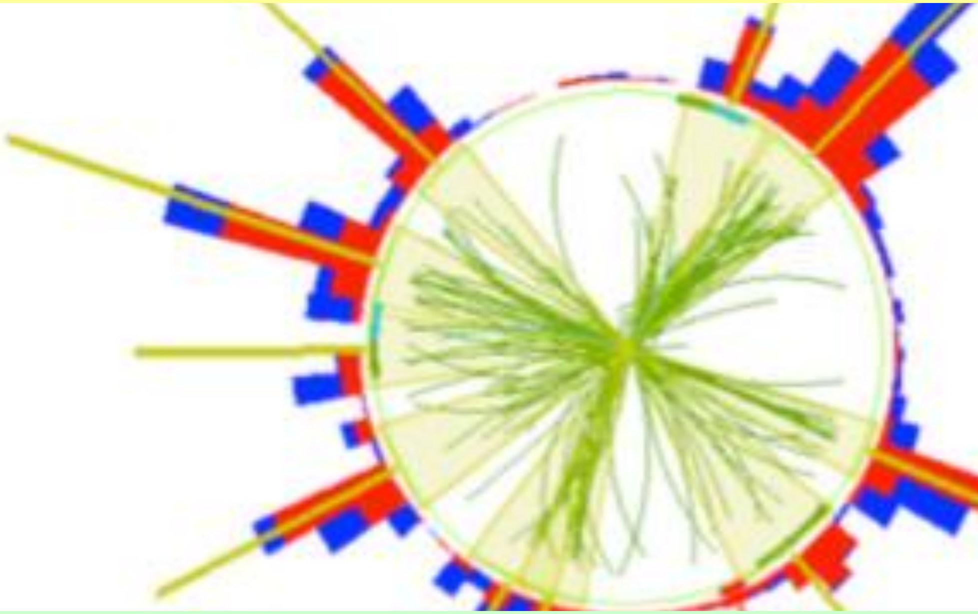
Beyond Feynman Diagrams



Lance Dixon

Academic Training Lectures

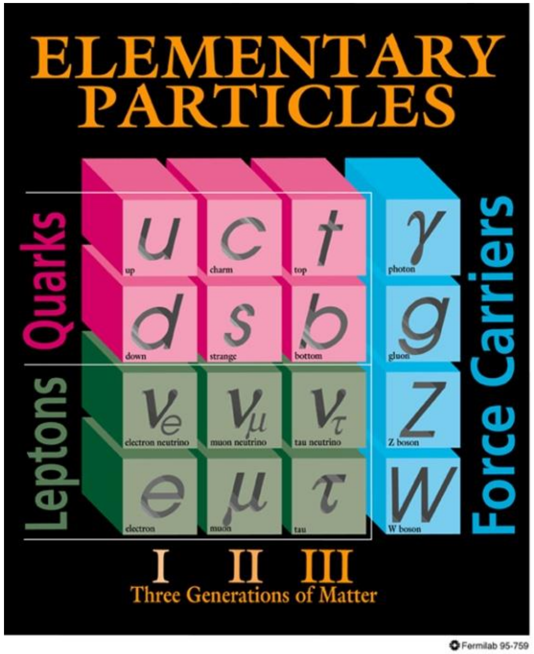
CERN

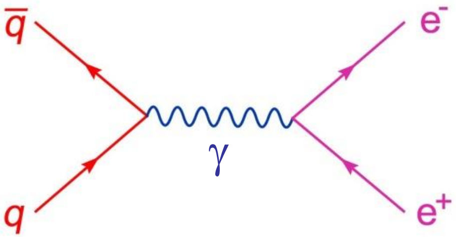
April 24-26, 2013

Standard Model

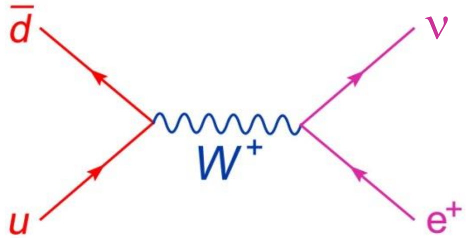
- All elementary forces except gravity in same basic framework
- Matter made of spin ½ fermions
- Forces carried by spin 1 vector bosons: γ $W^+ W^- Z^0$ g
- If we add a spin 0 Higgs boson *H* to explain masses of $W^+ W^- Z^0$

  → finite, testable predictions for all quantities – in principle

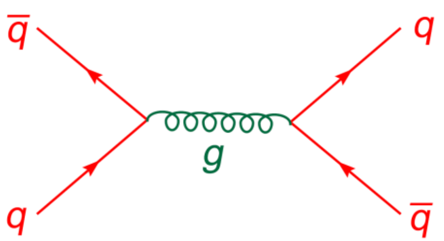




electromagnetism (QED)



weak



strong (QCD)

New Physics at LHC

- Many theories predict a host of new massive particles with masses similar to the W and Z bosons, within reach of the LHC, often including a **dark matter candidate:**
  - **supersymmetry**
  - **new dimensions of space-time**
  - **new forces**
  - etc.

- Most new massive particles decay rapidly to old, ~massless particles:
  quarks, gluons, charged leptons, neutrinos, photons

- **How to distinguish new physics from old (Standard Model)?**
- **From other types of new physics?**

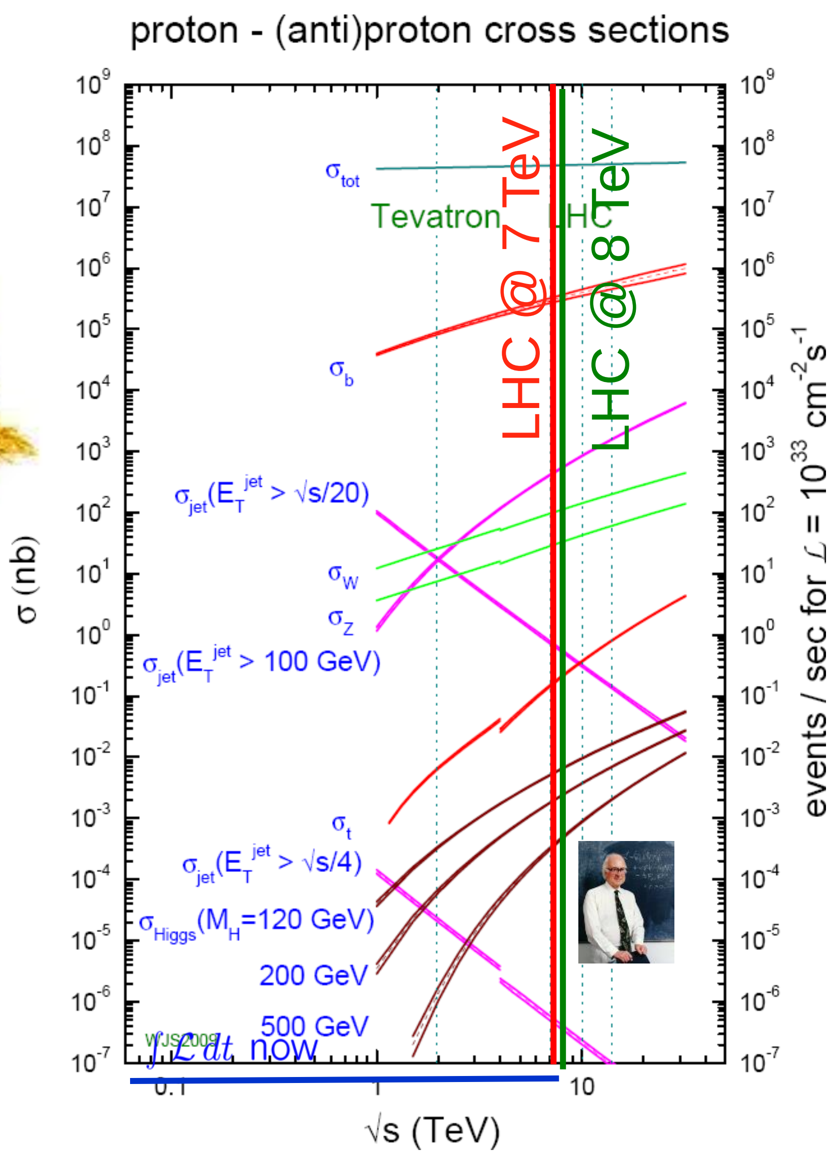
Signals vs. Backgrounds

electron-positron colliders
– small backgrounds

vs.

hadron colliders
– large backgrounds

LHC Data Dominated by Jets



Jets from quarks and gluons.

- *q,g* from decay of new particles?
- Or from old QCD?

- Every process shown also with one more jet at ~ 1/5 the rate
- Need **accurate production rates** for
  X + *1,2,3,…* jets
  in Standard Model

new physics →

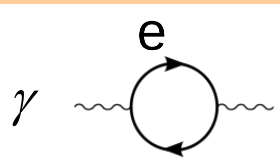
“New physics at the LHC is a riddle, wrapped in a mystery, inside an enigma; but perhaps there is a key.” -W. Churchill

The Key of Asymptotic Freedom

Gross, Wilczek, Politzer (1973)

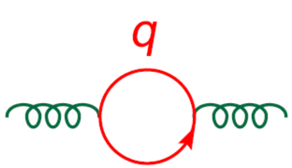
Quantum fluctuations of massless virtual particles polarize vacuum

QED: electrons screen charge (e larger at short distances)

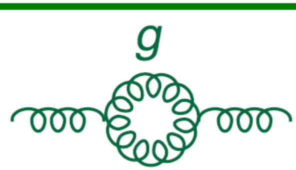


> 0 → $e^2(r) = \dfrac{e^2(r_0)}{1 + \frac{2e^2(r_0)}{3\pi} ln \frac{r}{r_0}}$

QCD: gluons anti-screen charge (g_s smaller at short distances)



> 0

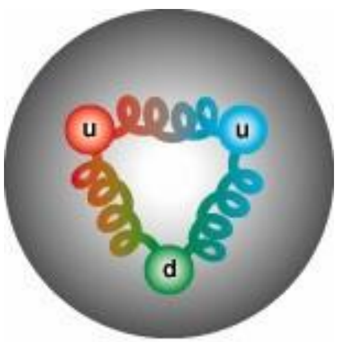


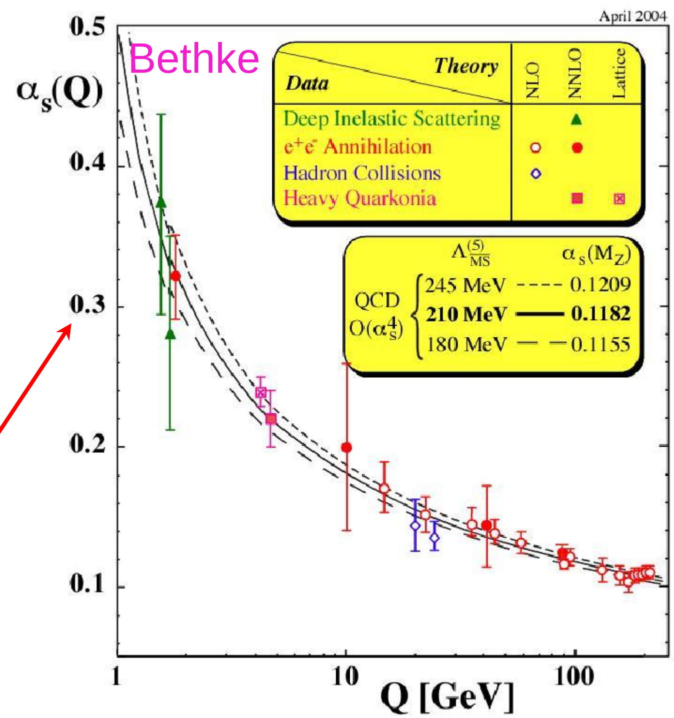
< 0

Gluon self-interactions make quarks almost free, and make QCD calculable at short distances (high energies): $\alpha_s \rightarrow$ **0** asymptotically

