Eidgenössische Technische Hochschule Zürich
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Perturbative QCD
Lecture 3

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Academic Training Lectures, CERN, May 2013

Outline of the third lecture

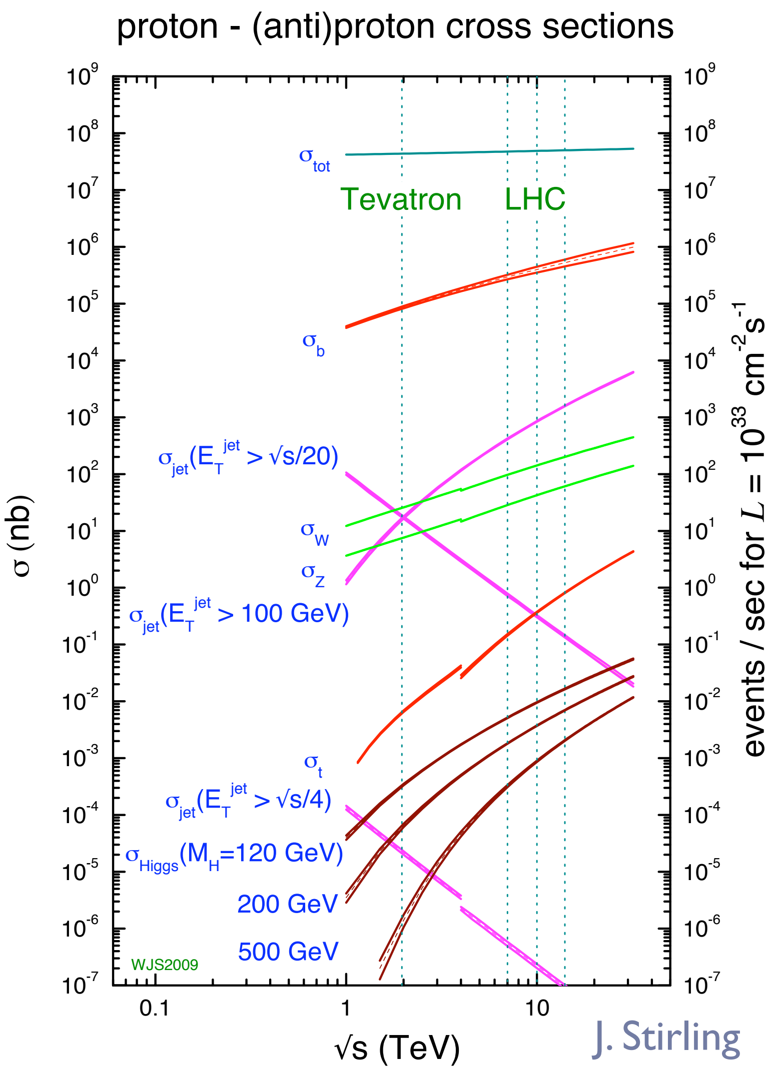
- Present essential characteristics of hard scattering observables for LHC

- Present results for hard scattering processes which are measured and calculated (at fixed order) with high accuracy
  - Z-boson production
  - Higgs production
  - Di-Photon production

- Study: perturbative convergence and phenomenology for these hadronic processes

Expectations at LHC

- Large production rates for processes with
  - Jets
  - Top-quark pairs
  - Vector bosons
- Allow precise determinations for
  - coupling constants
  - parton distributions
- Require theoretical description for hard scattering cross sections to be at least as precise as the experimental measurements



J. Stirling

Kinematics at LHC

- Hard scattering cross section

$$\mathrm{d}\sigma_{P_1 P_2 \to X}(s) = \sum_{i,j} \int \mathrm{d}x_1 \mathrm{d}x_2 f_{q_i/P_1}(x_1) f_{q_j/P_2}(x_2) \hat{\sigma}_{ij \to X}(\hat{s})$$

  with parton-parton centre-of-mass energy $\hat{s} = x_1 x_2 s$

  - Presence of longitudinal momentum fractions of partons: x_1,x_2

  →Parton-parton centre-of-mass system boosted along beam axis with respect to proton-proton centre-of-mass system

- Describe final state kinematics in terms of variables that are boost-invariant (or transform trivially)
  - e.g.: transverse momenta, polar angles, rapidities

Kinematics at LHC

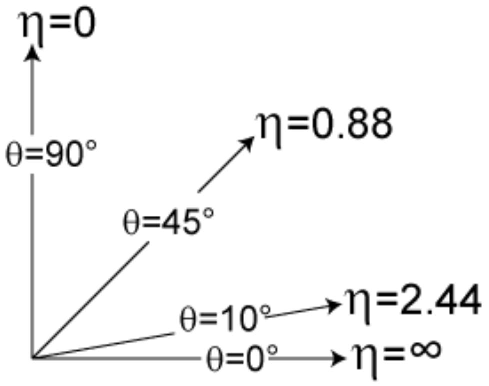
- Four-momentum of a massless particle

$$p^\mu = (E, p_x, p_y, p_z) = p_T(\cosh\eta, \sin\phi, \cos\phi, \sinh\eta)$$

  - Transverse momentum: $p_T = \sqrt{p_x^2 + p_y^2}$
  - Pseudo-Rapidity:

$$\eta = \frac{1}{2}\ln\left(\frac{E+p_z}{E-p_z}\right) = -\ln(\tan(\Theta_{cm}/2))$$

  - Polar angle: $\phi = \arctan(p_x/p_y)$



- Four-momentum of a massive particle

$$p^\mu = (E, p_x, p_y, p_z) = (m_T\cosh y, p_T\sin\phi, p_T\cos\phi, m_T\sinh y)$$

  - Transverse mass $m_T = \sqrt{m^2 + p_T^2}$ and rapidity $y = \frac{1}{2}\ln\left(\frac{E+p_z}{E-p_z}\right) \neq \eta$

Kinematics at LHC

- Transverse momentum and polar angle: boost-invariant
- Rapidity transformation from parton-parton (ab) frame to proton-proton (pp) frame:

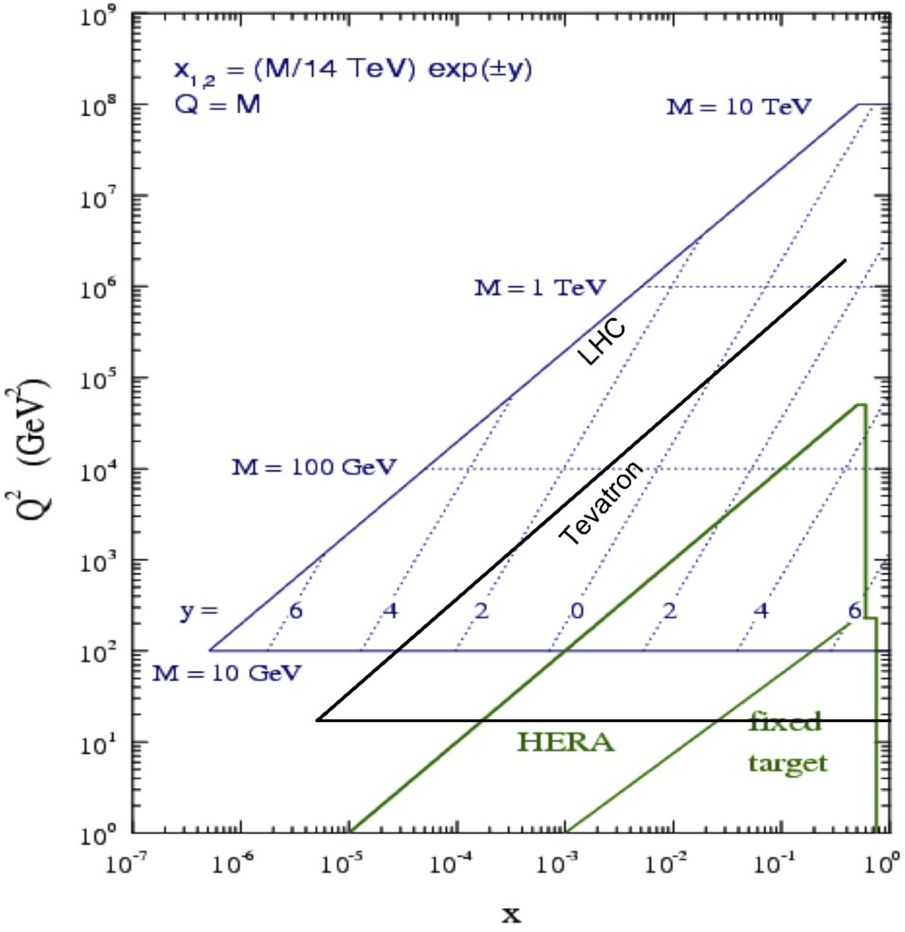
$$y_{pp} = y_{ab} + \frac{1}{2} \ln \left(\frac{x_1}{x_2} \right)$$

