

SO FAR:

CFT
entanglement
entropy

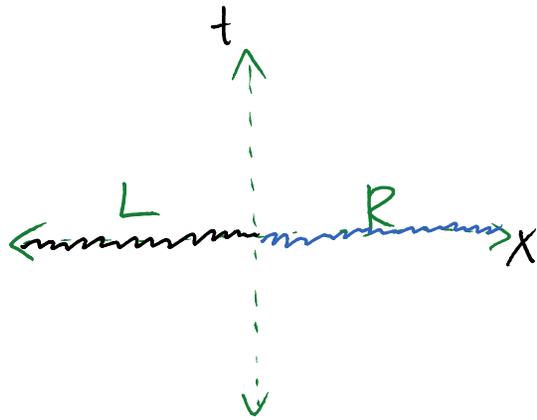


geometrical
observables
in gravity
picture

This lecture:

more general connections between
QFT entanglement structure &
spacetime structure / geometry.

Minkowski space QFT: how are d.o.f. entangled with each other in vacuum?

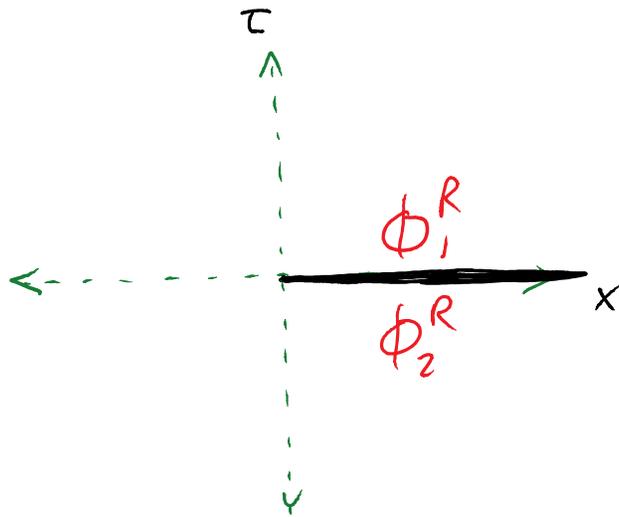


- define subsystems

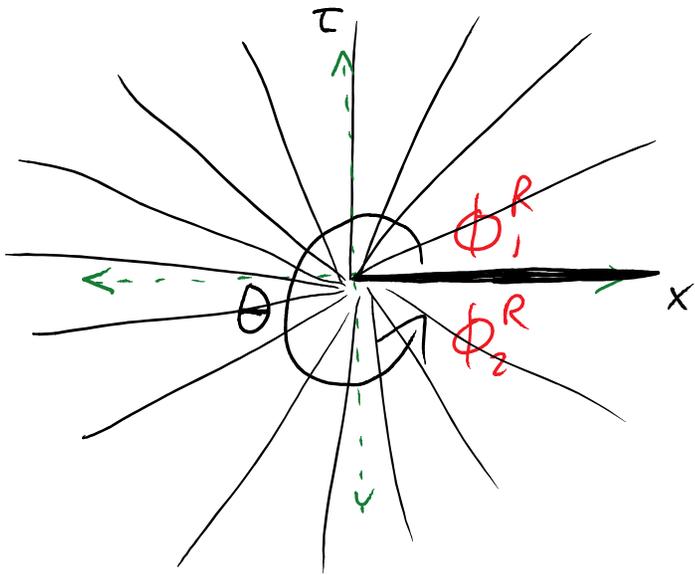
$$L: x < 0$$

$$R: x > 0$$

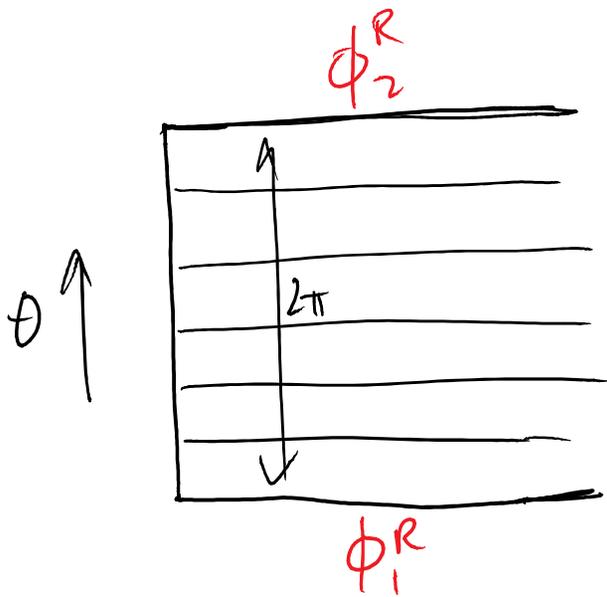
Density matrix for right side:

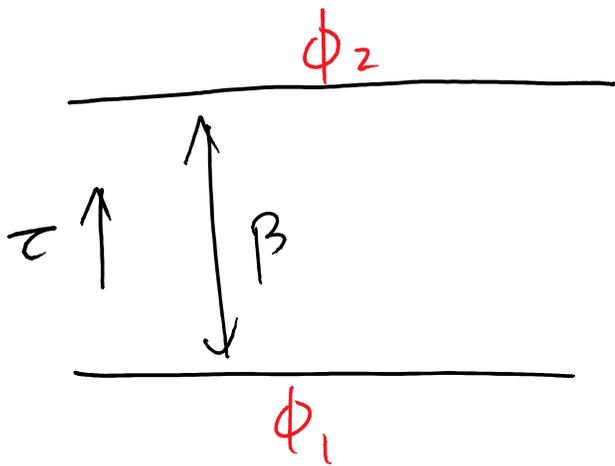


$$\langle \phi_2^R | \rho_R | \phi_1^R \rangle = \frac{1}{Z} \int [d\phi] e^{-S_{\text{Euc}}} \begin{matrix} \phi_R(0^+) = \phi_1^R \\ \phi_R(0^-) = \phi_2^R \end{matrix}$$



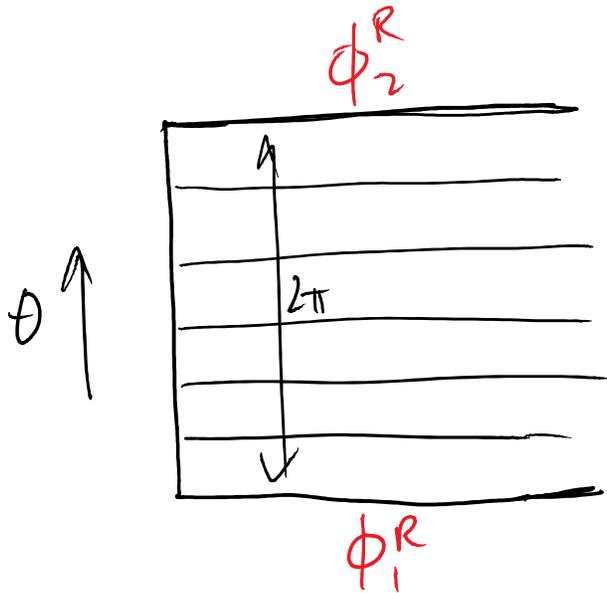
$$\langle \phi_2^R | \rho_R | \phi_1^R \rangle = \frac{1}{Z} \int [d\phi] e^{-S_{\text{Euc}}} \int_{\phi(\theta=0)=\phi_1^R}^{\phi(\theta=2\pi)=\phi_2^R}$$





$$\int_{\phi(0)=\phi_1}^{\phi(\beta)=\phi_2} [d\phi] e^{-S_{\text{Enc}}} = \langle \phi_2 | e^{-\beta H_t} | \phi_1 \rangle$$