Short-distance calculability

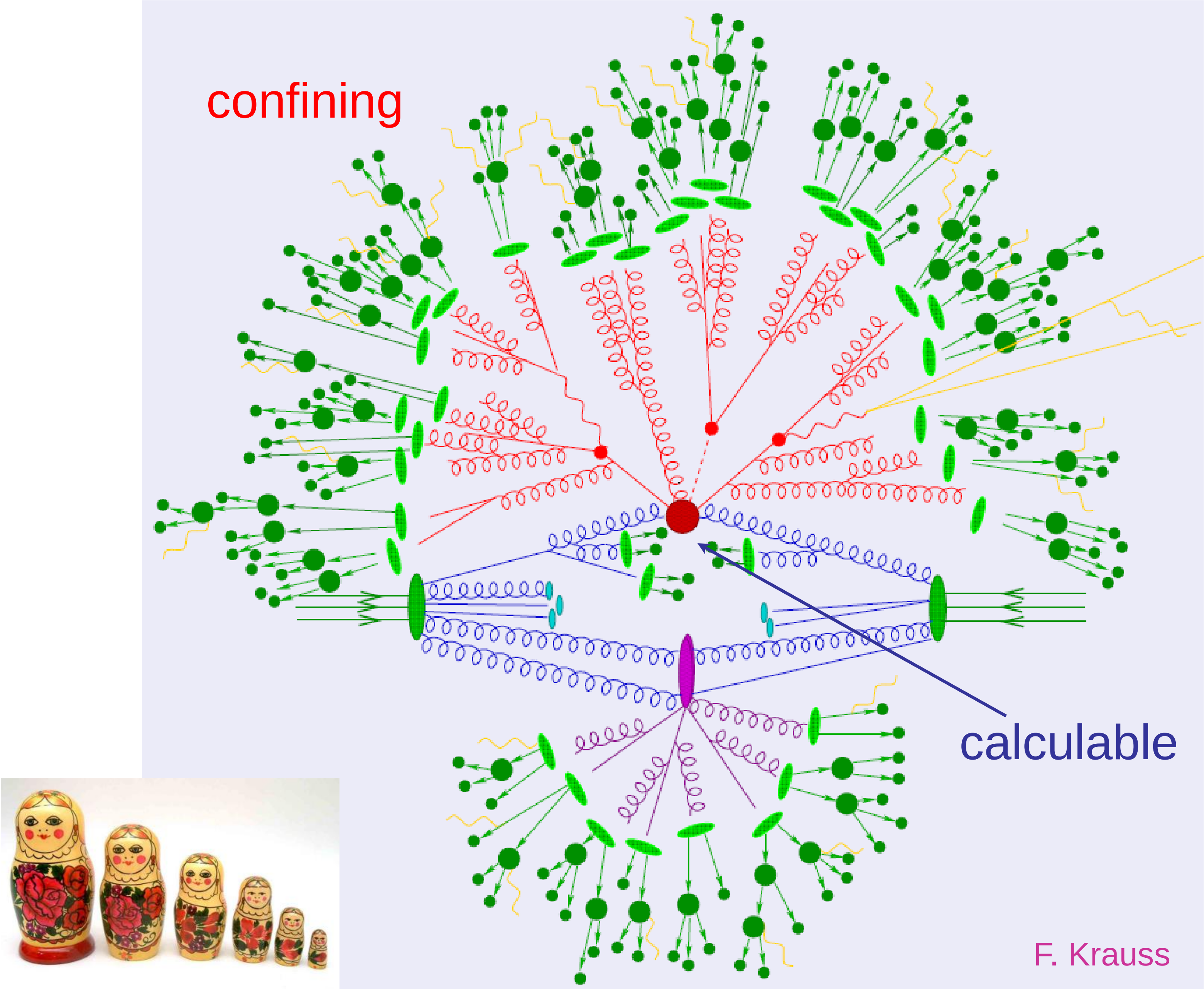
Running of α_s is *logarithmic*, *slow*
at short distances (large Q)





confining

calculable


confining
calculable
F. Krauss

QCD Factorization & Parton Model

Academic Training Lectures by Aude Gehrmann-de Ridder: May 22-24

At short distances, quarks and gluons (partons) in proton are almost free. Sampled "one at a time"

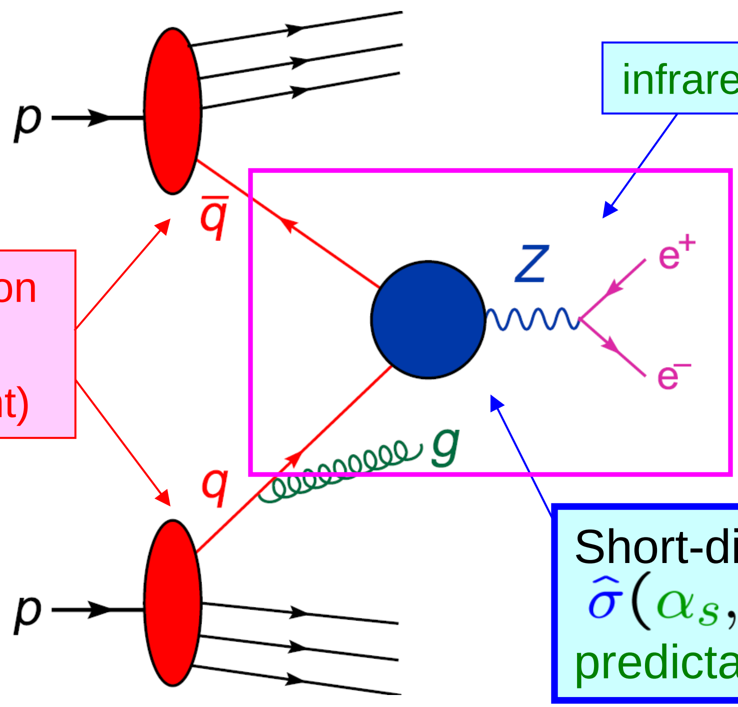
infrared safe final state

The "femto-universe"

size = factorization scale μ_F

("arbitrary")

Parton distribution functions (from experiment)



Short-distance cross section

$\hat{\sigma}(\alpha_s, \mu_F, \mu_R)$

predictable using perturbative QCD

Renorm. scale ("arbitrary")

Short-Distance Cross Section in Perturbation Theory

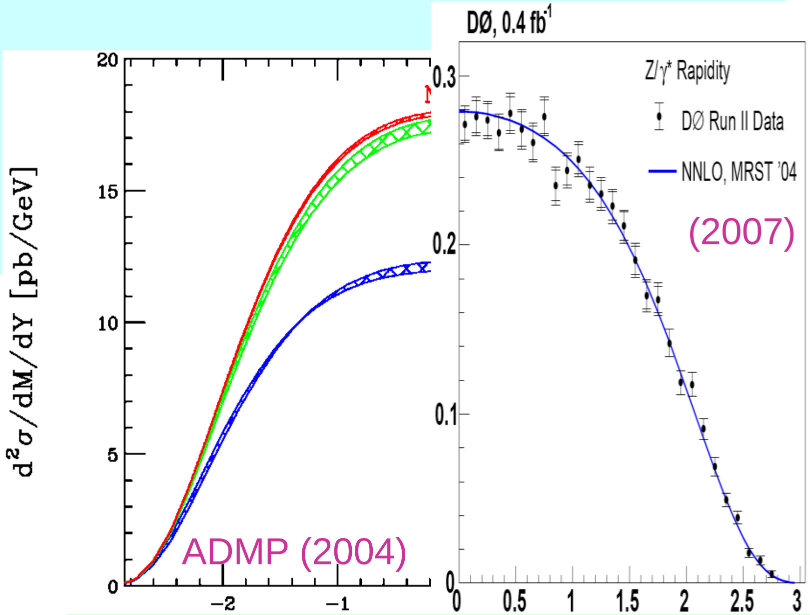
$$\hat{\sigma}(\alpha_s, \mu_F, \mu_R) \;=\; [\alpha_s(\mu_R)]^{n_\alpha}\left[\hat{\sigma}^{(0)} + \frac{\alpha_s}{2\pi}\hat{\sigma}^{(1)}(\mu_F,\mu_R) + \left(\frac{\alpha_s}{2\pi}\right)^2 \hat{\sigma}^{(2)}(\mu_F,\mu_R) + \cdots\right]$$

LO NLO NNLO

- **Problem:** Leading-order (LO) predictions only qualitative due to **poor convergence** of expansion in $\alpha_s(\mu)$
- Estimate "error" bands by varying $\mu_R = \mu_F = \mu$

Example: *Z* production at Tevatron as function of rapidity Y (~polar angle)

50% shift, LO → NLO

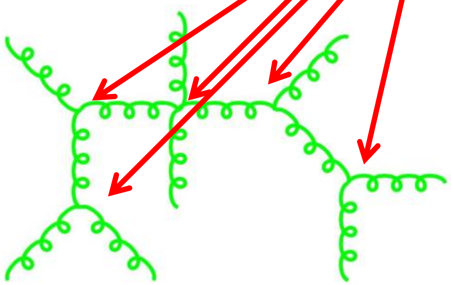


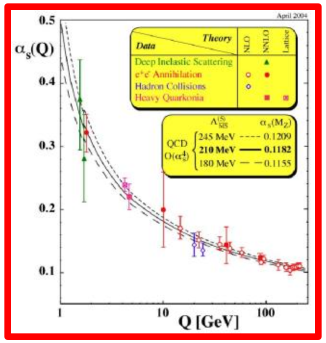
by NNLO, a precision observable

LO uncertainty increases with number of jets

$$\hat{\sigma}(\alpha_s, \mu_F, \mu_R) = [\alpha_s(\mu_R)]^{n_\alpha} \left[\hat{\sigma}^{(0)} + \frac{\alpha_s}{2\pi} \hat{\sigma}^{(1)}(\mu_F, \mu_R) + \left(\frac{\alpha_s}{2\pi}\right)^2 \hat{\sigma}^{(2)}(\mu_F, \mu_R) + \cdots \right]$$

LO NLO NNLO





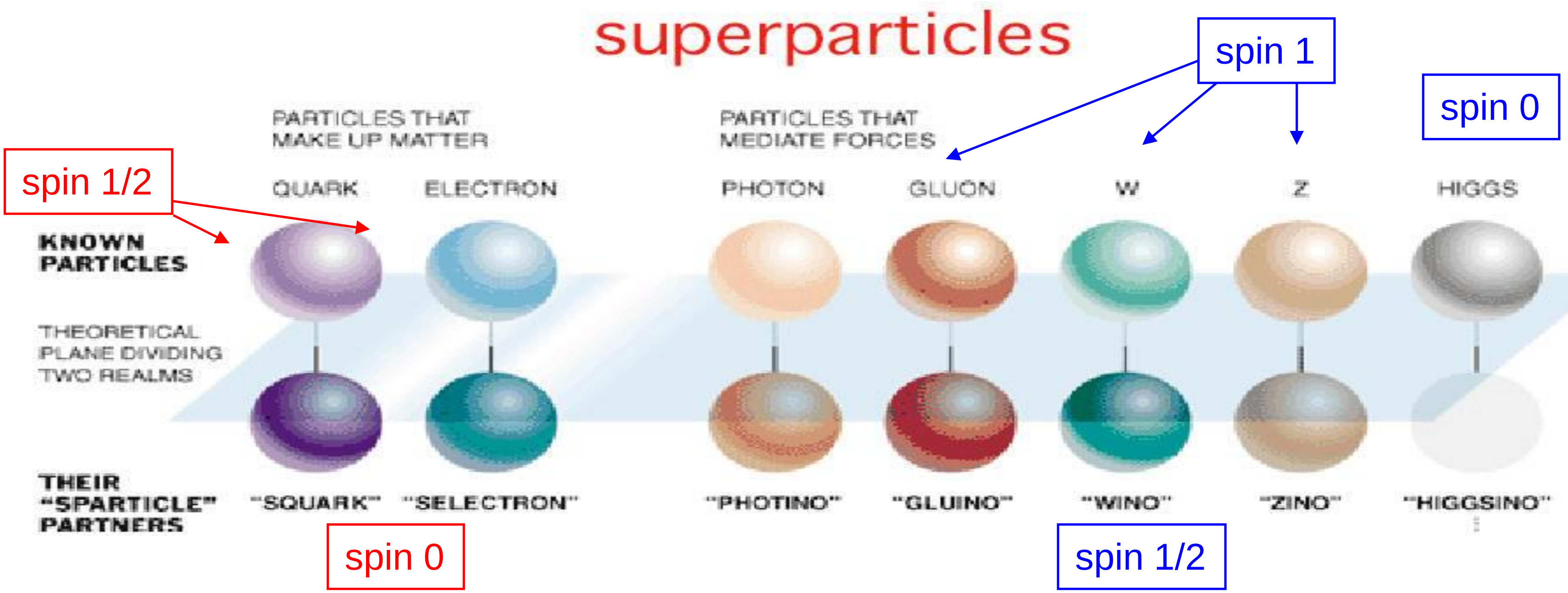
Uncertainty brought under much better control with NLO corrections: ~ 50% or more → ~ 15-20%

NLO required for quantitative control of multi-jet final states

Why Care About Multi-Jet Final States?

