Mass relations and extrapolations

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Update and extension of the work of A.S. Jensen, P.G. Hansen and B. Jonson, Nucl.Phys. A431 (1984) 395 — PR C87 (2013) 024319

Mass relations

- Systematic cancellation of smooth terms
- Differences $\leftrightarrow$ derivatives
- Determine pairing terms
- Signature of shell structure

Results

- The N=152 shell
- Extrapolations
- Examples: 2011 $\rightarrow$ 2012 mass tables
- Future extensions ?

Summary

Many thanks to Aksel Jensen and Dennis Hove !

Earlier relations

Garvey-Kelson relations very fruitful, all interactions between nucleons cancel to first order G.T. Garvey et al, Rev.Mod.Phys. 41 (69) S1

General overview of masses and other “local mass formulas” D. Lunney, J.M. Pearson and C. Thibault, Rev.Mod.Phys. 75 (03) 1021

Chaotic component in masses ?! e.g. PRL 88 (02) 092502; PL B637 (06) 48; PRL 96 (06) 042502

Recent activity (Van Isacker, Bertsch, Kirson and others)

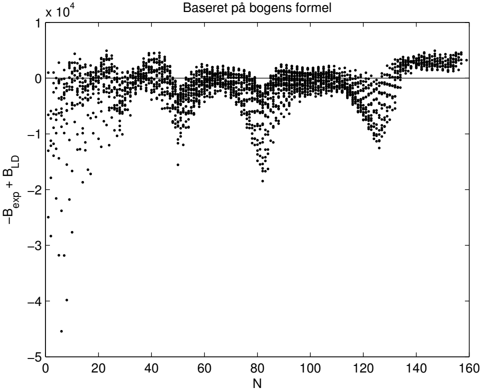
Nuclear binding energies

Divide the binding energy into three terms:

$$B(N,Z) = B_{LD}(N,Z) + B_{shell}(N,Z) + \Delta(N,Z)\,,$$

a smooth (liquid drop) term, a shell term and a “pairing” term.

The shell term:



Systematic derivations

Mass relations Q defined formally as

second order mass difference

$$Q(n_1, z_1; n_2, z_2) = -S(N-n_1, Z-z_1)+2S(N, Z)-S(N+n_1, Z+z_1)$$

between “separations” (first order difference)

$$S(N, Z) = B(N, Z) - B(N - n_2, Z - z_2)$$

Cancellations

If B were continous and smooth up to second order terms would cancel. By Taylor expansion one finds

$$Q = -\frac{\partial^3 B}{\partial N^3} n_1^2 n_2 - \frac{\partial^3 B}{\partial N^2 \partial Z} n_1(n_1 z_2 + 2 n_2 z_1)$$

$$-\frac{\partial^3 B}{\partial N \partial Z^2} z_1(n_2 z_1 + 2 n_1 z_2) - \frac{\partial^3 B}{\partial Z^3} z_1^2 z_2$$

I.e. expect only strong signals from abrupt or irregular changes:

Shells, pairing energies etc

Examples from SEMF

The semi-empirical mass formula contains terms like:

A (Volume) — all derivatives vanish

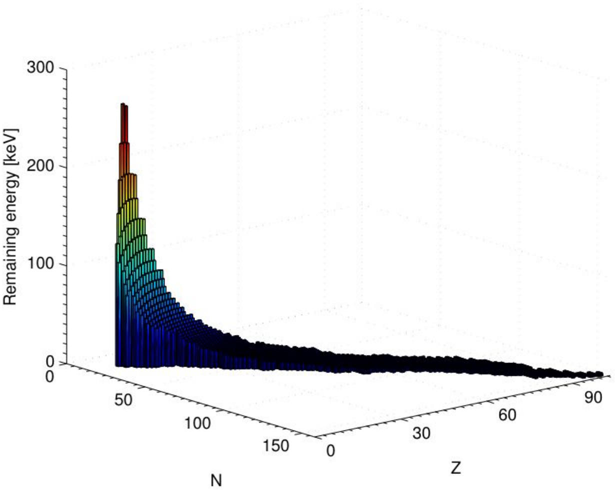
$A^{2/3}$ (Surface) — all derivatives equal $\frac{8}{27}A^{-7/3}$

$Z^2/A^{1/3}$ (Coulomb) — derivatives of order $A^{-4/3}$

$(N-Z)^2/A$ (Symmetry) — derivatives of order A^{-2}

Pairing goes as $(-1)^N$ or $(-1)^Z$ — so depends on n_i, z_i !

