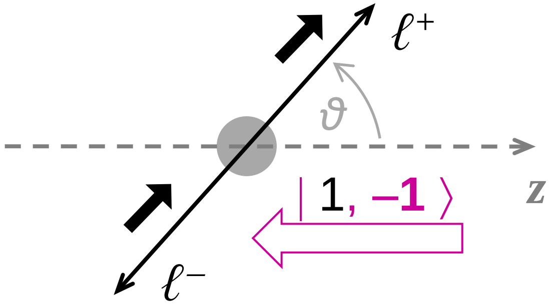

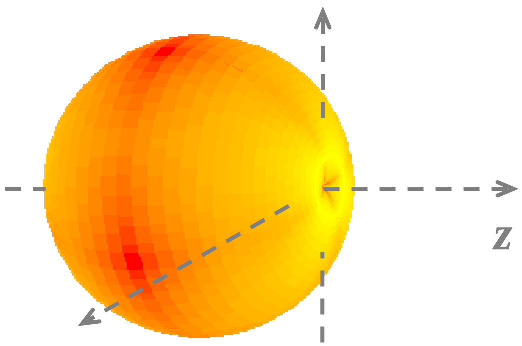

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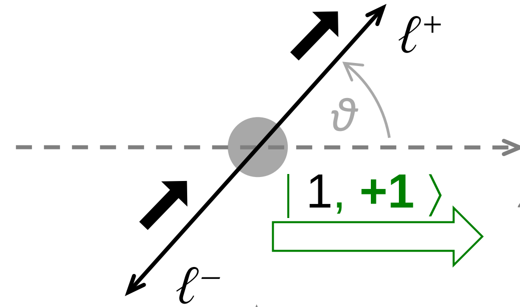

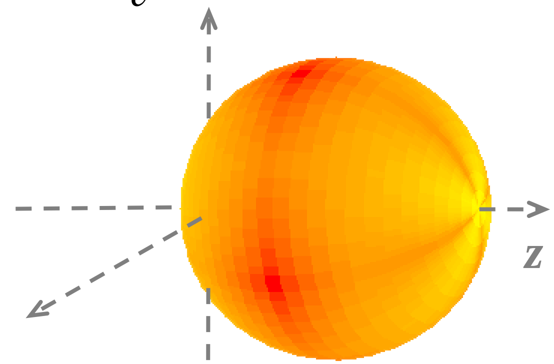

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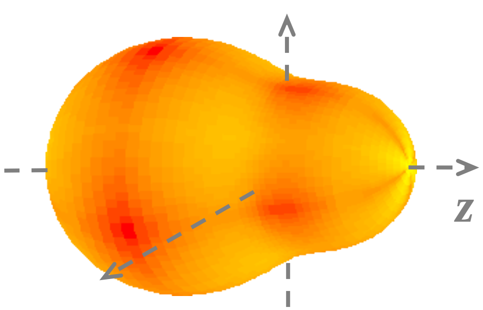

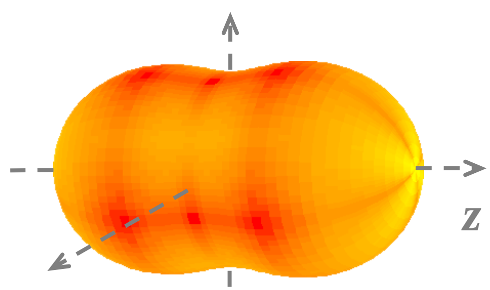

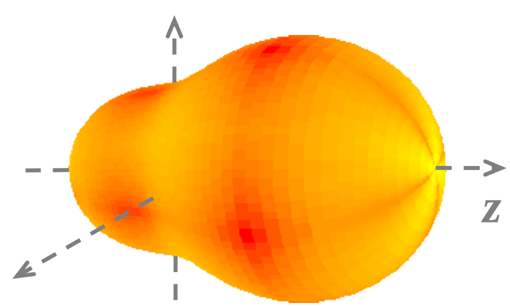

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$$\frac{dN}{d\Omega} \propto \mathbf{1 + \cos^2\vartheta + 2[\mathcal{P}(+1) - \mathcal{P}(-1)]\cos\vartheta}$$

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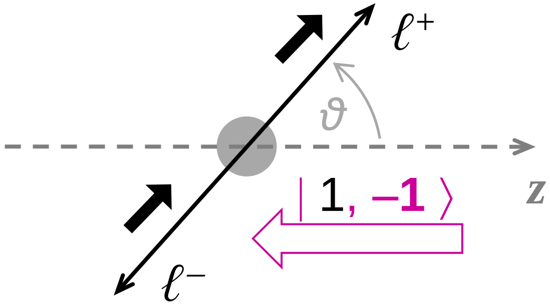

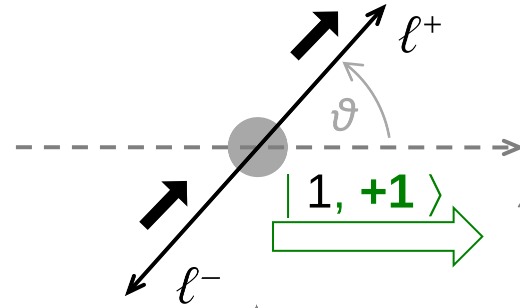

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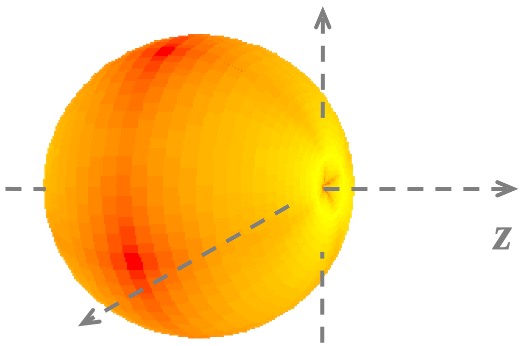

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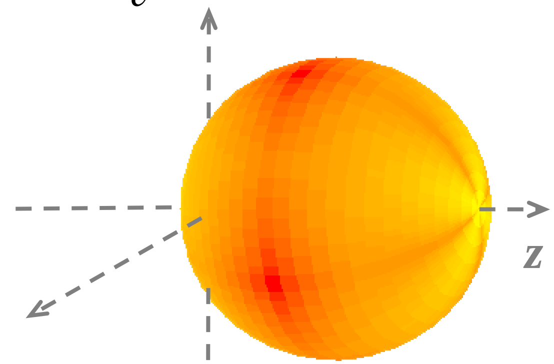

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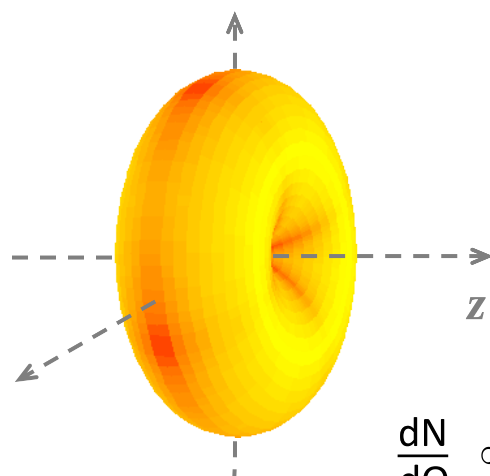
Decay distribution of $| 1, 0 \rangle$ state is always parity-symmetric:



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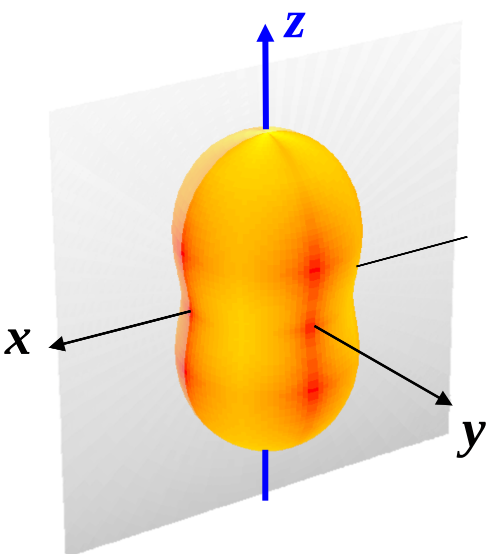
“Transverse” and “longitudinal”



$|\, J/\psi \,\rangle = |\, 1, \mathbf{+1} \,\rangle$

or $|\, 1, \mathbf{-1} \,\rangle$

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(parity-conserving case)

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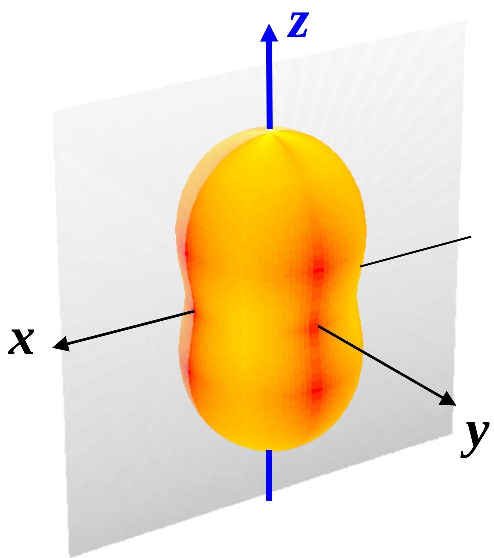

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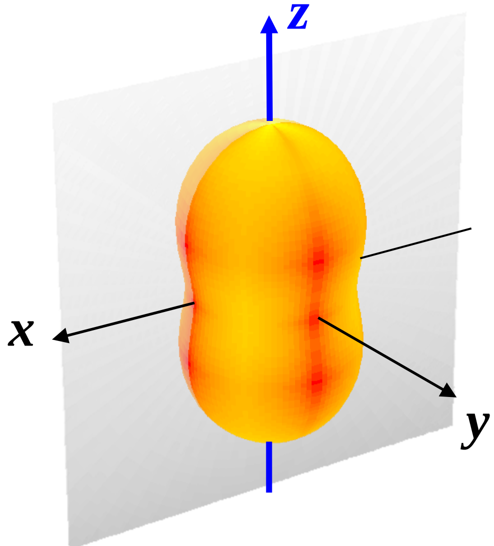

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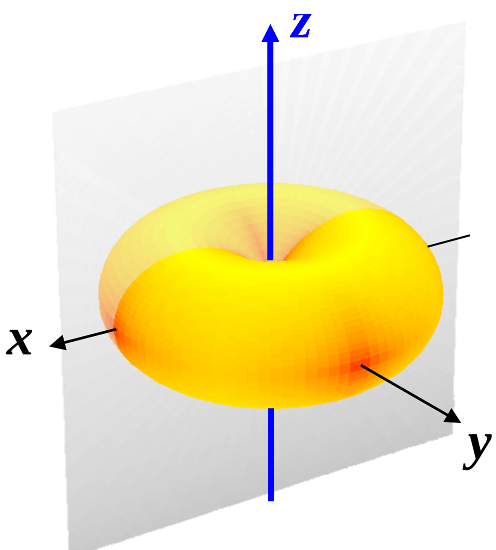

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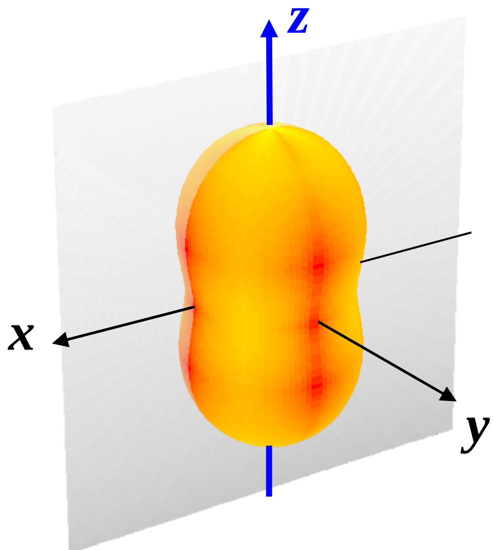

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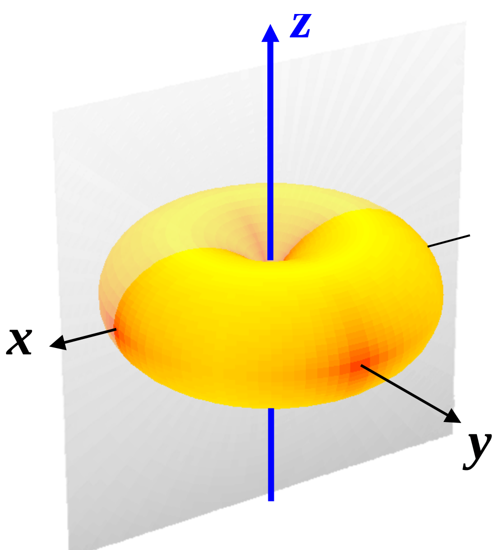

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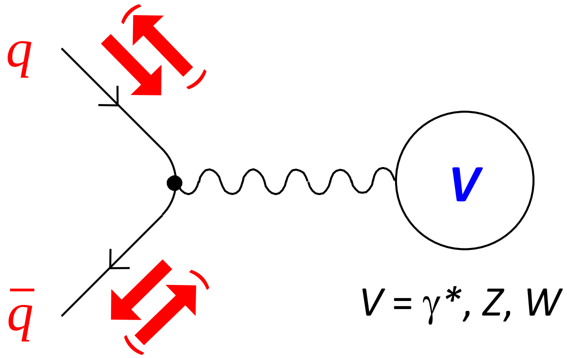
"Longitudinal" polarization

Why "photon-like" polarizations are common

We can apply **helicity conservation at the *production* vertex** to predict that all *vector* states produced in ***fermion-antifermion annihilations*** **(q-$\bar{q}$ or e^+e^-)** at Born level have *transverse* polarization



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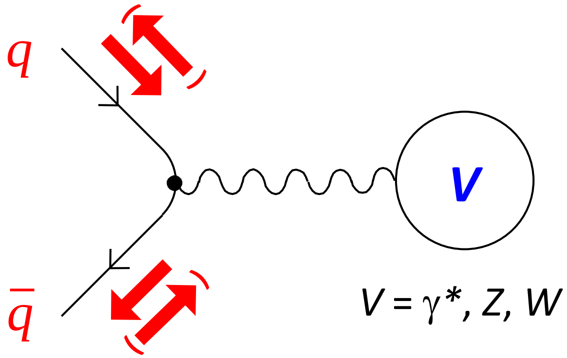

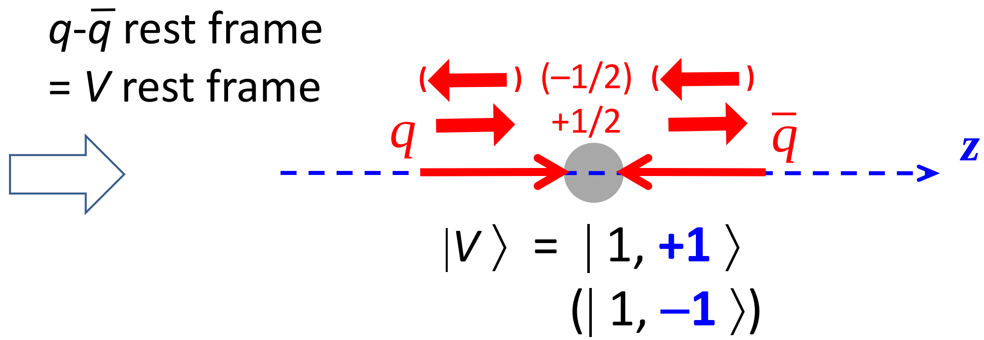

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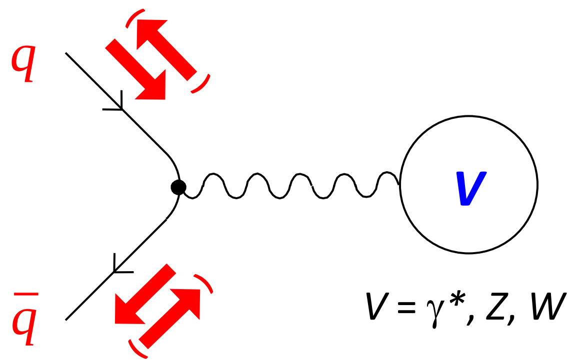
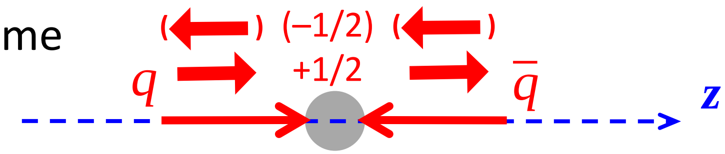


q-$\bar{q}$ rest frame
= V rest frame



$|V\rangle = |1, \mathbf{+1}\rangle$
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<u>The “natural” polarization axis in this case is the *relative direction of the colliding fermions* ***(Collins-Soper axis)***</u>

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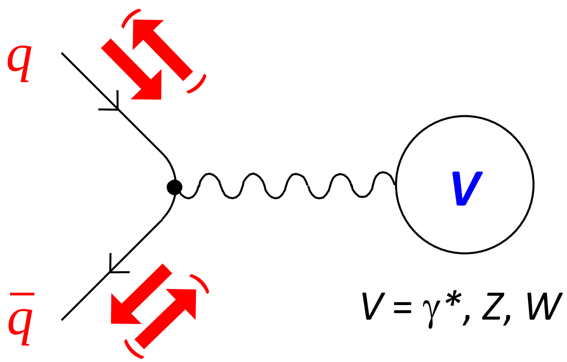

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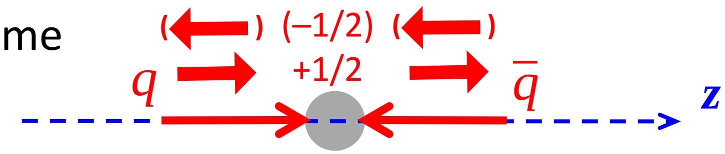

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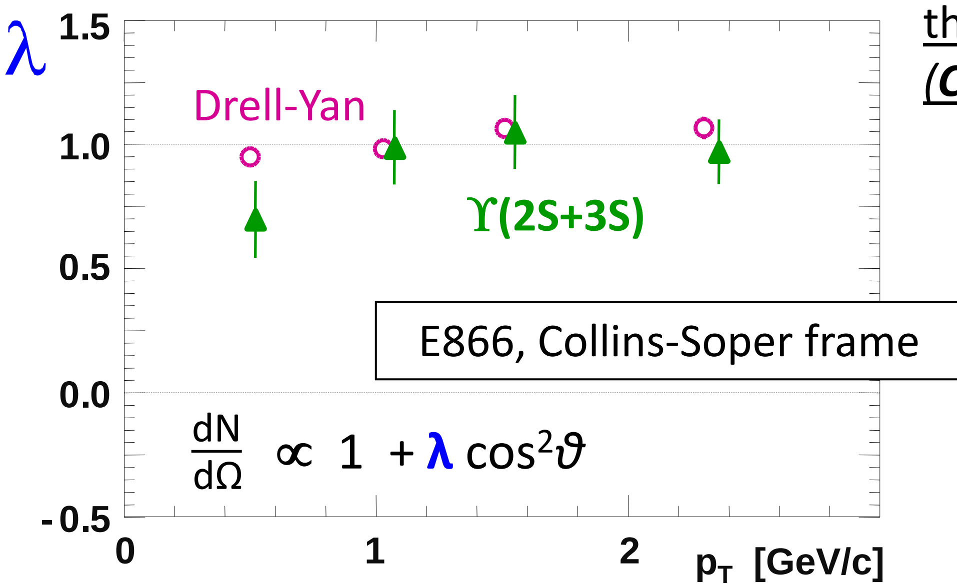

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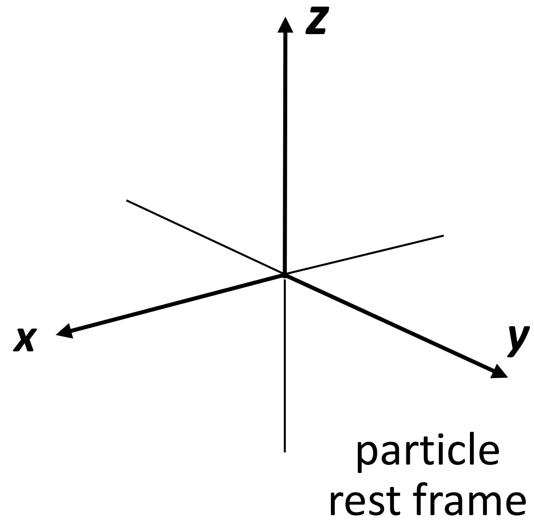
Drell-Yan is a paradigmatic case
But not the only one

The most general distribution



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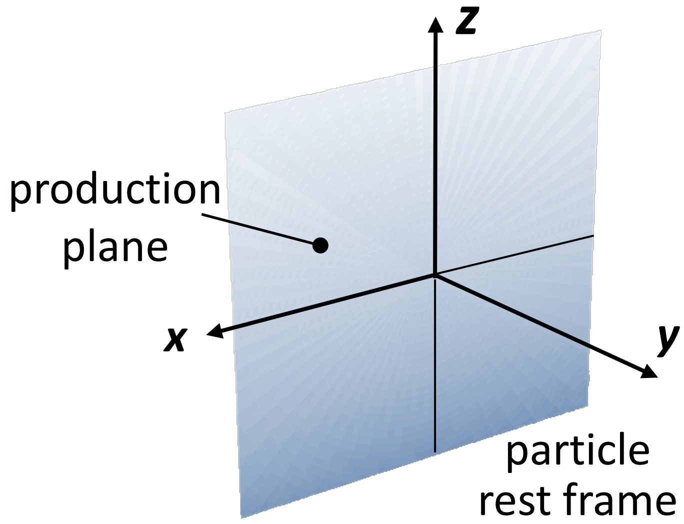

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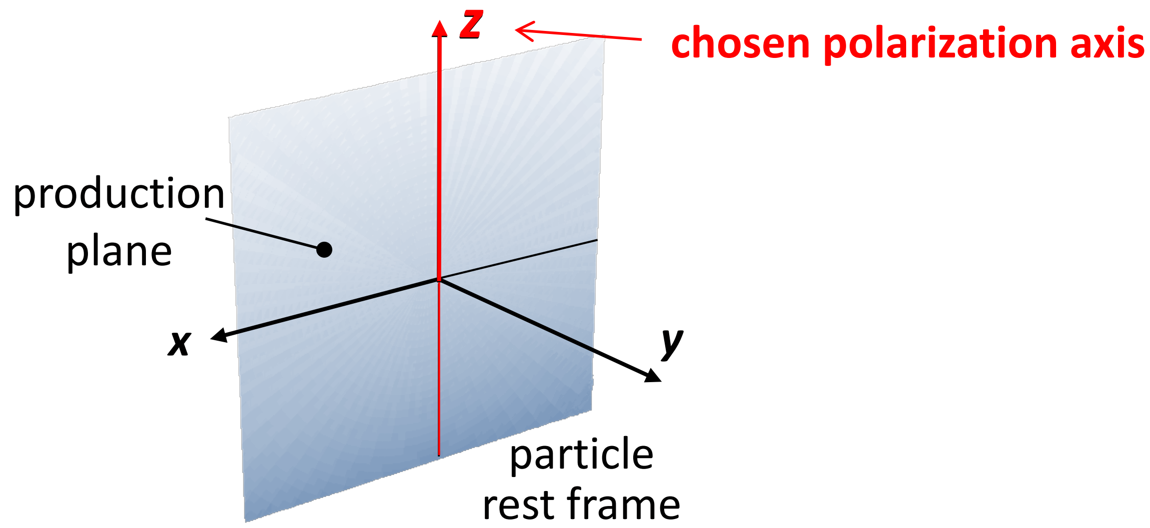

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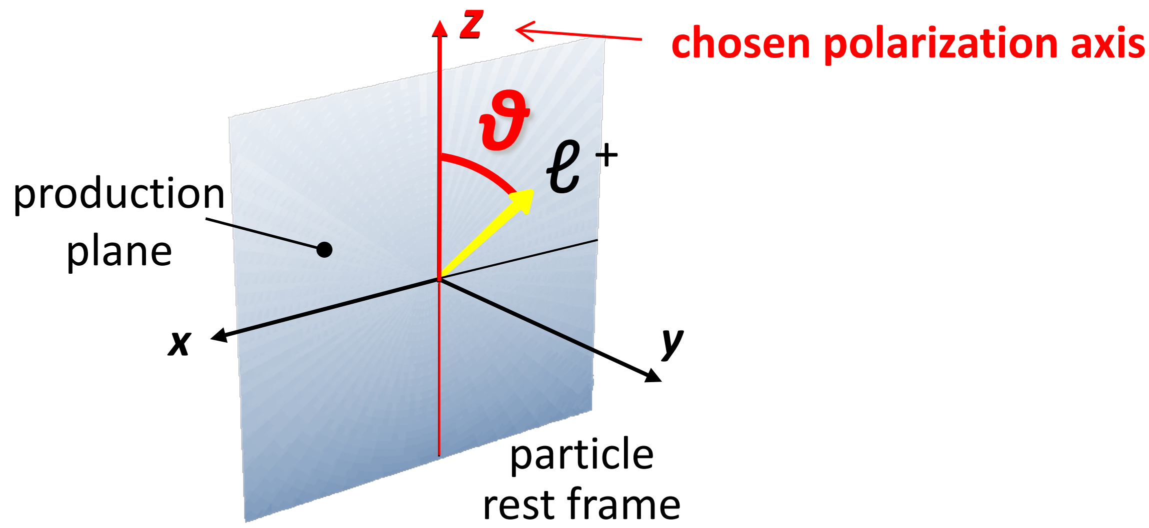