New Physics Example: Supersymmetry

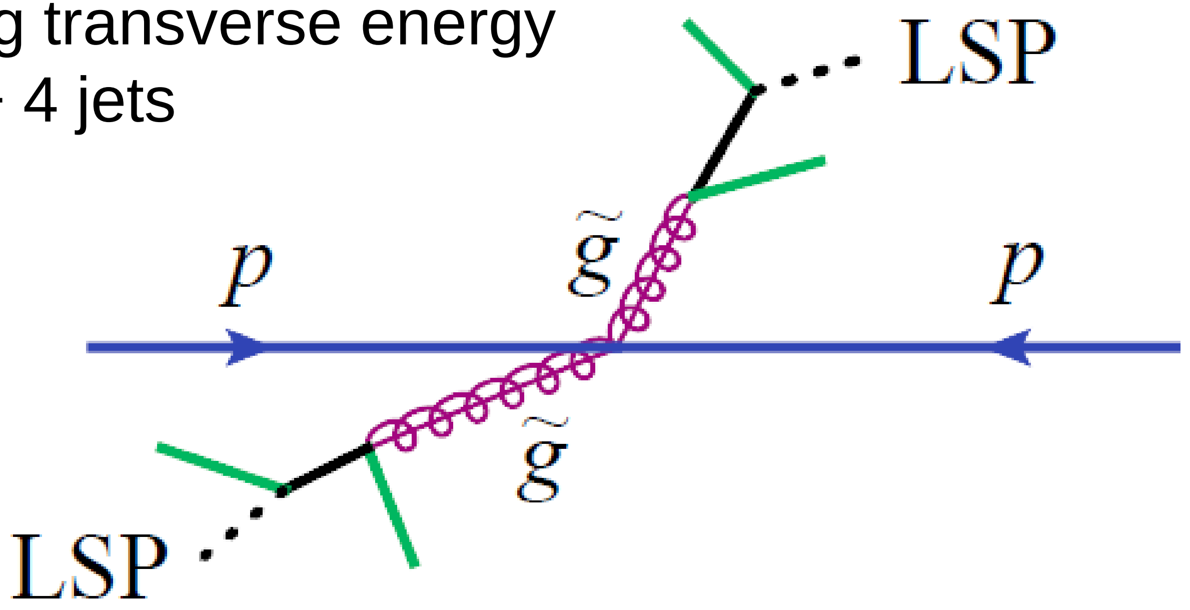
- Symmetry between fermions (matter) and bosons (forces)
- Very elegant, also solves theoretical puzzles
- Lightest supersymmetric particle (LSP) can be **dark matter**
- For every elementary particle already seen, another one should show up (soon?!?) at LHC



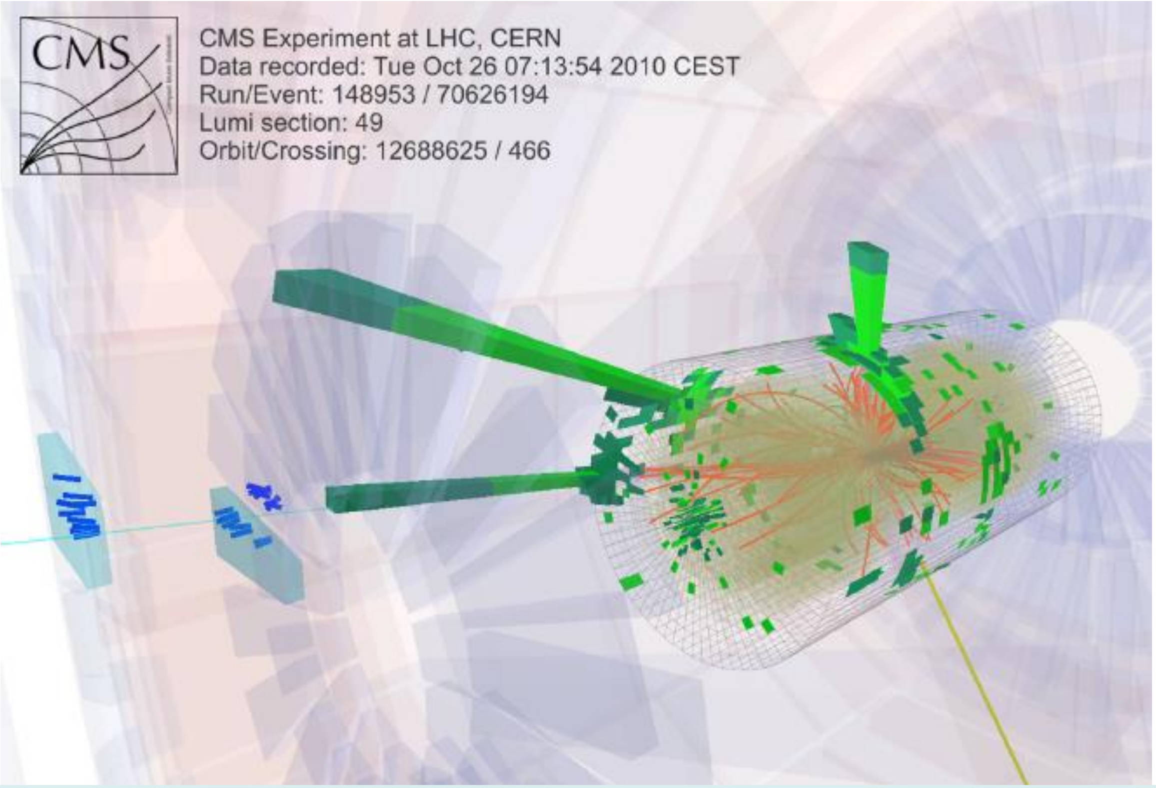
Classic SUSY dark matter signature

Heavy colored particles decay rapidly to stable Weakly Interacting Massive Particle (WIMP = LSP) plus jets

→ Missing transverse energy
MET + 4 jets



Is LHC already making dark matter?

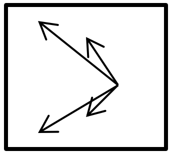


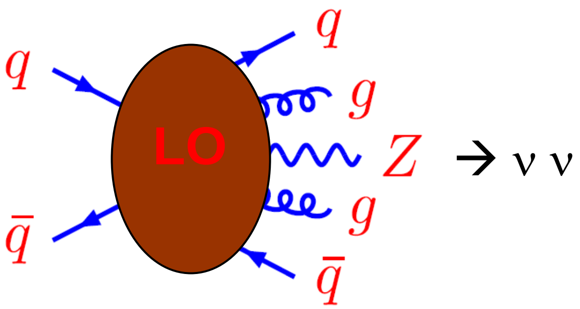
- 5 jets
- sum of jet transverse momenta H_T= 1132 GeV
- missing transverse energy H_{TMiss} = 693 GeV

No! Happens in Standard Model too

- **MET + 4 jets from**
  pp → Z + 4 jets,
  Z → neutrinos

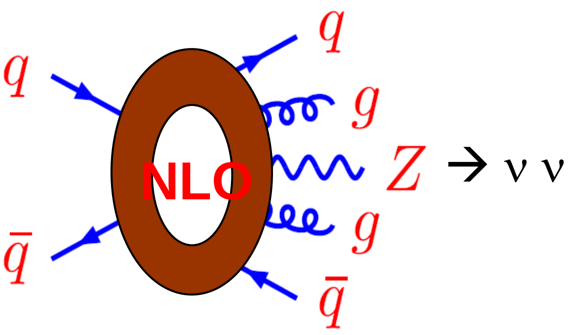
Neutrinos escape detector.
Irreducible background.

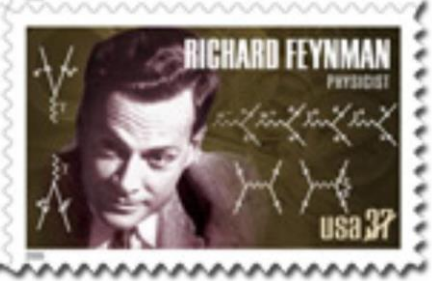




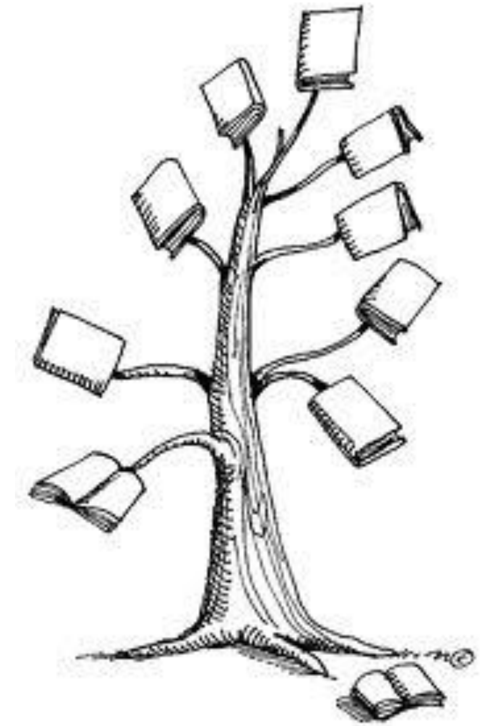
State of art for Z + 4 jets was based on **Leading Order (LO) approximation in QCD**
→ **normalization uncertain**

Now available at **Next to Leading Order**, greatly reducing theoretical uncertainties





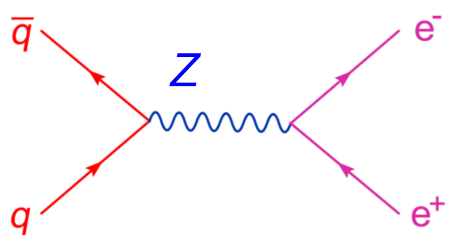
LO = Trees



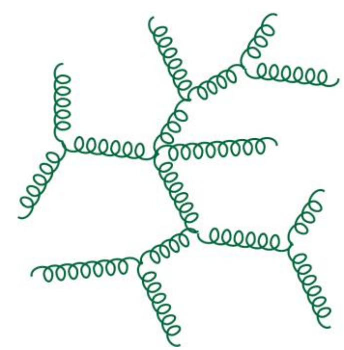
LO cross section uses only Feynman diagrams with no closed loops – tree diagrams.
Here's a very simple one:

$\hat{\sigma}^{(0)}$

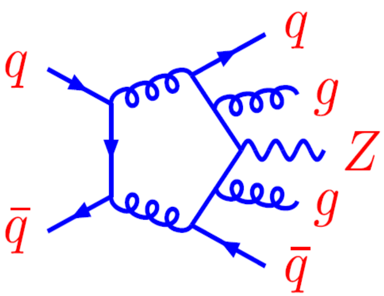
LO



Although there are many kinds of trees, some harder than others, "textbook" methods usually suffice



$+ \quad \ldots$



NLO = Loops

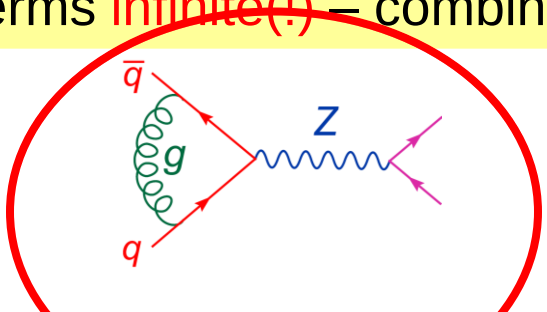
NLO cross section needs Feynman diagrams with exactly one closed loop

Where the fun really starts – textbook methods quickly fail, even with very powerful computers

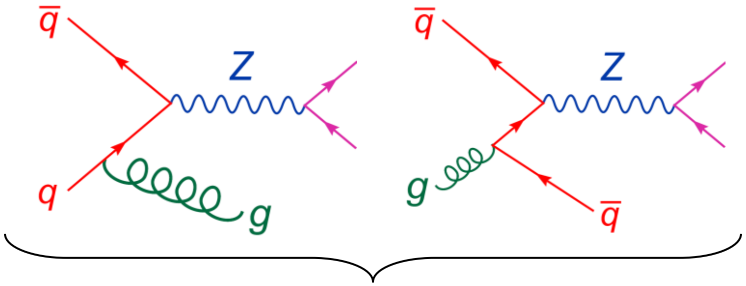
- NLO also needs tree-level amplitudes with one more parton
- Both terms infinite(!) – combine them to get a finite result

$\hat{\sigma}^{(1)}$

NLO



1 loop

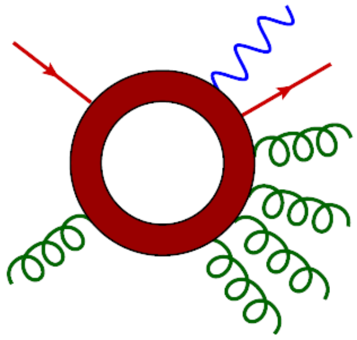


tree + 1 parton

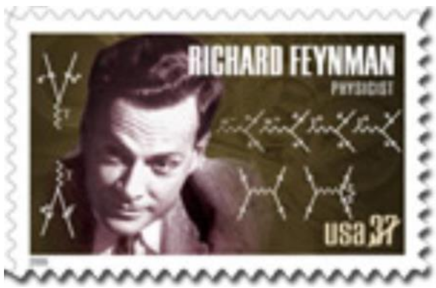
- One-loop amplitudes were the bottleneck for a long time
– focus today on this part of the problem

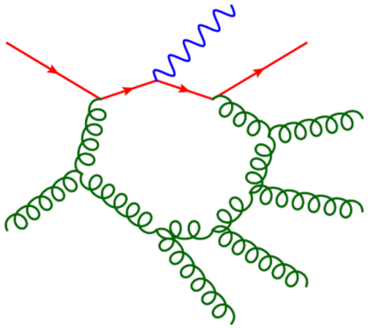
A Better Way to Compute?