  - Rapidity differences are boost-invariant
  - Highest parton-parton centre-of-mass energy obtained for production at central rapidity ($x_1 = x_2$)

Kinematics at LHC

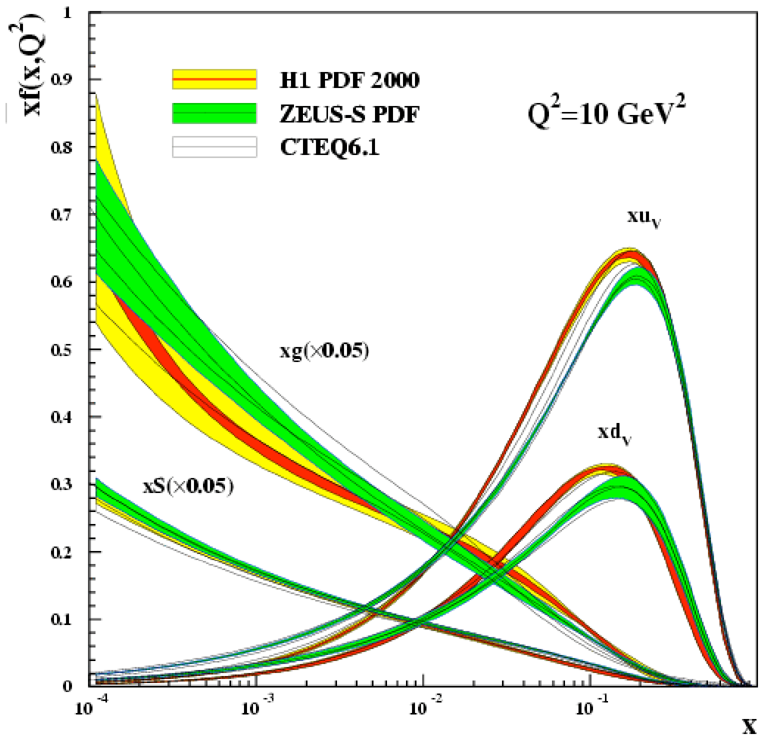
- LHC probes new range in (x, Q^2)
  - E.g. Z-boson production at central rapidity $(y=0)$:
    - Tevatron (2TeV) $x_1=x_2\simeq 0.05$
    - LHC (14TeV): $x_1=x_2\simeq 0.0065$
  - Different combinations of parton distributions dominate
    - Larger x: valence quarks
    - Smaller x: gluons and sea quarks
- Phenomenology of the same process can be substantially different for LHC and Tevatron

$$\hat{s} = x_1 x_2 s$$



Parton distributions at LHC

- Parton distributions determined from experimental data
  - Evolution determined by perturbative QCD (AP eqs.)
  - Input distributions are non-perturbative objects
  - Usually from global fit to multiple observables (HERA, fixed target, Tevatron, LHC)
  - Various groups (MRST, CT10, NNPDF, JR, ABM, HERAPDF,…) provide parametrizations of the distributions and errors on them



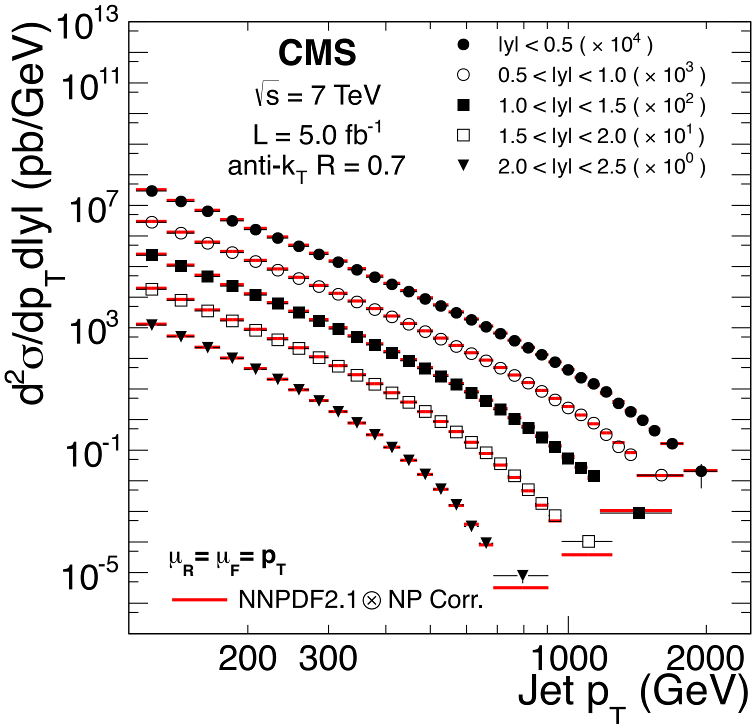
  - Challenge at LHC: Probe new territory for PDF extraction

Jet observables at LHC

- Jet production
  - Low multiplicities: large cross section, precision QCD study
  - High multiplicities: potential signature of BSM physics
- Use boost-invariant jet definition

$$y_{ij} = \min(k_{T,i}^{2p}, k_{t,j}^{2p}) \, \frac{(\eta_i - \eta_j)^2 + (\phi_i - \phi_j)^2}{R^2}$$

  - p=1: kT, p=0: Cambridge/Aachen, p=-1: anti-kT
- Single jet inclusive cross section
  - Double differential: (p_T, y)
  - Measured to per cent accuracy up to transverse momenta $p_T \simeq 2$ TeV



Theoretical uncertainties

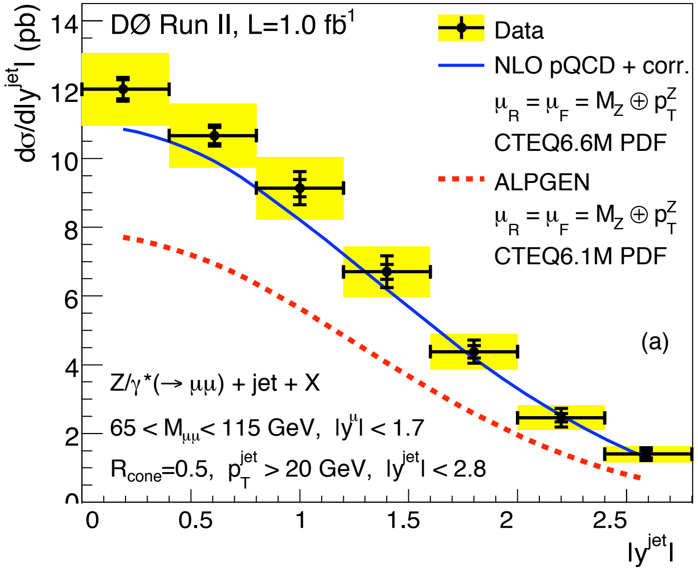
- Remember cross section formula for a hard scattering process

$$\boxed{d\sigma^{h_1h_2\to cd} = \int_0^1 dx_1 \int_0^1 dx_2 \sum_{a,b} f_{a/h_1}(x_1,\mu_F^2) f_{b/h_2}(x_2,\mu_F^2) d\hat{\sigma}^{ab\to cd}(Q^2,\mu_F^2)}$$