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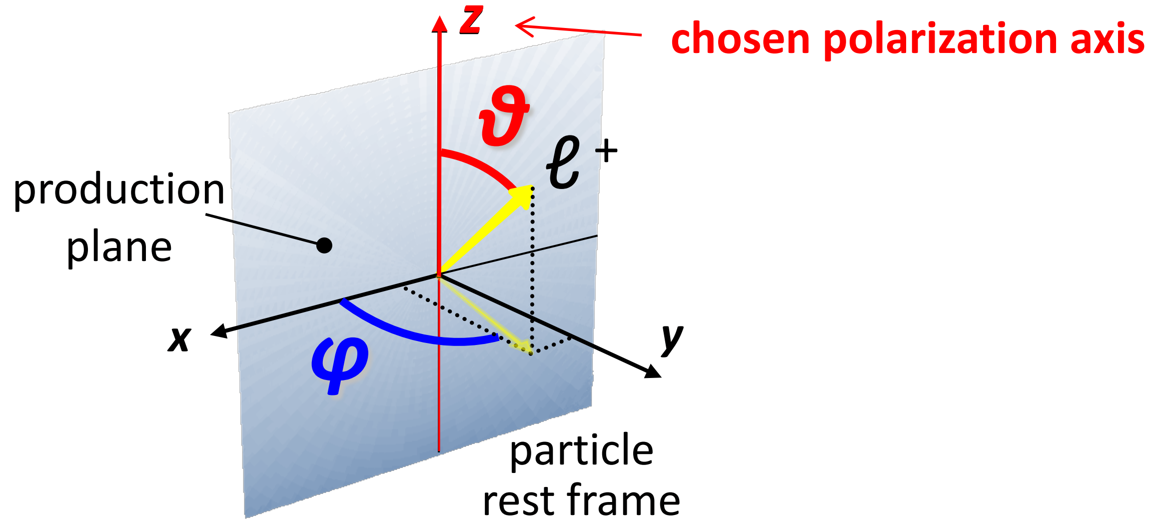

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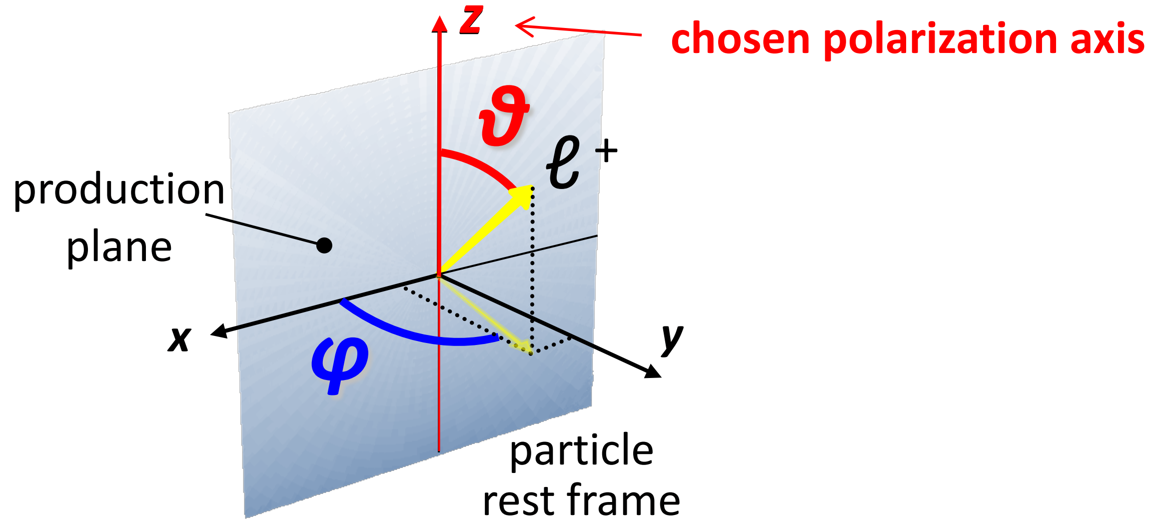

$$\frac{dN}{d\Omega} \propto 1 + \lambda_\theta \cos^2\theta + \lambda_\varphi \sin^2\theta \cos 2\varphi + \lambda_{\theta\varphi} \sin 2\theta \cos\varphi$$

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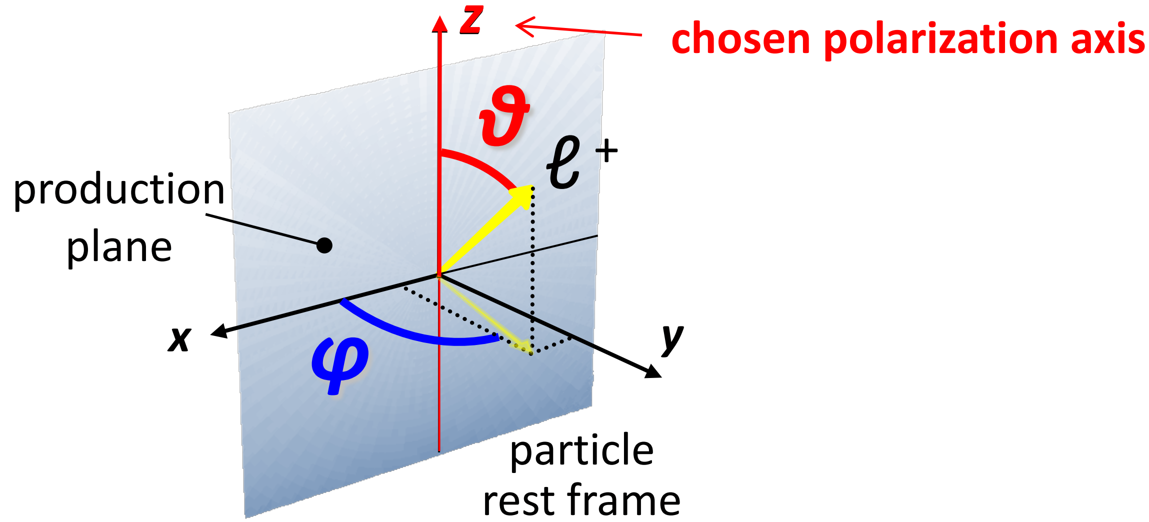

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parity violating

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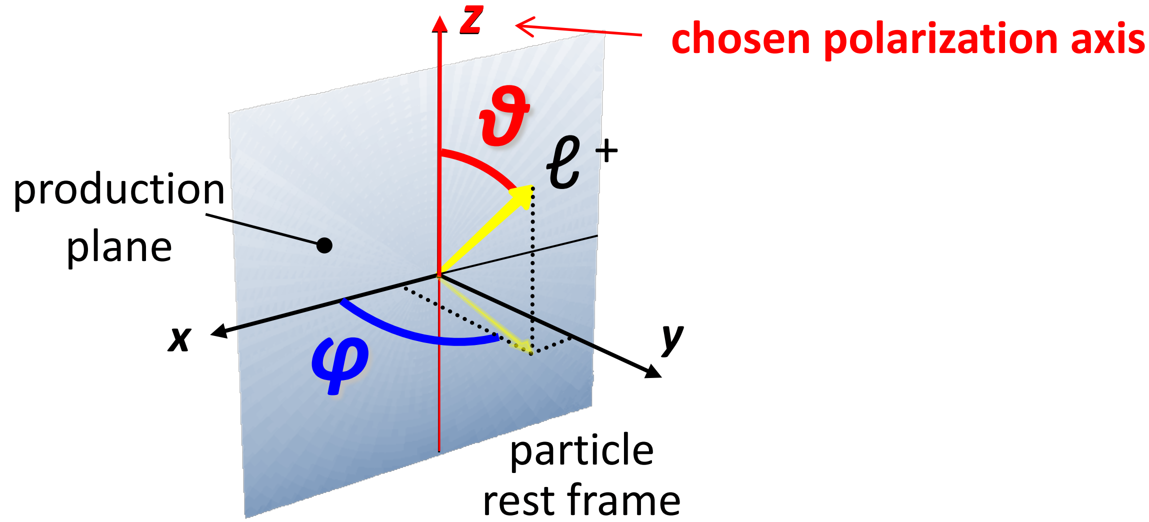

average polar anisotropy

average azimuthal anisotropy

correlation polar - azimuthal

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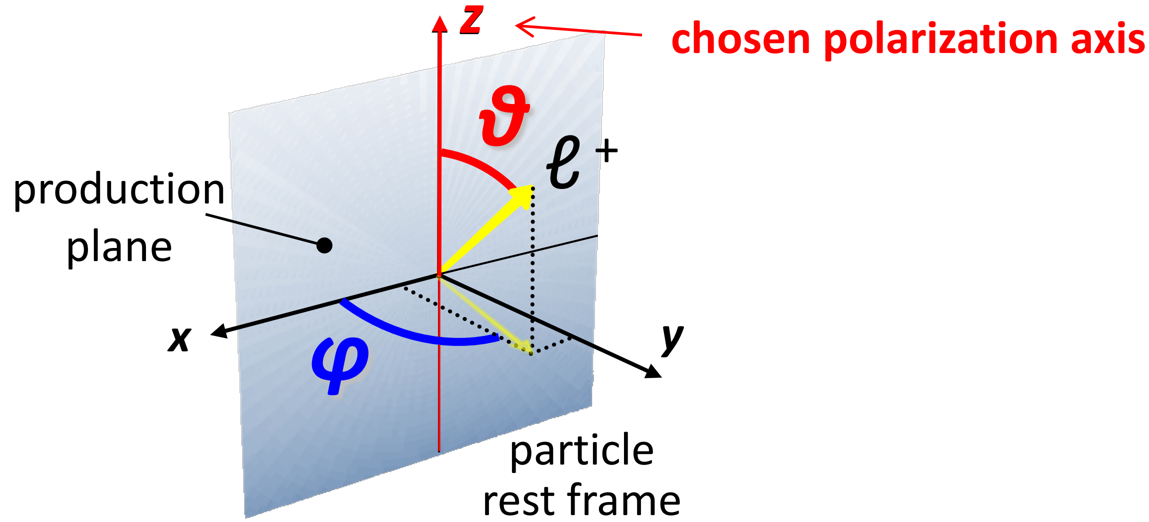

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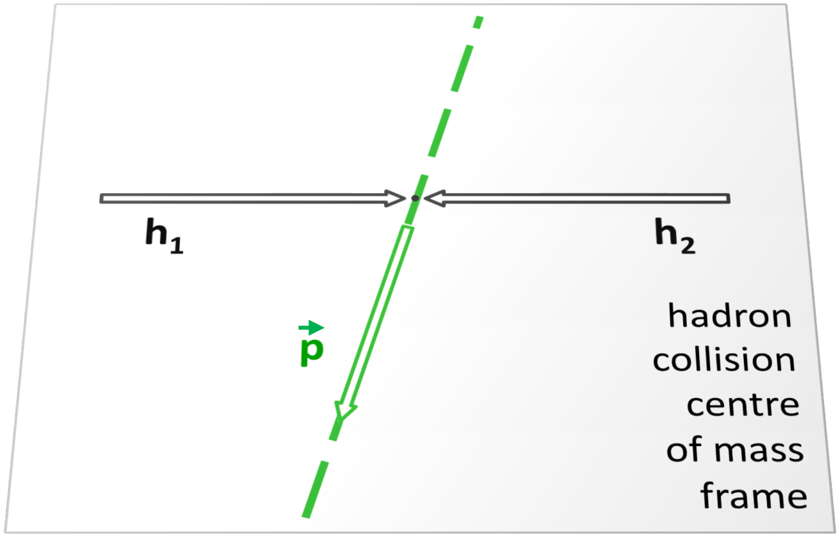
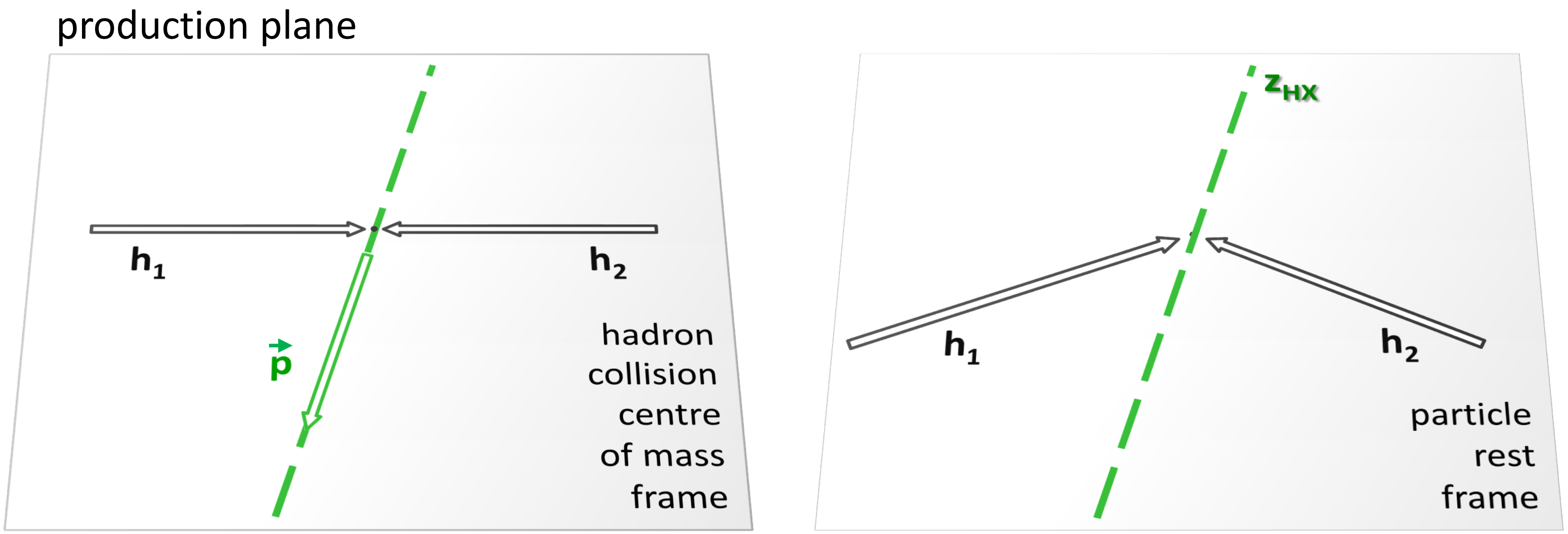
Polarization frames

production plane



Polarization frames

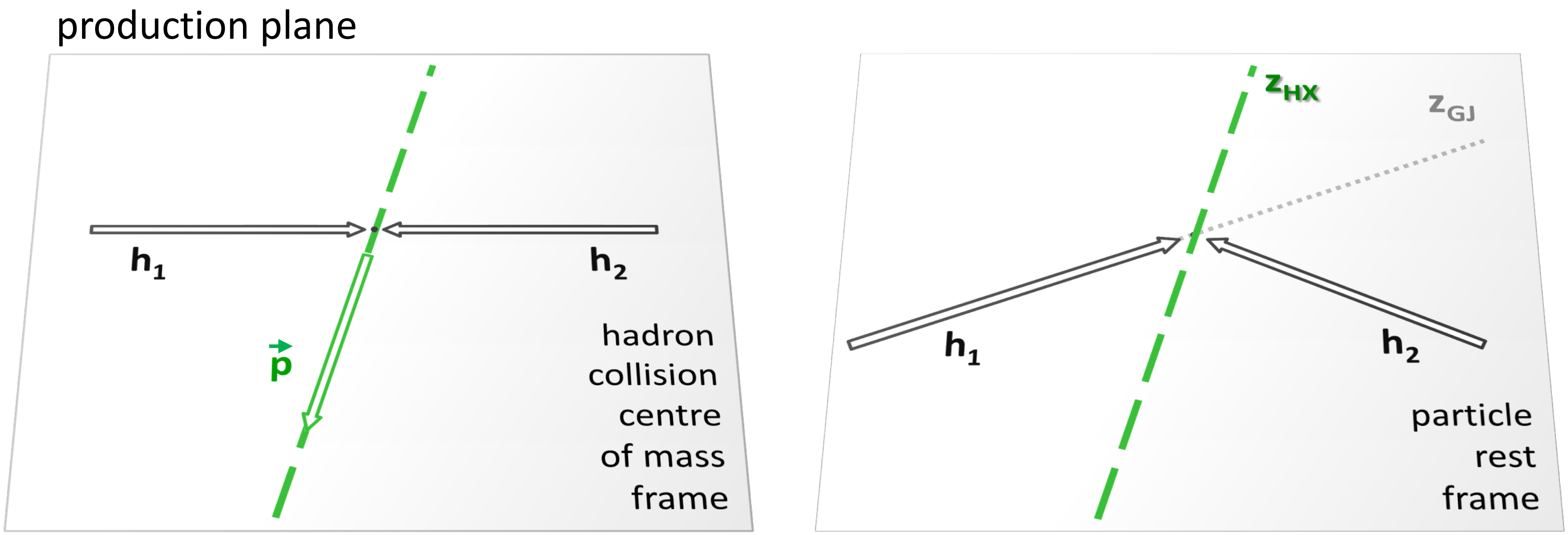
Helicity axis (HX): quarkonium momentum direction



Polarization frames

Helicity axis (HX): quarkonium momentum direction

Gottfried-Jackson axis (GJ): direction of one or the other beam

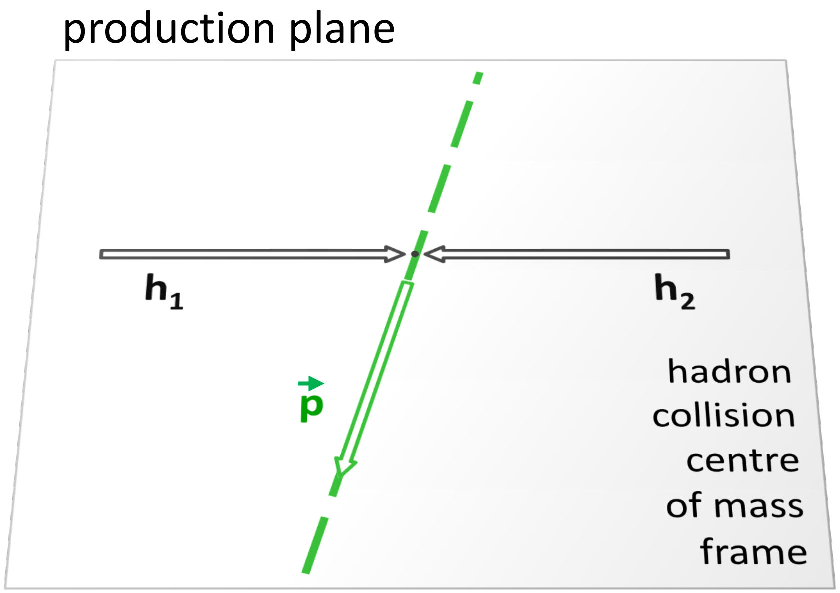


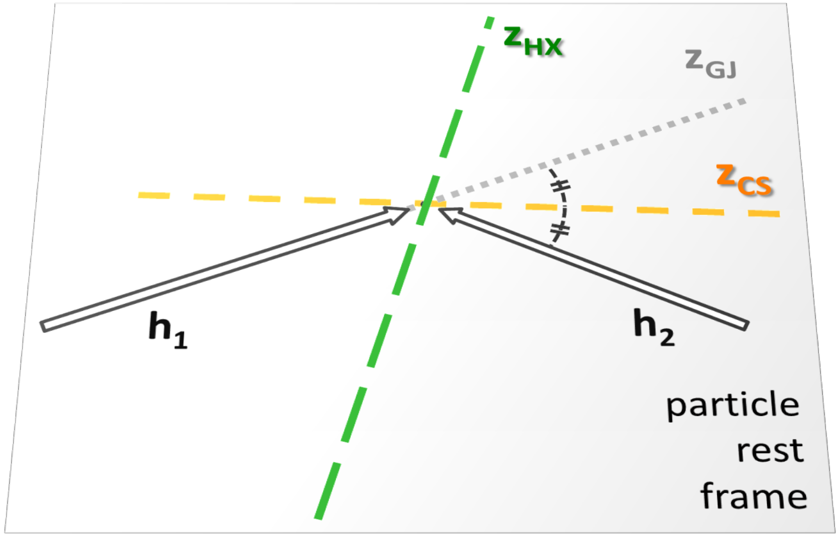
Polarization frames

Helicity axis (HX): quarkonium momentum direction

Gottfried-Jackson axis (GJ): direction of one or the other beam

Collins-Soper axis (CS): average of the two beam directions





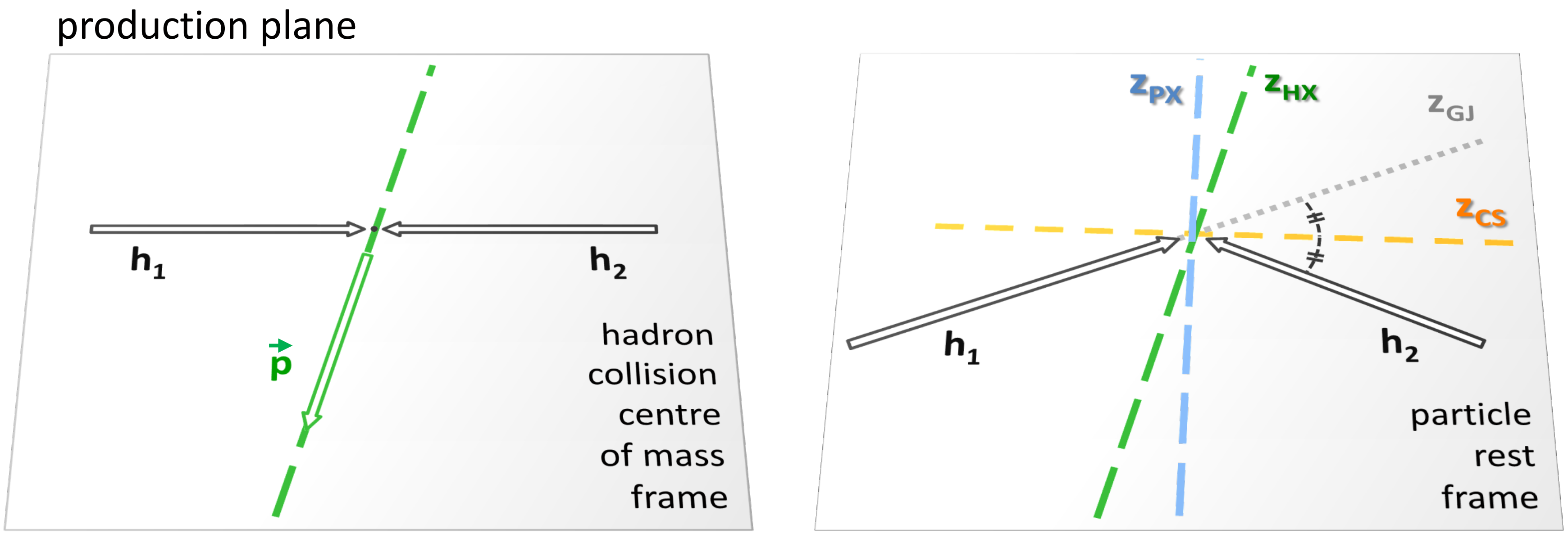
Polarization frames

Helicity axis (HX): quarkonium momentum direction

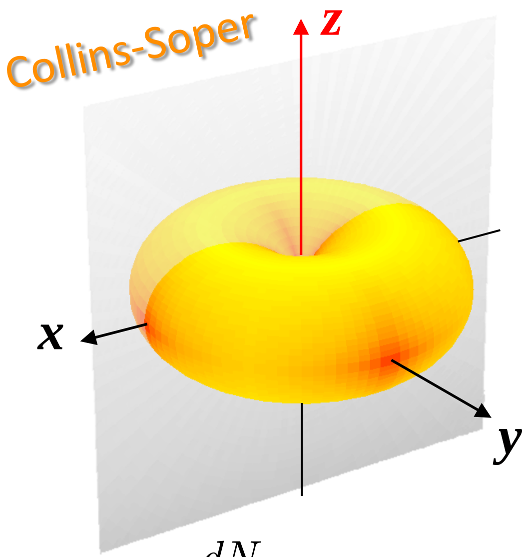
Gottfried-Jackson axis (GJ): direction of one or the other beam