- Backgrounds (and many signals) at NLO require one-loop scattering amplitudes for many ultra-relativistic ("massless") particles – especially quarks and gluons of QCD



- Long ago, **Feynman** told us how to do this **– in principle**



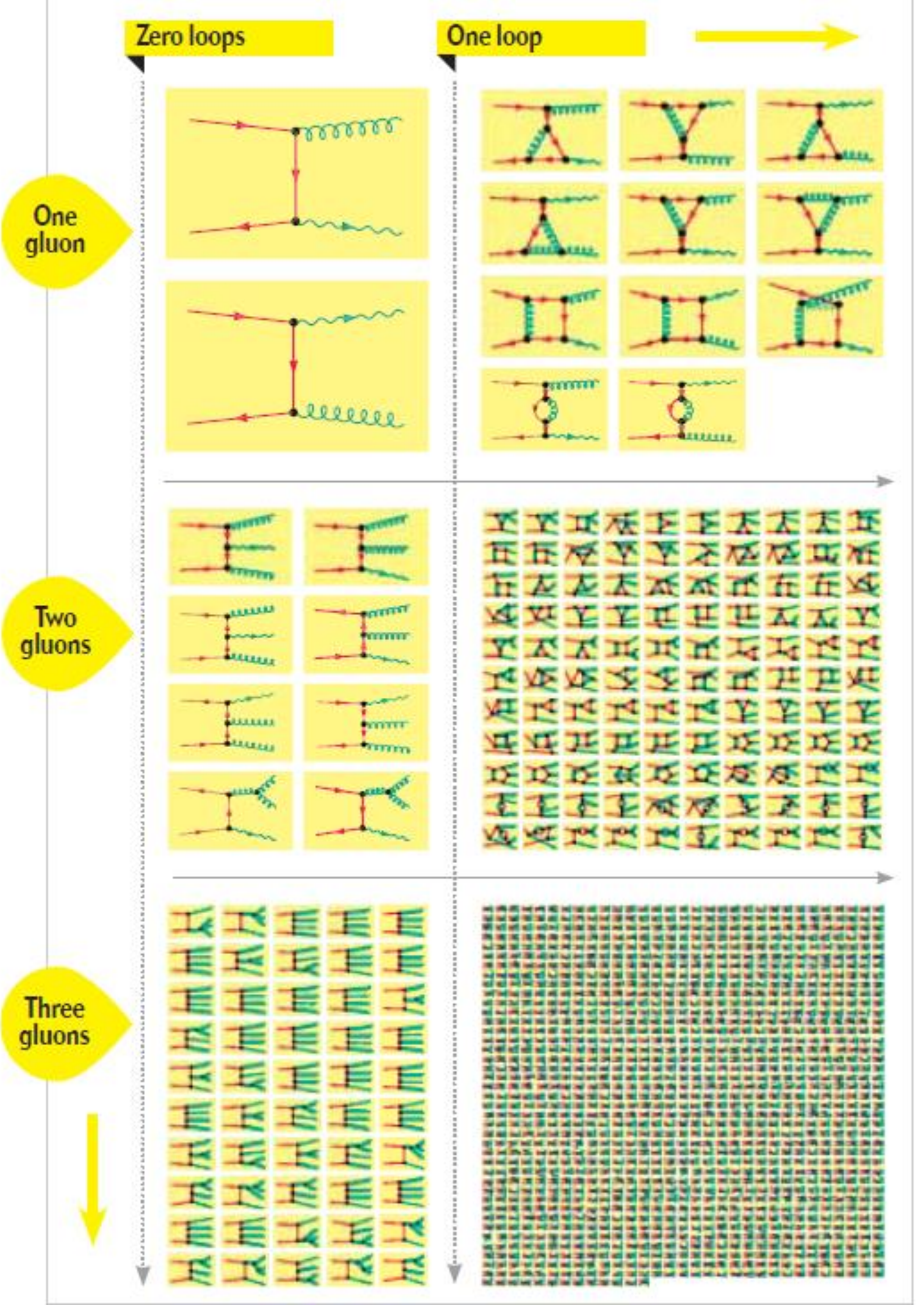


+ 256,264 more

- **Feynman diagrams**, while very general and **powerful**, are **not optimized** for these processes
- **There are much more efficient methods**

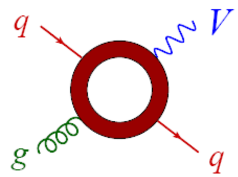
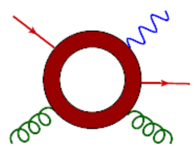
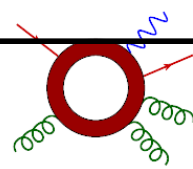
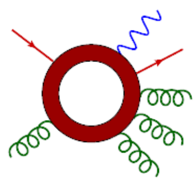
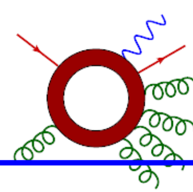
Just one QCD loop can be a challenge

$q\bar{q} \rightarrow W + n$ **gluons**



Quantifying the one-loop QCD challenge

***pp* → *W* + *n* jets** (amplitudes with most gluons)

# of jets		# 1-loop Feynman diagrams
1		11
2		110
3		1,253
4		16,648
5		256,265

Current limit with Feynman diagrams

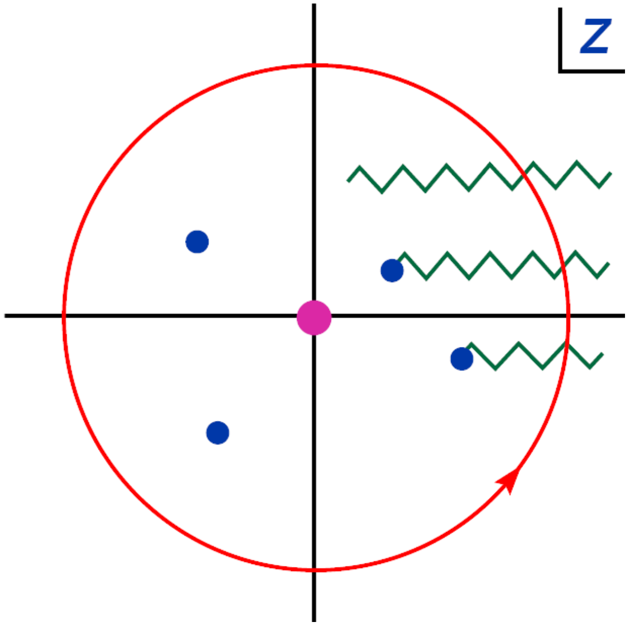
Current limit with on-shell methods

What can replace Feynman Diagrams?

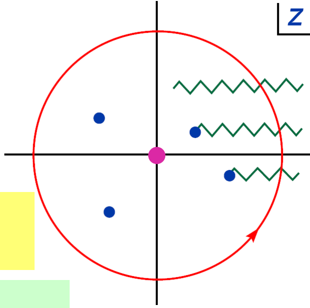
"One of the most remarkable discoveries in elementary particle physics has been that of the existence of the complex plane."

- Anonymous

(quoted by J. Schwinger)



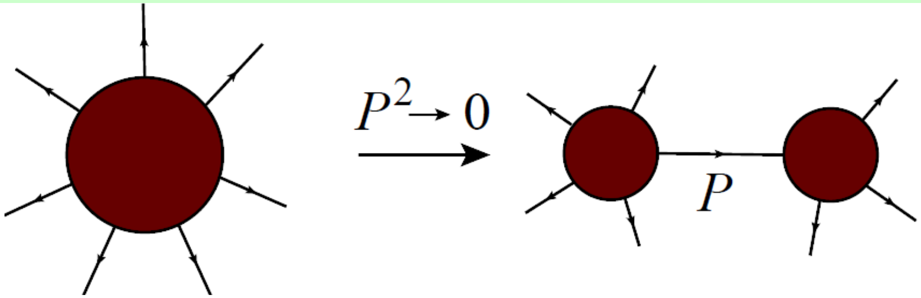
1960's Analytic *S*-Matrix



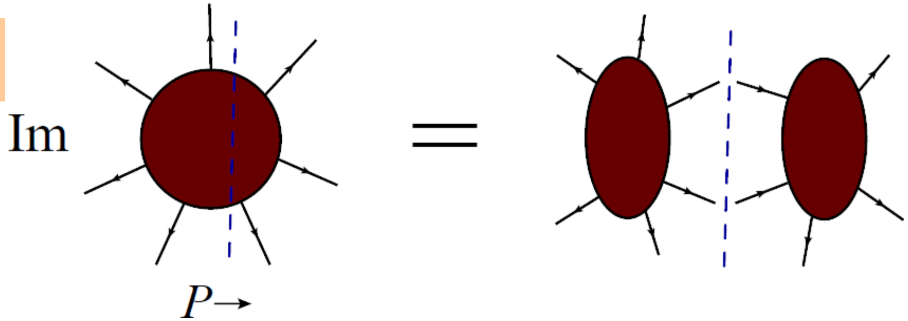
Strong interactions: No QCD, no Lagrangian or Feynman rules

Bootstrap program: Reconstruct scattering amplitudes **directly** from **analytic properties: "on-shell" information**

- Poles



- Branch cuts

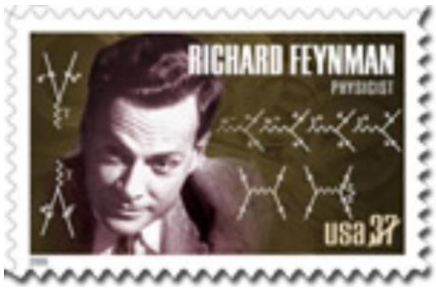


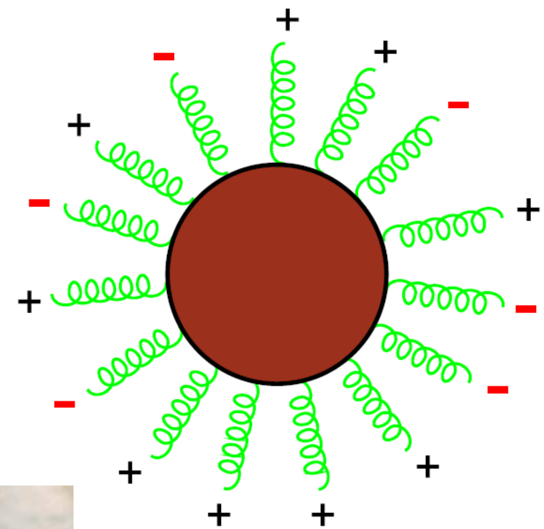
Landau; Cutkosky; Chew, Mandelstam, Frautschi; Eden, Landshoff, Olive, Polkinghorne; Veneziano; Virasoro, Shapiro; … (1960s)

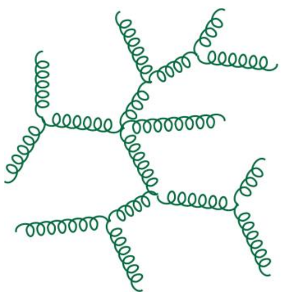
Analyticity fell out of favor in 1970s with the rise of QCD & Feynman rules

Now **resurrected** for computing amplitudes in **perturbative** QCD – as **alternative to Feynman diagrams!**
Perturbative information now assists analyticity.

Granularity vs. Fluidity



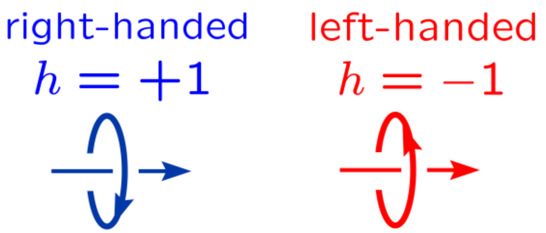


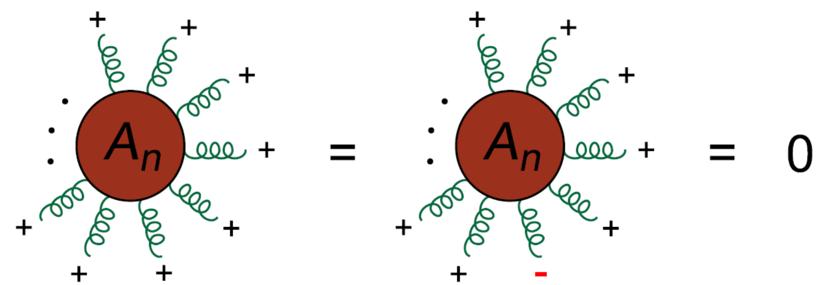


+ …

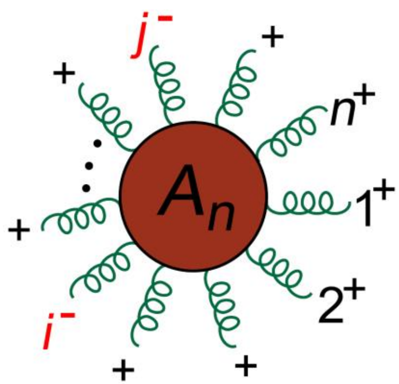
Helicity Formalism Exposes Tree-Level Simplicity in QCD

Many tree-level helicity amplitudes either vanish or are very short





Analyticity makes it possible to recycle this simplicity into loop amplitudes



$$= \frac{\langle i\, j\rangle^4}{\langle 1\, 2\rangle\langle 2\, 3\rangle\cdots\langle n\, 1\rangle}$$

Parke-Taylor formula (1986)

For Efficient Computation

Reduce
the number of "diagrams"