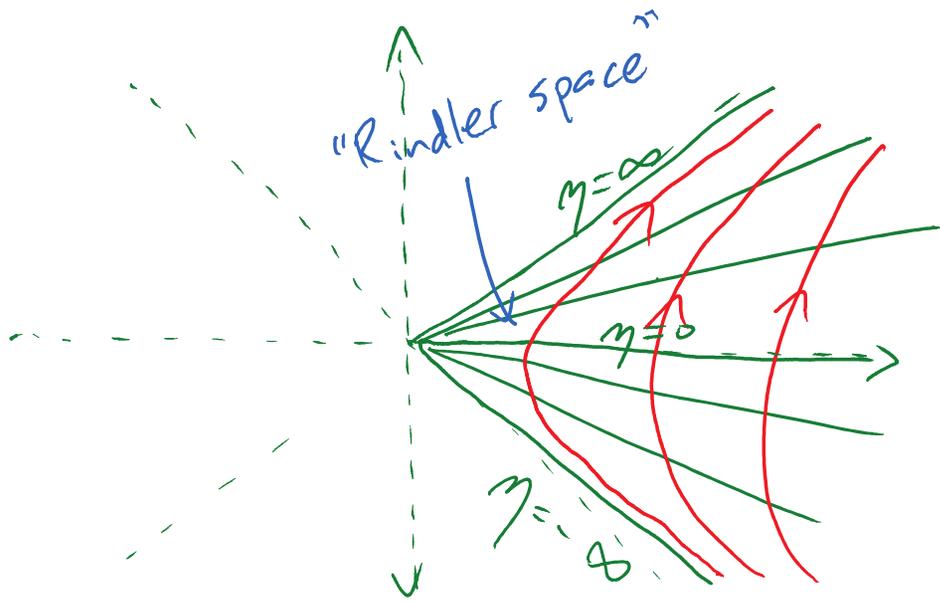
generator of evolution in $t = i\tau$



$$\int_{\phi(\theta=0)=\phi_1^R}^{\phi(\theta=2\pi)=\phi_2^R} [d\phi] e^{-S_{\text{Enc}}} = \langle \phi_2^R | e^{-2\pi H_\eta} | \phi_1^R \rangle$$

generator of evolution in $i\theta = \eta$

$$dr^2 + r^2 d\theta^2 \rightarrow dr^2 - r^2 d\eta^2$$



$$H_\eta = x H + t P$$

"boost generator" - generates accelerated trajectories

= "Rindler Hamiltonian"

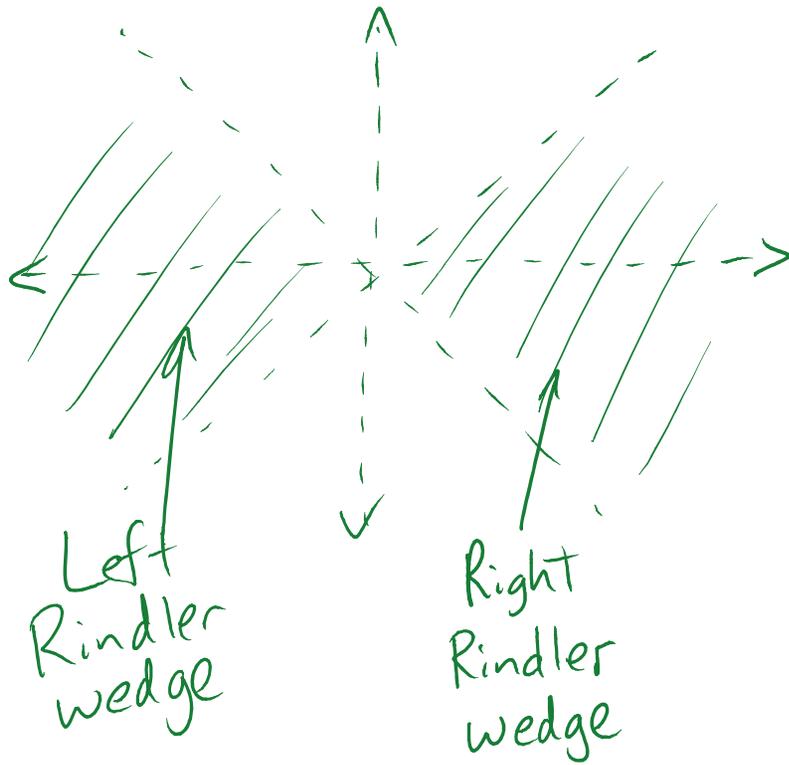
CONCLUSION: density matrix for R is:

$$\rho_R = \frac{1}{Z} e^{-2\pi H_\eta} = \frac{1}{Z} \sum e^{-2\pi E_i^R} |E_i^R\rangle \langle E_i^R|$$

$$t=0: H_\eta = \int_0^\infty dx \cdot x T_{00}$$

Accelerated observer sees thermal state (Unruh)

ENTANGLEMENT IN MINKOWSKI SPACE



$$\rho_R = \frac{1}{Z} \sum e^{-2\pi E_R^i} |E_R^i\rangle \langle E_R^i|$$

$$\rho_L = \frac{1}{Z} \sum e^{-2\pi E_L^i} |E_L^i\rangle \langle E_L^i|$$

Full state: $|vac\rangle = \frac{1}{Z} \sum_i e^{-\pi E^i} |E_L^i\rangle \otimes |E_R^i\rangle$

Free field theory: can find basis of modes
living in Rindler wedge

$$[H, b_\omega^\dagger] = \omega b_\omega^\dagger$$

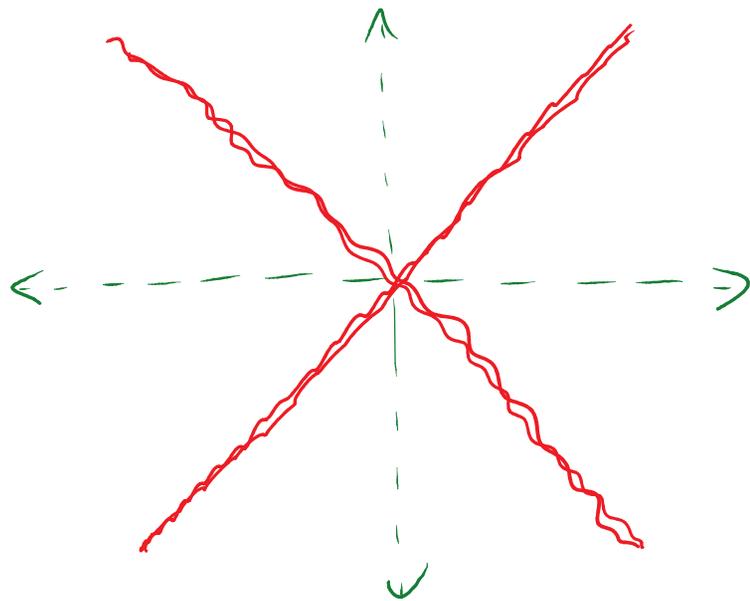
Basis of eigenstates:



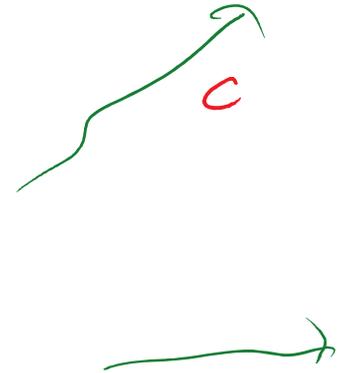
$$|E\rangle = \prod_{i=1}^N b_{\omega_i}^\dagger |0\rangle_{\text{Rindler}}$$

Each mode entangled w. corresponding
mode on L.S.