- Sources of uncertainty
  - Parton distributions $f_{a/h}(x, \mu_F)$: determined from data, inherent errors
  - Partonic cross section $d\sigma^{ab\to cd}$: expanded in perturbation theory to finite order, uncertainty from missing higher orders
- Quantify theoretical uncertainty through scale variations
  - Renormalization scale dependence: $\alpha_s(\mu_R)$
  - Factorization scale dependence: $f_{a/h}(x, \mu_F)$, $\sigma^{ab\to cd}(x, \mu_F)$
- Expect: Scale dependence decreases including more and more higher order terms

Fixed order predictions

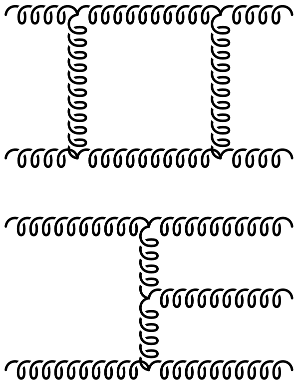
- LO predictions usually give a qualitative understanding of the behaviour of observables but are often not enough to describe the data accurately
- NLO needed to
  - reduce scale uncertainty of LO theory prediction
  - Have a reliable estimation of normalization and shape as it accounts for effects of extra radiation



- Example: Z+j at Tevatron
  - NLO error: ~15%
  - substantial NLO effect
  - correction not constant

NLO calculations

- Require two principal ingredients (here: pp →2j)
  - one-loop matrix elements
    - explicit infrared poles from loop integral
      - known for all 2 → 2 processes
      - known for many 2 → 3 processes
      - current frontier 2 → 7 (major challenge)
  - tree-level matrix elements
    - implicit poles from soft/collinear emission



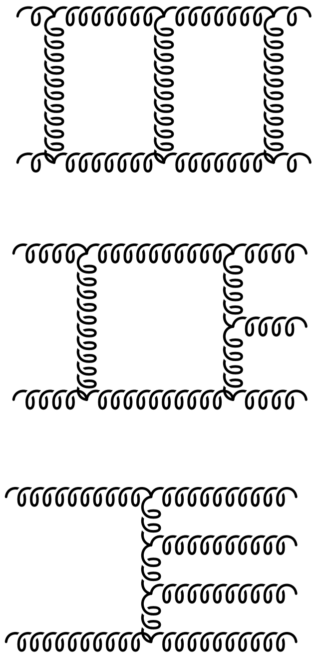
- Infrared poles cancel in the sum (KLN, factorization theorems)
- Subtraction methods used to extract infrared poles and combine contributions to evaluate NLO observables are well-established
- Several program packages for NLO:
  MCFM, MC@NLO, POWHEG, NLOJET++ …

NNLO predictions

- NNLO corrections needed:
  - For processes measured to few per cent accuracy
    - jet production
    - vector boson (+jet) production
    - top quark pair production
  - For processes with potentially large perturbative corrections as new channels or new phase space regions open up
    - Higgs or vector boson production
- Expectations for NNLO predictions:
  - Per-cent level accuracy (as required for a meaningful interpretation of collider data and extraction of parameters)
  - First reliable estimation of theoretical uncertainties

NNLO calculations

- Require three principal ingredients (here: pp → 2j)



  - two-loop matrix elements
    - explicit infrared poles from loop integral
      - known for all massless 2 → 2 processes
  - one-loop matrix elements
    - explicit infrared poles from loop integral
    - and implicit poles from single real emission
      - usually known from NLO calculations
  - tree-level matrix elements
    - implicit poles from double real emission
      - known from LO calculations
- Infrared poles cancel in the sum:
- **Challenge:** combine contributions into parton-level generator
  - Need methods at NNLO to extract implicit infrared poles

- Z-boson production at LHC

Z boson production at LHC

- Experimentally: Z bosons events (decaying into leptons) observed at large rate at LHC
  - At design luminosity (10^{34} cm^{-2}s^{-1}):100 Z bosons per second
  - Leptons yield clean final state signature
  - Measured with high accuracy (per cent level and below)
- Theoretically well understood
  - Perturbative corrections up to NNLO as fully differential event generator
  - Resummation of large logarithmic corrections
  - Process evaluated with per cent level accuracy
- Precision physics
  - Electroweak masses and couplings
  - Parton distributions

Z-boson production at LHC

- Only one partonic contribution at LO: $q\bar{q} \rightarrow Z$

$$\sigma^{\mathrm{LO}}_{pp\to Z} = \sum_i \int dx_1 dx_2\, f_{q_i}(x_1,\mu_{\mathrm{F}}^2)\, f_{\bar{q}_i}(x_2,\mu_{\mathrm{F}}^2)\ \hat{\sigma}_{0,q_i\bar{q}_i\to Z}(x_1p_1,x_2p_2)$$

- Including NLO and NNLO corrections

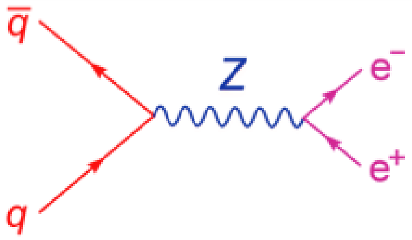
$$\sigma^{\mathrm{NNLO}}_{pp\to Z+X} = \sum_{i,j} \int dx_1 dx_2\, f_i(x_1,\mu_{\mathrm{F}}^2)\, f_j(x_2,\mu_{\mathrm{F}}^2) \Big[\hat{\sigma}_{0,ij\to Z}(x_1,x_2) \,+\, \alpha_{\mathrm{s}}(\mu_{\mathrm{R}})\hat{\sigma}_{1,ij\to Z+X}(x_1,x_2,\mu_{\mathrm{F}}) \\ +\, \alpha_{\mathrm{s}}^2(\mu_{\mathrm{R}})\hat{\sigma}_{2,ij\to Z+X}(x_1,x_2,\mu_{\mathrm{F}},\mu_{\mathrm{R}})\Big].$$

    - Sum over the flavours i,j of the initial partons
    - Include gluon-induced processes
- Inclusive cross section: Z+X production
    - Any number of jets is allowed with the Z boson

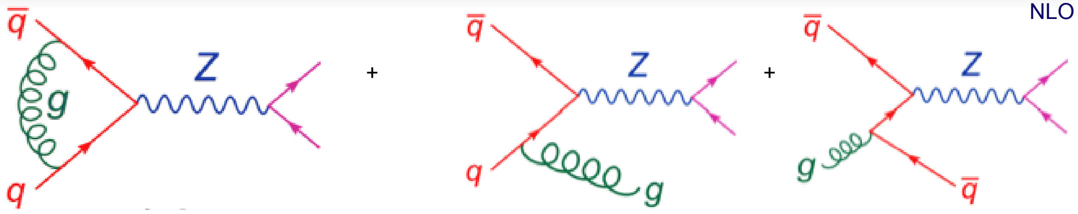
Z-boson production at LHC

- New channels open up at NLO and NNLO
  - qg-induced processes at NLO
  - gg-induced processes at NNLO