Collins-Soper axis (CS): average of the two beam directions

Perpendicular helicity axis (PX): perpendicular to CS



The observed polarization depends on the frame

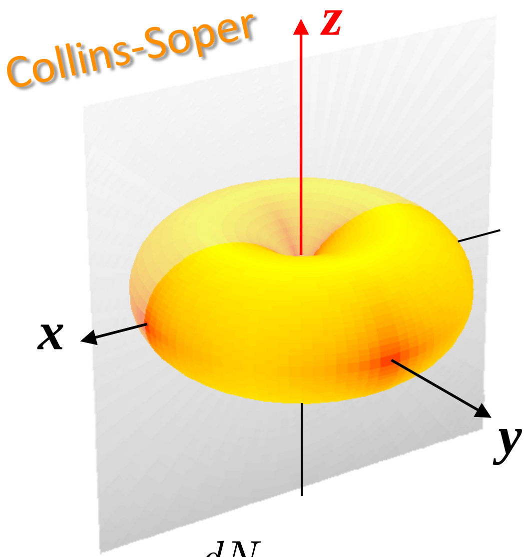


$$\frac{dN}{d\Omega} \propto 1 - \cos^2\theta$$

longitudinal

The observed polarization depends on the frame

For $|p_L| << p_T$, the CS and HX frames differ by a rotation of 90º



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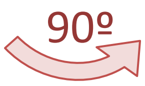

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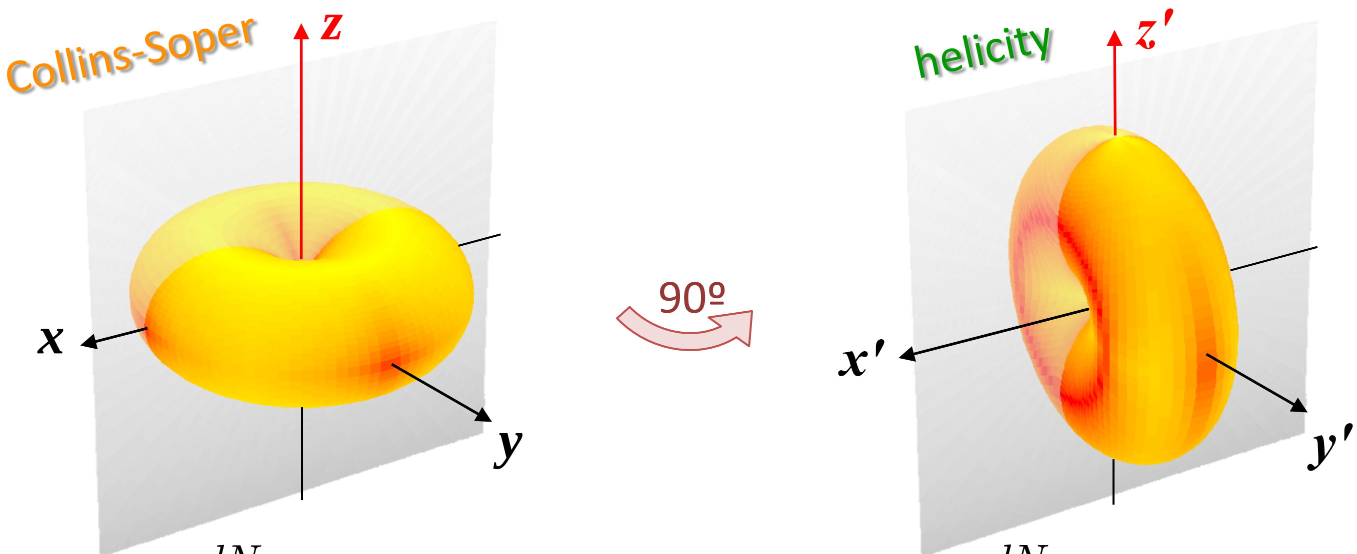


$$\frac{dN}{d\Omega} \propto 1 - \cos^2\theta$$

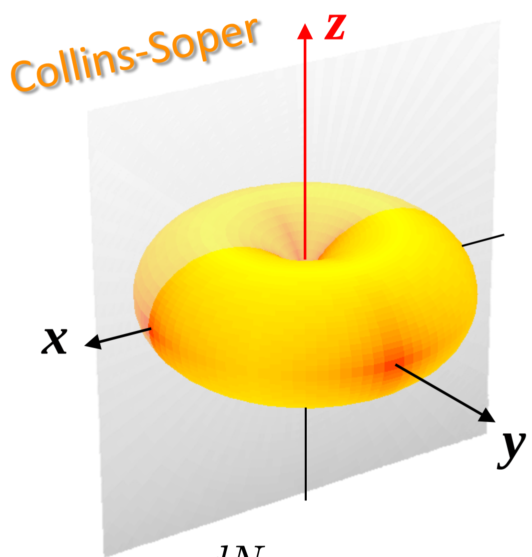
longitudinal

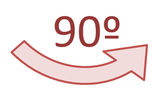
$$\frac{dN}{d\Omega} \propto 1 + \cos^2\theta - \sin^2\theta \cos 2\varphi$$

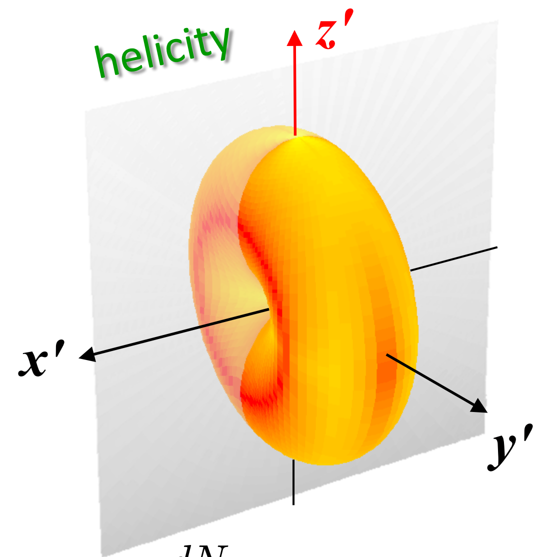
“transverse”

The observed polarization depends on the frame

For $|p_L| << p_T$, the CS and HX frames differ by a rotation of 90º







$$\frac{dN}{d\Omega} \propto 1 - \cos^2\theta$$

longitudinal

$$|\psi\rangle = |0\rangle$$

(pure state)

$$\frac{dN}{d\Omega} \propto 1 + \cos^2\theta - \sin^2\theta \cos 2\varphi$$

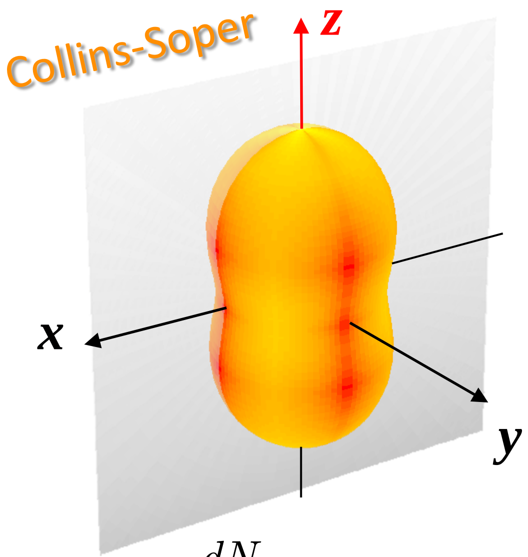
"transverse"

$$|\psi\rangle = \frac{1}{\sqrt{2}}|+1\rangle - \frac{1}{\sqrt{2}}|-1\rangle$$

(mixed state)

The observed polarization depends on the frame

For $|p_L| << p_T$, the CS and HX frames differ by a rotation of 90º



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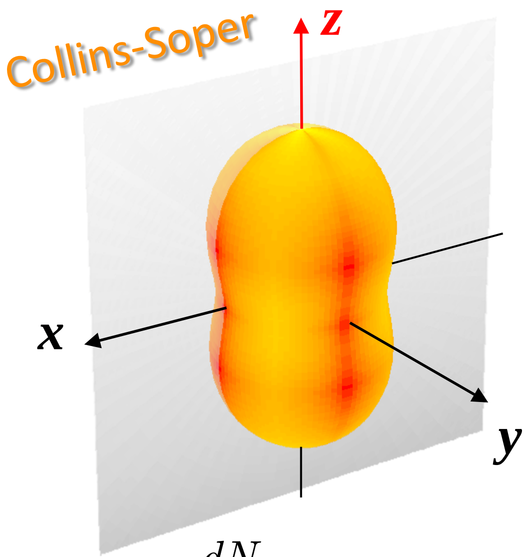
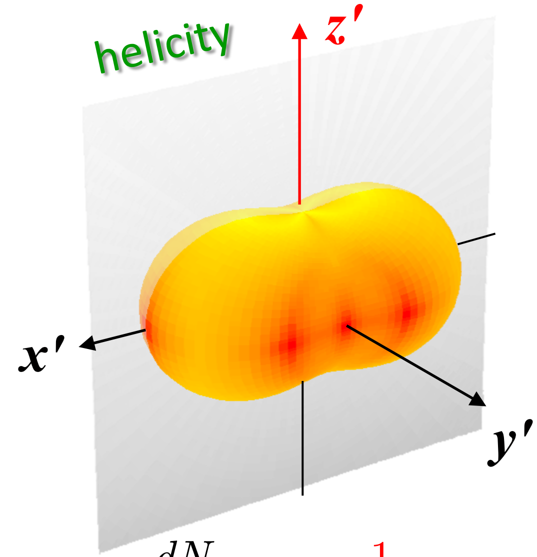


90º



$$\frac{dN}{d\Omega} \propto 1 + \cos^2\theta$$

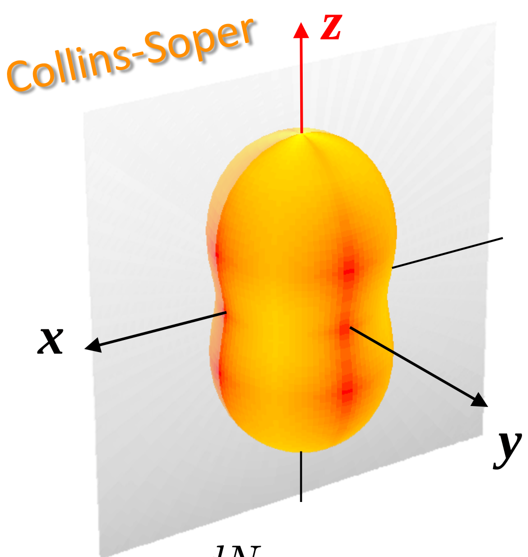
transverse

$$\frac{dN}{d\Omega} \propto 1 - \frac{1}{3}\cos^2\theta + \frac{1}{3}\sin^2\theta\,\cos 2\varphi$$

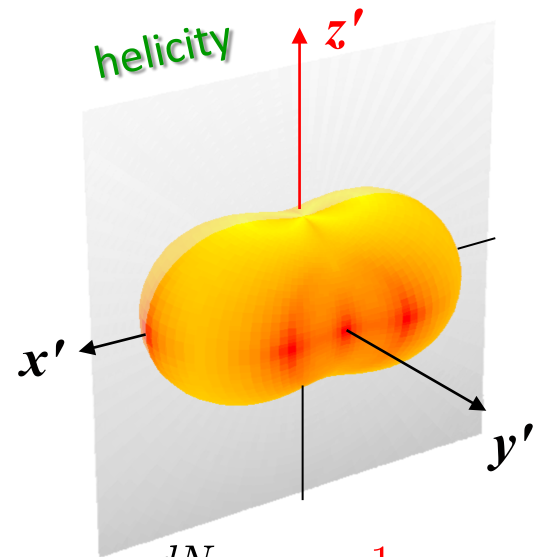
moderately “longitudinal”

The observed polarization depends on the frame

For $|p_L| << p_T$, the CS and HX frames differ by a rotation of 90º



90º



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$$\frac{dN}{d\Omega} \propto 1 - \frac{1}{3}\cos^2\theta + \frac{1}{3}\sin^2\theta\,\cos 2\varphi$$

moderately “longitudinal”

$$|\psi\rangle = |+1\rangle \; or \; |-1\rangle$$

(pure state)

$$|\psi\rangle = \frac{1}{2}|+1\rangle + \frac{1}{2}|-1\rangle \mp \frac{1}{\sqrt{2}}|0\rangle$$

(mixed state)

All reference frames are equal…
but some are more equal than others

What do different detectors measure with *arbitrary* frame choices?

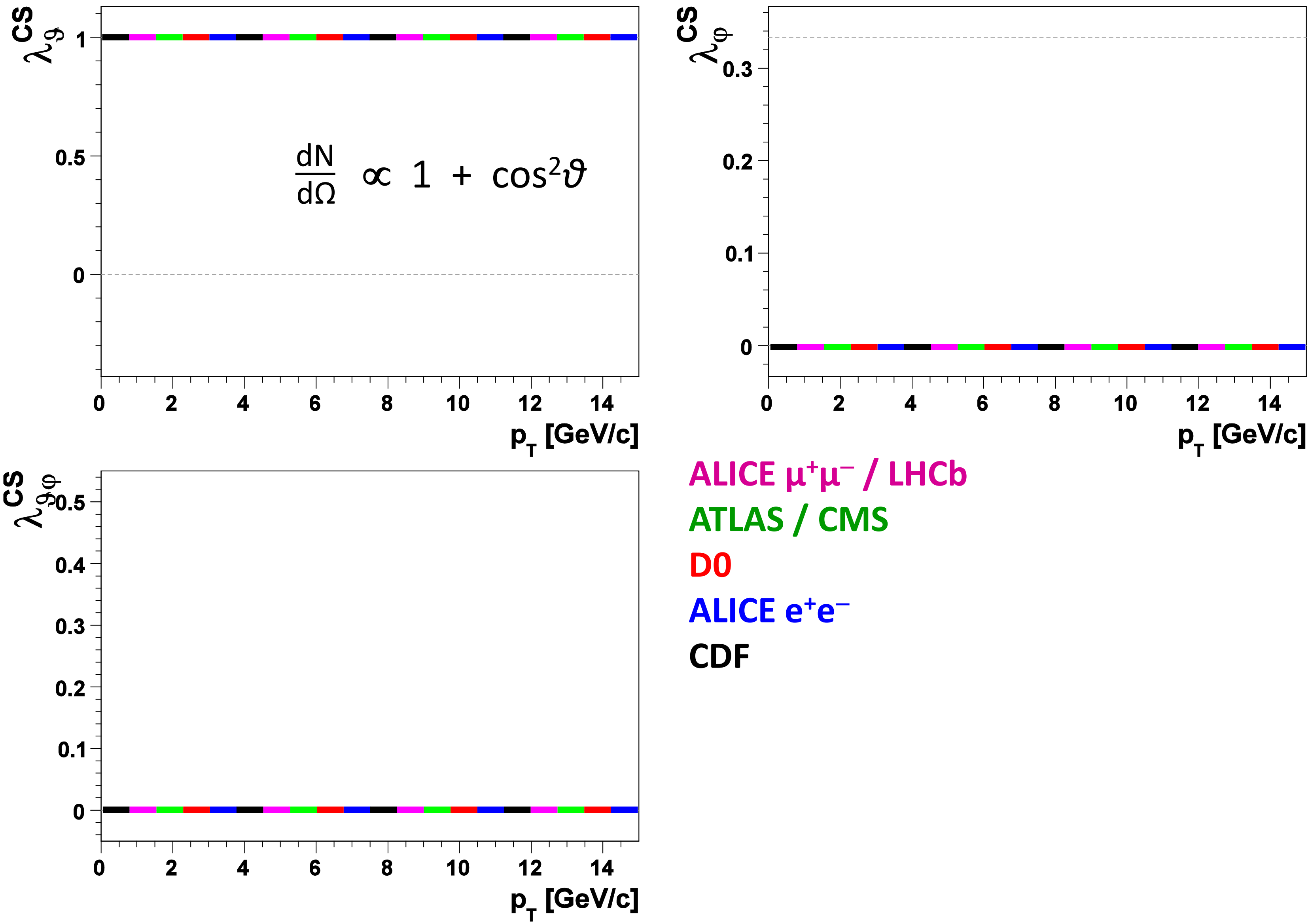
Gedankenscenario:

- **dileptons are fully transversely polarized in the CS frame**
- the decay distribution is measured at the Υ(1S) mass
  by 6 detectors with different **dilepton acceptances**:

CDF	$\|y\| < 0.6$
D0	$\|y\| < 1.8$
ATLAS & CMS	$\|y\| < 2.5$
ALICE e^+e^-	$\|y\| < 0.9$
ALICE $\mu^+\mu^-$	$2.5 < y < 4$
LHCb	$2 < y < 4.5$

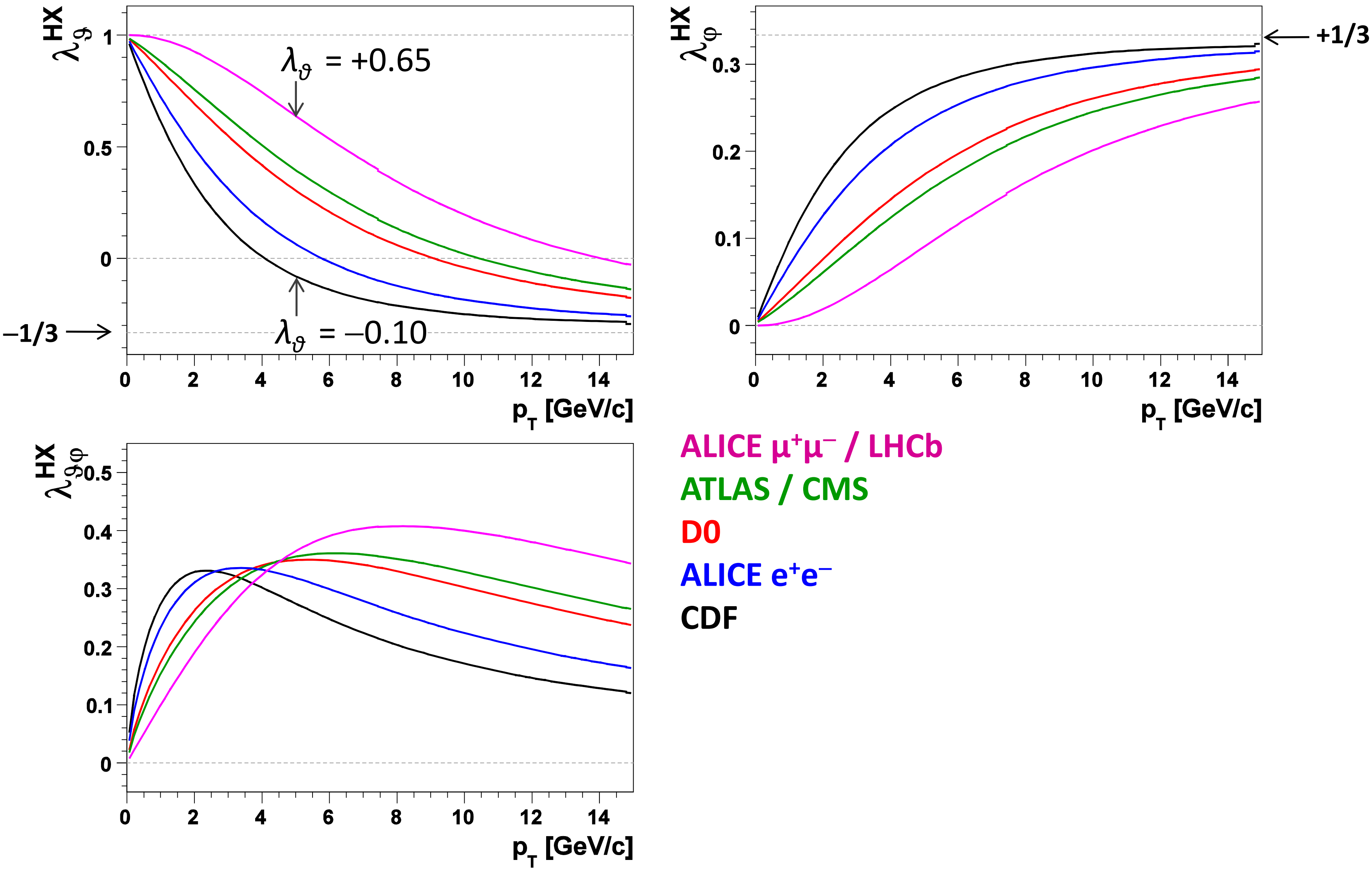
The lucky frame choice

(CS in this case)



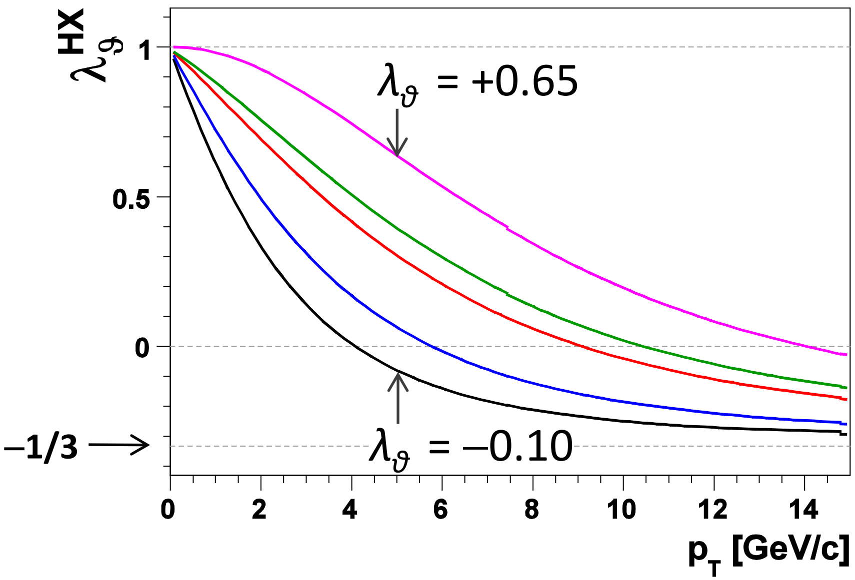
Less lucky choice

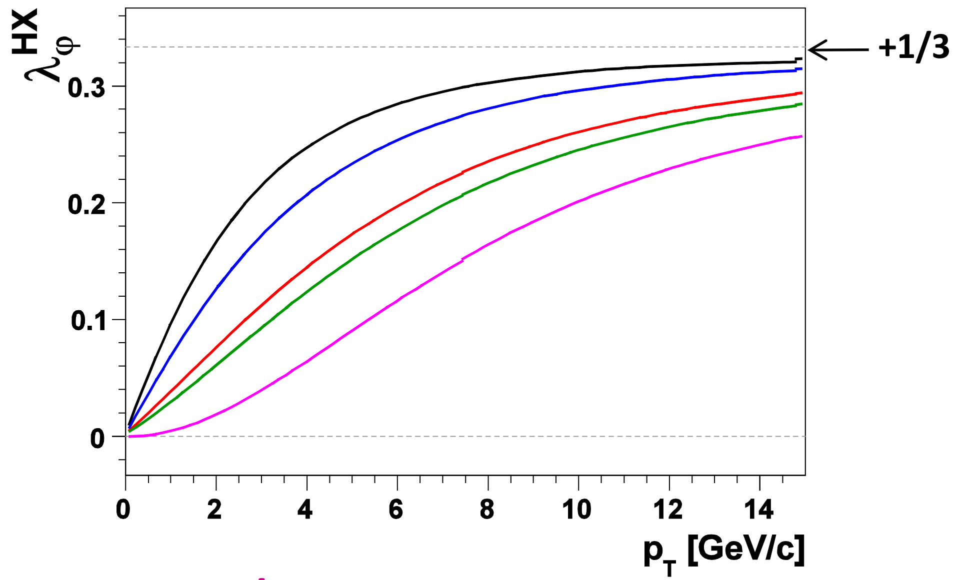
(HX in this case)

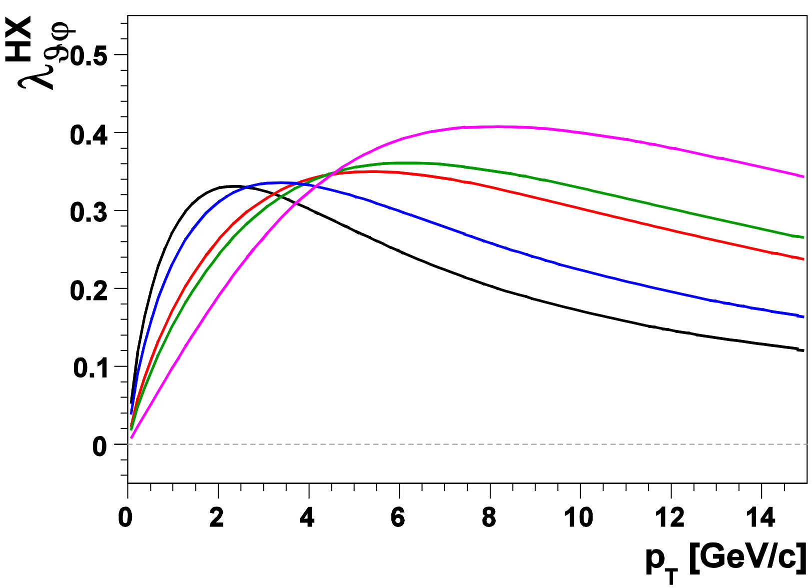


Less lucky choice

(HX in this case)







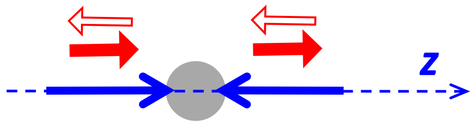
ALICE $\mu^+\mu^-$ / LHCb
ATLAS / CMS
D0
ALICE e^+e^-
CDF

artificial (experiment-dependent!) kinematic behaviour
→ measure in more than one frame!