Aside: states with no entanglement between L & R have a singular stress tensor:

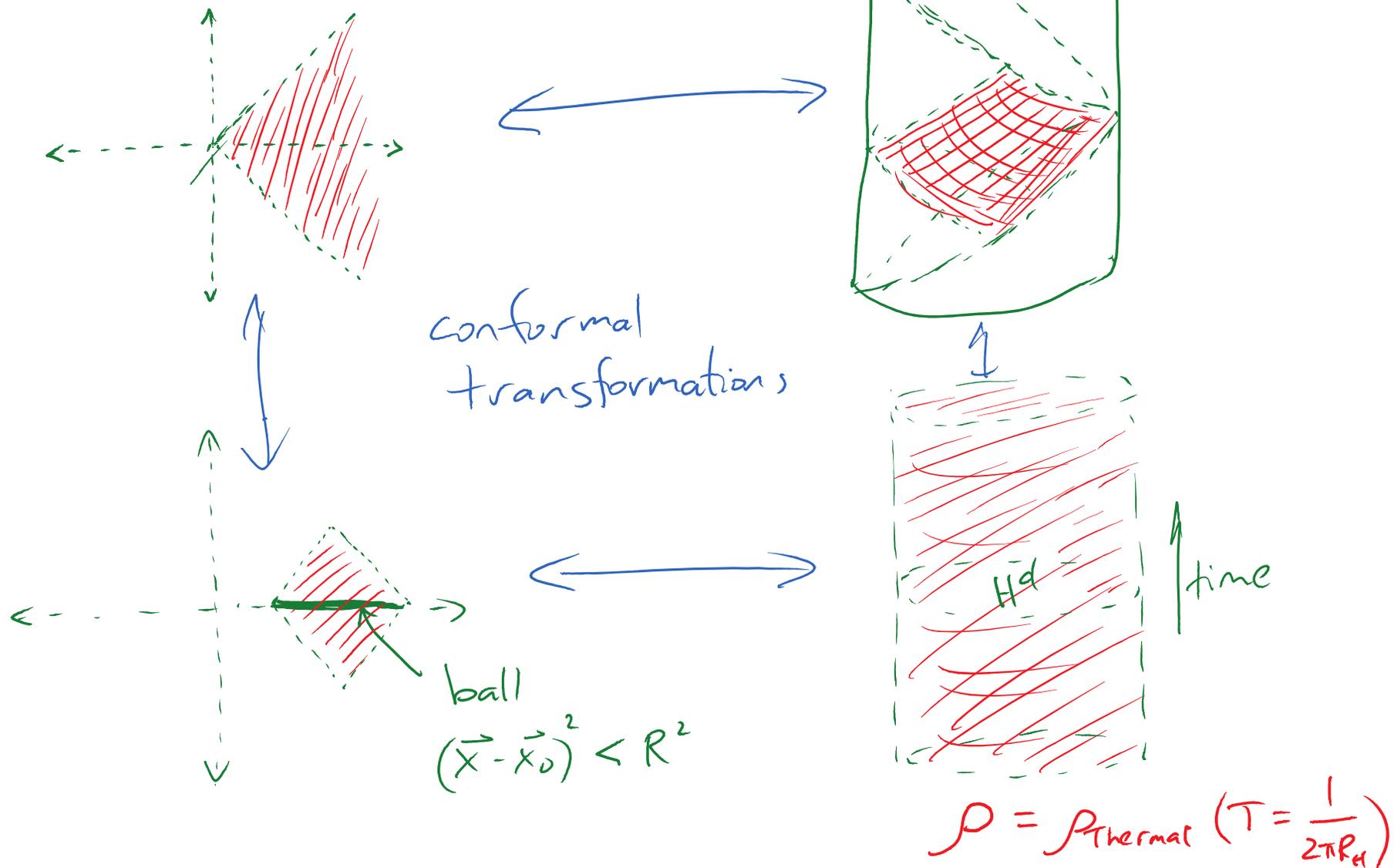


AMPS:

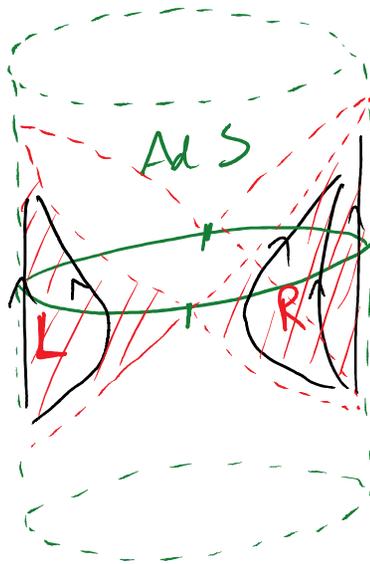


in old B.H.,
a entangled w. c,
not b
 \therefore FIREWALL

CFT case:



Back to AdS/CFT:



$$\longleftrightarrow |vac\rangle_{S^d}$$

↑
Entangled state of
A and \tilde{A}

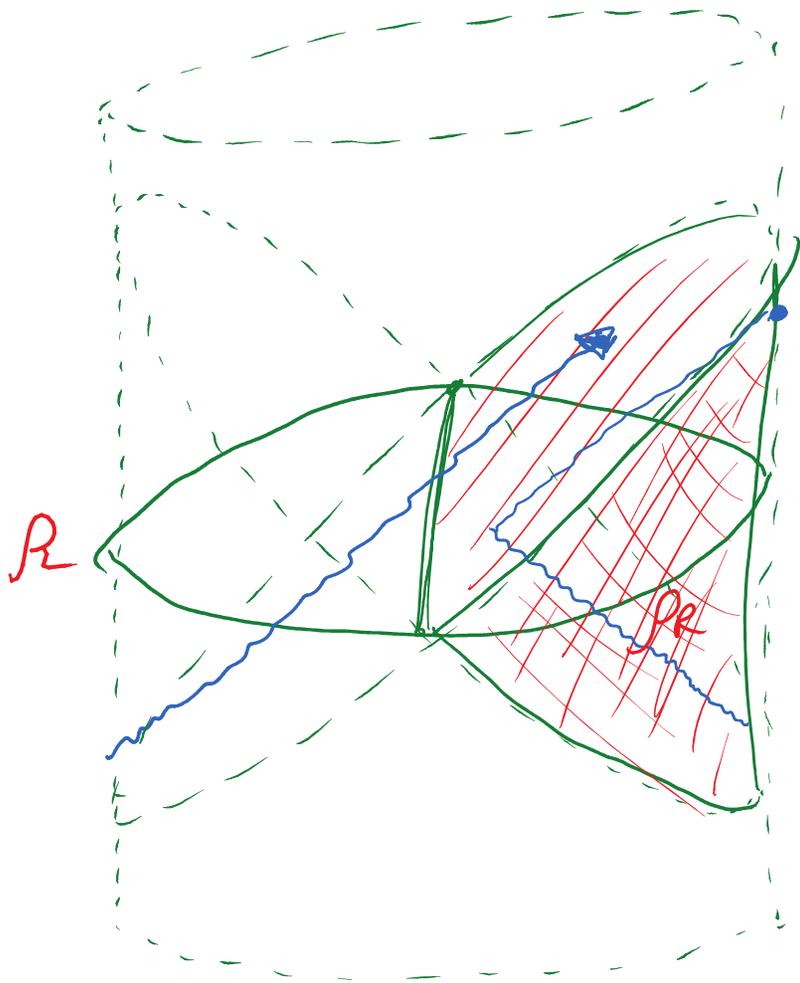


$$\sum_i e^{-E_i x/2} |E_i\rangle_L \otimes |E_i\rangle_R$$

$$\rho_L = \frac{1}{2} e^{-H_x^L}$$

$$\rho_R = \frac{1}{2} e^{-H_x^R}$$

What info about
geometry is contained
in ρ_L, ρ_R ?

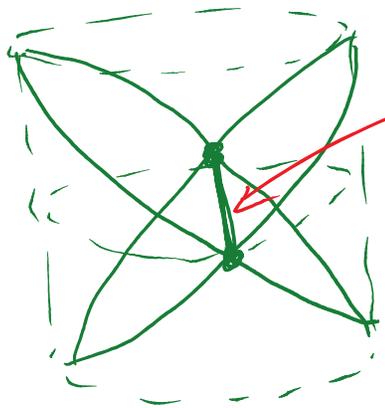


Plausible answer:

- ρ_R carries information about "Rindler wedge" of AdS
- Observables $\text{Tr}(\rho_R \rho_R)$ probe inside wedge region.
- States $u_L \otimes \mathbb{1}_R | \text{vac} \rangle$ have same ρ_R but ρ_L changed.
- effects of CFT perturbation in L can affect anywhere outside wedge.