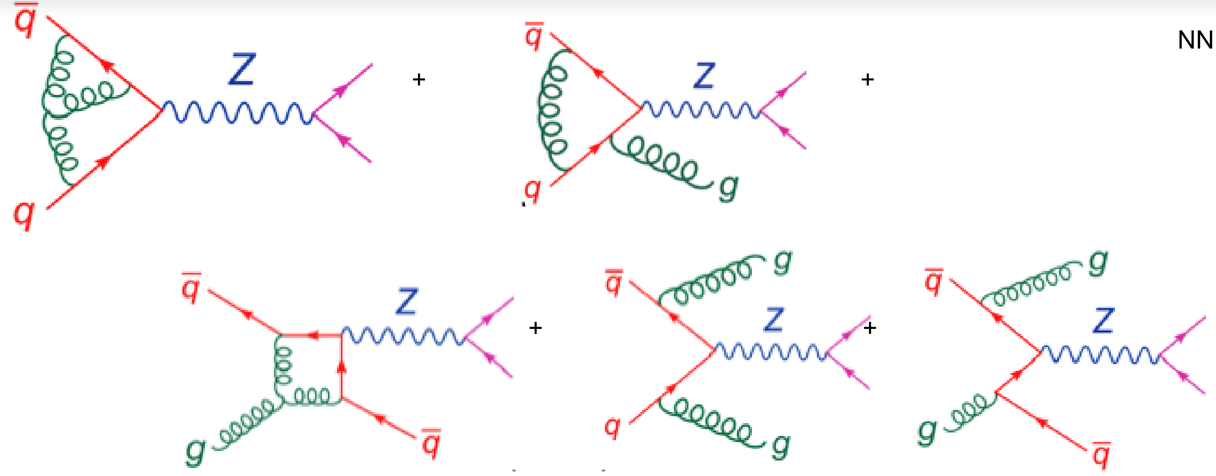
LO



NLO

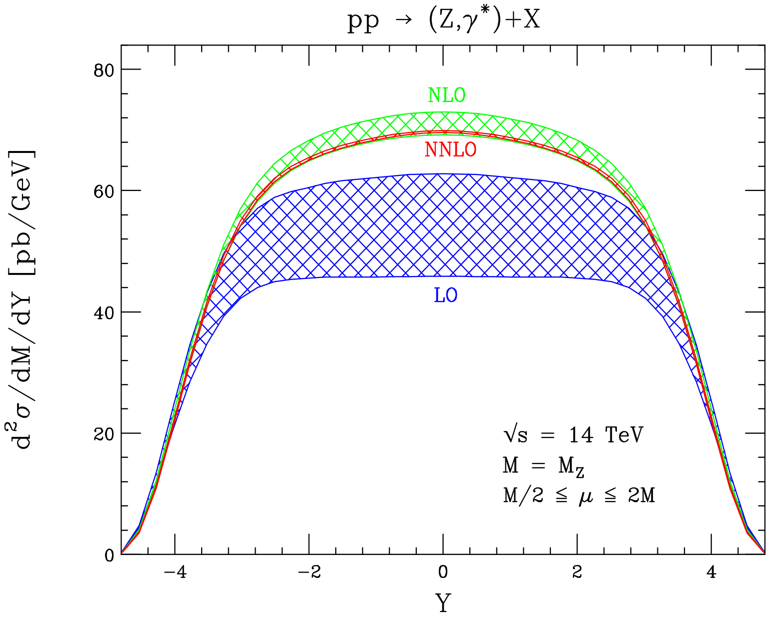


NNLO



Z-boson production at LHC

- Scale dependence
  - $\mu_F = \mu_R$ varied between $[M_Z/2;2M_Z]$
- Scale variation is reduced at each order
  - LO: 30% , NLO 6%, NNLO <1%
  - But: LO uncertainty band underestimates higher orders



Rapidity distribution in Z production
(C. Anastasiou, L. Dixon, K. Melnikov, F. Petriello)

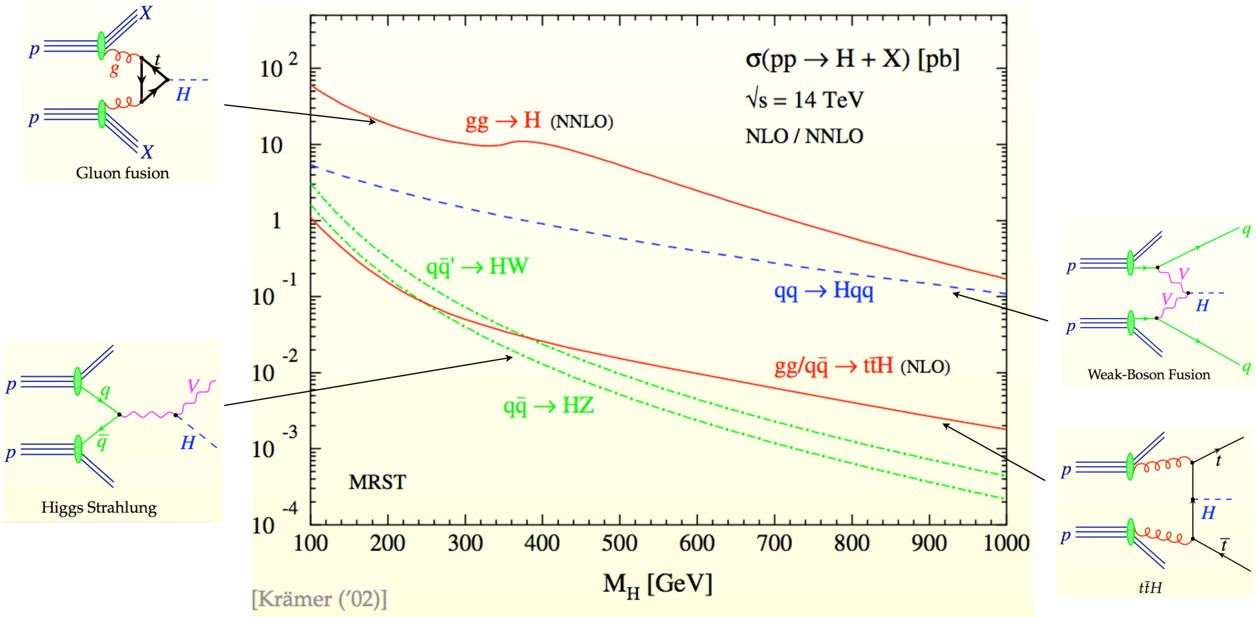
- Origin of large NLO corrections
  - New partonic channel $qg \to Zq$
  - Large gluon luminosity leads to NLO corrections of 15-30% (depending on rapidity)