Reuse
building blocks over & over

Recycle
lower-point (1-loop) & lower-loop (tree)
on-shell amplitudes

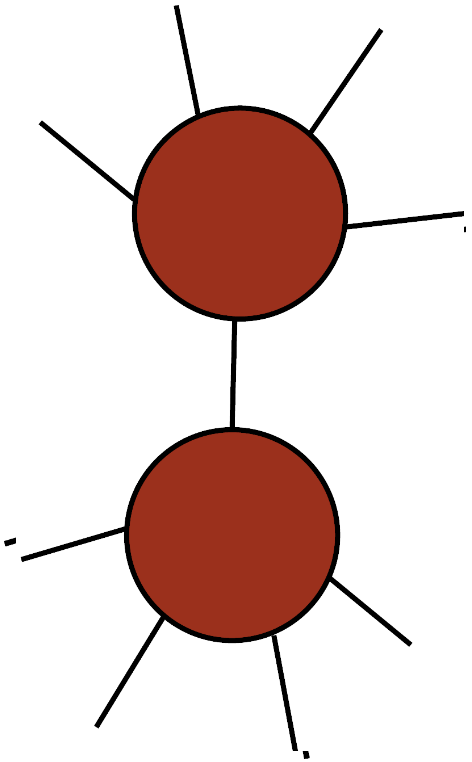
Recurse

Recycling “Plastic” Amplitudes

Amplitudes fall apart into simpler ones in special limits – pole information

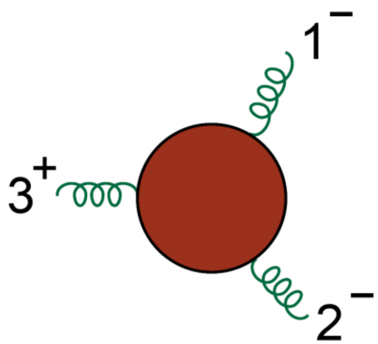
Picture leads directly to BCFW (on-shell) recursion relations

Britto, Cachazo, Feng, Witten, hep-th/0501052



Trees recycled into trees

All Gluon Tree Amplitudes Built From:



$$= \frac{\langle 1\,2\rangle^4}{\langle 1\,2\rangle\langle 2\,3\rangle\langle 3\,1\rangle}$$

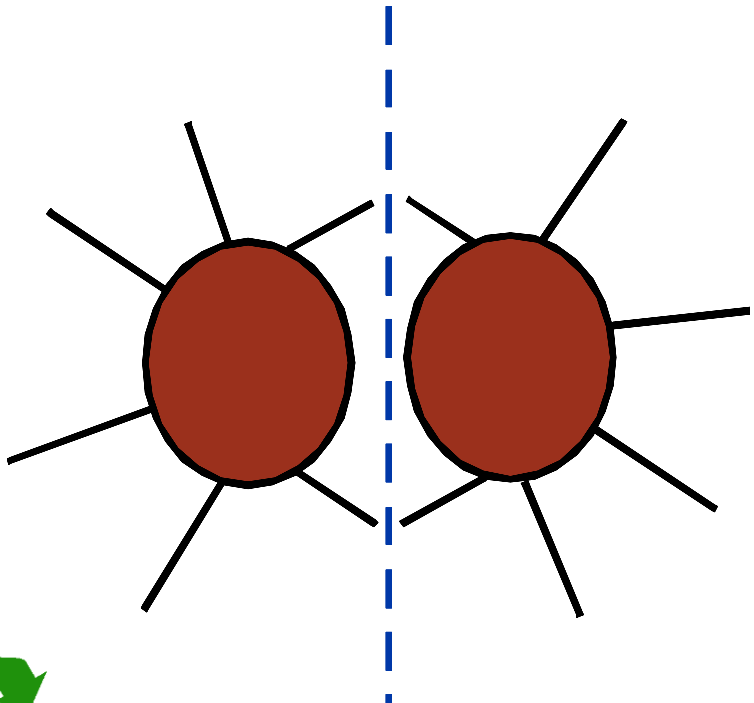
In contrast to Feynman vertices, it's on-shell, completely physical

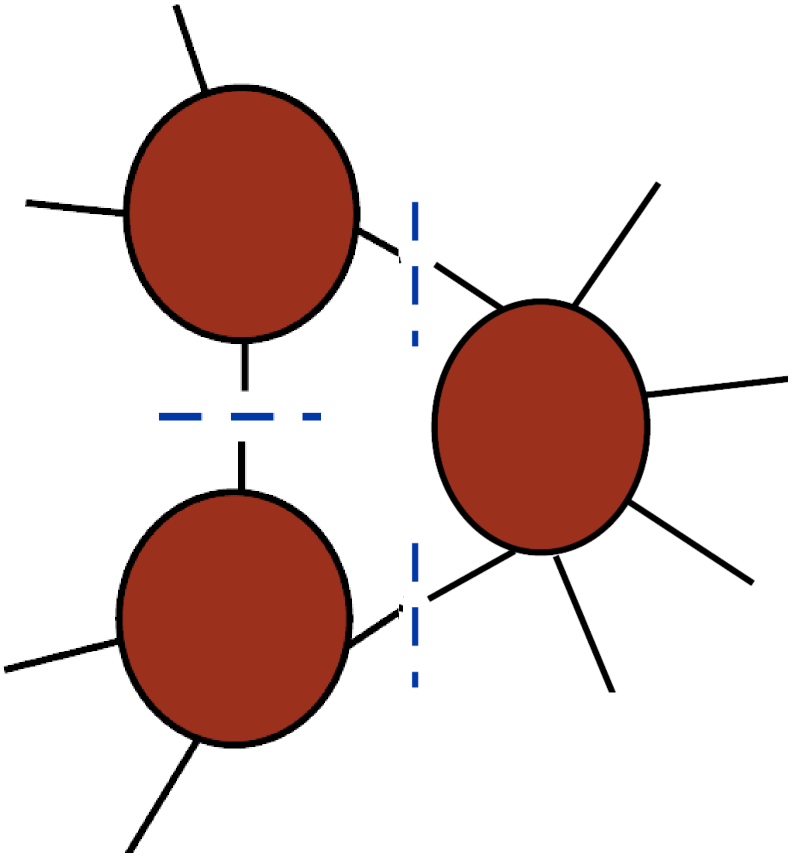
- On-shell recursion → very compact analytic formulae, fast numerical implementation.
- Can do same sort of thing at loop level.

Branch cut information → Generalized Unitarity (One-loop Plasticity)

Ordinary unitarity:
put 2 particles on shell

Generalized unitarity:
put 3 or 4 particles on shell

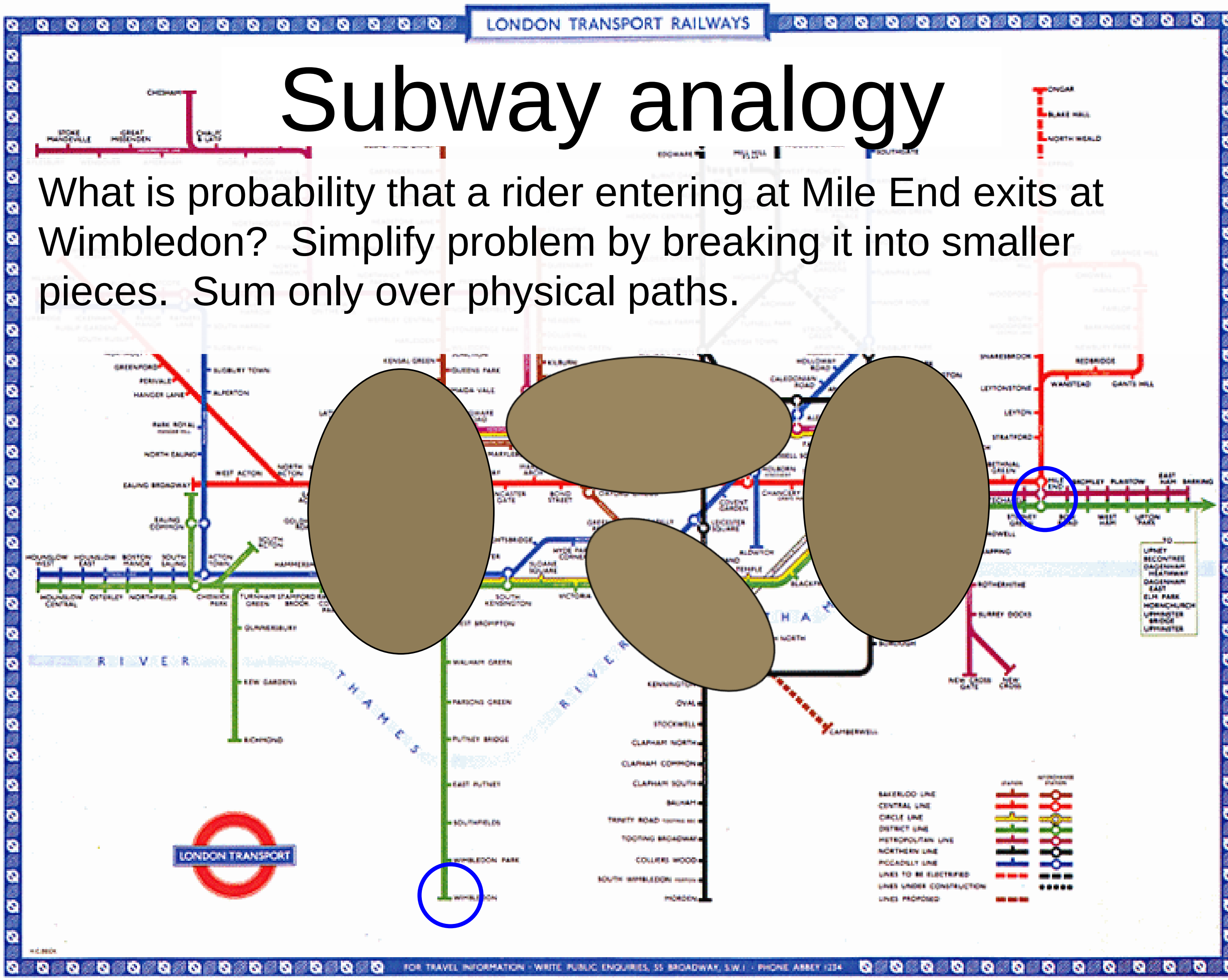




Trees recycled into loops!

Subway analogy

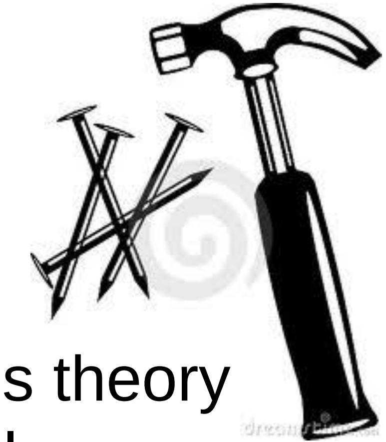
What is probability that a rider entering at Mile End exits at Wimbledon? Simplify problem by breaking it into smaller pieces. Sum only over physical paths.



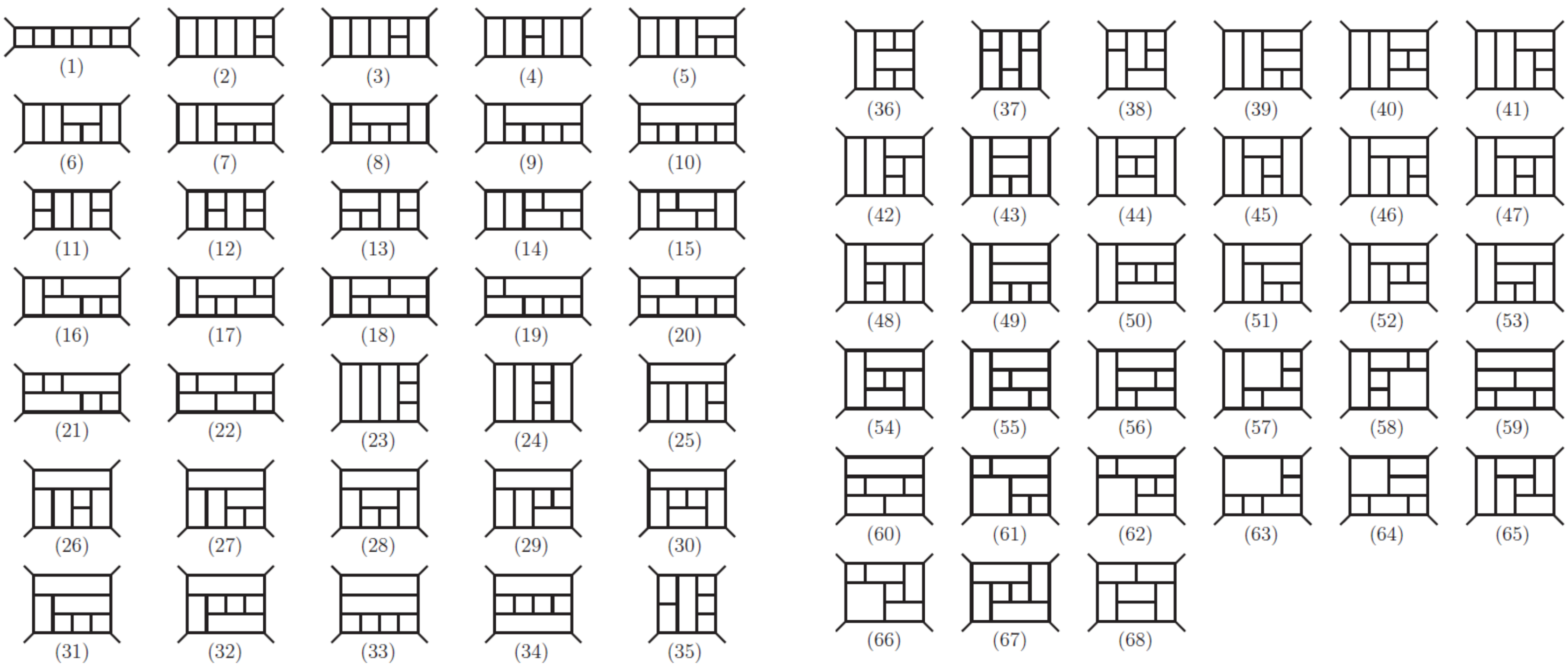
Bottom Line:

Trees recycled into loops!