Frames for Drell-Yan, *Z* and *W* polarizations

- polarization is *always fully* ***transverse***...

$V = \gamma^*, Z, W$



Due to helicity conservation at the q-$\bar{q}$-V (q-q^*-V) vertex,
$\boldsymbol{J_z = \pm 1}$ along the q-$\bar{q}$ (q-q^*) scattering direction $\boldsymbol{z}$

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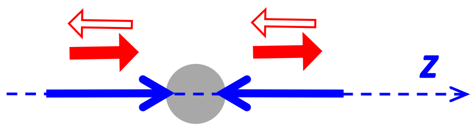
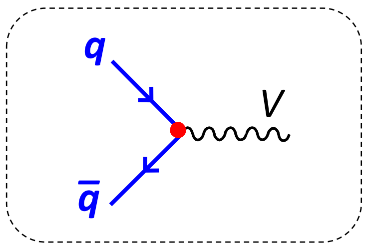


Due to helicity conservation at the q-$\bar{q}$-V (q-q^*-V) vertex,
$J_z = \pm 1$ along the q-$\bar{q}$ (q-q^*) scattering direction ***z***

- ...but with respect to a ***subprocess-dependent quantization axis***

$O(\alpha_S^0) \rightarrow$



z = relative dir. of incoming *q* and *q*bar
(~ **Collins-Soper frame**)

important only up to $p_T = \mathcal{O}$(parton k_T)

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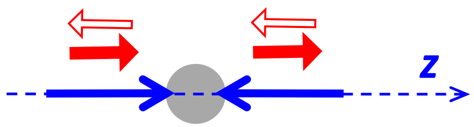

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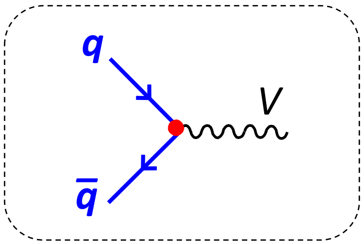
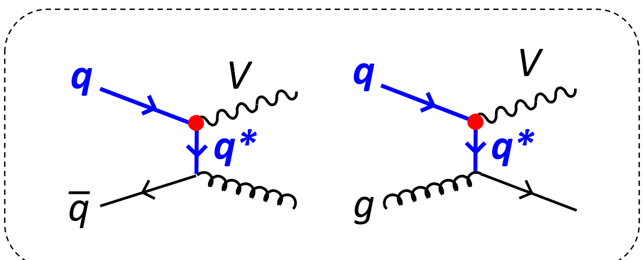
$O(\alpha_S^0)$



z = relative dir. of incoming *q* and *q*bar
(~ **Collins-Soper frame**)

important only up to $p_T = \mathcal{O}(\text{parton } k_T)$

$O(\alpha_S^1)$
QCD corrections

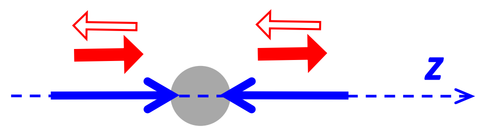


z = dir. of *one* incoming quark
(~ **Gottfried-Jackson frame**)

Frames for Drell-Yan, *Z* and *W* polarizations

- polarization is *always fully* ***transverse***...

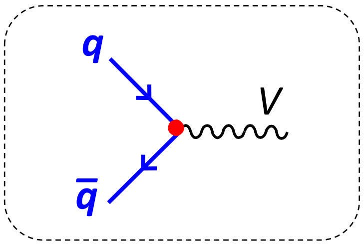
$V = \gamma^*, Z, W$



Due to helicity conservation at the q-$\bar{q}$-V (q-q^*-V) vertex, $J_z = \pm 1$ along the q-$\bar{q}$ (q-q^*) scattering direction ***z***

- ...but with respect to a ***subprocess-dependent quantization axis***

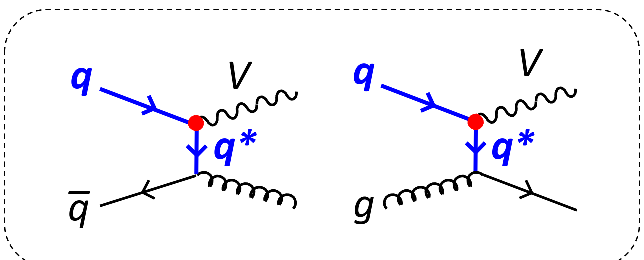
$O(\alpha_S^0)$



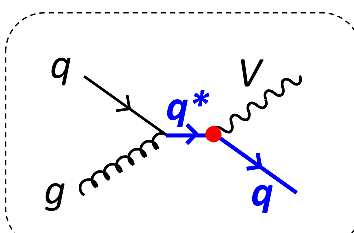
z = relative dir. of incoming q and $q$bar
(~ **Collins-Soper frame**)

important only up to $p_T = \mathcal{O}$(parton k_T)

$O(\alpha_S^1)$
QCD
corrections



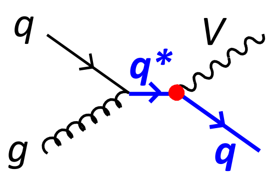
z = dir. of *one* incoming quark
(~ **Gottfried-Jackson frame**)



z = dir. of outgoing q
(= **parton-cms-helicity** $\approx$ **lab-cms-helicity**)

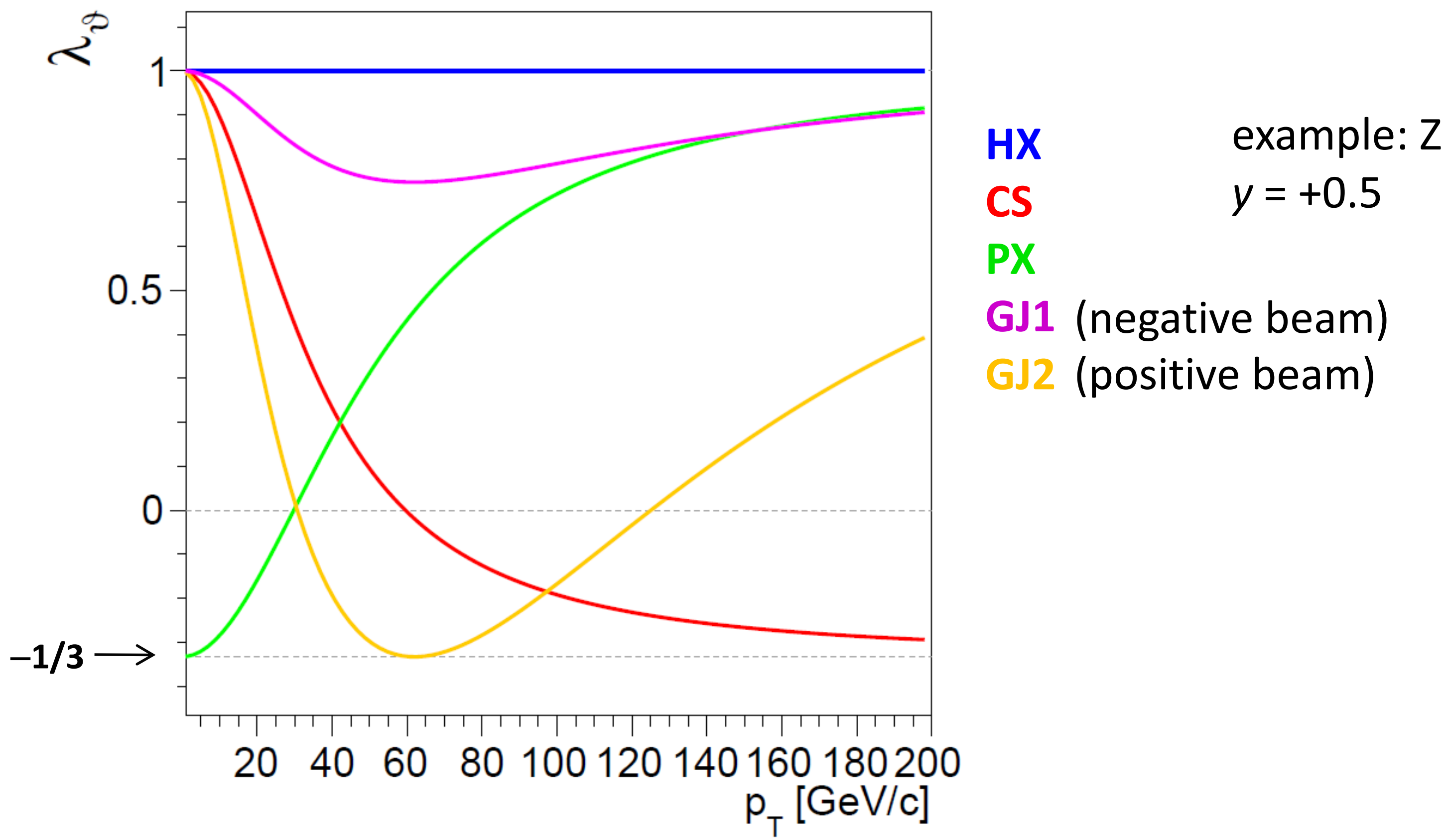
“Optimal” frames for Drell-Yan, *Z* and *W* polarizations

Different subprocesses have different “natural” quantization axes



For ***s*-channel processes** the natural axis is the direction of the outgoing quark (= direction of dilepton momentum)

→ optimal frame (= maximizing polar anisotropy): **HX** (neglecting parton-parton-cms vs proton-proton-cms difference!)



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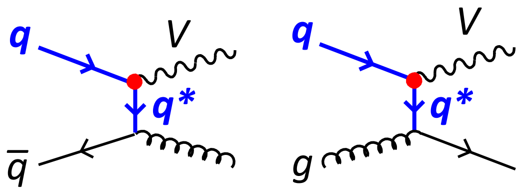
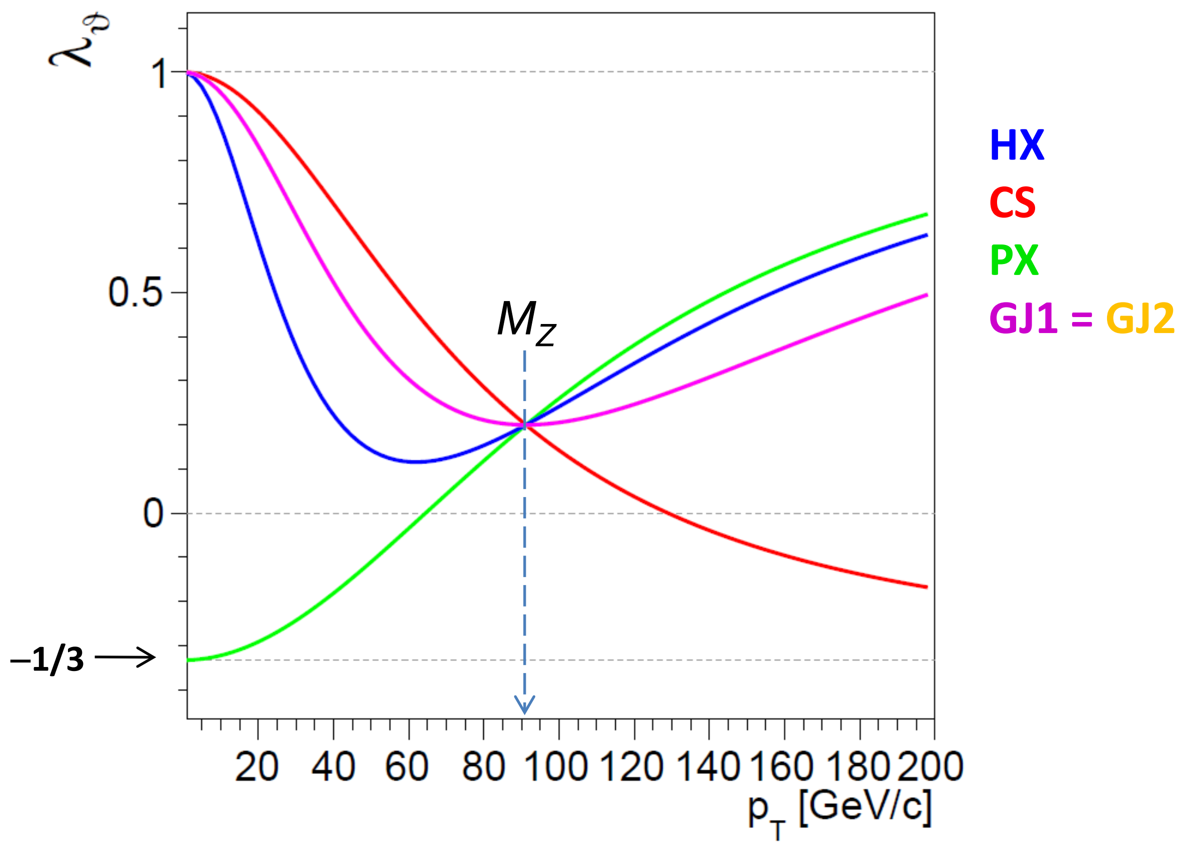


For ***t*- and *u*-channel processes** the natural axis is the direction of either one or the other incoming parton (~ “Gottfried-Jackson” axes)

$\rightarrow$ optimal frame: geometrical average of GJ1 and GJ2 axes = **CS ($p_T < M$)** and **PX ($p_T > M$)**



example: Z
y = +0.5

A complementary approach: frame-independent polarization

The *shape* of the distribution is (obviously) frame-invariant (= invariant by rotation)

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→ it can be characterized by frame-independent parameters:

$$\tilde{\lambda} = \frac{\lambda_\vartheta + 3\lambda_\varphi}{1 - \lambda_\varphi}$$

$$\lambda^* = \frac{\lambda_\vartheta - 3\Lambda^*}{1 + \Lambda^*} \qquad \Lambda^* = \frac{1}{4}\left\{\lambda_\vartheta - \lambda_\varphi \pm \sqrt{\left(\lambda_\vartheta - \lambda_\varphi\right)^2 + 4\lambda_{\vartheta\varphi}^2}\right\}$$

$$\tilde{\mathcal{A}} = \frac{\sqrt{A_\vartheta^2 + A_\varphi^2}}{3 + \lambda_\vartheta}$$

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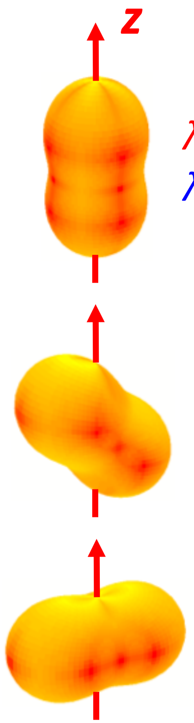

$\lambda_\vartheta = +1$
$\lambda_\varphi = 0$

$\lambda_\vartheta = +1/5$
$\lambda_\varphi = +1/5$

$\lambda_\vartheta = -1/3$
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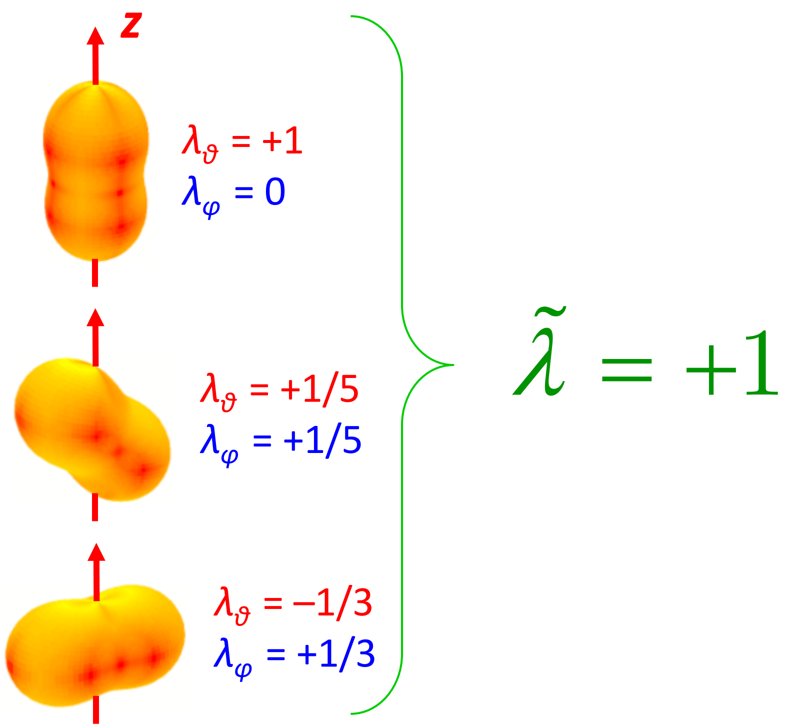

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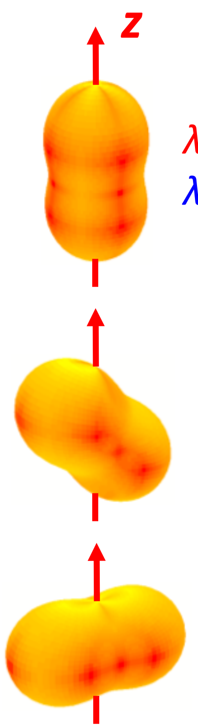
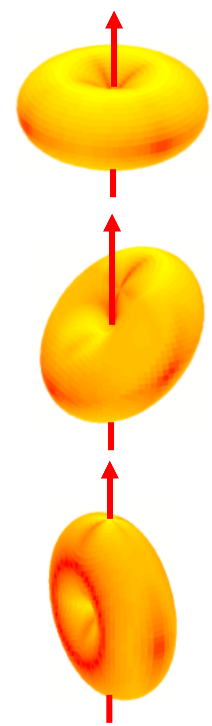


$\lambda_\vartheta = +1$
$\lambda_\varphi = 0$

$\lambda_\vartheta = +1/5$
$\lambda_\varphi = +1/5$

$\lambda_\vartheta = -1/3$
$\lambda_\varphi = +1/3$

$$\tilde{\lambda} = +1$$



$\lambda_\vartheta = -1$
$\lambda_\varphi = 0$

$\lambda_\vartheta = -1/3$
$\lambda_\varphi = -1/3$

$\lambda_\vartheta = +1$
$\lambda_\varphi = -1$

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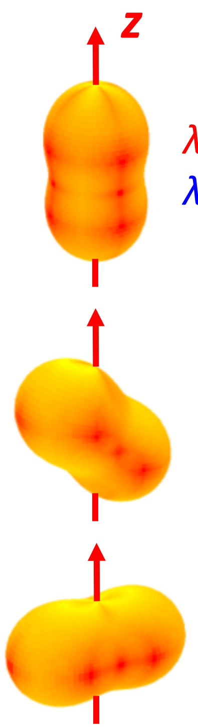

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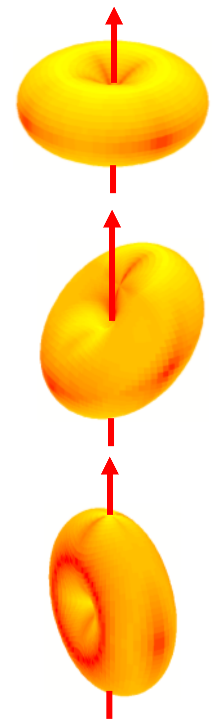

$\lambda_\vartheta = -1$, $\lambda_\varphi = 0$

$\lambda_\vartheta = -1/3$, $\lambda_\varphi = -1/3$

$\lambda_\vartheta = +1$, $\lambda_\varphi = -1$

$$\tilde{\lambda} = -1$$

rotations *in the production plane*

Reduces acceptance dependence

Gedankenscenario: vector state produced in this subprocess admixture:

- **60%** processes with natural **transverse** polarization in the **CS** frame
- **40%** processes with natural **transverse** polarization in the **HX** frame

assumed indep. of kinematics, for simplicity

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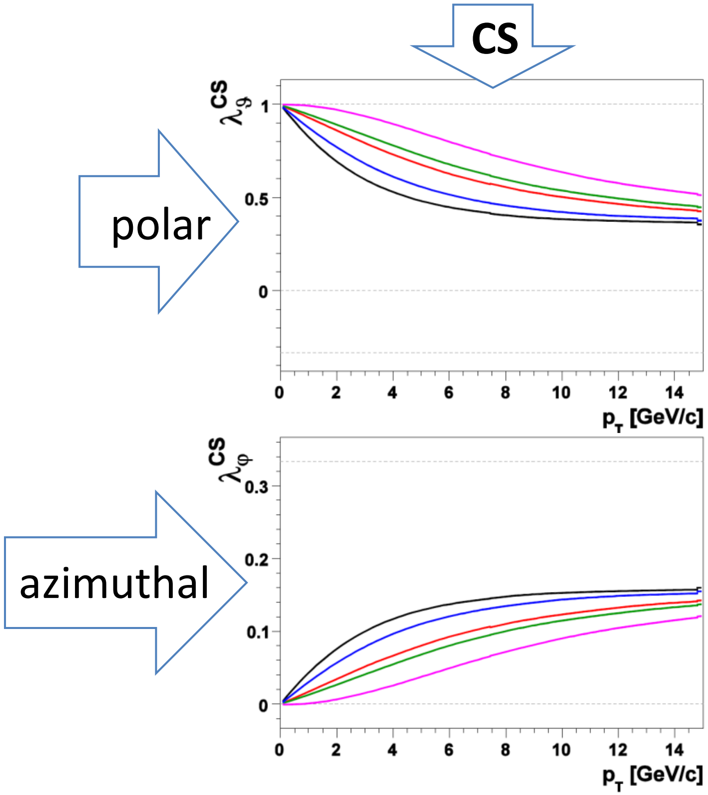

M = 10 GeV/c^2

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D0	**|y| < 1.8**
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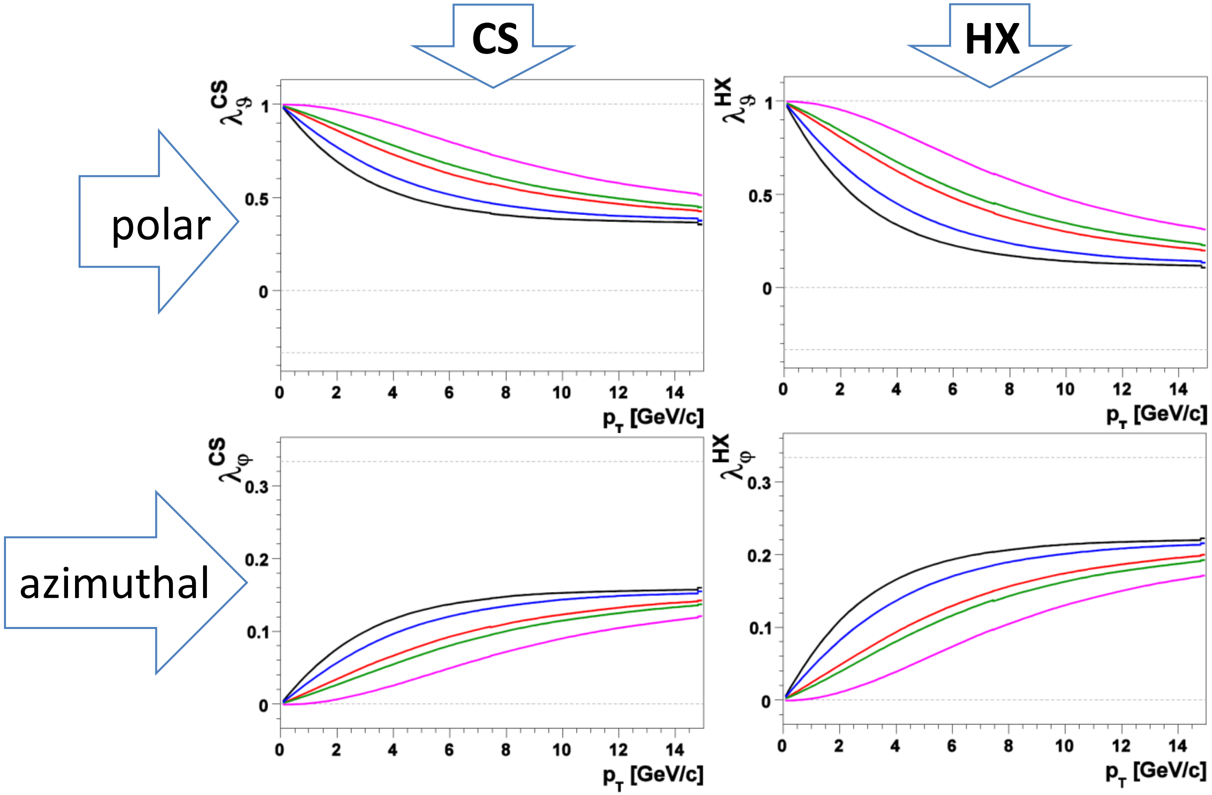