Recall: product state $|\psi_L\rangle \otimes |\psi_R\rangle$ in Minkowski
QFT has singularity on Rindler horizon
- bulk singularity also?

Bulk story: $\langle \psi_R | \mathcal{O}_R | \psi_R \rangle = \text{tr}(\rho_R \mathcal{O}_R)$ for
almost all observables, so inside of wedges
unaffected.

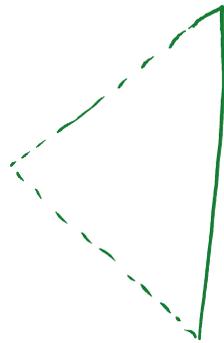


Ryu Takayanagi
surface

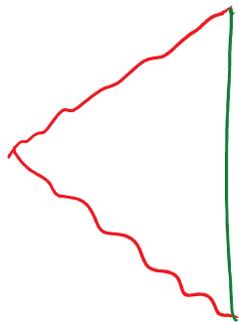
No entanglement: Area $\rightarrow 0$

Conjecture: no spacetime
outside wedges.

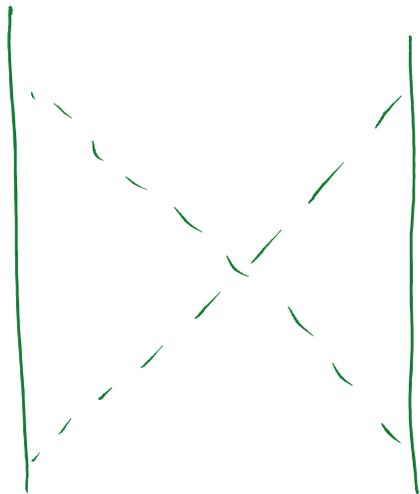
Summary:



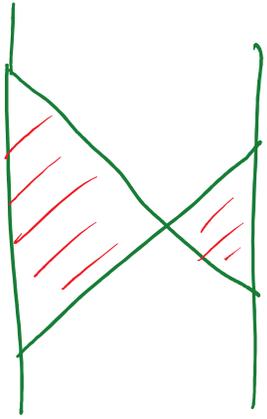
Density matrix ρ_R
describes wedge w .
smooth horizon



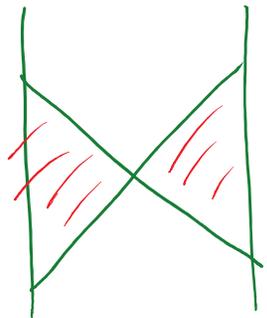
Pure state: $|E\rangle_R$: wedge w .
singular boundary



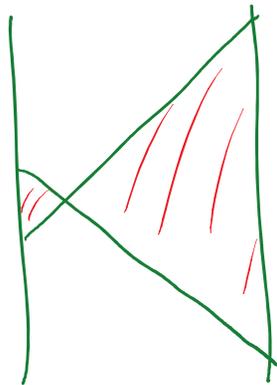
Entangled state (purification of ρ_R):
connected spacetime
- details of spacetime past
wedge depend on details of
purification.



Different ways to divide AdS into
Rindler wedges



Different ways to describe $|vac\rangle$
as entangled state of
subsystems.