In simpler theories can go to many loops

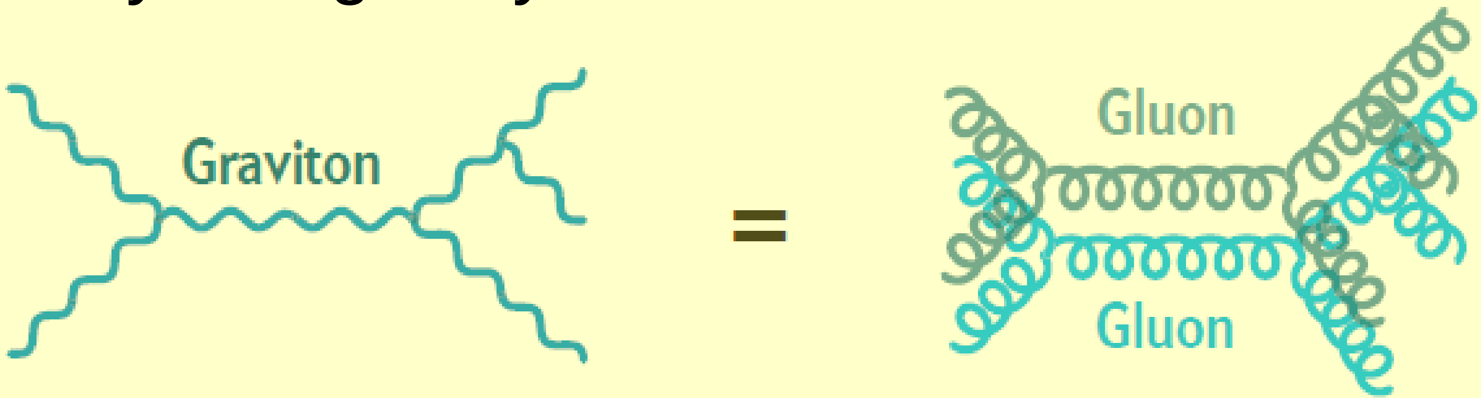


- 6 loop 4 gluon amplitude in N=4 super-Yang-Mills theory (QCD cousin) in the limit of a large number of colors:



Striking patterns emerge

- Including remarkable relations between gauge theory and gravity



- N=8 supergravity exceptionally well-behaved for a point-like theory of quantum gravity:
- Finite through at least 4 loops, probably until 7 loops.   No worse behaved through 4 loops than N=4 super-Yang-Mills (a finite theory).

Back to QCD for LHC: Need to Automate.
Many Automated On-Shell One Loop Programs

Blackhat: Berger, Bern, LD, Diana, Febres Cordero, Forde, Gleisberg, Höche, Ita, Kosower, Maître, Ozeren, 0803.4180, 0808.0941, 0907.1984, 1004.1659, 1009.2338…

+ **Sherpa** → NLO *W,Z* + 3,4,5 jets   pure QCD 4 jets

CutTools: Ossola, Papadopolous, Pittau, 0711.3596

NLO *WWW, WWZ*, ... Binoth+OPP, 0804.0350

NLO $t\bar{t}b\bar{b}$, $t\bar{t}$ + 2 jets,…

Bevilacqua, Czakon, Papadopoulos, Pittau, Worek, 0907.4723; 1002.4009

MadLoop: Hirschi, Frederix, Frixione, Garzelli, Maltoni, Pittau 1103.0621

HELAC-NLO: Bevilacqua et al, 1110.1499

Rocket: Giele, Zanderighi, 0805.2152

Ellis, Giele, Kunszt, Melnikov, Zanderighi, 0810.2762

NLO *W* + 3 jets Ellis, Melnikov, Zanderighi, 0901.4101, 0906.1445

$W^+W^\pm$ + 2 jets Melia, Melnikov, Rontsch, Zanderighi, 1007.5313, 1104.2327

SAMURAI: Mastrolia, Ossola, Reiter, Tramontano, 1006.0710

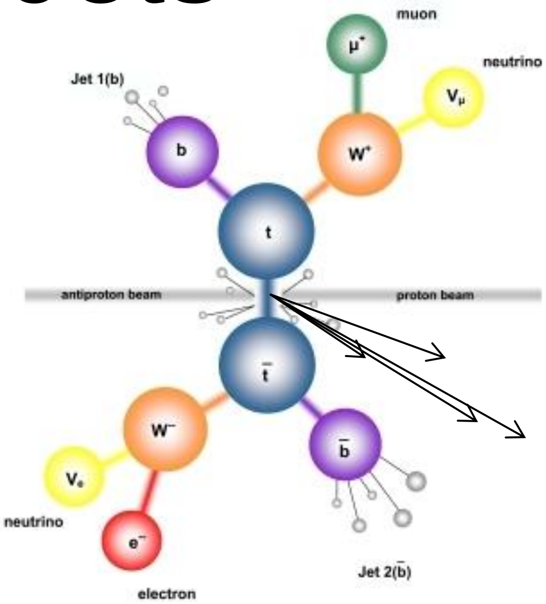
NGluon: Badger, Biedermann, Uwer, 1011.2900,1209.0098

Open Loops: Cascioli, Maierhofer, Pozzorini, 1111.5206

As a result…

Dramatic increase recently
in rate of NLO QCD predictions
for new processes!

Top Quark Pairs + Jets



- Like (W,Z) + jets, very important bkgd
- Cross sections large

– no electroweak couplings

- Jets boost $t\,\bar{t}$ system, increase MET,

provide jets to pass various signal cuts.

- State of art [Feynman diagrams, new methods]:
- **NLO *tt* + 1 jet:** Dittmaier, Uwer, Weinzierl, hep-ph/0703120,…
- **+ top decays:** Melnikov, Schulze, 1004.3284
- **+ NLO parton shower:** Kardos, Papadopoulos, Trócsányi, 1101.2672

- **NLO *tt* + *bb*:** Bredenstein, Denner, Dittmaier, Pozzorini, 0905.0110, 1001.4006; Bevilacqua, Czakon, Papadopoulos, Pittau, Worek, 0907.4723
- **NLO *tt* + 2 jets:** Bevilacqua, Czakon, Papadopoulos, Worek, 1002.4009

NLO $pp \to t\bar{t}\,b\bar{b}$ at LHC

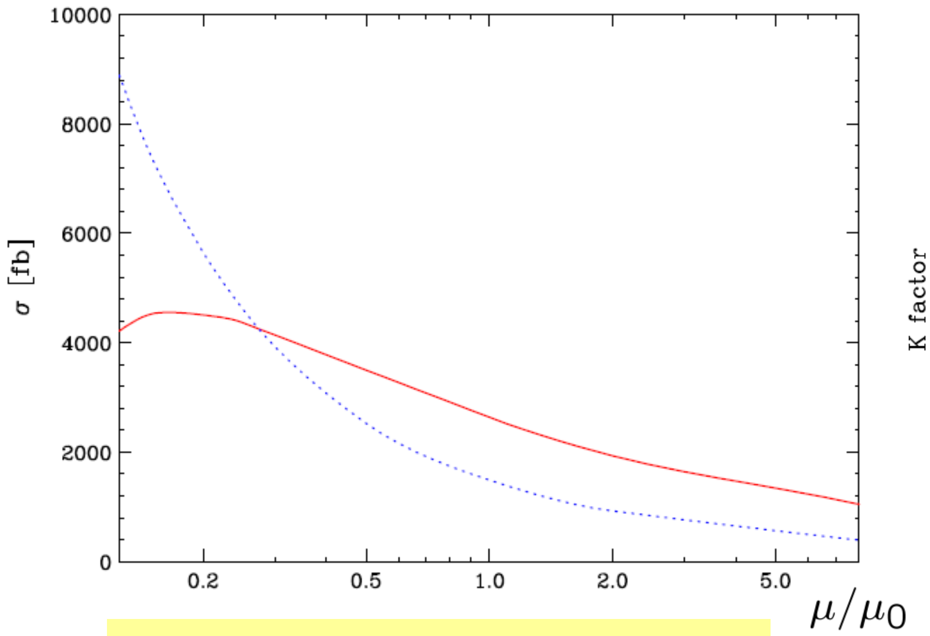
Background to $t\,\bar{t}$ + *Higgs*, $H \to b\bar{b}$ at LHC (for λ_t)

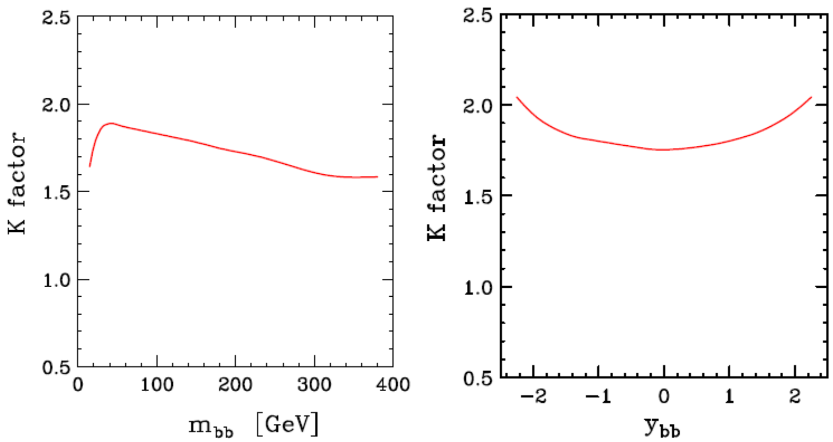
First done using Feynman diagrams

Recomputed using unitarity (**CutTools)**

Bredenstein et al., 0807.1248, 0905.0110

Bevilacqua et al., 0907.4723





much improved
scale uncertainties at NLO

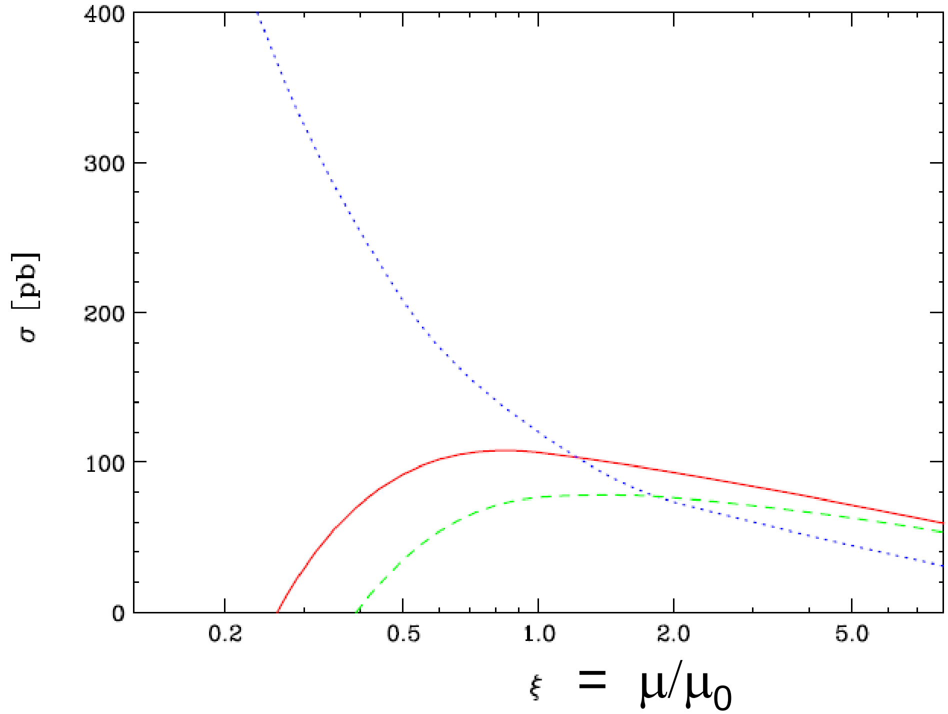
shape changes in *bb* distributions from LO to NLO (K=NLO/LO)

$$pp \to t\bar{t}jj$$

Like $pp \to t\bar{t}b\bar{b}$, a background to $pp \to t\bar{t}H$, $H \to b\bar{b}$