- reliable estimate of theoretical uncertainty only at NNLO

- Higgs Boson production at the LHC

Higgs boson production at LHC

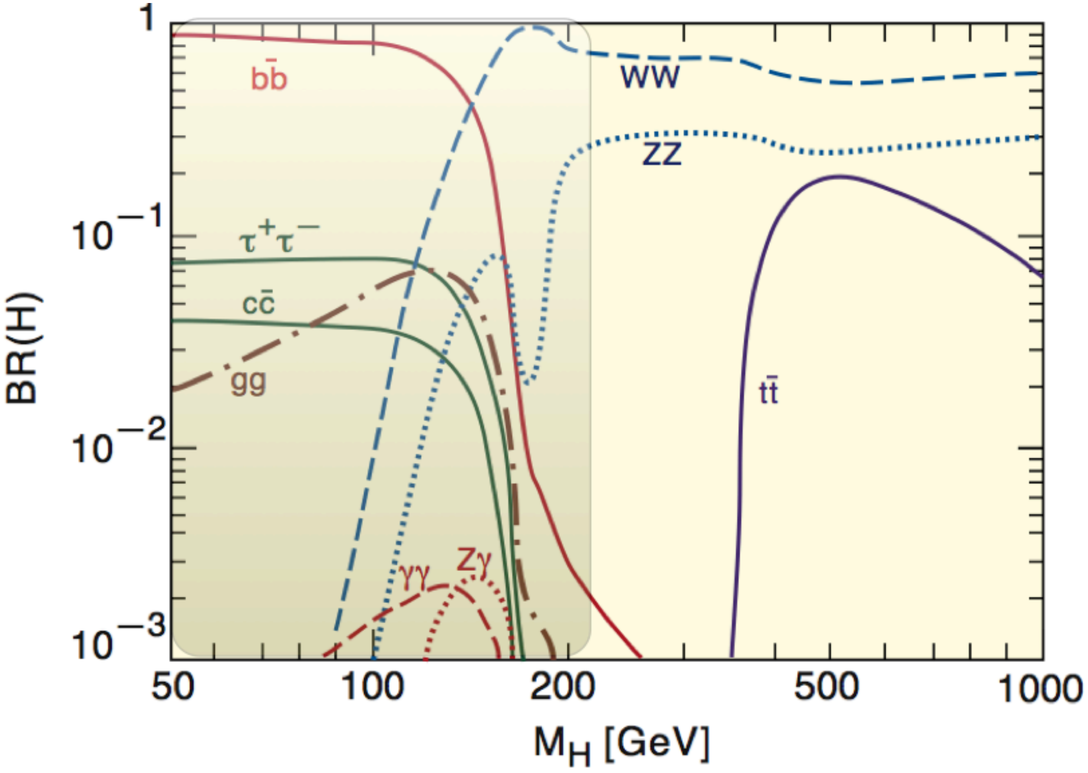
- Standard model Higgs particle
  - scalar boson, m_H = 125 GeV
  - couples directly to massive particles, proportional to mass
  - couples indirectly to gluons and photons via loops
  - Dominant production mechanism: gluon fusion gg → H



[Krämer ('02)]

Higgs boson production at LHC

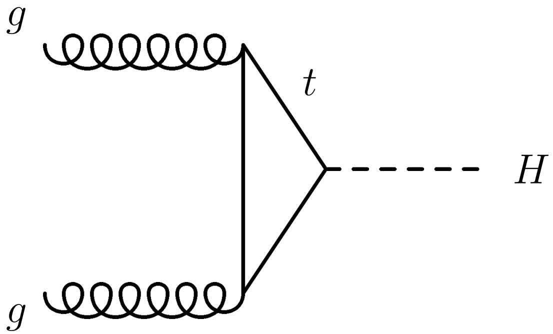
- Higgs discovery (2012) in multiple decay channels
  - BR($\gamma\gamma$) $\simeq 10^{-3}$
  - BR(WW*) $\simeq$ 0.1
  - BR(ZZ*) $\simeq$ 0.02
  - BR($\tau\tau$) $\simeq$ 0.08
- Dominant channel: not yet observed due to large background
  - BR($b\bar{b}$) $\simeq$ 0.8



Higgs boson production at leading order

- For gluon fusion process: gg → H
  - LO cross section: already 1 loop!
  - 2→1 process: very simple kinematics

$$PS \sim \delta(\hat{s} - M_H^2) \qquad \hat{s} = x_1 x_2 s_{pp}$$



- Parton level cross section

$$\sigma^{LO}_{gg\to H}(z) = \frac{G_F \alpha_s^2}{288\pi\sqrt{2}} \left| \frac{3}{4} \sum_Q \mathcal{F}_{1/2}(\tau_Q) \right|^2 \delta(1-z)$$

$$\tau_Q = \frac{M_H^2}{4m_Q^2}$$

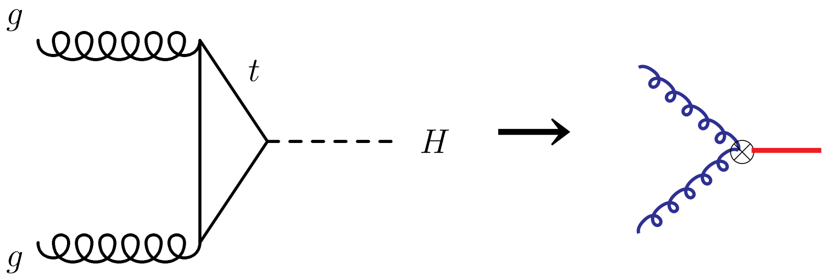
$$z = \frac{m_H^2}{\hat{s}}$$

  - Quark loop factor $\mathcal{F}(\tau)$: smooth and finite function

Higgs production at NLO

- Effective Lagrangian approach ($m_{top} \to \infty$ limit)
  - Top-quark loop reduces to a point-like interaction
  - Coupling to gluons described by an effective Lagrangian:

$$\mathcal{L} = \frac{1}{4v} C_1 Z_1 G^a_{\mu\nu} G^{a\mu\nu} H,$$

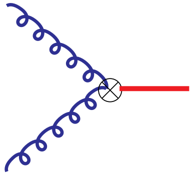


  - gluon field strength $G^a_{\mu\nu}$
  - Higgs field H
  - Wilson coefficient C_1, renormalization Z_1, both have series in α_s
  - Higgs field vacuum expectation value $v \simeq 246$ GeV
- Feynman rules for the ggH, gggH, ggggH vertices can be deduced from this effective Lagrangian

Effective Lagrangian approach

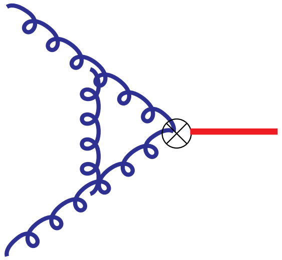
- Factorises QCD effects (dynamics of gluons) from heavy particle effects
  - heavy quark loop described with Wilson coefficient C_1
- Simplifies calculation of QCD corrections considerably
  - Reduces the number of loops by one at each order
  - Turns a two-scale problem (m_t, m_H) into a one-scale problem
- LO cross section: only proportional to ggH vertex

$$\hat{\sigma_0}(z) = \sigma_0 \delta(1-z) = \frac{\pi}{576 v^2}\left(\frac{\alpha_s}{\pi}\right)^2 \delta(1-z)$$



Higgs production at NLO

- Virtual corrections to gg → H
  - Matrix-element $2\mathrm{Re}(M_0^* M_1)$
  - After UV renormalisation of coupling α_s and Wilson coefficient C_1



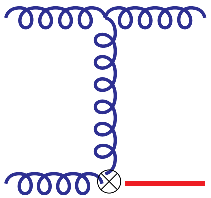
$$\hat{\sigma}_V = \sigma_0^{(d)} \left(\frac{\mu^2}{m_H^2}\right)^{\epsilon} \frac{\alpha_s}{\pi} \left\{ \left[C_A \left(-\frac{1}{\epsilon^2}\right) + Finite \right] - \frac{1}{\epsilon} \left(C_A \frac{11}{6} - n_f \frac{1}{3} \right) + \mathcal{O}(\epsilon) \right\} \delta(1-z)$$

  - Pole terms: $1/\varepsilon^2$ (soft) and $1/\varepsilon$ (collinear, ∝ β_0)
  - Cancel partly with real contributions ($1/\varepsilon^2$, $1/\varepsilon$) and partly with mass factorization of incoming gluon distribution involving

$$P_{gg}(z) = 2C_A \left(\frac{z}{(1-z)_+} + \frac{1-z}{z} \right) + \left(\frac{11}{6} C_A - \frac{2}{3} T_F n_f \right) \delta(1-z)$$