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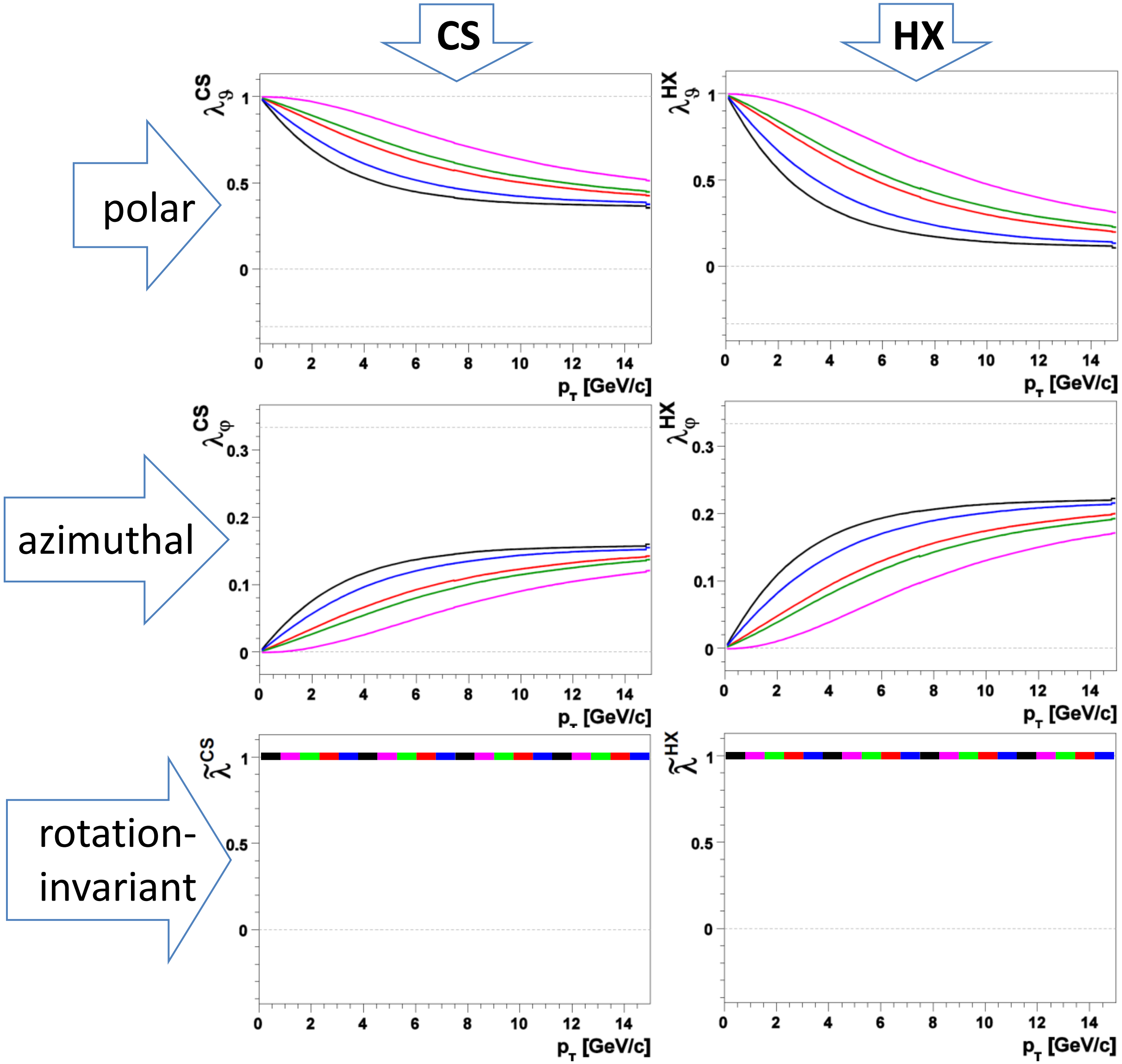
CDF	**$\vert y\vert < 0.6$**
D0	**$\vert y\vert < 1.8$**
ATLAS/CMS	**$\vert y\vert < 2.5$**
ALICE e^+e^-	**$\vert y\vert < 0.9$**
ALICE $\mu^+\mu^-$	**$2.5 < y < 4$**
LHCb	**$2 < y < 4.5$**

Reduces acceptance dependence

Gedankenscenario: vector state produced in this subprocess admixture:

- **60%** processes with natural **transverse** polarization in the **CS** frame
- **40%** processes with natural **transverse** polarization in the **HX** frame

assumed indep. of kinematics, for simplicity



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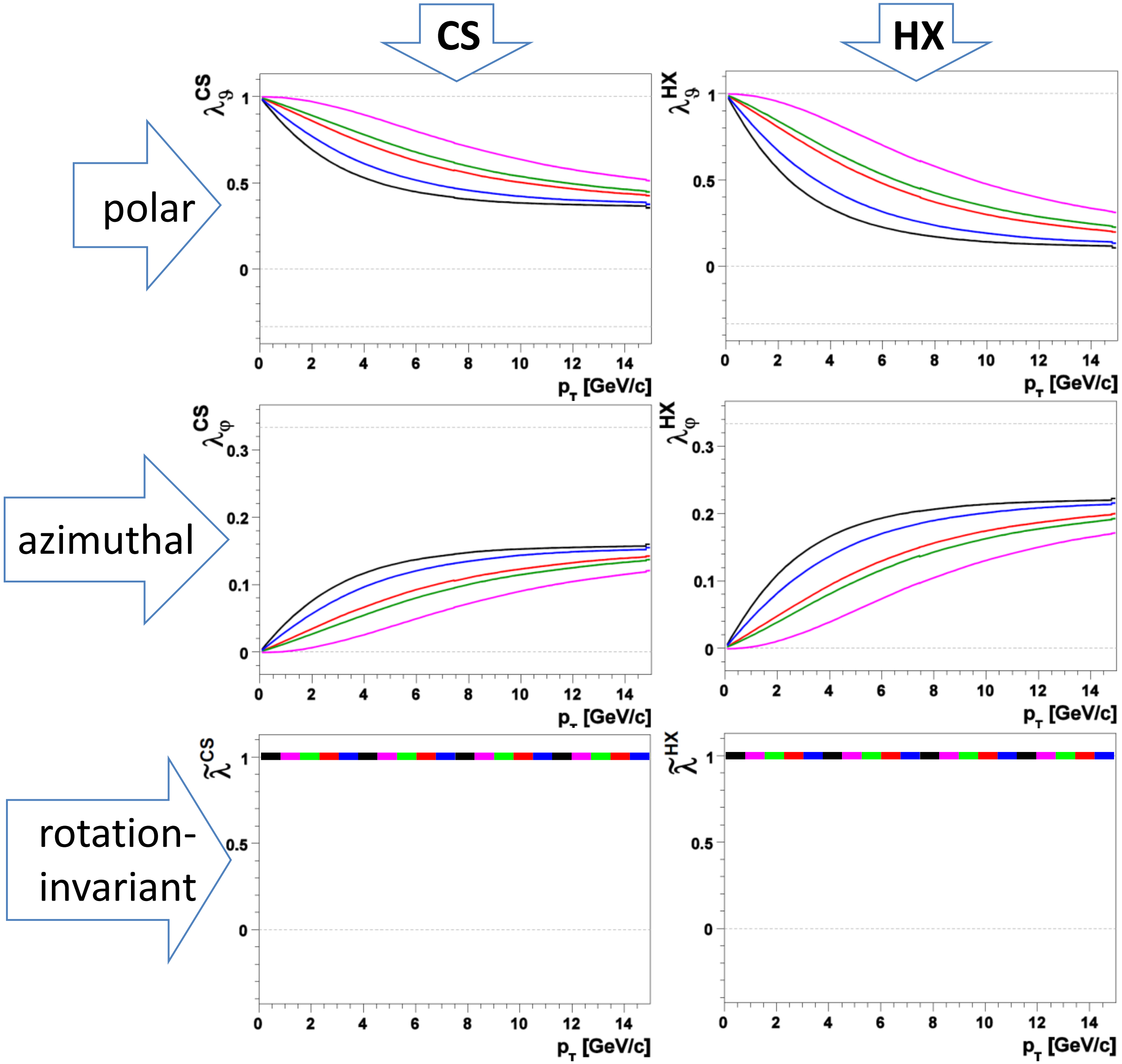

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- Immune to "extrinsic" kinematic dependencies

→ *less acceptance-dependent*

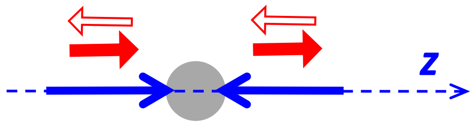
→ *facilitates comparisons*

- *useful as closure test*

Physical meaning: Drell-Yan, *Z* and *W* polarizations

- polarization is *always fully* ***transverse***...

$V = \gamma^*, Z, W$



Due to helicity conservation at the q-$\bar{q}$-V (q-q^*-V) vertex,
$\mathbf{J_z = \pm 1}$ along the q-$\bar{q}$ (q-q^*) scattering direction ***z***

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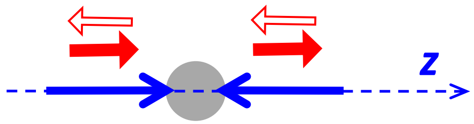

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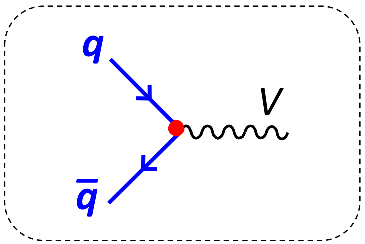
$O(\alpha_S^0) \longrightarrow$



"natural" z = relative dir. of *q* and *q*bar
$\rightarrow$ $\boldsymbol{\lambda_\vartheta}$**("CS") = +1**

wrt ***any*** axis: $\tilde{\lambda} = +1$

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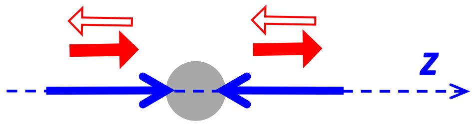

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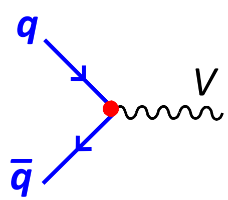

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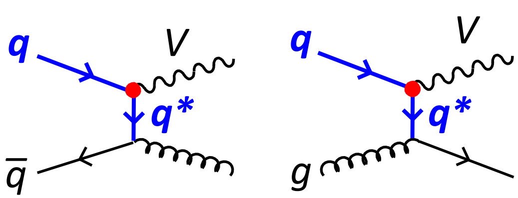
(LO) QCD corrections



z = dir. of *one* incoming quark
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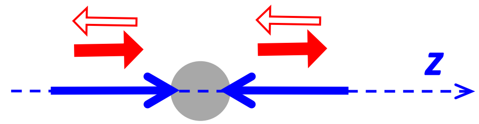

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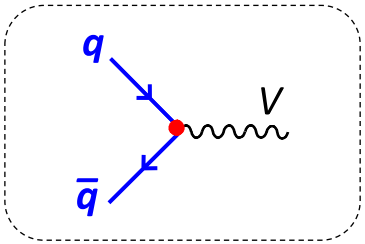

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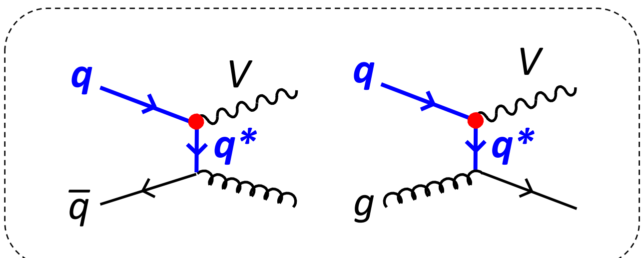

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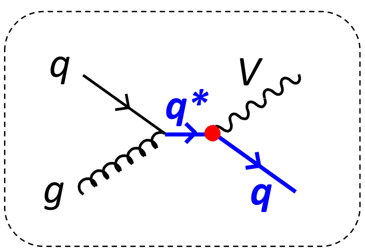

z = dir. of outgoing *q*

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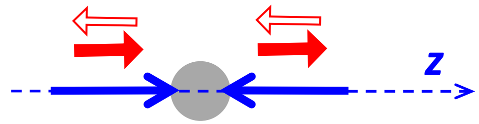

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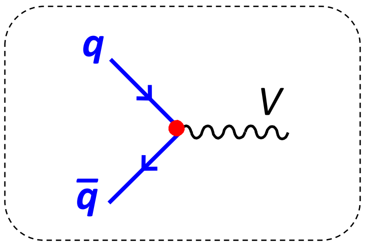

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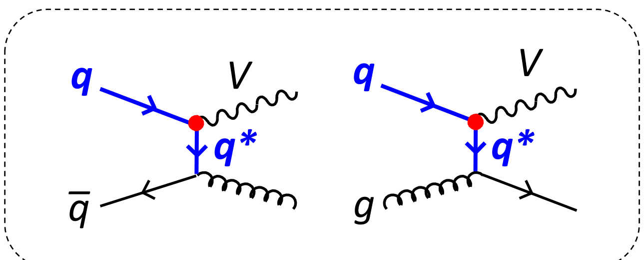

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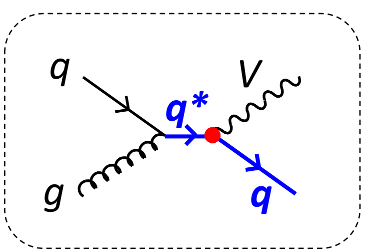

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$\tilde{\lambda}$ = +1
any frame

In all these cases the *q*-*q*-*V* lines are in the production plane (planar processes);
The CS, GJ, pp-HX and qg-HX axes only differ by a rotation in the production plane

λ_ϑ vs $\widetilde{\lambda}$

Example: ***Z/γ*/W* polarization** (CS frame) as a function of contribution of LO QCD corrections:

λ_ϑ vs $\tilde{\lambda}$

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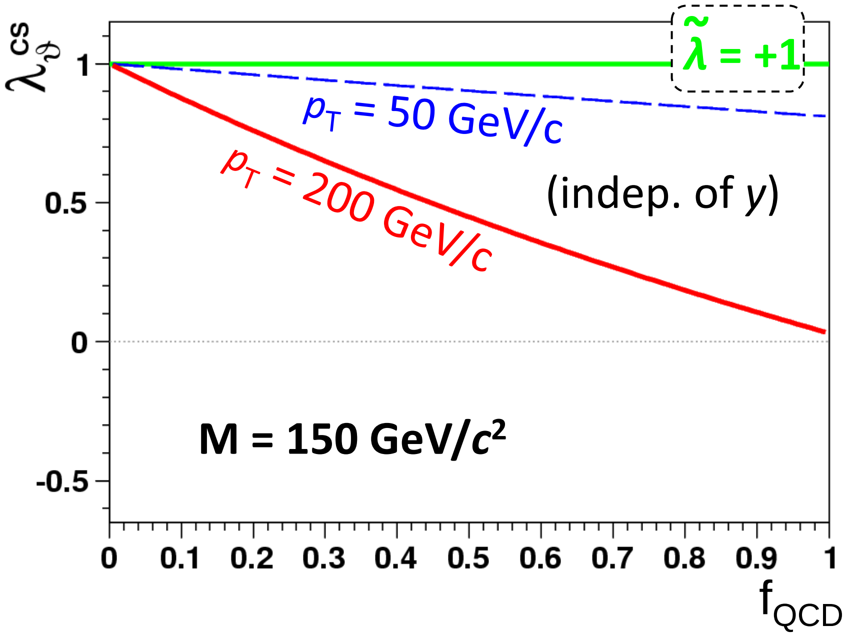

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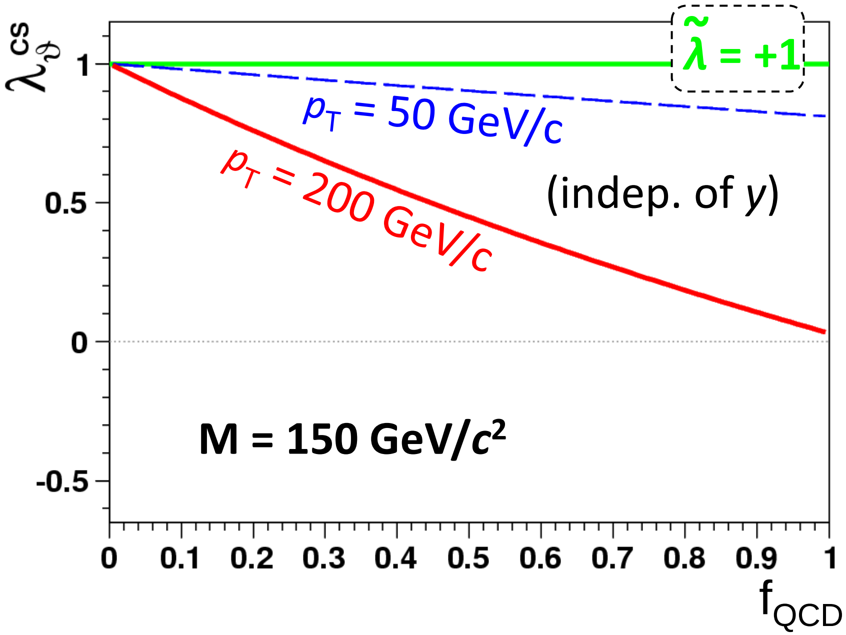
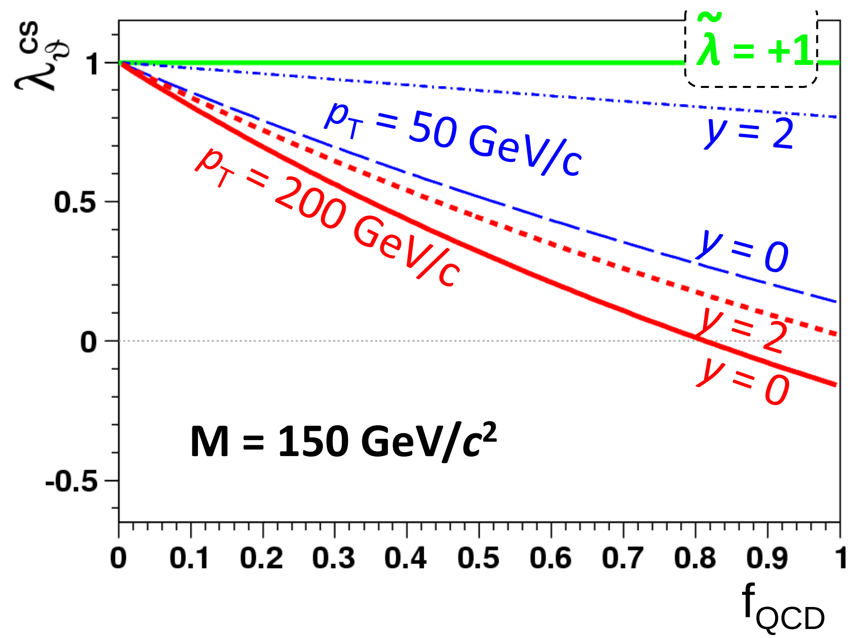


Case 2: dominating ***q-g*** QCD corrections



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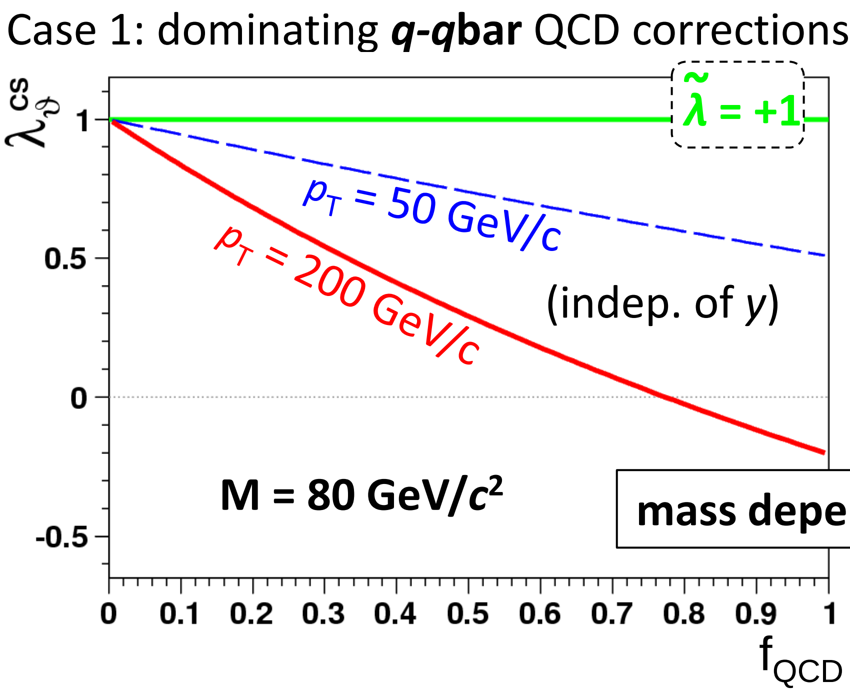
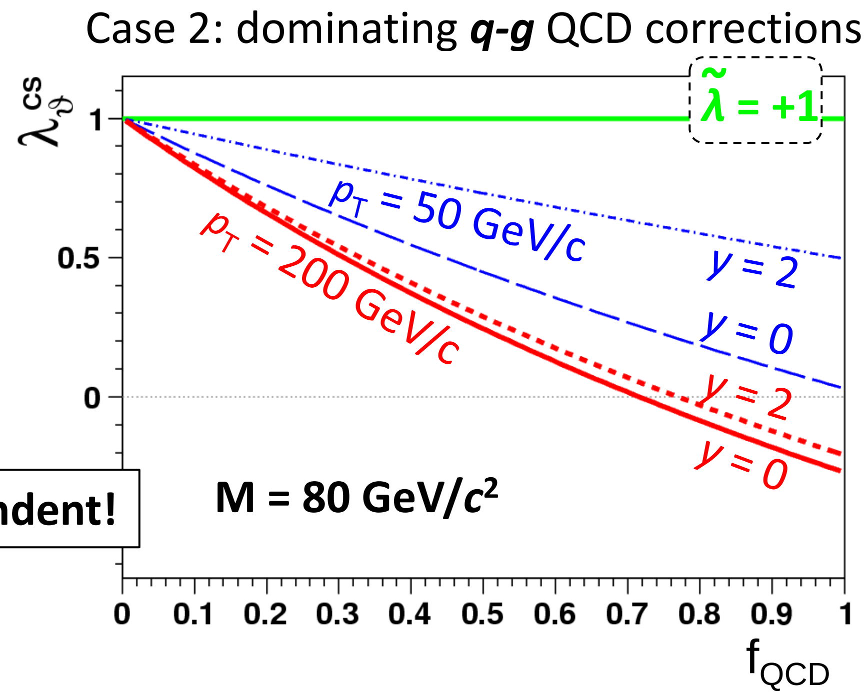


mass dependent!