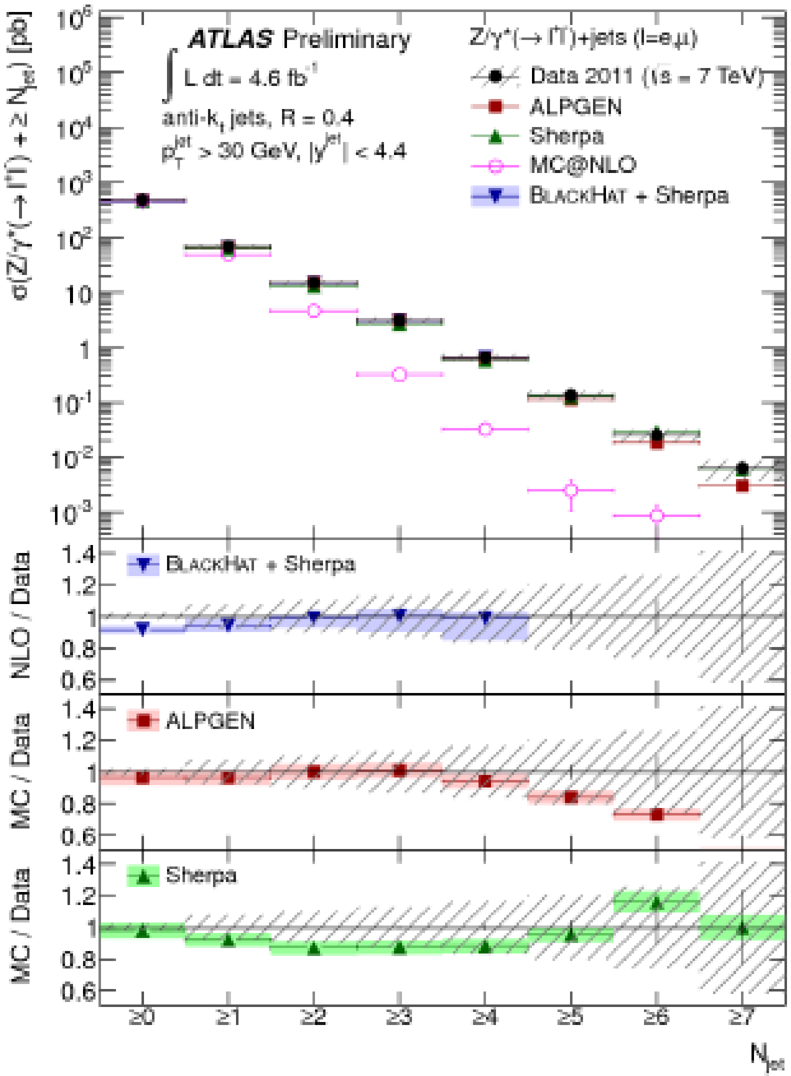
Only computed via unitarity (**CutTools)**

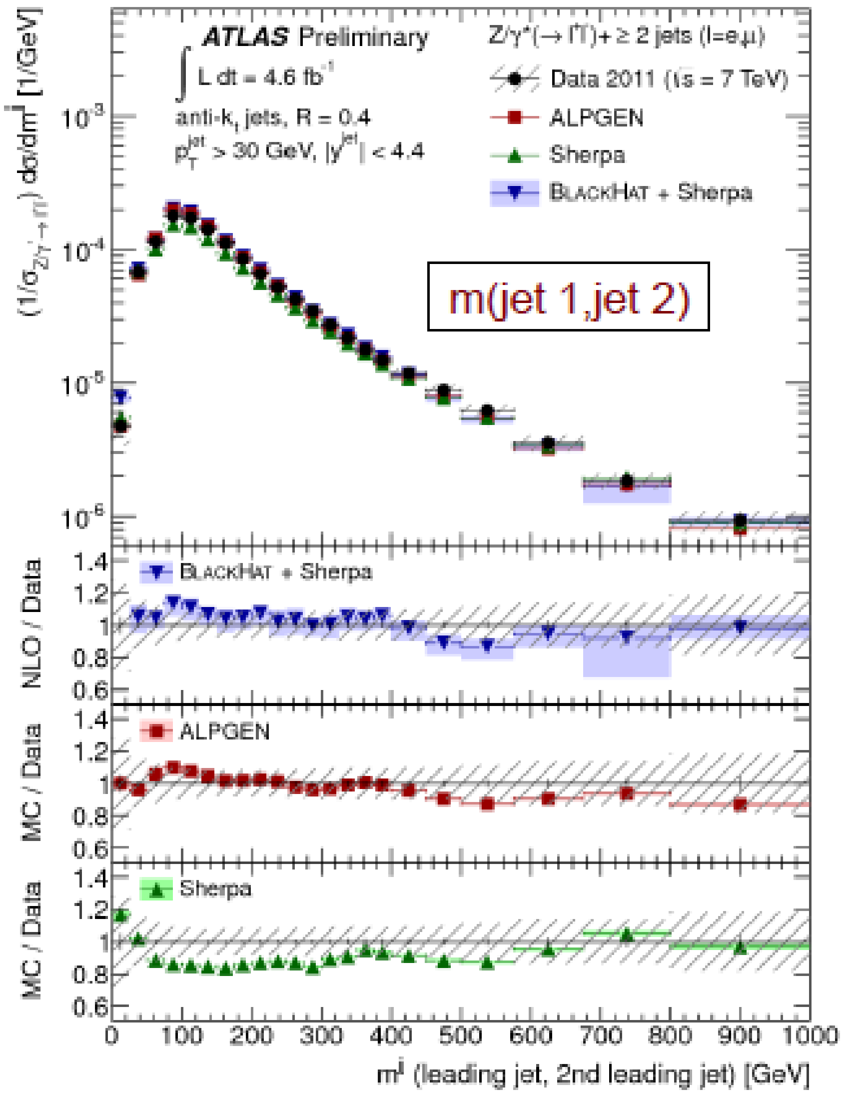
Bevilacqua, Czakon, Papadopoulos, Worek, 1002.4009



Again large reduction in scale dependence from LO → NLO

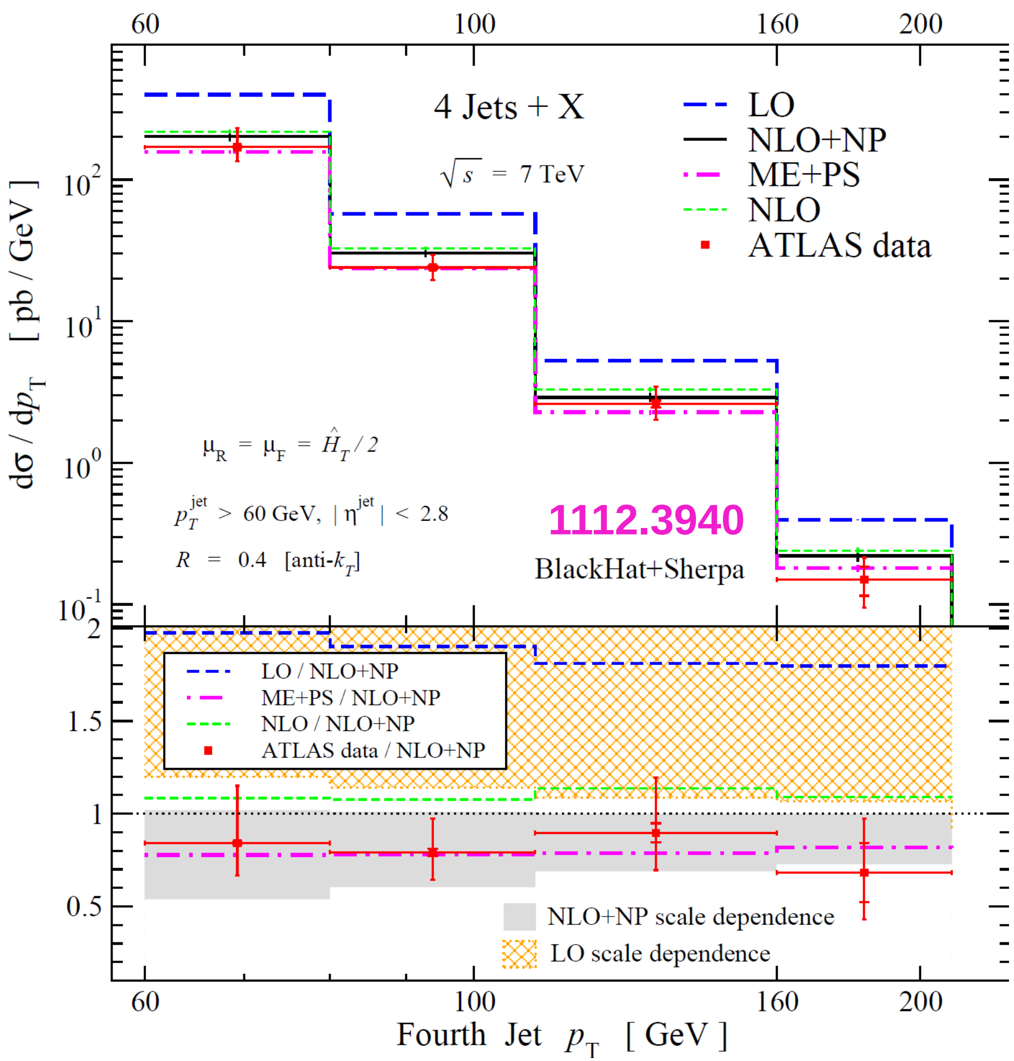
NLO $pp \to Z + 1,2,3,4$ jets vs. ATLAS 2011 data





NLO Z+4: Ita et al., 1108.2229

Pure QCD: *pp* → 4 jets vs. ATLAS data

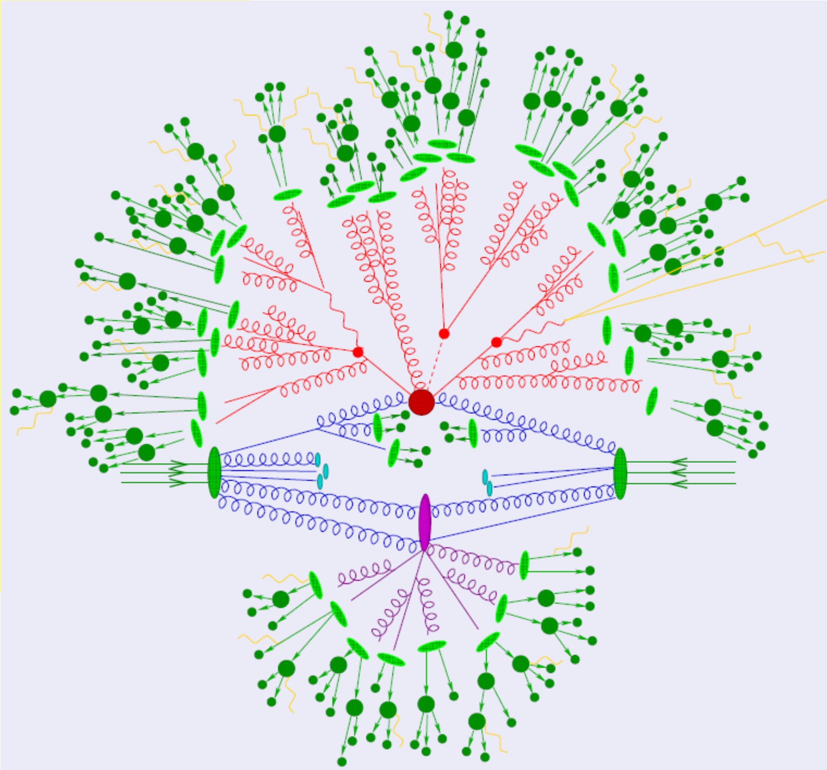


4 jet events might hide pair production of 2 colored particles, each decaying to a pair of jets

Detailed study of multi-jet QCD dynamics may help understand other channels

Fixed order vs. Monte Carlo

- Previous plots NLO but fixed-order, few partons: no model of long-distance effects included; cannot pass through a detector simulation
- Methods available for matching NLO parton-level results to parton showers, with NLO accuracy:
  - **MC@NLO** Frixione, Webber (2002) ; …; SHERPA implementation
  - **POWHEG** Nason (2004); Frixione, Nason, Oleari (2007)
- Recently implemented for increasingly complex final states, e.g.
  W + 1,2,3 jets Höche et al, 1201.5882



Fixed order vs. Monte Carlo (cont.)