Higgs production at NLO

- Real corrections gg → Hg



  - Matrix element singular if the outgoing gluon is either soft or collinear to one of the incoming gluons
  - Parametrise final state phase space to map these singularities in variables (z, λ)(Soft limit $z \to 1$, collinear limits $\lambda \to (0,1)$)

$$p_g = \frac{\hat{s}(1-z)}{2}\left(1, 2\sqrt{\lambda(1-\lambda)}, 0, 1-2\lambda\right)$$

  - yields phase space regulator $(1-z)^{1-2\epsilon}\int_0^1 \mathrm{d}\lambda\,[\lambda(1-\lambda)]^{-\epsilon}$
  - Extraction of poles: expansion in distributions (like in DIS)

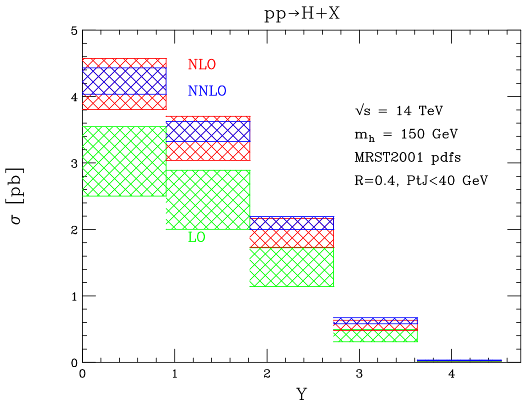
$$\hat{\sigma}_R = \sigma_0^{(d)}\left(\frac{\mu^2}{m_H^2}\right)^{\epsilon}\frac{\alpha_s}{\pi}C_A\left\{\left(+\frac{1}{\epsilon^2}\right) - \frac{2}{\epsilon}\left(\frac{z}{(1-z)_+} + \frac{1-z}{z}\right) + Finite + \mathcal{O}(\epsilon)\right\}$$

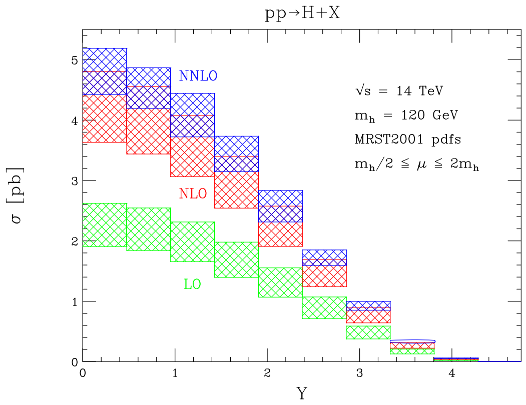
  - Real + virtual corrections: universal left-over $1/\varepsilon\, P_{gg}(z)$:

mass factorisation of initial-state collinear gluon singularity used

Higgs production at NNLO

- State-of-the-art theory for Higgs production: NNLO
  (FeHIP: C. Anastasiou, K. Melnikov, F. Petriello; HNNLO: S. Catani, M. Grazzini)
- Bin integrated Higgs boson rapidity distributions
  - with and without a jet veto

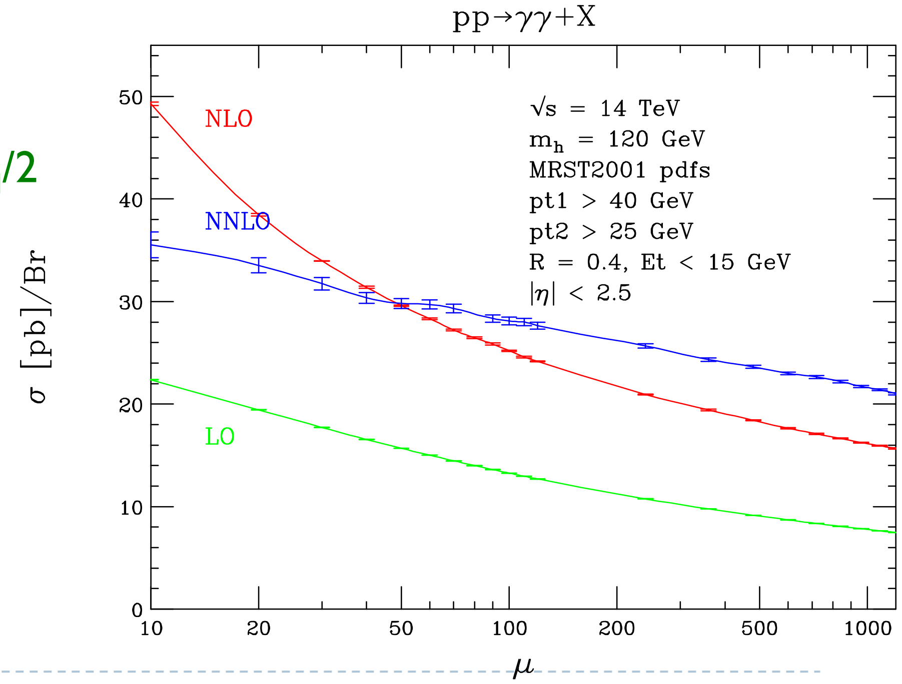




  - Hard radiation at higher orders suppressed with a jet veto
  - large NLO corrections, perturbative convergence observed at NNLO
  - scale variation at LO underestimates missing higher orders

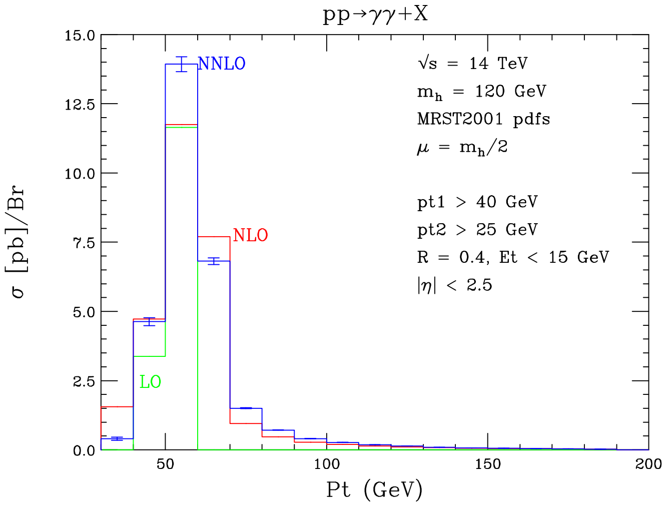
Results for pp → H → $\gamma\gamma$

- Total di-photon production cross section normalized to branching fraction
  - with cuts on the two photons, to improve signal/background ratio
  - Observe: perturbative corrections to di-photon signal follow same pattern as the inclusive Higgs production cross section
    - NLO/LO large, NNLO/NLO moderate
    - scale dependence: stabilized at NNLO
    - NNLO corrections smallest for $\mu = M_H/2$



Results for pp $\rightarrow$H $\rightarrow$ $\gamma\gamma$

- Di-photon transverse momentum ($p_{T\gamma}$) distribution
  - consider average p_T of the two photons
  - Observe: large perturbative corrections close to the leading order kinematical bound: $p_{T\gamma} < m_H/2$



- General feature of fixed order predictions: not accurate at phase space edges
  - At low $p_{T\gamma}$: resummation is required
- Background has completely different kinematical features
  - can use transverse momentum distribution as discriminator