SEMF contribution, numerically



Alternative to binding energy: Q_α

The Q_α is almost similar to S:

$$S(N,Z) - B(^4\mathrm{He}) = -Q_\alpha$$

and can therefore be used in its place

$$Q(n_1,z_1;n_2,z_2) = Q_\alpha(N-n_1,Z-z_1) - 2Q_\alpha(N,Z) + Q_\alpha(N+n_1,Z+z_1)$$

NB! Should use ground state $\rightarrow$ ground state values...

Some examples

$(n_1, z_1; n_2, z_2) = (1, 0; 1, 0)$ $\quad (0, 1; 0, 1)$ $\quad (1, 0; 0, 1)$

Δ_n

1	-3	3	-1

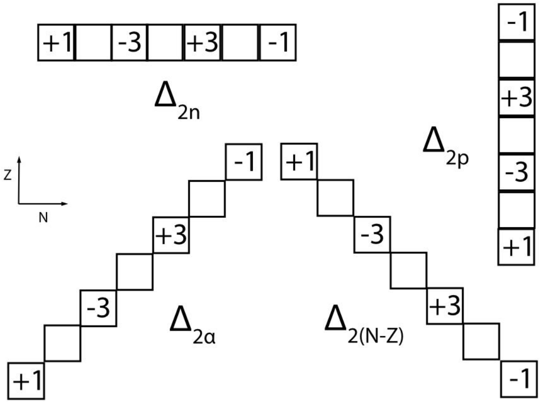
Δ_p

-1
3
-3
1

Δ_{np}

-1	2	-1
1	-2	1

The "double" structures



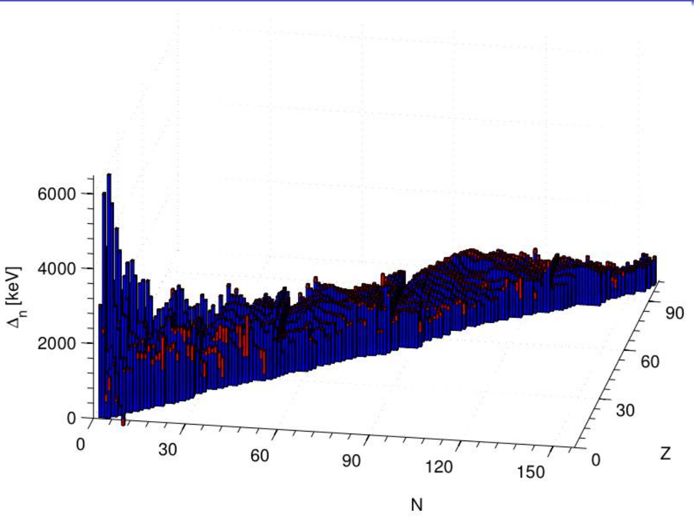
Source of mass data

2012 mass table: G. Audi, M. Wang, A.H. Wapstra, F.G. Kondev, M. MacCormick, X. Xu, and B. Pfeiffer, Chinese Phys. C36 (2012) 1287

2011 prerelease: Georges Audi and Wang Meng, private communication, April 2011

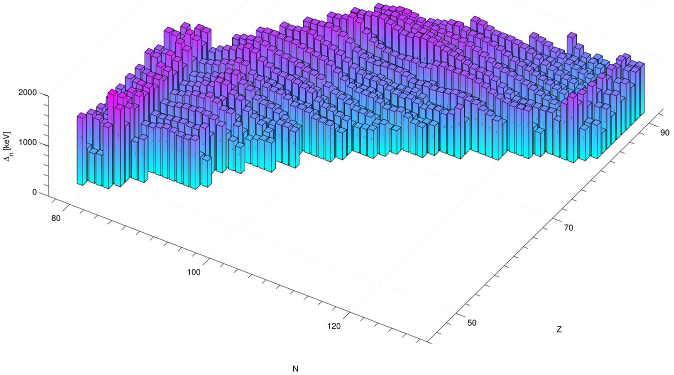
Results from Δ_n

Δ_n (keV)