- Most recently, several groups have produced methods for matching/merging NLO and parton showers with the NLO accuracy maintained for events in the sample with more than the minimum number of jets.
- i.e., a NLO version of ALPGEN/Pythia or SHERPA

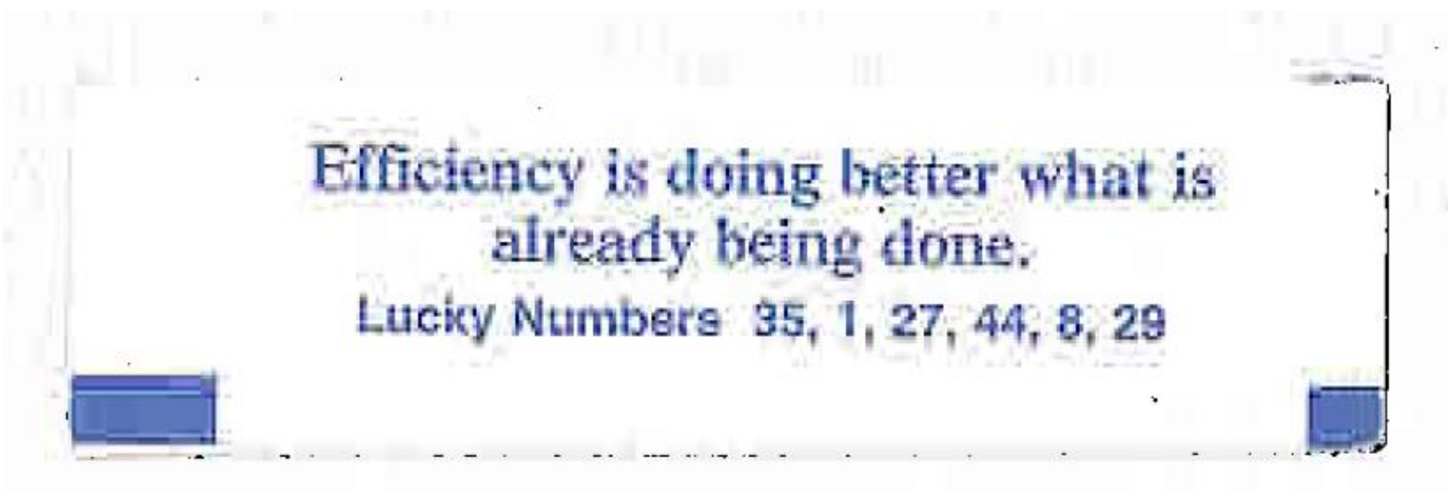
Lavesson, Lönnblad, 0811.2912;
Höche, Krauss, Schönherr, Siegert, 1207.5030;
Gehrmann, Höche, Krauss, Schönherr, Siegert, 1207.5031;
Frederix, Frixione, 1209.6125;
Lönnblad, Prestel, 1211.4827. 1211.7278; Platzer, 1211.5467;
Alioli et al, 1211.7049; Hamilton, Nason, Oleari, Zanderighi, 1212.4504;
Hartging, Laenen, Skands, 1303.4974

One indicator of NLO progress

pp → W + 0 jet	1978	Altarelli, Ellis, Martinelli
pp → W + 1 jet	1989	Arnold, Ellis, Reno
pp → W + 2 jets	2002	Campbell, Ellis
pp → W + 3 jets	2009	BH+Sherpa Ellis, Melnikov, Zanderighi
pp → W + 4 jets	2010	BH+Sherpa
pp → W + 5 jets	2013	BH+Sherpa

Next Two Lectures

- Understand in more detail how the new methods work
- First at tree level, then at one loop



Further Reading

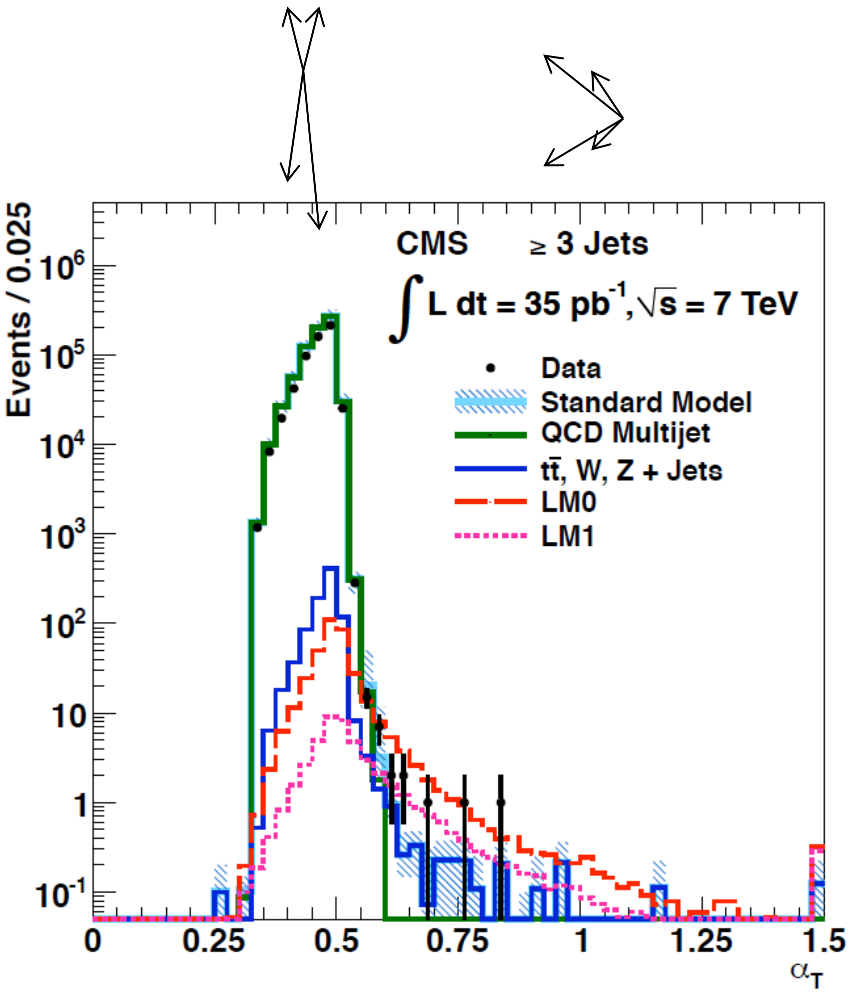
- Bern, LD, Kosower, 0704.2798 [hep-ph]
- Ossola, Papadopoulos, Pittau, hep-ph/0609007
- Ellis, Kunszt, Melnikov, Zanderighi, 1105.4319
- **Special volume of J.Phys. A44 (2011)**

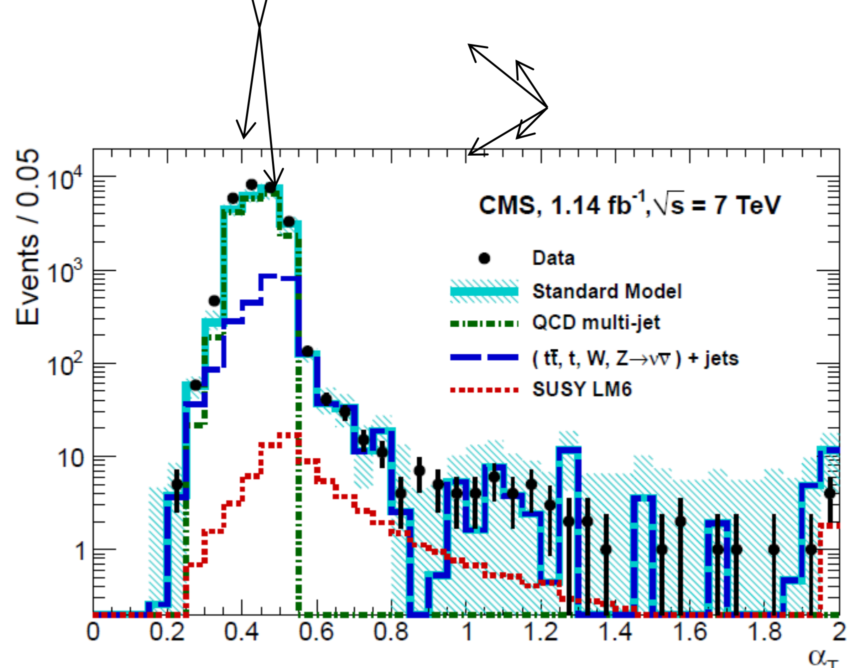
  - LD, 1105.0771
  - Britto, 1012.4493
  - Bern, Huang, 1103.1869
  - Brandhuber, Spence, Travaglini, 1103.3477
  - Ita, 1109.6527

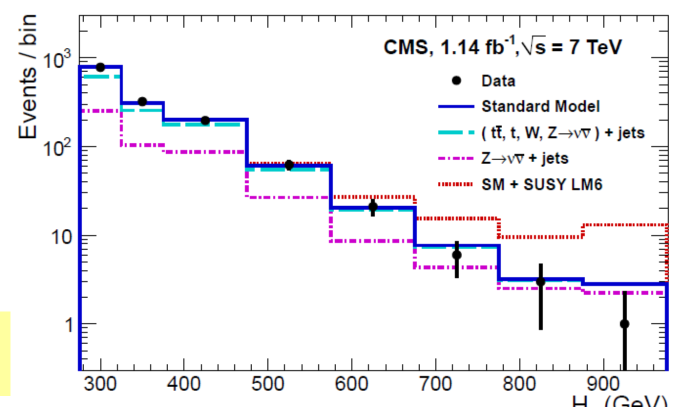
Extra Slides

MET + jets search at CMS (circa 2011)

1101.1628, 1109.2352

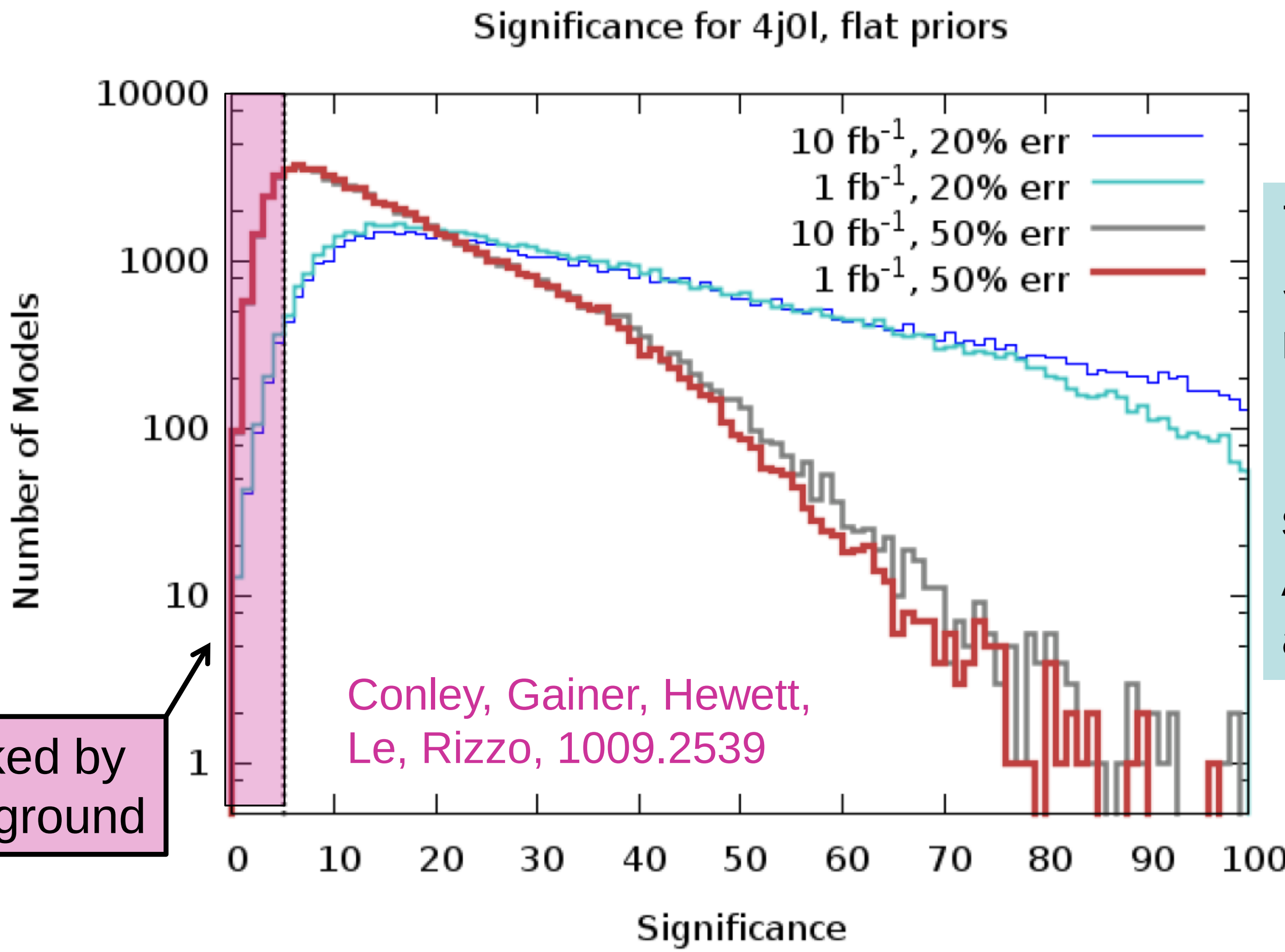






α_T ~ misalignment of MET with main jet axis

Reducing Background Systematics Improves SUSY Search Sensitivity



masked by background

70,000 SUSY models

MET + 4 jet search, ATLAS-style analysis