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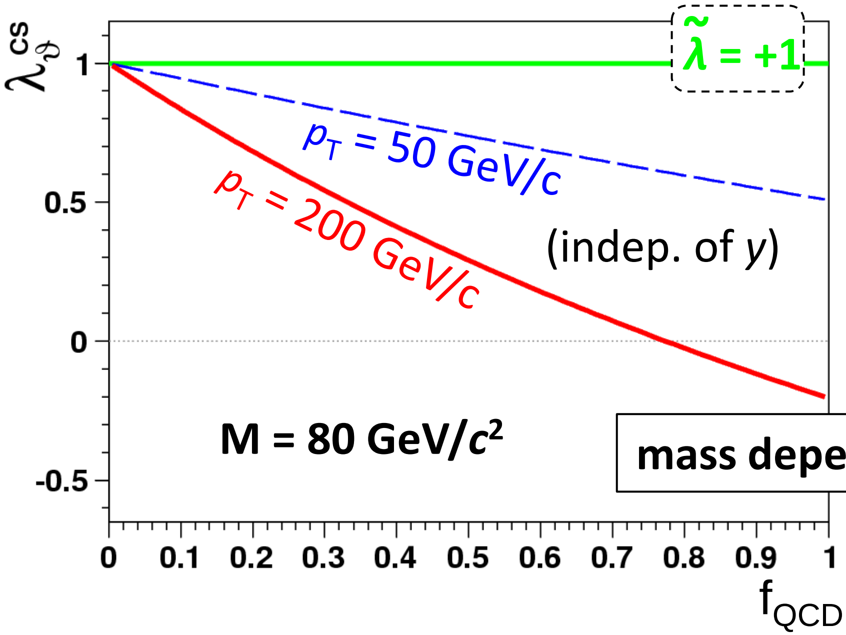

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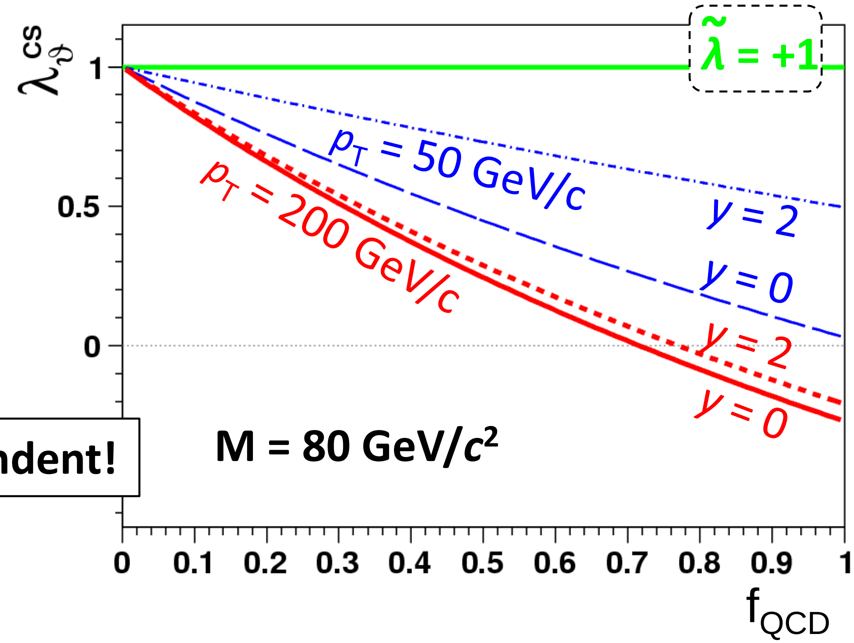

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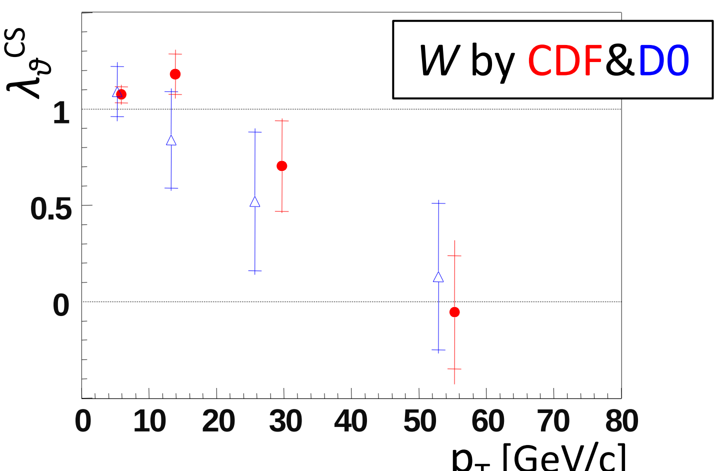

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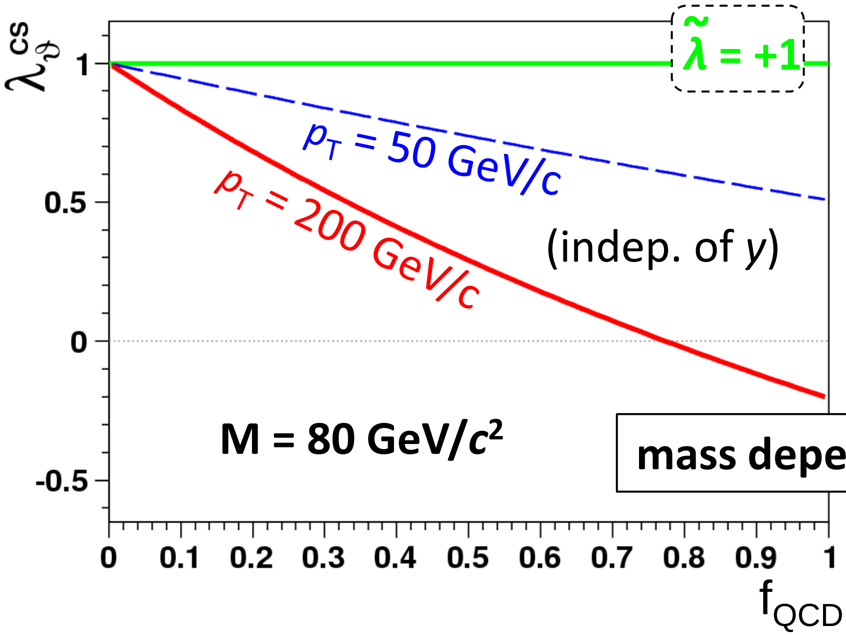

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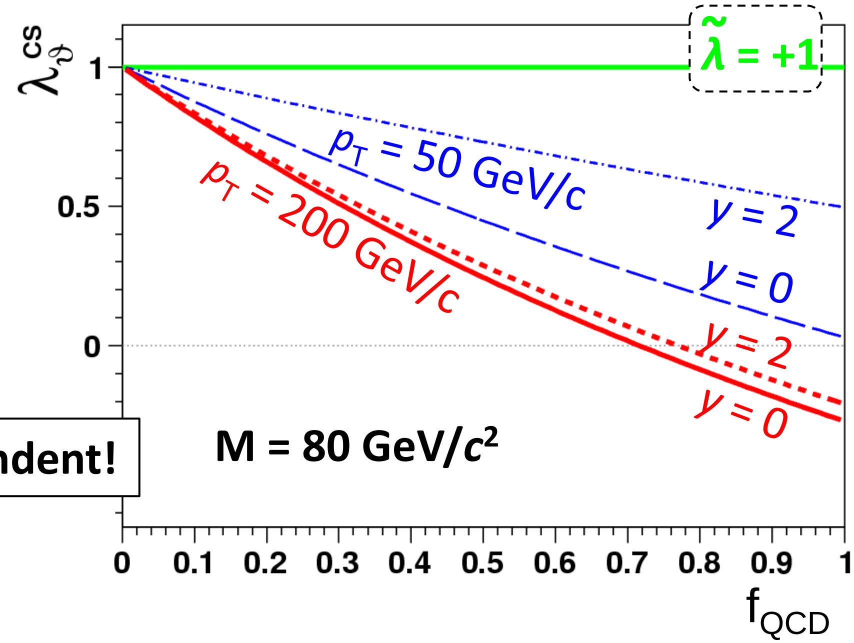

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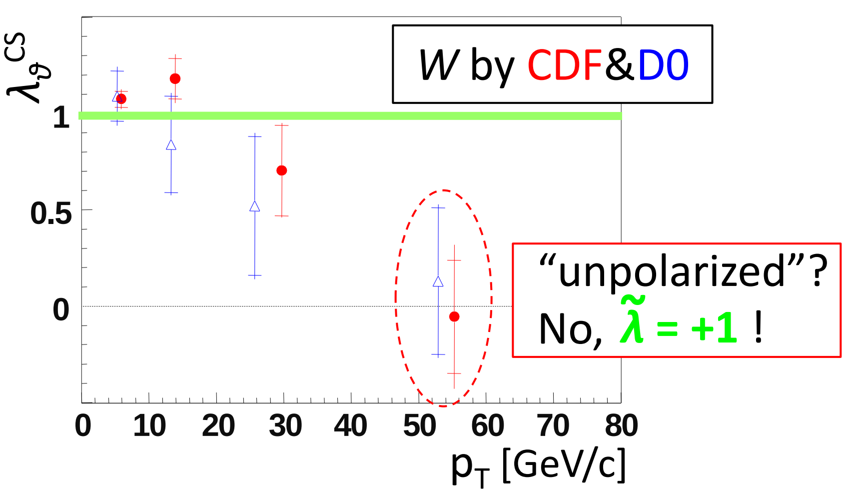

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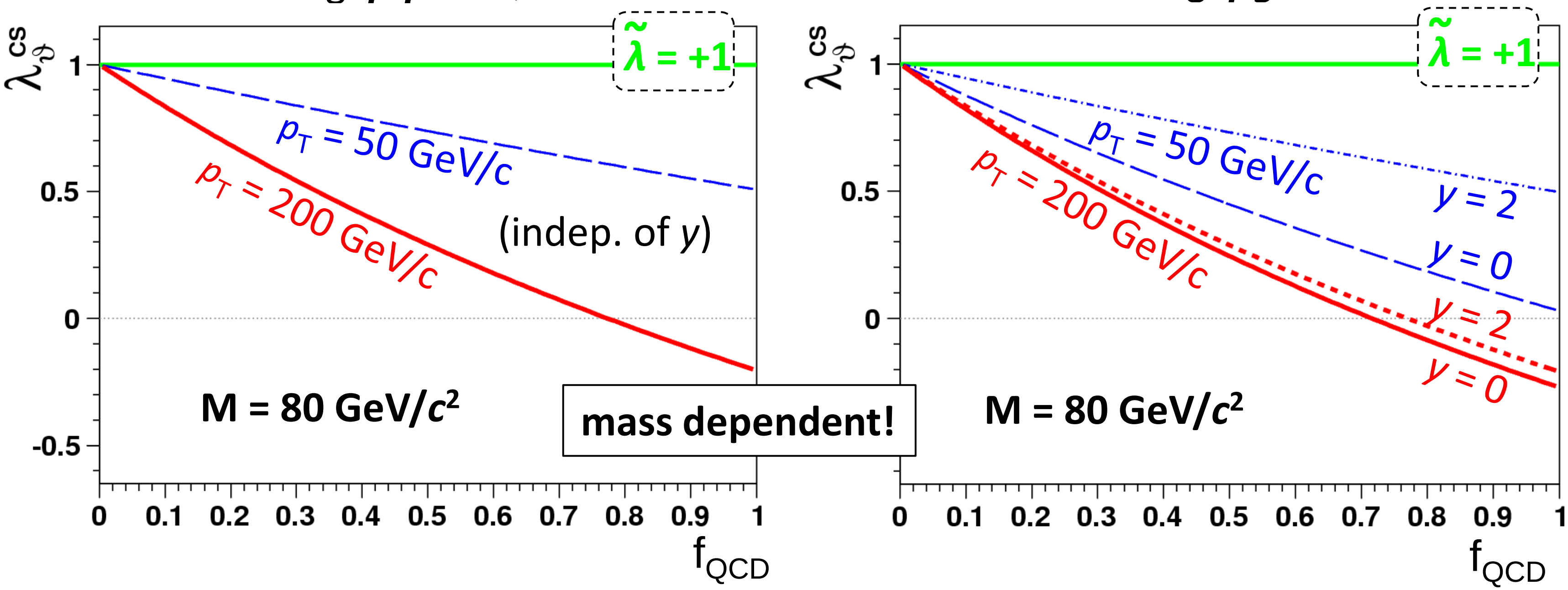

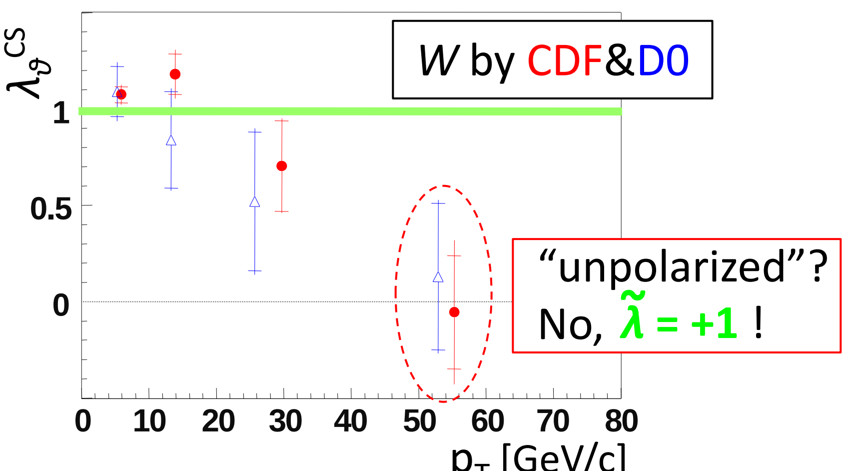

λ_ϑ

- depends on p_T , y and mass
  → by integrating we lose significance
- is far from being maximal
- depends on process admixture
  → need pQCD and PDFs

λ_ϑ vs $\tilde{\lambda}$

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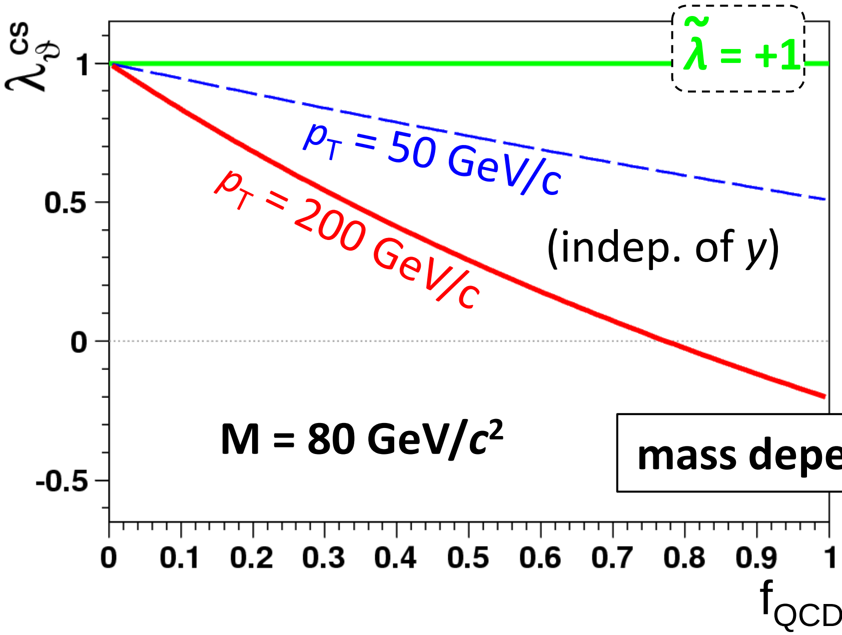

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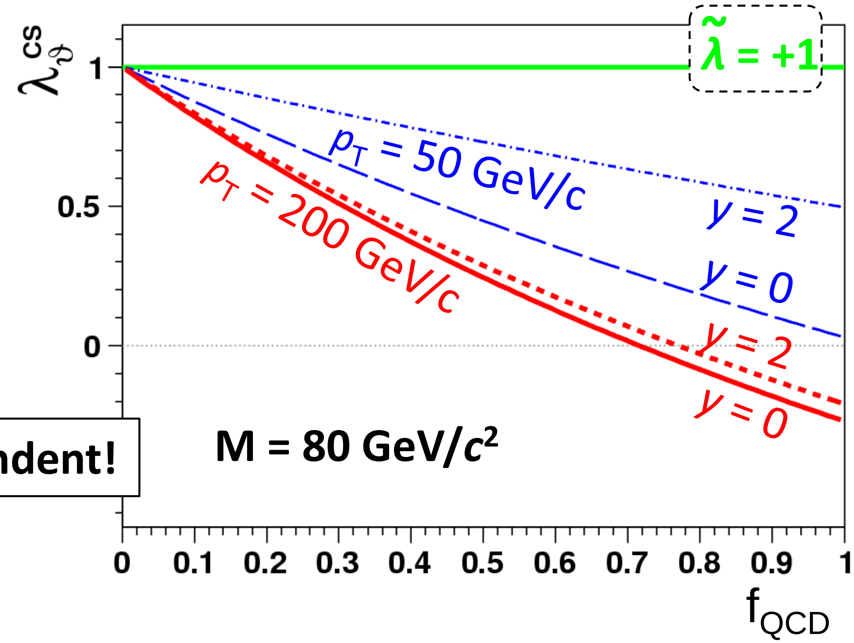

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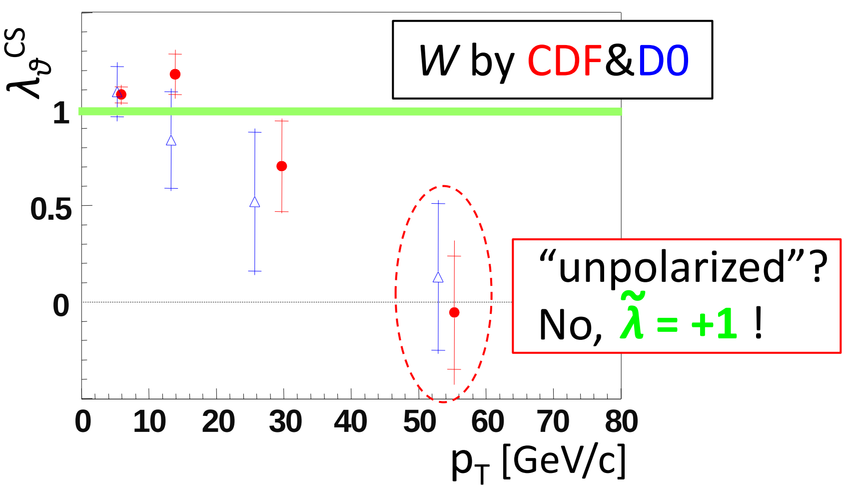

λ_ϑ
- depends on p_T , y and mass
  $\rightarrow$ by integrating we lose significance
- is far from being maximal
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  $\rightarrow$ need pQCD and PDFs

$\tilde{\lambda}$ is constant, maximal and independent of process admixture

λ_ϑ vs $\tilde{\lambda}$

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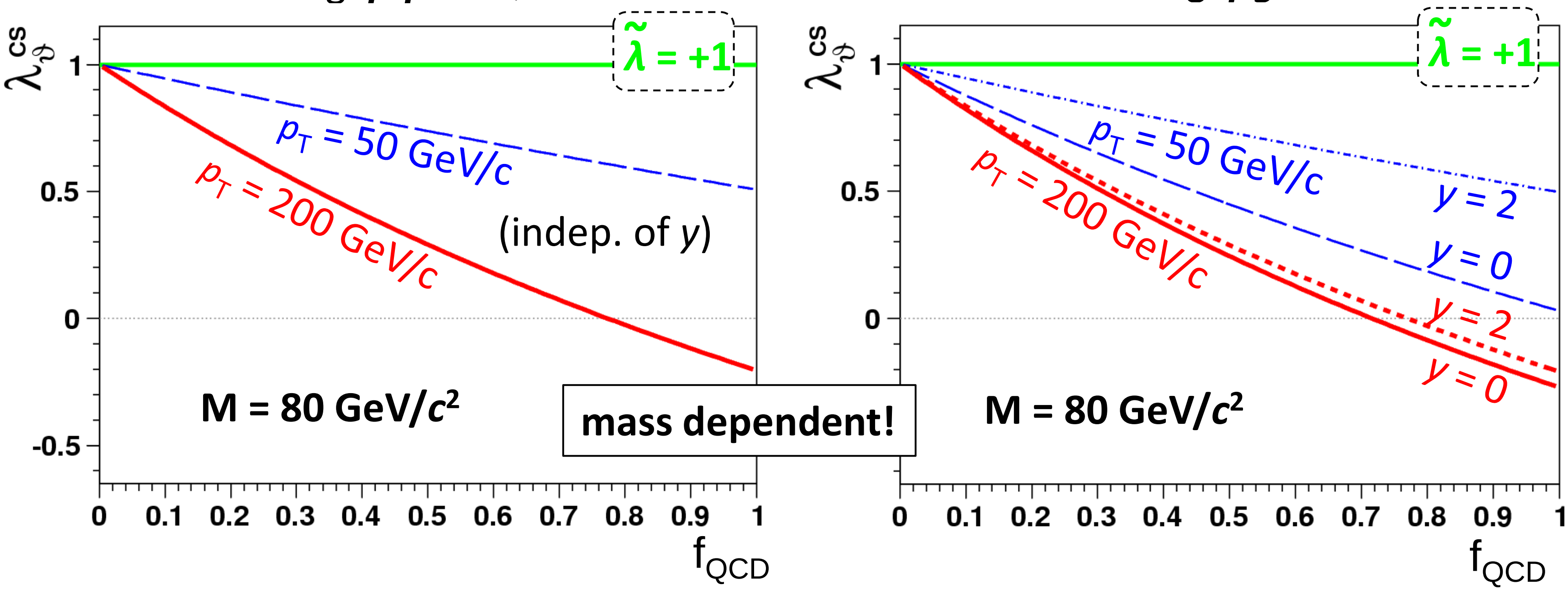
Case 1: dominating ***q-qbar*** QCD corrections   Case 2: dominating ***q-g*** QCD corrections



On the other hand, $\tilde{\lambda}$ forgets about the direction of the quantization axis.
This information is crucial if we want to **disentangle the *qg* contribution**,
the only one resulting in a ***rapidity-dependent*** $\boldsymbol{\lambda_\vartheta}$

Measuring λ_ϑ(CS) *as a function of rapidity* gives information on the gluon content of the proton

The Lam-Tung relation

A fundamental result of the theory of vector-boson polarizations (Drell-Yan, directly produced *Z* and *W*) is that, at leading order in perturbative QCD,

$$\lambda_\vartheta + 4\lambda_\varphi = 1$$ independently of the polarization frame

Lam-Tung relation, Pysical Review D 18, 2447 (1978)

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Today we know that it is only a *special* case of general frame-independent polarization relations, corresponding to a *transverse* intrinsic polarization:

$$\tilde{\lambda} = \frac{\lambda_\vartheta + 3\lambda_\varphi}{1 - \lambda_\varphi} = +1 \quad \Rightarrow \lambda_\vartheta + 4\lambda_\varphi = 1$$

It is, therefore, not a “QCD” relation, but a consequence of

1) rotational invariance

2) properties of the **quark-photon/Z/W couplings** (helicity conservation)

Beyond the Lam-Tung relation

Even when the Lam-Tung relation is violated,
$\tilde{\lambda}$ can always be defined and is always frame-independent

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$\rightarrow +1 \text{ for } p_T \rightarrow 0$

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→ vector-boson – quark – quark couplings in ***non-planar* processes** (higher-order contributions)

$$\left.\begin{array}{l}\tilde{\lambda} \ll +1 \\ \tilde{\lambda} > +1\end{array}\right\}$$

→ contribution of ***different/new couplings or processes***
(e.g.: *Z* from Higgs, *W* from top, triple *ZZ*γ coupling,
higher-twist effects in DY production, etc…)

Polarization can be used to distinguish
between different kinds of physics signals,
or between “signal” and “background” processes
($\rightarrow$improve significance of new-physics searches)

Example: *W* from top ↔ *W* from *q*-*q*bar and *q*-*g*

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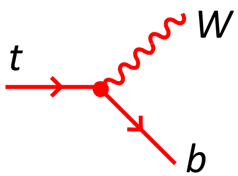
longitudinally polarized:

$\lambda_{\vartheta}^{\mathrm{SM}} \cong -0.65$
$\lambda_{\varphi}^{\mathrm{SM}} \cong 0$

wrt *W direction in the top rest frame* (top-frame helicity)



independently of top production mechanism

The top quark decays almost always to *W*+*b*
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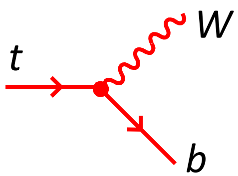

independently of top production mechanism

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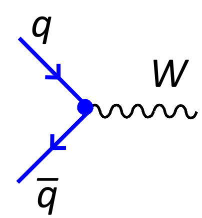
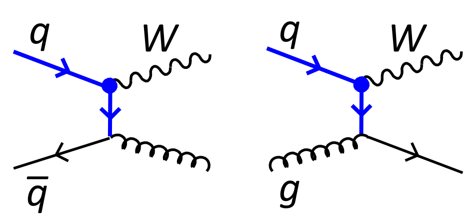
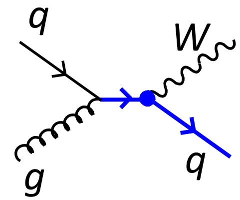
transversely polarized,
$\lambda_{\vartheta} = +1$ & $\lambda_{\varphi} = 0$ *wrt 3 different axes:*



relative direction of *q* and *q*bar ("Collins-Soper")



direction of *q* or *q*bar ("Gottfried-Jackson")



direction of outgoing *q* (cms-helicity)

a) Frame-dependent approach

We measure λ_ϑ choosing the helicity axis

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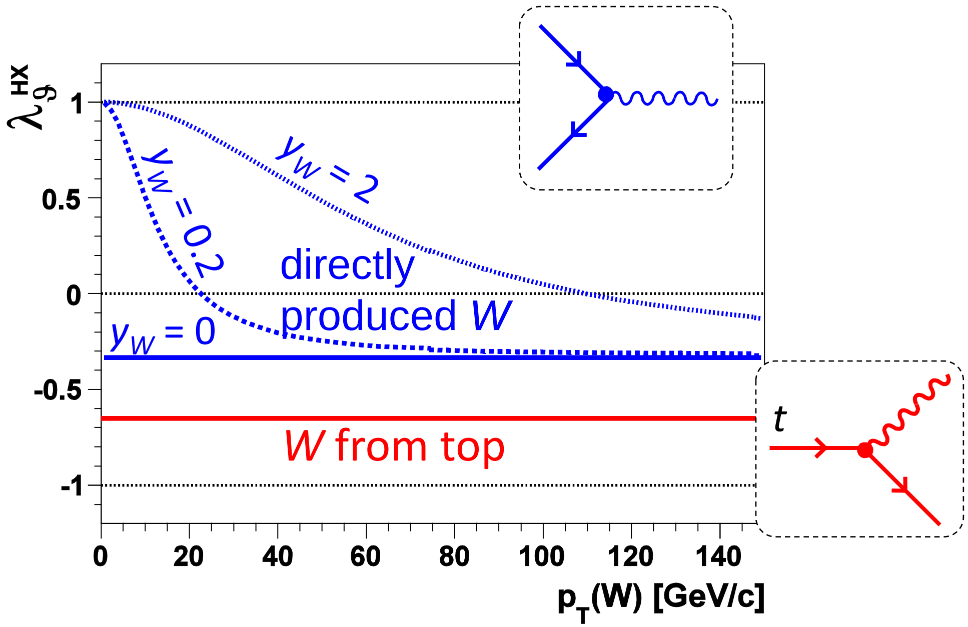
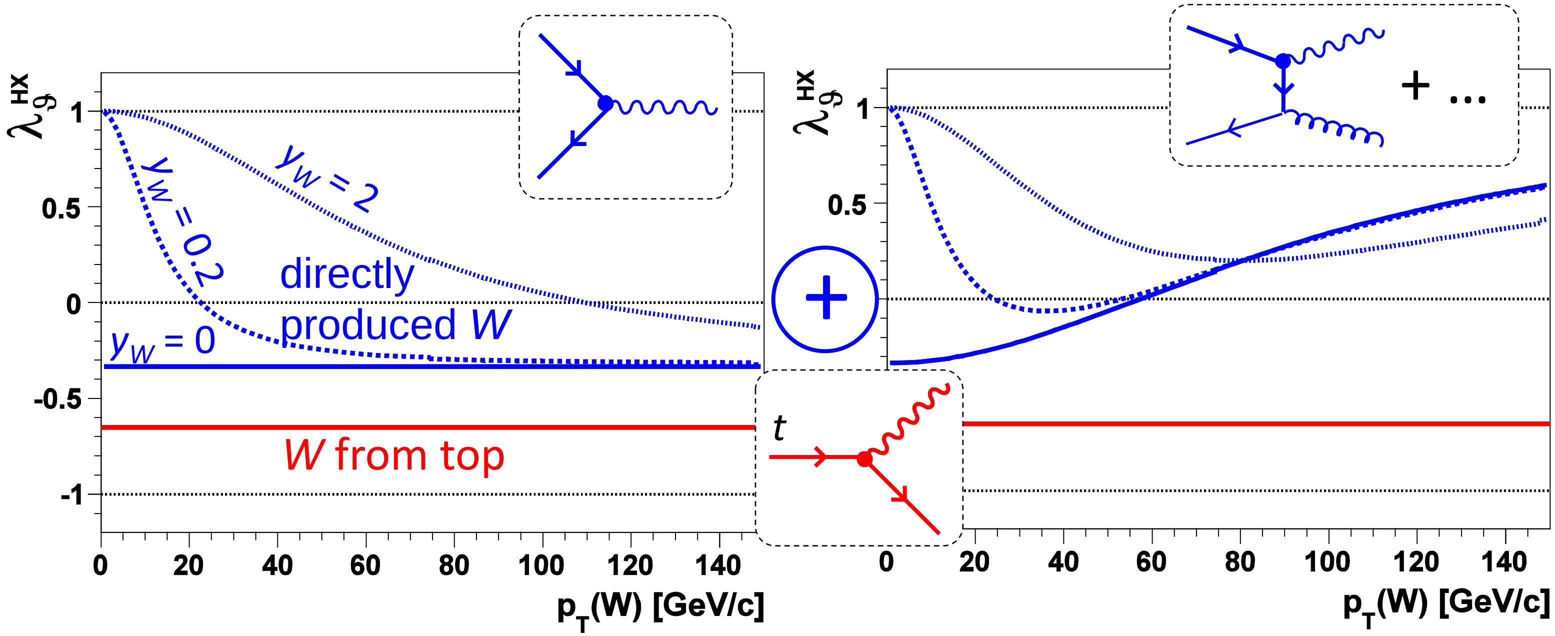


a) Frame-dependent approach

We measure λ_ϑ choosing the helicity axis



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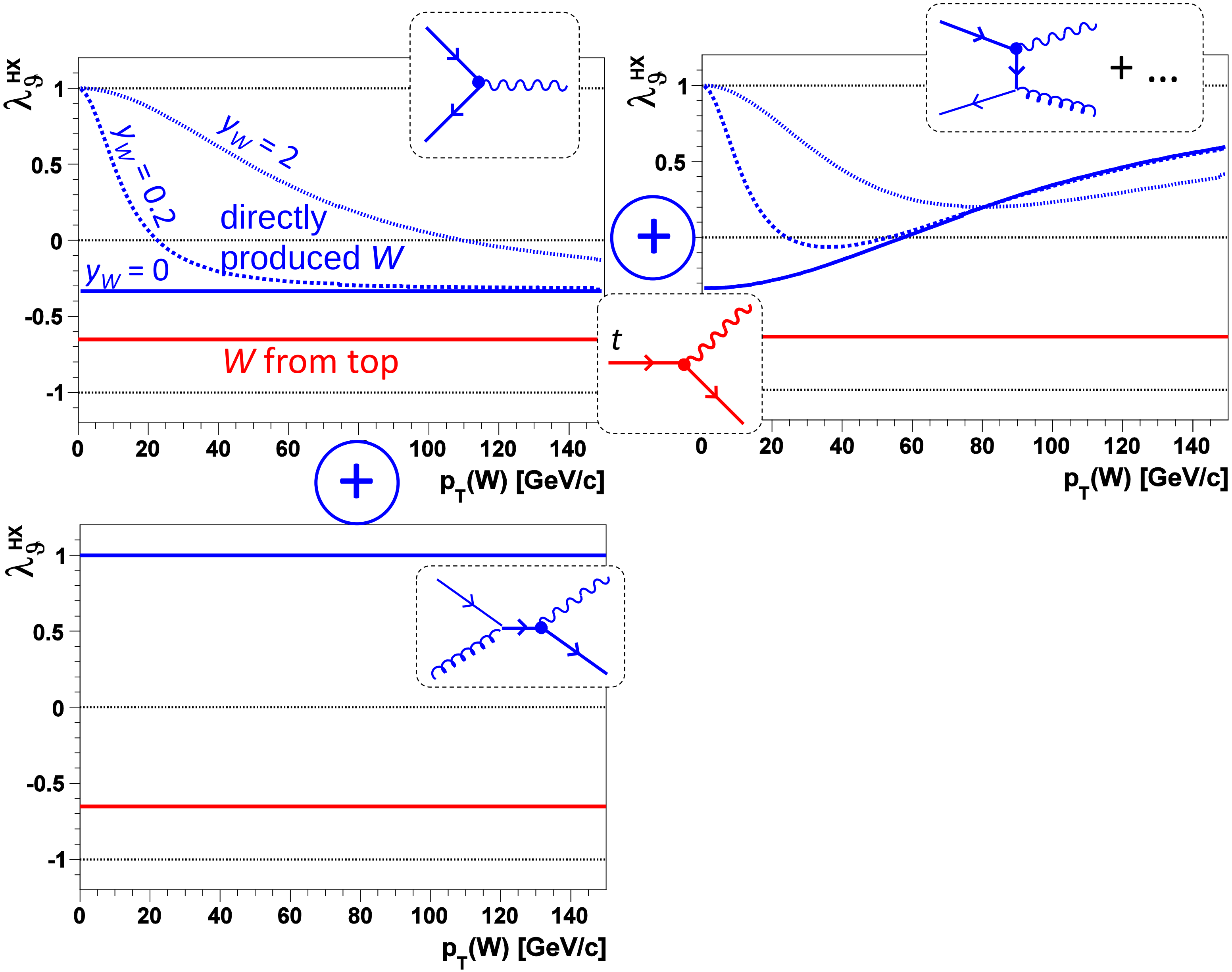

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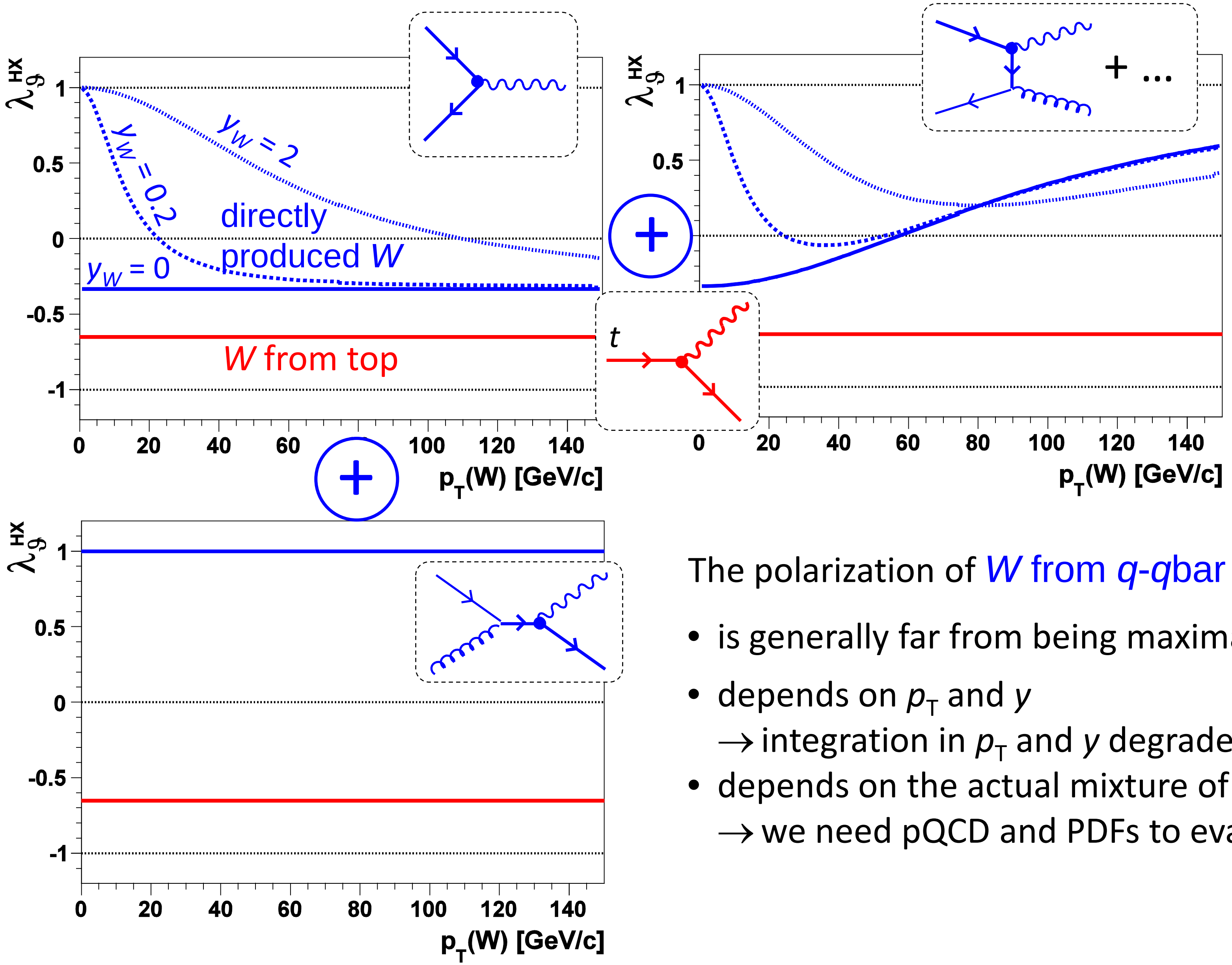
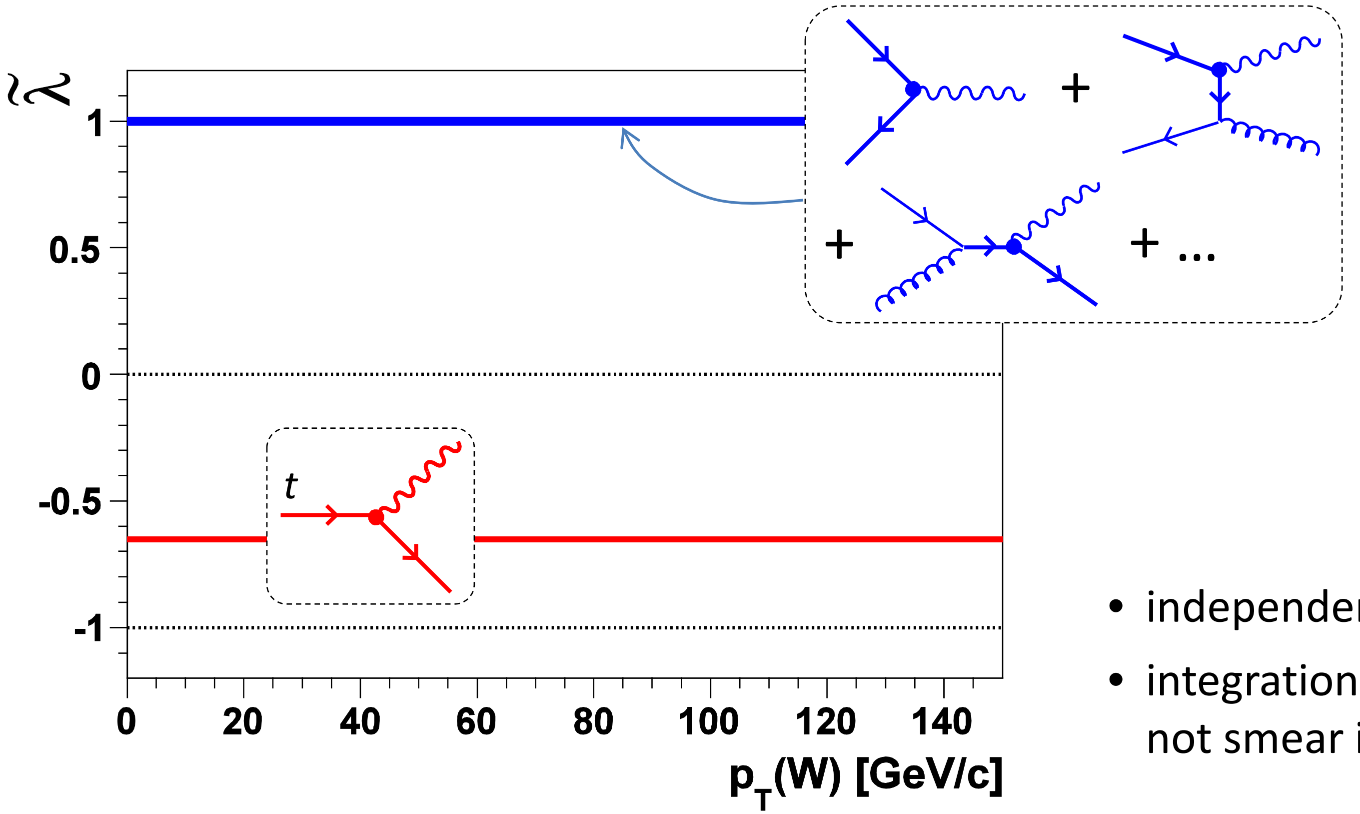


The polarization of *W from q-qbar / q-g*

- is generally far from being maximal
- depends on p_T and y
  → integration in p_T and y degrades significance
- depends on the actual mixture of processes
  → we need pQCD and PDFs to evaluate it

b) Rotation-invariant approach



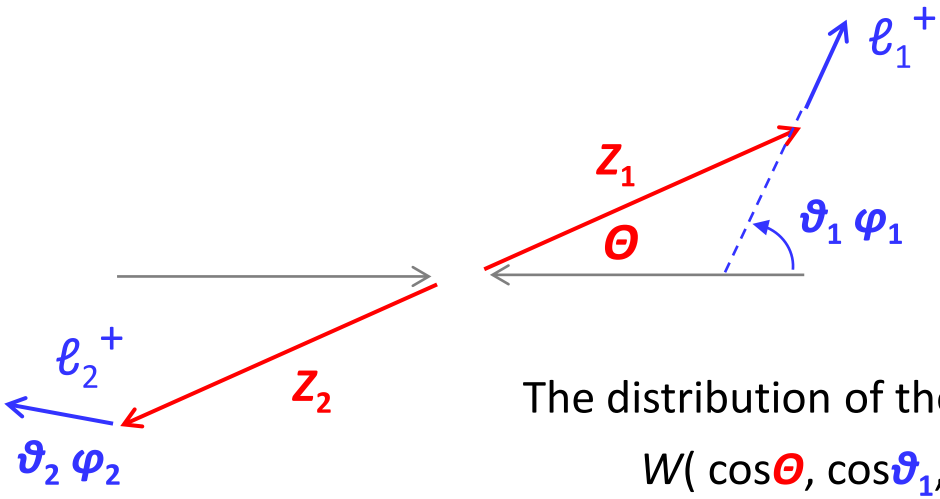
The *invariant* polarization of *W* from *q-qbar* / *q-g* is **constant** and fully **transverse**

- independent of PDFs
- integration over kinematics does not smear it

Example: the *q*-*q*bar → *ZZ* continuum background

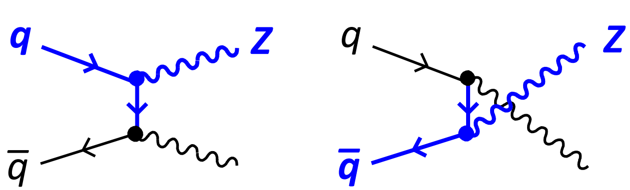
dominant Standard Model background for new-signal searches
in the $ZZ \to 4\ell$ channel with $m(ZZ) > 200$ GeV/c^2

The new Higgs-like resonance was discovered also thanks to these techniques



The distribution of the **5 angles** depends on the **kinematics**

$$W(\cos\Theta, \cos\vartheta_1, \varphi_1, \cos\vartheta_2, \varphi_2 \mid M_{ZZ}, \vec{p}(Z_1), \vec{p}(Z_2))$$

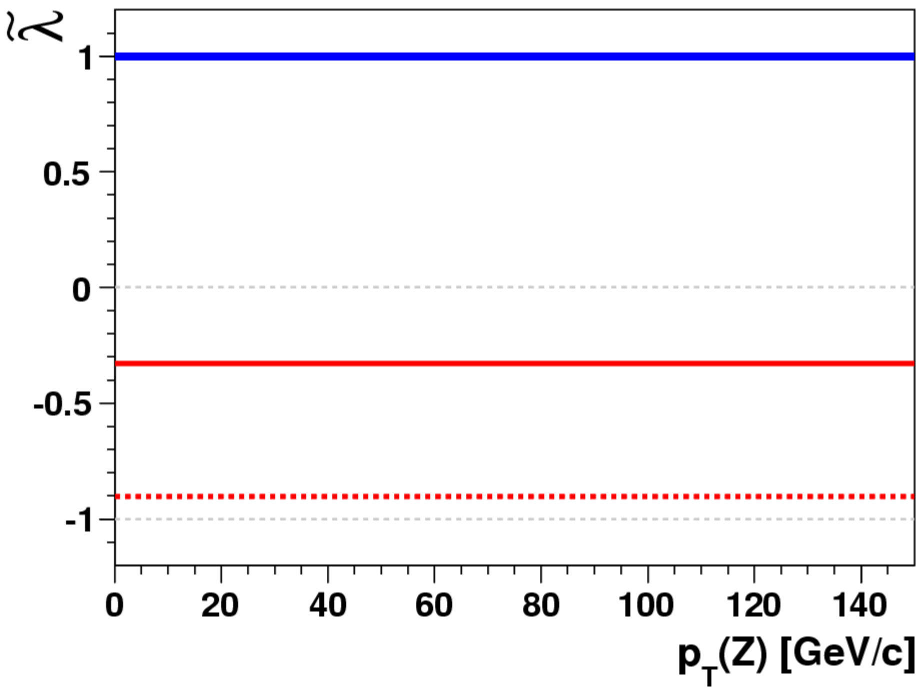


t-channel u-channel

- for helicity conservation each of the two *Z's* is transverse along the direction of one or the other incoming quark
- t-channel and u-channel amplitudes are proportional to $\frac{1}{1-\cos\Theta}$ and $\frac{1}{1+\cos\Theta}$ for $M_Z/M_{ZZ} \to 0$

Z Z from Higgs ↔ *Z Z* from *q*-*q*bar

Discriminant nº1: ***Z* polarization**



Z Z from Higgs ↔ Z Z from q-qbar

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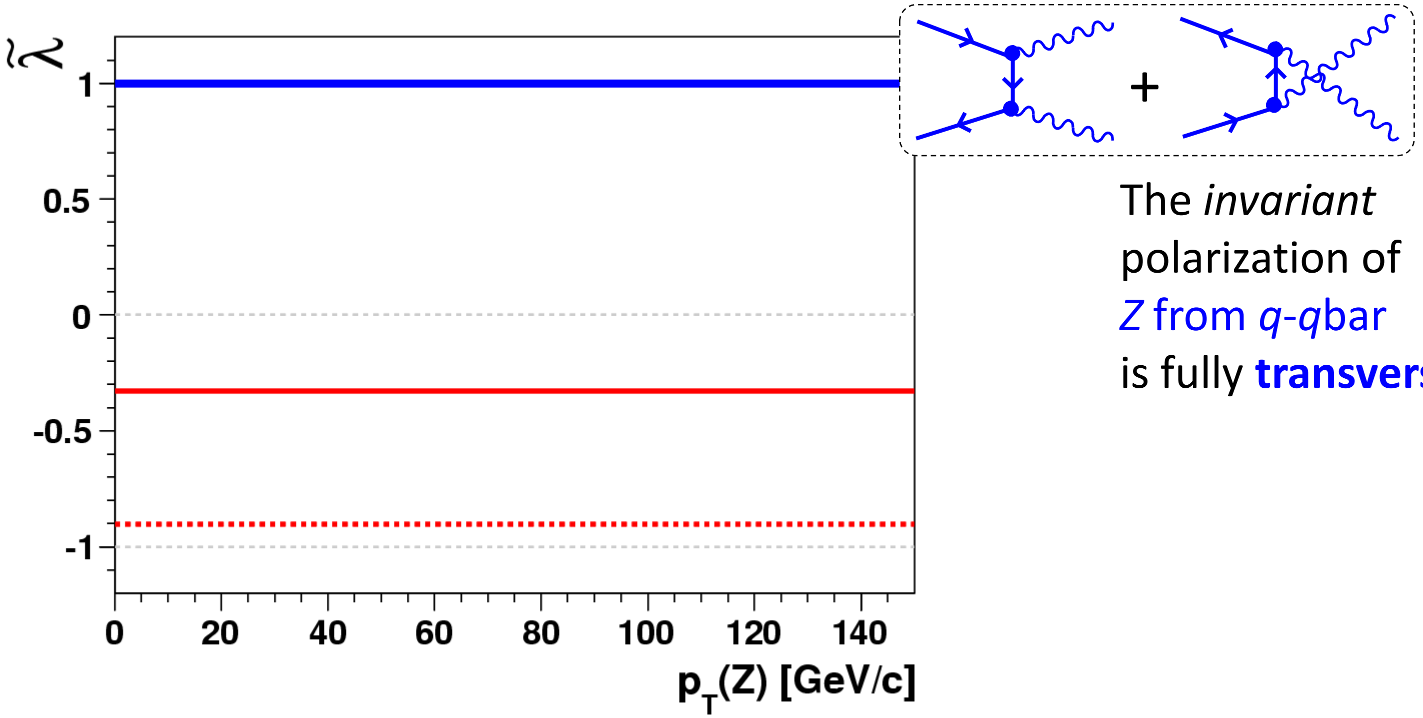

The *invariant* polarization of *Z* from *q*-*q*bar is fully **transverse**

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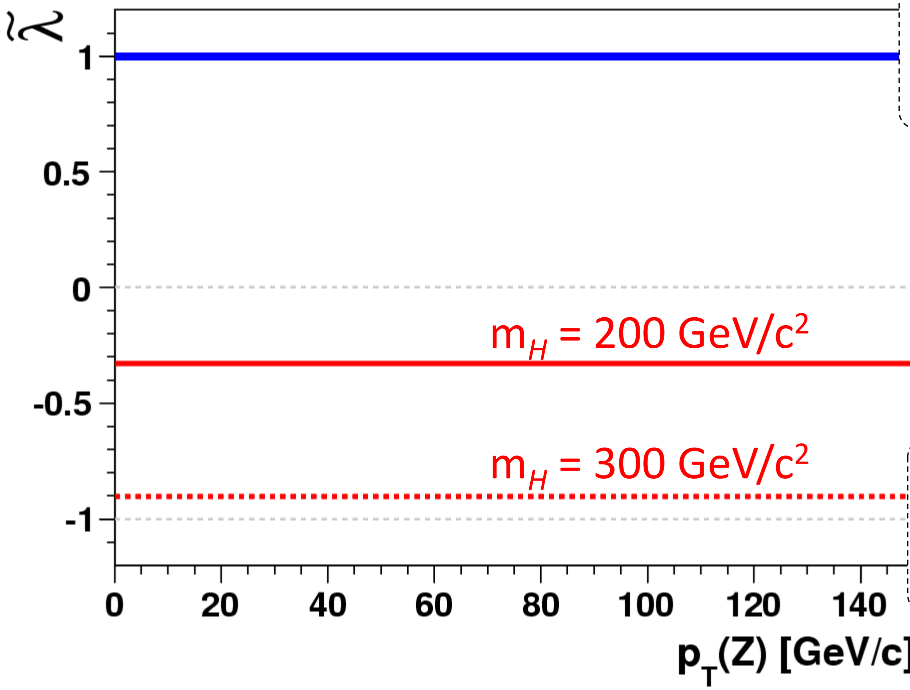