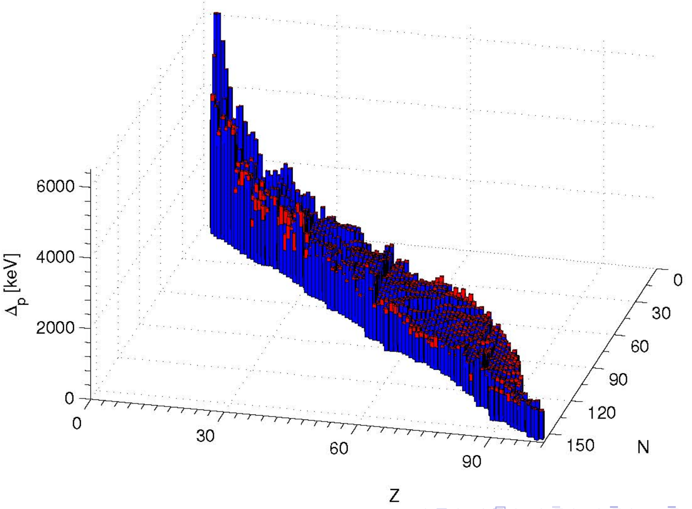
Zoom of Δ_n

Δ_n (keV)



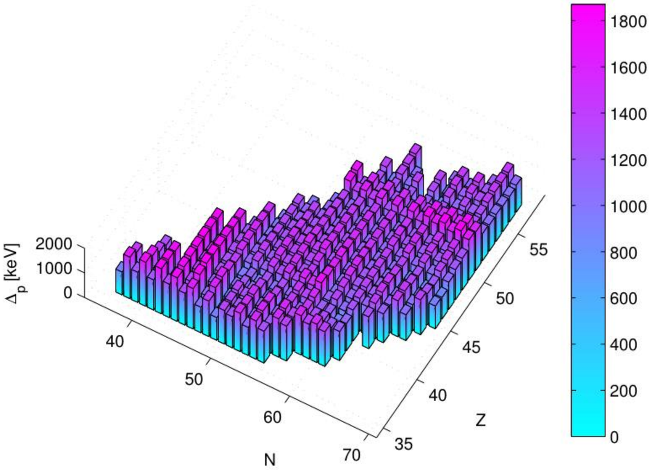
Results from Δ_p

Δ_p (keV)

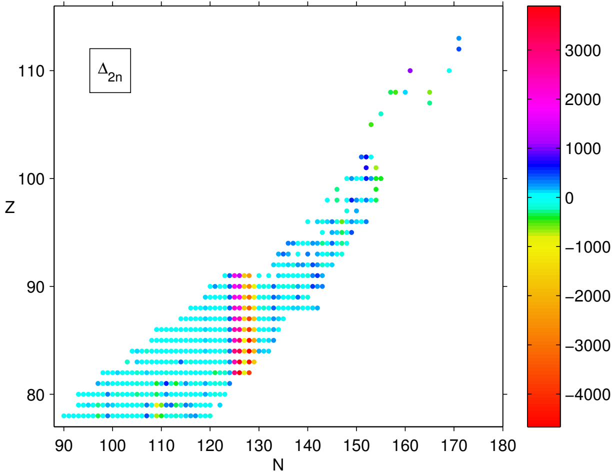


Zoom of Δ_p

Δ_p (keV)