Each case: wedges connected by
entanglement

$$\sum e^{-E_i} |E_i\rangle_L \otimes |E_i\rangle_R$$

product state: 2 disconnected wedges



$$\sum e^{-E_i} \left[\text{Diagram 1} \right] = \left[\text{Diagram 2} \right]$$

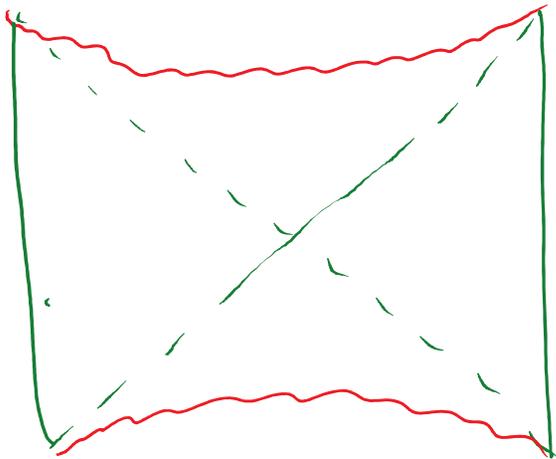
The diagram shows an equality between two spacetime diagrams. On the left, a red wavy line forms a triangle pointing right, and a red straight line forms a triangle pointing left, meeting at a central point. On the right, a green dashed line forms a triangle pointing right, and a green dashed line forms a triangle pointing left, also meeting at a central point. The two diagrams are enclosed in vertical lines representing boundaries.

Quantum superposition of disconnected spacetimes
= connected

Another example: 2 CFTs

- states $|E^1\rangle \otimes |E^2\rangle$ describe 2 separate asymptotically AdS spacetimes

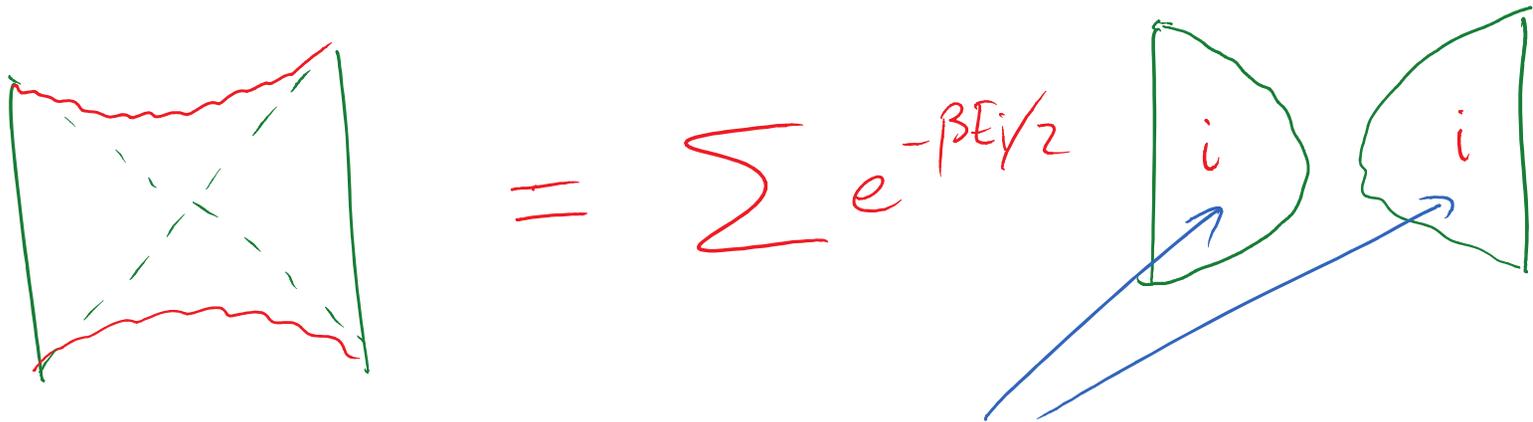
- Witten: thermal state $\sum e^{-\beta E_i} |E_i\rangle \langle E_i|$ describes black hole.



Maldacena: maximally extended BH w. 2 asymptotic regions described by:

$$\sum e^{-\beta E_i/2} |E_i\rangle_L \otimes |E_i\rangle_R$$

Can connect 2 asymptotically AdS spacetimes
by entangling 2 CFTs:



Black hole
microstates: are these singular?