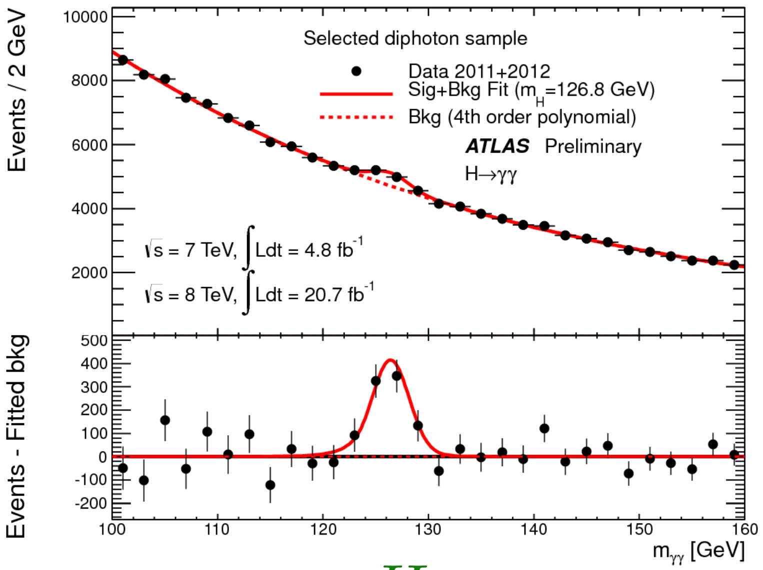
Higgs+jet production at the LHC

- Essential to establish the properties of the newly discovered Higgs boson
- Experiments select events according to number of jets
  - Different backgrounds for different jet multiplicities
  - H+0jet and inclusive H production known at NNLO
    (C. Anastasiou, K. Melnikov, F. Petriello; S.Catani,M. Grazini)
  - H+1jet and H+2jet known at NLO
  - H+0jet and H+1jet samples of comparable sizes
- NNLO for H+1jet needed
  - gluons-only total cross section just completed recently
    (R. Boughezal, F. Caola, K. Melnikov, F. Petriello, M. Schulze)
  - Full calculation and differential distributions in progress

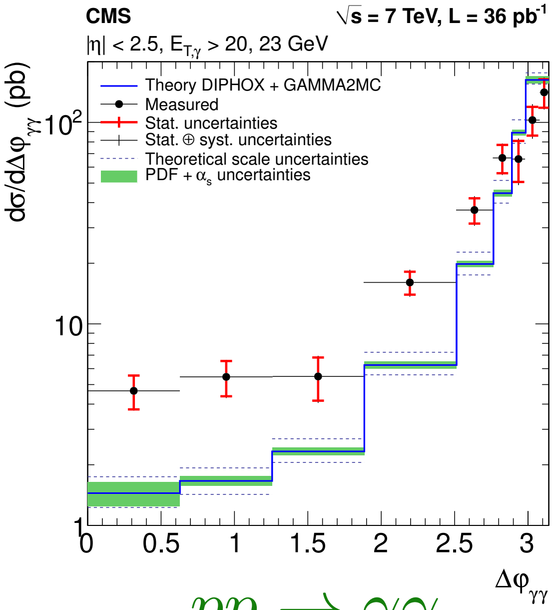
- Di-Photon production at the LHC

Di-Photon production at the LHC

- Di-photon production: irreducible background for H $\rightarrow \gamma\gamma$
  - at present determined from sideband data fits
- Discrepancy between NLO theory and data in some distributions



$pp \rightarrow H \rightarrow \gamma\gamma$



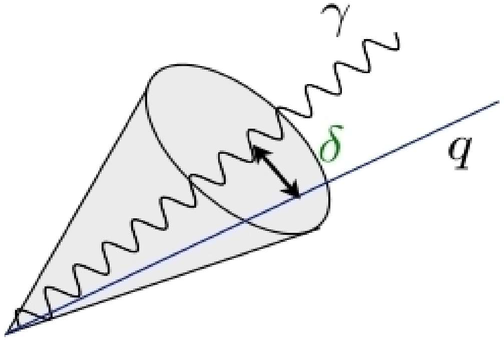
$pp \rightarrow \gamma\gamma$

- Require precise theoretical predictions (NNLO)

Photon isolation

- Photons need to be isolated from hadrons in events
  - Suppress secondary photons from hadron decays
  - Complete isolation not infrared safe, nor exp. well-defined
- Isolation criteria
  - Fixed cone isolation

$$\sum_{\delta<R} E_T^h < E_T^{max}$$

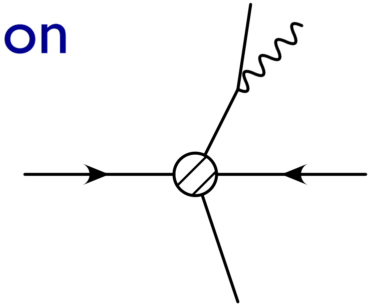


  - Smooth cone isolation (S.Frixione)

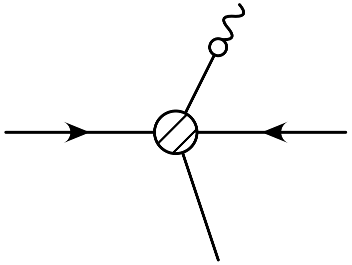
$$\sum_{\delta<R} E_T^h < E_T^{max} \left(\frac{1-\cos(\delta)}{1-\cos(R)} \right)^n$$

    - only soft radiation allowed close to photon
    - experimental implementation difficult (finite detector resolution)

Photon production mechanisms



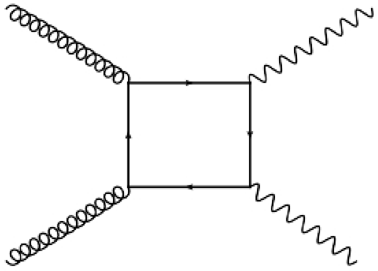
- Direct process: photon produced in hard interaction
  - perturbatively calculable
  - collinear quark-photon contributions present
- Fragmentation of parton into photon:
  - described by a non-perturbative parton-to-photon fragmentation function
  - absorbs collinear singularities from direct process
  - requires non-perturbative input



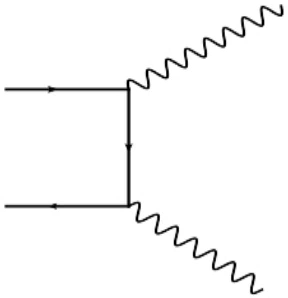
- Fixed cone isolation
  - both processes contribute
  - fragmentation contributions reduced but not eliminated
- Smooth cone isolation
  - No fragmentation contribution, direct process contribute without collinear part

Di-photon production at the LHC

- NNLO calculation: 2γNNLO
  (S. Catani, L. Cieri, D. de Florian, G. Ferrera, M. Grazzini)
  - parton-level event generator, based on q_T-subtraction
    - □ Analytic cancellation of infrared poles
    - using a smooth isolation criterion to define photons
    - includes all O(α_s^2) corrections to direct photon production pp → $\gamma\gamma$
  - First fully consistent inclusion of the gluonic box contribution



O(α_s^2), gluon luminosity comparable size to



O(α_s^0), qq luminosity

- Box also included in NLO-type codes (DIPHOX+gamma2MC, MCFM)
  (T. Binoth, J.P. Guillet, E. Pilon, M. Werlen; Z. Bern, L. Dixon, C. Schmidt; J. Campbell et al.)