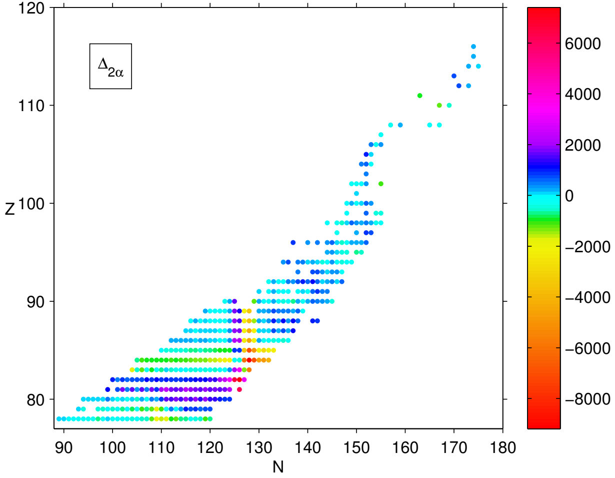


Results from Δ_{2n}



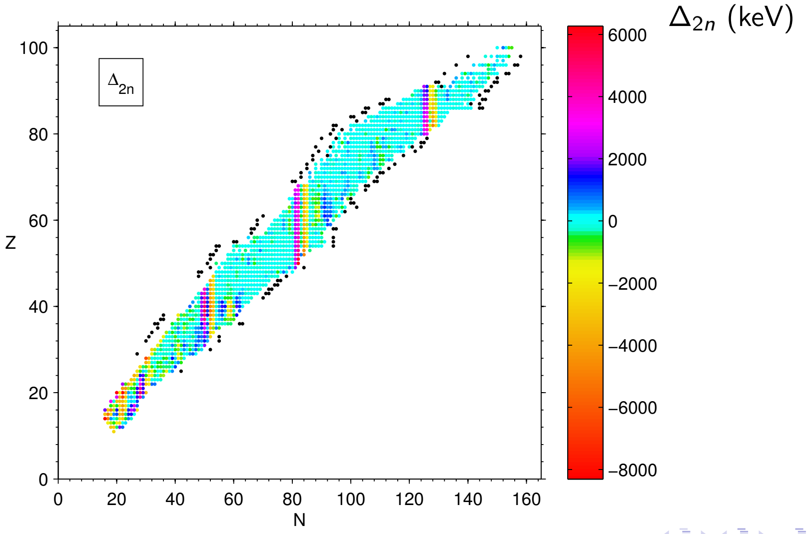
Δ_{2n} (keV), Q_α

Results from $\Delta_{2\alpha}$

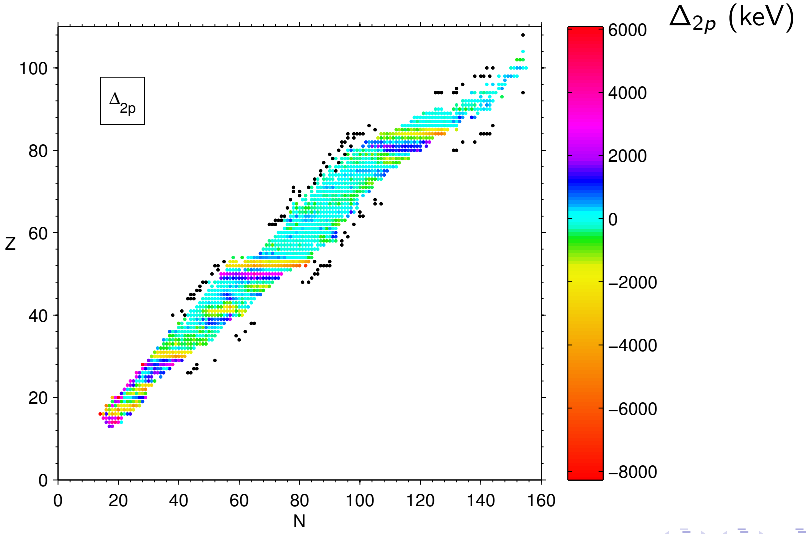


$\Delta_{2\alpha}$ (keV), Q_α

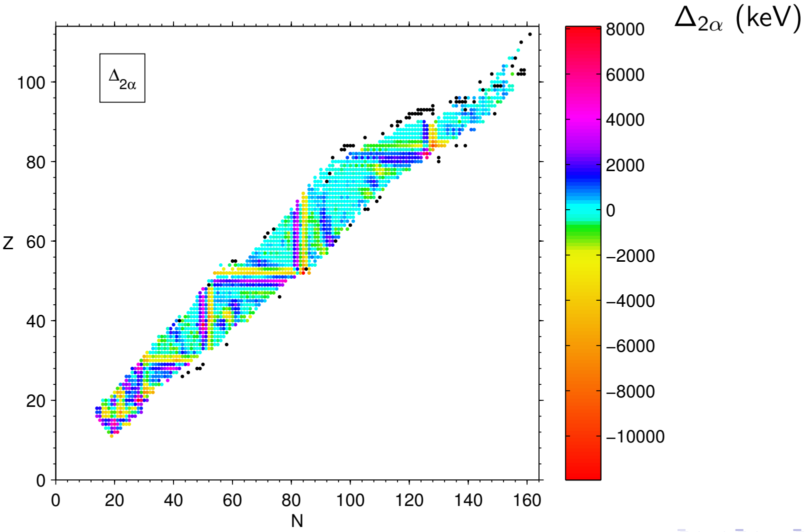
Results from Δ_{2n}



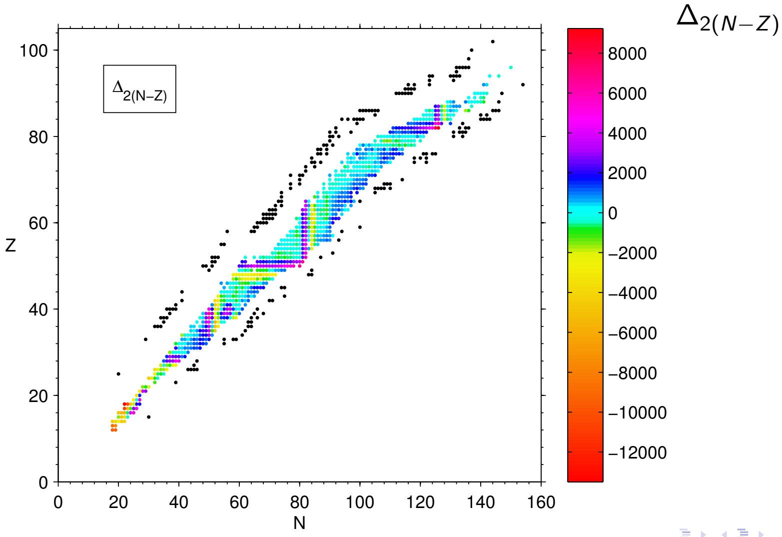
Results from Δ_{2p}



Results from $\Delta_{2\alpha}$



Results from $\Delta_{2(N-Z)}$



Use for extrapolations

If **(1)** the average behaviour of a mass relation is known in a region and **(2)** all but one masses in one specific relation is known then one can estimate the missing mass

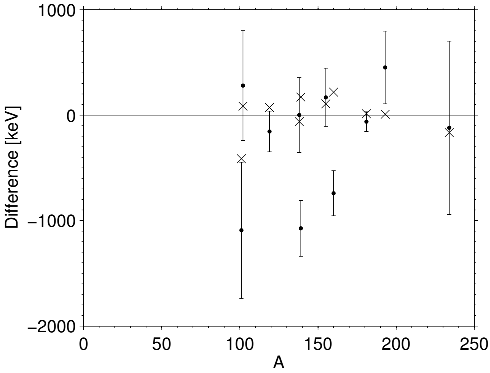
(1) typically gives bias and scatter a few 100 keV

Estimates from “different directions” can be compared to test the accuracy independent determinations !

Accuracy of extrapolations

Combine several Δ's to improve reliability.

Compare final results (with extrapolation uncertainty) to new mass values, i.e. values in 2012 tables but not in 2011 preview:



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- Systematic derivation of mass relations
- Extendable to light nuclei ?
- Indicator for shells
- Extract local structures = pairing
- New extrapolations possible, accuracy typically 2-300 keV