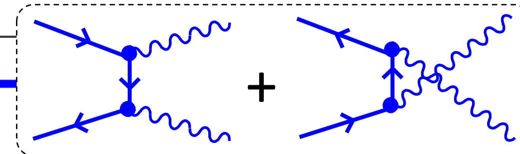

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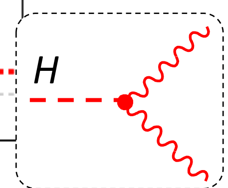
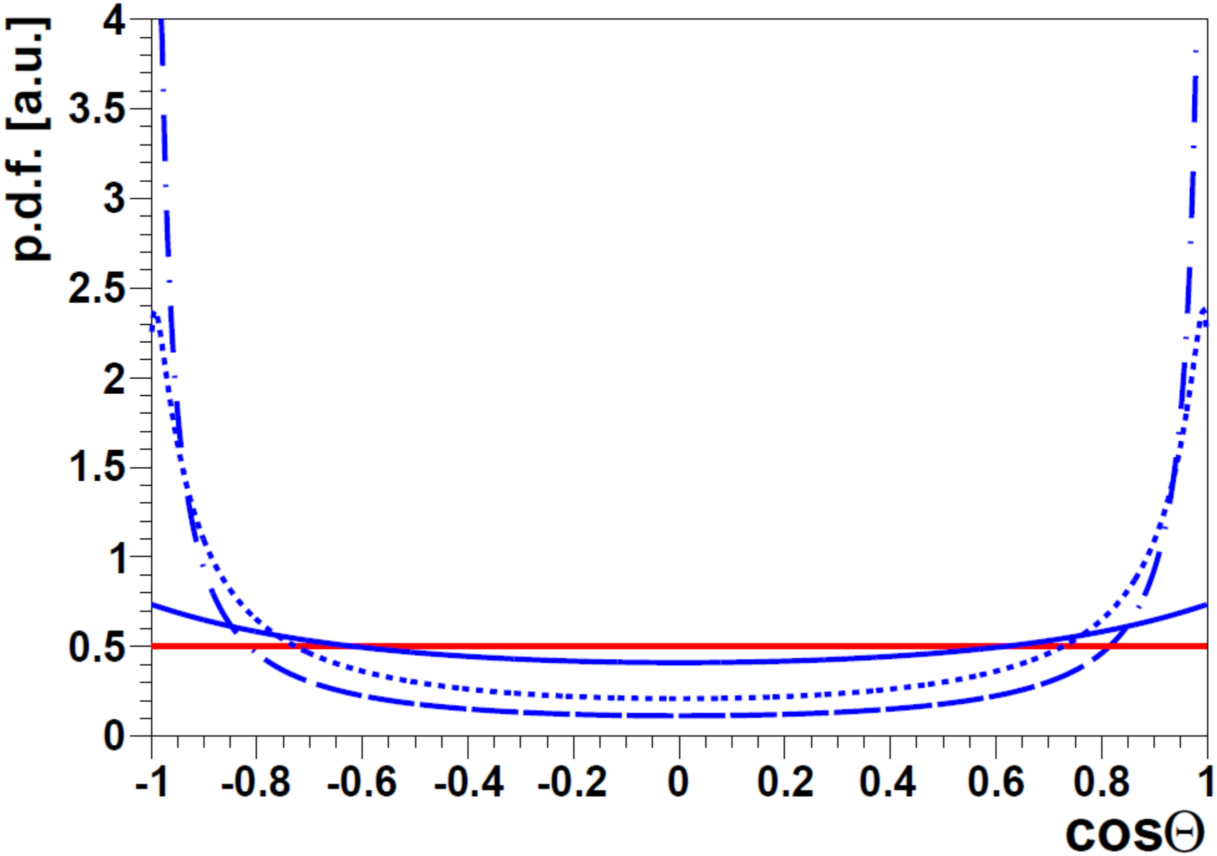


Z bosons from *H* → *ZZ* are **longitudinally** polarized (stronger polarization for heavier *H*)

Z Z from Higgs ↔ *Z Z* from *q*-*q*bar

Discriminant nº2: ***Z* emission direction**



Z Z from Higgs ↔ Z Z from *q*-*q*bar

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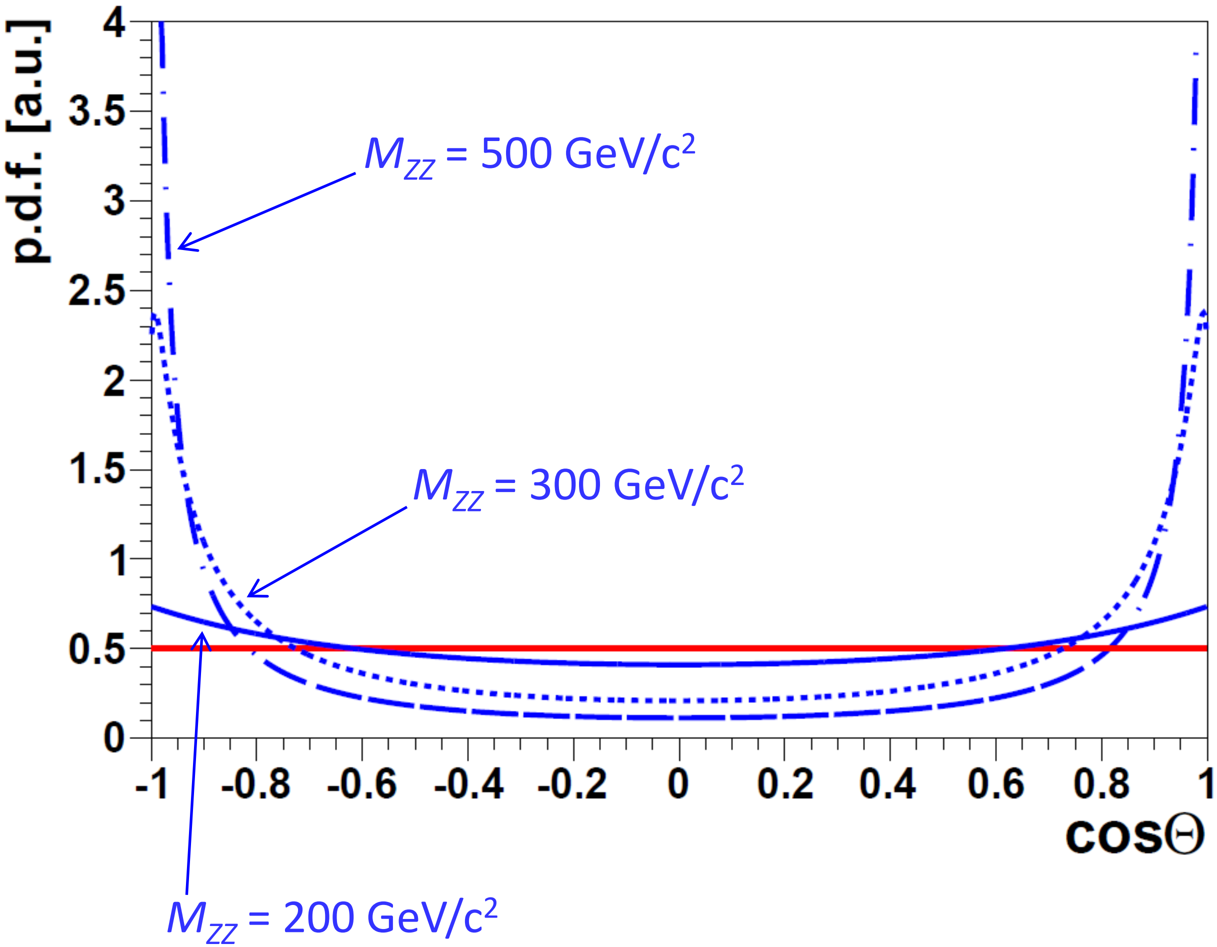
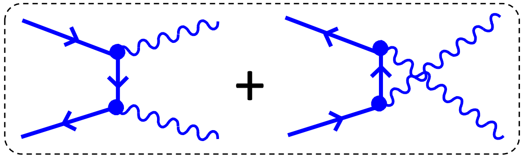
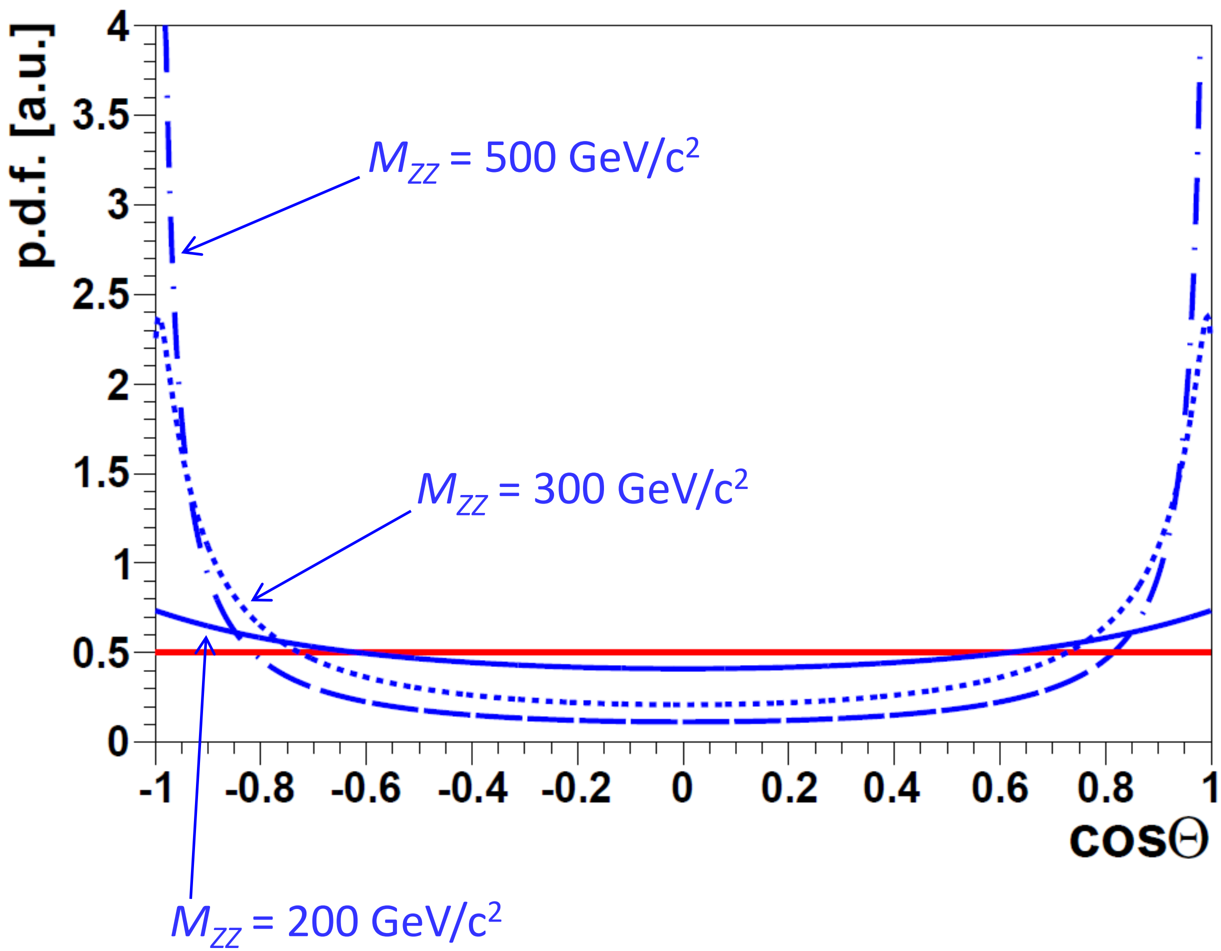


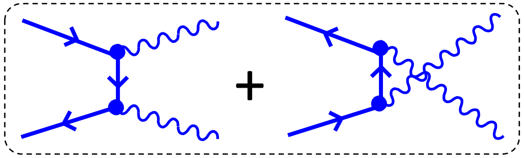


Z from *q*-*q*bar
is emitted mainly
close to the beam
if M_{ZZ}/M_Z is large

Z Z from Higgs ↔ Z Z from q-qbar

Discriminant nº2: ***Z* emission direction**





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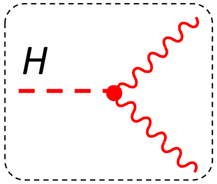

Z bosons from *H* decay are emitted isotropically

Putting everything together

5 angles ($\boldsymbol{\Theta}$, $\boldsymbol{\vartheta_1}$, $\boldsymbol{\varphi_1}$, $\boldsymbol{\vartheta_2}$, $\boldsymbol{\varphi_2}$), with distribution depending on
5 kinematic variables (M_{ZZ}, $p_T(Z_1)$, $y(Z_1)$, $p_T(Z_2)$, $y(Z_2)$)

1 shape discriminant: $$\xi = \ln \frac{\mathcal{P}_{H\to ZZ}}{\mathcal{P}_{q\bar{q}\to ZZ}}$$

event probabilities, including detector acceptance and efficiency effects

Putting everything together

5 angles (**Θ, ϑ_1, φ_1, ϑ_2, φ_2**), with distribution depending on
5 kinematic variables (M_{ZZ}, $p_T(Z_1)$, $y(Z_1)$, $p_T(Z_2)$, $y(Z_2)$)

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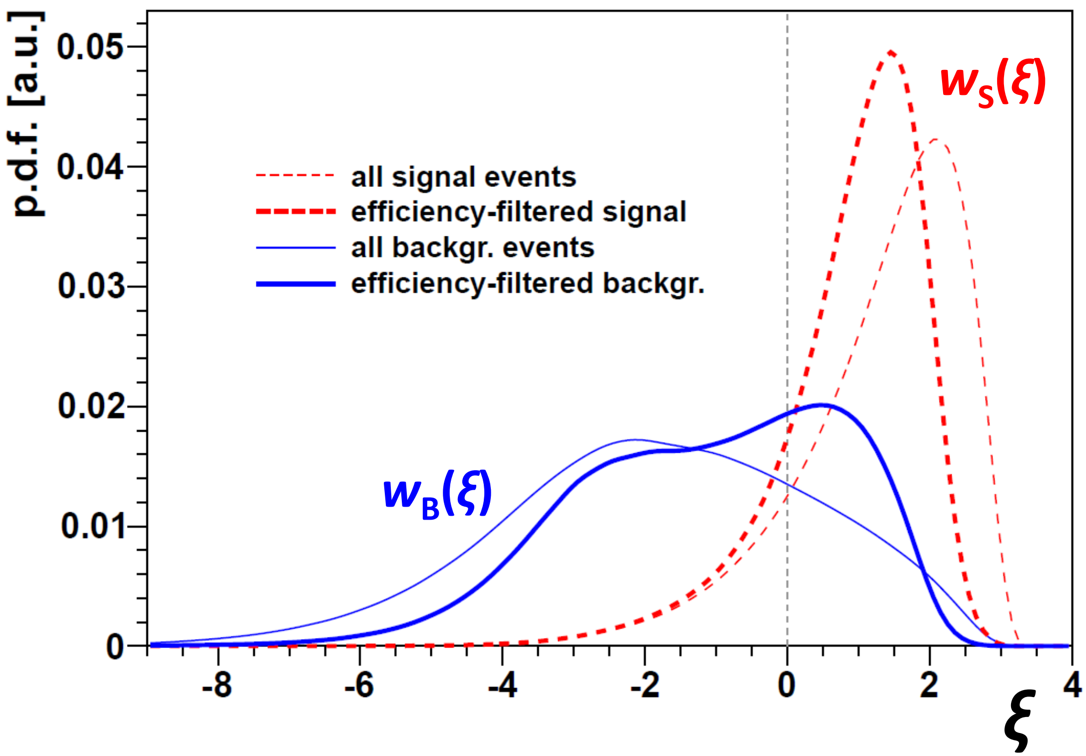

$\sqrt{s}$ = 14 TeV
500 < M_{ZZ} < 900 GeV/c^2
M_H = 700 GeV/c^2
$|y_{ZZ}|$ < 2.5

lepton selection:
p_T > 15 GeV/c
$|\eta|$ < 2.5

β = ratio of observed / expected signal events

$\beta > 0 \longrightarrow$ observation of something new

$\beta < 1 \longrightarrow$ exclusion of expected hypothetical signal

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"integrated yield" constraint: signal = excess yield wrt expected **number of BG events**

1) $$\mathcal{P}_{\text{BGnorm}}(\boldsymbol{\beta}) \propto \frac{e^{-(\mu_B + \boldsymbol{\beta}\,\mu_S)}\,(\mu_B + \boldsymbol{\beta}\,\mu_S)^N}{N!}$$

crucially dependent on the expected BG normalization

μ_B = avg. number of BG events expected for the given luminosity
μ_S = avg. number of Higgs events expected for the given luminosity
N = total number of events in the sample

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signal = deviation from the ***shape* of the BG angular distribution**

2) $$\mathcal{P}_{\text{angular}}(\boldsymbol{\beta}) \propto \prod_{i=1}^{N} \left(\frac{\mu_B}{\mu_B + \boldsymbol{\beta}\,\mu_S}\, w_B(\xi_i) + \frac{\boldsymbol{\beta}\,\mu_S}{\mu_B + \boldsymbol{\beta}\,\mu_S}\, w_S(\xi_i) \right)$$

*in*dependent of luminosity and cross-section uncertainties!

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*in*dependent of luminosity and cross-section uncertainties!

3) $$\mathcal{P}_{\text{tot}}(\boldsymbol{\beta}) = \mathcal{P}_{\text{angular}}(\boldsymbol{\beta}) \times \mathcal{P}_{\text{BGnorm}}(\boldsymbol{\beta})$$

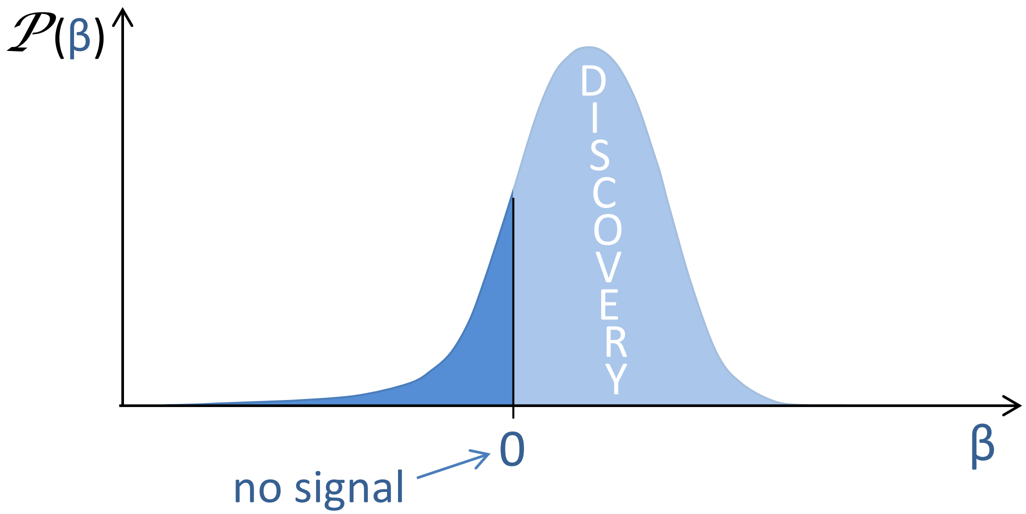
combination of the two methods

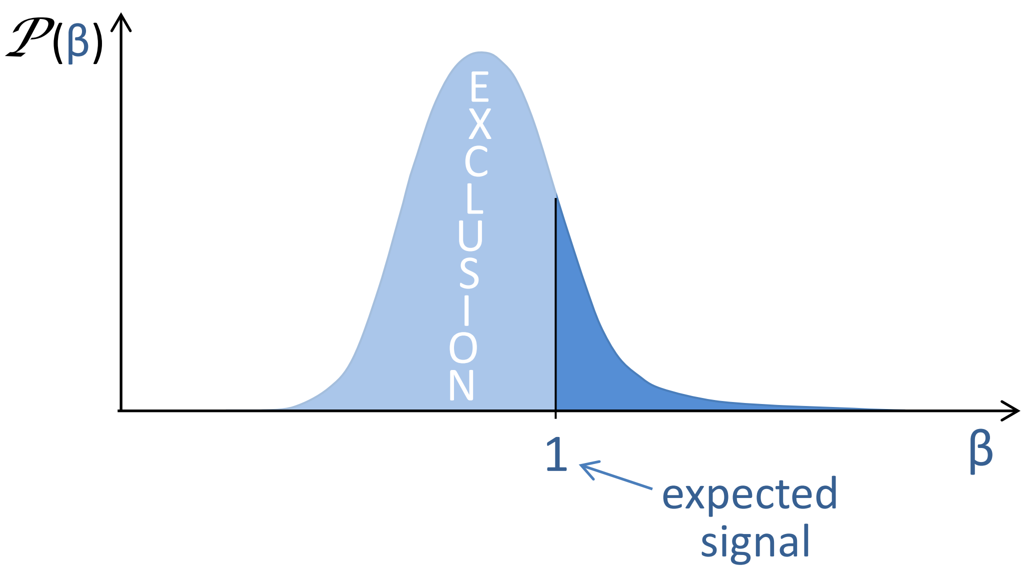
μ_B = avg. number of BG events expected for the given luminosity

μ_S = avg. number of Higgs events expected for the given luminosity

N = total number of events in the sample

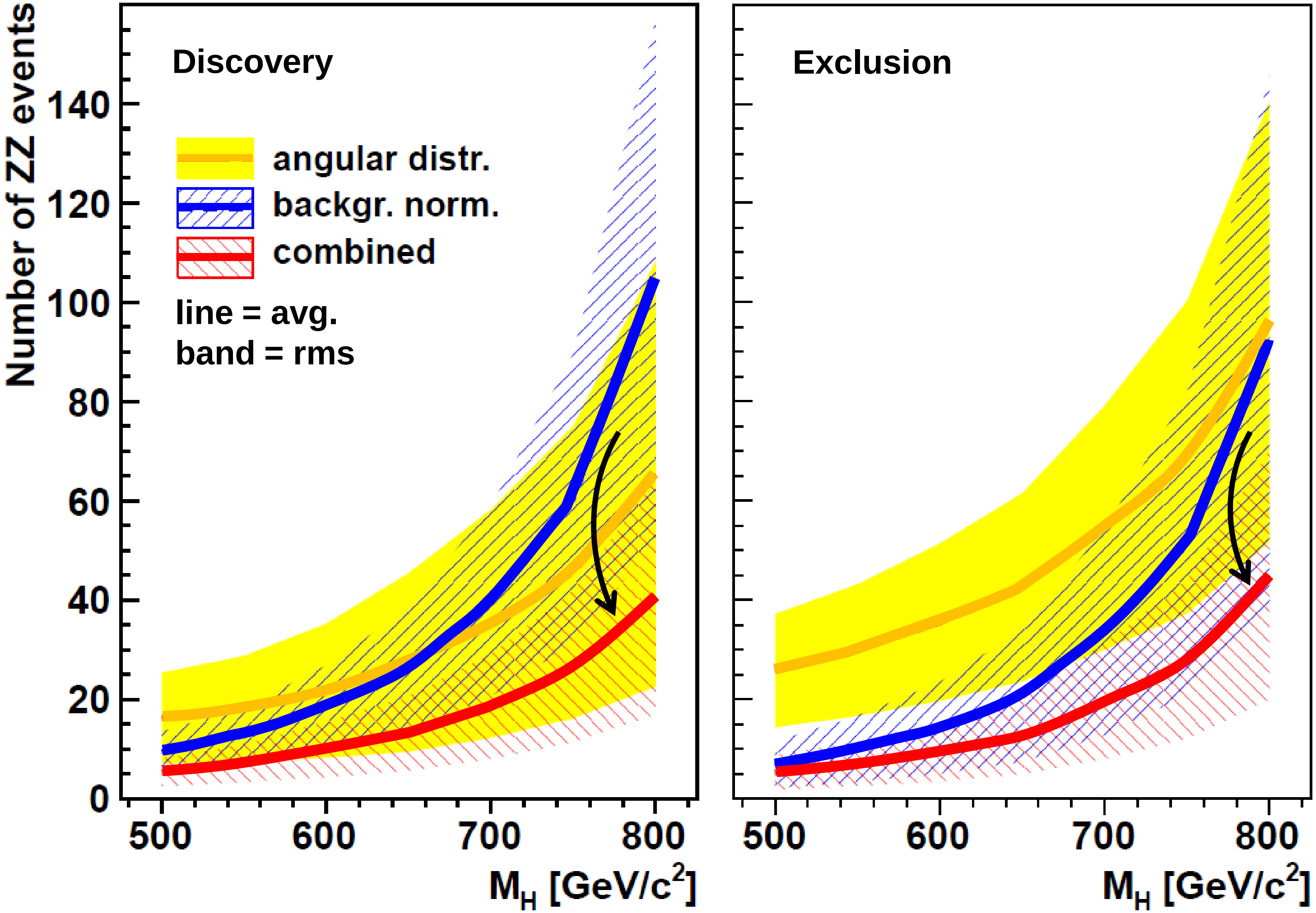
Confidence levels





Limits vs m_H

"3σ" level



Variation with mass essentially due to varying BG level:
30% for m_H = 500 GeV/c^2 → 70% for m_H = 800 GeV/c^2
Angular method more advantageous with higher BG levels

Further reading

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- P. Faccioli and J. Seixas, *Observation of χ_c and χ_b nuclear suppression via dilepton polarization measurements*, Phys. Rev. D 85, 074005 (2012)
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