Di-photon production at the LHC

- Partonic contributions to pp→ $\gamma\gamma$ up to NNLO

LO : $q\bar{q} \to \gamma\gamma$

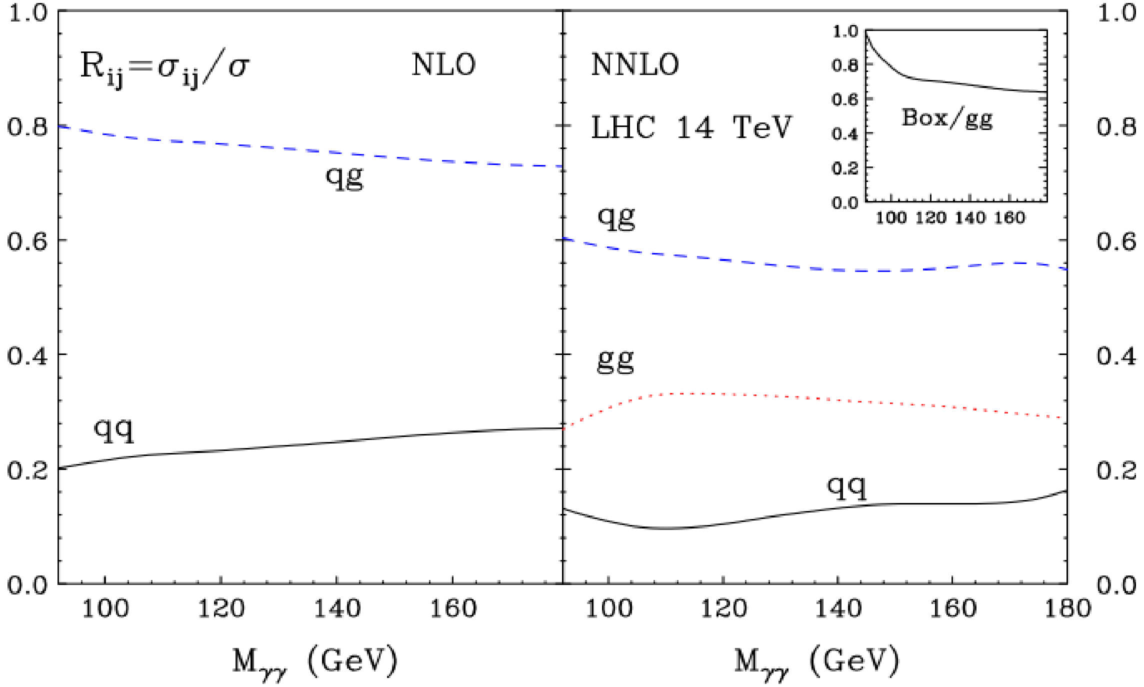
NLO : $q\bar{q} \to \gamma\gamma(+g)$

$qg \to \gamma\gamma q$

NNLO : $q\bar{q} \to \gamma\gamma(+gg)$

$qg \to \gamma\gamma q(+g)$

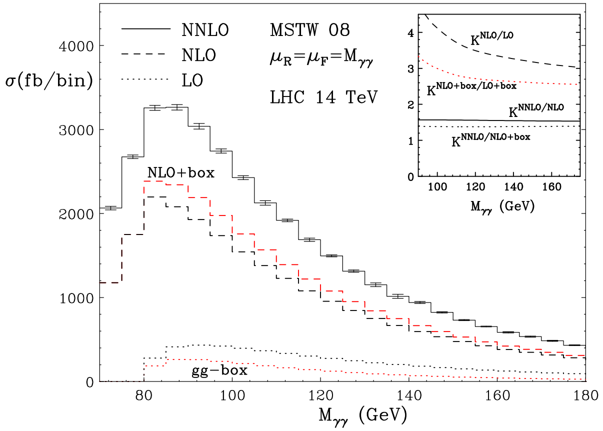
$gg \to \gamma\gamma$



- At NLO and NNLO: qg channel dominant

Di-photon production at NNLO

- Invariant-mass distribution with staggered photon cuts



$$p_T^{\gamma\,hard} \geq 40\,\mathrm{GeV}$$

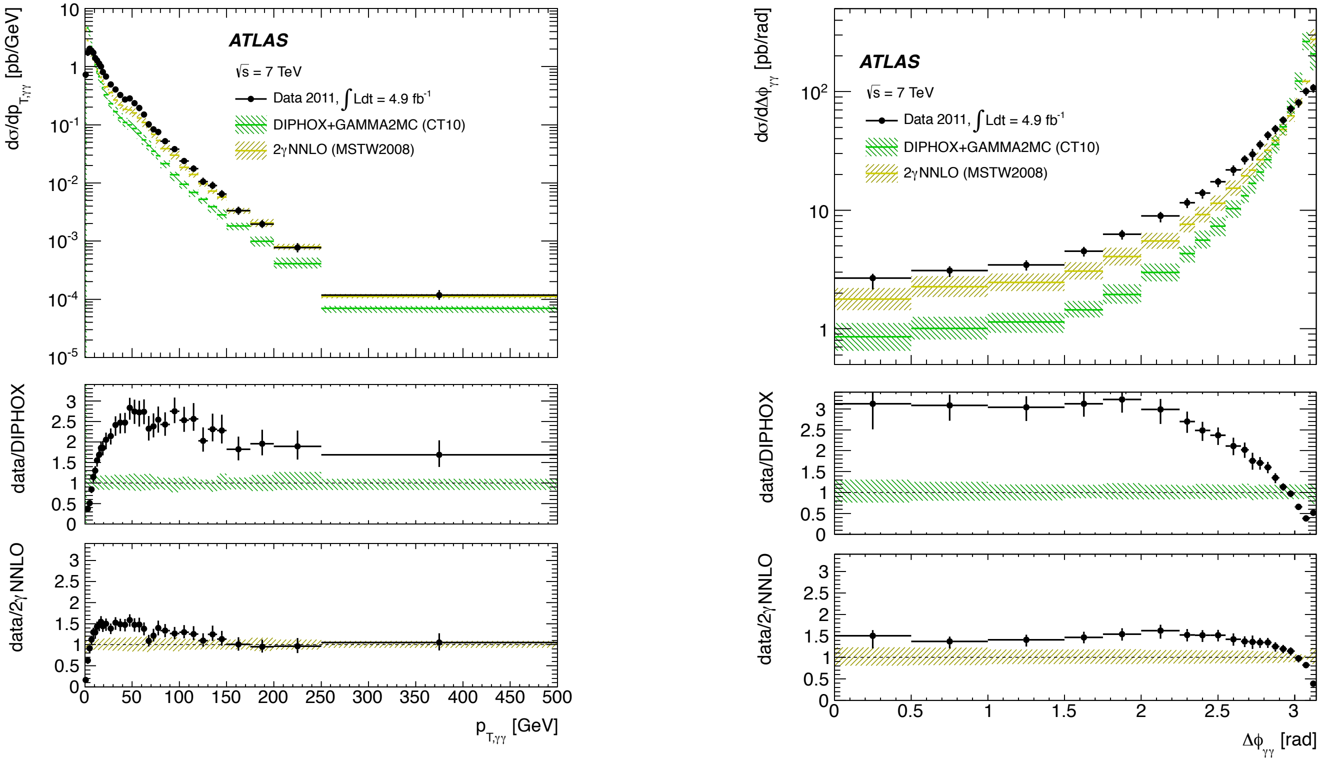
$$p_T^{\gamma\,soft} \geq 25\,\mathrm{GeV}$$

$$|\eta^{\gamma}| \leq 2.5$$

$$20\,\mathrm{GeV} \leq M_{\gamma\gamma} \leq 250\,\mathrm{GeV}$$

- At LO: both γ produced at equal p_T (40 GeV)
  - New phase space region (25GeV < $p_{T\gamma}^{soft}$< 40 GeV) opens at NLO
- NNLO corrections large for low $M_{\gamma\gamma}$ region
  - Main contribution from qg channel (dominant channel at NLO)
  - Box contribution of similar size than LO

ATLAS di-photon results



- Inclusion of NNLO corrections resolves discrepancy between NLO-type prediction and data

- End of Lecture 3